AN INTRODUCTION TO QCD

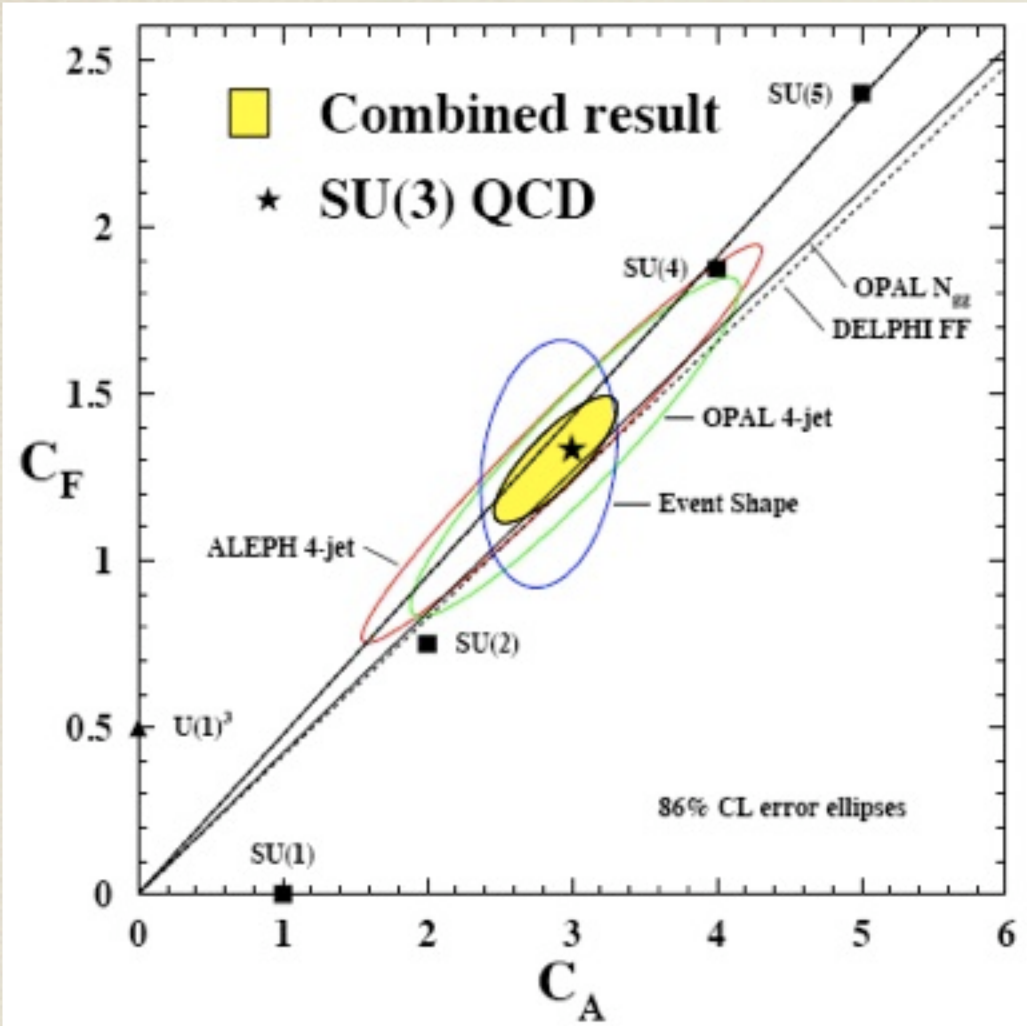
Frank Petriello
Northwestern U. & ANL

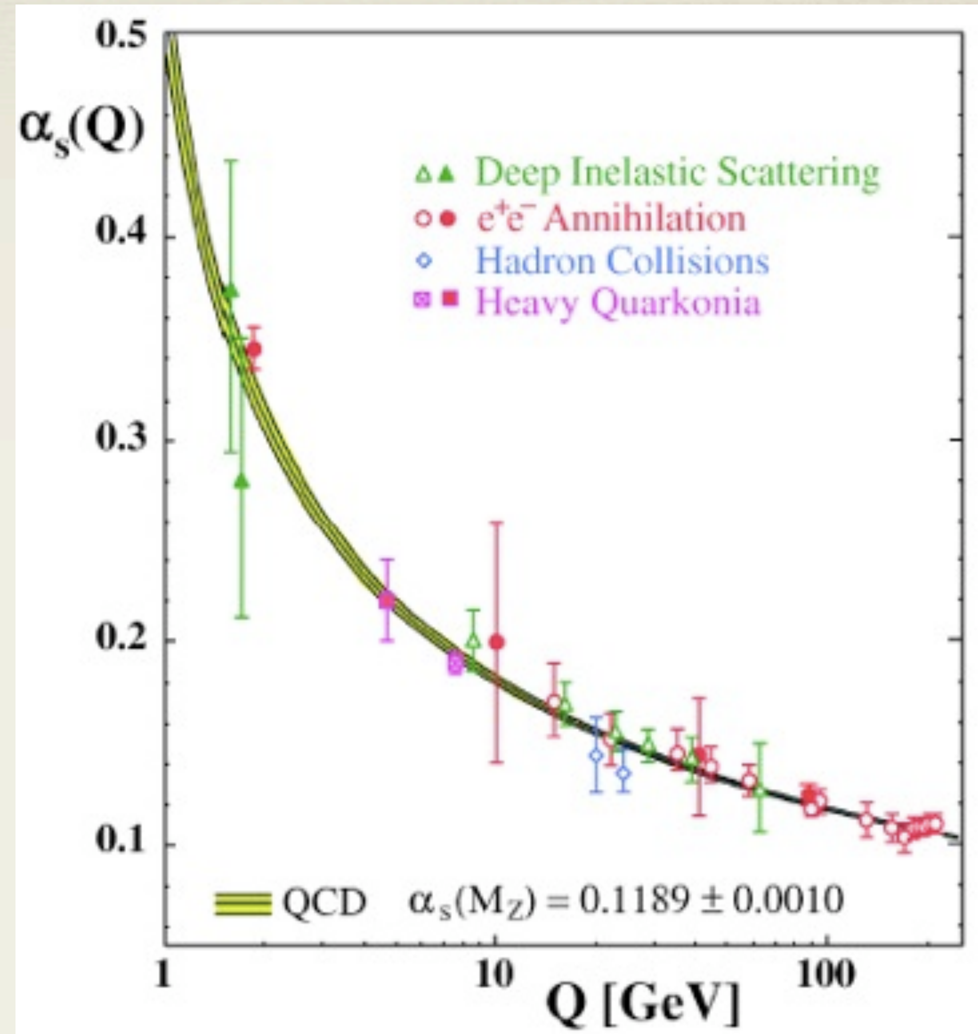
Hadron Collider Physics Summer School
August 6-17, 2012

Outline

- We'll begin with motivation for the continued study of QCD, especially in the ongoing LHC era
- Framework for QCD at colliders: the basic framework, asymptotic freedom and confinement, factorization and universality
- Learning by doing: the lectures will be structured around three examples that illustrate the important features of QCD
- Example #1: $e^+e^- \rightarrow$ hadrons at NLO; infrared singularities; scale dependence; jets
- Example #2: deep-inelastic scattering; initial-state collinear singularities; DGLAP evolution; PDFs and their errors
- Example #3: Higgs production in gluon fusion; why NLO corrections can be large; effective field theory
- Advanced topics (time permitting)

