Quantum Mechanics

Saturday Morning Physics

Patrick Fox

Fermilab

Qu ics

S s

"Actually I started out in quantum mechanics, but somewhere along the way I took a wrong turn."

Please, please ask questions

My (our?) Challenge

We live in a "classical" world

Our everyday experiences are those of Newton, not Einstein (relativity) or Schrödinger (QM)

Nothing prepares you for the *weirdness* of quantum mechanics

My (our?) Challenge

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--Niels Bohr

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-- Richard Feynman

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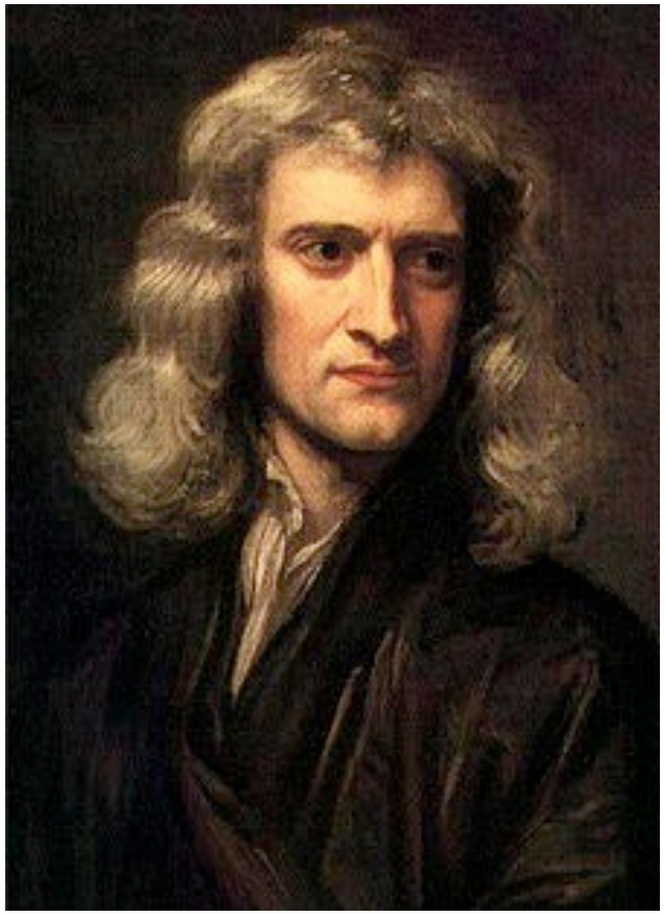
My (our?) Challenge

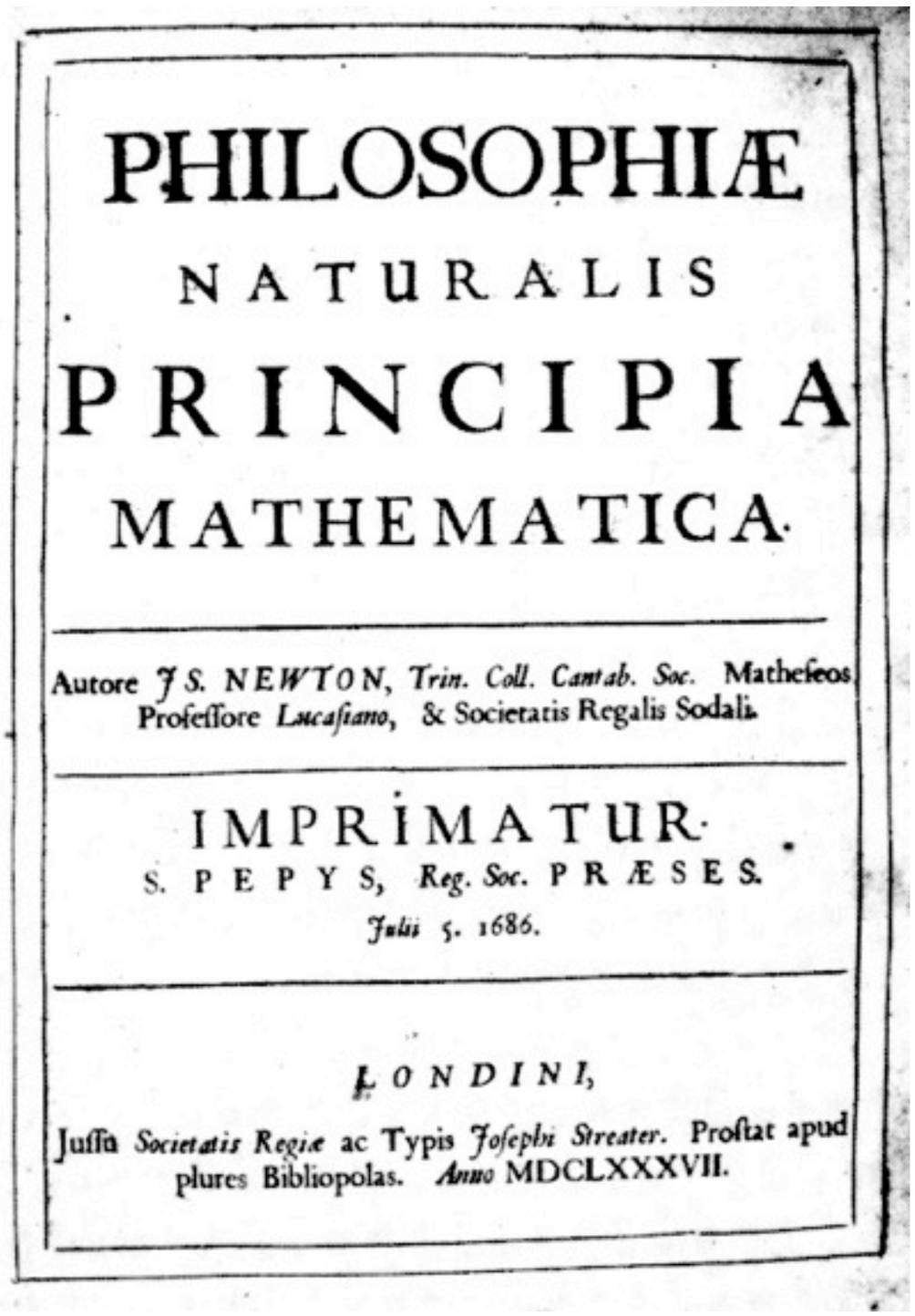
How did we discover it?
What is it?
How do we know it is true?

Classical Physics



Explains all of physics up to ~1900

Deterministic

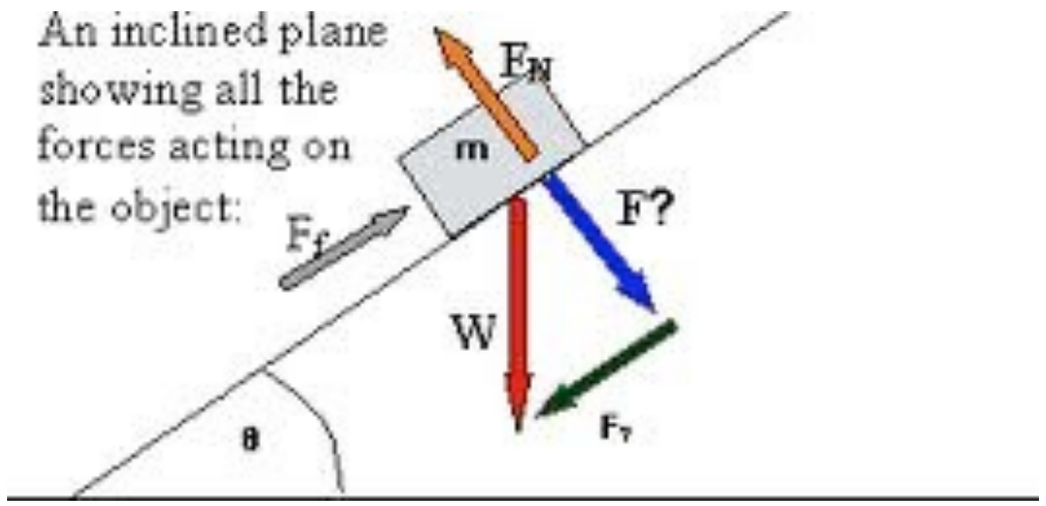


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore *J S. NEWTON, Trin. Coll. Cantab. Soc.* Matheseos Professore *Lucasiano,* & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu *Societatis Regiæ* ac Typis *Josephi Streater.* Prostat apud plures Bibliopolas. *Anno* MDCLXXXVII.



$$\oint \mathbf{E} \cdot d\mathbf{A} = q / \varepsilon_0$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

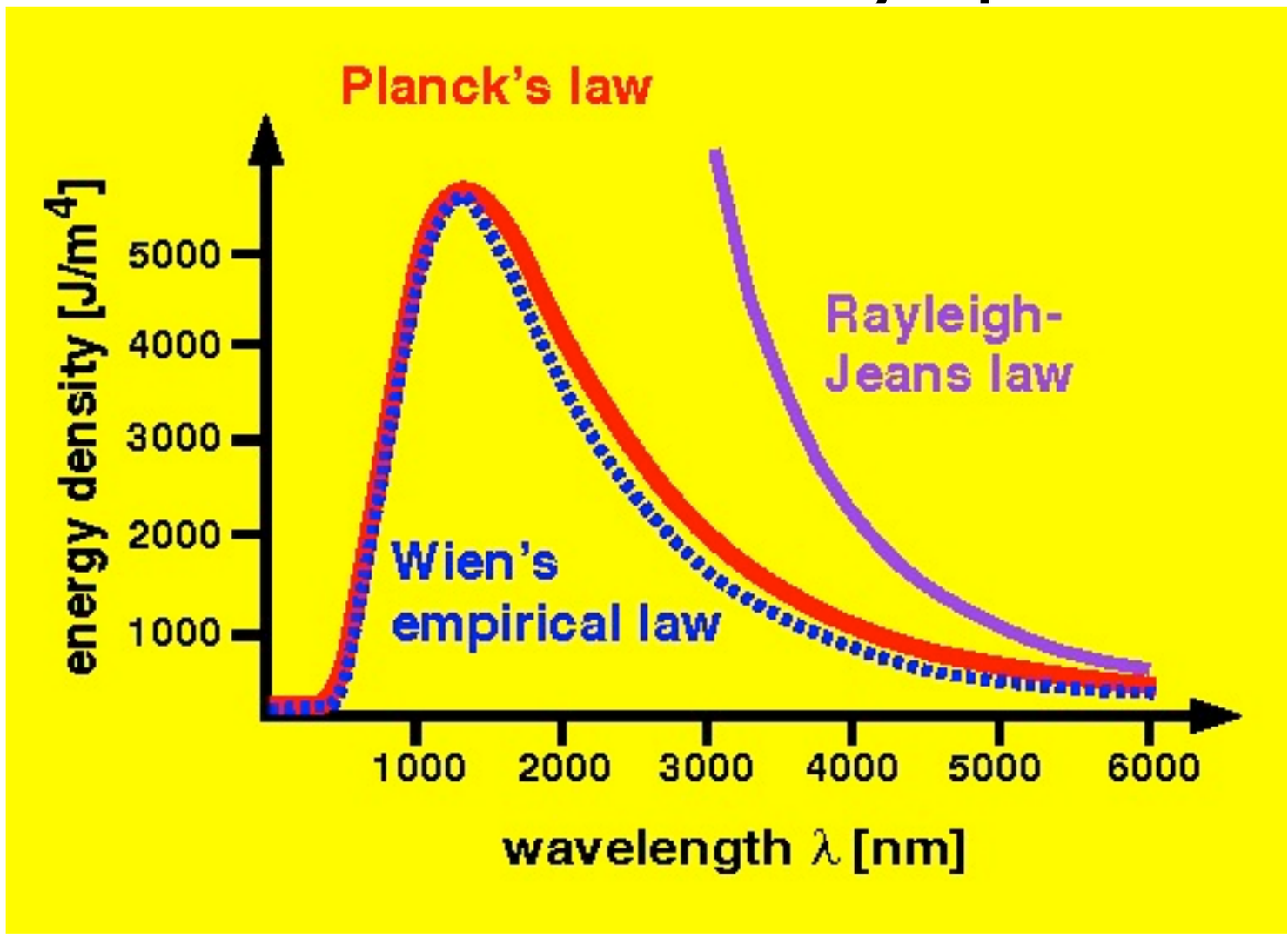
$$\oint \mathbf{E} \cdot d\mathbf{S} = -d\Phi_B / dt$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 i + \mu_0 \varepsilon_0 d\Phi_E / dt$$

The beginning of the end of classical physics

Max Planck

Black Body spectrum



Light can only be emitted and absorbed in discrete units of energy, QUANTA

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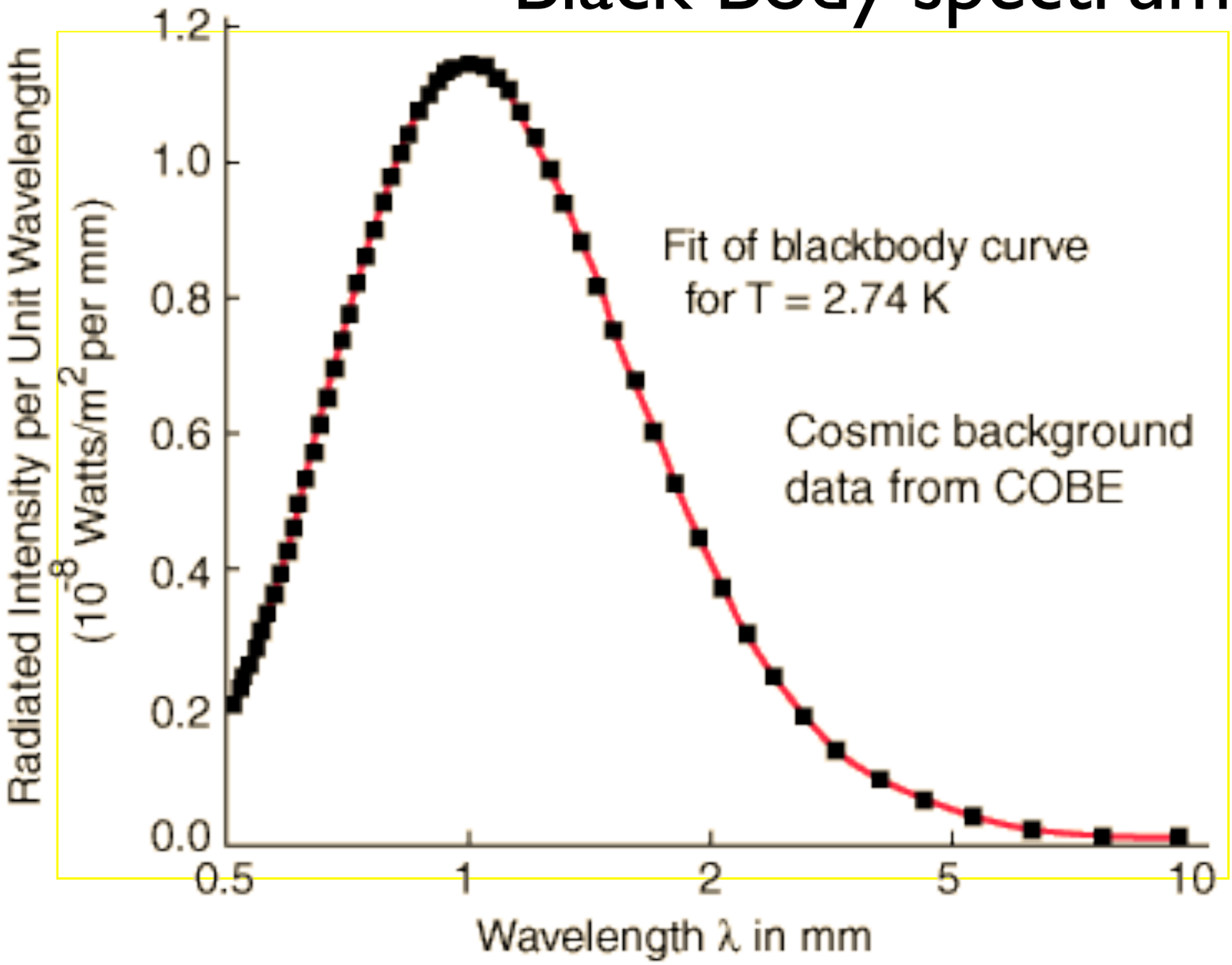

Light can only be emitted and absorbed in discrete units of energy, QUANTA

Planck's constant

Light of frequency ν can only be emitted and absorbed in units (quanta) of $h\nu$

$$E = h\nu$$

$$h = 6.626068 \times 10^{-34} m^2 kg/s = 6.626068 \times 10^{-34} Js$$

A new fundamental constant of nature: Planck's constant

Why it took so long to notice quanta

How many quanta of light are emitted from the light bulb above us?

$$E = h\nu$$

$$h = 6.626068 \times 10^{-34} kg\, m^2/s = 6.62608 \times 10^{-34} Js$$

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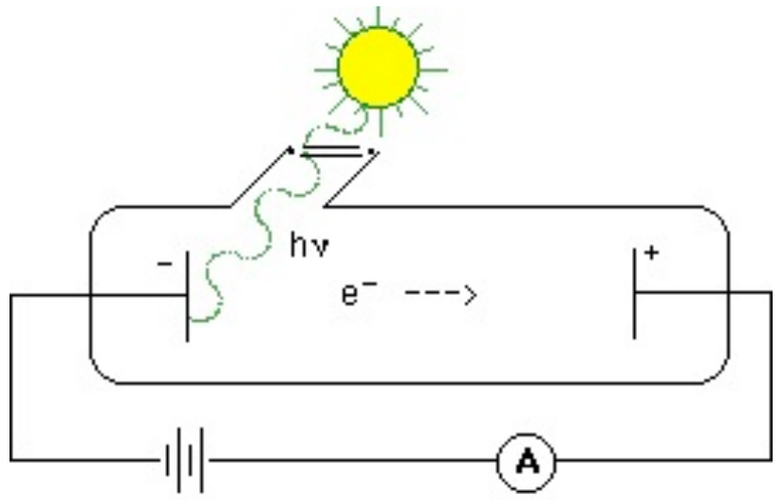
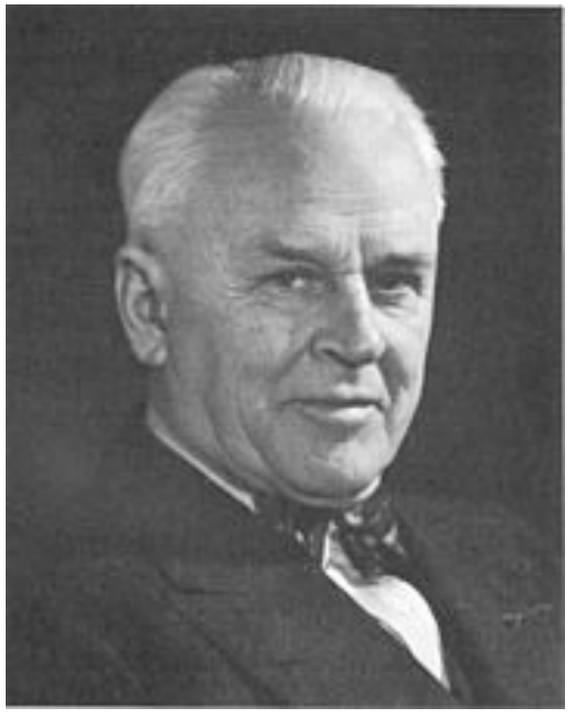
$$N = \frac{100 J/s}{E_q} = \frac{100 J/s}{3.3 \times 10^{-19} J} \approx 3 \times 10^{20}/s$$!!!

