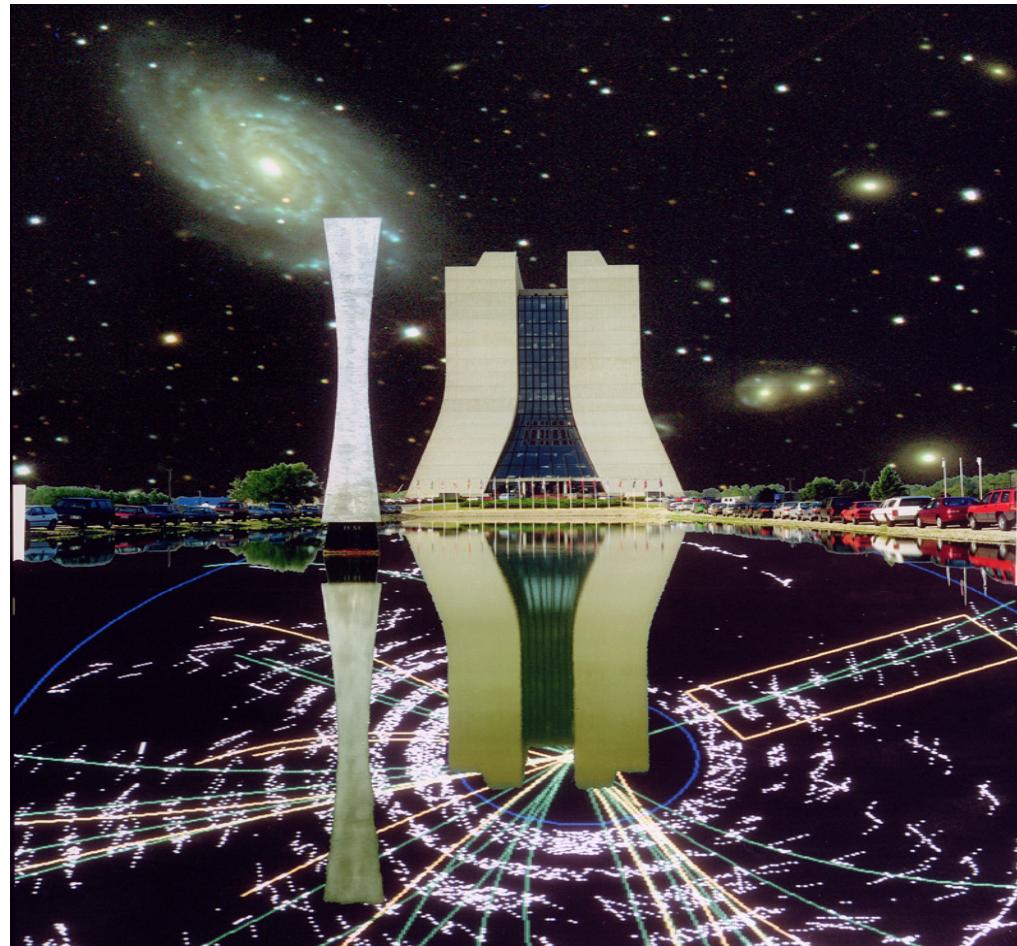


# Measuring the Mass of the W Boson

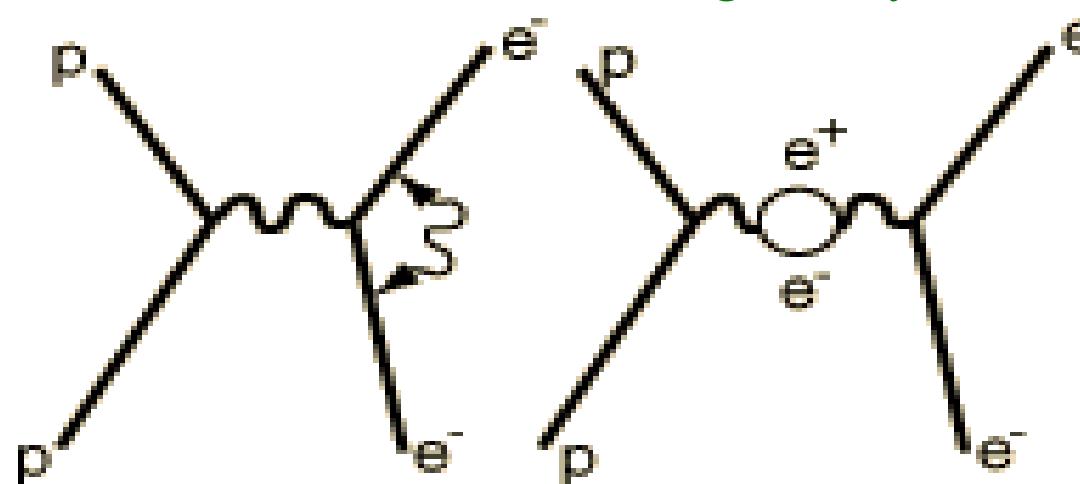
Ashutosh Kotwal  
Duke University



Hadron Collider Physics Summer School  
Fermilab  
August 16, 2012

# Detecting New Physics through Precision Measurements

- Willis Lamb (Nobel Prize 1955) measured the difference between energies of  $^2S_{\frac{1}{2}}$  and  $^2P_{\frac{1}{2}}$  states of hydrogen atom
  - 4 micro electron volts difference compared to few electron volts binding energy
  - States should be degenerate in energy according to tree-level calculation
- Harbinger of vacuum fluctuations to be calculated by Feynman diagrams containing quantum loops
  - Modern quantum field theory of electrodynamics followed ( Nobel Prize 1965 for Schwinger, Feynman, Tomonaga)



# Parameters of Electro-Weak Interactions

- Gauge symmetries related to the electromagnetic and weak forces in the standard model, extension of QED
  - U(1) gauge group with gauge coupling  $g$
  - SU(2) gauge group with gauge coupling  $g'$
- And gauge symmetry-breaking via vacuum expectation value of Higgs field  $v \neq 0$
- Another interesting phenomenon in nature: the U(1) generator and the neutral generator of SU(2) get mixed (linear combination) to yield the observed gauge bosons
  - Photon for electromagnetism
  - Z boson as one of the three gauge bosons of weak interaction
- Linear combination is given by Weinberg mixing angle  $\vartheta_W$

# Parameters of Electro-Weak Interactions

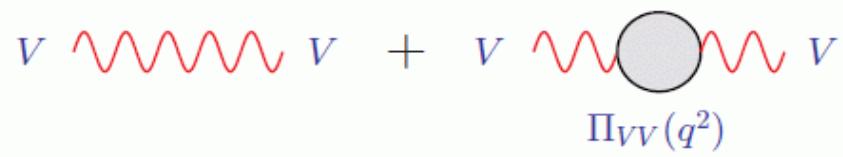
At **tree level**, all of the observables can be expressed in terms of *three* parameters of the SM Lagrangian:  $v$ ,  $g$ ,  $g'$  or, equivalently,  $v$ ,  $e$ ,  $s \equiv \sin \theta_W$  (also  $c \equiv \cos \theta_W$ )

$$\alpha = \frac{e^2}{4\pi}, \quad G_F = \frac{1}{2\sqrt{2}v^2}, \quad m_Z = \frac{ev}{\sqrt{2}sc}, \quad m_W = \frac{ev}{\sqrt{2}s}, \quad s_{\text{eff}}^2 = s^2,$$

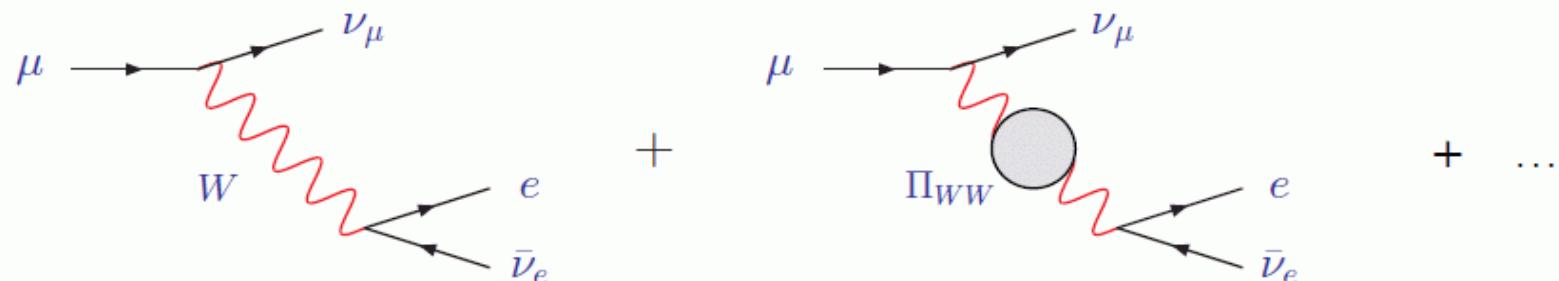
Radiative corrections to the relations between physical observables and Lagrangian params:

$$m_Z^2 = \frac{e^2 v^2}{2 s^2 c^2} + \Pi_{ZZ}(m_Z^2)$$

$$m_W^2 = \frac{e^2 v^2}{2 s^2} + \Pi_{WW}(m_W^2)$$

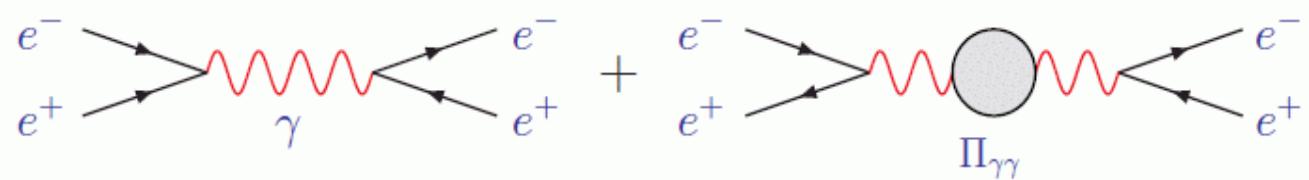


$$G_F = \frac{1}{2\sqrt{2}v^2} \left[ 1 - \frac{\Pi_{WW}(0)}{m_W^2} + \delta_{\text{VB}} \right]$$



# Radiative Corrections to Electromagnetic Coupling

$$\alpha = \frac{e^2}{4\pi} \left[ 1 + \lim_{q^2 \rightarrow 0} \frac{\Pi_{\gamma\gamma}(q^2)}{q^2} \right]$$



this one is tricky: the hadronic contribution to  $\Pi'_{\gamma\gamma}(0)$  cannot be computed perturbatively

We can however trade it for another experimental observable:  $R_{\text{had}}(q^2) = \frac{\sigma_{\text{had}}(q^2)}{\sigma_{\ell^+\ell^-}(q^2)}$

$$\alpha(m_Z) = \frac{e^2}{4\pi} \left[ 1 + \frac{\Pi_{\gamma\gamma}(m_Z)}{m_Z^2} \right] = \frac{\alpha}{1 - \Delta\alpha(m_Z)}$$

$$\Delta\alpha(m_Z) = \underbrace{\Delta\alpha_\ell(m_Z) + \Delta\alpha_{\text{top}}(m_Z)}_{\text{calculable}} + \Delta\alpha_{\text{had}}^{(5)}(m_Z)$$

$$\Delta\alpha_{\text{had}}^{(5)}(m_Z) = -\frac{m_Z^2}{3\pi} \int_{4m_\pi^2}^\infty \frac{R_{\text{had}}(q^2) dq^2}{q^2(q^2 - m_Z^2)} = 0.02758 \pm 0.00035$$

(This hadronic contribution is one of the biggest sources of uncertainty in EW studies)

# Radiative Corrections to W Boson Mass

All these corrections can be combined into relations among physical observables, e.g.:

$$m_W^2 = m_Z^2 \left[ \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{2\sqrt{2}\pi\alpha}{G_F m_Z^2}} (1 + \Delta r) \right]$$

$\Delta r$  can be parametrized in terms of two universal corrections and a remainder:

$$\Delta r = \Delta\alpha(m_Z) - \frac{c^2}{s^2} \Delta\rho + \Delta r_{\text{rem}}$$

The leading corrections depend quadratically on  $m_t$  but only logarithmically on  $m_H$ :

$$\Delta\rho = \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2} \approx \frac{3\alpha}{16\pi c^2} \left( \frac{m_t^2}{s^2 m_Z^2} + \log \frac{m_H^2}{m_W^2} + \dots \right)$$

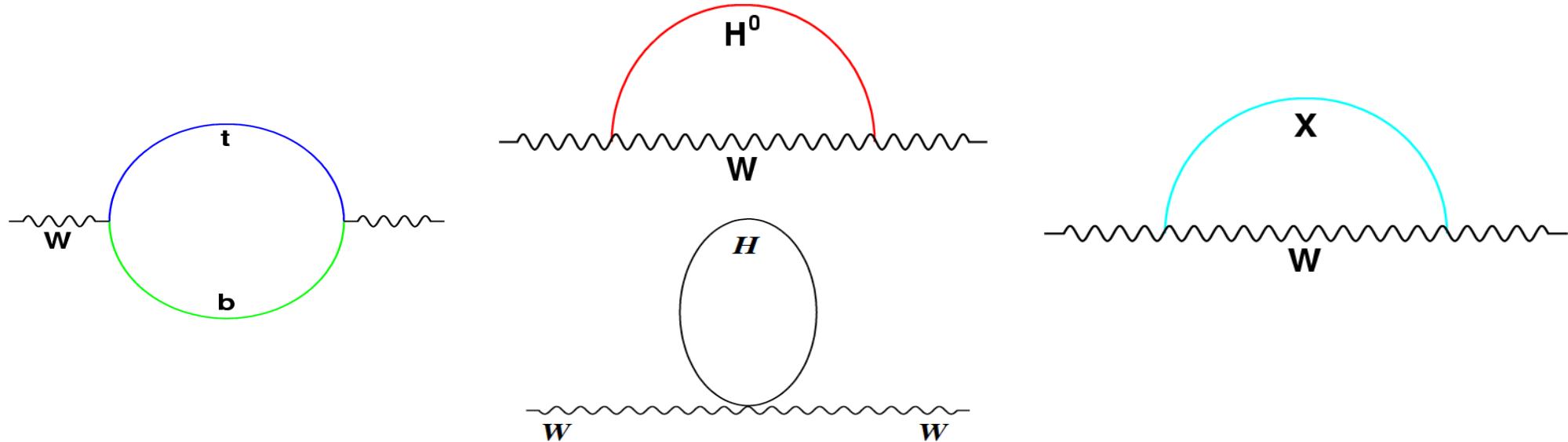
$$\frac{\delta m_W^2}{m_W^2} \approx \frac{c^2}{c^2 - s^2} \Delta\rho, \quad \delta \sin^2 \theta_{\text{eff}} \approx -\frac{c^2 s^2}{c^2 - s^2} \Delta\rho$$

# Motivation for Precision Measurements

- The electroweak gauge sector of the standard model is constrained by three precisely known parameters
  - $\alpha_{\text{EM}}(M_Z) = 1 / 127.918(18)$
  - $G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$
  - $M_Z = 91.1876(21) \text{ GeV}$
- At tree-level, these parameters are related to other electroweak observables, *e.g.*  $M_W$ 
  - $M_W^2 = \pi \alpha_{\text{EM}} / \sqrt{2} G_F \sin^2 \vartheta_W$ 
    - Where  $\vartheta_W$  is the Weinberg mixing angle, defined by  
$$\cos \vartheta_W = M_W/M_Z$$

# Motivation for Precision Measurements

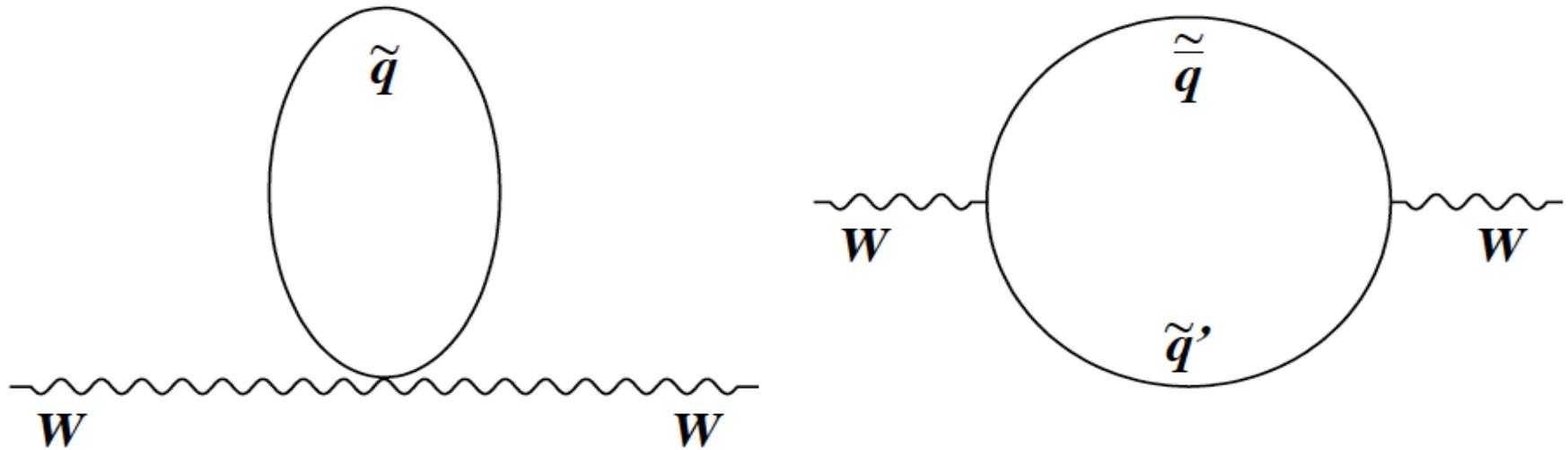
- Radiative corrections due to heavy quark and Higgs loops and exotica



Motivate the introduction of the  $\rho$  parameter:  $M_W^2 = \rho [M_W(\text{tree})]^2$  with the predictions  $\Delta\rho = (\rho - 1) \sim M_{\text{top}}^2$  and  $\Delta\rho \sim \ln M_H$

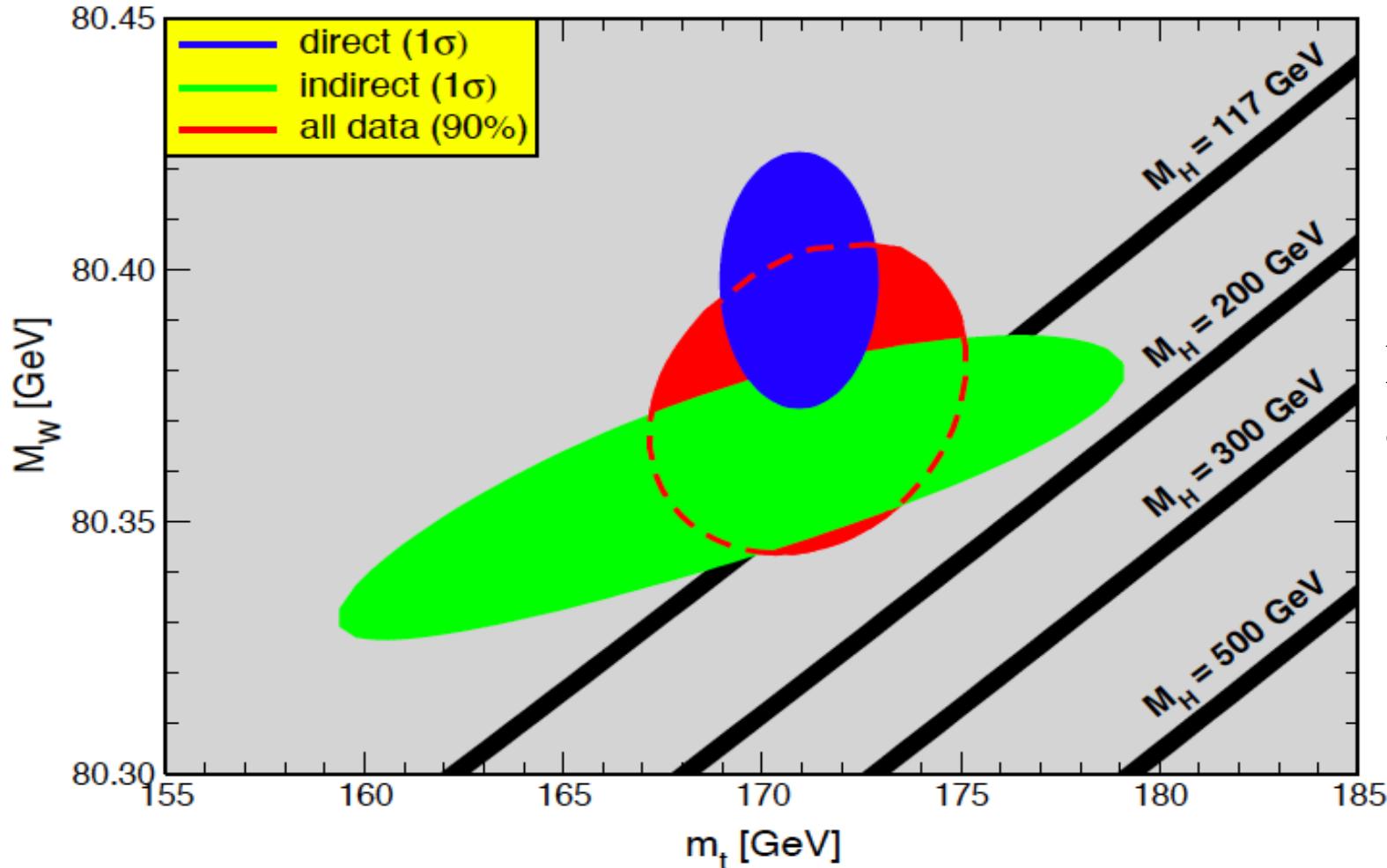
- In conjunction with  $M_{\text{top}}$ , the W boson mass constrains the mass of the Higgs boson, and possibly new particles beyond the standard model

# Contributions from Supersymmetric Particles



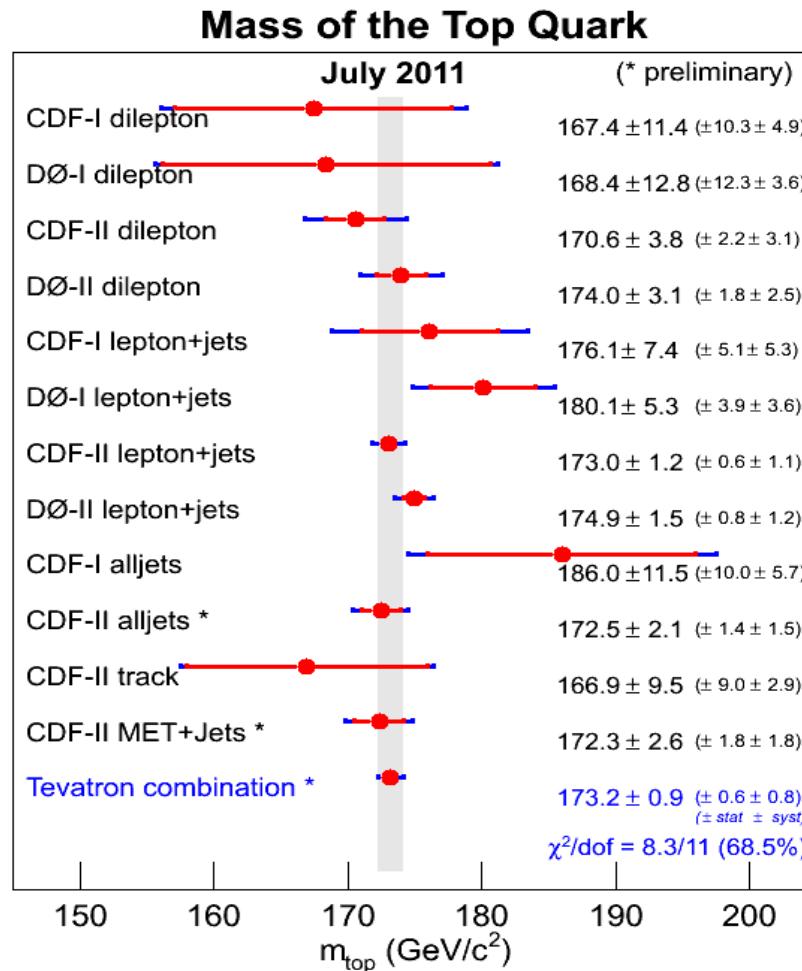
- Radiative correction depends on mass splitting ( $\Delta m^2$ ) between squarks in SU(2) doublet
- After folding in limits on SUSY particles from direct searches, SUSY loops can contribute  $\sim 100$  MeV to  $M_W$

# Uncertainty from $\alpha_{\text{EM}}(M_Z)$



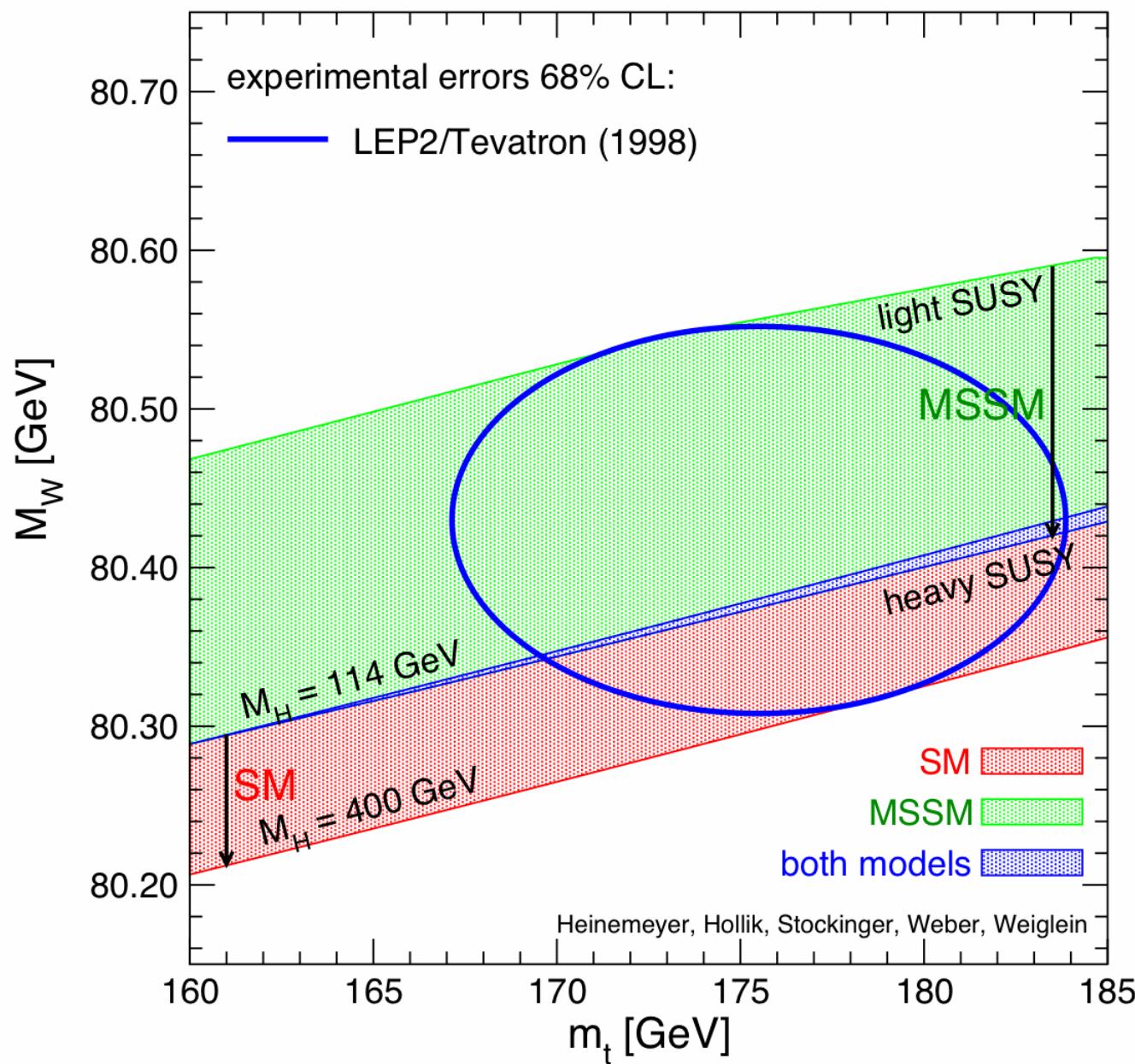
- $\delta\alpha_{\text{EM}}$  dominated by uncertainty from non-perturbative contributions: hadronic loops in photon propagator at low  $Q^2$
- equivalent  $\delta M_W \approx 4 \text{ MeV}$  for the same Higgs mass constraint
  - Was equivalent  $\delta M_W \approx 15 \text{ MeV}$  a decade ago !