Status of pQCD

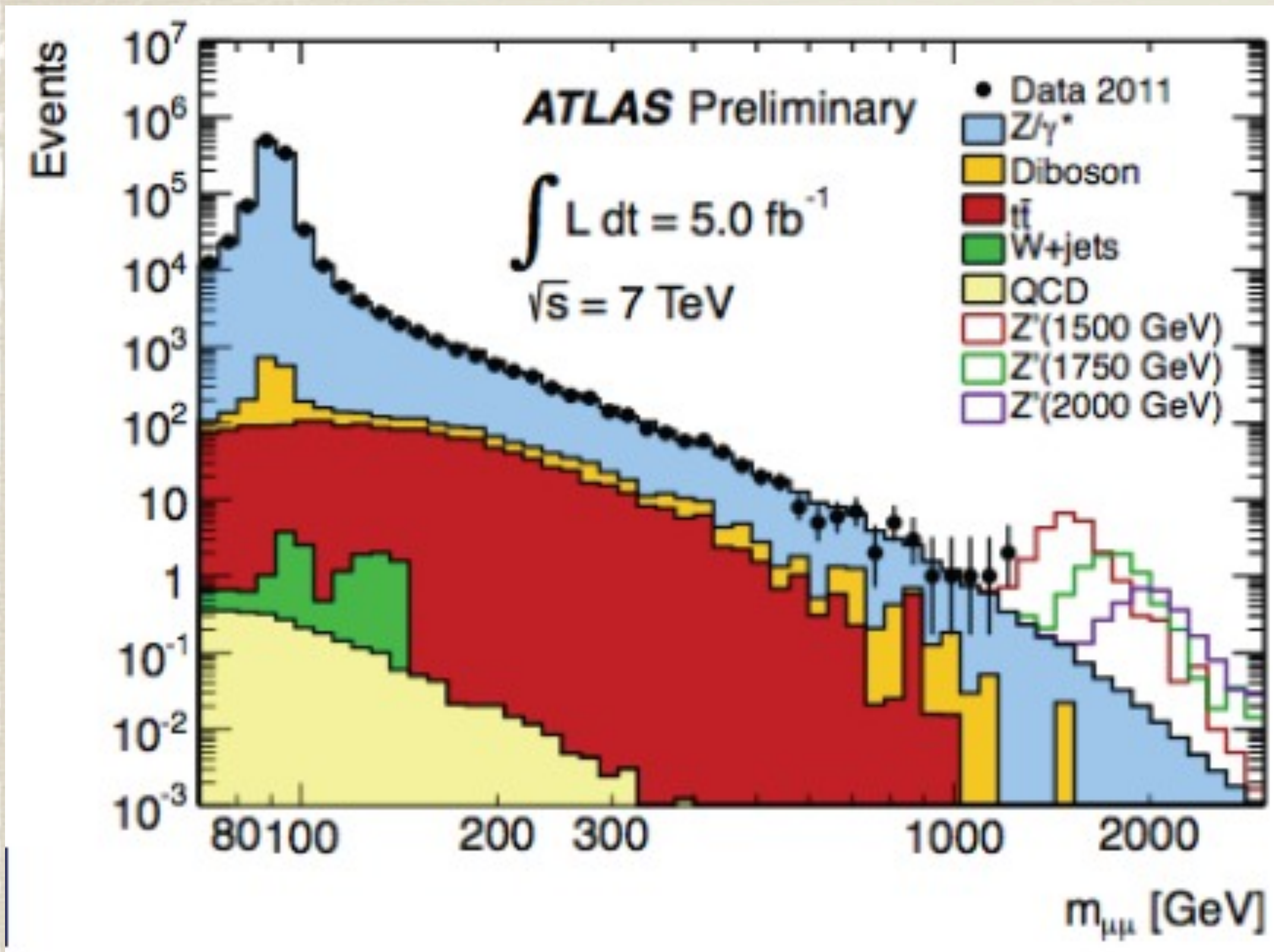


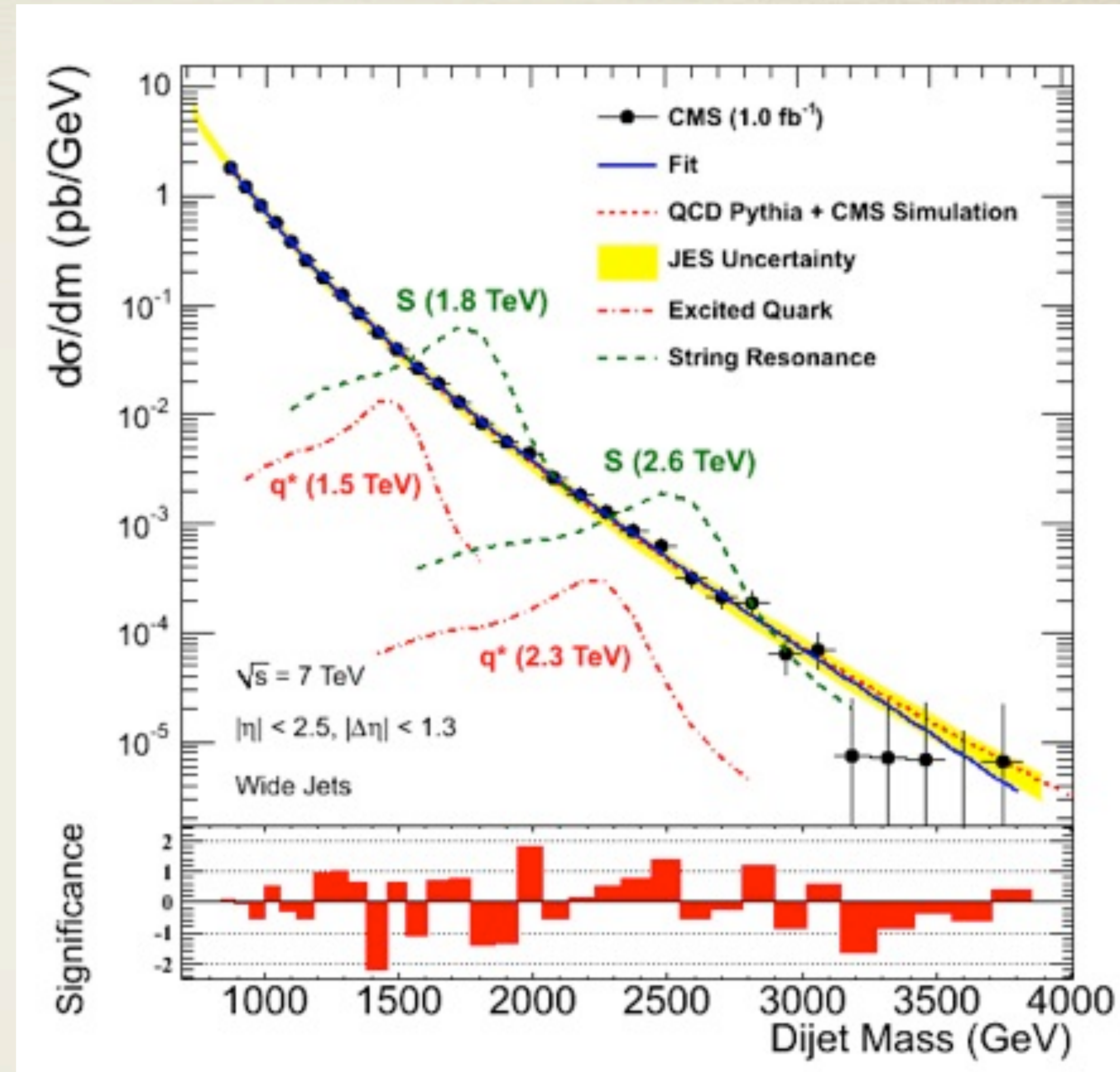


- SU(3) gauge theory of QCD established as theory of Nature
- Predicted running of α_s established in numerous experiments over several orders of magnitude
- Why do we still care about QCD?