The end of classical physics

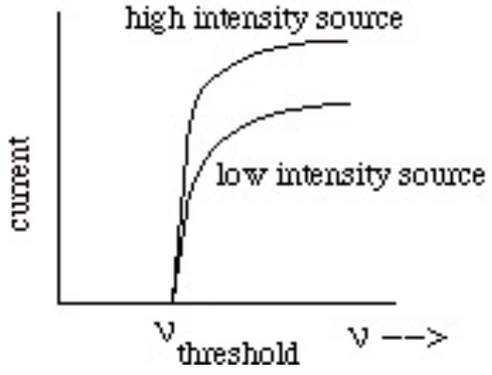
Heinrich Hertz

Photoelectric effect





R A Millikan

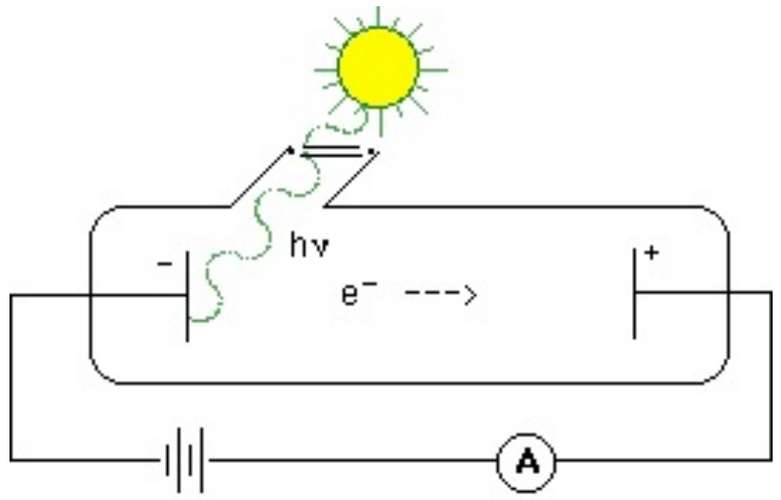


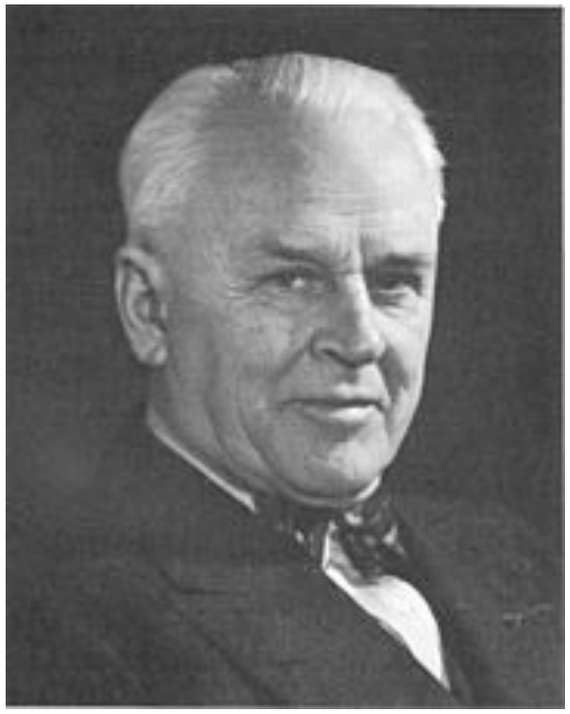
1. Metals emit electrons when irradiated
2. Threshold, depends on frequency
3. Current ∝ intensity
4. Energy ∝ frequency

The end of classical physics

Heinrich Hertz

Photoelectric effect





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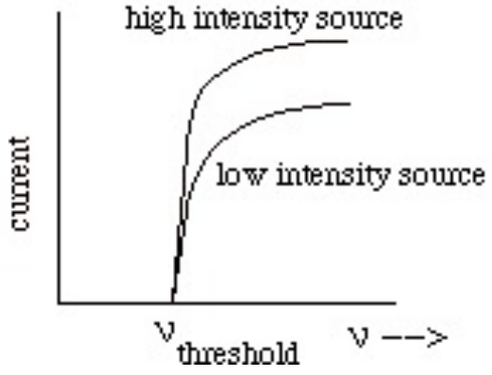

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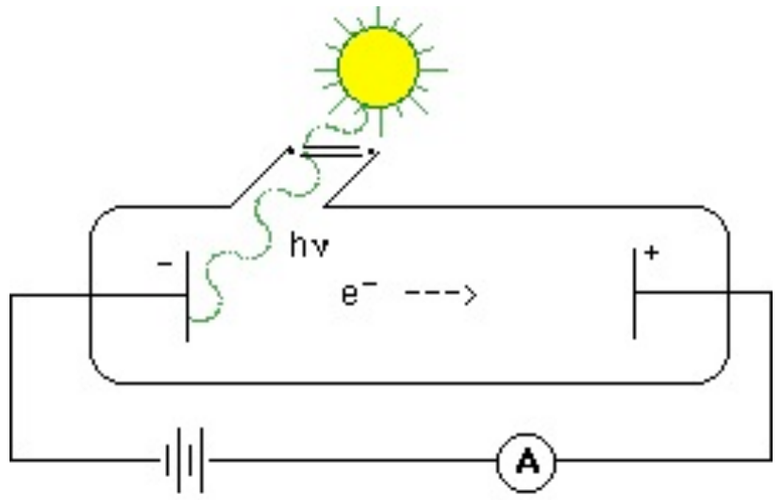

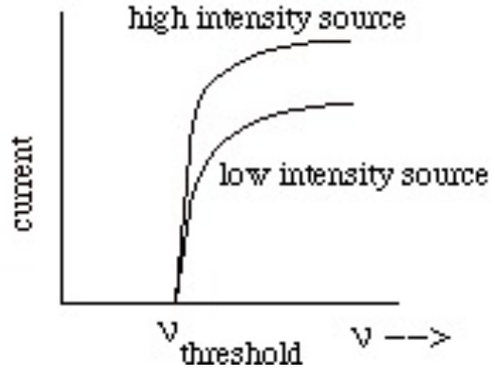

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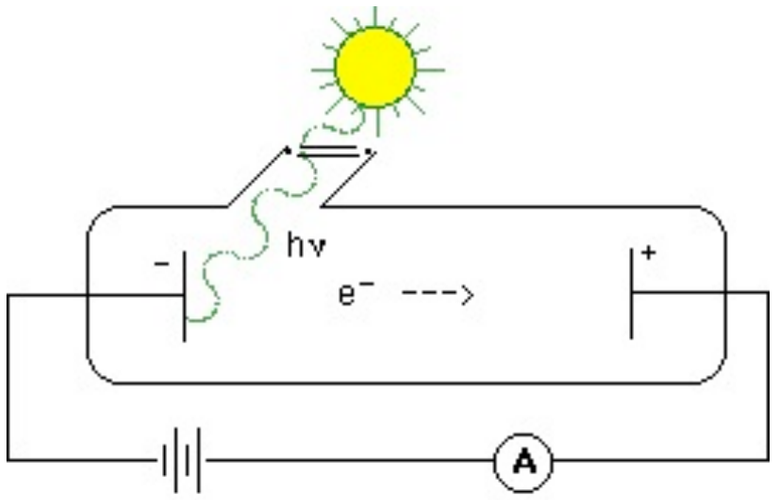

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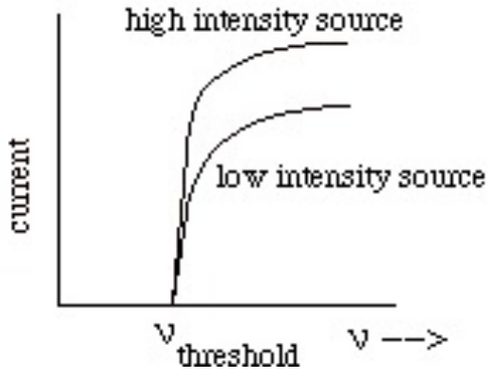
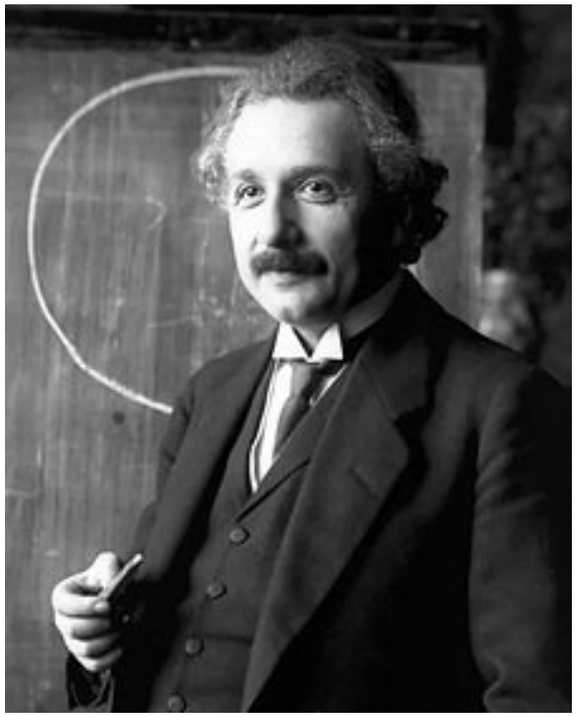


Photons as waves cannot explain this!

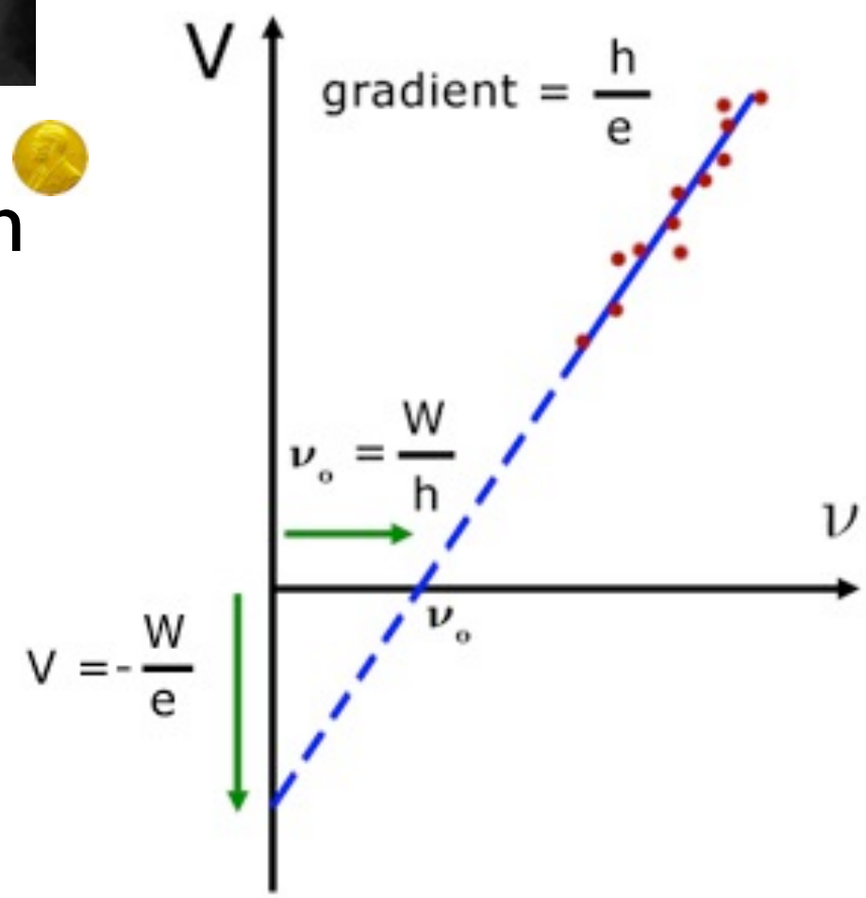
The end of classical physics



Albert Einstein

Light behaves as if it is discrete bundles of energy (photons) of energy $h\nu$

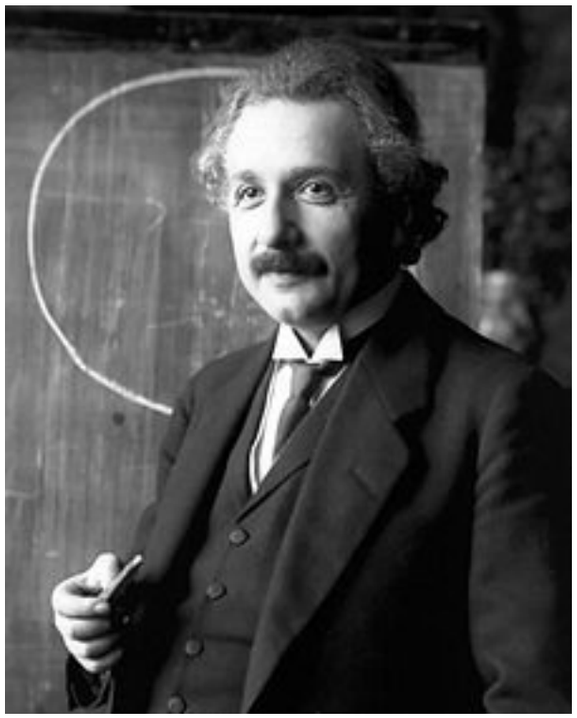
$$\frac{1}{2}mv^2 = h\nu - W$$



Explains photoelectric effect

Light is a particle!

The end of classical physics



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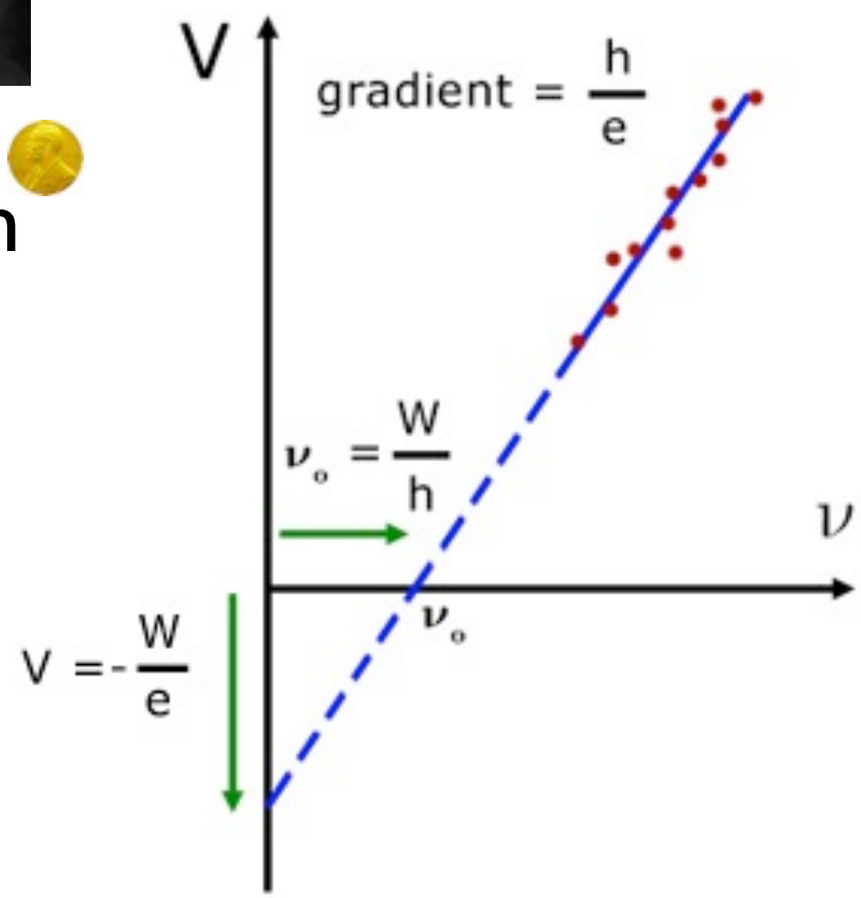

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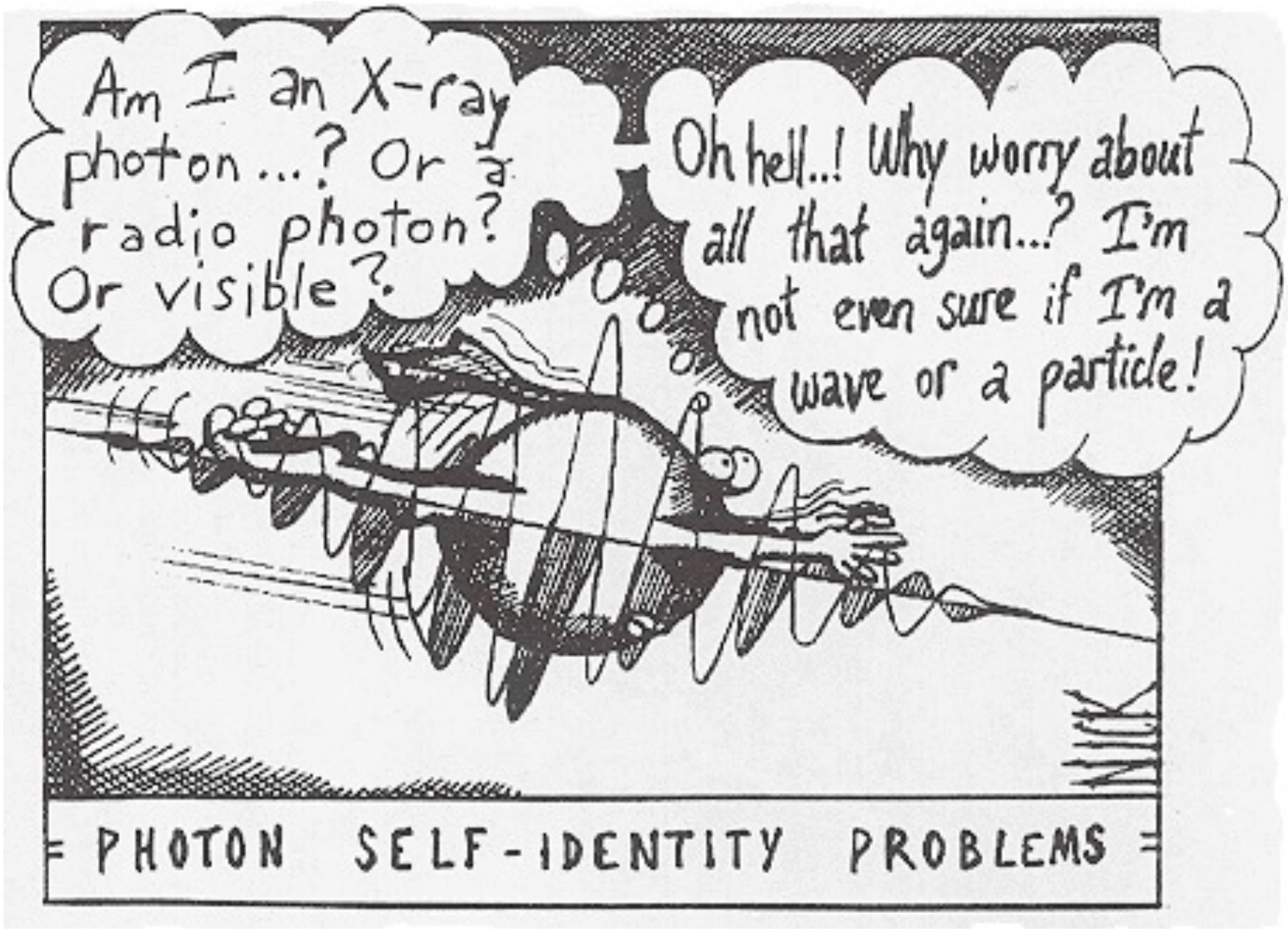
Explains photoelectric effect

Light is a particle!

and a wave!


Am I an X-ray photon...? Or a radio photon? Or visible?
Oh hell..! Why worry about all that again..? I'm not even sure if I'm a wave or a particle!
NICK
PHOTON SELF-IDENTITY PROBLEMS

Wave-particle duality

Louis de Broglie

If a wave can be a particle, can a particle be a wave?

For light,

$$E \quad h\nu \quad h$$

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

What is your de Broglie wavelength?

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$$10^{-36} m$$

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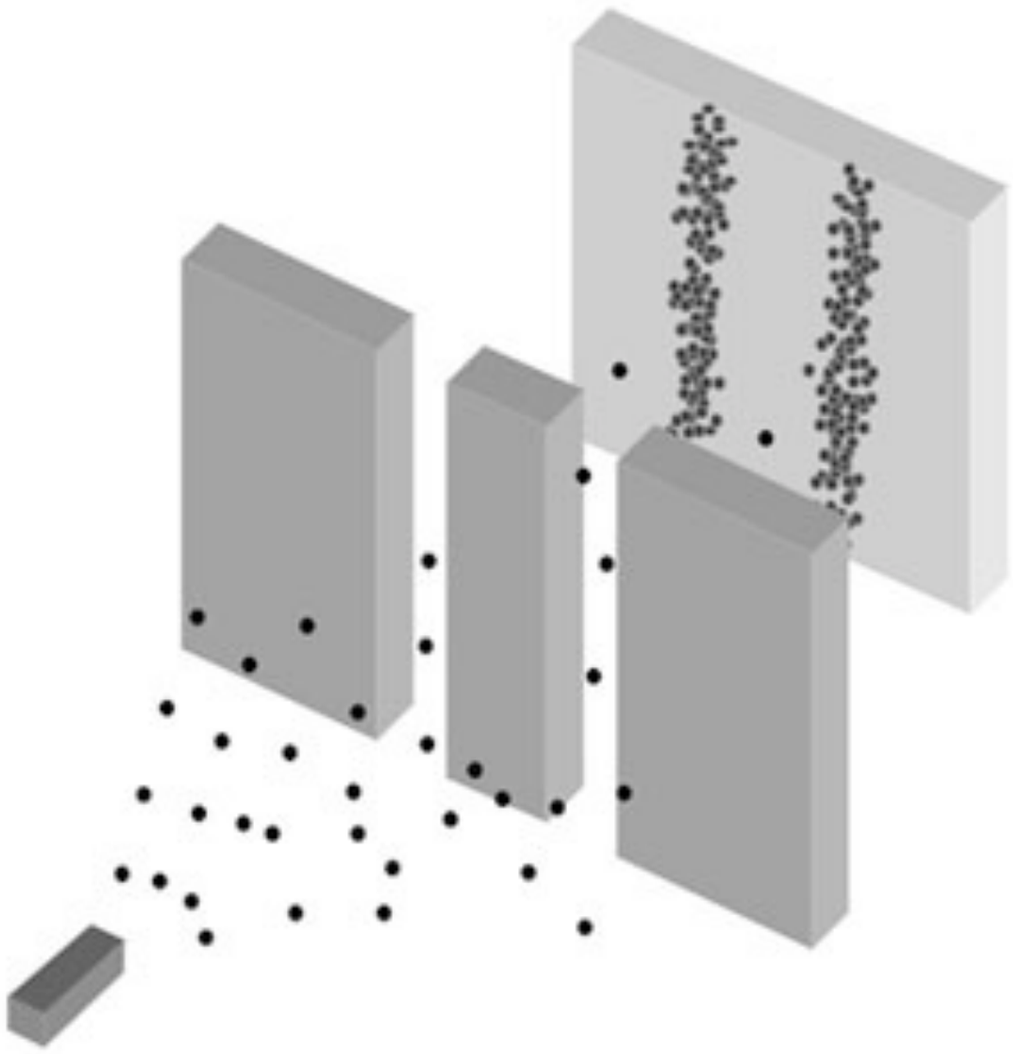
$10^{-36} m$