# Progress on $M_{\text{top}}$ at the Tevatron

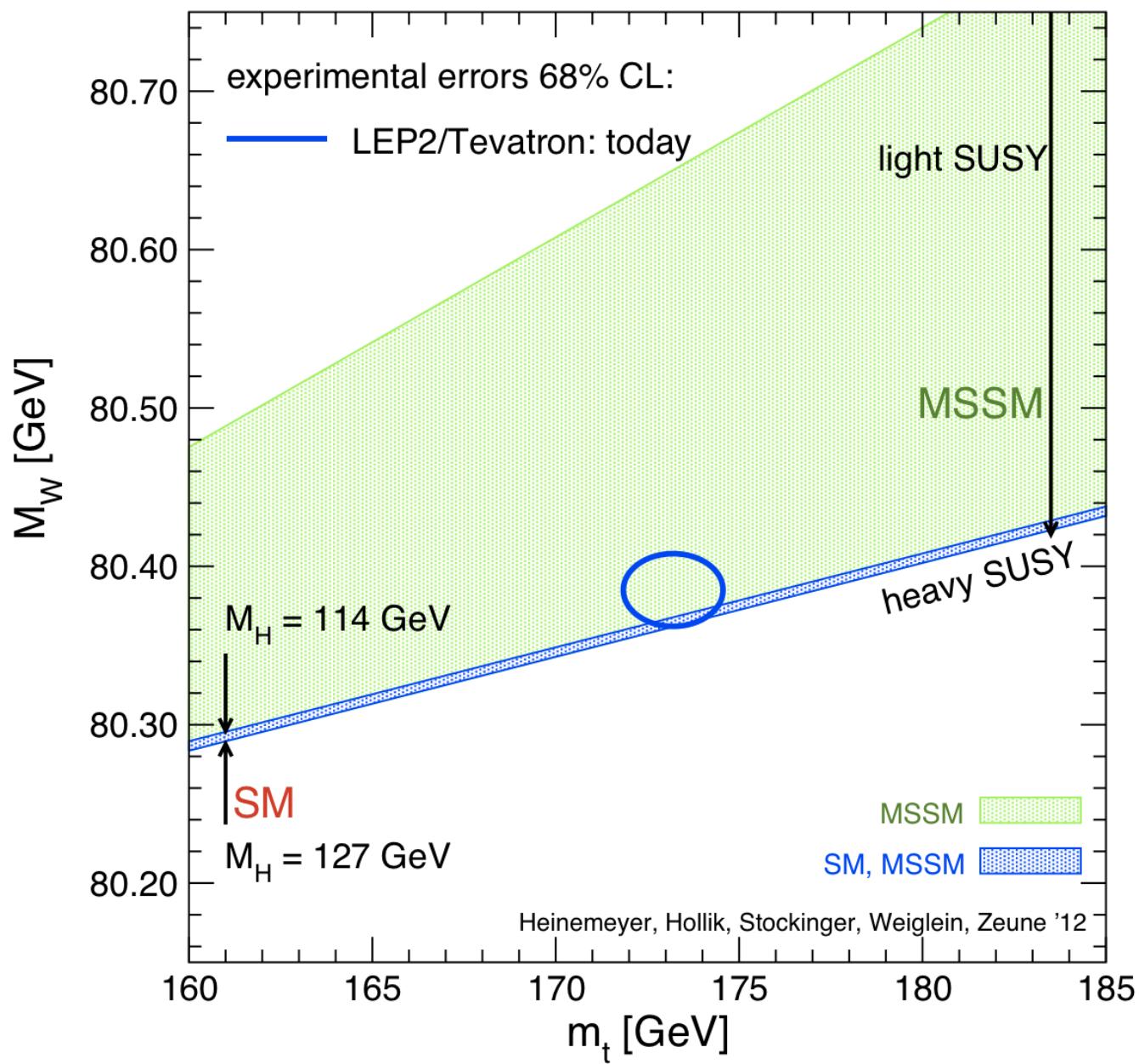


- From the Tevatron,  $\Delta M_{\text{top}} = 0.9 \text{ GeV} \Rightarrow \Delta M_H / M_H = 8\%$
- equivalent  $\Delta M_W = 6 \text{ MeV}$  for the same Higgs mass constraint
- Current world average  $\Delta M_W = 15 \text{ MeV}$ 
  - progress on  $\Delta M_W$  has the biggest impact on Higgs constraint

# 1998 Status of $M_W$ vs $m_t$

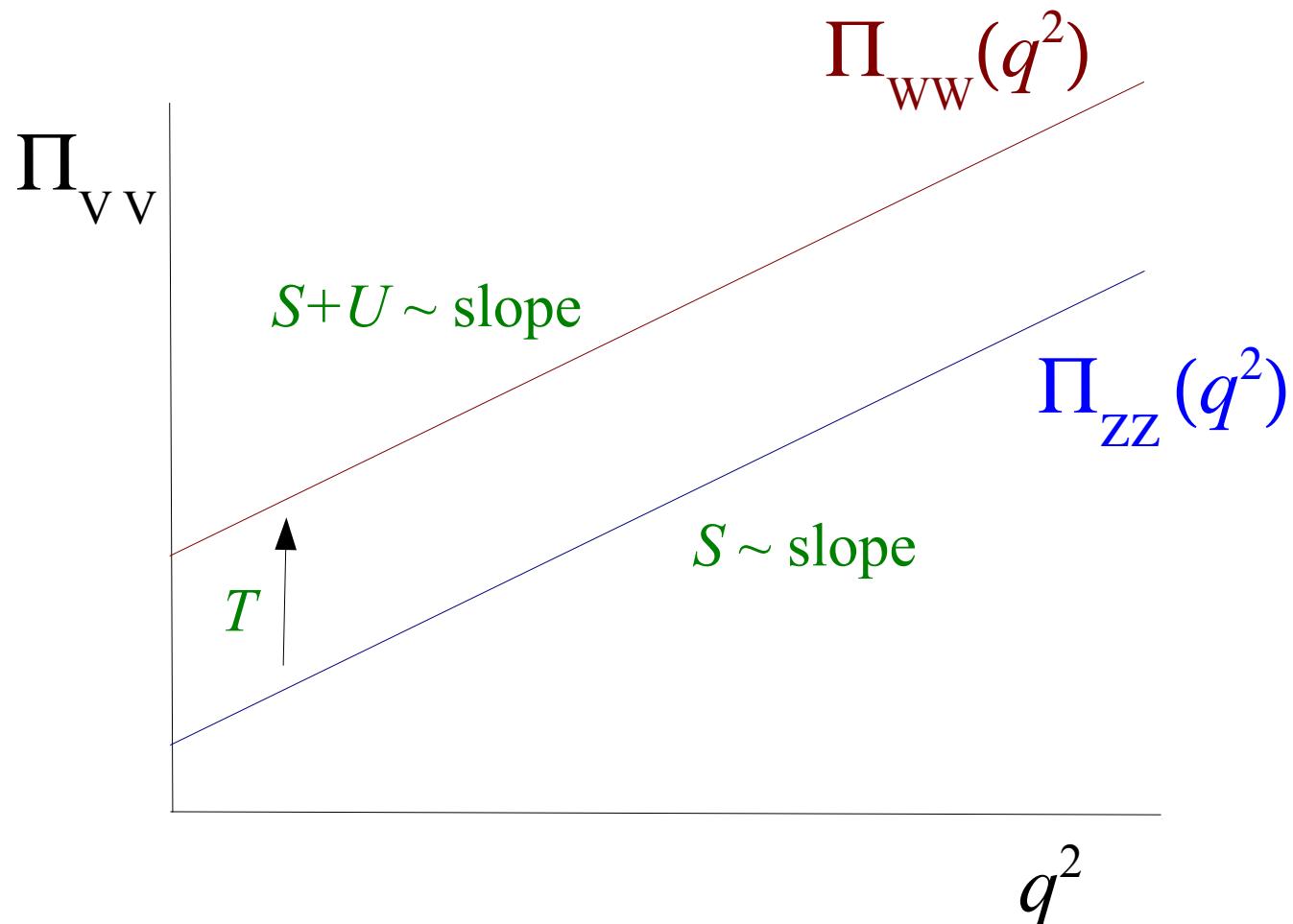


# 2012 Status of $M_W$ vs $m_t$



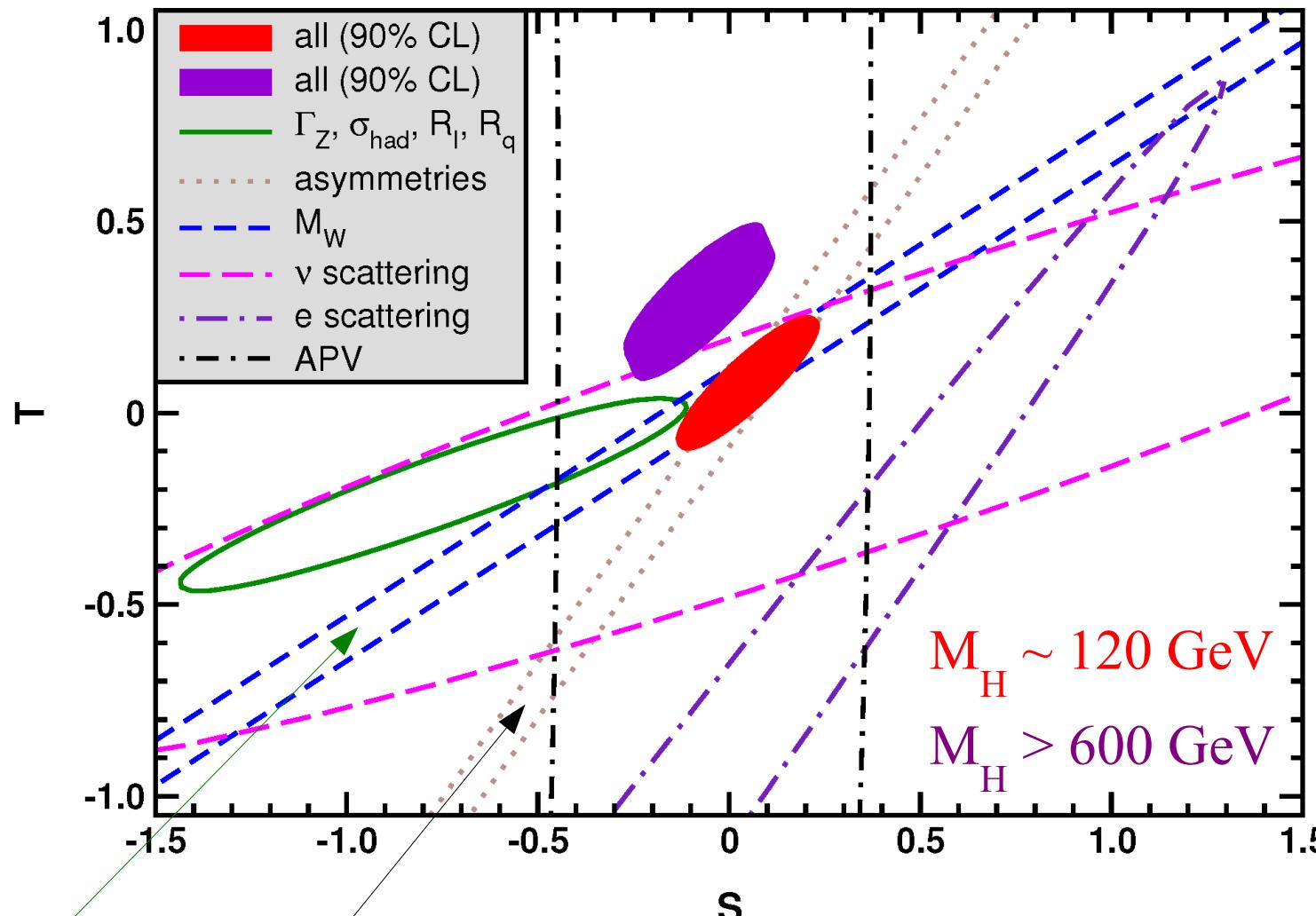
# Motivation

- Generic parameterization of new physics contributing to W and Z boson self-energies through radiative corrections in propagators
  - $S, T, U$  parameters (Peskin & Takeuchi, Marciano & Rosner, Kennedy & Langacker, Kennedy & Lynn)



# Motivation

- Generic parameterization of new physics contributing to W and Z boson self-energies:  $S$ ,  $T$ ,  $U$  parameters (Peskin & Takeuchi)



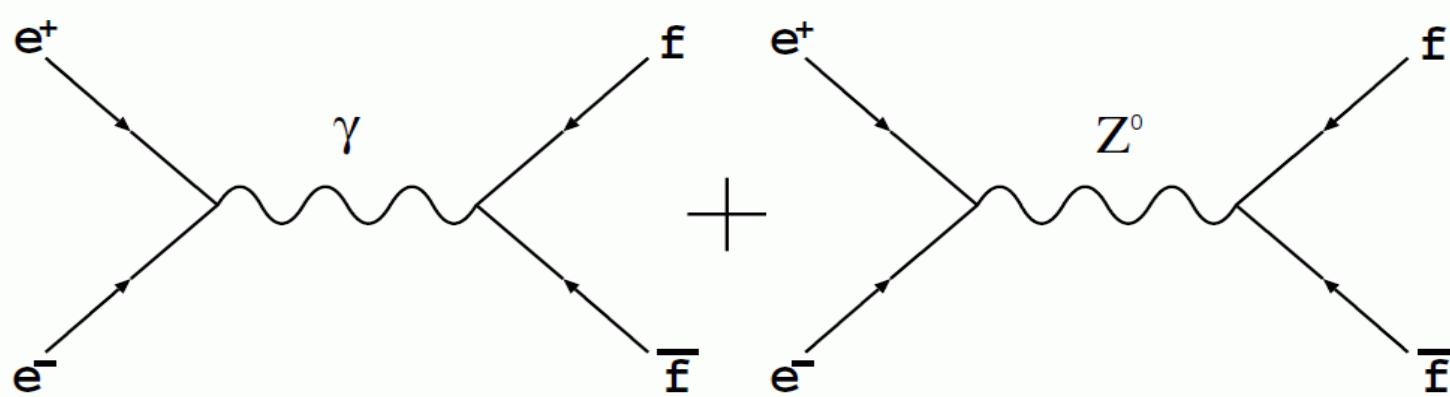
Additionally,  $M_W$  is the only measurement which constrains  $U$

(from P. Langacker, 2012)

$M_W$  and Asymmetries are the most powerful observables in this parameterization

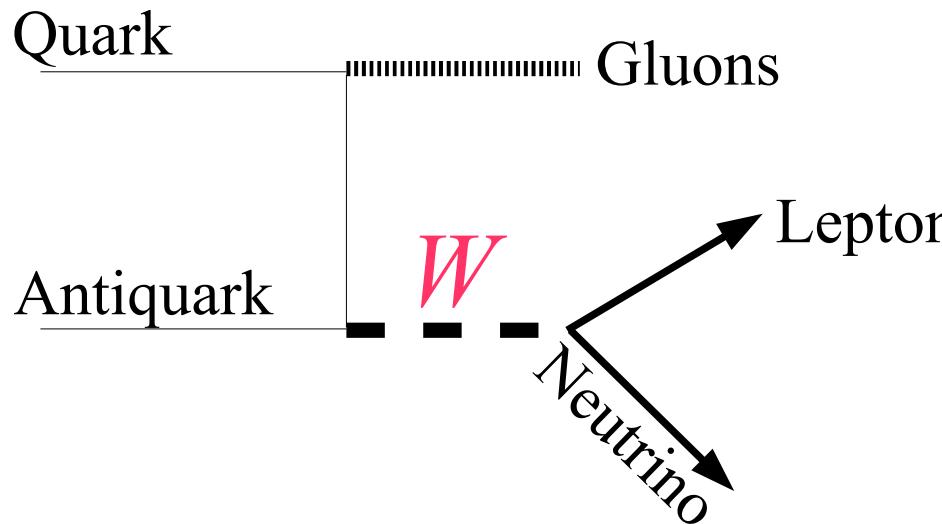
## $A_{FB}$ and $A_{LR}$ Observables

- Asymmetries definable in electron-positron scattering sensitive to Weinberg mixing angle  $\vartheta_W$



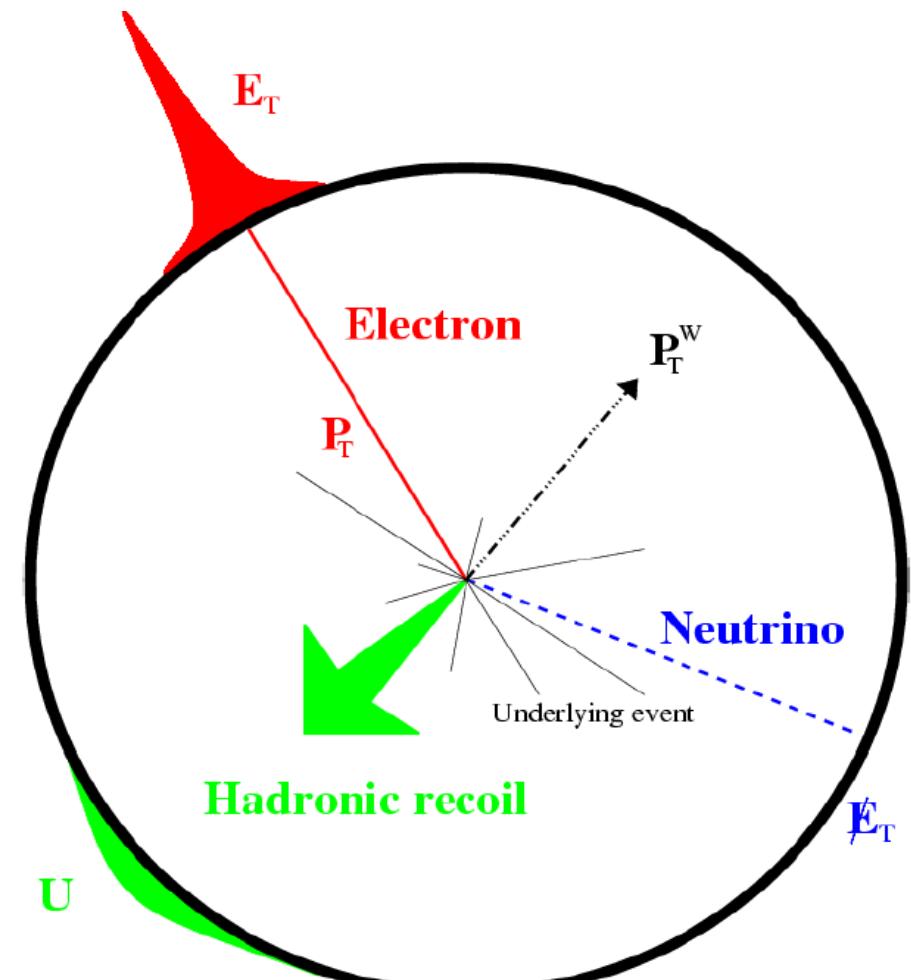
- Higgs and Supersymmetry also contribute radiative corrections to  $\vartheta_W$  via quantum loops
- $A_{FB}$  is the angular (forward – backward) asymmetry of the final state
- $A_{LR}$  is the asymmetry in the total scattering probability for different polarizations of the initial state

# W Boson Production at the Tevatron



Quark-antiquark annihilation  
dominates (80%)

Lepton  $p_T$  carries most of  $W$  mass  
information, can be measured precisely (achieved 0.01%)



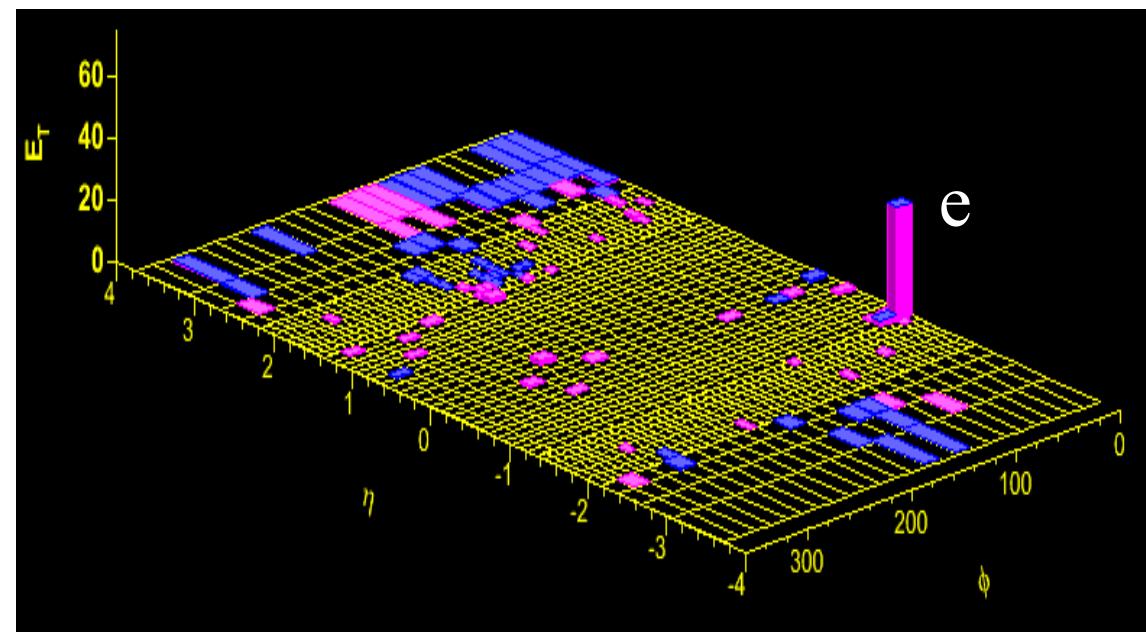
Initial state QCD radiation is  $O(10 \text{ GeV})$ , measure as soft 'hadronic recoil' in calorimeter (calibrated to  $\sim 0.5\%$ )

Pollutes  $W$  mass information, fortunately  $p_T(W) \ll M_W$

# W Boson Production at the Tevatron



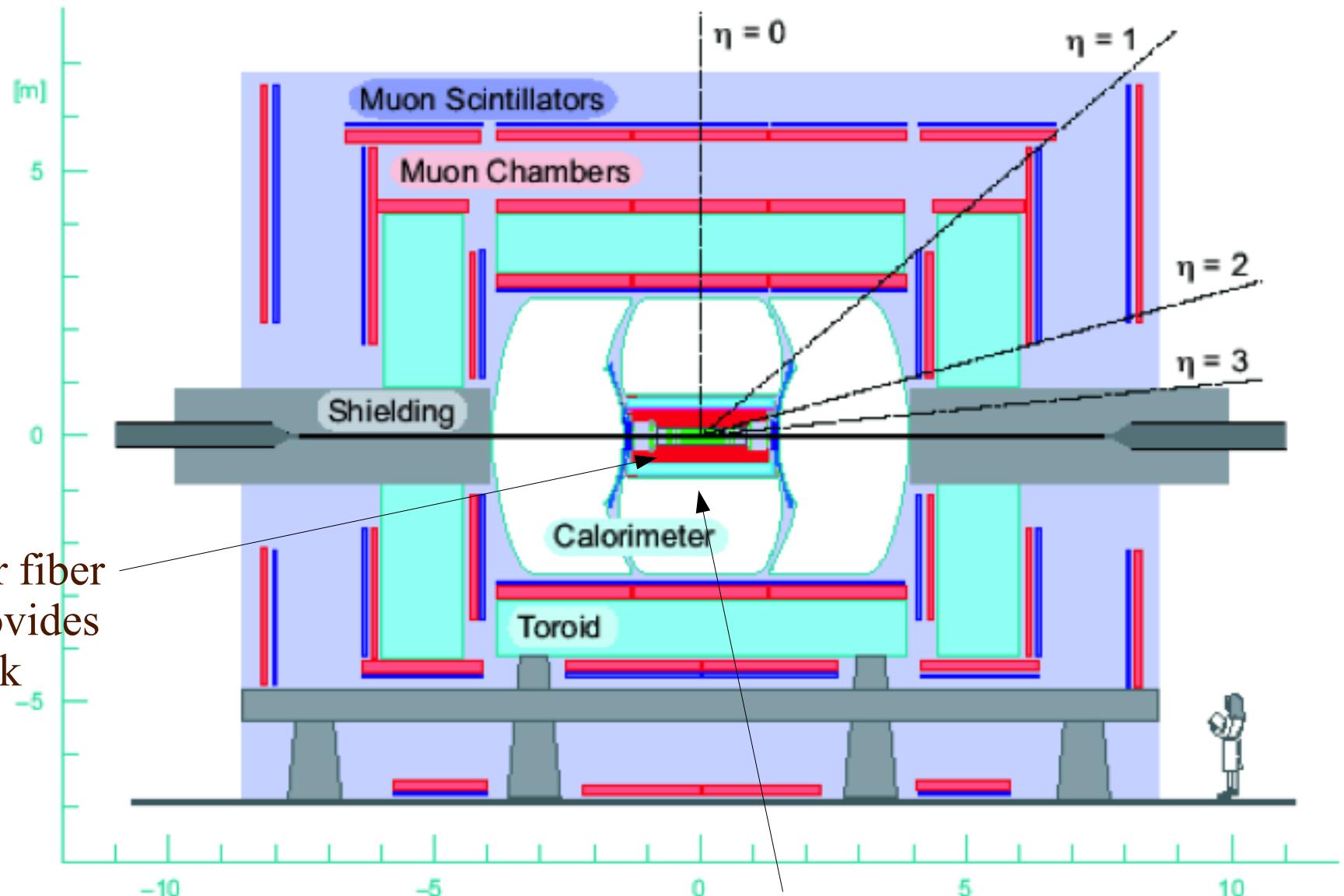
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# D0 Detector at Fermilab



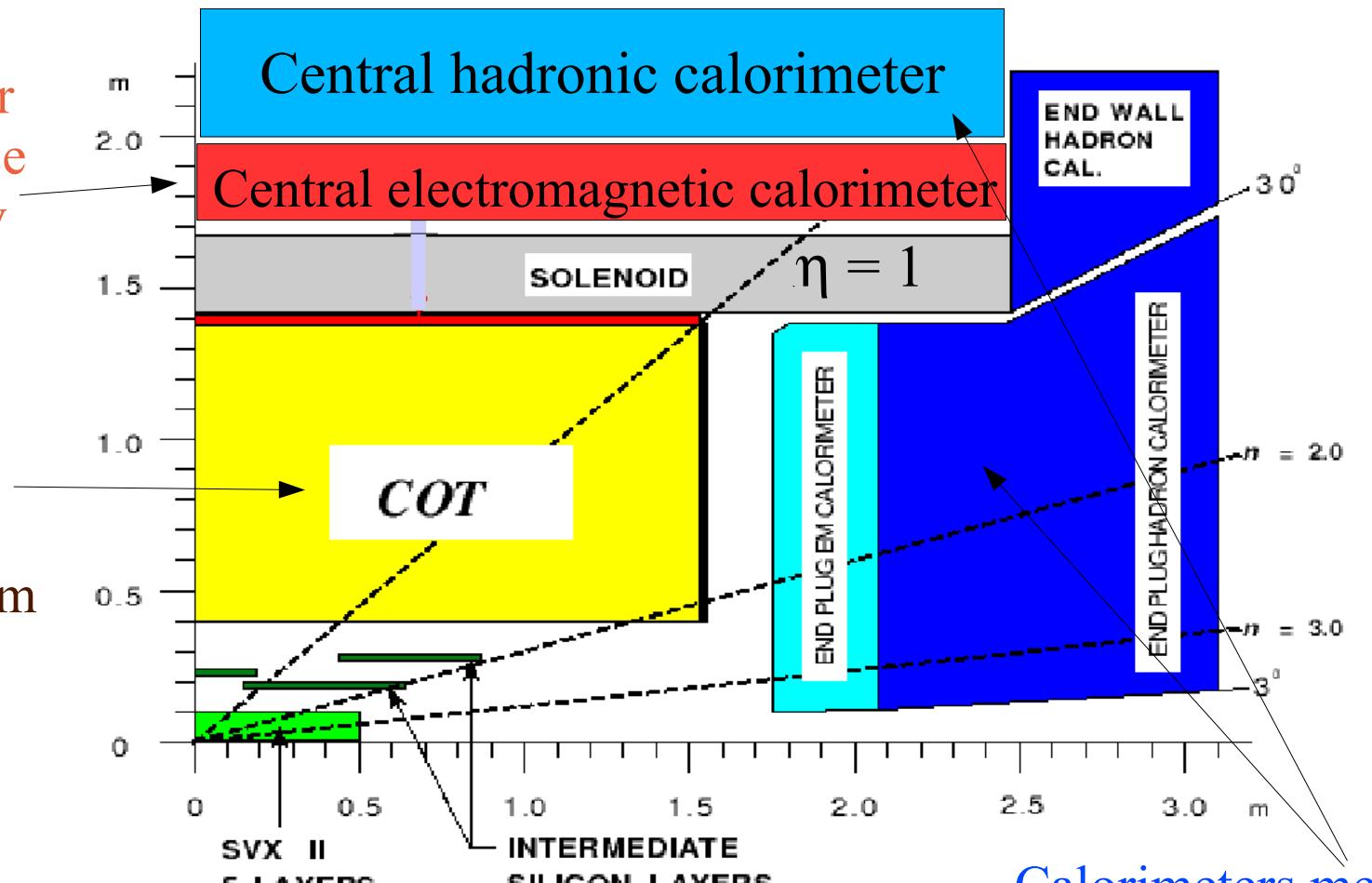
Scintillator fiber  
tracker provides  
lepton track  
direction

Electromagnetic Calorimeter measures electron energy  
Hadronic calorimeters measure recoil particles