2004: Gross, Politzer, Wilczek

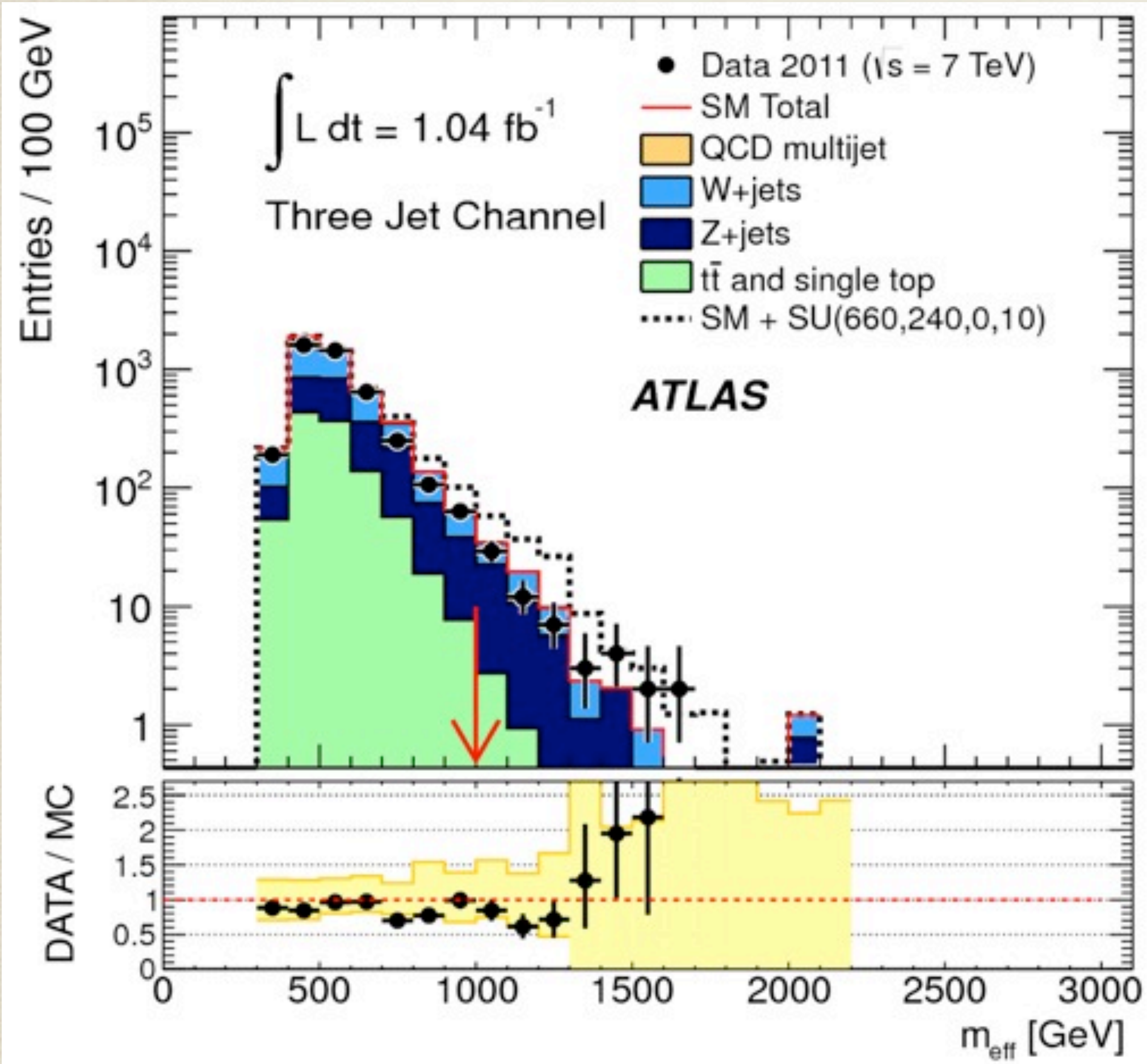
Discoveries at the LHC I





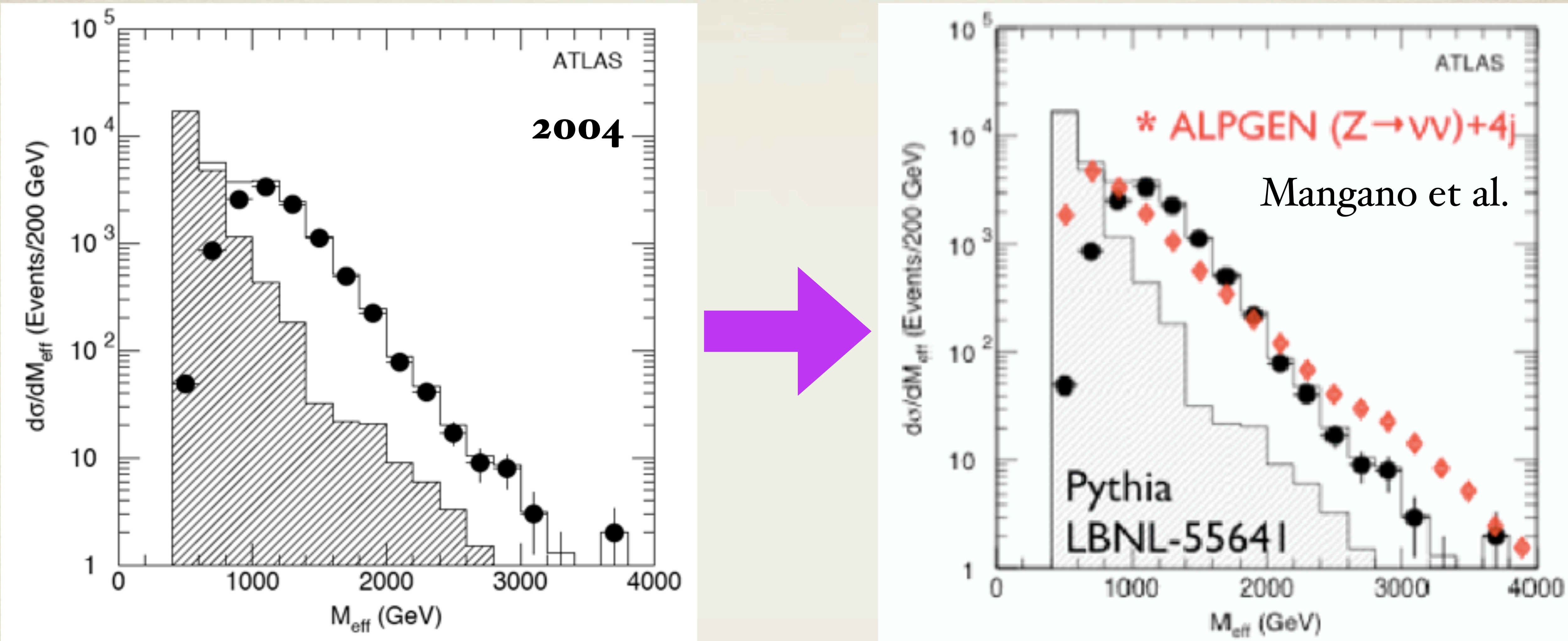
- Some discoveries at the LHC require little to no QCD input, such as resonance searches in the l^+l^- or dijet channels

Discoveries at the LHC II



- Others rely upon shape differences between signal and background
- Measure background in control region, extrapolate to signal region using theory
- Care must be taken in both choice of tool and variable used for extrapolation

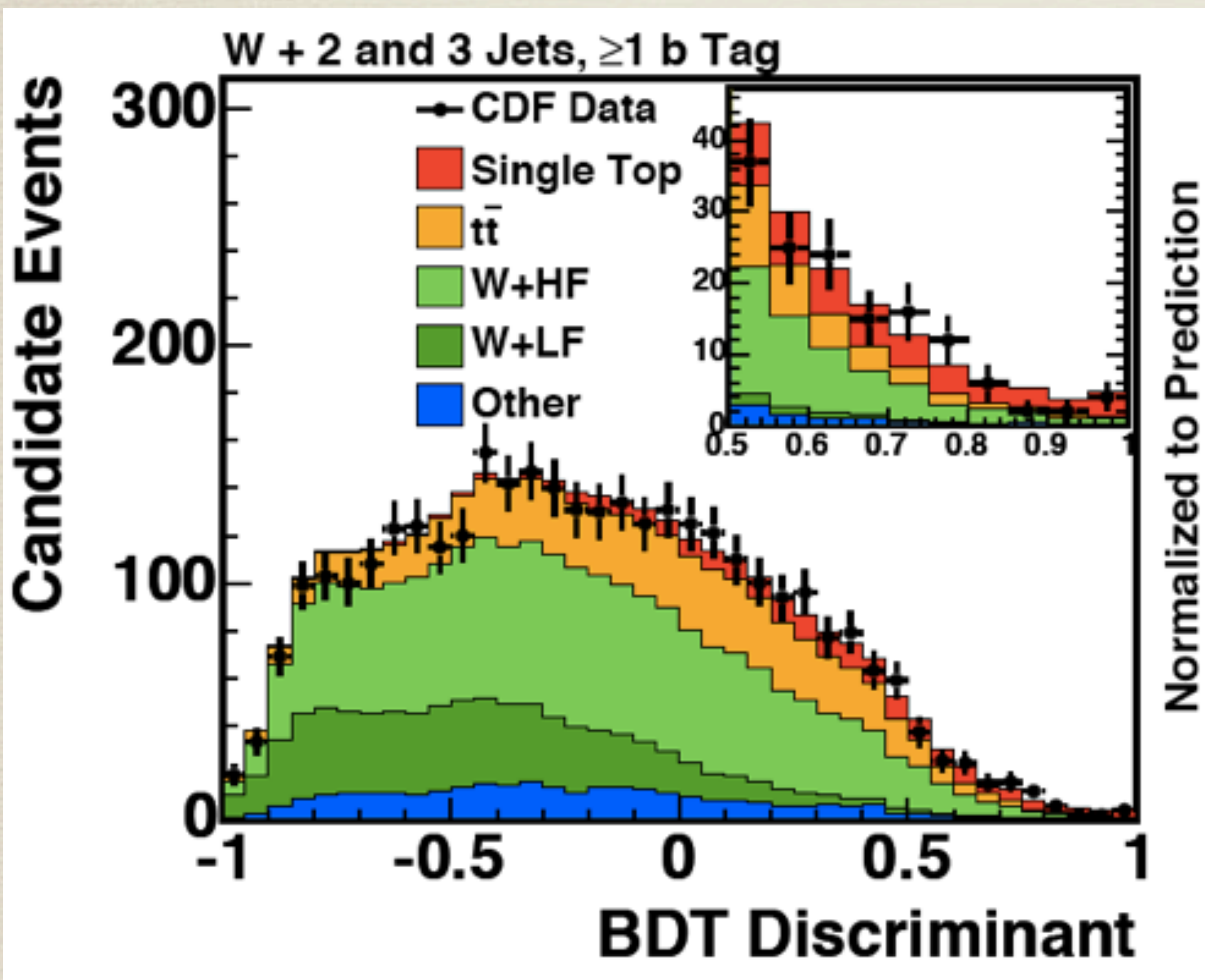
Discoveries at the LHC II

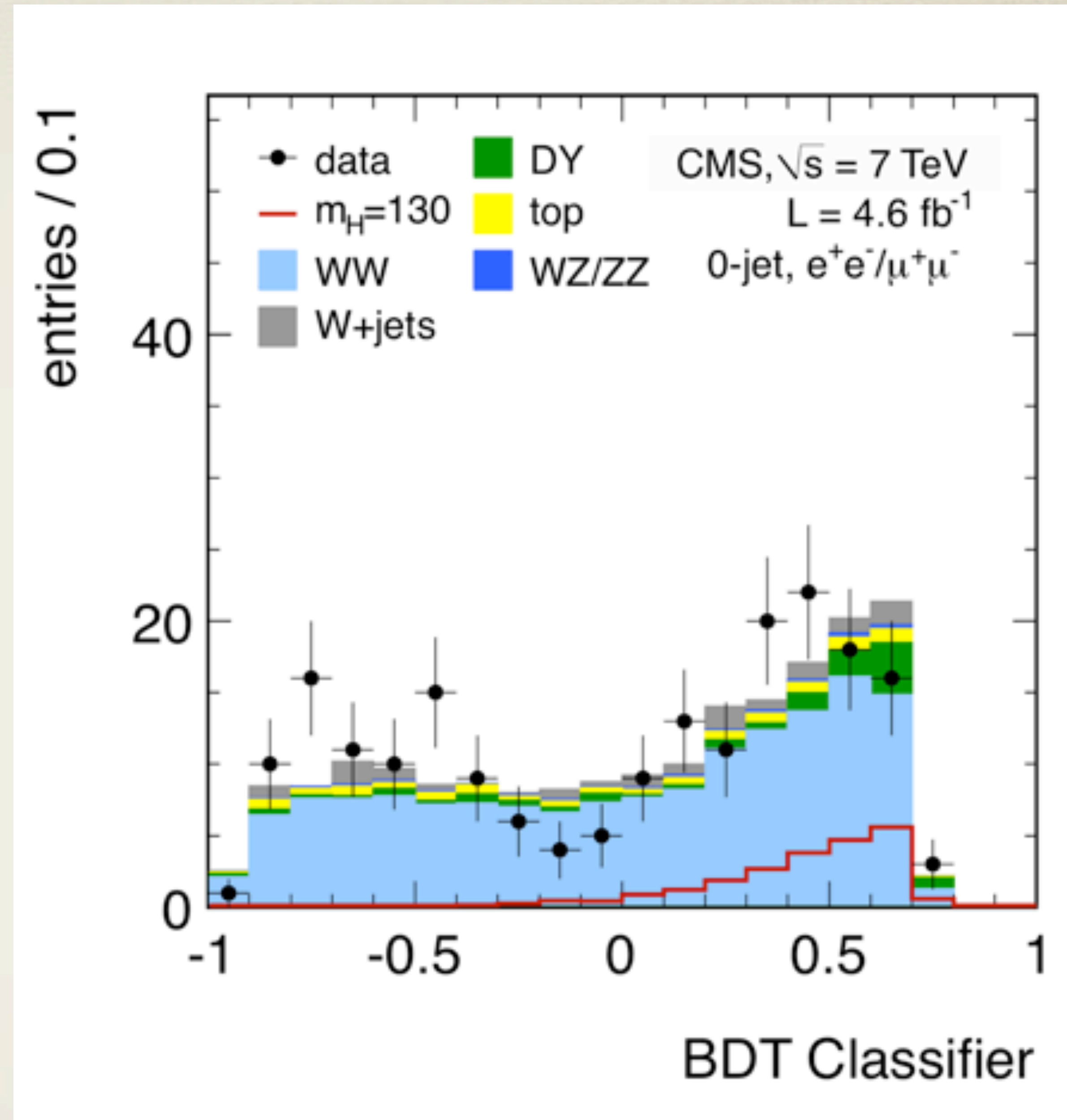


- Crucial to merge parton-shower simulations with exact multi-parton matrix elements, especially in energetic phase space regions