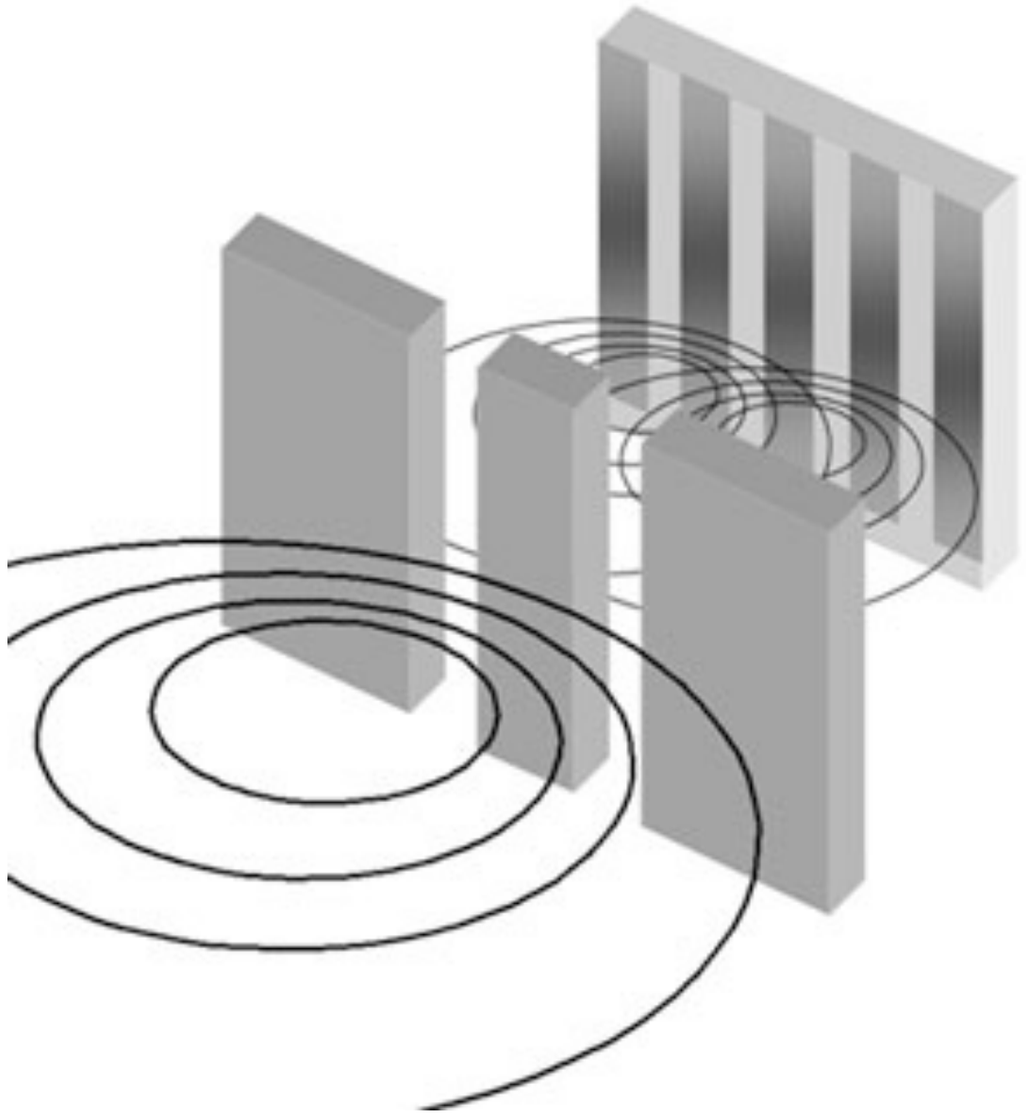
proton is about

$10^{-15} m$

The double slit experiment

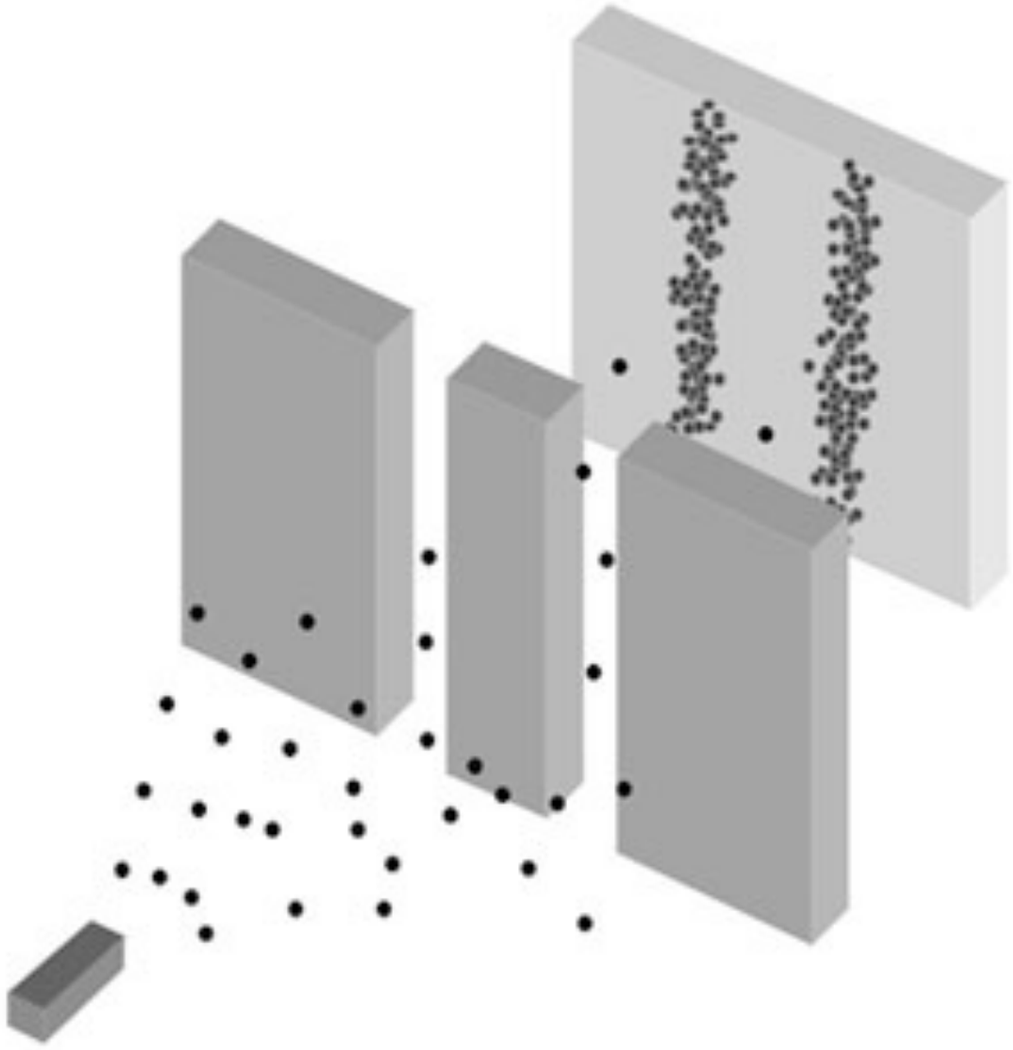


Classical particles, e.g. bullets



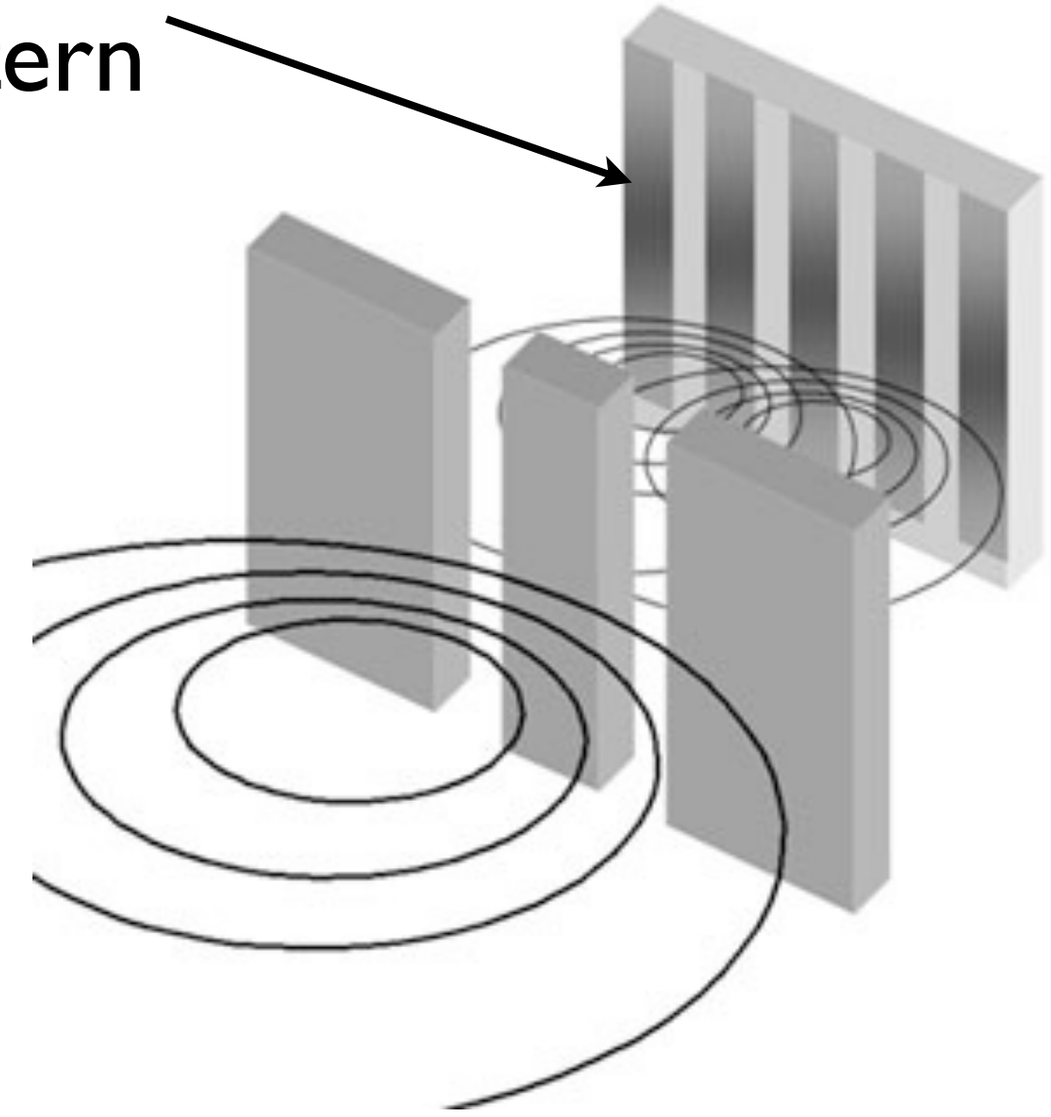
Waves, e.g. on water surface

The double slit experiment



Classical particles, e.g. bullets

Interference pattern



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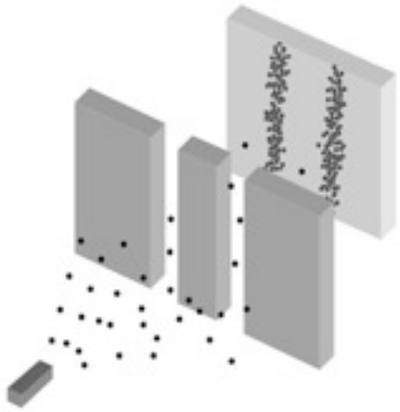

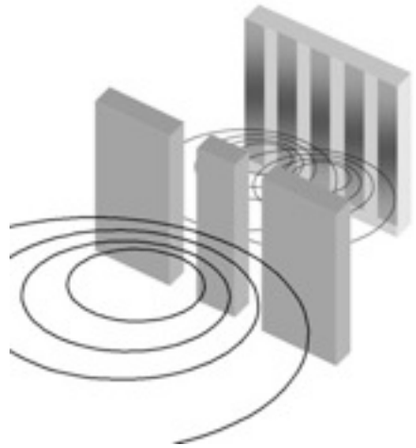

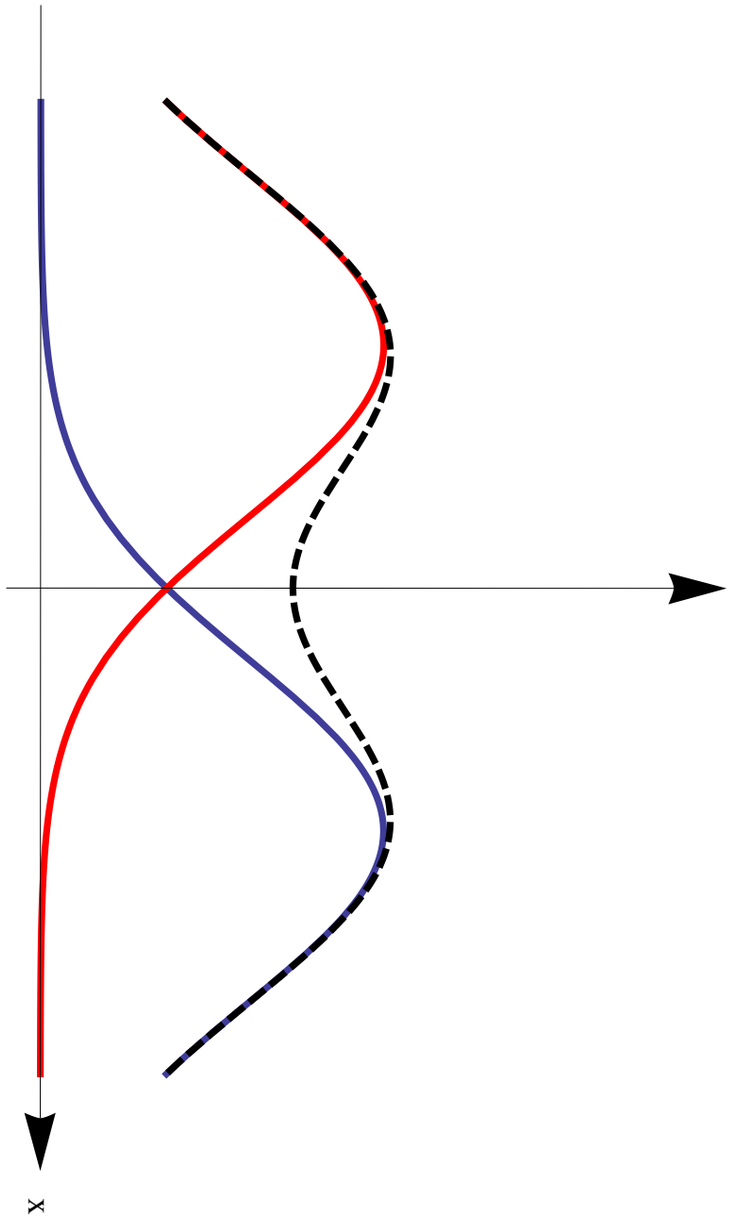
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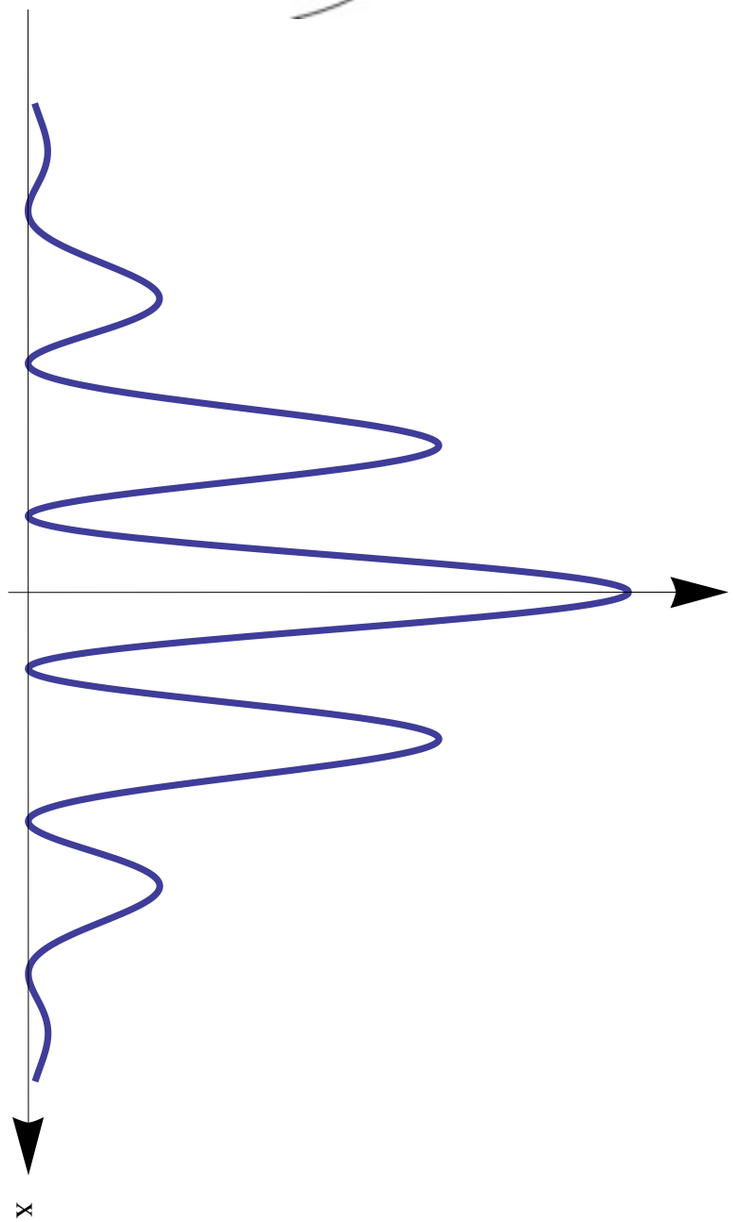
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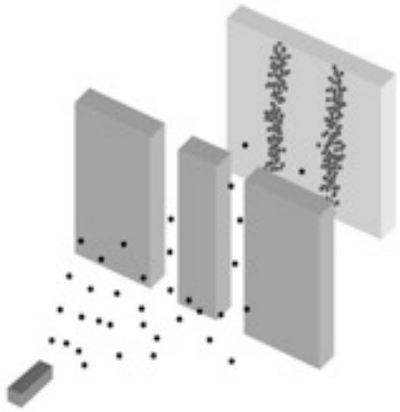
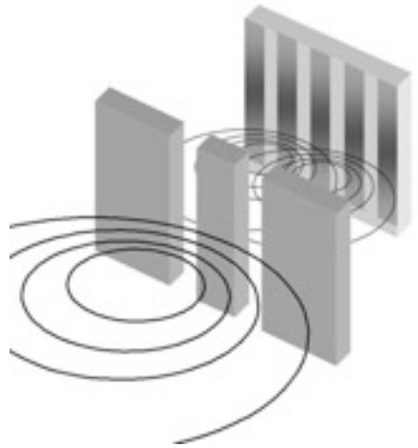
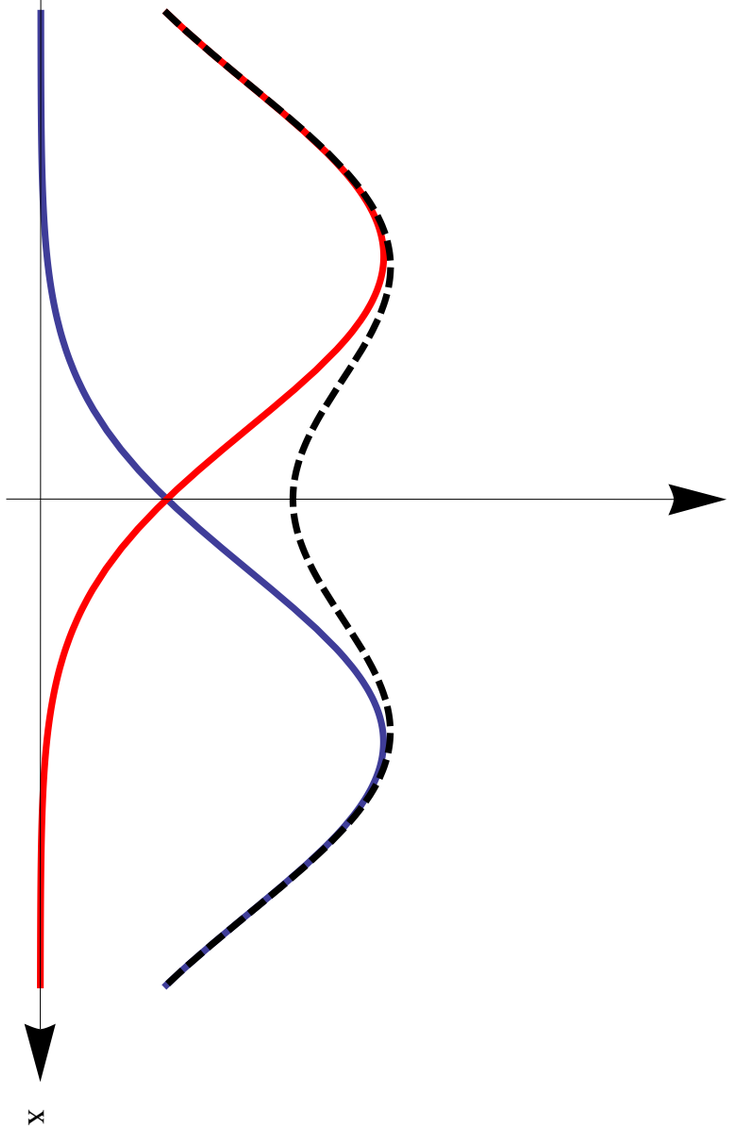
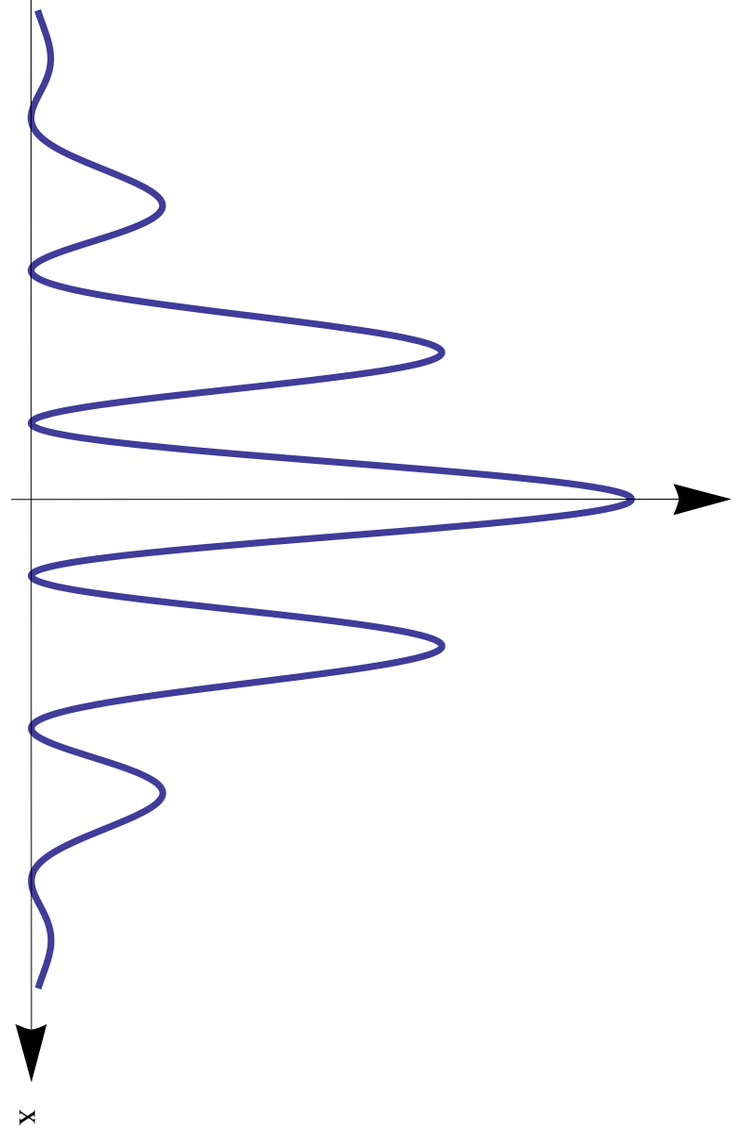
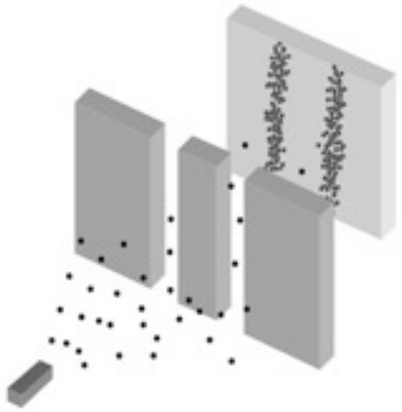