# Quadrant of Collider Detector at Fermilab (CDF)

EM calorimeter provides precise electron energy measurement

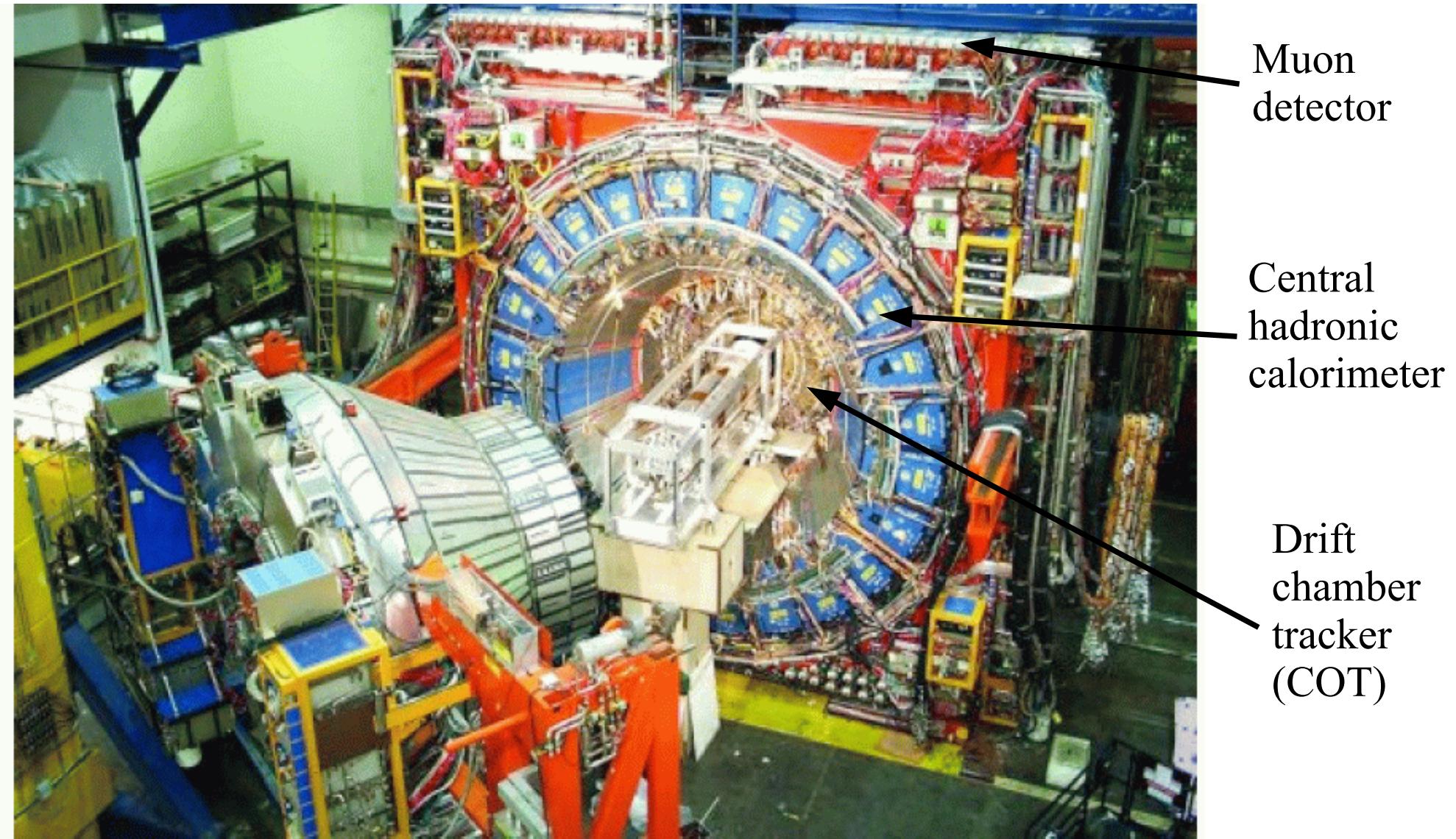
Drift chamber provides precise lepton track momentum measurement



Calorimeters measure hadronic recoil particles

Select W and Z bosons with central ( $| \eta | < 1$ ) leptons

# Collider Detector at Fermilab (CDF)

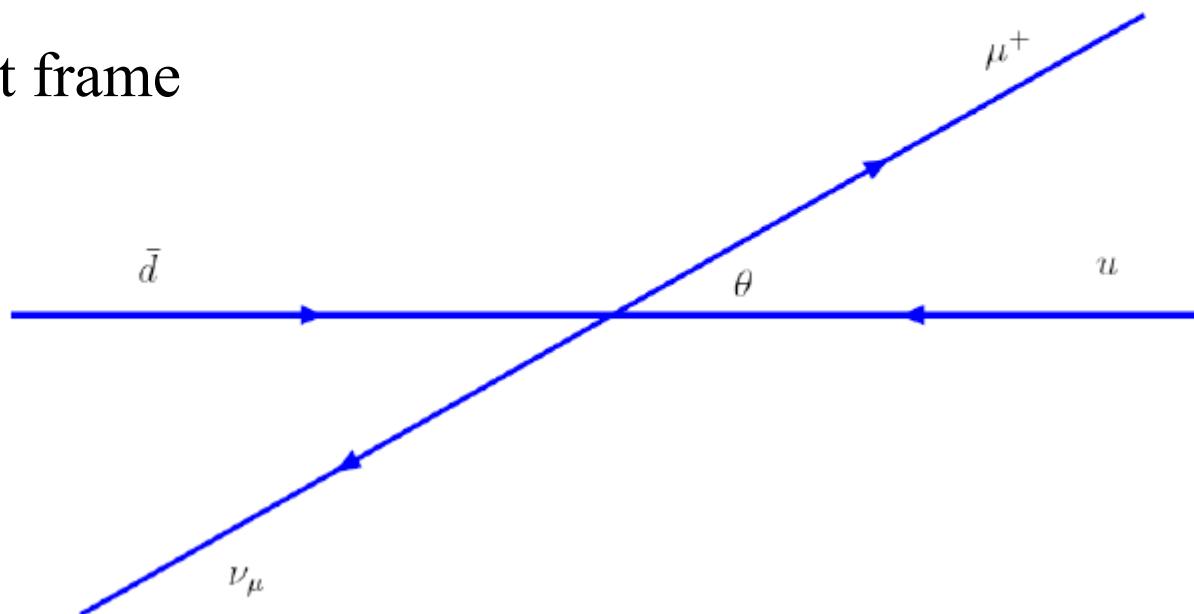


# W mass measurement – decay kinematics

- Main complication: invariant mass cannot be reconstructed from 2-body leptonic decay mode
  - Because neutrino is not detectable directly
- Exploit the “Jacobian edge” in lepton transverse momentum spectrum

$$\begin{aligned}\frac{d\sigma}{d \cos \hat{\theta}} &= \sigma_0(\hat{s}) \left[ \frac{1}{2}(1 + \cos \hat{\theta})^2 + \frac{1}{2}(1 - \cos \hat{\theta})^2 \right] \\ &= \sigma_0(\hat{s})(1 + \cos^2 \hat{\theta})\end{aligned}$$

W boson rest frame



# W mass measurement – decay kinematics

- Main complication: invariant mass cannot be reconstructed from 2-body leptonic decay mode
  - Because neutrino is not detectable directly
- Exploit the “Jacobian edge” in lepton transverse momentum spectrum

$$\begin{aligned}\frac{d\sigma}{dp_T} &= \frac{d\sigma}{d((m_W/2) \sin \hat{\theta})} \\ &= \frac{2}{m_W} \frac{d\sigma}{d \sin \hat{\theta}} \\ &= \frac{2}{m_W} \frac{d\sigma}{d \cos \hat{\theta}} \left| \frac{d \cos \hat{\theta}}{d \sin \hat{\theta}} \right| \\ &= \frac{2}{m_W} \sigma_0(\hat{s})(1 + \cos^2 \theta) |\tan \hat{\theta}| \\ &= \sigma_0(\hat{s}) \frac{4p_T}{m_W^2} (2 - 4p_T^2/m_W^2) \left( \frac{1}{\sqrt{1 - 4p_T^2/m_W^2}} \right)\end{aligned}$$

Invariant under  
longitudinal boost



## W mass measurement – decay kinematics

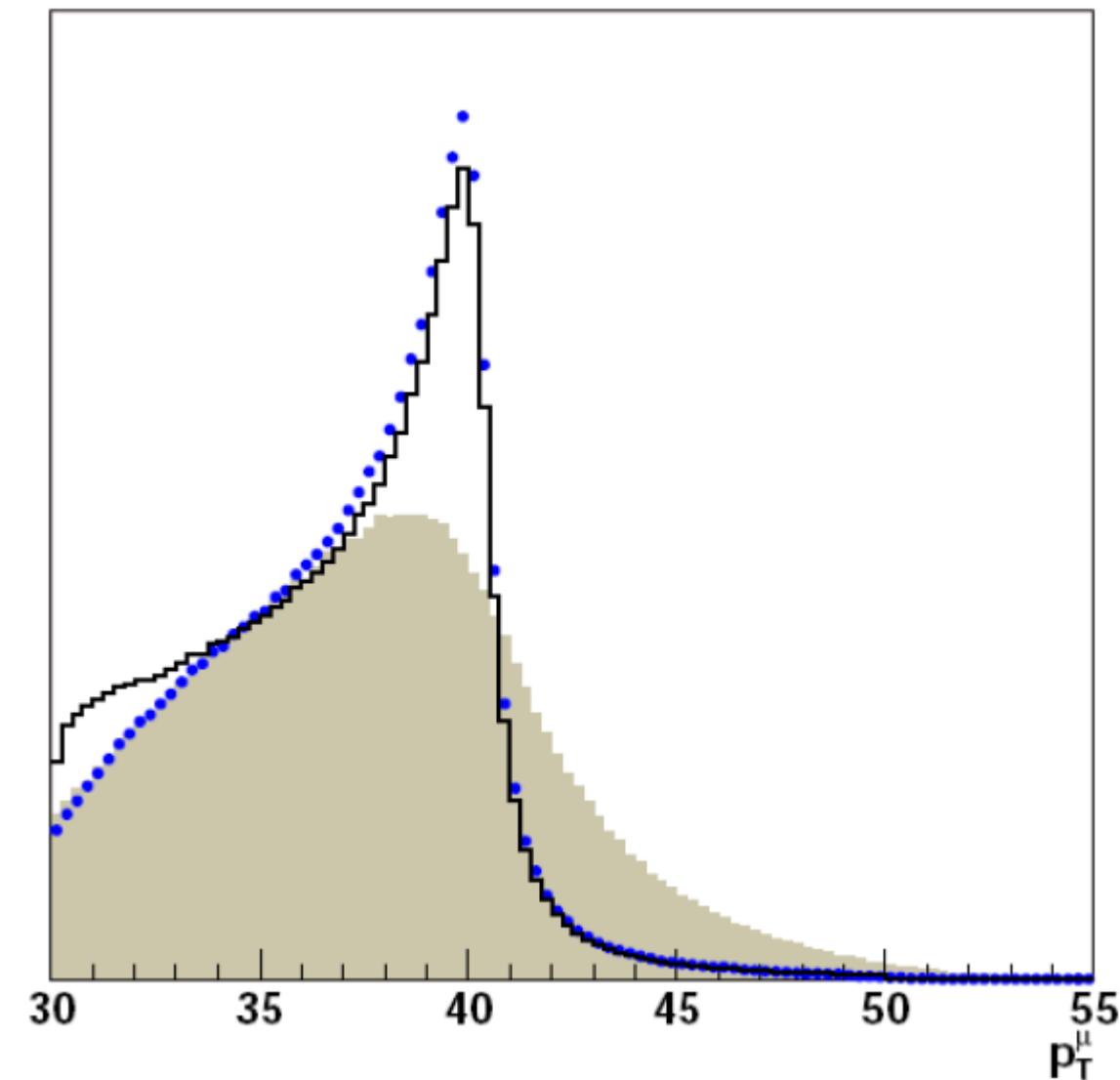
- Main complication: invariant mass cannot be reconstructed from 2-body leptonic decay mode
  - Because neutrino is not detectable directly
- Exploit the “Jacobian edge” in lepton transverse momentum spectrum

We can transfer  $\frac{d\sigma}{dp_T}$  to  $\frac{d\sigma}{dm_T}$  by using  $m_T = 2p_T$ :

$$\begin{aligned}\frac{d\sigma}{dm_T} &= \frac{1}{2} \frac{d\sigma}{dp_T} \\ &= \sigma_0(\hat{s}) \frac{m_T}{m_W} \left(2 - \frac{m_T^2}{m_W^2}\right) \left(\frac{1}{\sqrt{1 - m_T^2/m_W^2}}\right)\end{aligned}$$

# W mass measurement – decay kinematics

- Lepton transverse momentum not invariant under transverse boost
- But measurement resolution on leptons is good



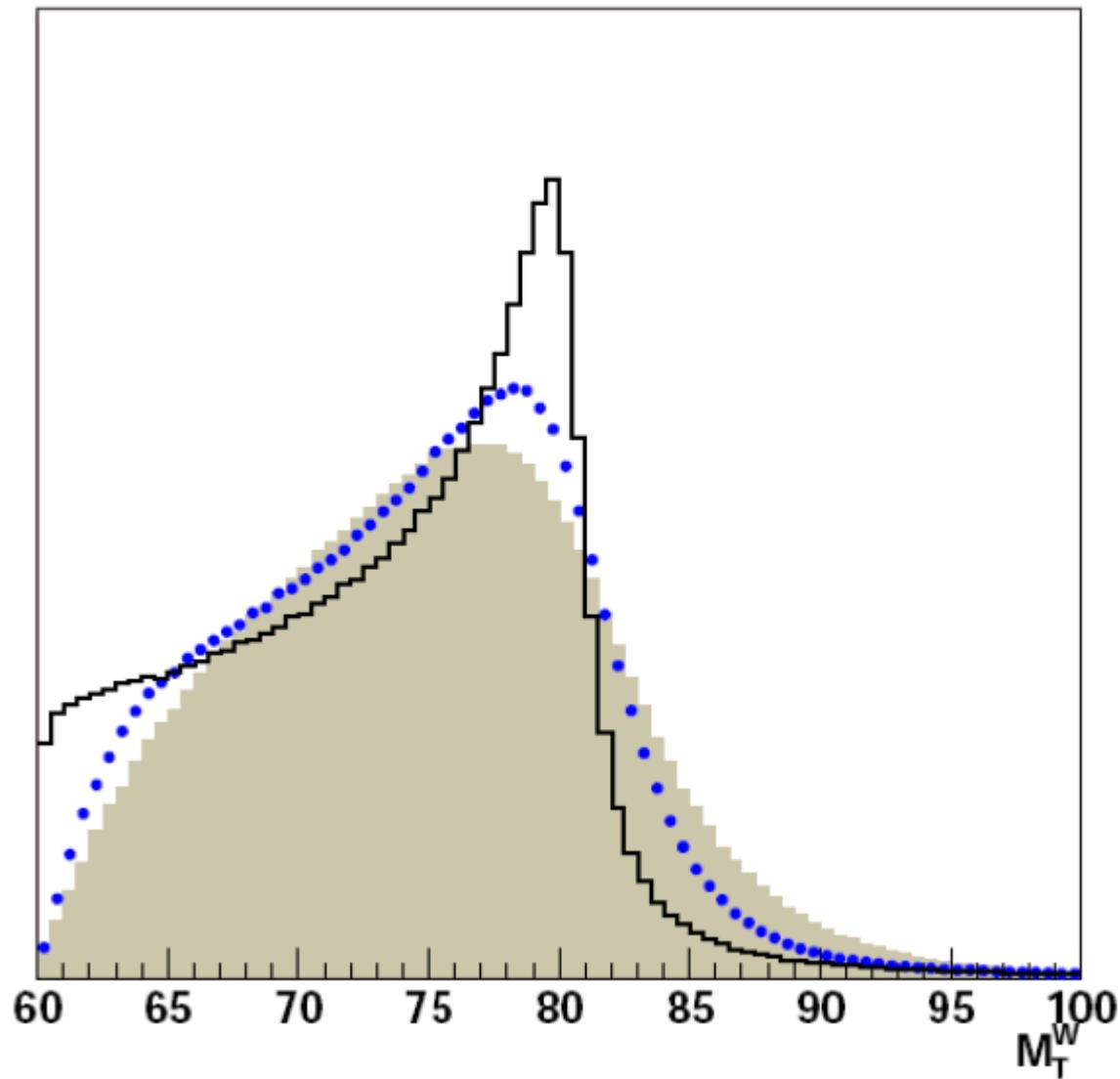
Black curve: truth level, no  $p_T(W)$

Blue points: detector-level with  
lepton resolution and selection,  
But no  $p_T(W)$

Shaded histogram: with  $p_T(W)$

# W mass measurement – decay kinematics

- Define “transverse mass” → approximately invariant under transverse boost
- But measurement resolution of “neutrino” is not as good due to recoil



Black curve: truth level, no  $p_T(W)$

Blue points: detector-level with  
lepton resolution and selection,  
But no  $p_T(W)$

Shaded histogram: with  $p_T(W)$

$$\begin{aligned} m_T &= \sqrt{(E_T^l + E_T^\nu)^2 - (\vec{p}_T^l + \vec{p}_T^\nu)^2} \\ &= \sqrt{2p_T^\mu p_T^\nu (1 - \cos \Delta\phi)} \end{aligned}$$

# CDF Event Selection

- Goal: Select events with high  $p_T$  leptons and small hadronic recoil activity
  - to maximize  $W$  mass information content and minimize backgrounds
- Inclusive lepton triggers: loose lepton track and muon stub / calorimeter cluster requirements, with lepton  $p_T > 18 \text{ GeV}$ 
  - Kinematic efficiency of trigger  $\sim 100\%$  for offline selection
- Offline selection requirements:
  - Electron cluster  $E_T > 30 \text{ GeV}$ , track  $p_T > 18 \text{ GeV}$
  - Muon track  $p_T > 30 \text{ GeV}$
  - Loose identification requirements to minimize selection bias
- $W$  boson event selection: one selected lepton,  $|u| < 15 \text{ GeV}$  &  $p_T(v) > 30 \text{ GeV}$ 
  - $Z$  boson event selection: two selected leptons

# CDF W & Z Data Samples

Sample	Candidates
$W \rightarrow e\nu$	470126
$W \rightarrow \mu\nu$	624708
$Z \rightarrow e^+e^-$	16134
$Z \rightarrow \mu^+\mu^-$	59738

- Integrated Luminosity (collected between February 2002 – August 2007):
  - Electron and muon channels:  $\mathcal{L} = 2.2 \text{ fb}^{-1}$
  - Identical running conditions for both channels, guarantees cross-calibration
- Event selection gives fairly clean samples
  - Mis-identification backgrounds  $\sim 0.5\%$

# Analysis Strategy

# Strategy

*Maximize the number of internal constraints and cross-checks*

*Driven by two goals:*

- 1) *Robustness: constrain the same parameters in as many different ways as possible*
- 2) *Precision: combine independent measurements after showing consistency*

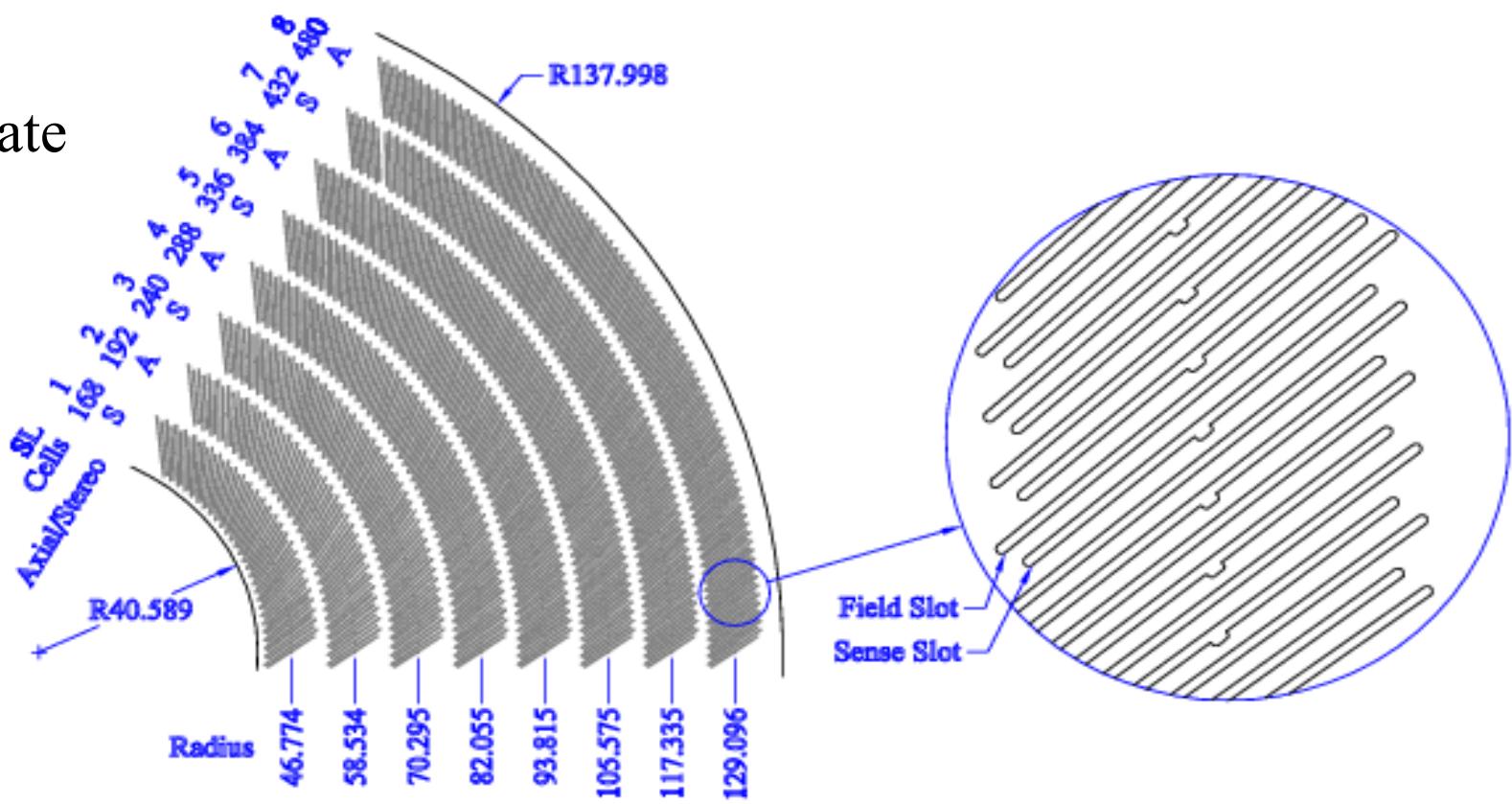
# Outline of Analysis

*Energy scale measurements drive the W mass measurement*

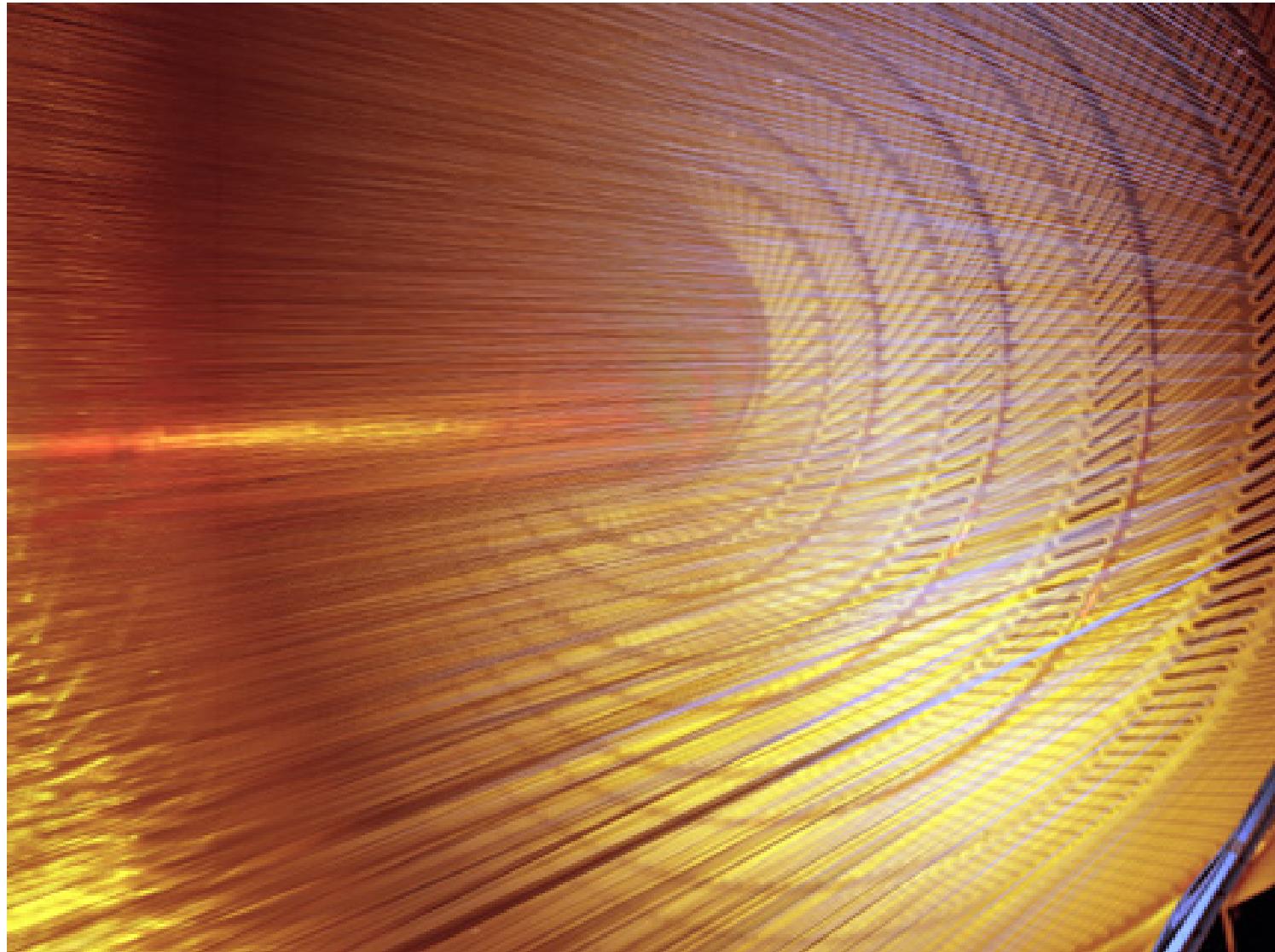
- Tracker Calibration
  - alignment of the COT ( $\sim 2400$  cells) using cosmic rays
  - COT momentum scale and tracker non-linearity constrained using  $J/\psi \rightarrow \mu\mu$  and  $\Upsilon \rightarrow \mu\mu$  mass fits
  - Confirmed using  $Z \rightarrow \mu\mu$  mass fit
- EM Calorimeter Calibration
  - COT momentum scale transferred to EM calorimeter using a fit to the peak of the  $E/p$  spectrum, around  $E/p \sim 1$
  - Calorimeter energy scale confirmed using  $Z \rightarrow ee$  mass fit
- Tracker and EM Calorimeter resolutions
- Hadronic recoil modelling
  - Characterized using  $p_T$ -balance in  $Z \rightarrow ll$  events

# Drift Chamber (COT) Alignment

COT endplate  
geometry



# CDF Particle Tracking Chamber



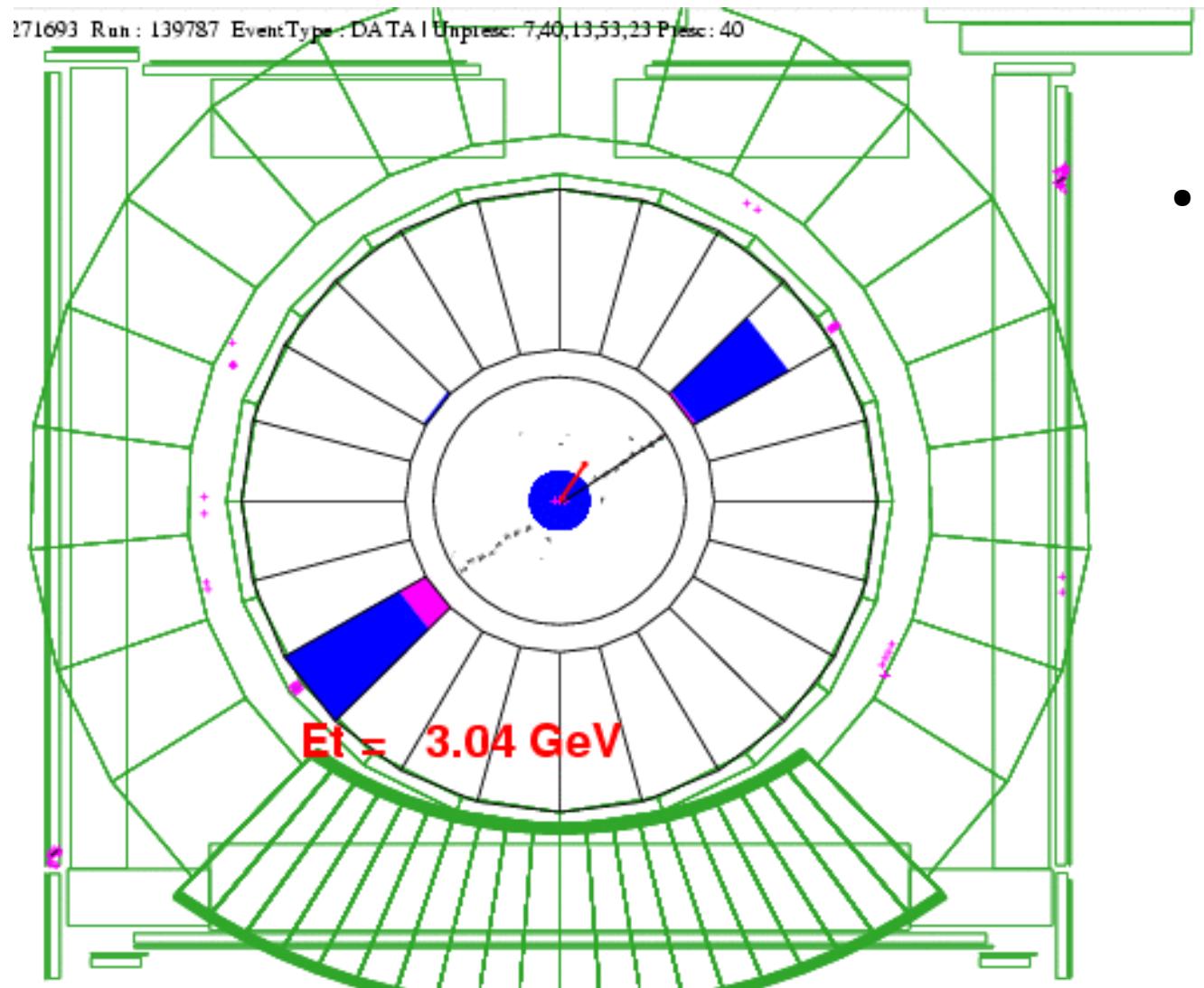
Reconstruction of particle trajectories, calibration to  $\sim 2 \mu\text{m}$  accuracy:

A. Kotwal, H. Gerberich and C. Hays, NIM A506, 110 (2003)

C. Hays et al, NIM A538, 249 (2005)

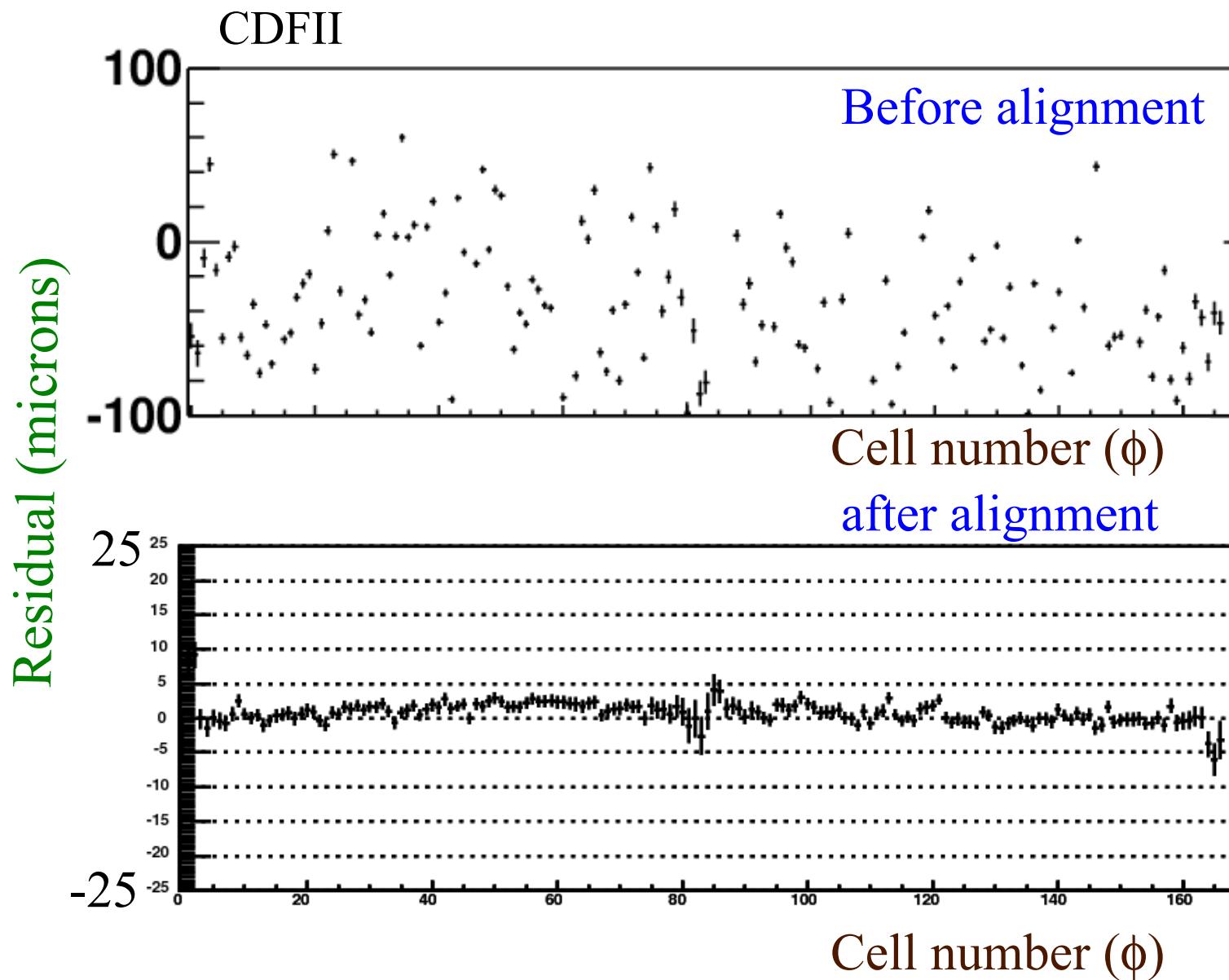
# Internal Alignment of COT

- Use a clean sample of  $\sim 400k$  cosmic rays for cell-by-cell internal alignment



- Fit COT hits on both sides simultaneously to a single helix (AK, H. Gerberich and C. Hays, NIMA 506, 110 (2003))
  - Time of incidence is a floated parameter in this 'dicosmic fit'

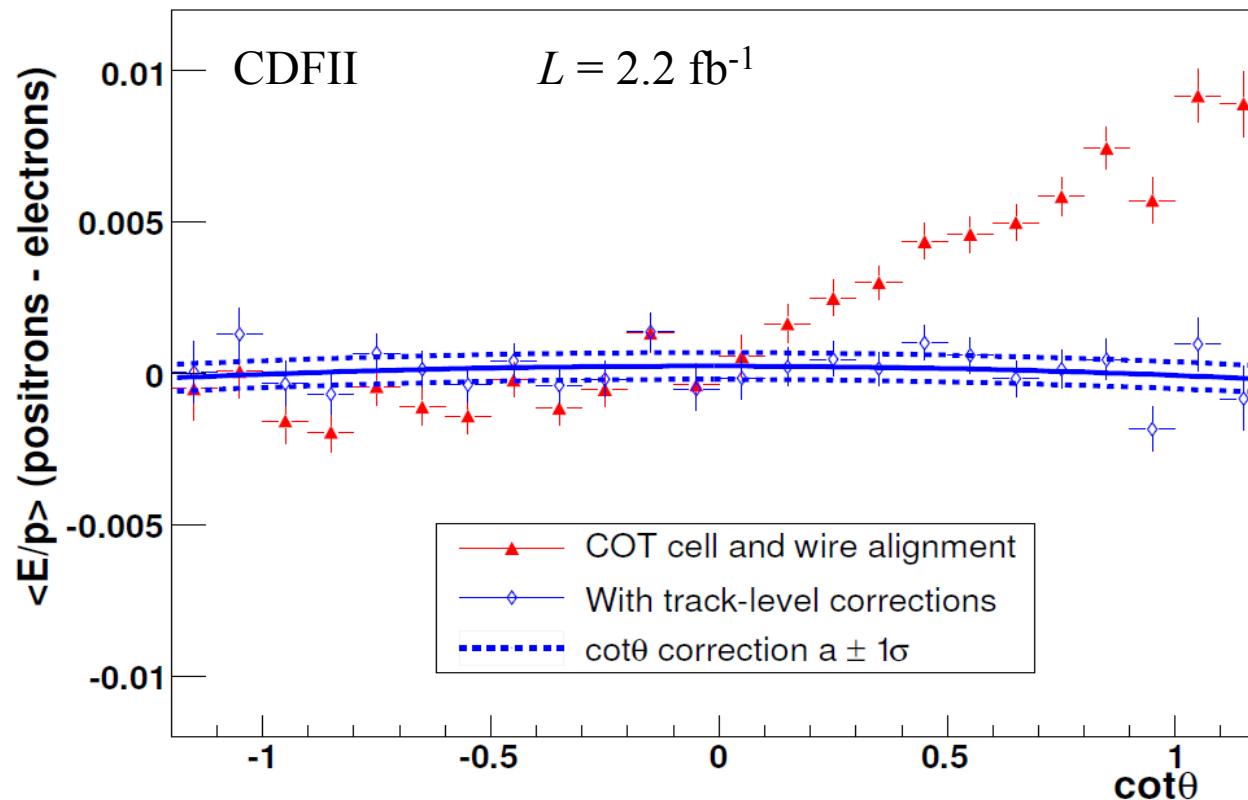
# Residuals of COT cells after alignment



Final relative alignment of cells  $\sim 2 \mu\text{m}$  (initial alignment  $\sim 50 \mu\text{m}$ )

# Cross-check of COT alignment

- Cosmic ray alignment removes most deformation degrees of freedom, but “weakly constrained modes” remain
- Final cross-check and correction to beam-constrained track curvature based on difference of  $\langle E/p \rangle$  for positrons *vs* electrons
- Smooth ad-hoc curvature corrections as a function of polar and azimuthal angle: statistical errors  $\Rightarrow \Delta M_W = 2 \text{ MeV}$

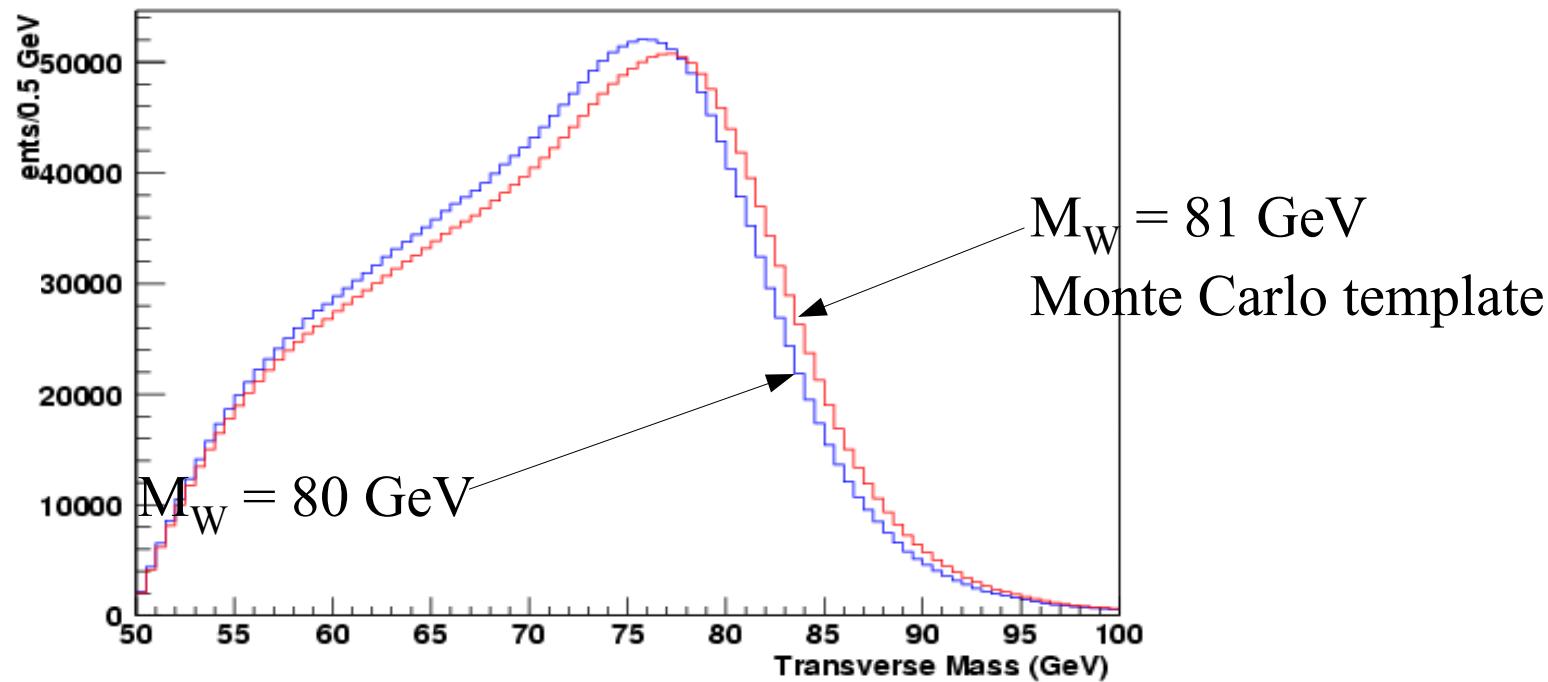


# Signal Simulation and Fitting

# Signal Simulation and Template Fitting

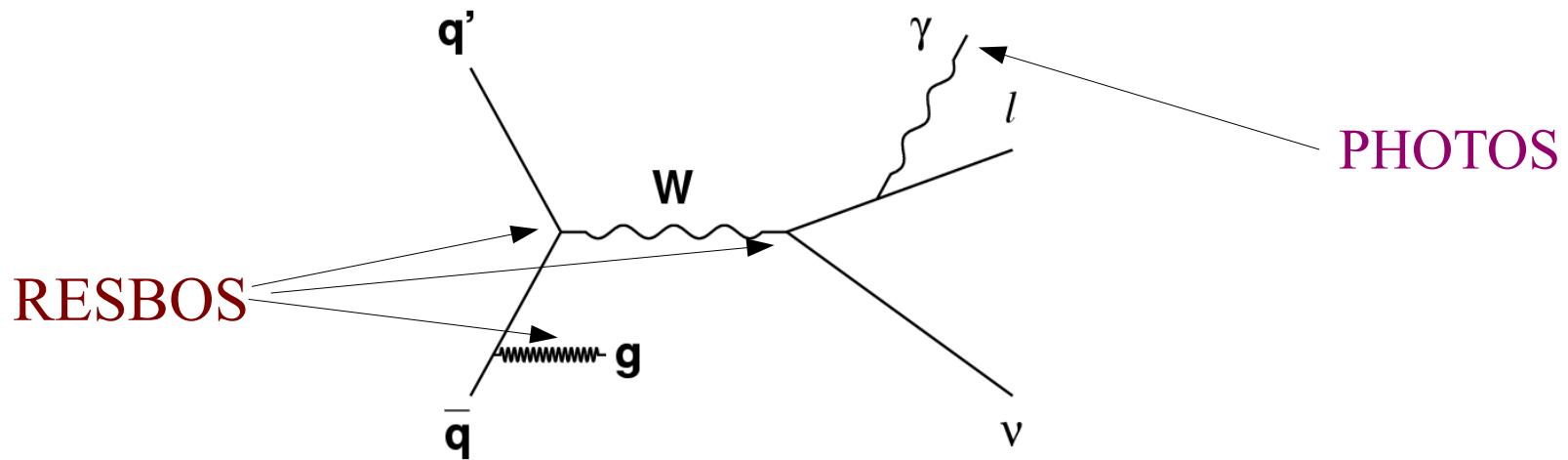
- All signals simulated using a Custom Monte Carlo
  - Generate finely-spaced templates as a function of the fit variable
  - perform binned maximum-likelihood fits to the data
- Custom fast Monte Carlo makes smooth, high statistics templates
  - And provides analysis control over key components of the simulation

$$L = \prod_{i=1}^N \frac{e^{-m_i} m_i^{n_i}}{n_i!}$$



- We will extract the W mass from six kinematic distributions: Transverse mass, charged lepton  $p_T$  and missing  $E_T$  using both electron and muon channels

# Generator-level Signal Simulation



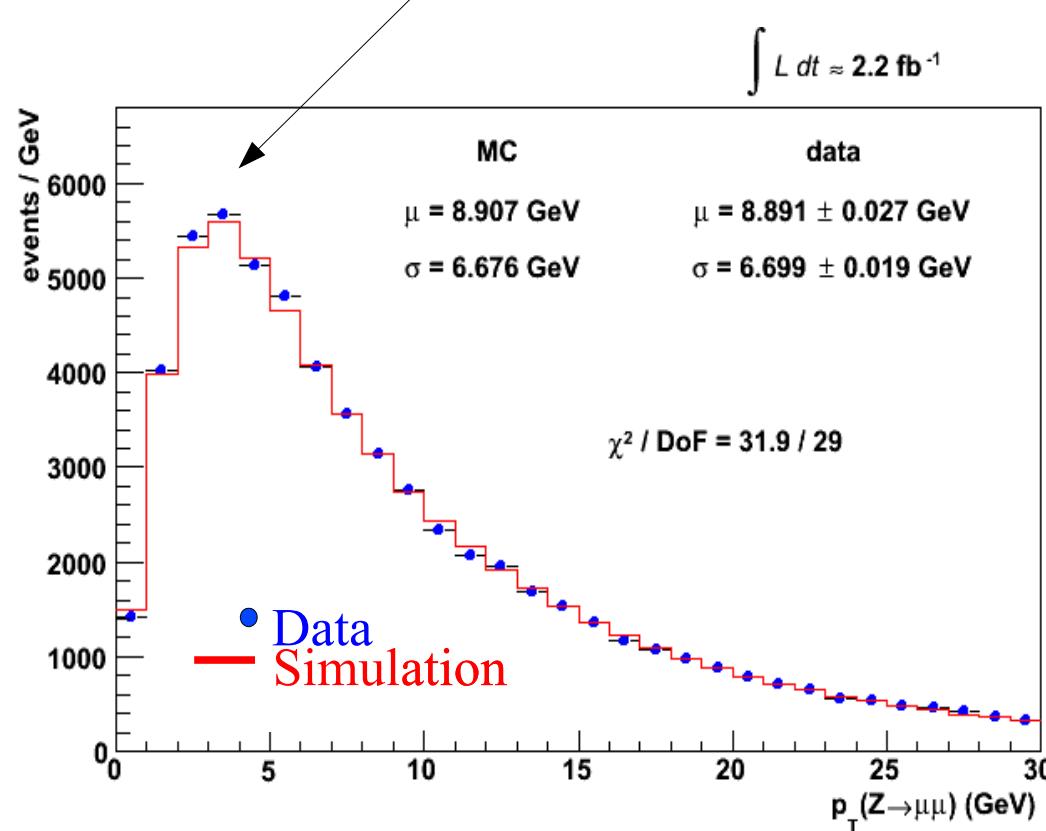
- Generator-level input for  $W$  &  $Z$  simulation provided by RESBOS (C. Balazs & C.-P. Yuan, PRD56, 5558 (1997) and references therein), which
  - Calculates triple-differential production cross section, and  $p_T$ -dependent double-differential decay angular distribution
  - calculates boson  $p_T$  spectrum reliably over the relevant  $p_T$  range: includes tunable parameters in the non-perturbative regime at low  $p_T$
- Multiple radiative photons generated according to PHOTOS (P. Golonka and Z. Was, Eur. J. Phys. C 45, 97 (2006) and references therein)

# Constraining Boson $p_T$ Spectrum

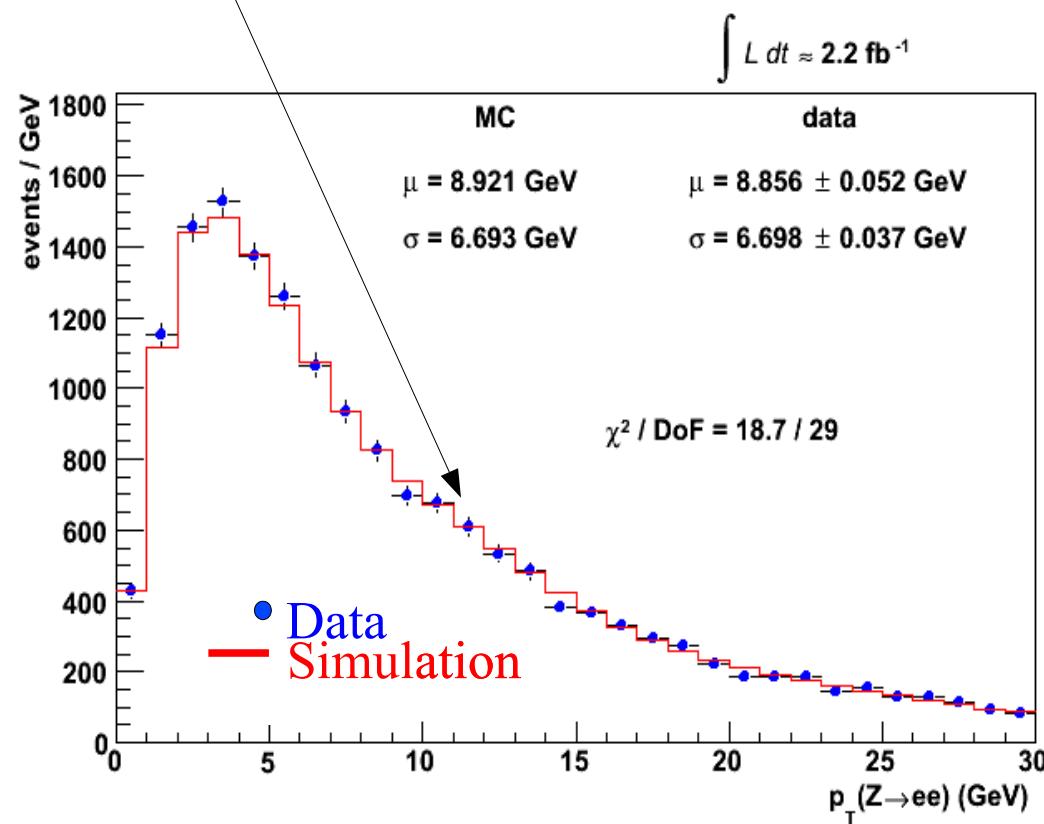
- Fit the non-perturbative parameter  $g_2$  and QCD coupling  $\alpha_s$  in RESBOS to  $p_T(l\bar{l})$  spectra:

$$\Delta M_W = 5 \text{ MeV}$$

Position of peak in boson  $p_T$  spectrum depends on  $g_2$



Tail to peak ratio depends on  $\alpha_s$



# Outline of Analysis

*Energy scale measurements drive the W mass measurement*

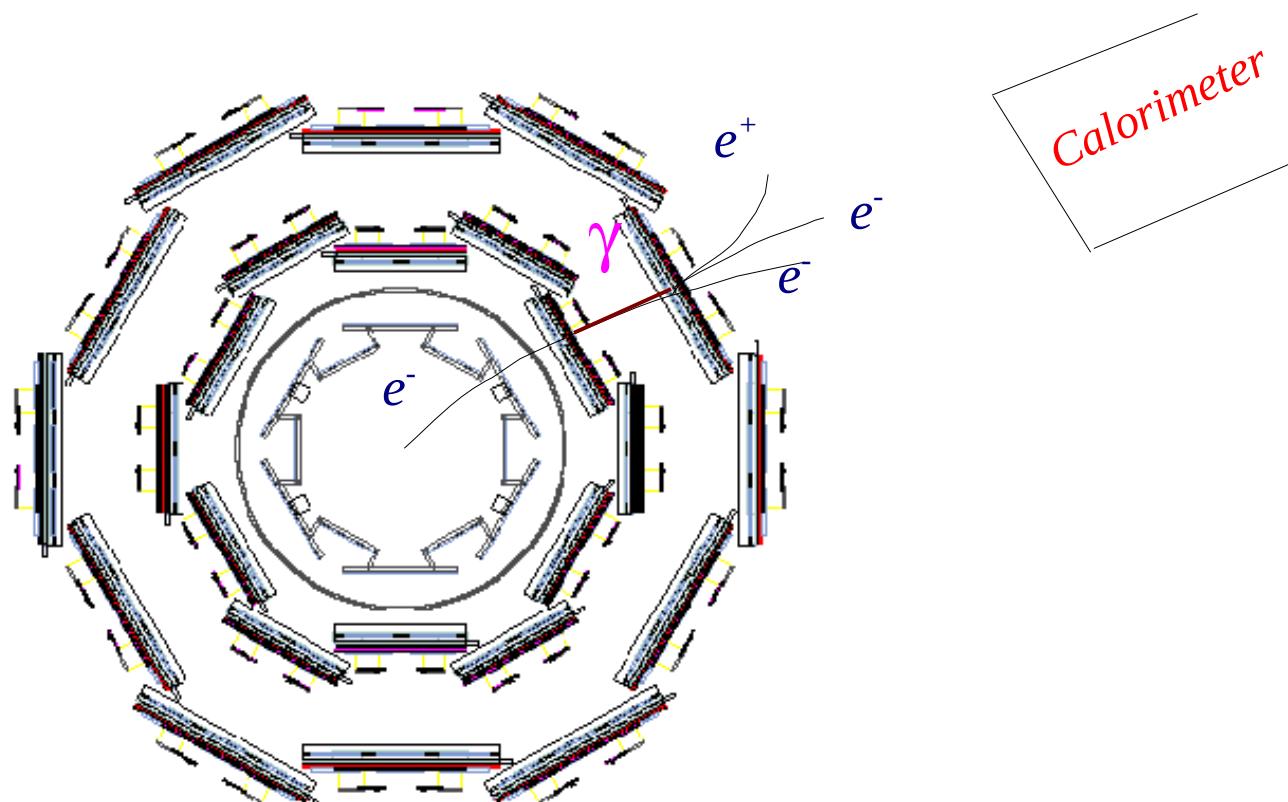
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  - Confirmed using  $Z \rightarrow \mu\mu$  mass fit
- EM Calorimeter Calibration
  - COT momentum scale transferred to EM calorimeter using a fit to the peak of the  $E/p$  spectrum, around  $E/p \sim 1$
  - Calorimeter energy scale confirmed using  $Z \rightarrow ee$  mass fit
- Tracker and EM Calorimeter resolutions
- Hadronic recoil modelling
  - Characterized using  $p_T$ -balance in  $Z \rightarrow ll$  events

# Custom Monte Carlo Detector Simulation

- A complete detector simulation of all quantities measured in the data
- First-principles simulation of tracking
  - Tracks and photons propagated through a high-resolution 3-D lookup table of material properties for silicon detector and COT
  - At each material interaction, calculate
    - Ionization energy loss according to detailed formulae and Landau distribution
    - Generate bremsstrahlung photons down to 0.4 MeV, using detailed cross section and spectrum calculations
    - Simulate photon conversion and compton scattering
    - Propagate bremsstrahlung photons and conversion electrons
    - Simulate multiple Coulomb scattering, including non-Gaussian tail
  - Deposit and smear hits on COT wires, perform full helix fit including optional beam-constraint

# Custom Monte Carlo Detector Simulation

- A complete detector simulation of all quantities measured in the data
- First-principles simulation of tracking
  - Tracks and photons propagated through a high-resolution 3-D lookup table of material properties for silicon detector and COT

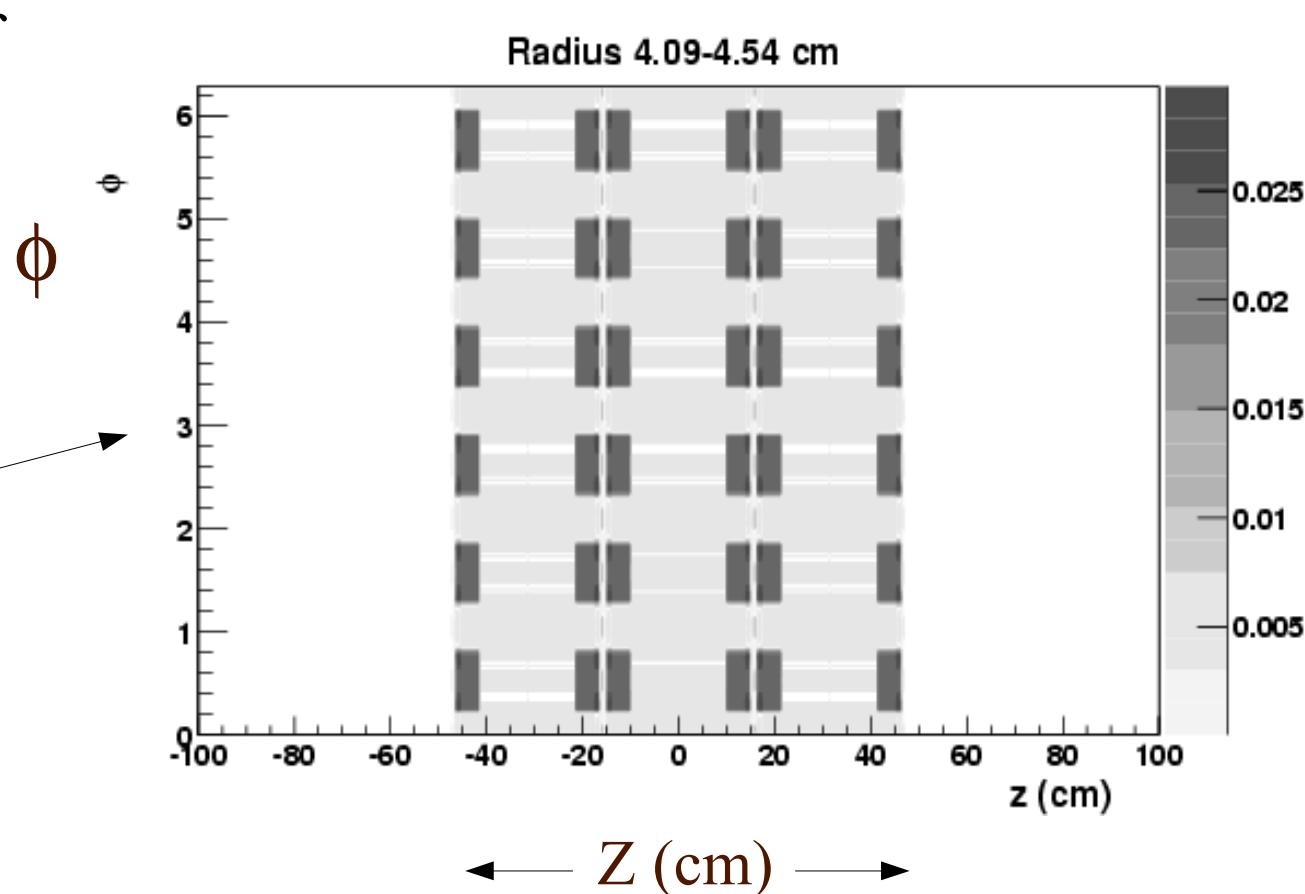


# 3-D Material Map in Simulation

- Built from detailed construction-level knowledge of inner tracker: silicon ladders, bulkheads, port-cards etc.