More in John Campbell's lectures

Discoveries at the LHC III

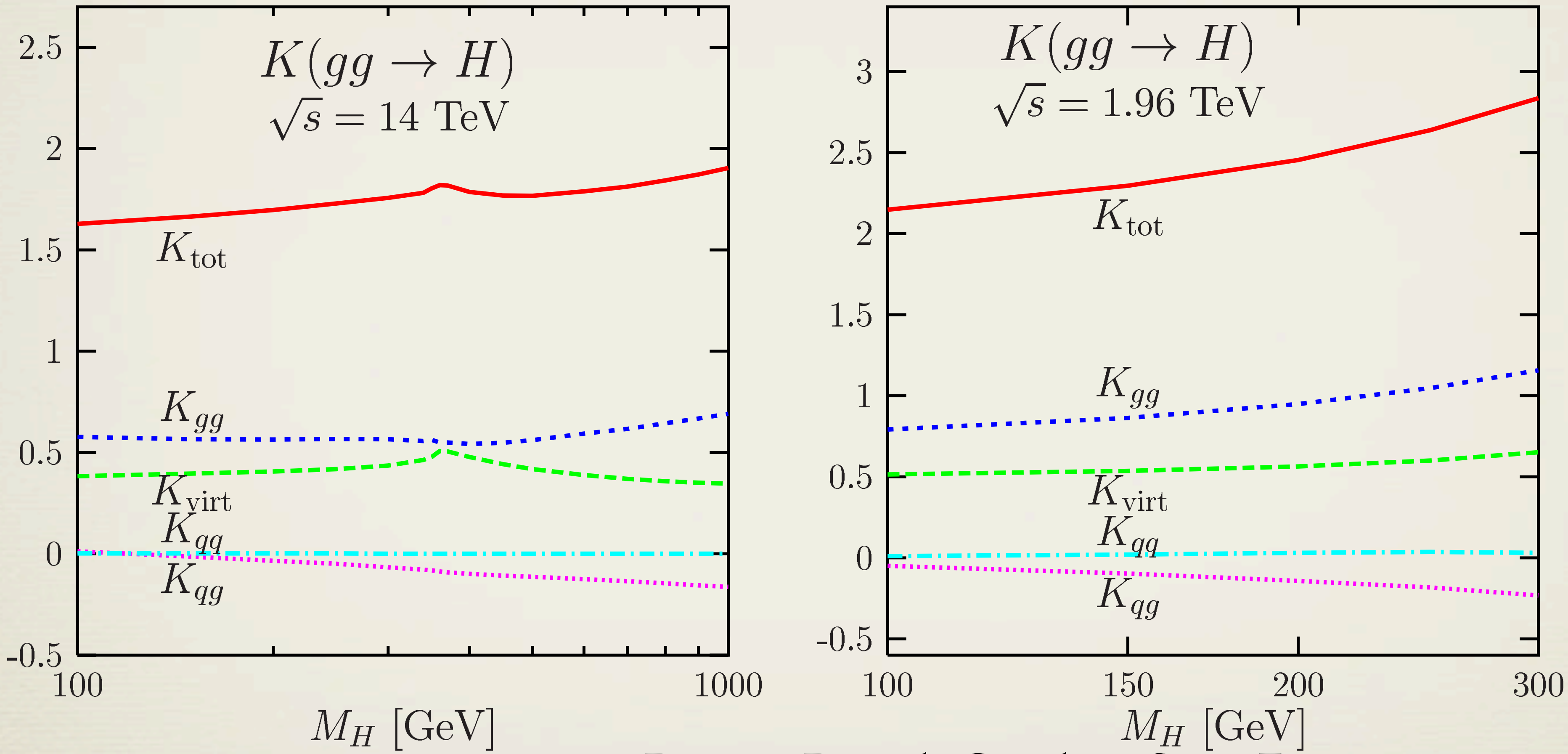




- For some searches with overwhelming backgrounds, detailed knowledge of signal and background distributions is crucial for discovery. QCD predictions become crucial

What can happen in a QCD prediction?

- Theoretical predictions for collider observables are usually made as expansions in α_s, the strong coupling constant. $\alpha_s(10^2\ \text{GeV}) \sim 0.1$