$$P_{12} = |A_1|^2 + |A_2|^2$$







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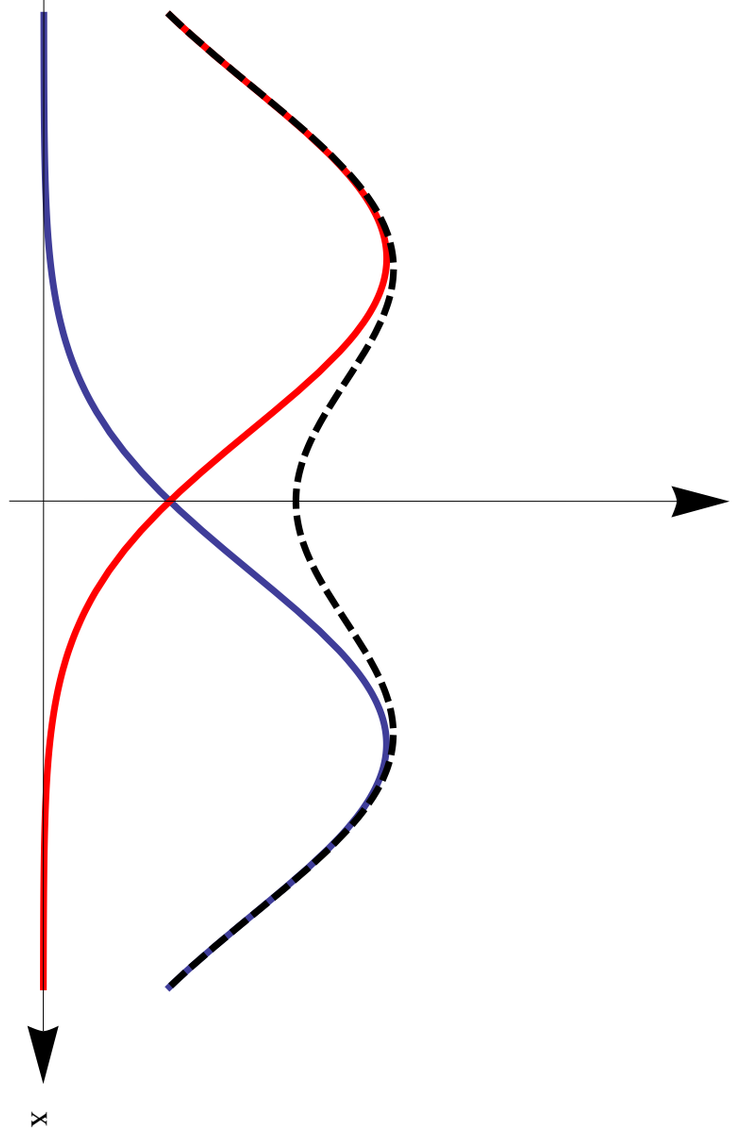
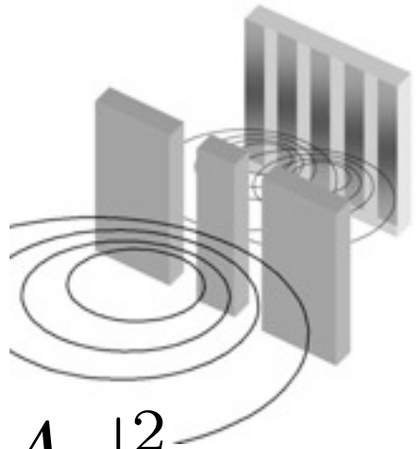
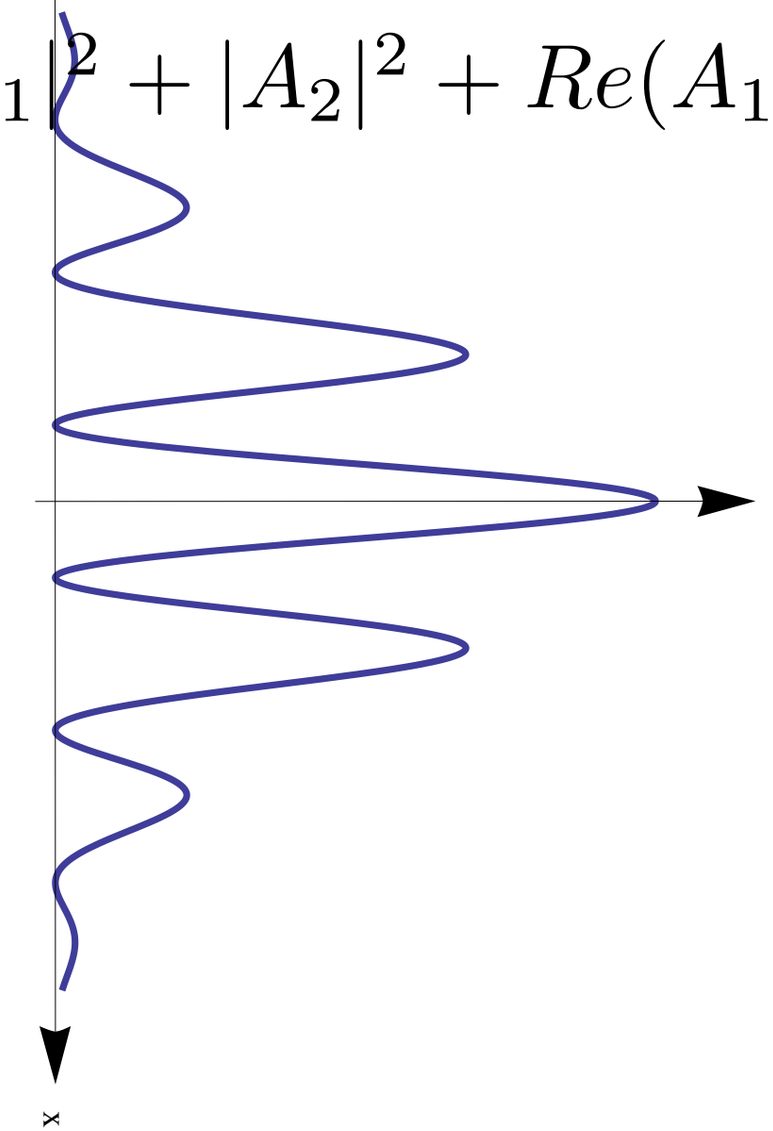




$$P_{12} = |A_1 + A_2|^2$$

$$= |A_1|^2 + |A_2|^2 + Re(A_1 A_2^*)$$



The double slit experiment

Repeat with electrons

Interference pattern!

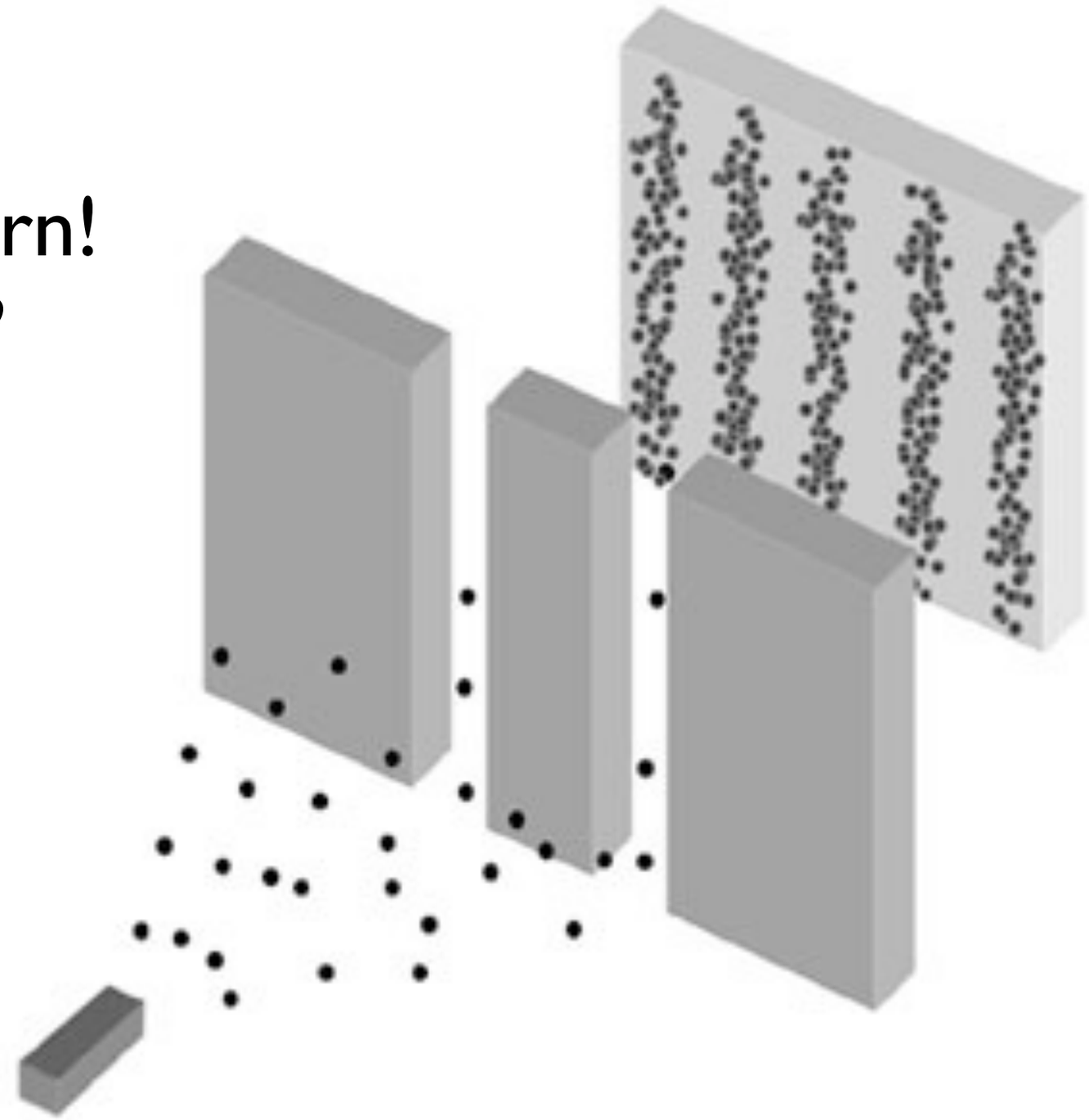
Waves of what?

The double slit experiment

Repeat with electrons

Interference pattern!

Waves of what?



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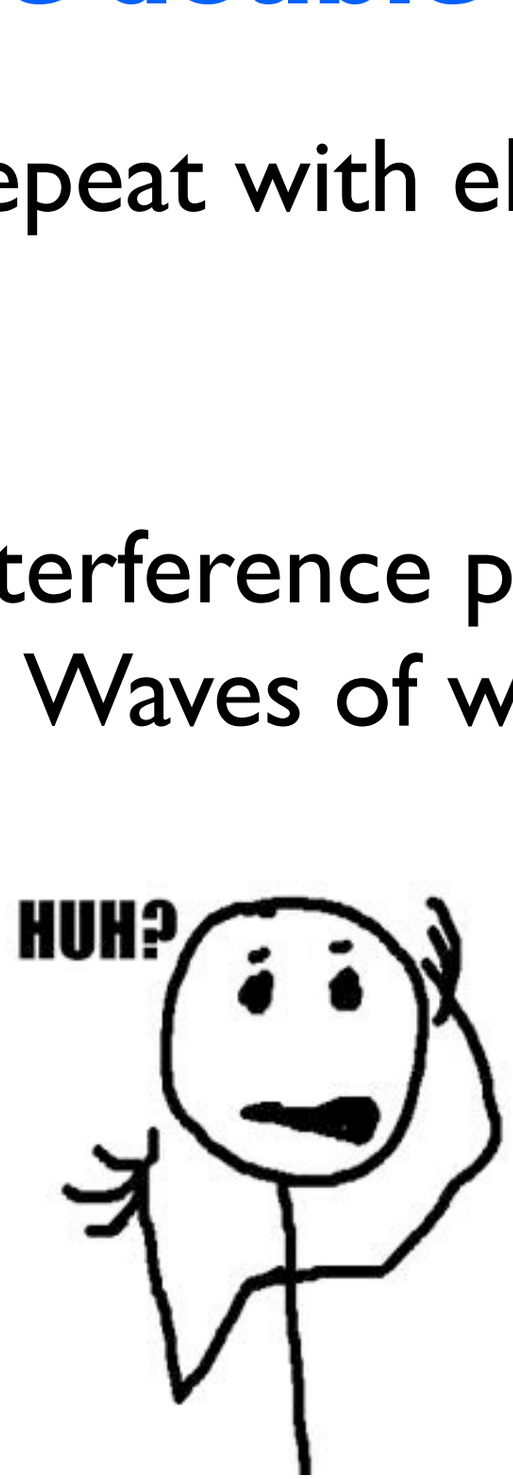

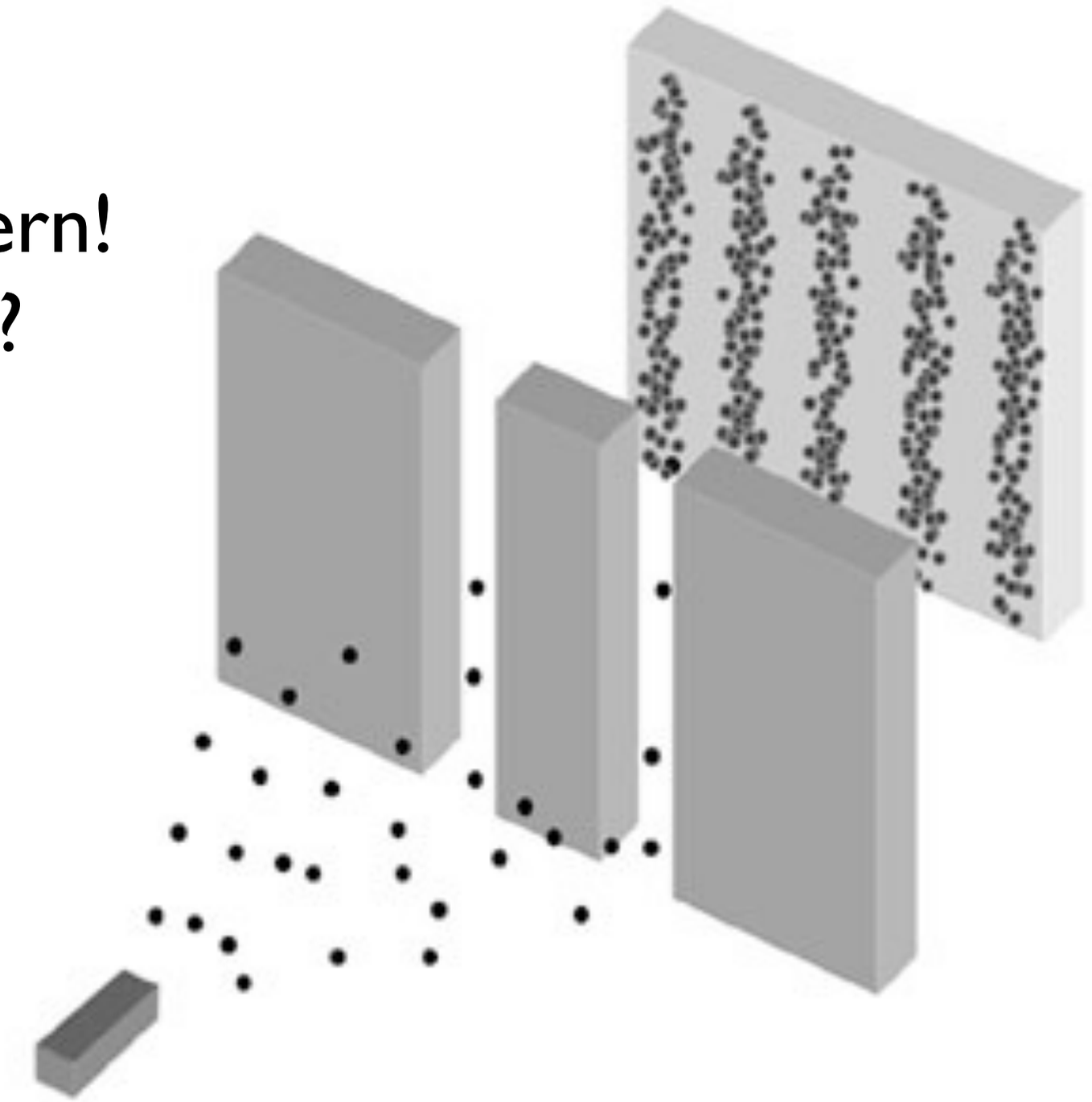


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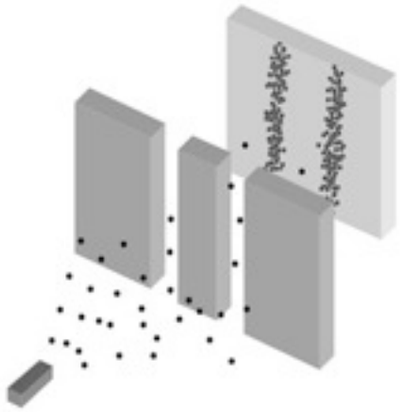