- Tuned based on studies of inclusive photon conversions

- Radiation lengths  $\text{vs } (\phi, z)$  at different radii shows localized nature of material distribution



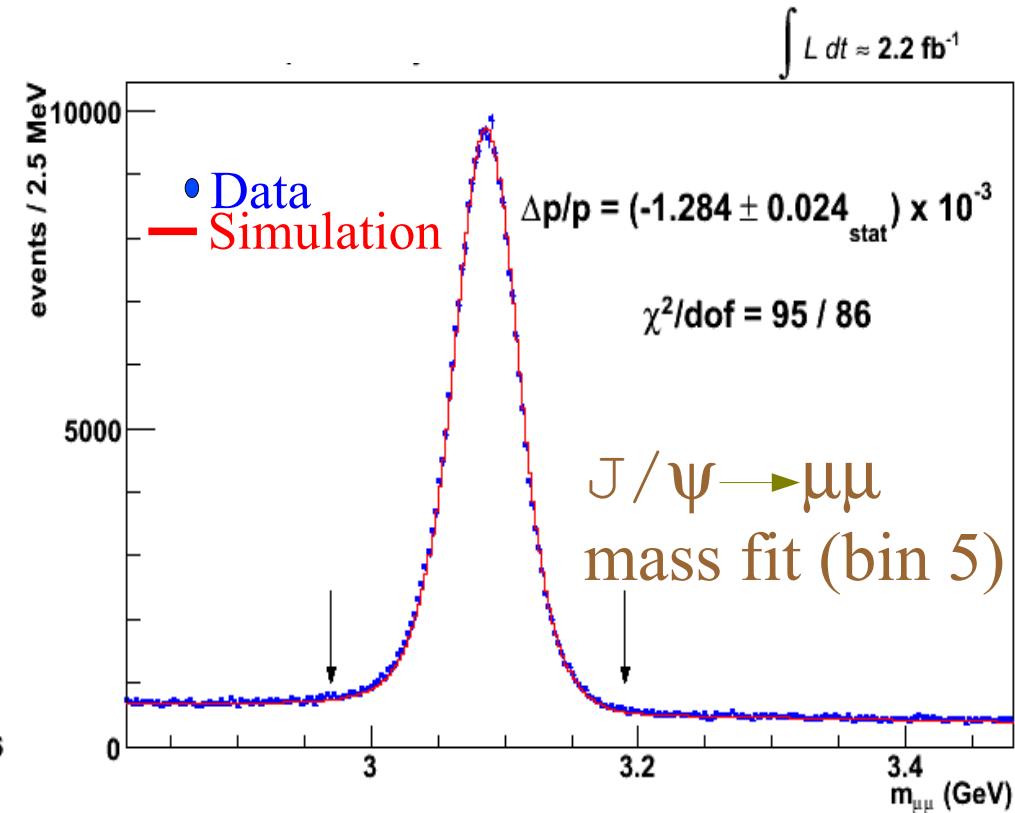
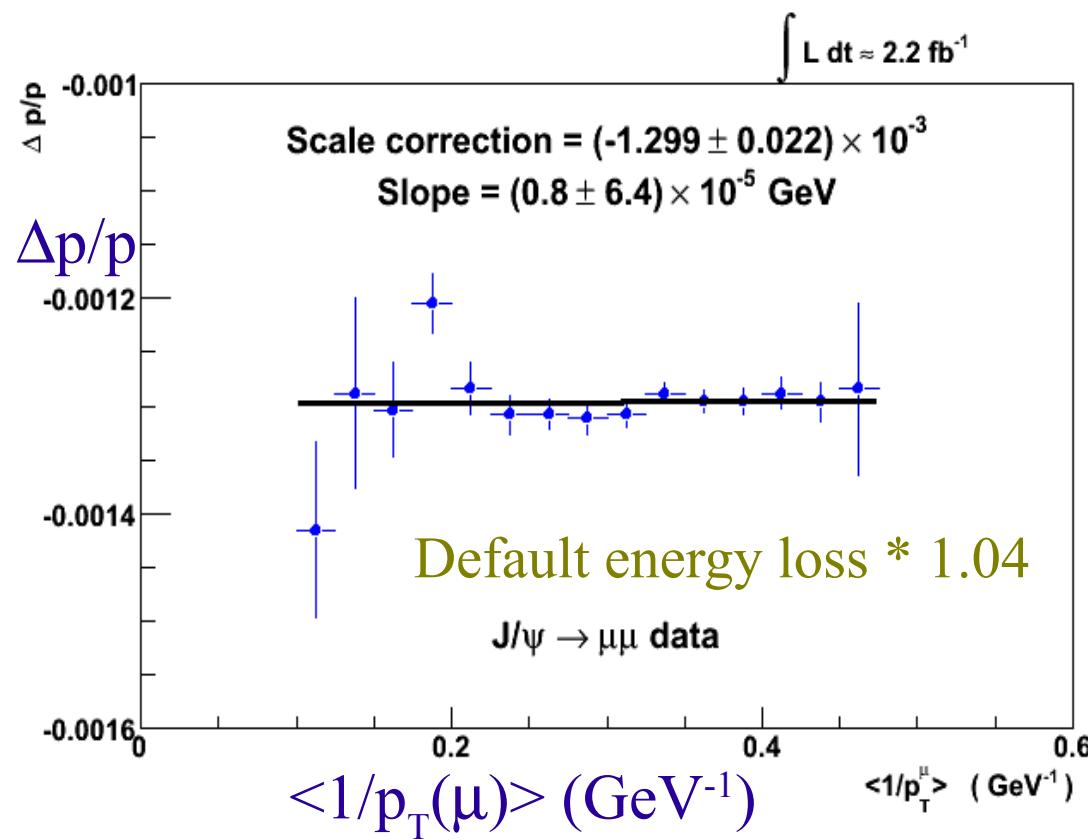
- Include dependence on type of material via Landau-Pomeranchuk-Migdal suppression of soft bremsstrahlung

Tracking Momentum Scale

# Tracking Momentum Scale

Set using  $J/\psi \rightarrow \mu\mu$  and  $\Upsilon \rightarrow \mu\mu$  resonance and  $Z \rightarrow \mu\mu$  masses

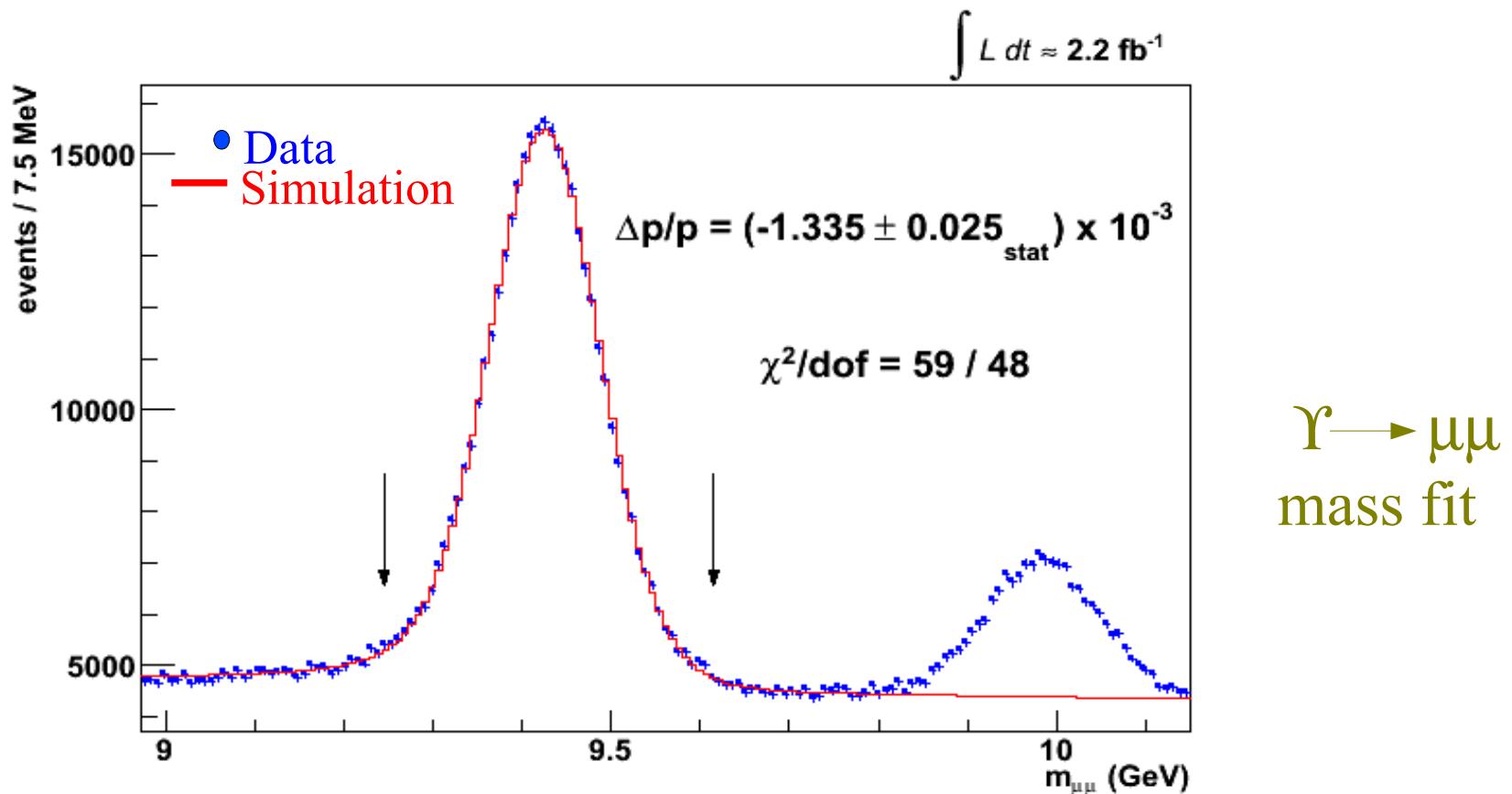
- Extracted by fitting  $J/\psi$  mass in bins of  $1/p_T(\mu)$ , and extrapolating momentum scale to zero curvature
- $J/\psi \rightarrow \mu\mu$  mass independent of  $p_T(\mu)$  after 4% tuning of energy loss



# Tracking Momentum Scale

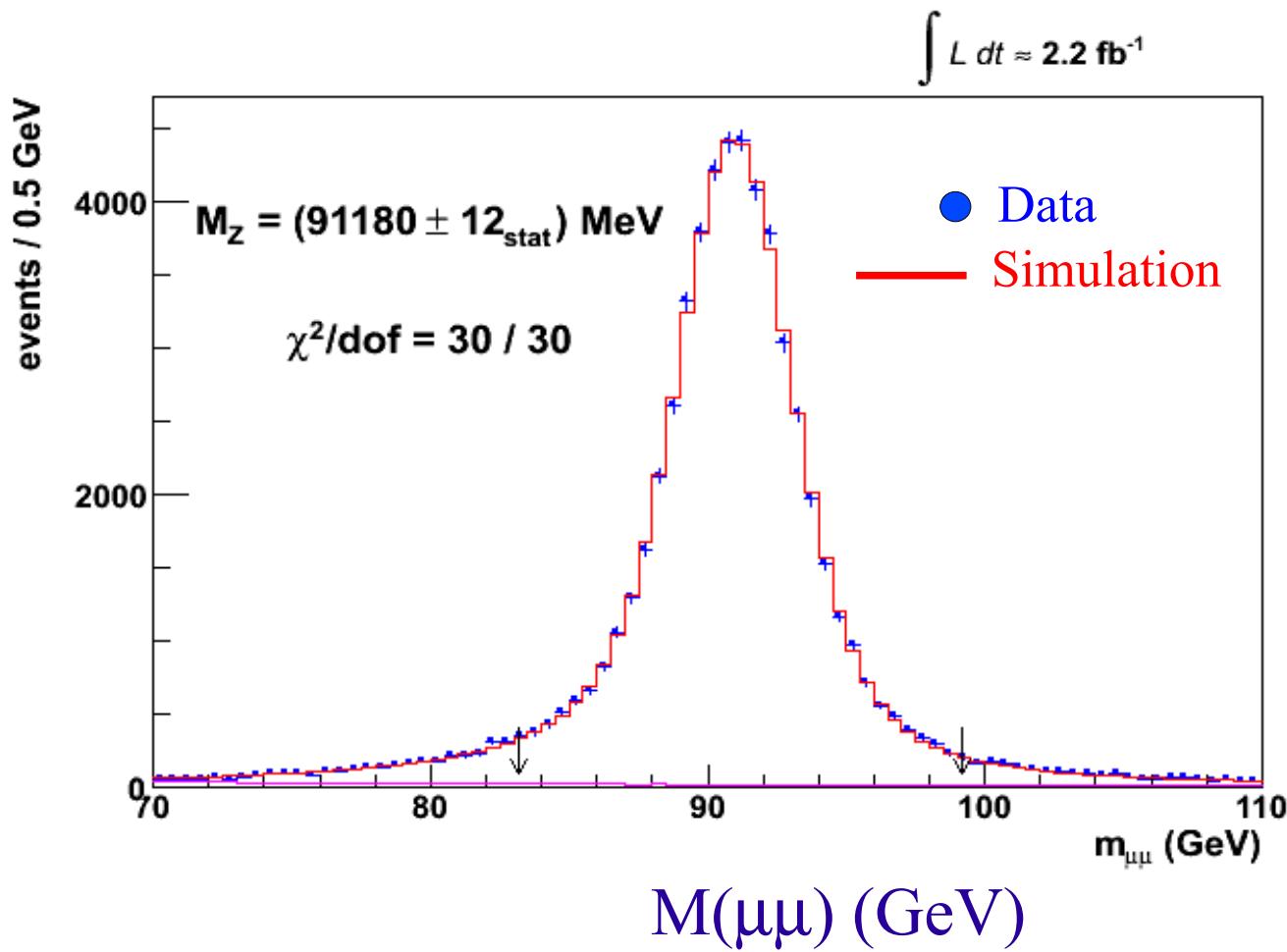
$\Upsilon \rightarrow \mu\mu$  resonance provides

- Momentum scale measurement at higher  $p_T$



## $Z \rightarrow \mu\mu$ Mass Cross-check & Combination

- Using the  $J/\psi$  and  $\Upsilon$  momentum scale, performed “blinded” measurement of  $Z$  mass
  - $Z$  mass consistent with PDG value (91188 MeV) ( $0.7\sigma$  statistical)
  - $M_Z = 91180 \pm 12_{\text{stat}} \pm 9_{\text{momentum}} \pm 5_{\text{QED}} \pm 2_{\text{alignment}}$  MeV

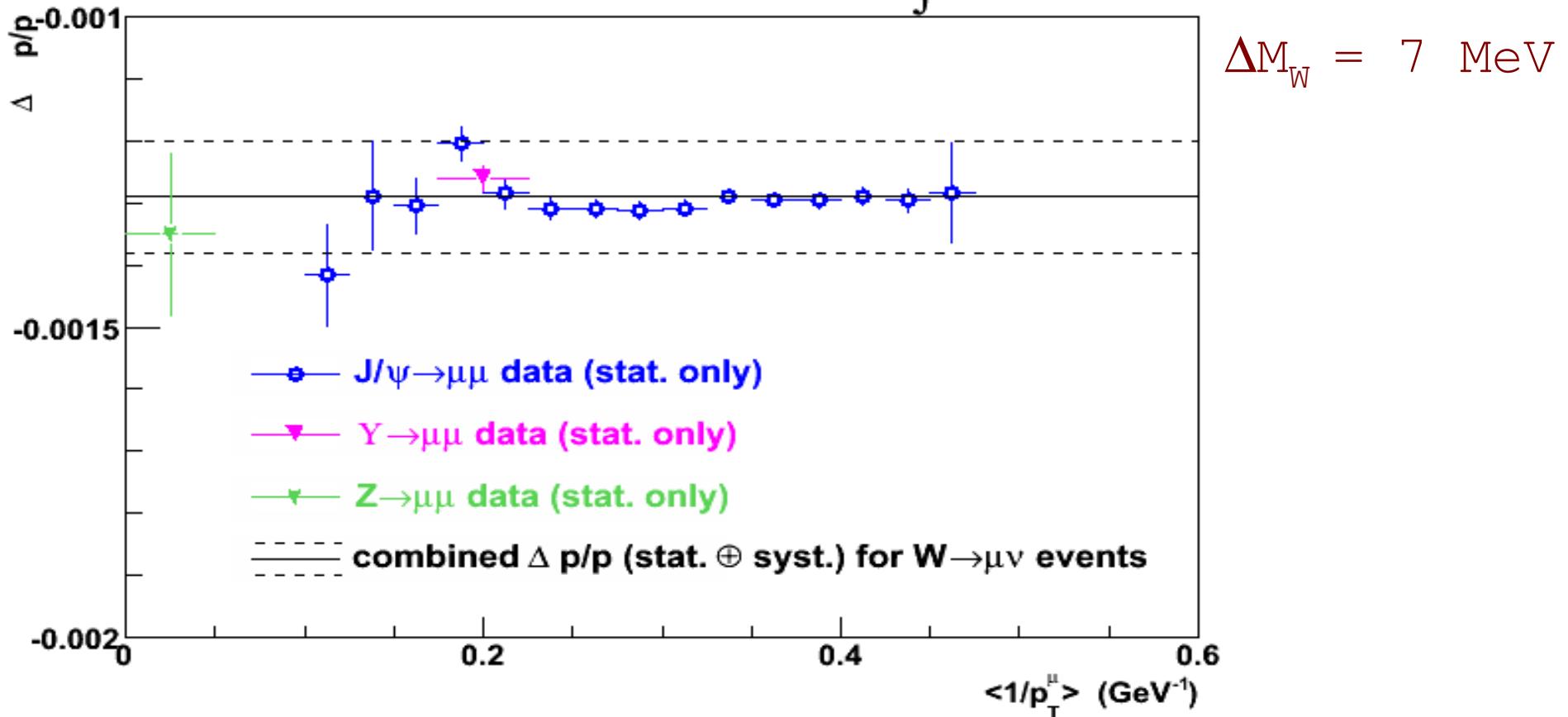


# Tracker Linearity Cross-check & Combination

- Final calibration using the J/ $\psi$ ,  $\Upsilon$  and Z bosons for calibration
- Combined momentum scale correction :

$$\Delta p/p = (-1.29 \pm 0.07_{\text{independent}} \pm 0.05_{\text{QED}} \pm 0.02_{\text{align}}) \times 10^{-3}$$

$$\int L dt \approx 2.2 \text{ fb}^{-1}$$

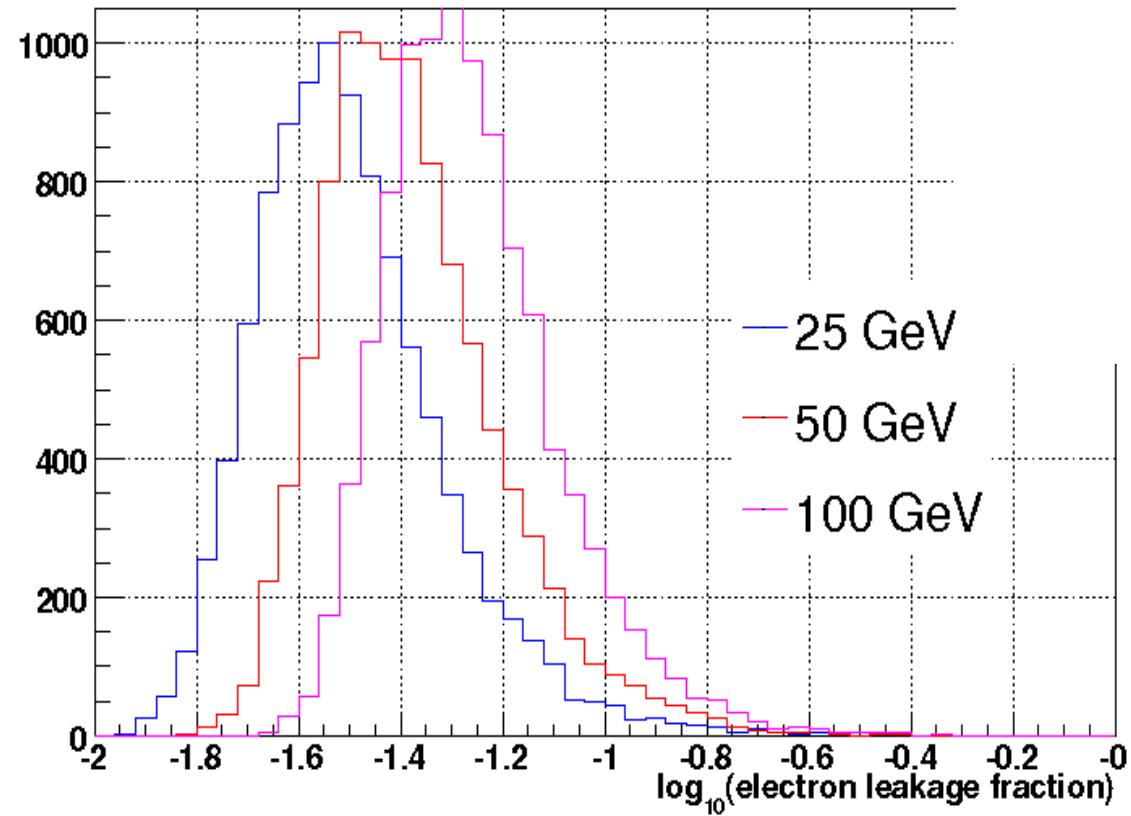


# EM Calorimeter Response

# Calorimeter Simulation for Electrons and Photons

- Distributions of lost energy calculated using detailed GEANT4 simulation of calorimeter

- Leakage into hadronic calorimeter
  - Absorption in the coil
  - Dependence on incident angle and  $E_T$



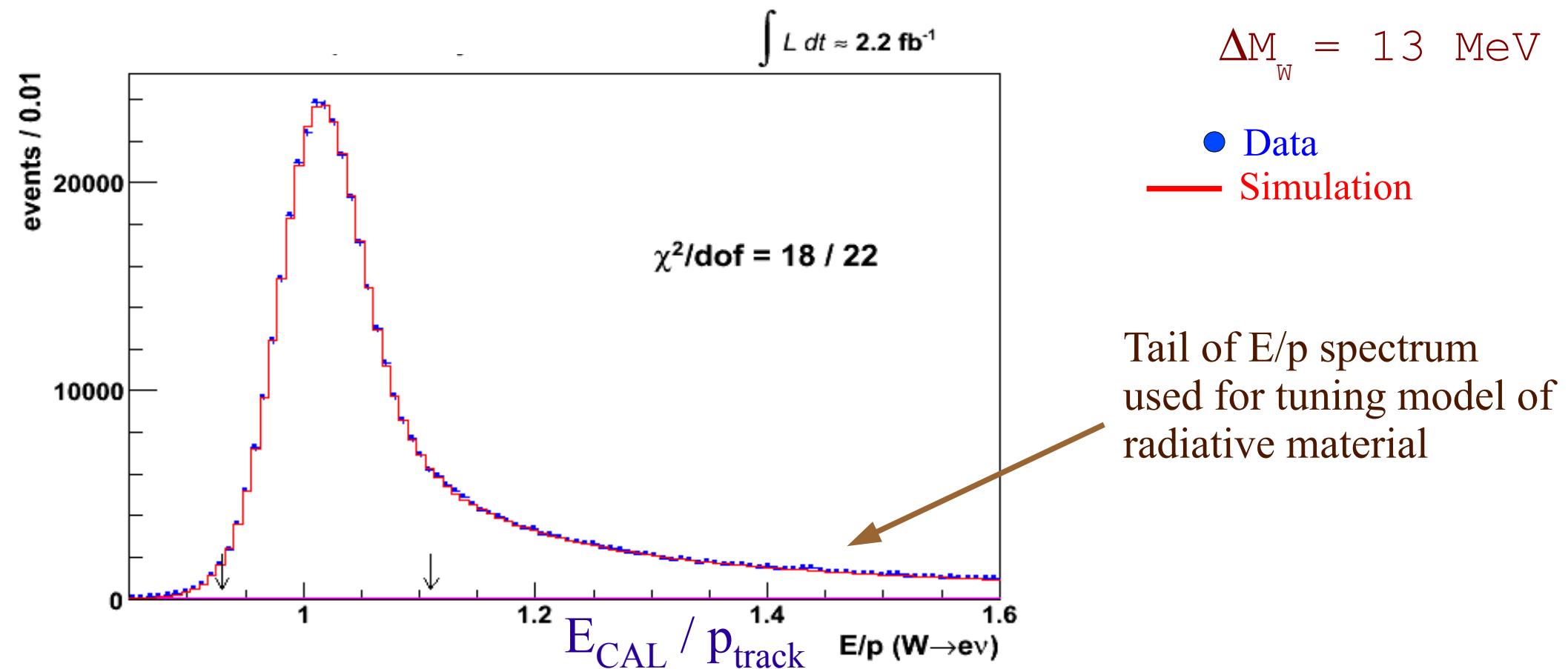
- Energy-dependent gain (non-linearity) parameterized and fit from data
- Energy resolution parameterized as fixed sampling term and tunable constant term
  - Constant terms are fit from the width of  $E/p$  peak and  $Z \rightarrow ee$  mass peak

## EM Calorimeter Scale

- E/p peak from  $W \rightarrow e\nu$  decays provides measurements of EM calorimeter scale and its ( $E_T$ -dependent) non-linearity

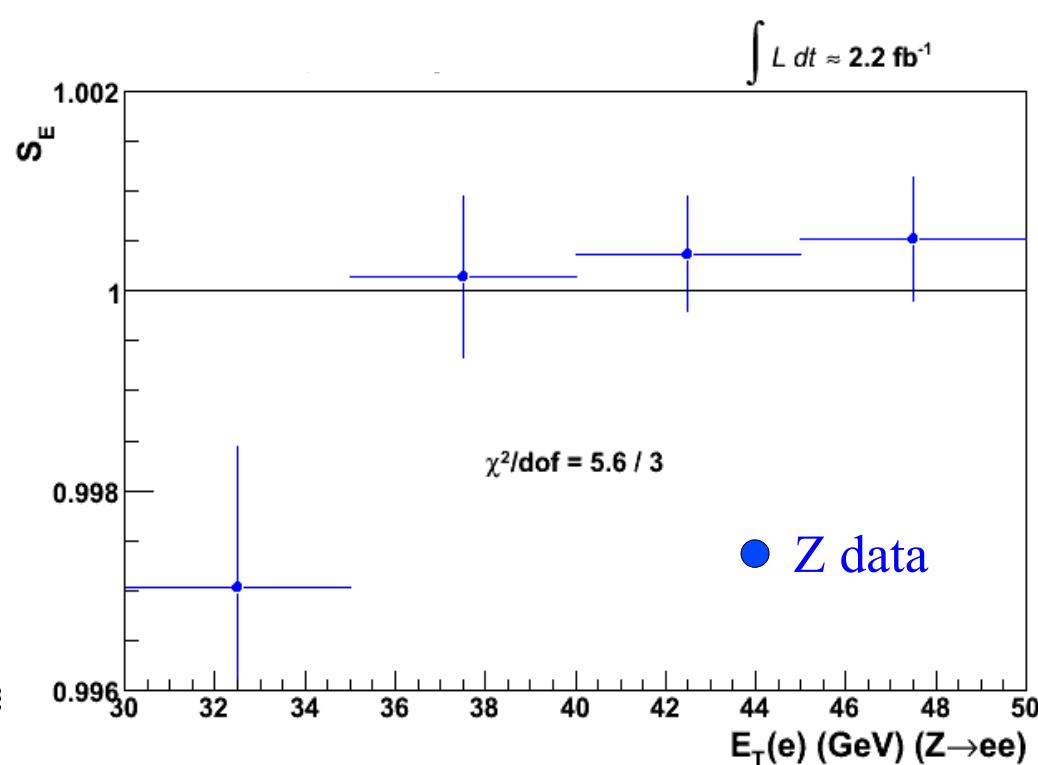
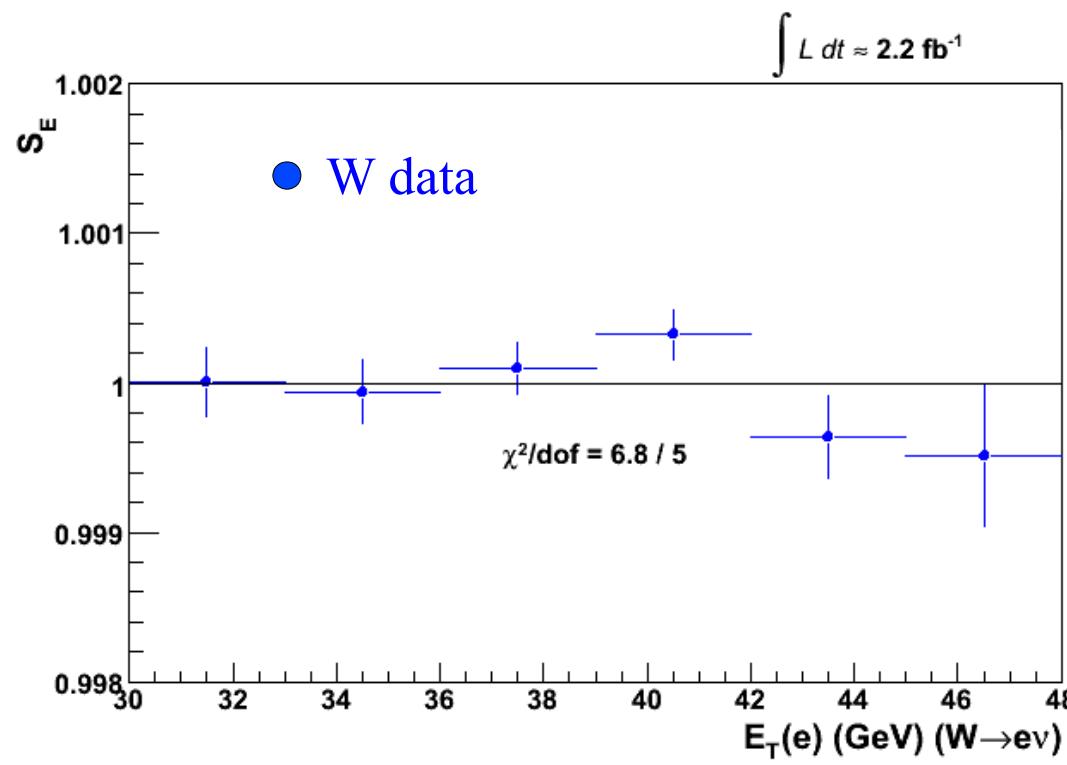
$$\Delta S_E = (9_{\text{stat}} \pm 5_{\text{non-linearity}} \pm 5_{X0} \pm 9_{\text{Tracker}}) \times 10^{-5}$$