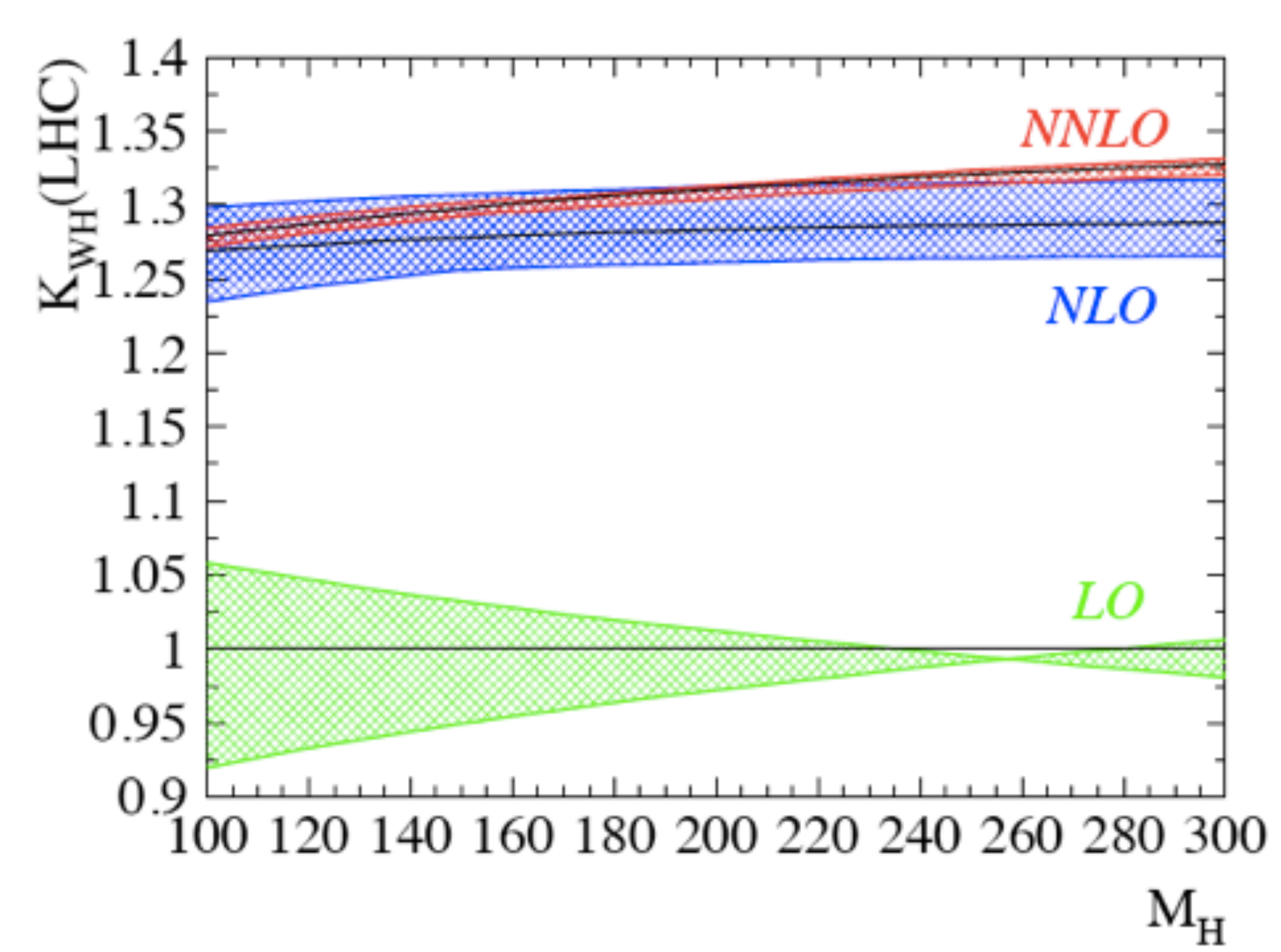


Dawson; Djouadi, Graudenz, Spira, Zerwas 1991, 1995

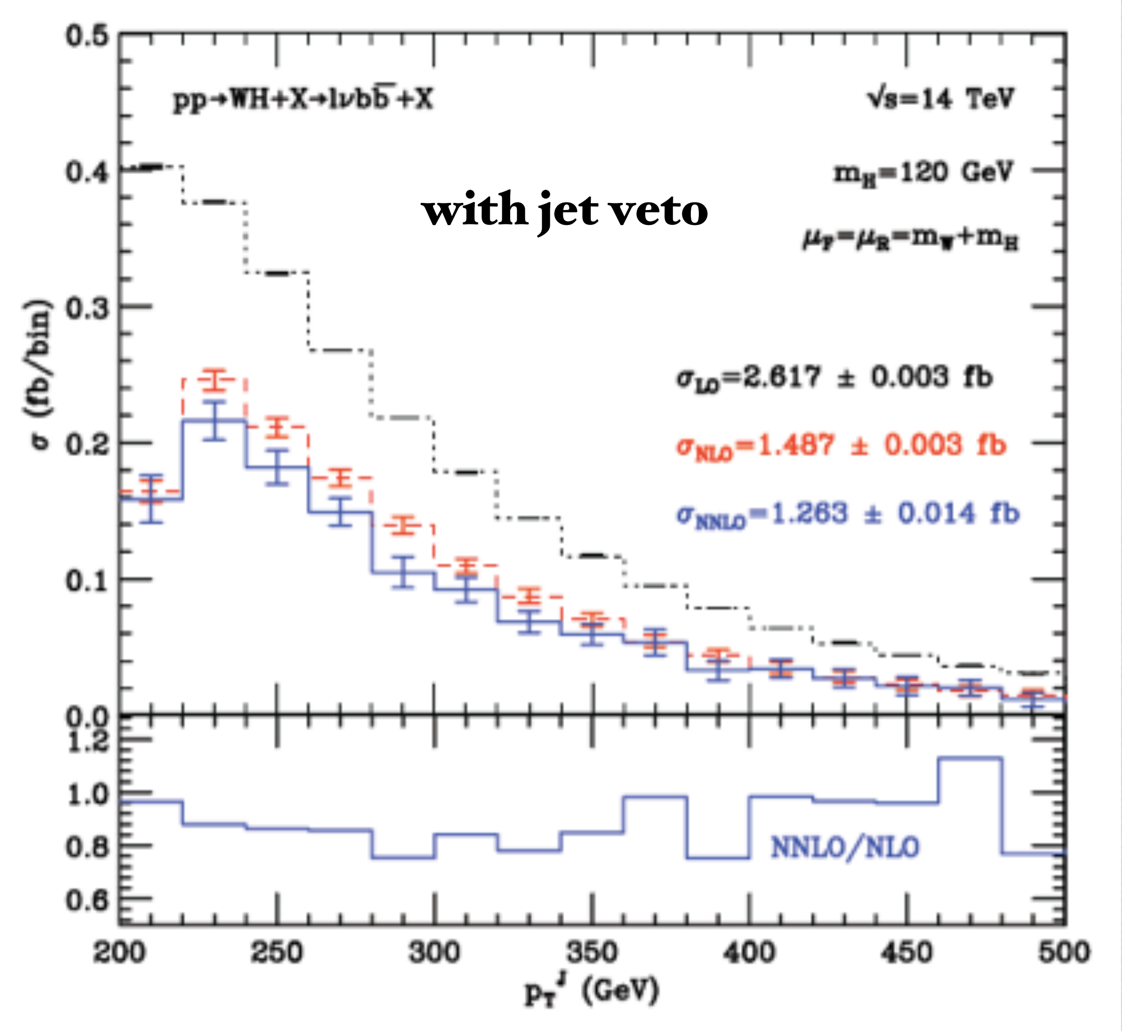
- Size of corrections can be much larger than expected

What can happen in a QCD prediction?

- Theoretical predictions for collider observables are usually made as expansions in α_s, the strong coupling constant. $\alpha_s(10^2\ \mathrm{GeV}) \sim 0.1$



Brein, Djouadi, Harlander 2003



Ferrera, Grazzini, Tramontano 2011

- Experimental cuts can dramatically change the expansion

Why study QCD?

- Many other reasons to study QCD, aesthetic (mathematical structure of scattering amplitudes in SQCD) and monetary ($\$10^6$ for proving Yang-Mills theories confine)
- But a very practical consideration that will motivate us here is that we can't make sense of LHC physics at the quantitative level without QCD beyond the leading order of perturbation theory

What is QCD?

- The birth of QCD has a long and interesting history (Gell-Mann and Zweig propose quarks; Han, Nambu, Greenberg propose color to explain the Δ^{++} baryon; SLAC deep-inelastic scattering experiments discover real quarks)
- We will just start with QCD as an SU(3) gauge theory

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4} F^A_{\alpha\beta} F^{\alpha\beta}_A + \sum_{flavors} \bar{q}_a \left(i \not{D} - m \right)_{ab} q + \mathcal{L}_{gauge} + \mathcal{L}_{ghost} \\
F^A_{\alpha\beta} &= \partial_\alpha A^A_\beta - \partial_\beta A^B_\alpha - g_s f^{ABC} A^B_\alpha A^C_\beta \\
(D_\alpha)_{ab} &= \partial_\alpha \delta_{ab} + i g_s t^C_{ab} A^C_\alpha
\end{aligned}
$$

gluon self-interactions distinguish QCD from QED

- a=1,...,3; quark in **fundamental representation**
- A=1,...,8; gluon in **adjoint representation**

Gauges and ghosts

- Like in QED, can't invert the quadratic part for the gluon to obtain the propagator. Need to add a gauge fixing term.

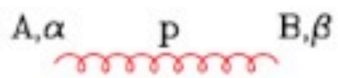
$$\mathcal{L}_{gauge} = \frac{1}{2\lambda}\left(\partial_\alpha A_\alpha^A\right)^2$$

- Unlike in QED, the resulting ghost fields interact with the gluons and can't be neglected

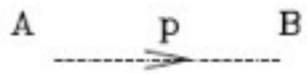
$$\mathcal{L}_{ghost} = \partial_\mu \bar{c}_a \partial^\mu c_a - g_s f^{abc} \bar{c}_a \partial^\mu \left(A_\mu^b c_c\right)$$

- Certain "physical" gauges (axial, light-like) remove the ghosts. We will use Feynman gauge, λ=1, for our calculations.

Feynman rules

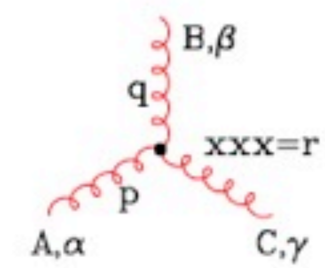


$$\delta^{AB}\left[-g^{\alpha\beta}+(1-\lambda)\frac{p^{\alpha}p^{\beta}}{p^{2}+i\epsilon}\right]\frac{i}{p^{2}+i\epsilon}$$



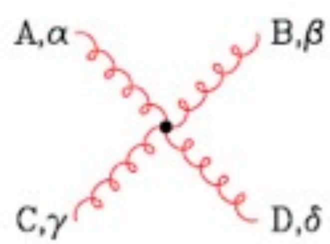
$$\delta^{AB}\frac{i}{(p^{2}+i\epsilon)}$$

$$\delta^{ab}\frac{i}{(\not{p}-m+i\epsilon)_{ji}}$$

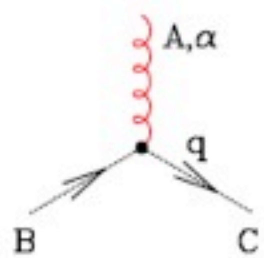


$$-g\ f^{ABC}[(p-q)^{\gamma}g^{\alpha\beta}+(q-r)^{\alpha}g^{\beta\gamma}+(r-p)^{\beta}g^{\gamma\alpha}]$$

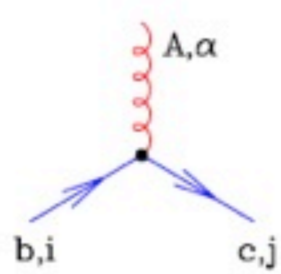
(all momenta incoming)



$$-ig^{2}\ f^{XAC}f^{XBD}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$
$$-ig^{2}\ f^{XAD}f^{XBC}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}]$$
$$-ig^{2}\ f^{XAB}f^{XCD}\ [g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$



$$g\ f^{ABC}q^{\alpha}$$

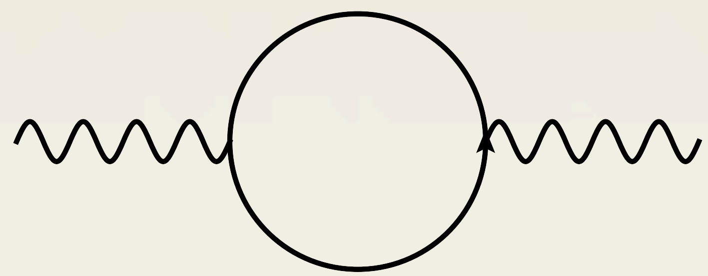


$$-ig\ (t^{A})_{cb}\ (\gamma^{\alpha})_{ji}$$

Useful reference: Ellis, Stirling, Webber, *QCD and Collider Physics*

The QED beta function

- Gluon self-couplings lead to a profound difference from QED. Consider the QED beta function (just the electron contribution).



$$Q^2 \frac{d\alpha}{dQ^2} = \beta_{QED}(\alpha), \quad \beta_{QED} = \frac{\alpha^2}{3\pi} + \mathcal{O}(\alpha^3)$$

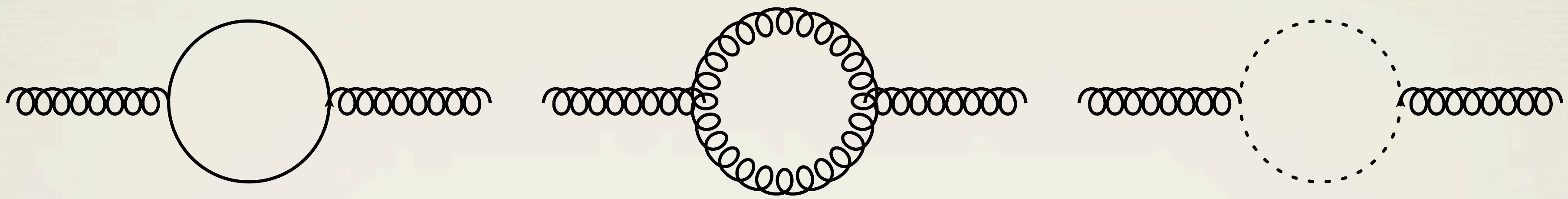
$$\alpha(Q^2) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi} \ln\left(\frac{Q^2}{m_e^2}\right)}$$

$\alpha_0 \approx 1/137$

Coupling constant grows with energy; hits a *Landau pole* when denominator vanishes. QED becomes strongly-coupled at high energies.