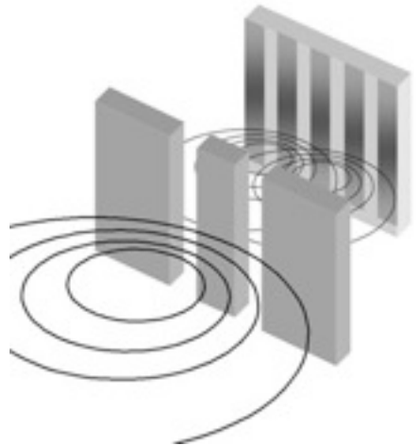

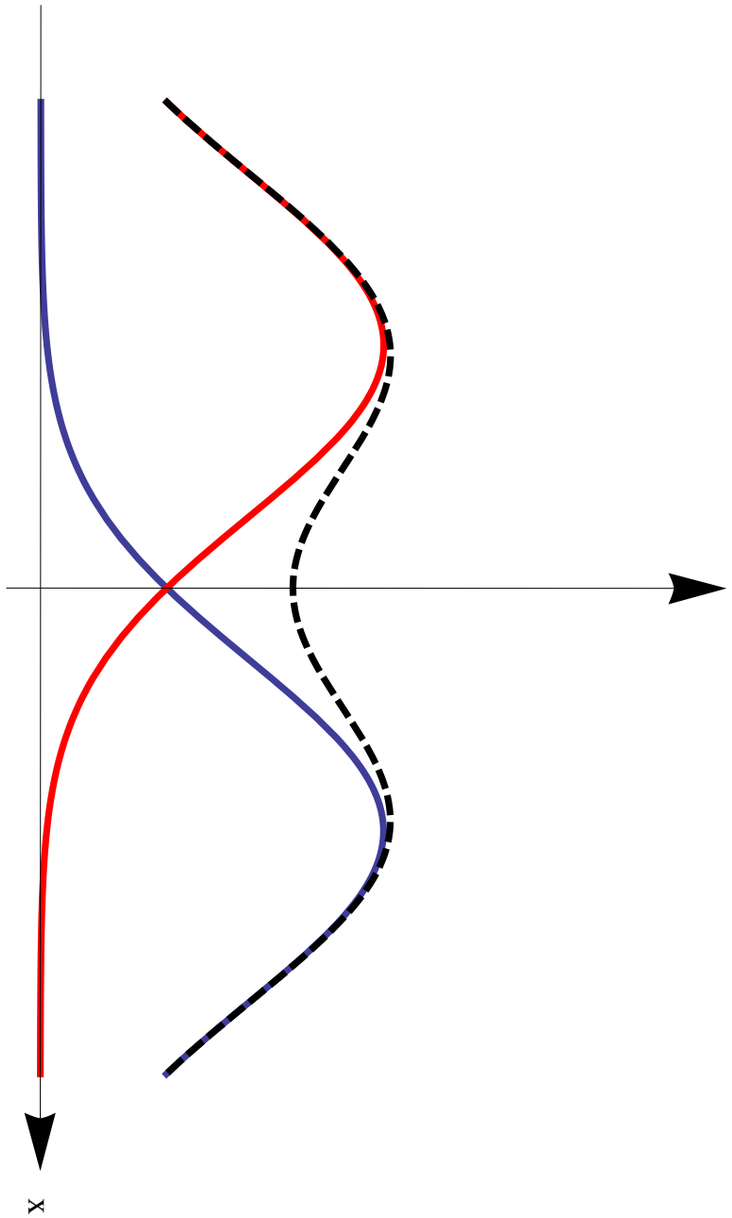
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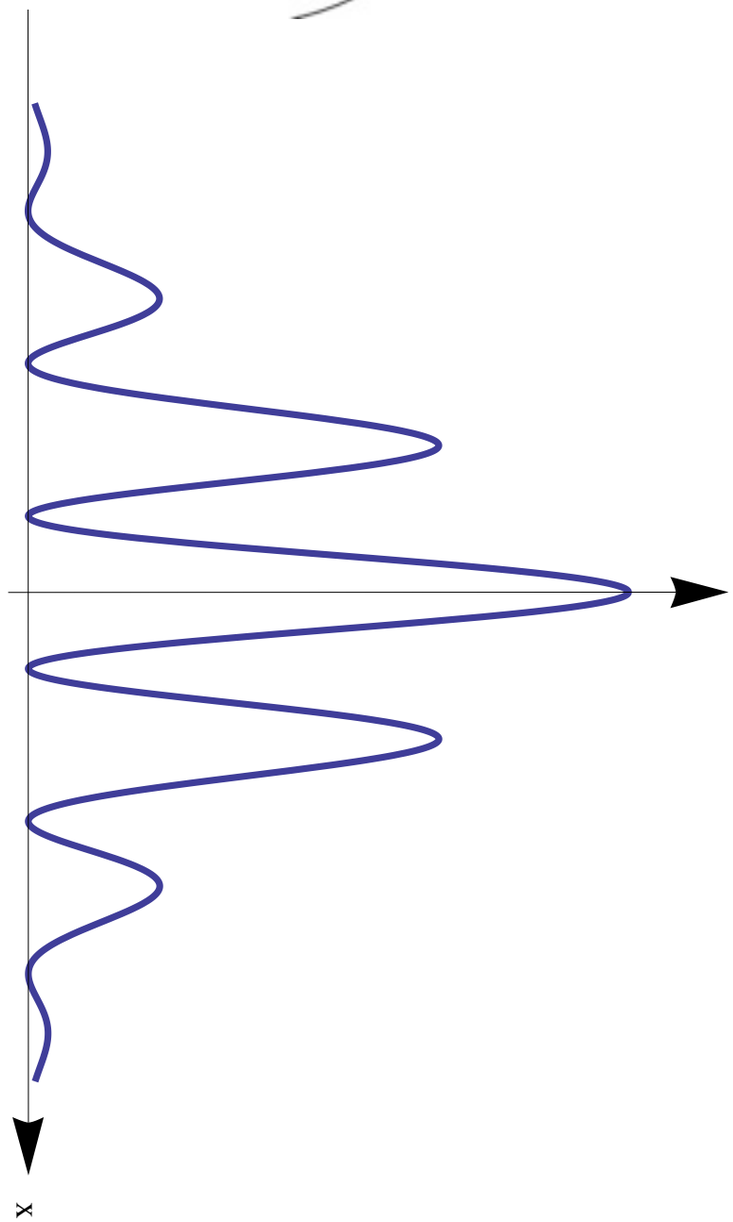
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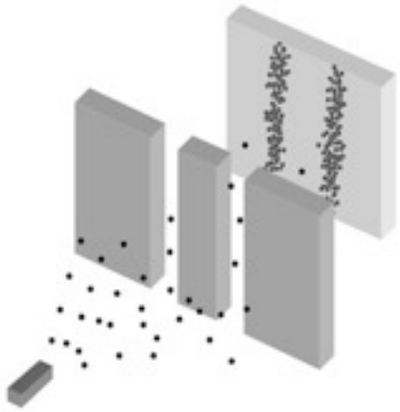
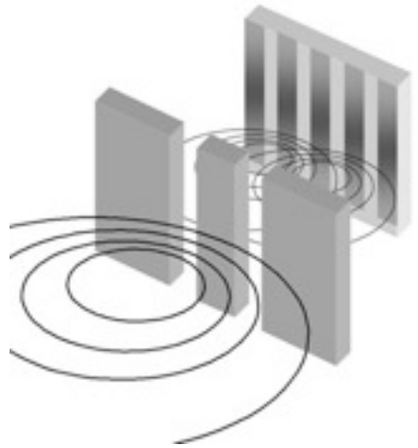
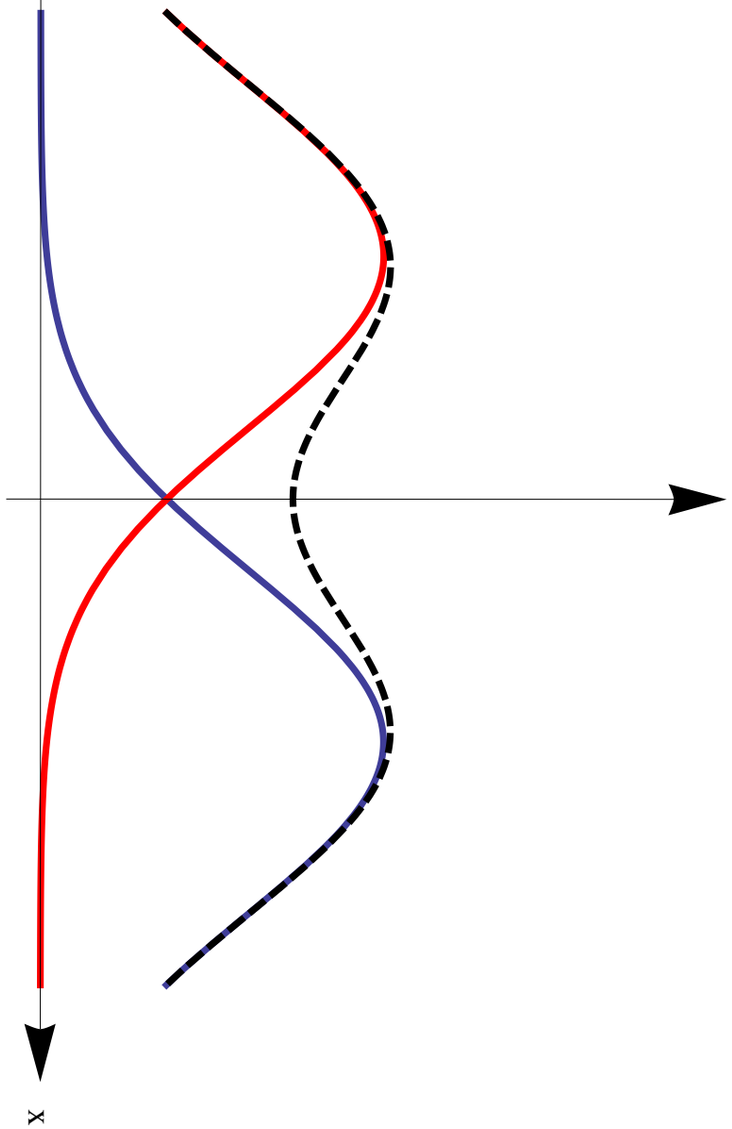
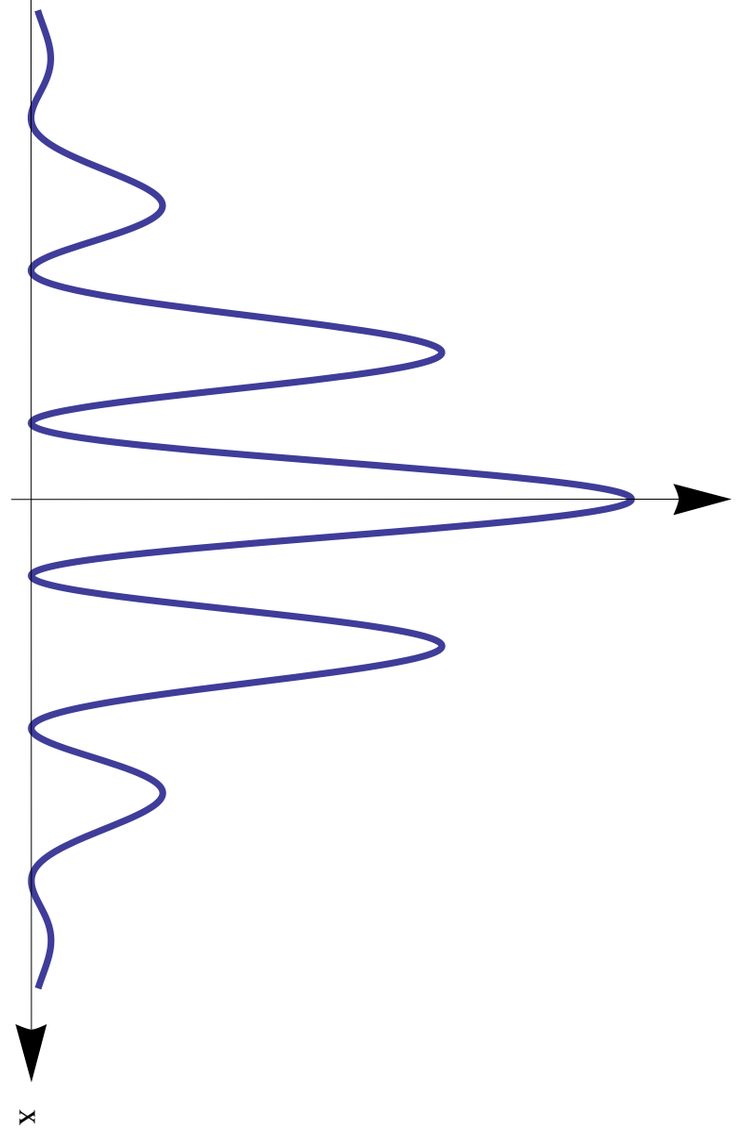
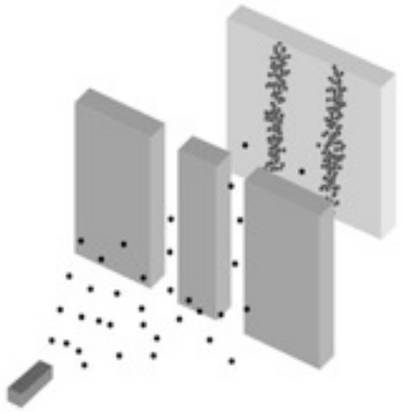




$$P_{12} = |A_1|^2 + |A_2|^2$$







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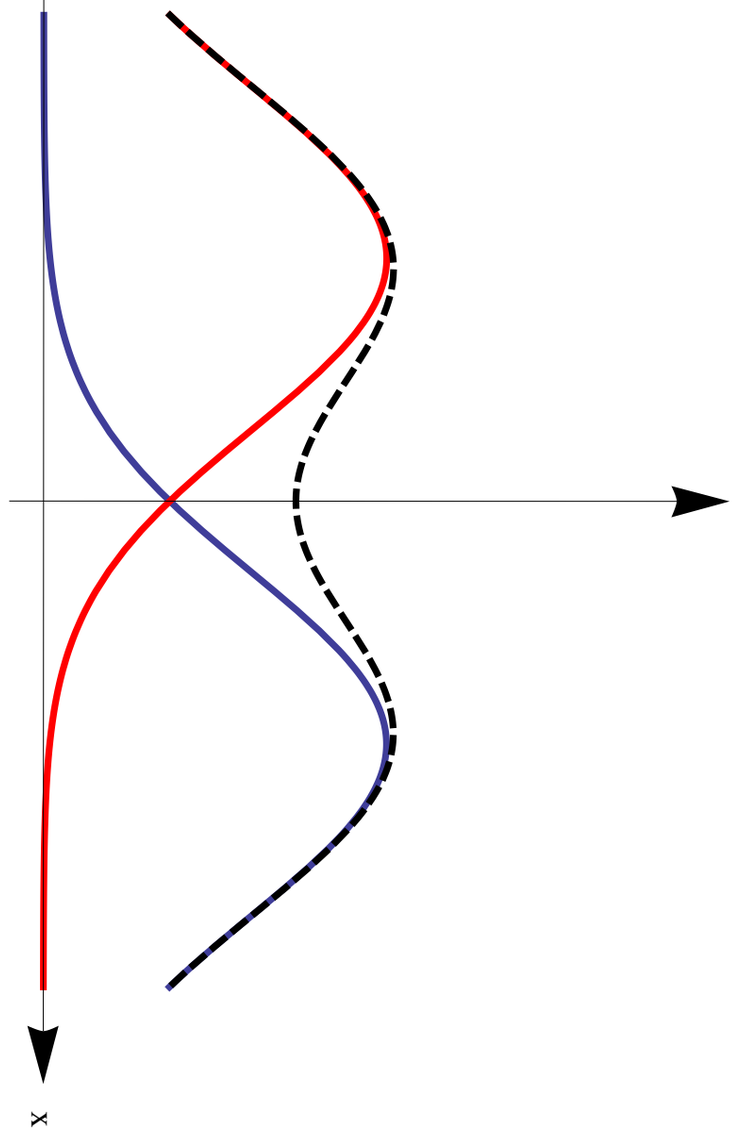
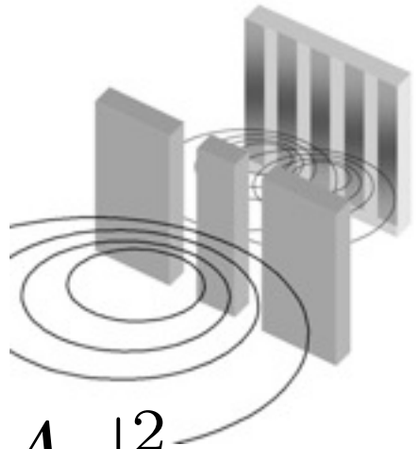
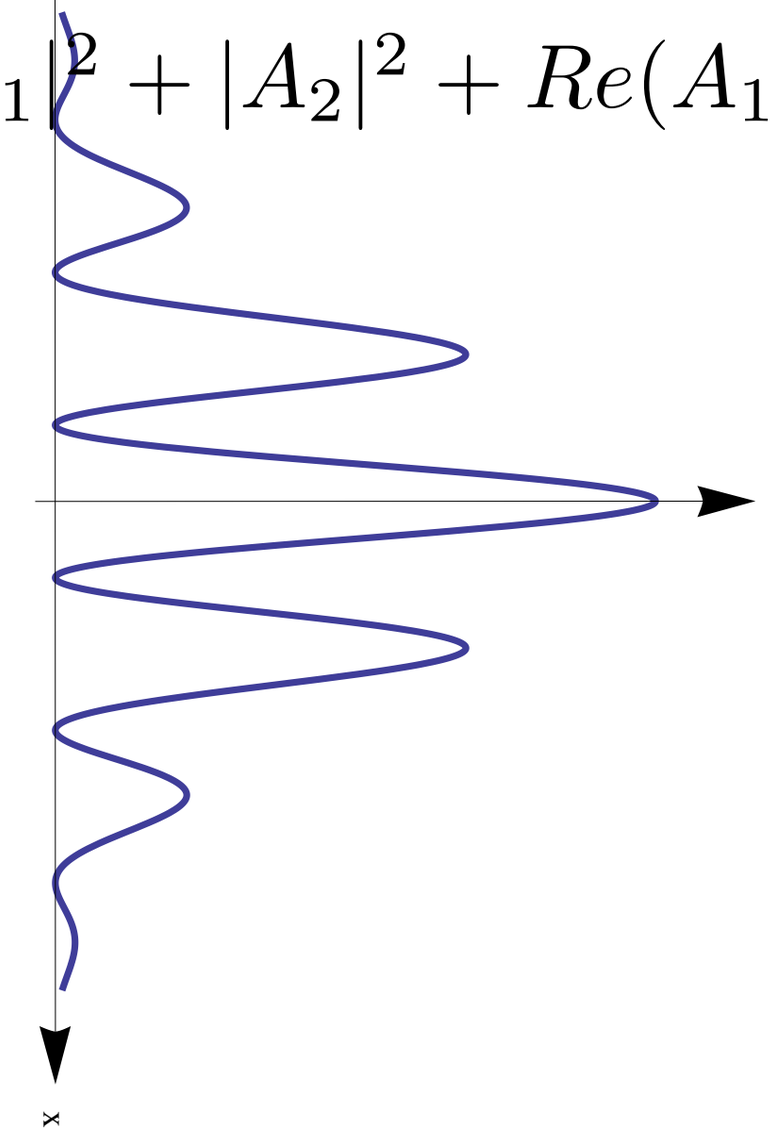




$$P_{12} = |A_1 + A_2|^2$$

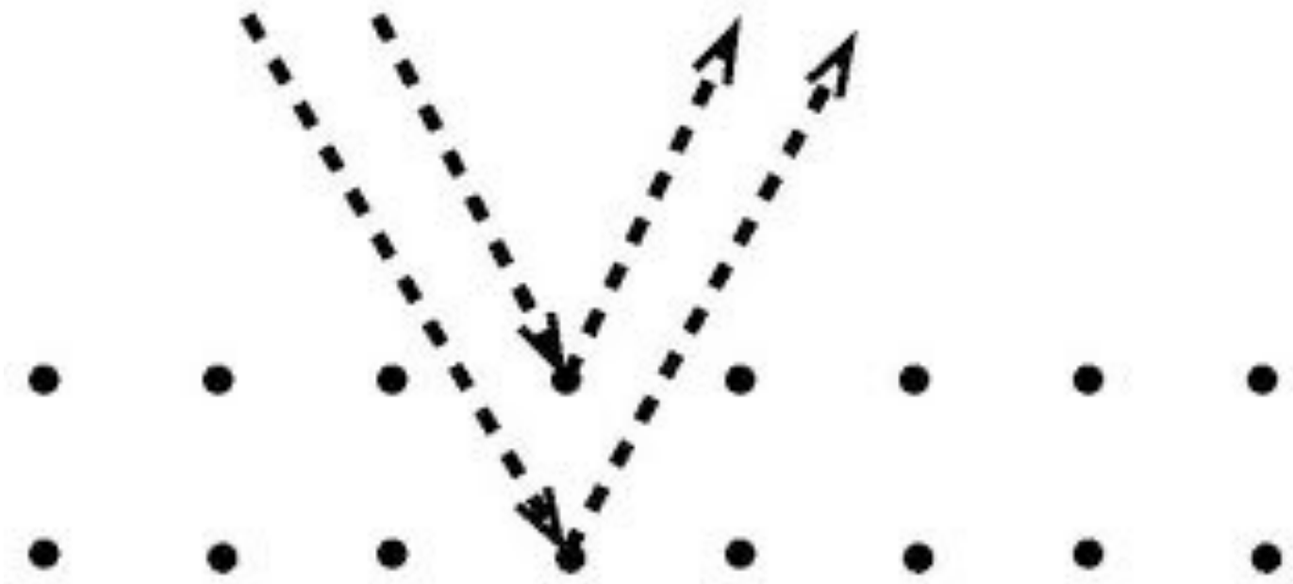
$$= |A_1|^2 + |A_2|^2 + Re(A_1 A_2^*)$$



Gedanken vs real experiments

Interference occurs if wavelength ~ slit separation

de Broglie wavelength for an electron at 60 km/s is comparable to the atomic spacing in a metal



The quantum mechanics wave function

$$\Psi(x,t)$$

The probability for a particle to be at a point (x,t) is $|\Psi(x,t)|^2$

Quantum mechanics is a probabilistic theory
Evolution **is** deterministic but predictions are only statistical

The Schrödinger equation

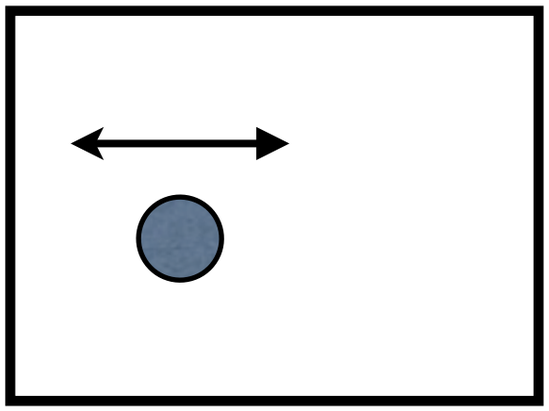
Erwin Schrödinger

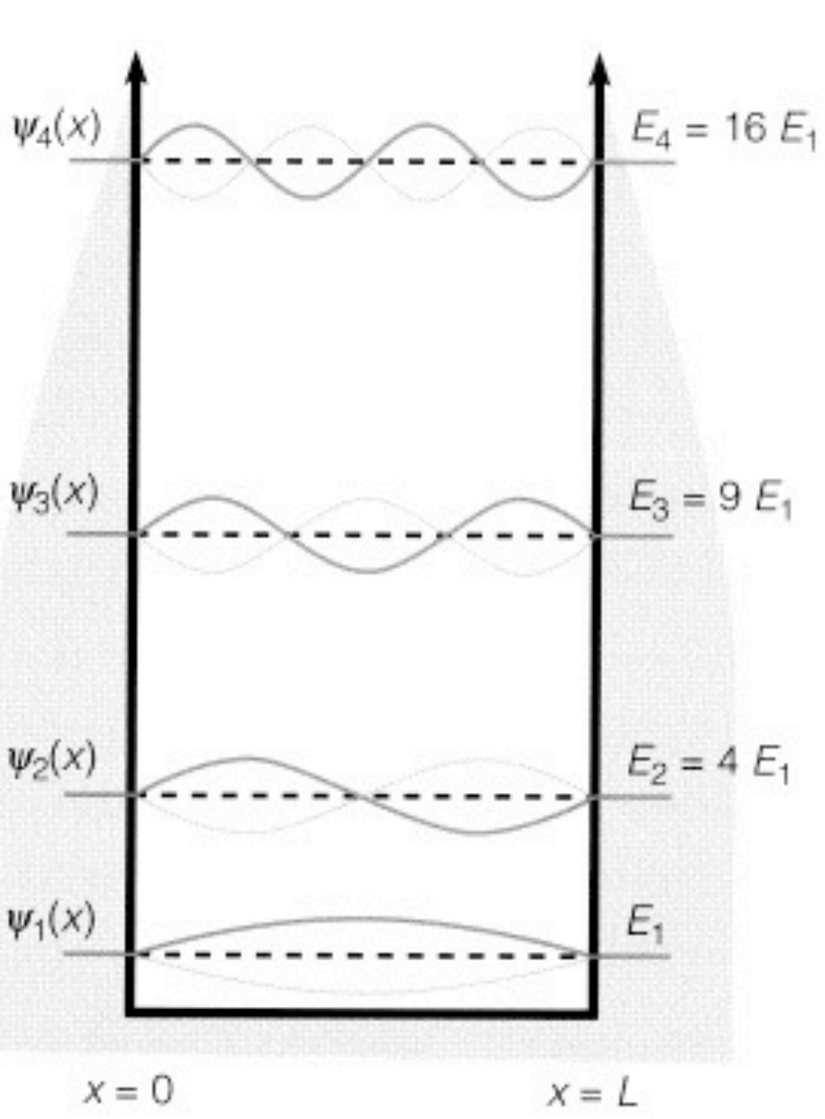
There is an equation that governs the evolution of the wavefunction

Can only ask questions like: what is the probability of seeing X?