Setting  $S_E$  to 1 using E/p calibration from combined  $W \rightarrow e\nu$  and  $Z \rightarrow ee$  samples



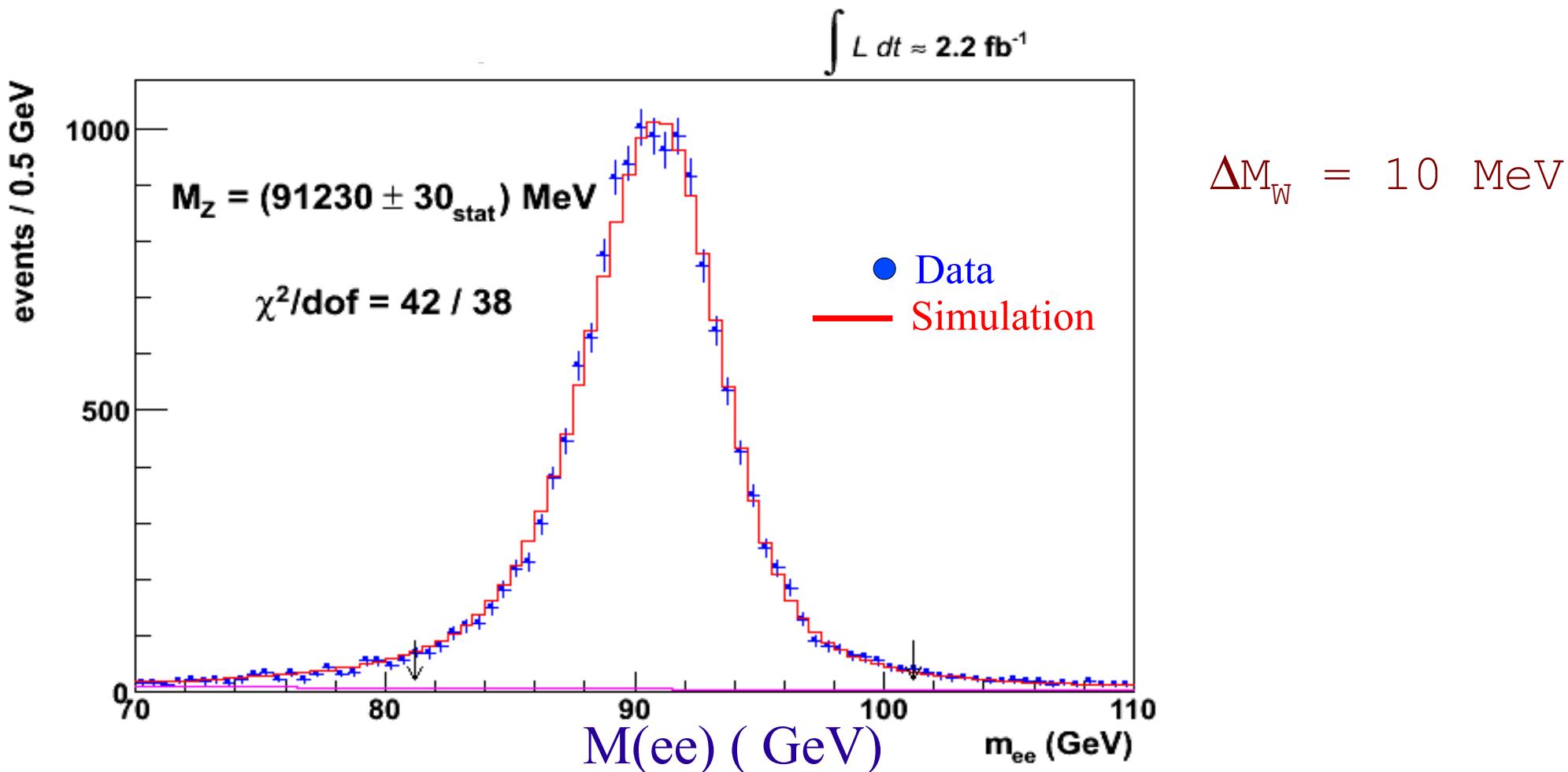
# Measurement of EM Calorimeter Non-linearity

- Perform E/p fit-based calibration in bins of electron  $E_T$
- GEANT-motivated parameterization of non-linear response:  
$$S_E = 1 + \beta \log(E_T / 39 \text{ GeV})$$
- Tune on W and Z data:  $\beta = (5.2 \pm 0.7_{\text{stat}}) \times 10^{-3}$   
 $\Rightarrow \Delta M_W = 4 \text{ MeV}$



# $Z \rightarrow ee$ Mass Cross-check and Combination

- Performed “blind” measurement of  $Z$  mass using  $E/p$ -based calibration
  - Consistent with PDG value (91188 MeV) within  $1.4\sigma$  (statistical)
  - $M_Z = 91230 \pm 30_{\text{stat}} \pm 10_{\text{calorimeter}} \pm 8_{\text{momentum}} \pm 5_{\text{QED}} \pm 2_{\text{alignment}}$  MeV
- Combine  $E/p$ -based calibration with  $Z \rightarrow ee$  mass for maximum precision

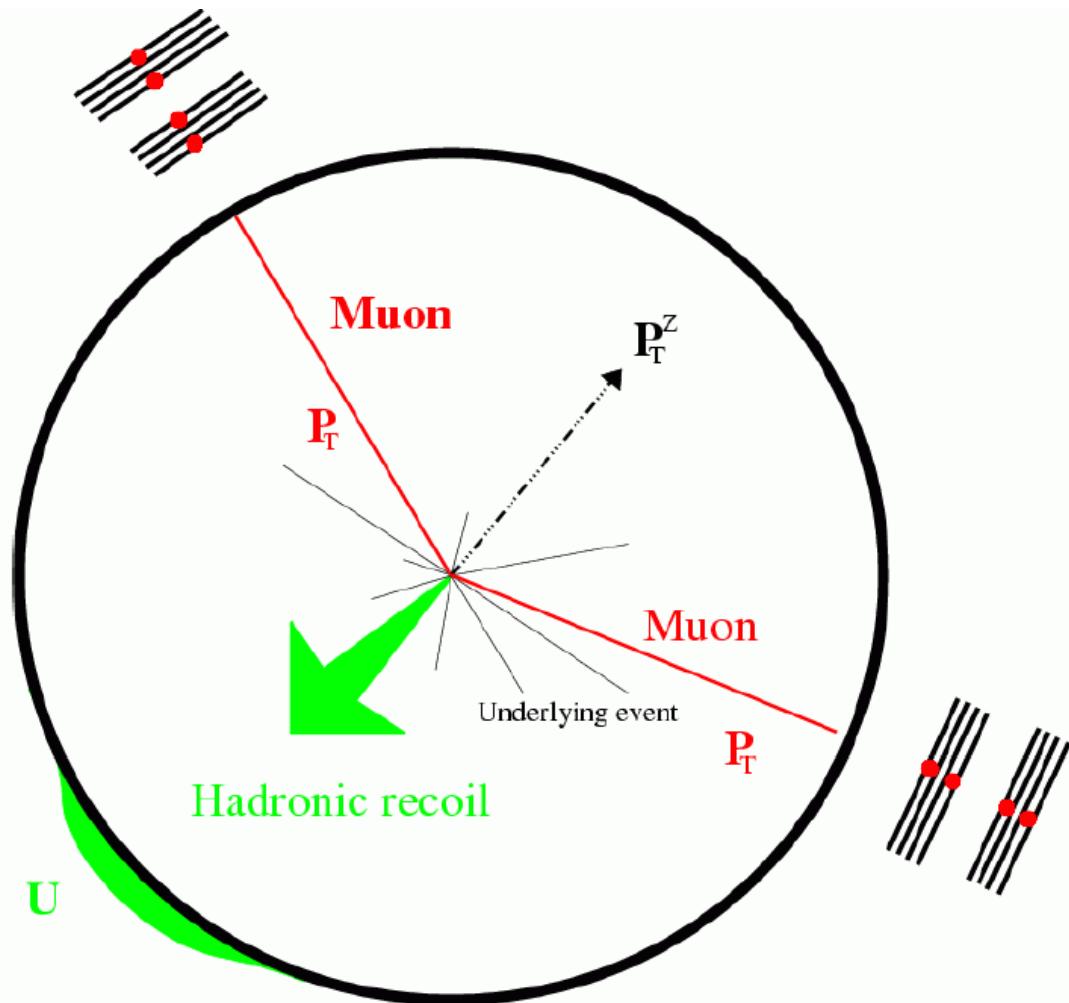


# Hadronic Recoil Model

# Constraining the Hadronic Recoil Model

Exploit similarity in production  
and decay of  $W$  and  $Z$  bosons

Detector response model for  
hadronic recoil tuned using  
 $p_T$ -balance in  $Z \rightarrow ll$  events

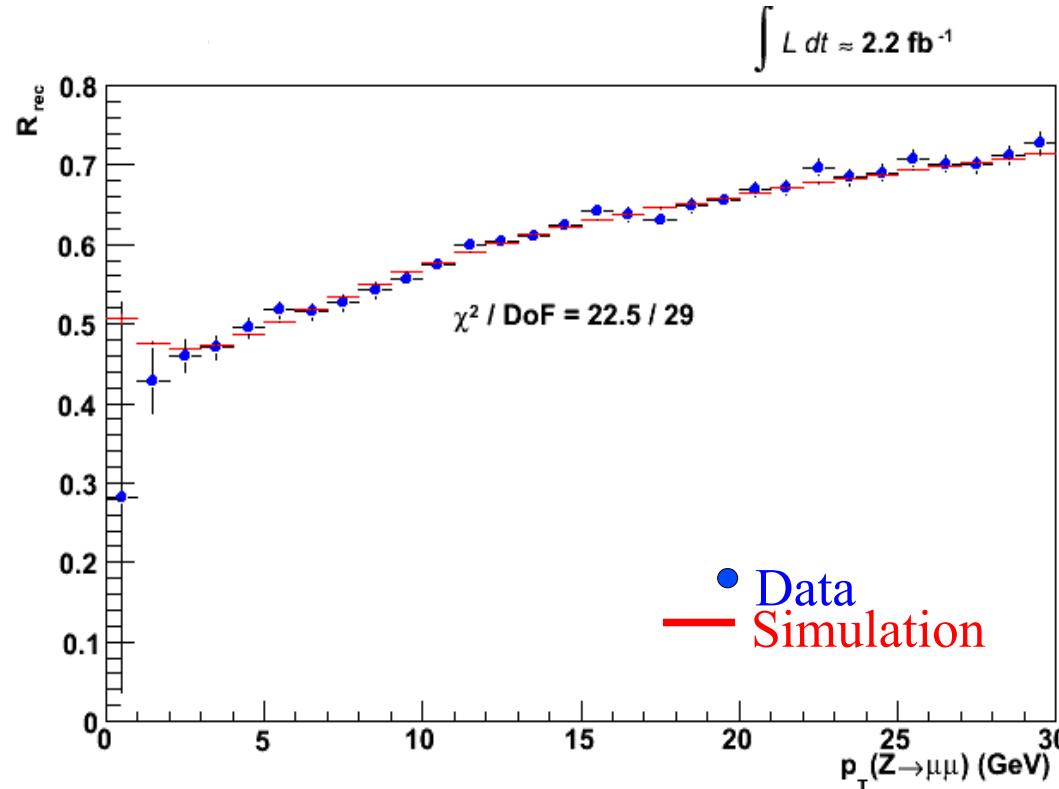


Transverse momentum of Hadronic recoil ( $u$ ) calculated as 2-vector-sum over calorimeter towers

# Hadronic Recoil Simulation

Recoil momentum 2-vector  $\mathbf{u}$  has

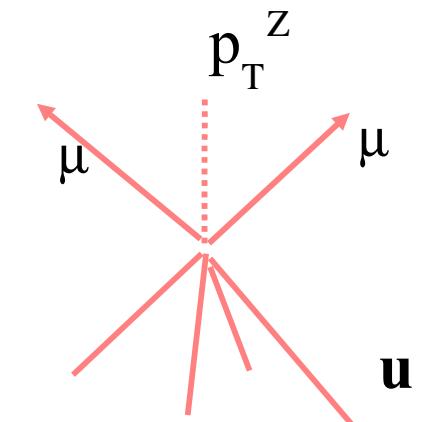
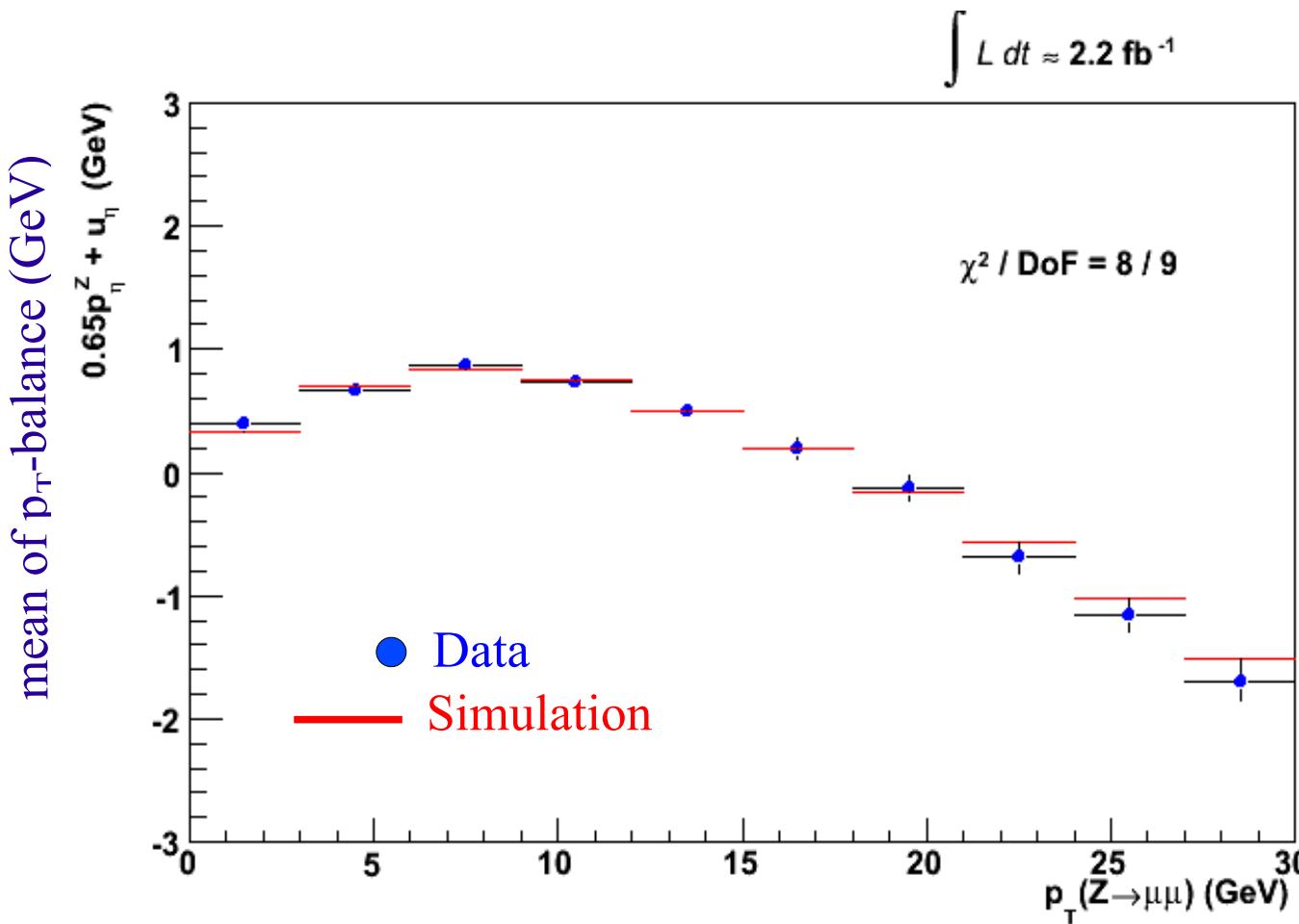
- a soft 'spectator interaction' component, randomly oriented
  - Modelled using minimum-bias data with tunable magnitude
- A hard 'jet' component, directed opposite the boson  $\mathbf{p}_T$ 
  - $\mathbf{p}_T$ -dependent response and resolution parameterizations
  - Hadronic response  $R = \mathbf{u}_{\text{reconstructed}} / \mathbf{u}_{\text{true}}$  parameterized as a logarithmically increasing function of boson  $\mathbf{p}_T$  motivated by Z boson data



# Tuning Recoil Response Model with Z events

Project the vector sum of  $p_T(l\bar{l})$  and  $\mathbf{u}$  on a set of orthogonal axes defined by boson  $p_T$

Mean and rms of projections as a function of  $p_T(l\bar{l})$  provide information on hadronic model parameters

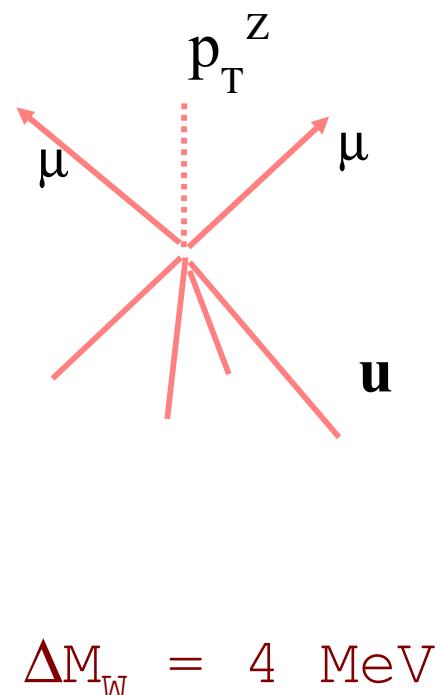
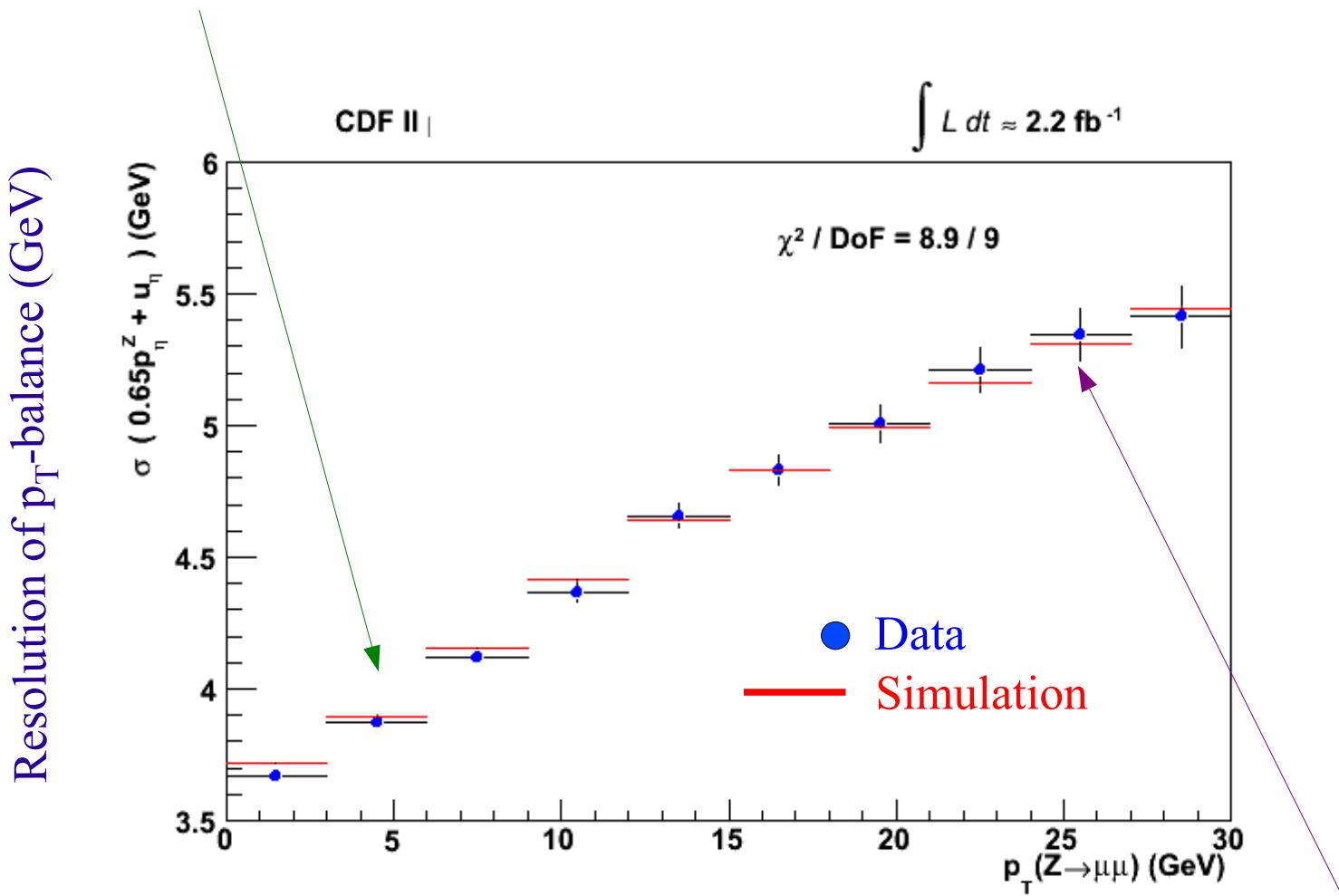


Hadronic model parameters tuned by minimizing  $\chi^2$  between data and simulation

$$\Delta M_W = 4 \text{ MeV}$$

# Tuning Recoil Resolution Model with Z events

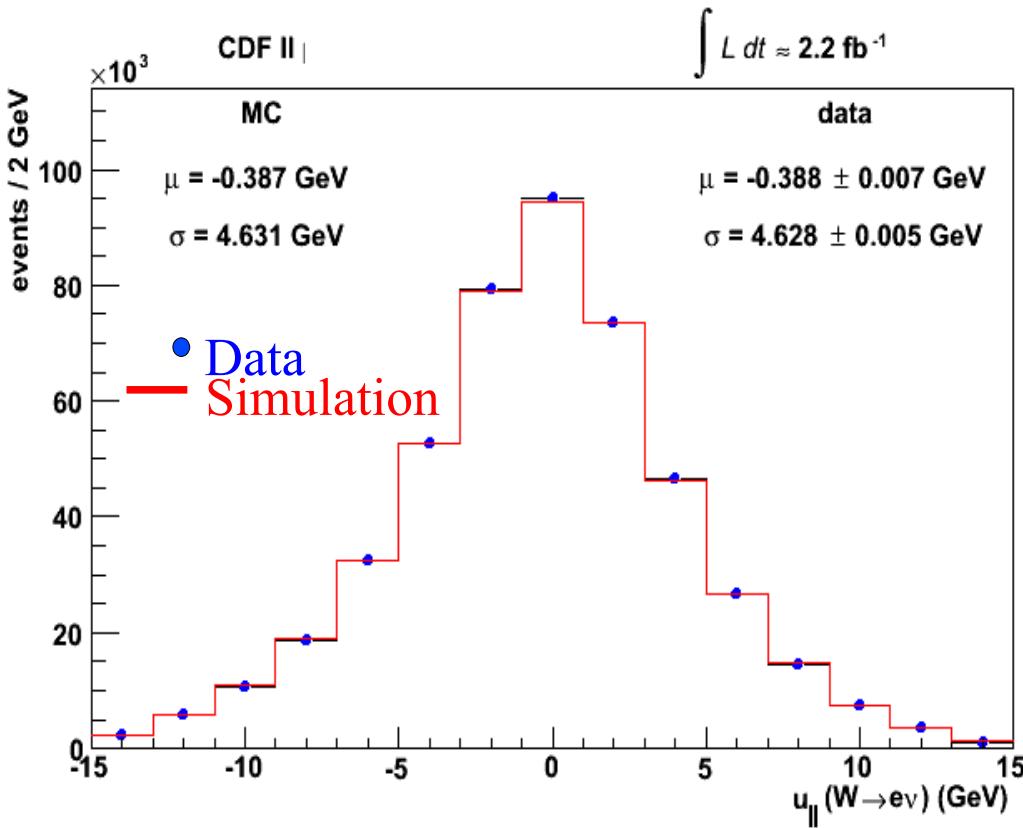
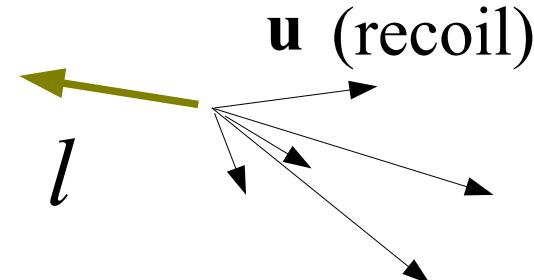
At low  $p_T(Z)$ ,  $p_T$ -balance constrains hadronic resolution due to underlying event



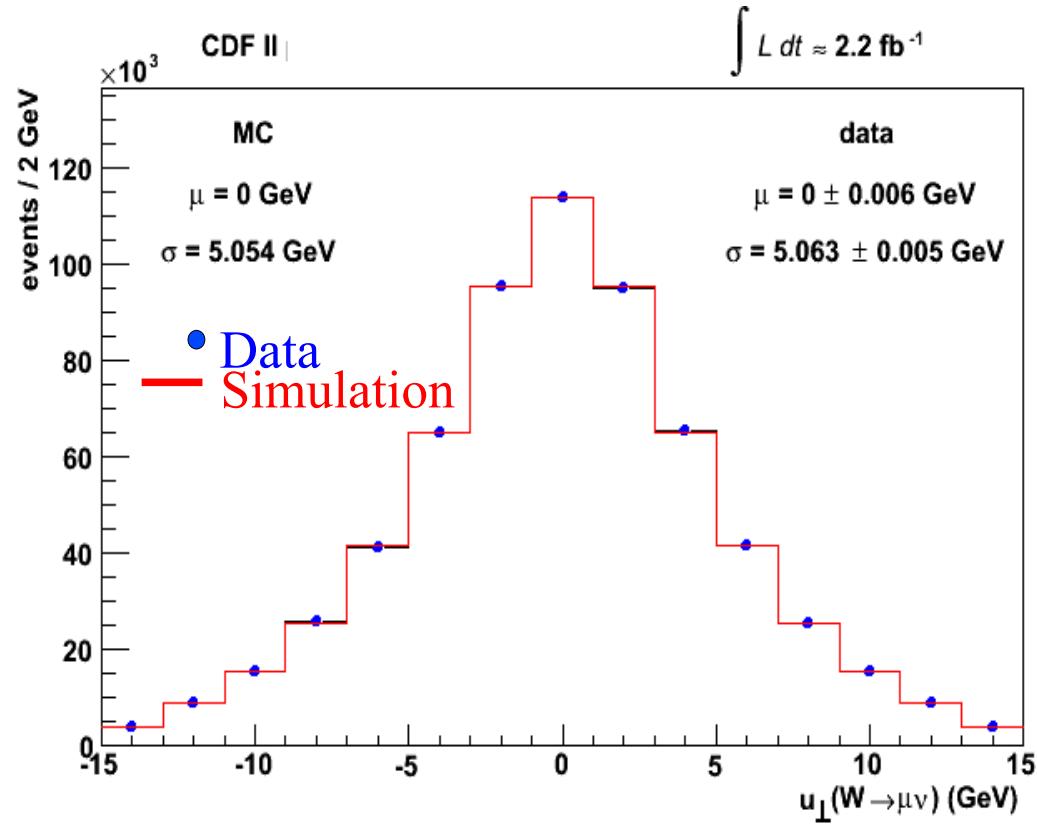
At high  $p_T(Z)$ ,  $p_T$ -balance constrains jet resolution