The QCD beta function

- Gluon self-couplings reverse the sign of the beta function



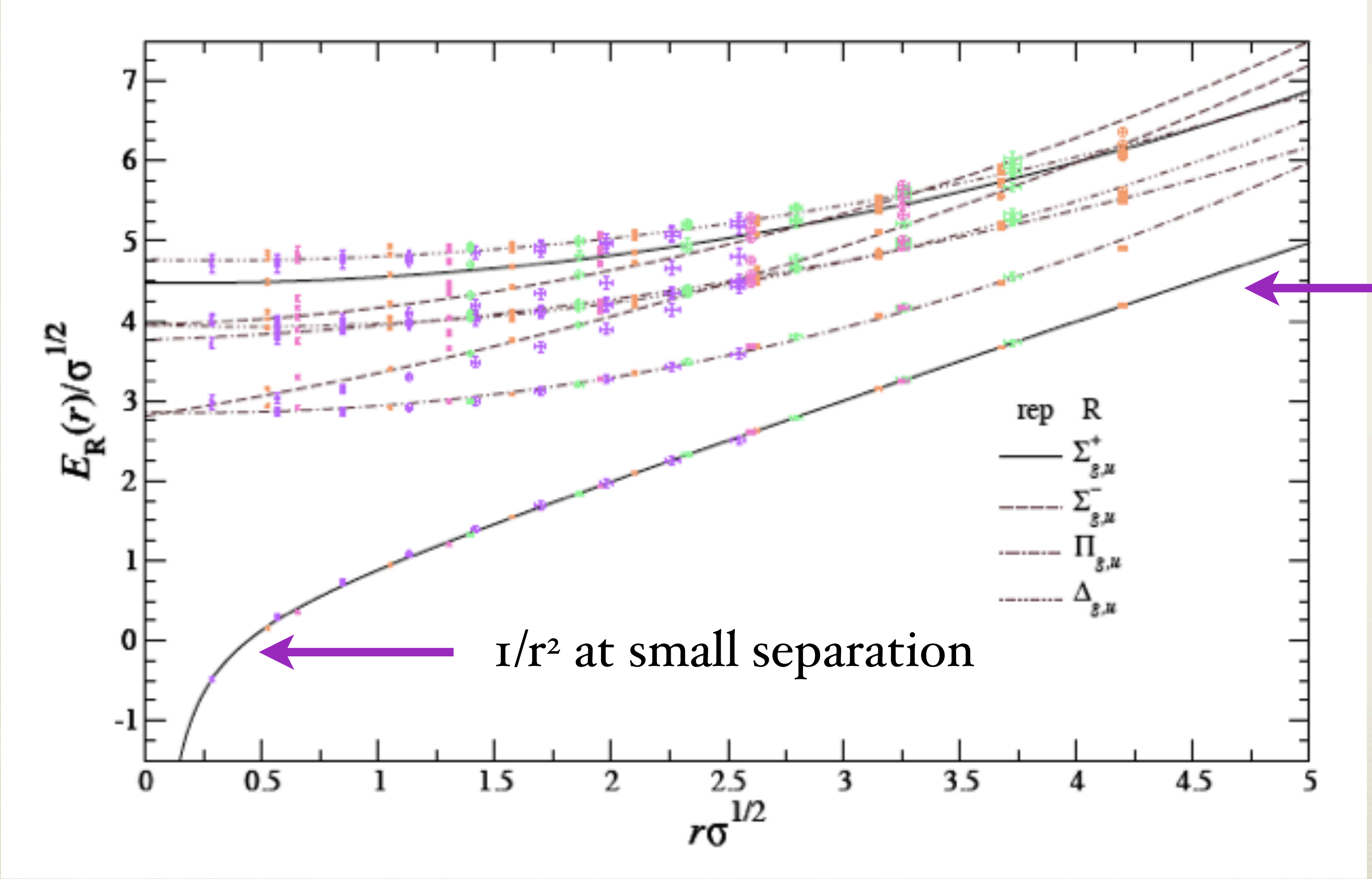
$$\beta_{QCD}(\alpha_s) = -\frac{\beta_0}{4\pi}\alpha_s^2, \quad \beta_0 = 11 - \frac{2}{3}N_F$$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\frac{\beta_0}{4\pi}\ln\left(\frac{Q^2}{\mu^2}\right)}$$

- Asymptotic freedom; coupling constant decreases at high energies and the perturbative expansion improves

Confinement in QCD

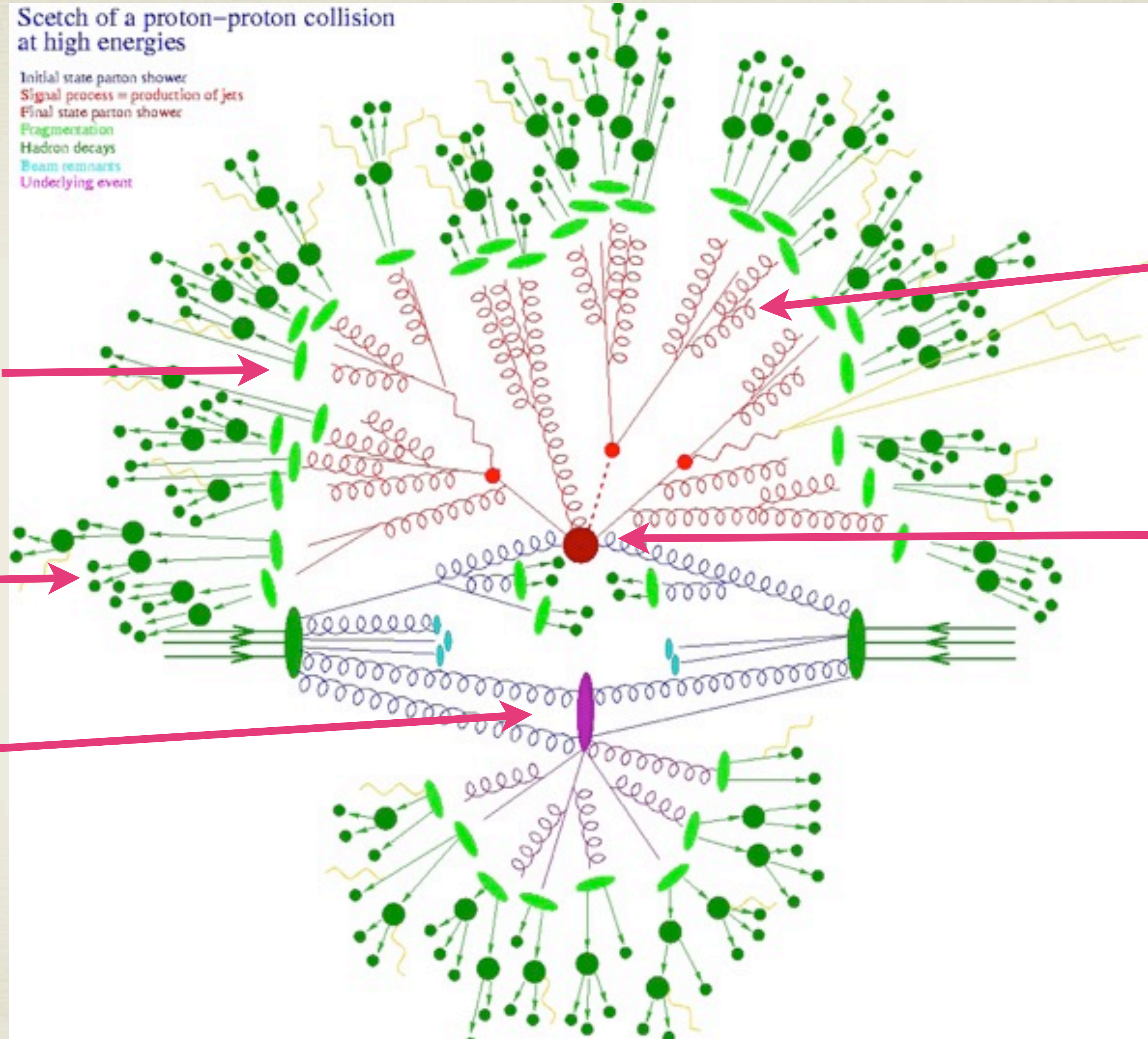
- QCD becomes strongly coupled at low energies. We *think* this leads to the experimentally observed confinement of quarks and gluons into hadrons.



quark-antiquark potential grows linearly at large separation, suggesting confinement