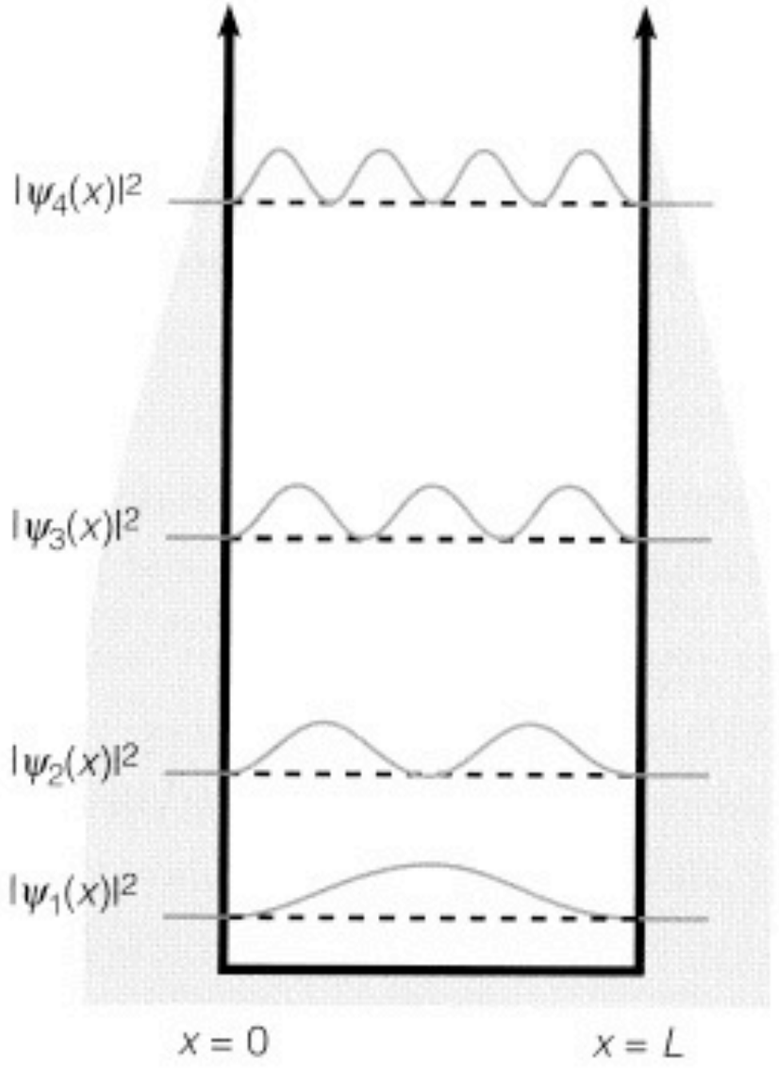
$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

Particle trapped in a box





Infinite-well wave functions.

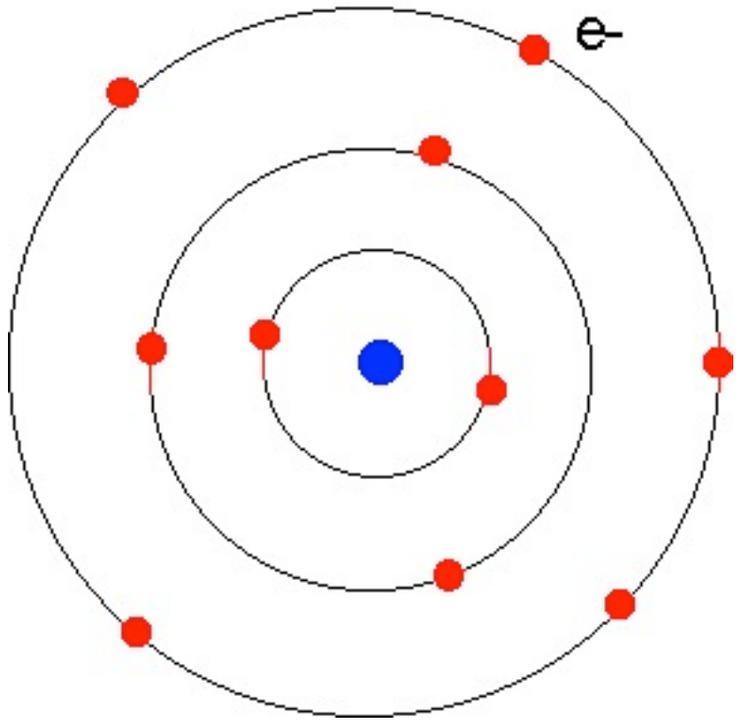


Infinite-well probability densities.

faster

In a classical world you wouldn't be here!

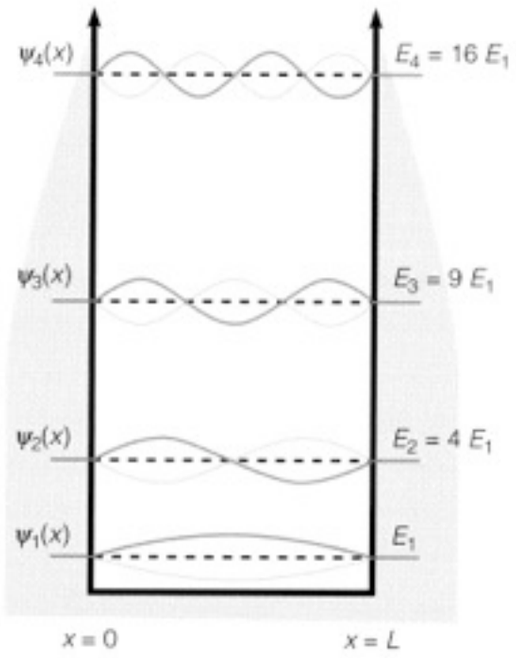
Neils Bohr



Unstable!

The Bohr atom, with quantum mechanics is stable

Electrons as standing waves



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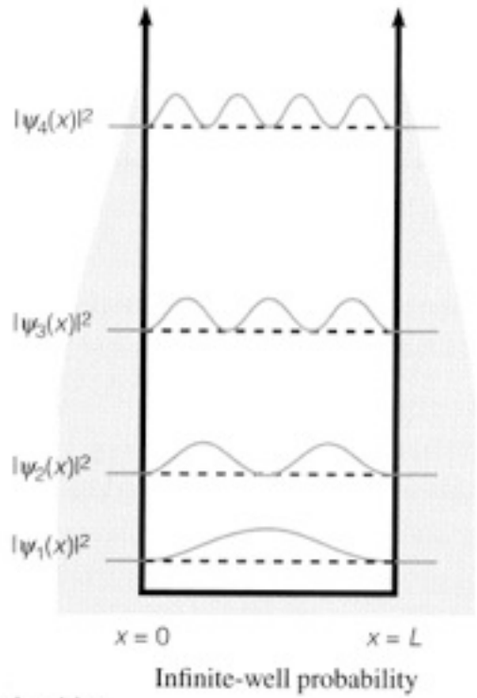
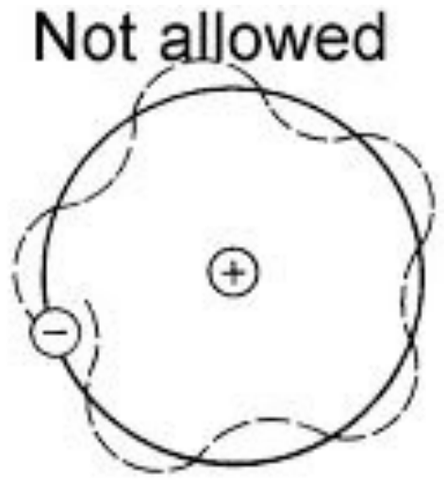
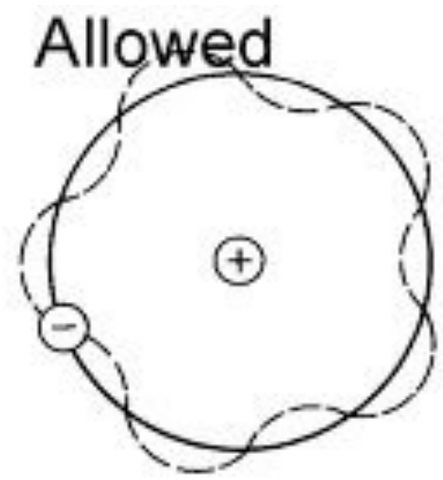


densities.

Infinite-well probability

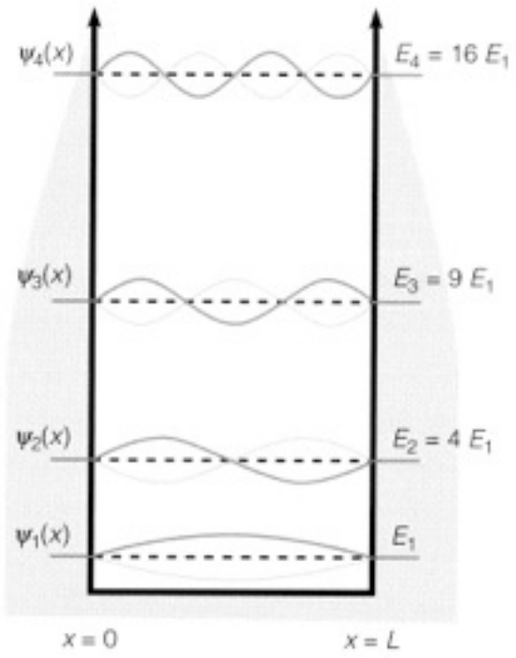




Angular momentum is quantized

Lowest orbital has L=1, no where to decay to

Electrons as standing waves



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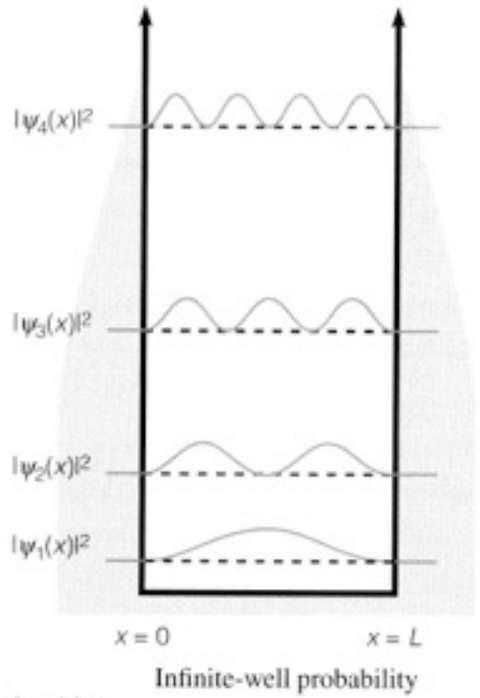

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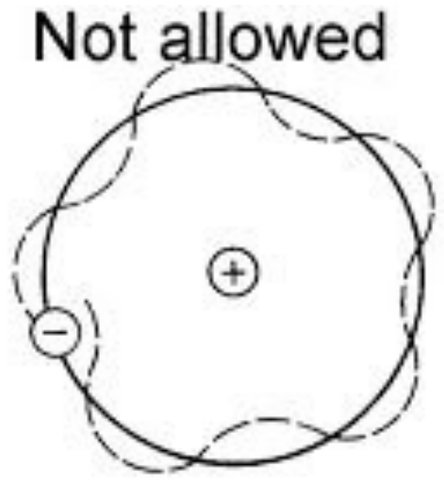

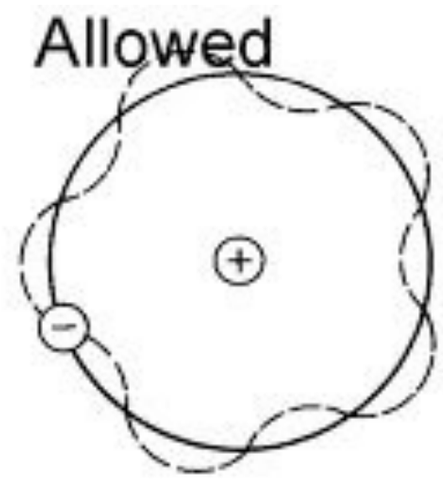

Angular momentum is quantized

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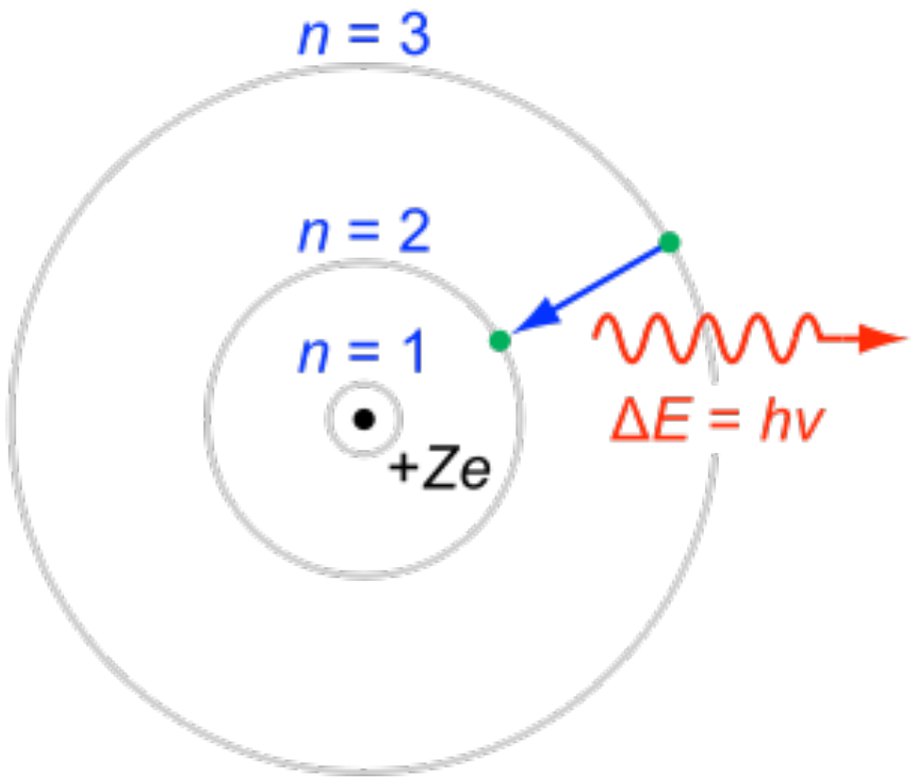
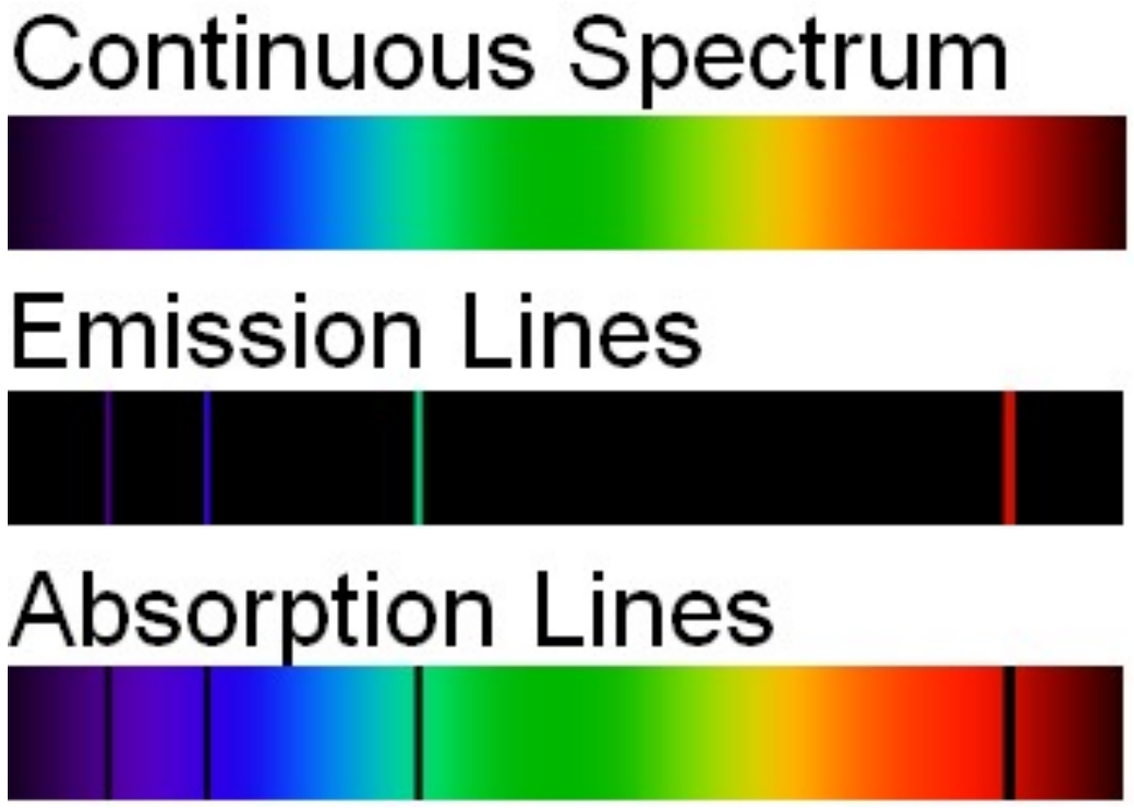
Atoms are stable

The Bohr Atom



Light emitted/absorbed at fixed frequencies



$$\nu \sim \frac{1}{n_1^2} - \frac{1}{n_2^2}$$

Heisenberg Uncertainty relation

Werner Heisenberg

$$\Delta p > \frac{\hbar}{a}$$

"The more precisely the position is determined, the less precisely the momentum is known in this instant, and vice versa."