# Testing Hadronic Recoil Model with $W$ events

Compare recoil distributions  
between simulation and data



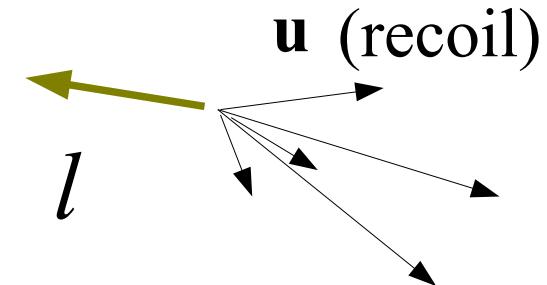
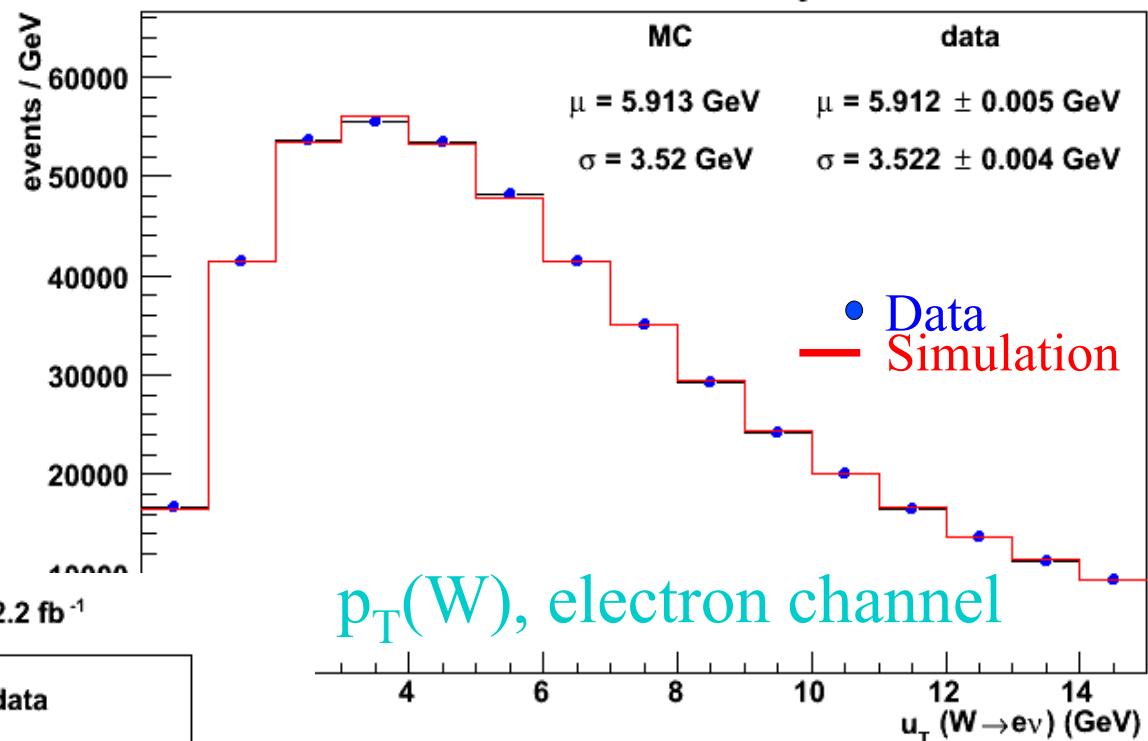
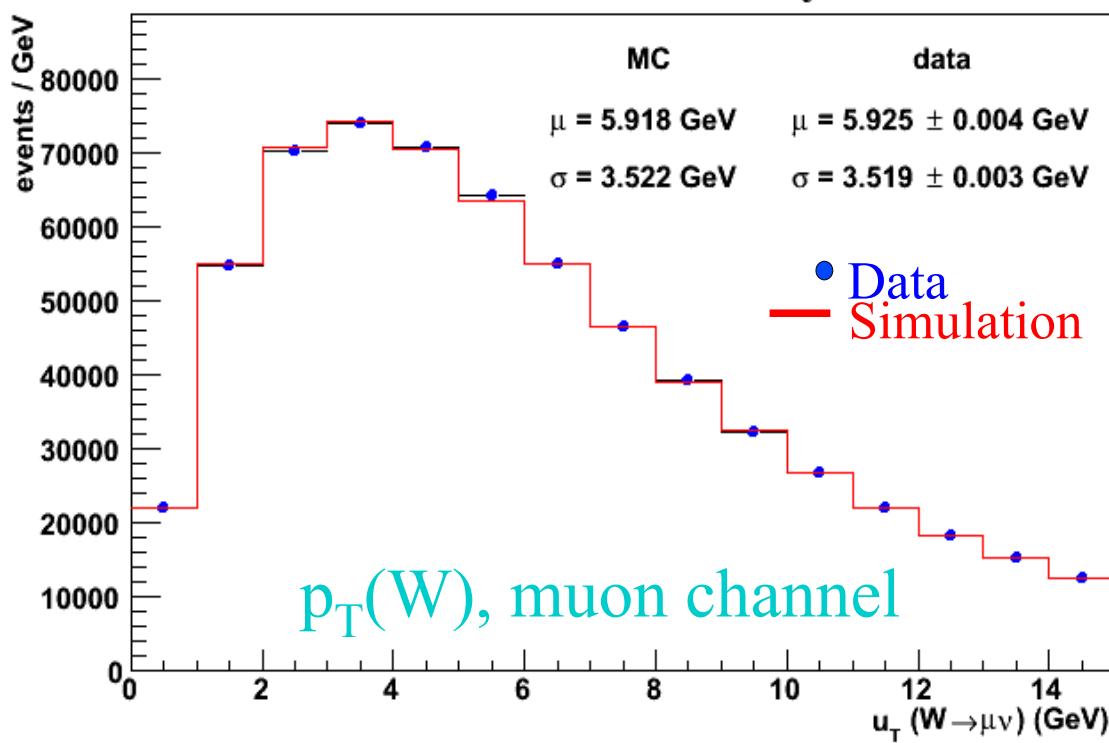
Recoil projection (GeV) on lepton direction



Recoil projection (GeV) perpendicular to lepton

# Testing Hadronic Recoil Model with $W$ events

Recoil model validation  
plots confirm the consistency  
of the model



# Parton Distribution Functions

- Affect W kinematic lineshapes through acceptance cuts
- In the rest frame,  $p_T = m \sin \theta^* / 2$
- Longitudinal cuts on lepton in the lab frame sculpt the distribution of  $\theta^*$ , hence biases the distribution of lepton  $p_T$ 
  - Relationship between lab frame and rest frame depends on the boost of the W boson along the beam axis
- Parton distribution functions control the longitudinal boost
- Uncertainty due to parton distribution functions evaluated by fitting pseudo-experiments (simulated samples with the same statistics and selection as data) with varied parton distribution functions
  - Current uncertainty 10 MeV
  - Largest source of systematic uncertainty
  - Expected to reduce with lepton and boson rapidity measurements at Tevatron and LHC

# W Mass Fits

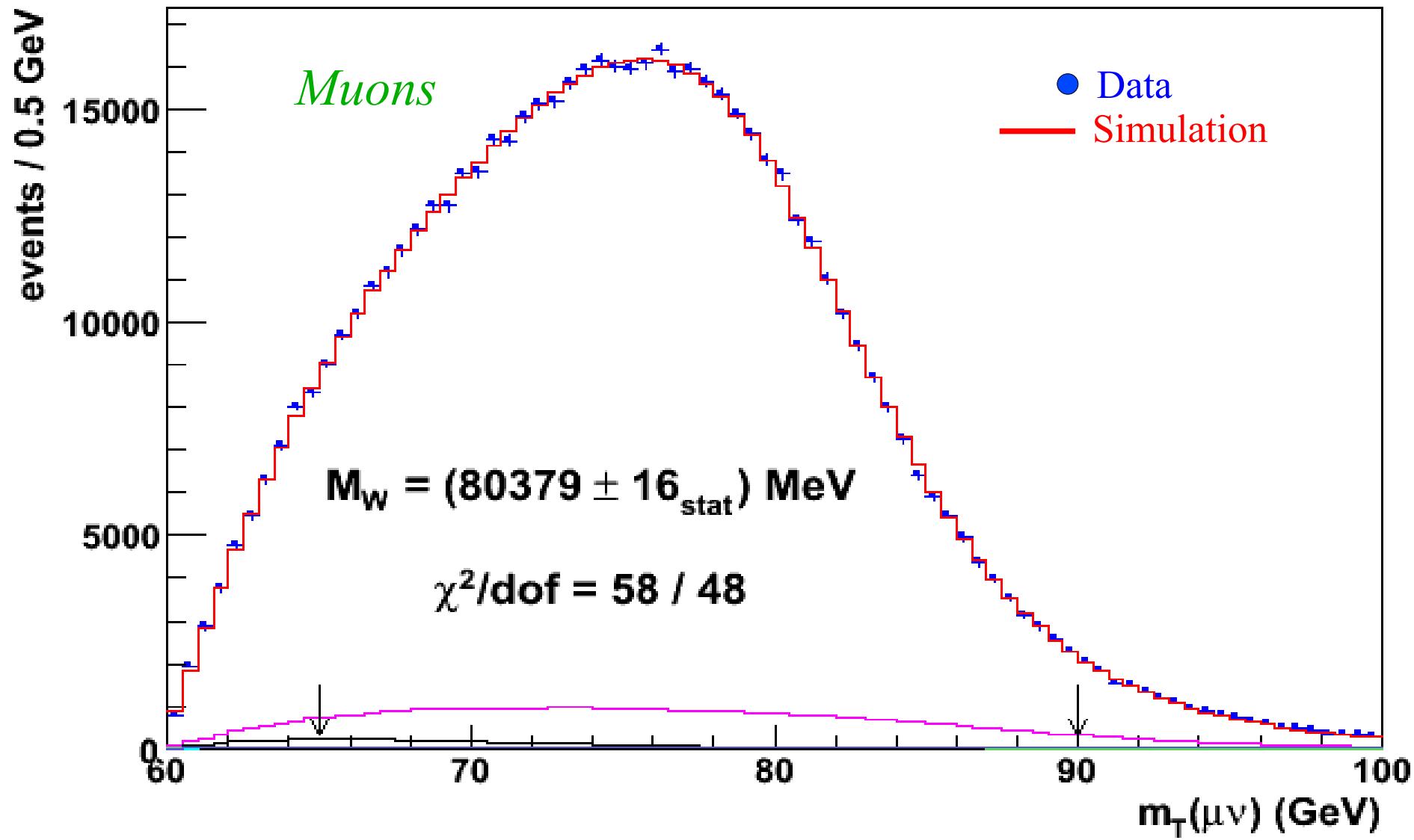
# Blind Analysis Technique

- All W and Z mass fit results were blinded with a random [-75,75] MeV offset hidden in the likelihood fitter
- Blinding offset removed after the analysis was declared frozen
- Technique allows to study all aspects of data while keeping Z mass and W mass result unknown within 75 MeV

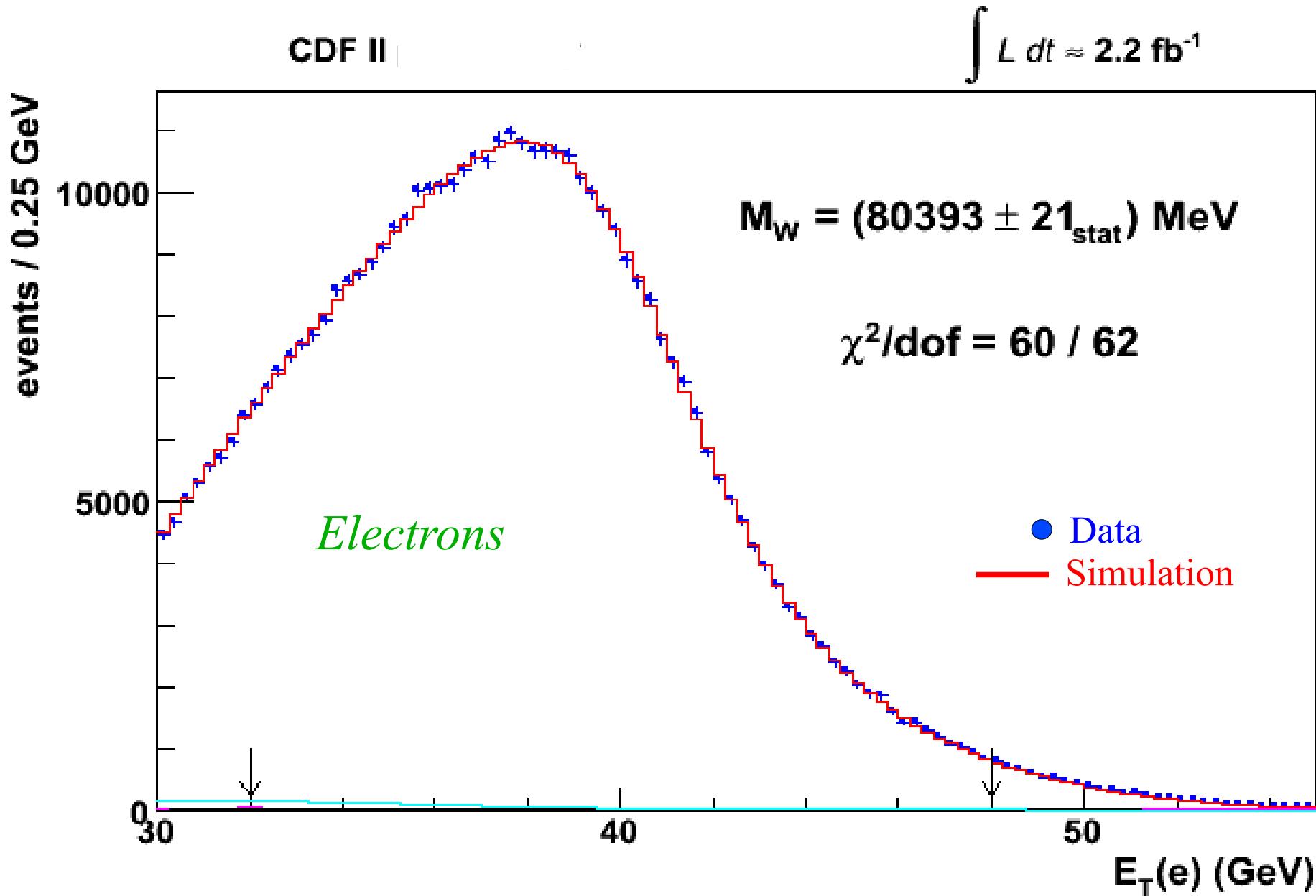
# $W$ Transverse Mass Fit

CDF II

$\int L dt = 2.2 \text{ fb}^{-1}$

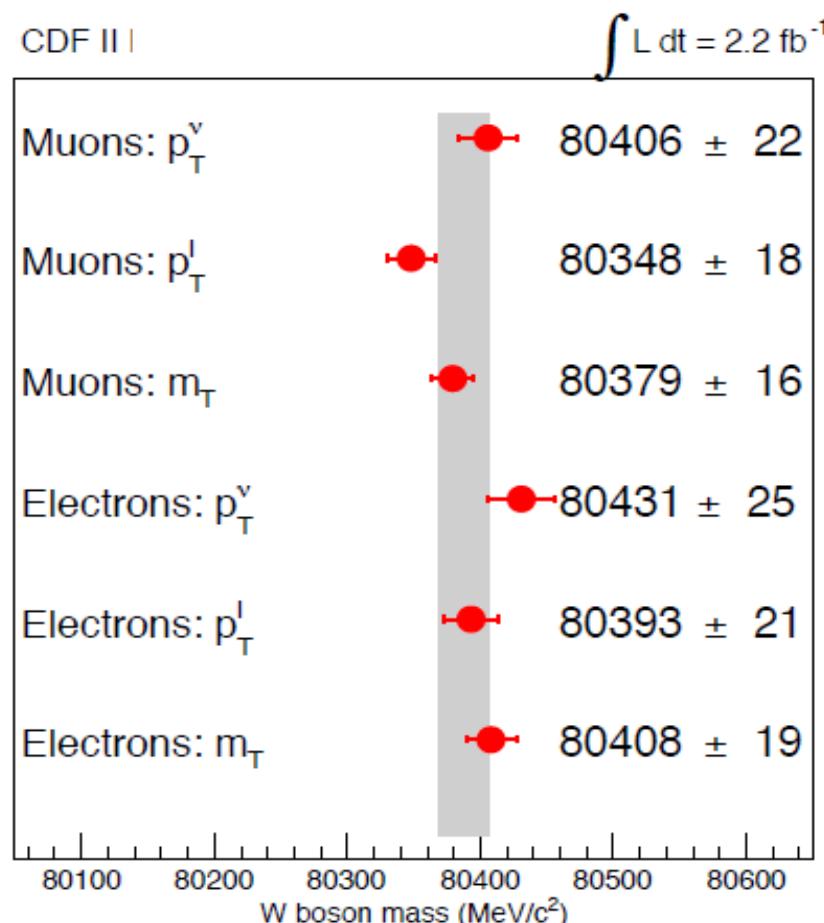


# $W$ Mass Fit using Lepton $p_T$



# Summary of $W$ Mass Fits

Charged Lepton	Kinematic Distribution	Fit Result (MeV)	$\chi^2/\text{DoF}$
Electron	Transverse mass	$80408 \pm 19$	52/48
Electron	Charged lepton $p_T$	$80393 \pm 21$	60/62
Electron	Neutrino $p_T$	$80431 \pm 25$	71/62
Muon	Transverse mass	$80379 \pm 16$	57/48
Muon	Charged lepton $p_T$	$80348 \pm 18$	58/62
Muon	Neutrino $p_T$	$80406 \pm 22$	82/62



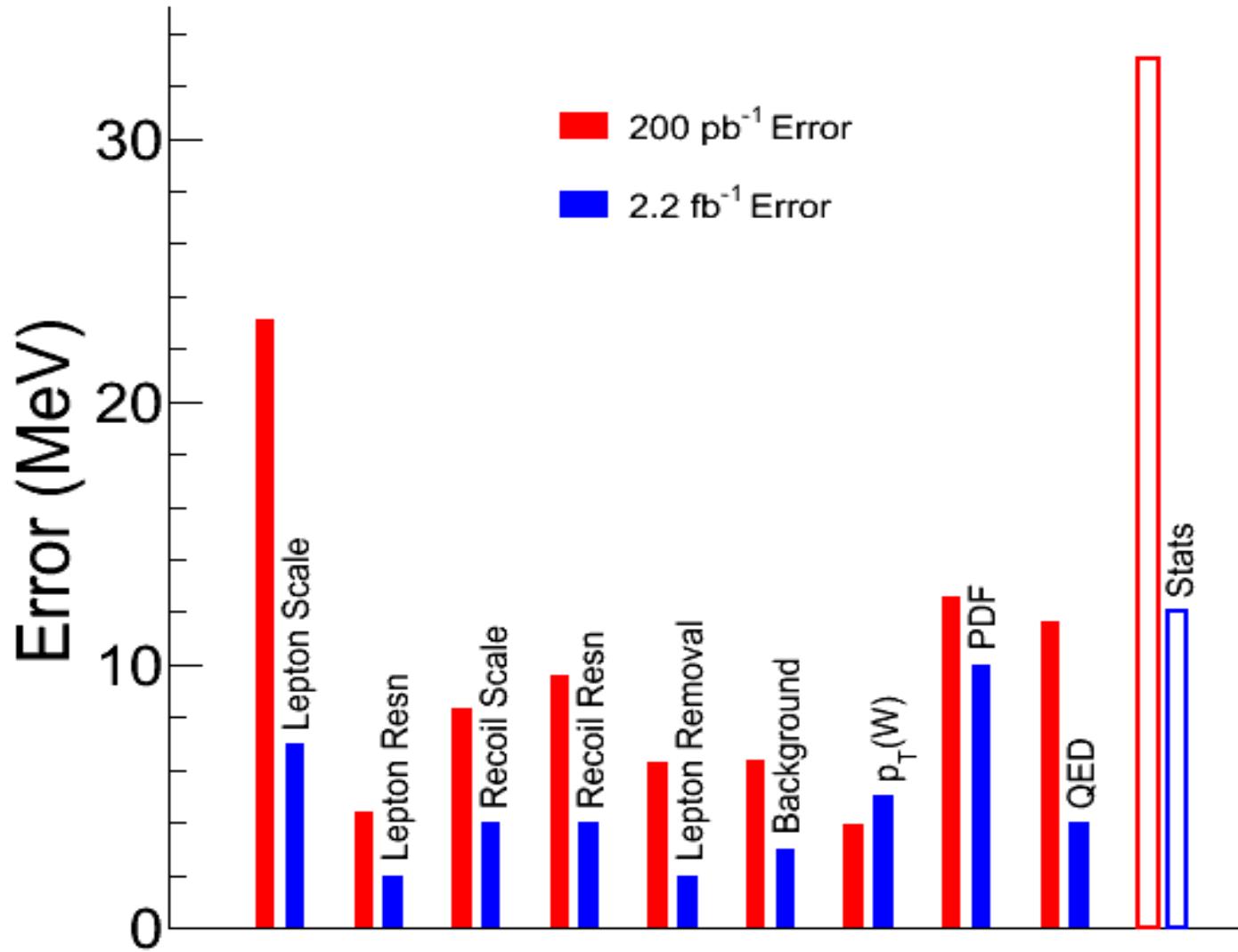
# CDF Result ( $2.2 \text{ fb}^{-1}$ )

## Transverse Mass Fit Uncertainties (MeV)

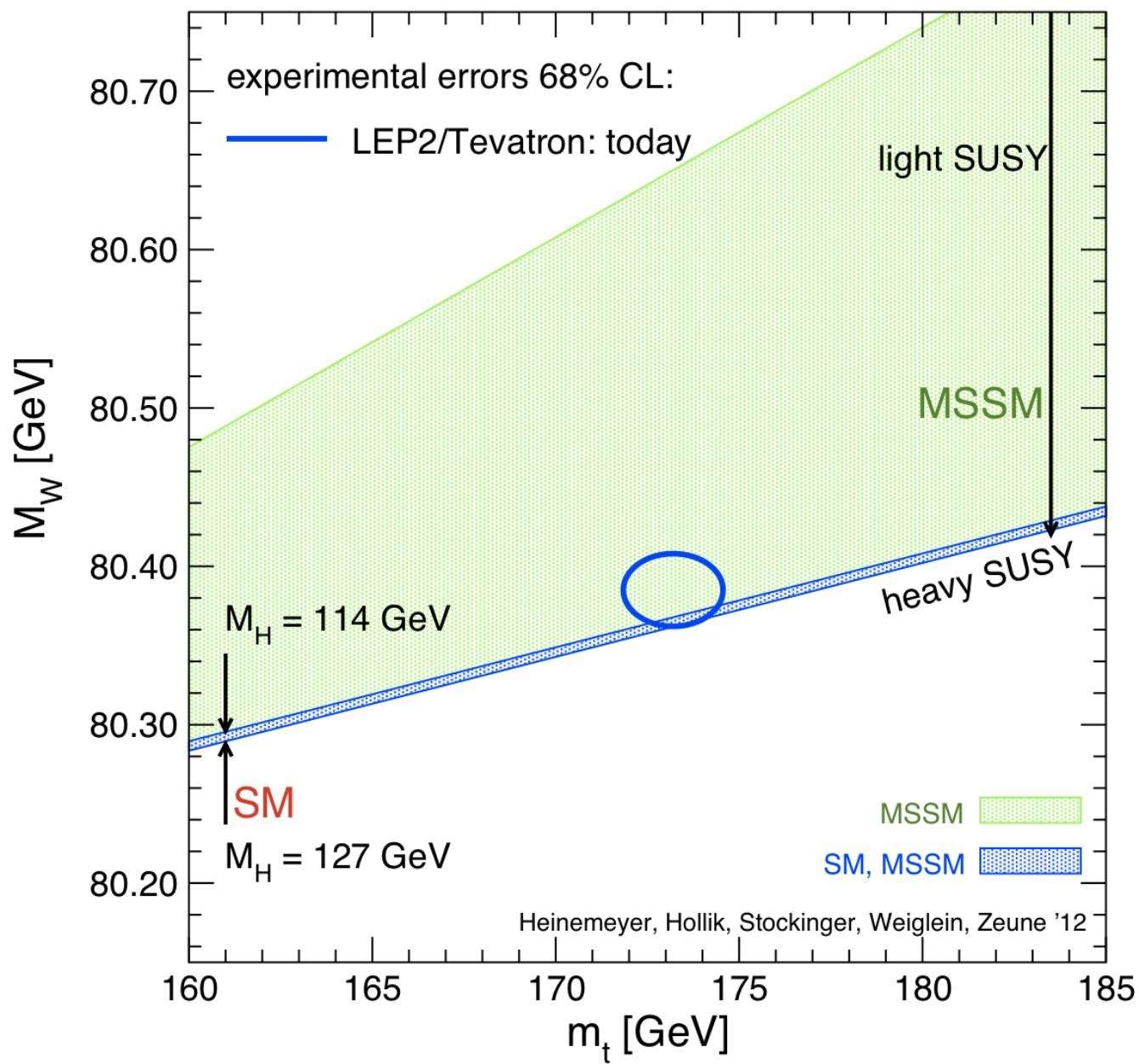
	<i>electrons</i>	<i>muons</i>	<i>common</i>
W statistics	19	16	0
Lepton energy scale	10	7	5
Lepton resolution	4	1	0
Recoil energy scale	5	5	5
Recoil energy resolution	7	7	7
Selection bias	0	0	0
Lepton removal	3	2	2
Backgrounds	4	3	0
pT(W) model	3	3	3
Parton dist. Functions	10	10	10
QED rad. Corrections	4	4	4
Total systematic	18	16	15
Total	26	23	

Systematic uncertainties shown in green: statistics-limited by control data samples

# Combined W Mass Result, Error Scaling



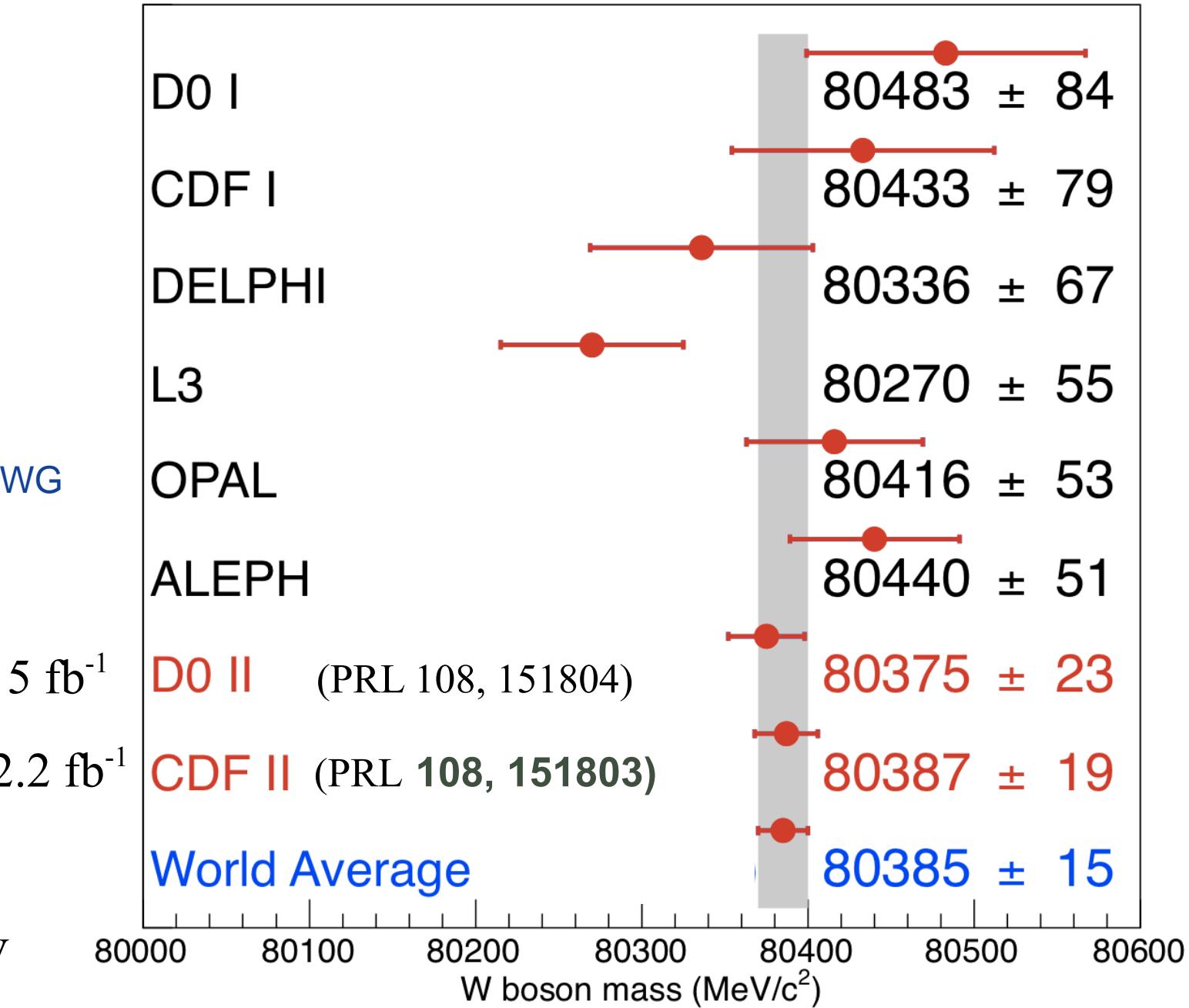
# 2012 Status of $M_W$ vs $m_t$



# W Boson Mass Measurements from Different Experiments

World average  
computed by TeVEWWG  
ArXiv: 1204.0042

Previous world  
average  
 $= 80399 \pm 23 \text{ MeV}$



# Future $M_W$ Measurements at Tevatron and LHC

- Factor of 2-5 bigger samples of W and Z bosons available at Tevatron
- Huge samples at LHC
- For most of the sources of systematic uncertainties, we have demonstrated that we can find ways to constrain them with data and scale systematic uncertainties with data statistics
- Exception is the PDF uncertainty, where we have not made a dedicated effort to constrain the PDFs within the analysis
- We need to address specific PDF degrees of freedom to answer the question:
  - Can we approach total uncertainty on  $M_W \sim 10$  MeV at the Tevatron?
- (A.V. Kotwal and J. Stark, Ann. Rev. Nucl. Part. Sci., vol. 58, Nov 2008)