Juge, Kuti, Morningstar; review by Kronfeld, 1203.1204

Picture of a hadronic collision



Parton-shower evolution to low energies

Hadronization at Λ_{QCD}

Hard collision (Higgs production) at short distances/ high energies

Hadron decays

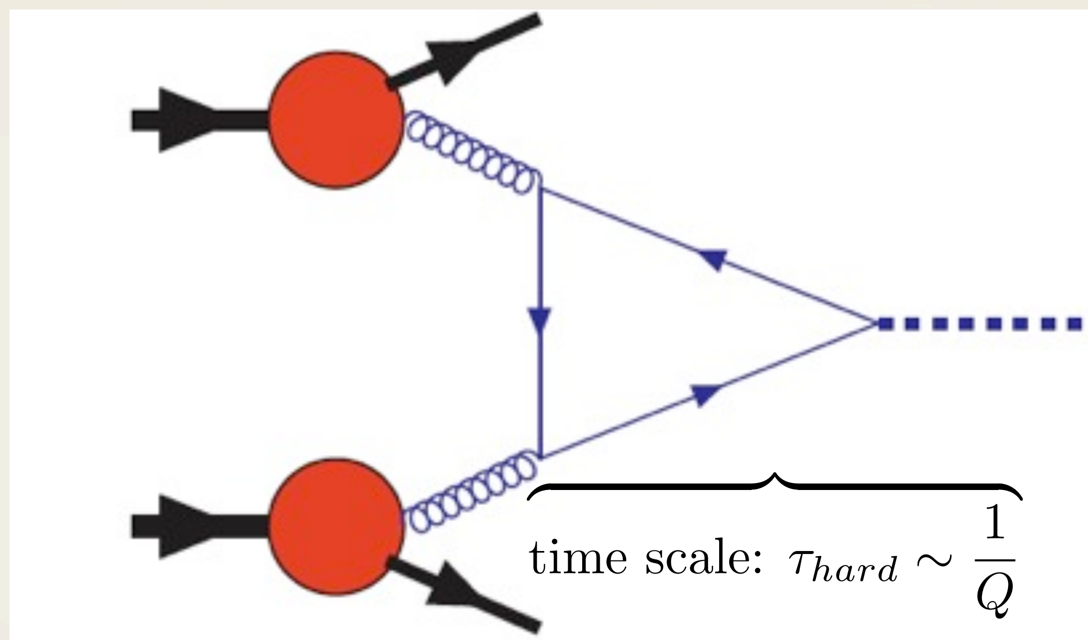
Multiple parton interactions

How does one make a prediction for such an event?

Divide and conquer

Make sense of this with *factorization*: separate hard and soft scales

$$\underbrace{\text{time scale: } \tau_{proton} \sim \frac{1}{\Lambda_{QCD}}}$$



$$\underbrace{\text{time scale: } \tau_{hard} \sim \frac{1}{Q}}$$

Review of factorization theorems:
Collins, Soper, Sterman
hep-ph/0409313

$$\sigma_{h_1 h_2 \to X} = \int dx_1 dx_2 \underbrace{f_{h_1/i}(x_1; \overbrace{\mu_F^2}^{\text{factorization scale}}) f_{h_1/j}(x_2; \mu_F^2)}_{PDFs} \underbrace{\sigma_{ij\to X}(x_1, x_2, \mu_F^2, \{q_k\})}_{\text{partonic cross section}} + \underbrace{\mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)^n}_{\text{power corrections}}$$

Non-perturbative but *universal*; measure in DIS, fixed-target, apply to Tevatron, LHC

Process dependent but calculable in pQCD

Small for sufficiently inclusive observables

Recipe for a QCD prediction

- Calculate $\sigma_{ij \to X}$
- Evolve initial, final states to Λ_{QCD} using parton shower
- Connect initial state to PDFs, final state to hadronization

Recipe for a QCD prediction

- Calculate $\sigma_{ij\to X}$
- Evolve initial, final states to Λ_{QCD} using parton shower
- Connect initial state to PDFs, final state to hadronization

How precisely must we know σ?

Do we know how to combine σ, parton shower?

Are our observables inclusive or must we worry about large logarithms?

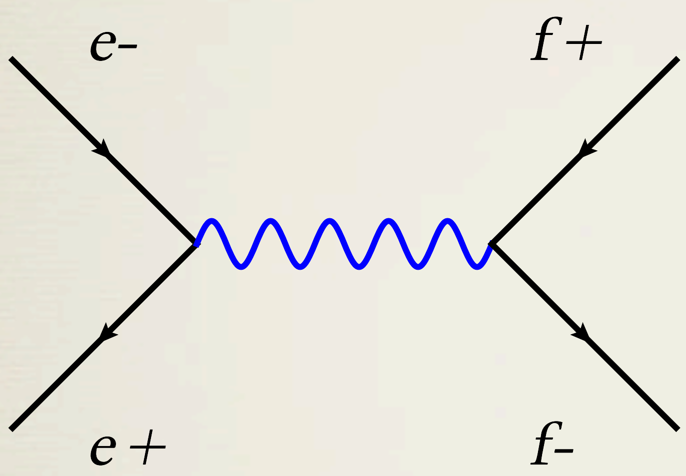
Do we have hard jets? Parton showers assume soft/ collinear radiation

Do we know the PDFs in the relevant kinematic regions?

Example 1: e^+e^- to hadrons at NLO

The basics: the R ratio in e^+e^-

- Many QCD issues relevant to hadronic collisions appear here.

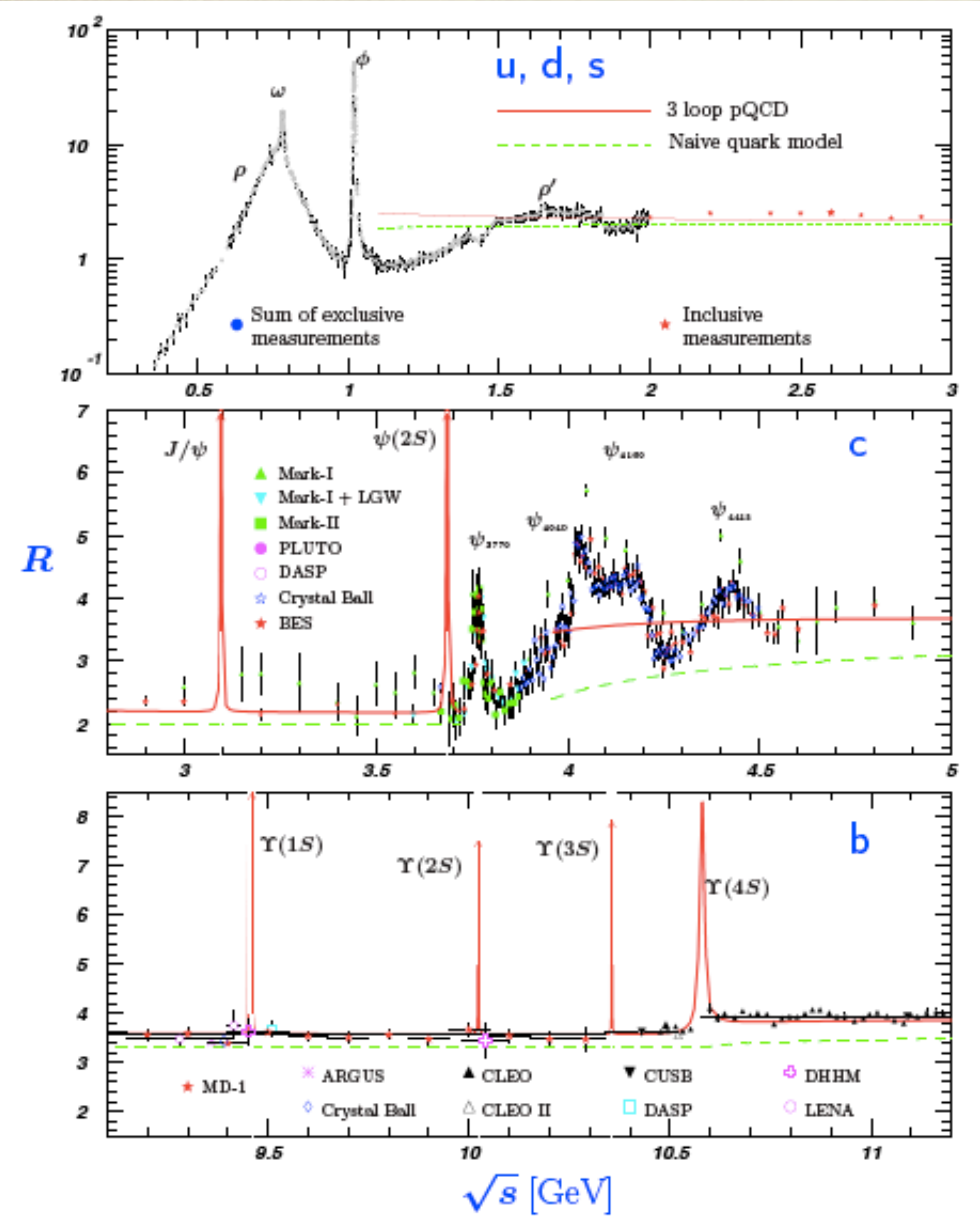


$$R = \frac{\sigma\left(e^+e^- \to \text{hadrons}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)}$$