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

The act of measurement disturbs the system

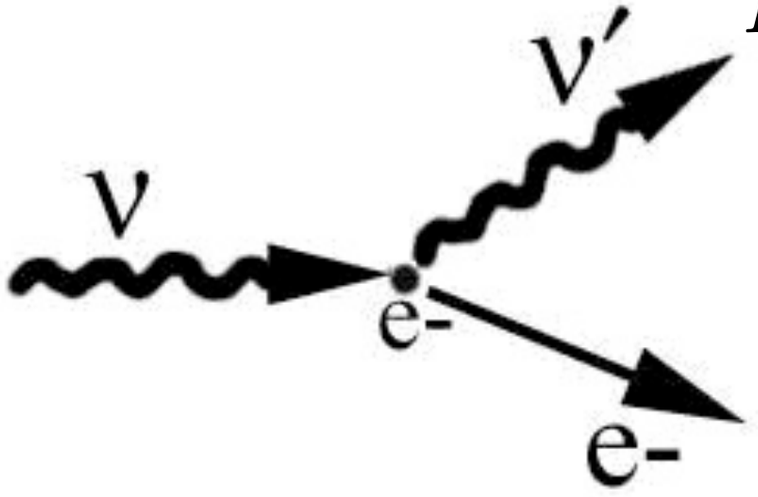
Measuring a particle's position

Use a "microscope" - shine light on an electron

Shorter wavelength means better precision $\Delta x \sim \lambda$

Compton scattering

$p \sim h/\lambda$



$\nu' < \nu$

Electron is initially at rest

e- gains energy

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Use a "microscope" - shine light on an electron

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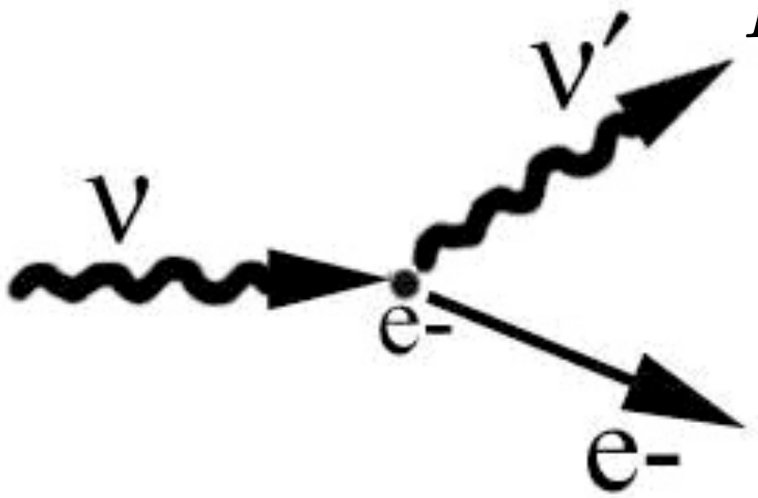

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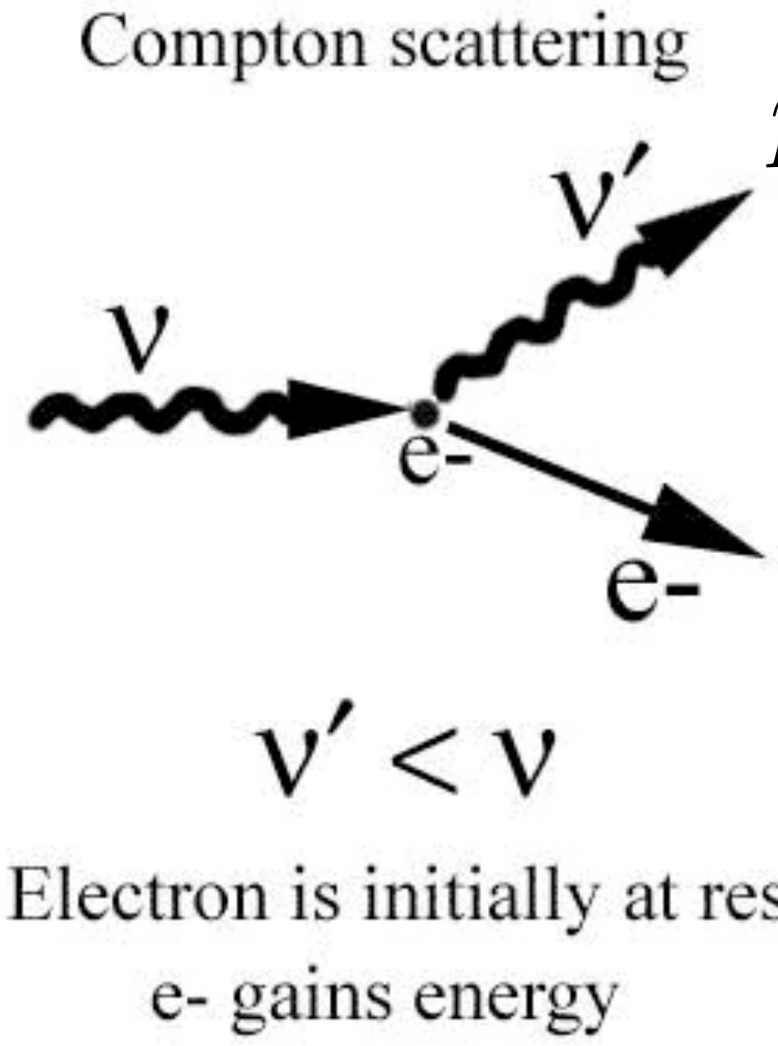


$p \sim h/\lambda$

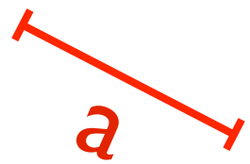
$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Similar argument for Young's double slit

Monitor two slits, need $\Delta x < \frac{a}{2}$

But then $\Delta p > \frac{\hbar}{a}$

The fractional change in electrons momentum parallel to screen is $\frac{\Delta p}{p} > \frac{\lambda}{a}$



Smears out interference pattern!

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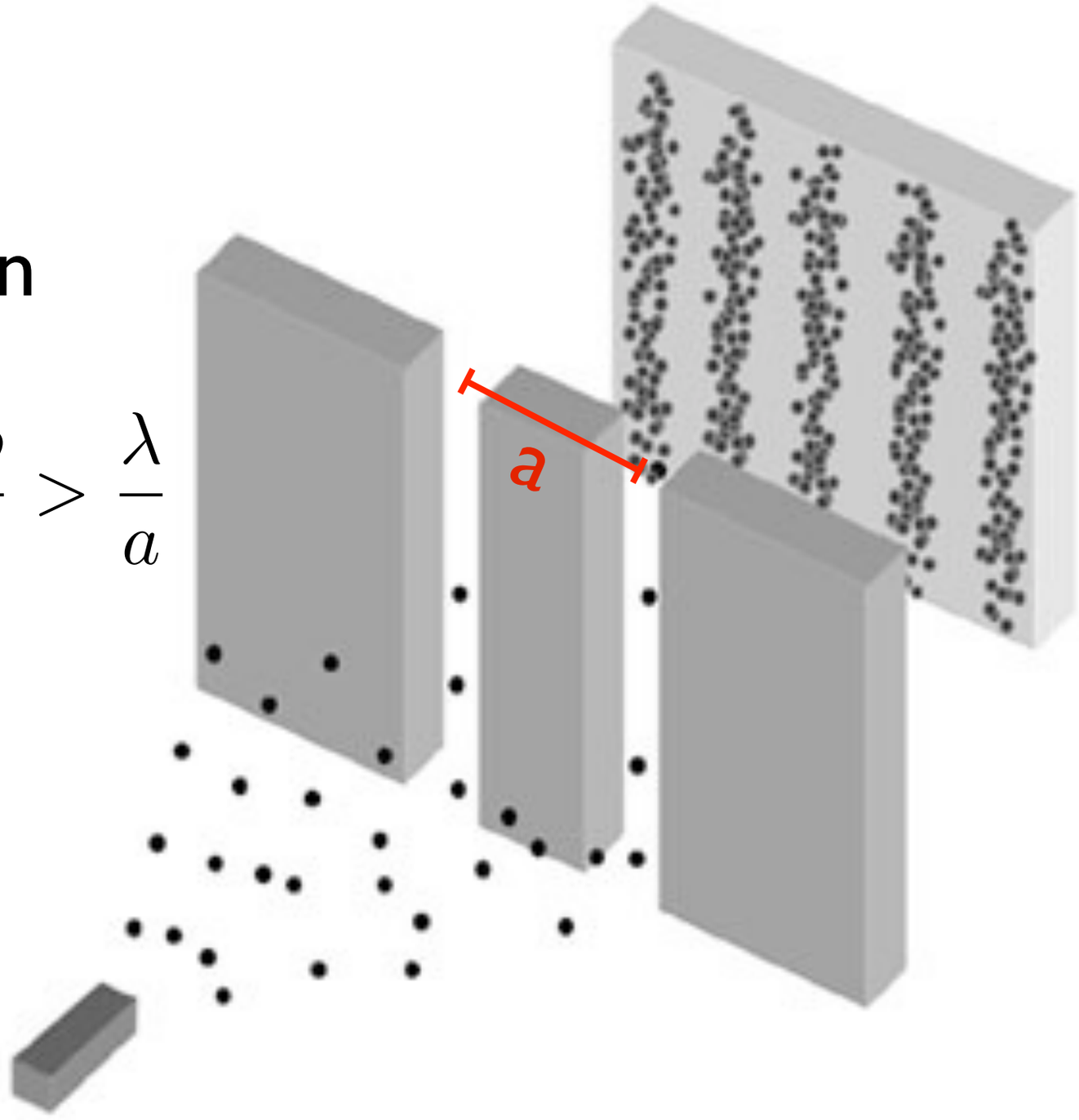

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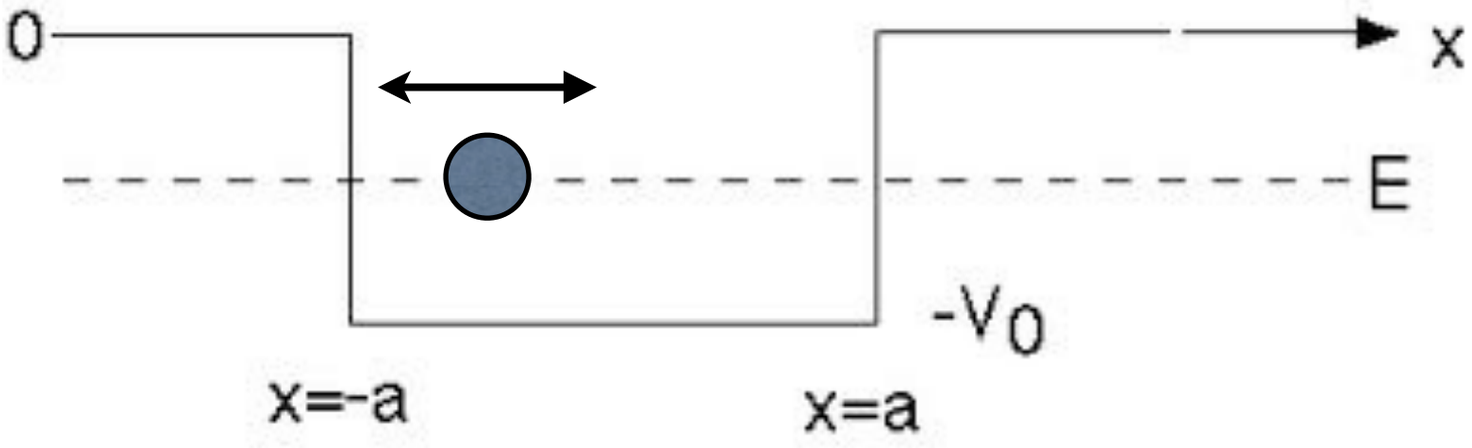
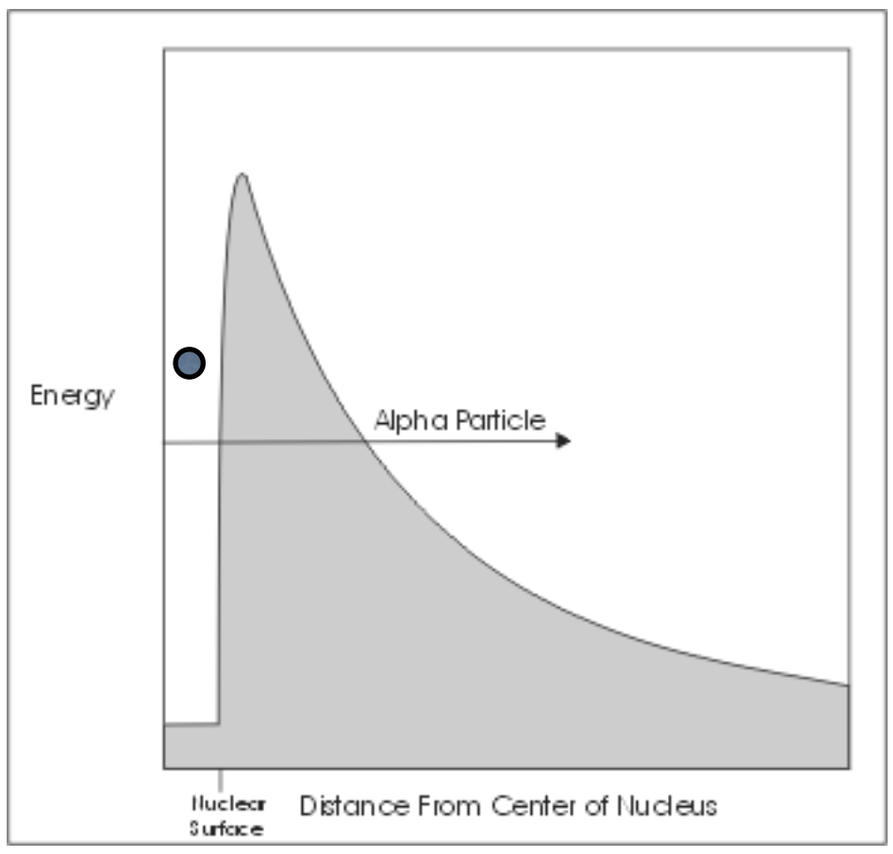
Quantum tunneling

In QM the classically impossible is now just very unlikely



Uncertainty principle means it can be outside well



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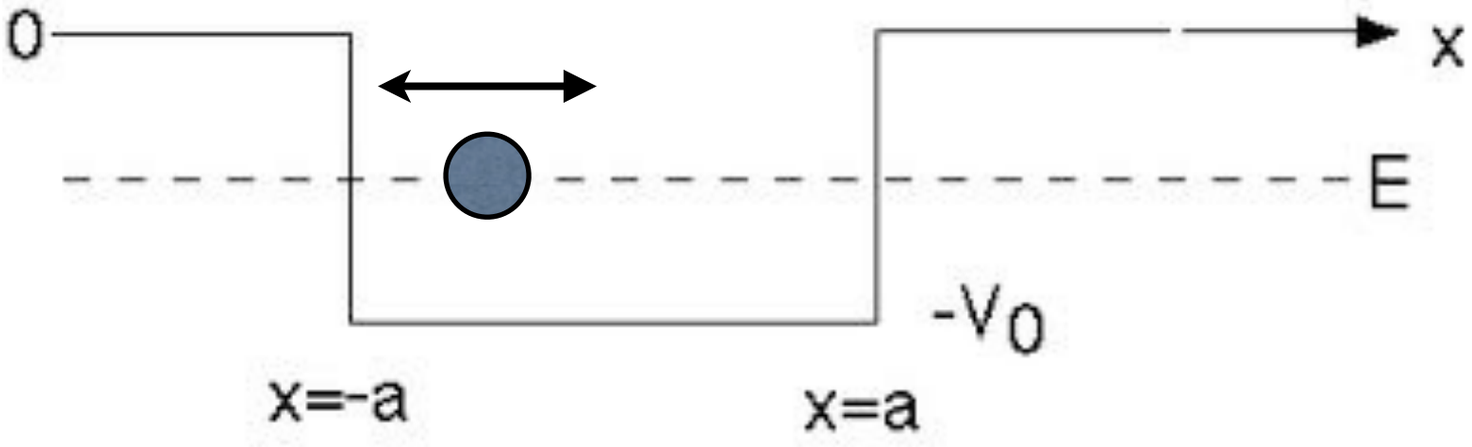

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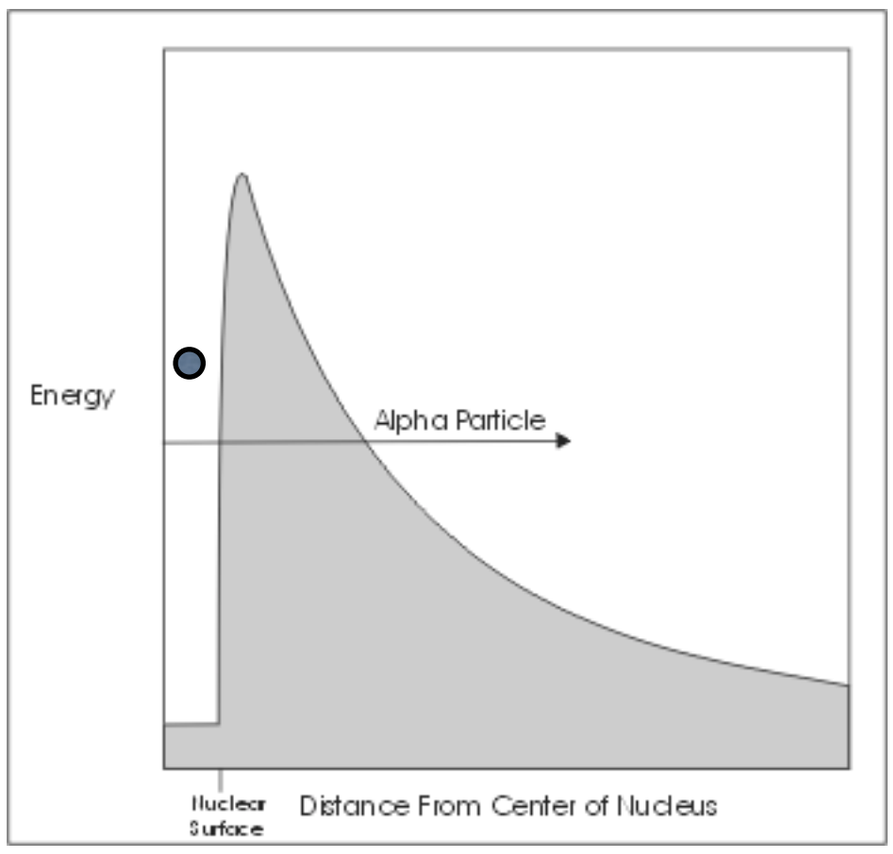

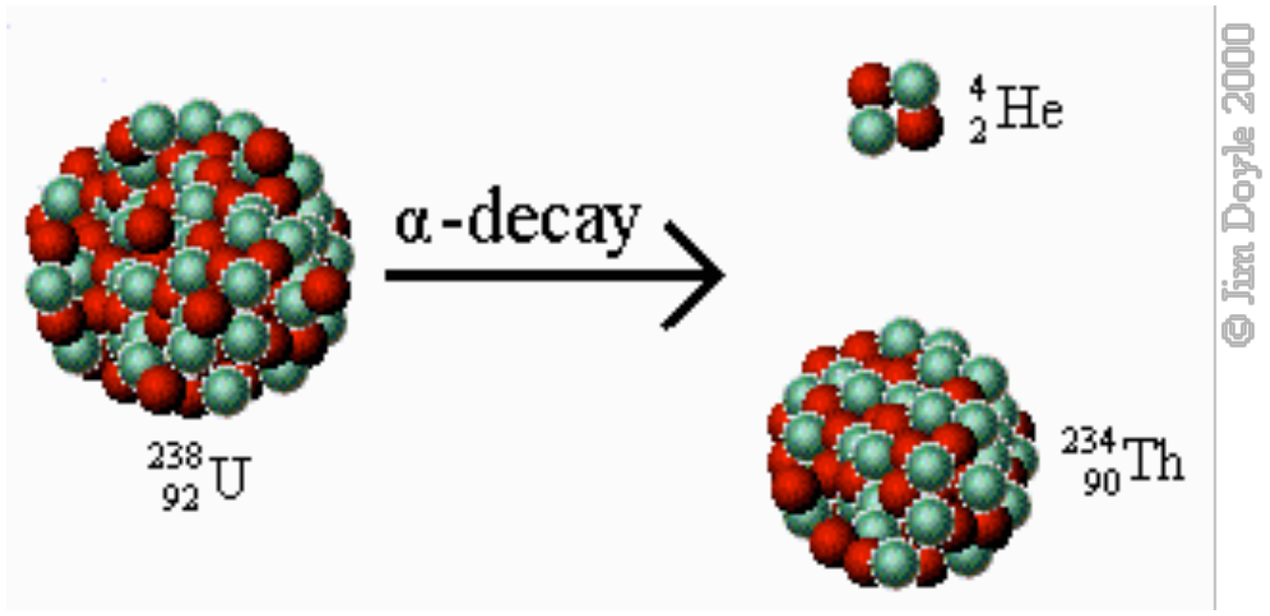

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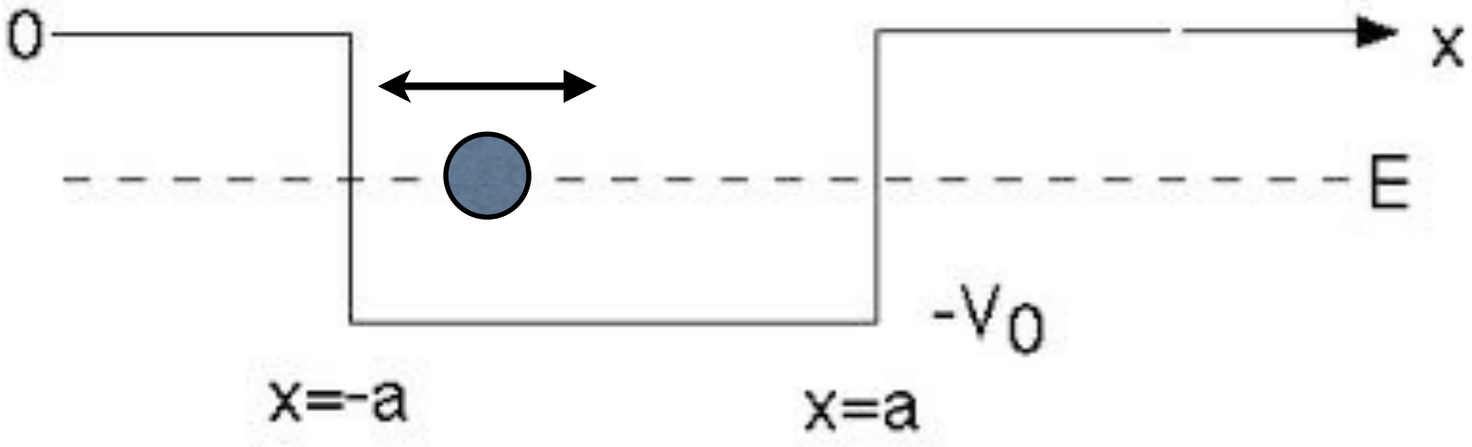