# PDF Uncertainties – scope for improvement

- Newer PDF sets, *e.g.* CT10W include more recent data, such as Tevatron W charge asymmetry data
- Dominant sources of W mass uncertainty are the  $d_{\text{valence}}$  and  $\bar{d}-\bar{u}$  degrees of freedom
  - Understand consistency of data constraining these d.o.f.
  - PDF fitters increase tolerance to accommodate inconsistent datasets
- Tevatron and LHC measurements that can further constrain PDFs:
  - Z boson rapidity distribution
  - $W \rightarrow l\nu$  lepton rapidity distribution
  - W boson charge asymmetry

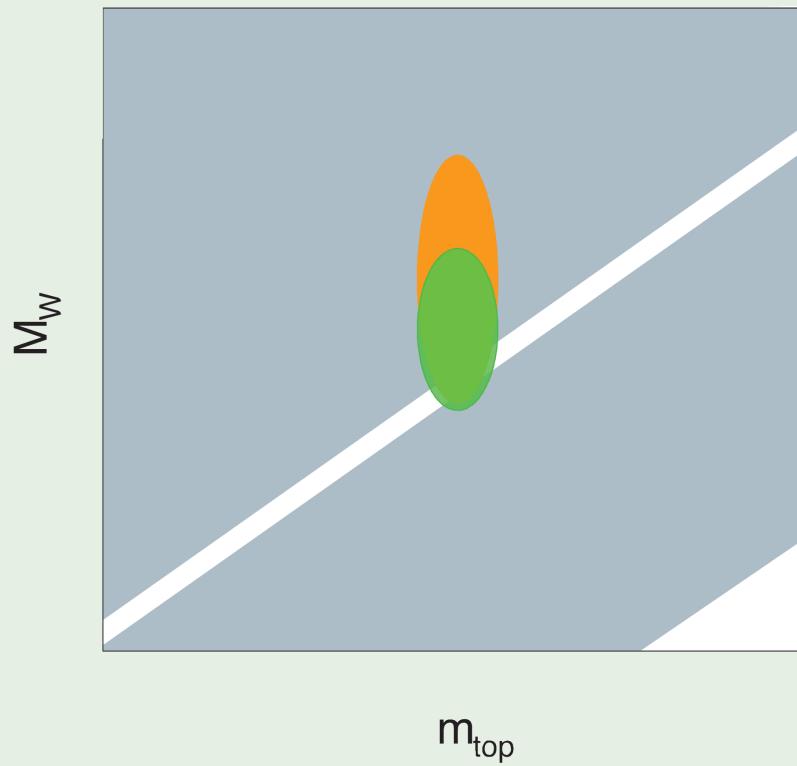
# Summary

- The W boson mass is a very interesting parameter to measure with increasing precision
- New Tevatron W mass results are very precise:
  - $M_W = 80387 \pm 19 \text{ MeV}$  (CDF)
  - =  $80375 \pm 23 \text{ MeV}$  (D0)
  - =  $80385 \pm 15 \text{ MeV}$  (world average)
- New global electroweak fit  $M_H = 94^{+29}_{-24} \text{ GeV}$  @ 68% CL (LEPEWWG)
  - SM Higgs prediction is pinned in the low-mass range
  - confront mass of new particle from direct search result  $\sim 125 \text{ GeV}$
- Looking forward to  $\Delta M_W < 10 \text{ MeV}$  from full Tevatron dataset  
goal of  $\Delta M_W < 5 \text{ MeV}$  from LHC data

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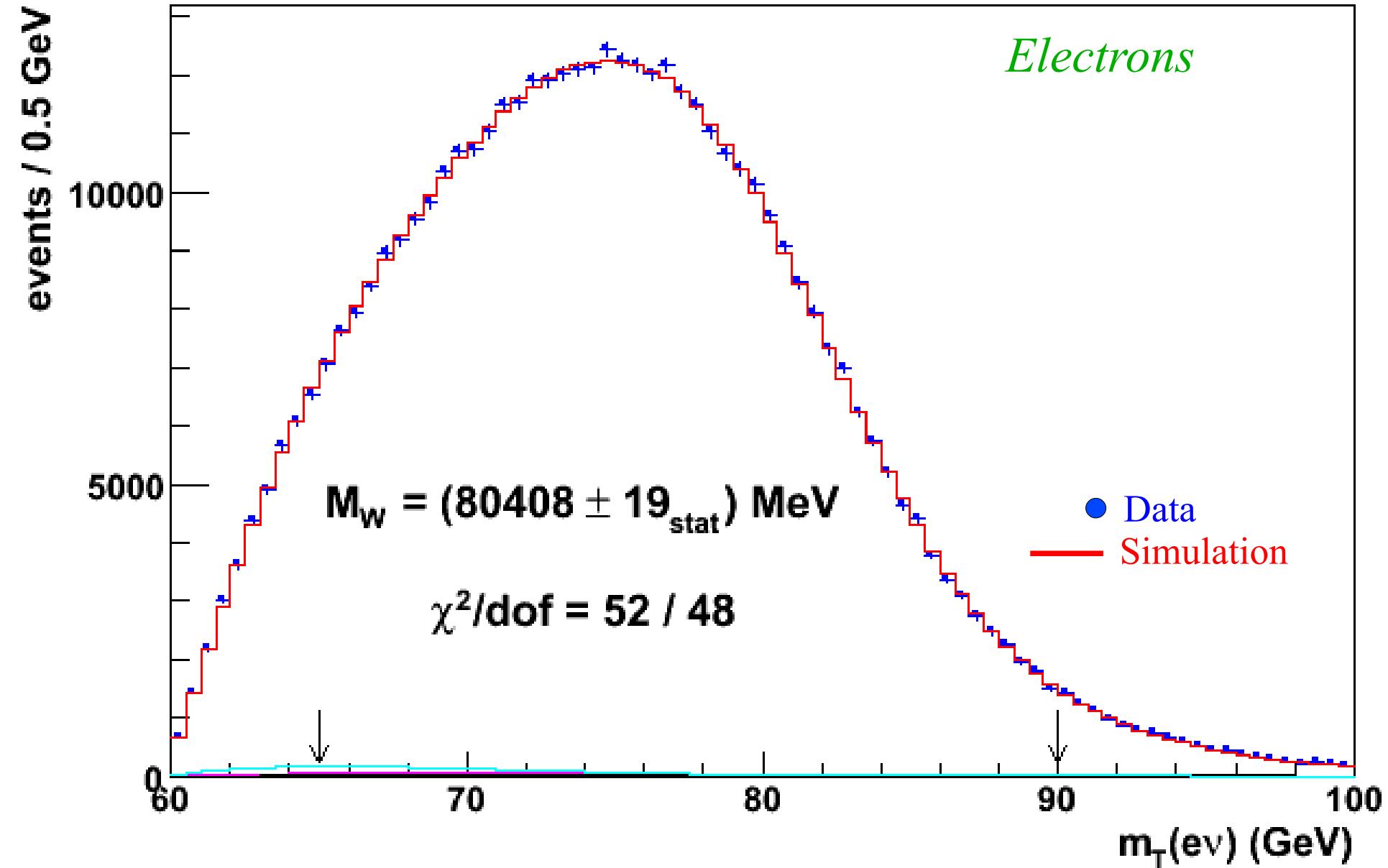


Volume 108, Number 15

# $W$ Transverse Mass Fit

CDF II

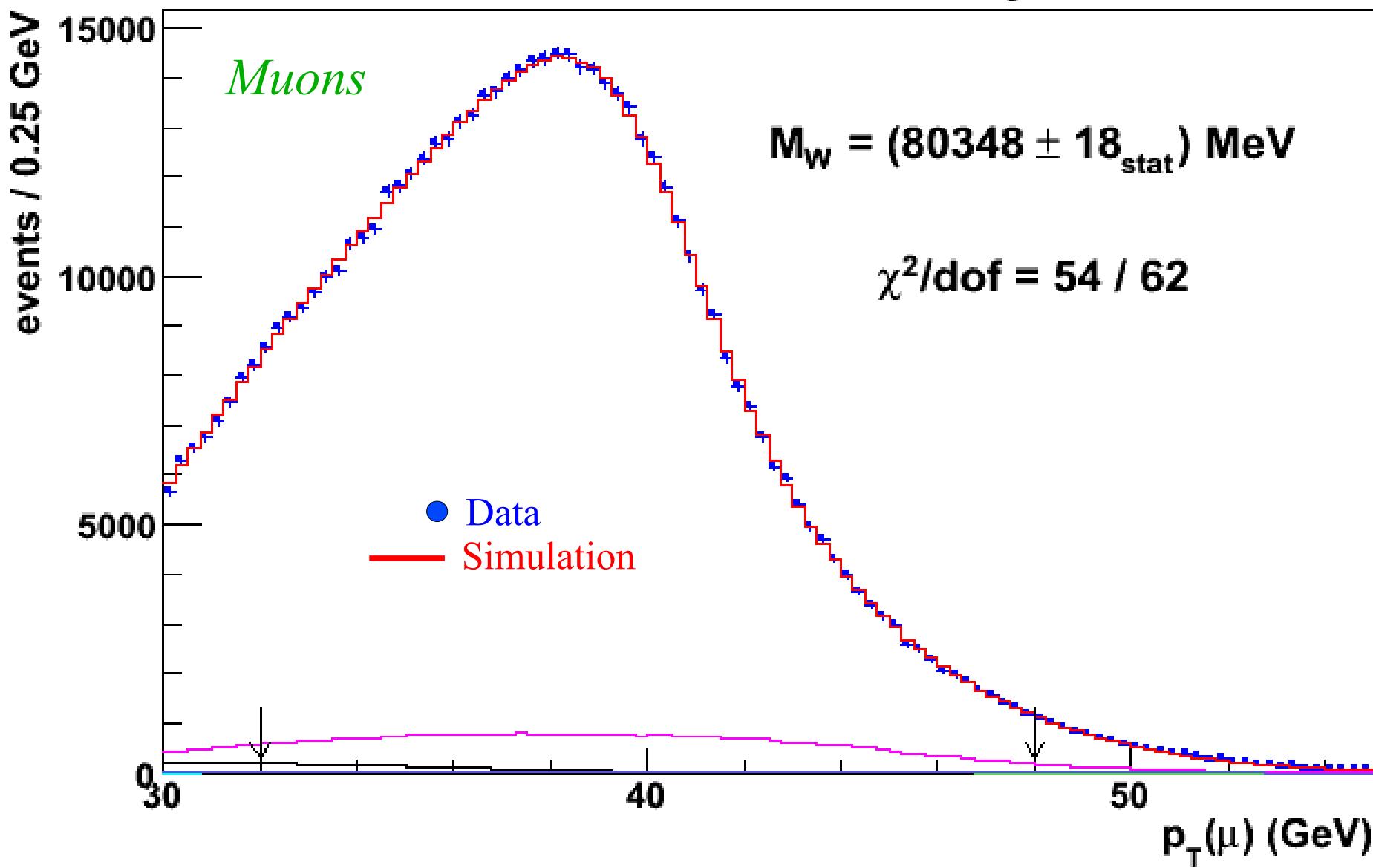
$$\int L dt \approx 2.2 \text{ fb}^{-1}$$



# *W* Lepton p<sub>T</sub> Fit

CDF II |

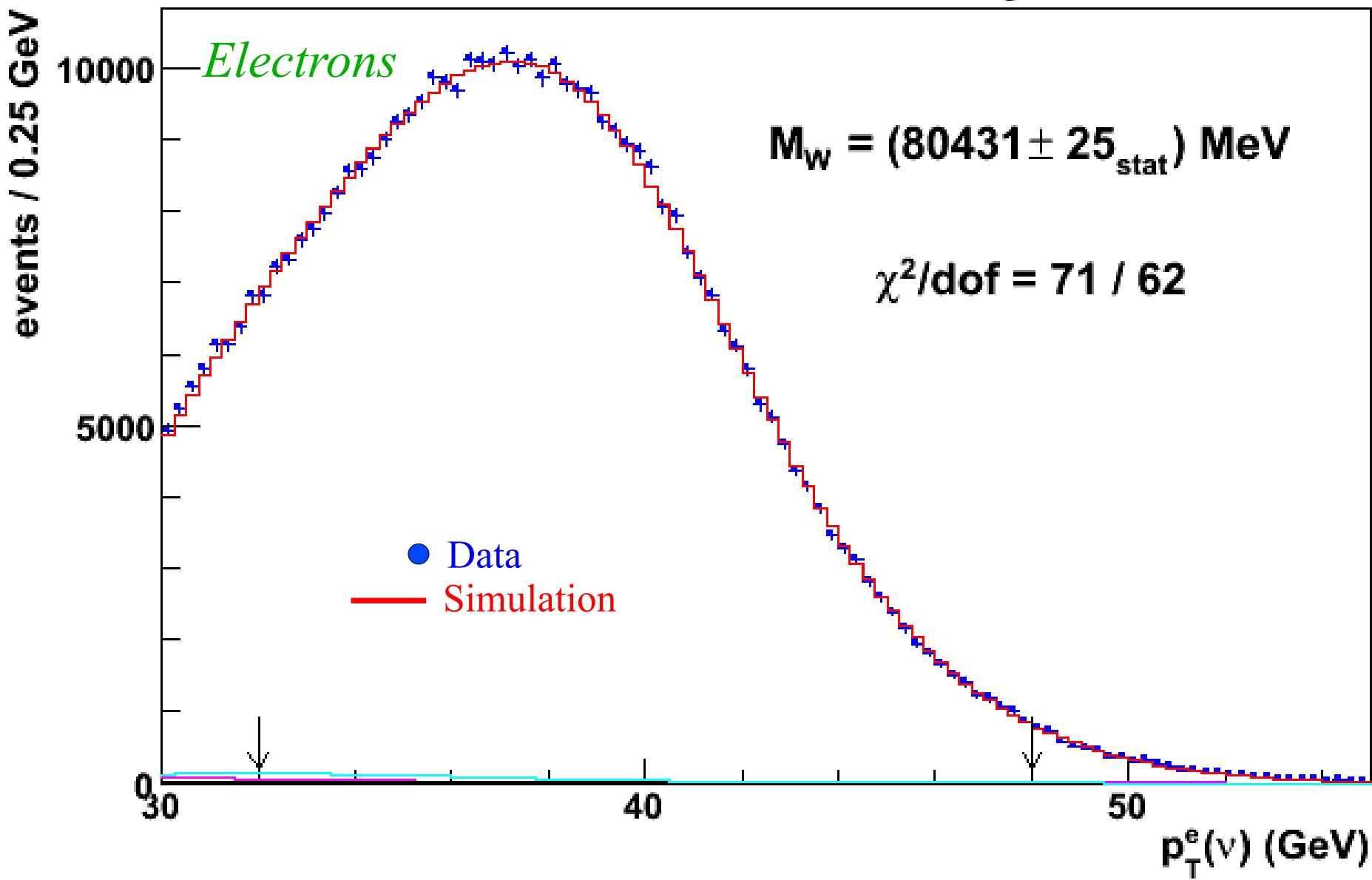
$$\int L dt = 2.2 \text{ fb}^{-1}$$



# $W$ Missing $E_T$ Fit

CDF II

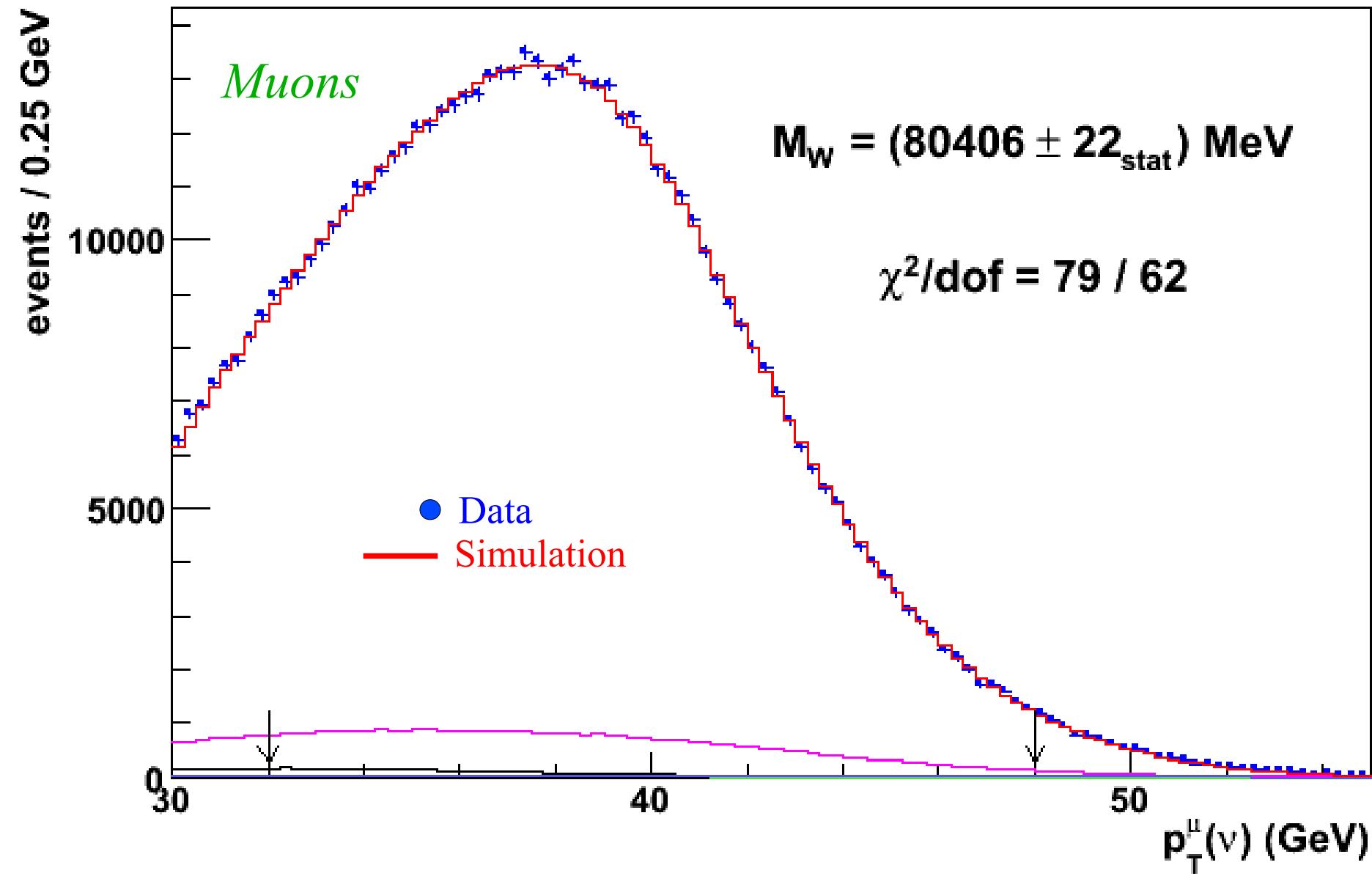
$$\int L dt = 2.2 \text{ fb}^{-1}$$



# $W$ Missing $E_T$ Fit

CDF II

$\int L dt = 2.2 \text{ fb}^{-1}$



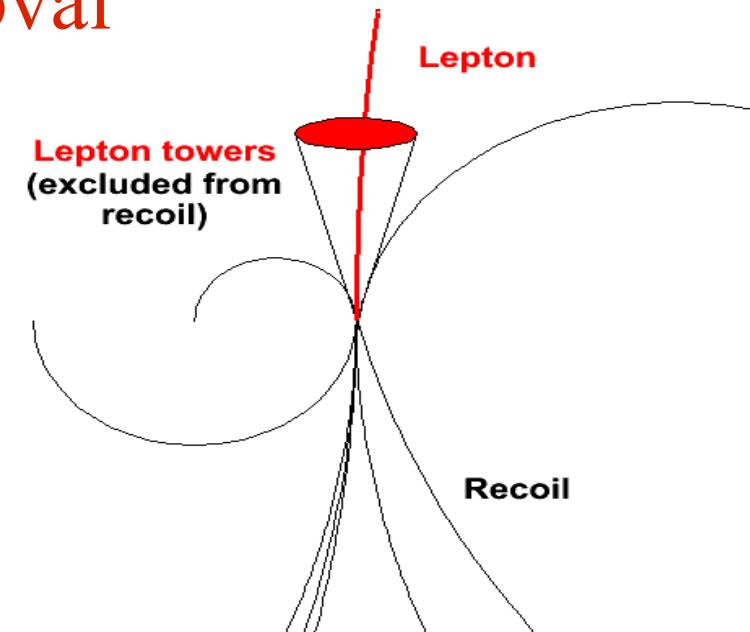
# Lepton Resolutions

- Tracking resolution parameterized in the custom simulation by
  - Radius-dependent drift chamber hit resolution  $\sigma_h \sim (150 \pm 1_{\text{stat}}) \mu\text{m}$
  - Beamspot size  $\sigma_b = (35 \pm 1_{\text{stat}}) \mu\text{m}$
  - Tuned on the widths of the  $Z \rightarrow \mu\mu$  (beam-constrained) and  $\Upsilon \rightarrow \mu\mu$  (both beam constrained and non-beam constrained) mass peaks
    - $\Rightarrow \Delta M_W = 1 \text{ MeV} (\text{muons})$
- Electron cluster resolution parameterized in the custom simulation by
  - $12.6\% / \sqrt{E_T}$  (sampling term)
  - Primary constant term  $\kappa = (0.68 \pm 0.05_{\text{stat}}) \%$
  - Secondary photon resolution  $\kappa_\gamma = (7.4 \pm 1.8_{\text{stat}}) \%$
  - Tuned on the widths of the  $E/p$  peak and the  $Z \rightarrow ee$  peak (selecting radiative electrons)
    - $\Rightarrow \Delta M_W = 4 \text{ MeV} (\text{electrons})$

# Lepton Tower Removal

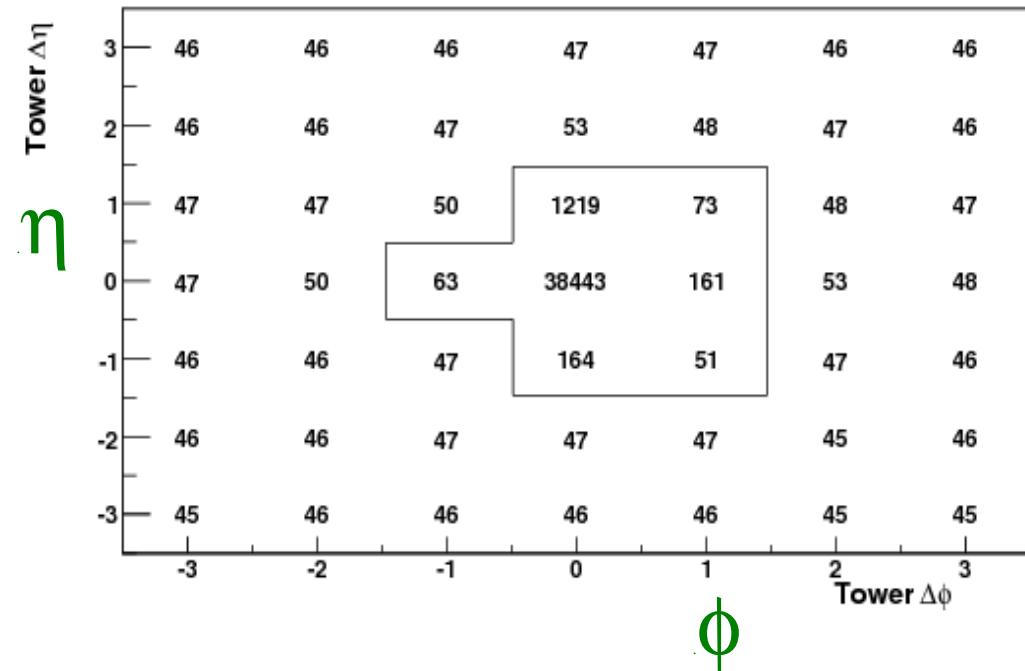
- We remove the calorimeter towers containing lepton energy from the hadronic recoil calculation
  - Lost underlying event energy is measured in  $\phi$ -rotated windows

$$\Delta M_W = 2 \text{ MeV}$$



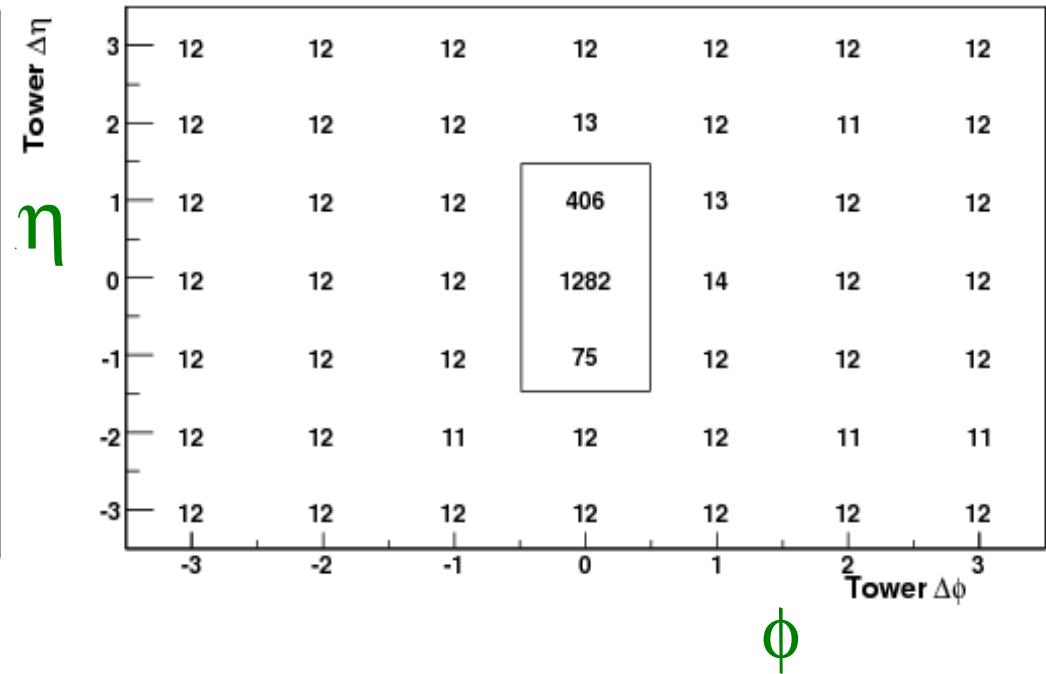
Electron channel W data

Electron Electromagnetic  $E_T$ (MeV)



Muon channel W data

Muon Hadronic  $E_T$  (MeV)



# Backgrounds in the W sample

## Muons

Background	% of $W \rightarrow \mu\nu$ data	$\delta m_W$ (MeV)		
		$m_T$ fit	$p_T^\mu$ fit	$p_T^\nu$ fit
$Z \rightarrow \mu\mu$	$7.35 \pm 0.09$	2	4	5
$W \rightarrow \tau\nu$	$0.880 \pm 0.004$	0	0	0
QCD	$0.035 \pm 0.025$	1	1	1
DIF	$0.24 \pm 0.08$	1	3	1
Cosmic rays	$0.02 \pm 0.02$	1	1	1
Total		3	5	6

## Electrons

Background	% of $W \rightarrow e\nu$ data	$\delta m_W$ (MeV)		
		$m_T$ fit	$p_T^e$ fit	$p_T^\nu$ fit
$Z \rightarrow ee$	$0.139 \pm 0.014$	1	2	1
$W \rightarrow \tau\nu$	$0.93 \pm 0.01$	1	1	1
QCD	$0.39 \pm 0.14$	4	2	4
Total		4	3	4

Backgrounds are small (except  $Z \rightarrow \mu\mu$  with a forward muon)

## *W* Mass Fit Results

- Electron and muon  $m_T$  fits combined  
 $m_W = 80390 \pm 20 \text{ MeV}$ ,  $\chi^2/\text{dof} = 1.2/1$  (28%)
- Electron and muon  $p_T$  fits combined  
 $m_W = 80366 \pm 22 \text{ MeV}$ ,  $\chi^2/\text{dof} = 2.3/1$  (13%)
- Electron and muon MET fits combined  
 $m_W = 80416 \pm 25 \text{ MeV}$ ,  $\chi^2/\text{dof} = 0.5/1$  (49%)
- All electron fits combined  
 $m_W = 80406 \pm 25 \text{ MeV}$ ,  $\chi^2/\text{dof} = 1.4/2$  (49%)
- All muon fits combined  
 $m_W = 80374 \pm 22 \text{ MeV}$ ,  $\chi^2/\text{dof} = 4/2$  (12%)
- All fits combined  
 $m_W = 80387 \pm 19 \text{ MeV}$ ,  $\chi^2/\text{dof} = 6.6/5$  (25%)

# $p_T(l)$ Fit Systematic Uncertainties

Systematic (MeV/c <sup>2</sup> )	Electrons	Muons	Common
Lepton Energy Scale	10	7	5
Lepton Energy Resolution	4	1	0
Recoil Energy Scale	6	6	6
Recoil Energy Resolution	5	5	5
$u_{  }$ efficiency	2	1	0
Lepton Removal	0	0	0
Backgrounds	3	5	0
$p_T(W)$ model	9	9	9
Parton Distributions	9	9	9
QED radiation	4	4	4
Total	19	18	16