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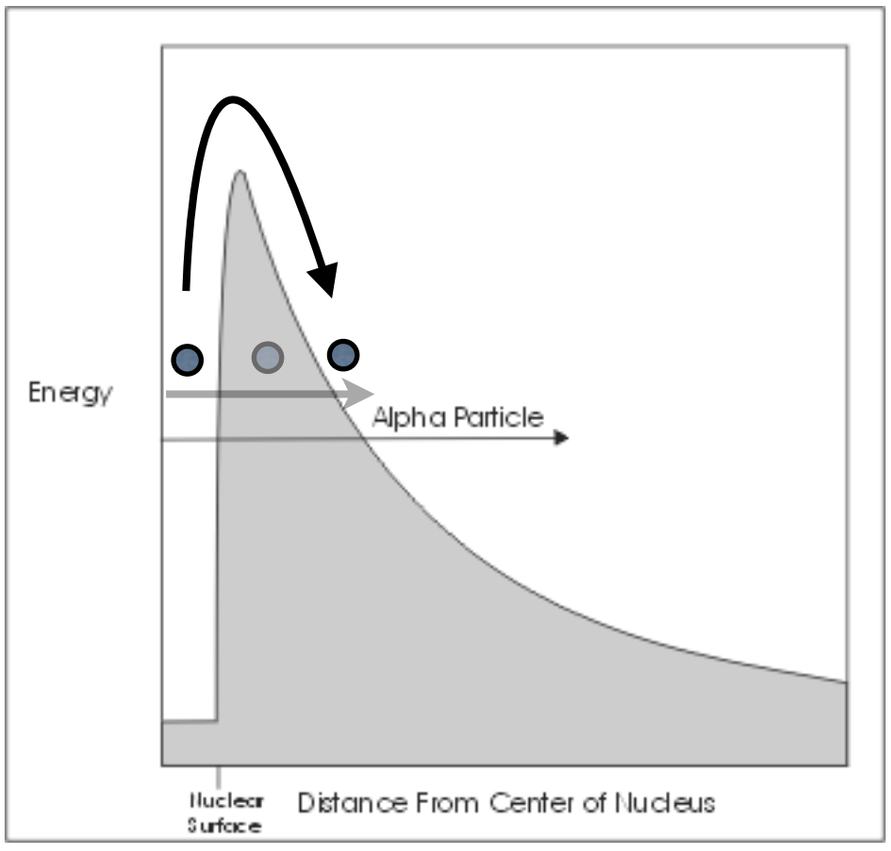
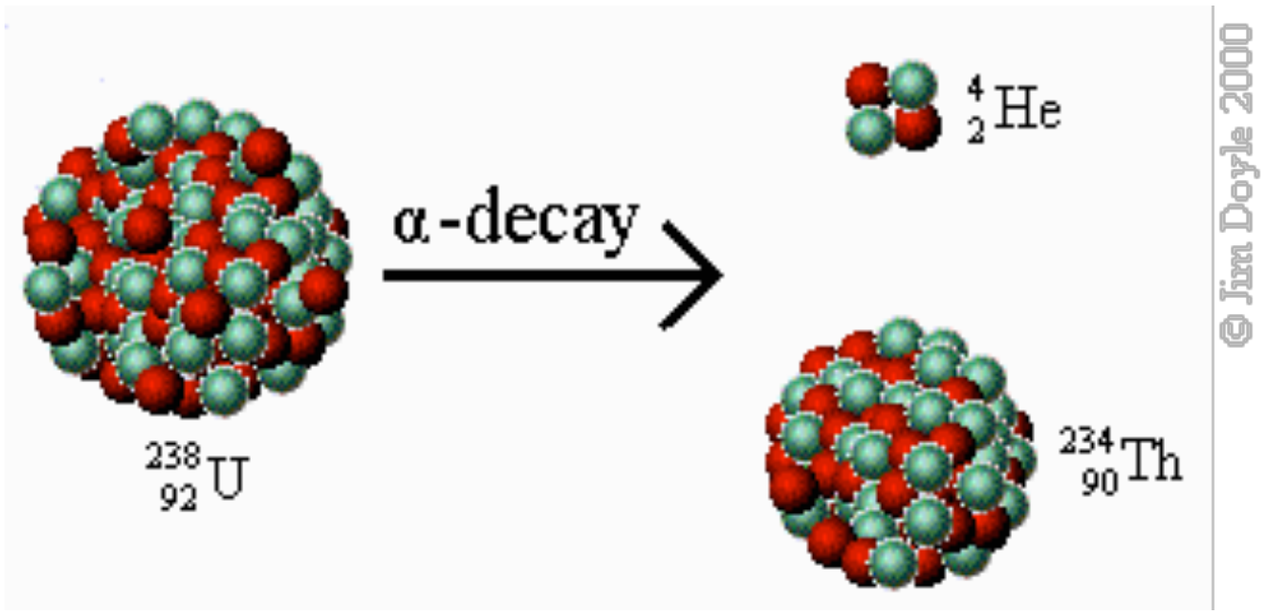




Superpositions

$$\Psi = \sum_i \psi_i$$

Wavefunction is a superposition of all possibilities

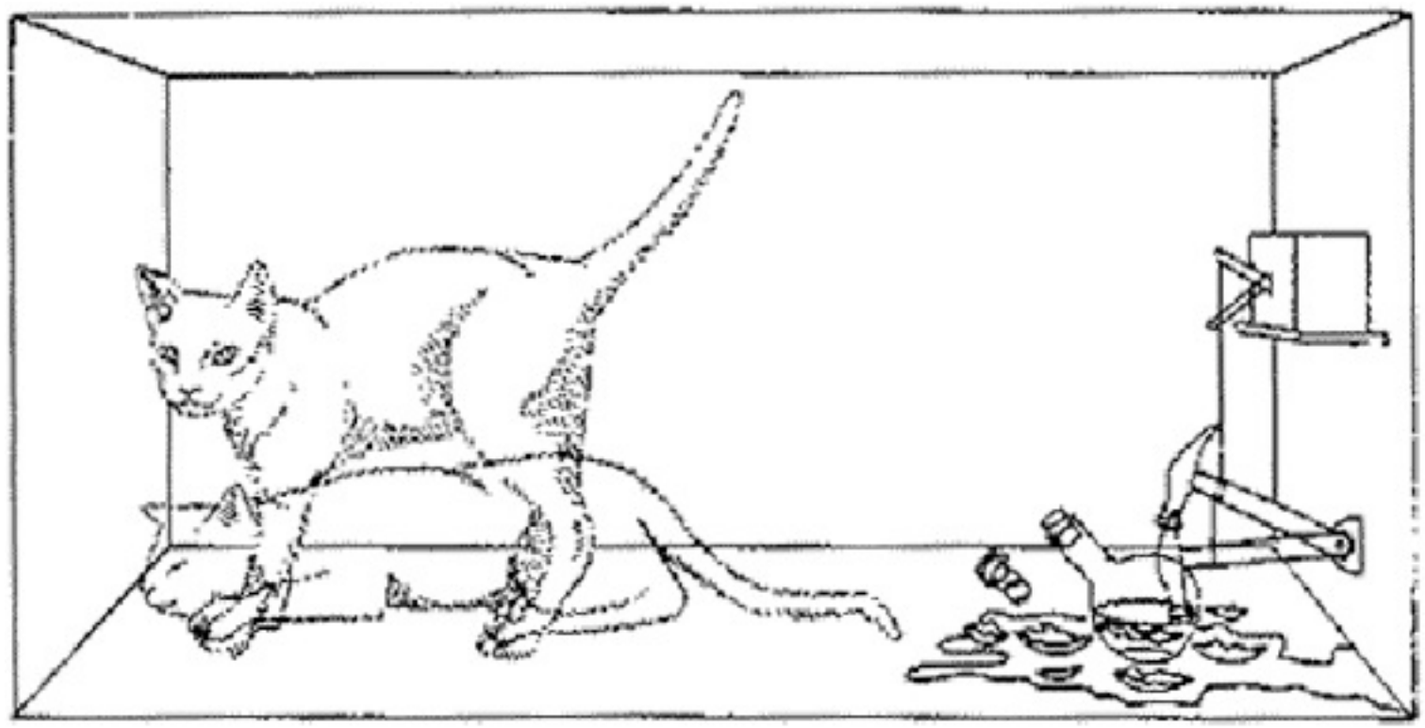
QM predicts outcomes if expt. is repeated many times

Measurement "collapses" the wavefunction

Schrödinger's cat

Copenhagen interpretation of QM

Cat is in a superposition of alive and dead until box is opened!



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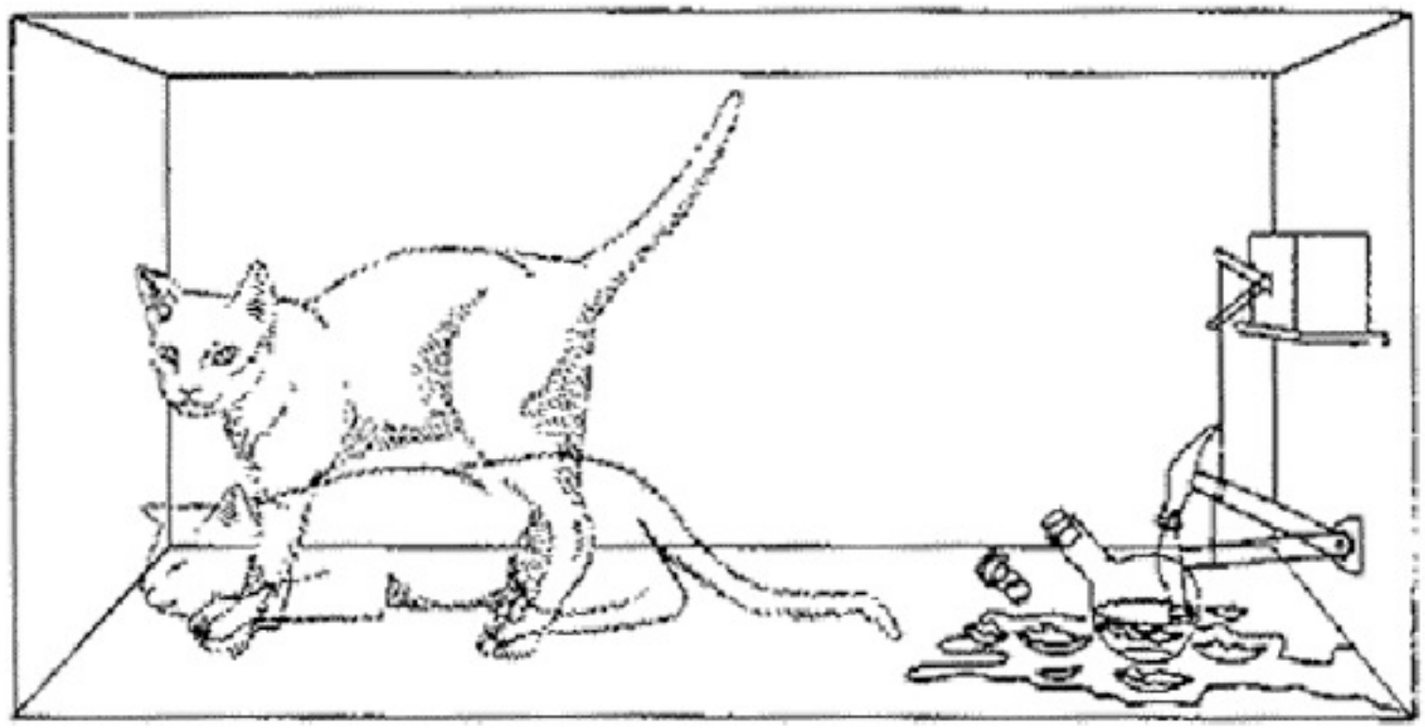


"Shut up and calculate"
--Richard Feynman

Applications of Quantum Mechanics

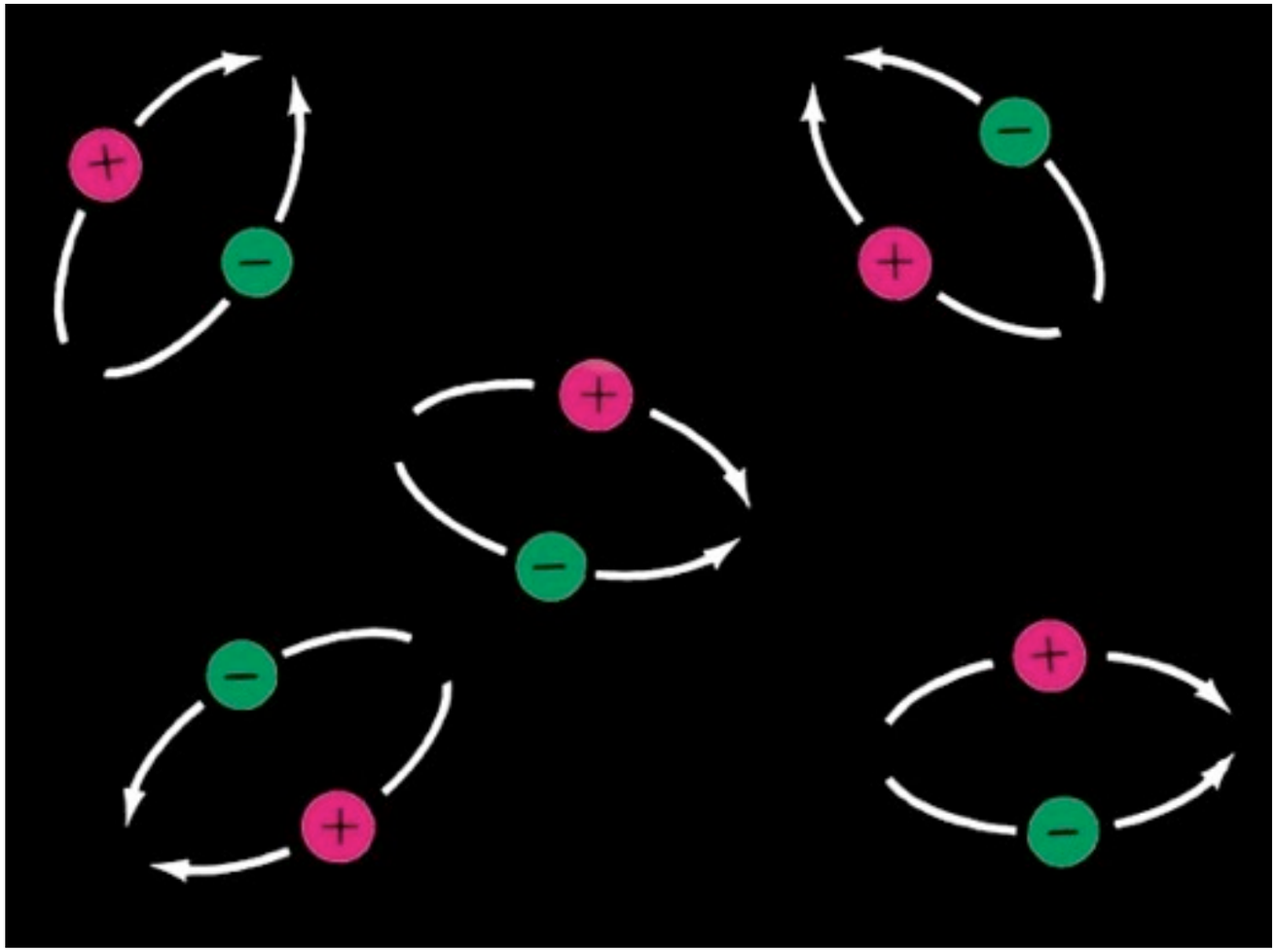
The birth of QM in the early 1900's lead to a profound change in our understanding of nature at short distances

Without this understanding we would not have transistors (and all semi-conductors), lasers, medical imaging technology, superconductors,....Fermilab!

Quantum Mechanics is not the final story.....

The loss of empty space

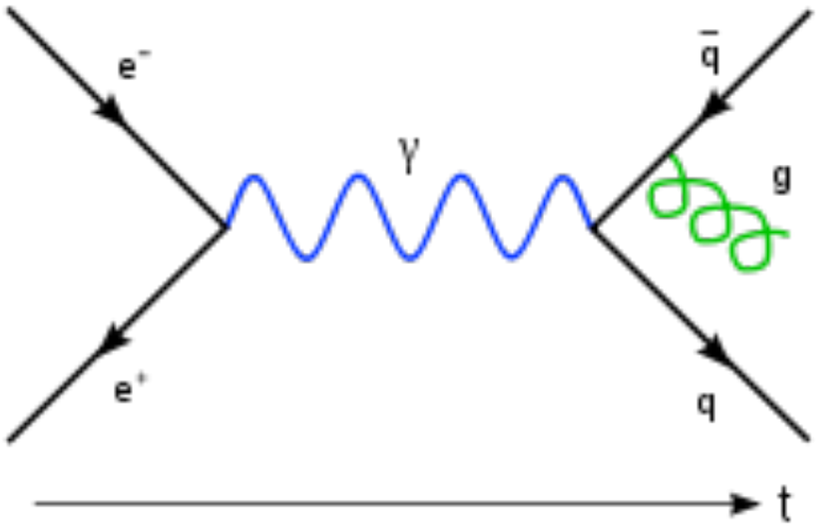
Paul A.M. Dirac



Dirac combined relativity and QM

Predicted antiparticles

Quantum Electrodynamics (QED)

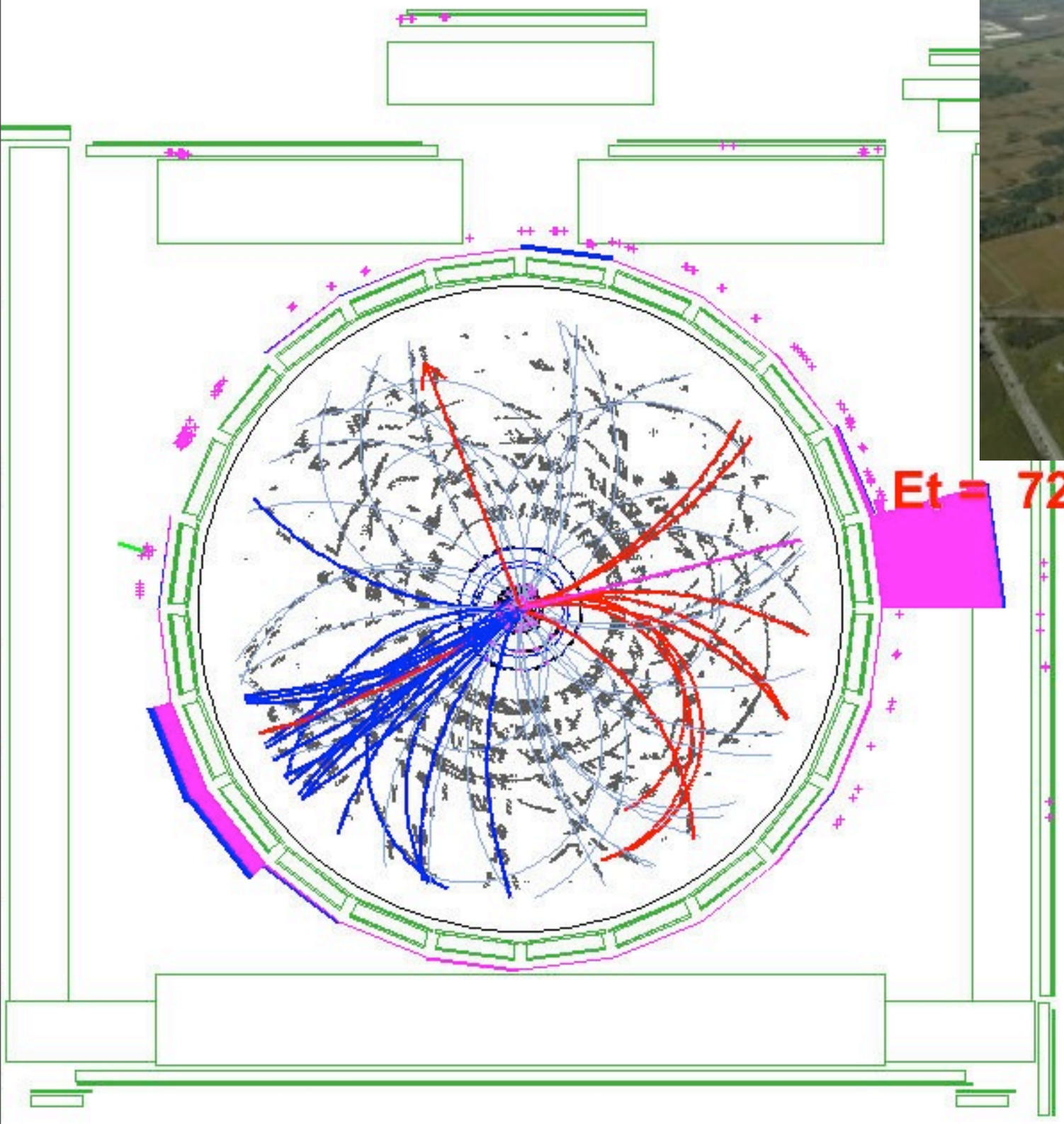


Feynman diagrams

Richard Feynman

Describes physics at short distances, and high energies

Like at Fermilab


Et = 72

Are there new laws governing nature at even shorter distances?