- Time scale for f^+f^- production: $\tau \sim 1/Q$
- Time scale for hadronization: $\tau \sim 1/\Lambda$

$$R \to 3\sum_q Q_q^2$$

The basics: the R ratio in e^+e^-

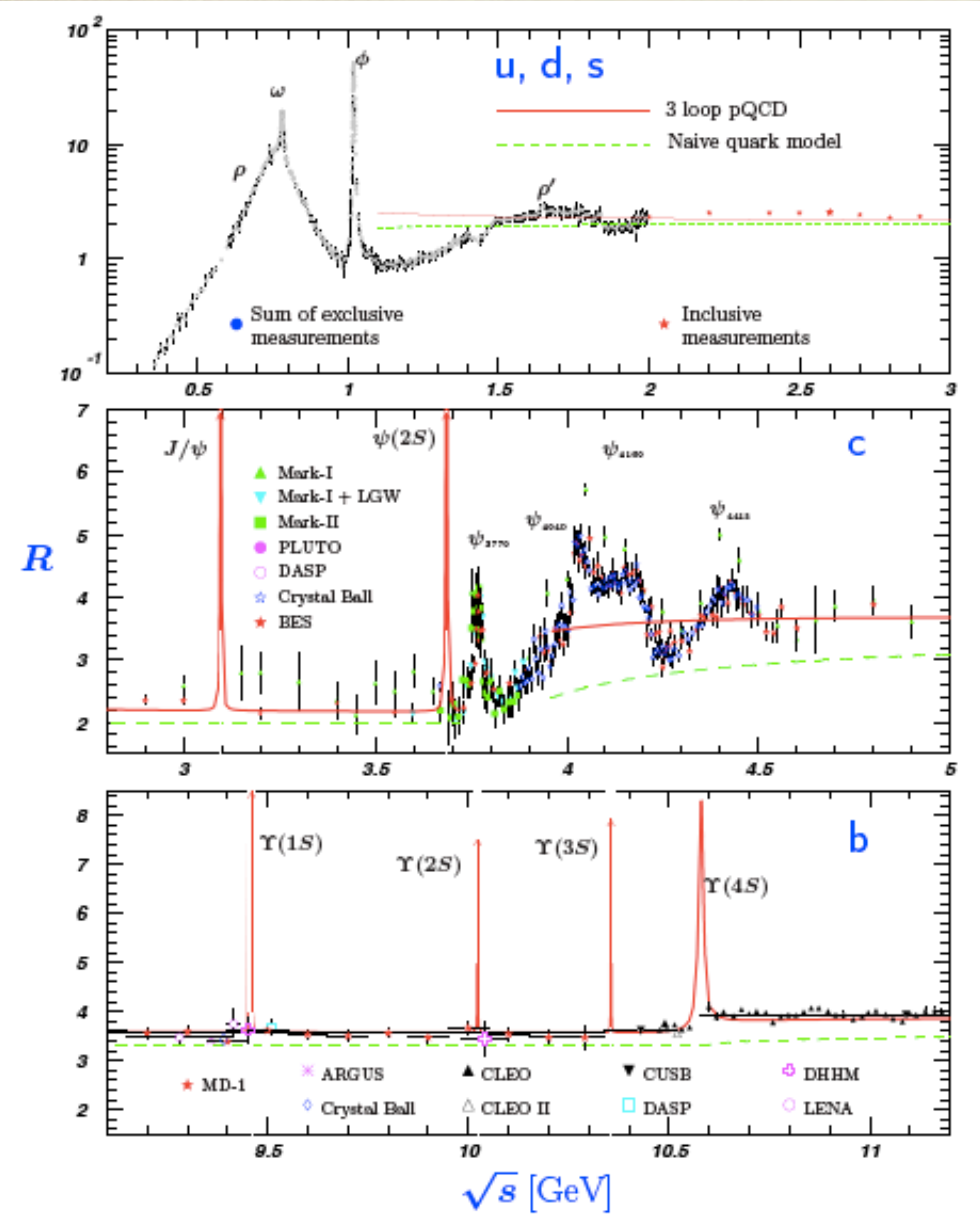


$$3 \times \left\{ \left(\frac{2}{3}\right)^2 + 2\left(\frac{1}{3}\right)^2 \right\} = 2$$

$$3 \times \left\{ 2\left(\frac{2}{3}\right)^2 + 2\left(\frac{1}{3}\right)^2 \right\} = \frac{10}{3}$$

$$3 \times \left\{ 2\left(\frac{2}{3}\right)^2 + 3\left(\frac{1}{3}\right)^2 \right\} = \frac{11}{3}$$

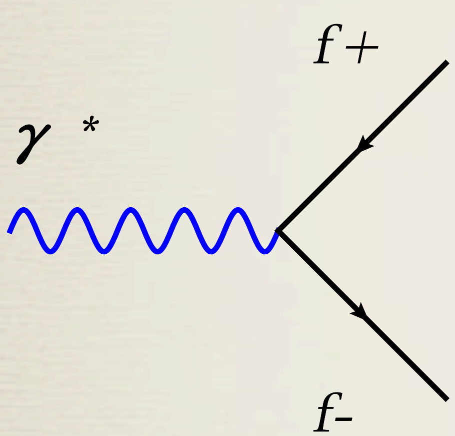
The basics: the R ratio in e^+e^-



- Note that even though we measure hadrons, summing over the accessible quarks gives the correct result (away from the resonance regions): *parton-hadron duality*
- Note also that there are pQCD corrections that are needed to accurately predict this ratio
- Our goal will be to calculate the next-to-leading order (NLO) QCD corrections to R

Leading order result

- Work through this; since production part of $e^+e^-\rightarrow$hadrons, $\mu^+\mu^-$ identical, can just consider $\gamma^*\rightarrow$hadrons, $\mu^+\mu^-$ and form ratio



- Leading-order matrix elements:

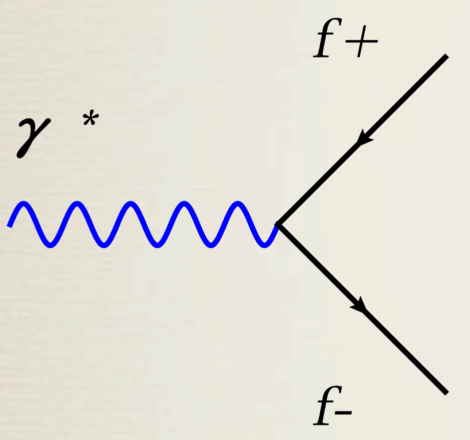
CM energy²

$$|\bar{\mathcal{M}}_0|^2 = \frac{1}{3}|\mathcal{M}_0|^2 = \frac{4e^2Q_F^2N_c\, s}{3}$$

Polarization averaging for off-shell photon

Leading order result

- Work through this; since production part of $e^+e^- \to$hadrons, $\mu^+\mu^-$ identical, can just consider $\gamma^* \to$hadrons, $\mu^+\mu^-$ and form ratio



- Leading-order phase space:

Go to 4 dimensions at the end

$$PS_0 = \frac{1}{2\sqrt{s}}\frac{1}{(2\pi)^2}\int d^dp_1 d^dp_2 \delta(p_1^2)\delta(p_2^2)\delta^{(d)}(p_\gamma - p_1 - p_2)$$

- Matrix elements don't depend on momentum directions, so we can simply parameterize:

$$p_1 = (E, 0, 0, E)$$

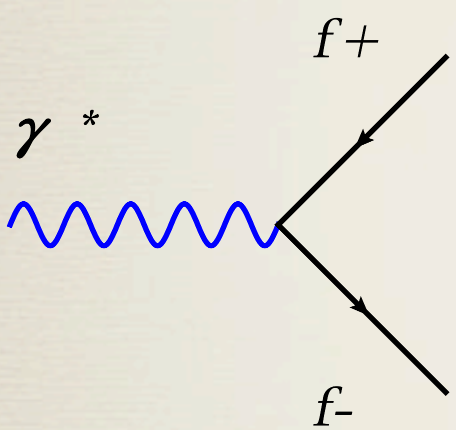
- Use delta functions to do integrals; get:

$$PS_0 = \frac{\Omega(3)}{64\pi^2\sqrt{s}}$$

Solid angle is 4π

Leading order result

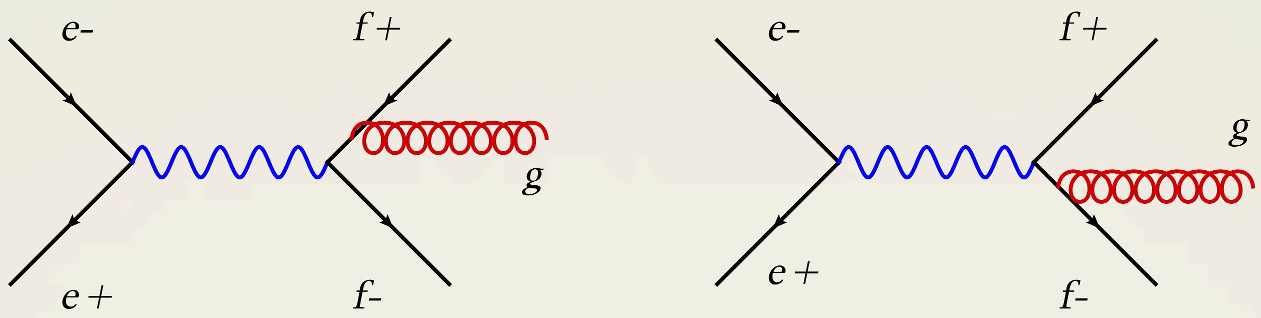
- Work through this; since production part of $e^+e^- \rightarrow$hadrons, $\mu^+\mu^-$ identical, can just consider $\gamma^* \rightarrow$hadrons, $\mu^+\mu^-$ and form ratio



$$R_0 = \frac{\sigma_{hadrons}}{\sigma_{\mu^+\mu^-}} = \frac{\left[|\bar{\mathcal{M}}_0|^2 \times PS_0\right]_{hadrons}}{\left[|\bar{\mathcal{M}}_0|^2 \times PS_0\right]_{\mu^+\mu^-}} = N_c \sum_q Q_q^2$$

Real emission corrections

- What can happen in field theory? Can emit additional gluon.



- Work out the phase space:

$$PS_1 = \frac{1}{2\sqrt{s}} \frac{1}{(2\pi)^5} \int d^dp_1 d^dp_2 d^dp_g \delta(p_1^2)\delta(p_2^2)\delta(p_g^2)\delta^{(d)}(p_\gamma - p_1 - p_2 - p_g)$$

$$p_\gamma = \sqrt{s}\,(1,0,0,0)$$
$$p_1 = E_1\,(1,0,0,1)$$
$$p_2 = E_2\,(1,s_1,0,c_1)$$

shorthand for cosine

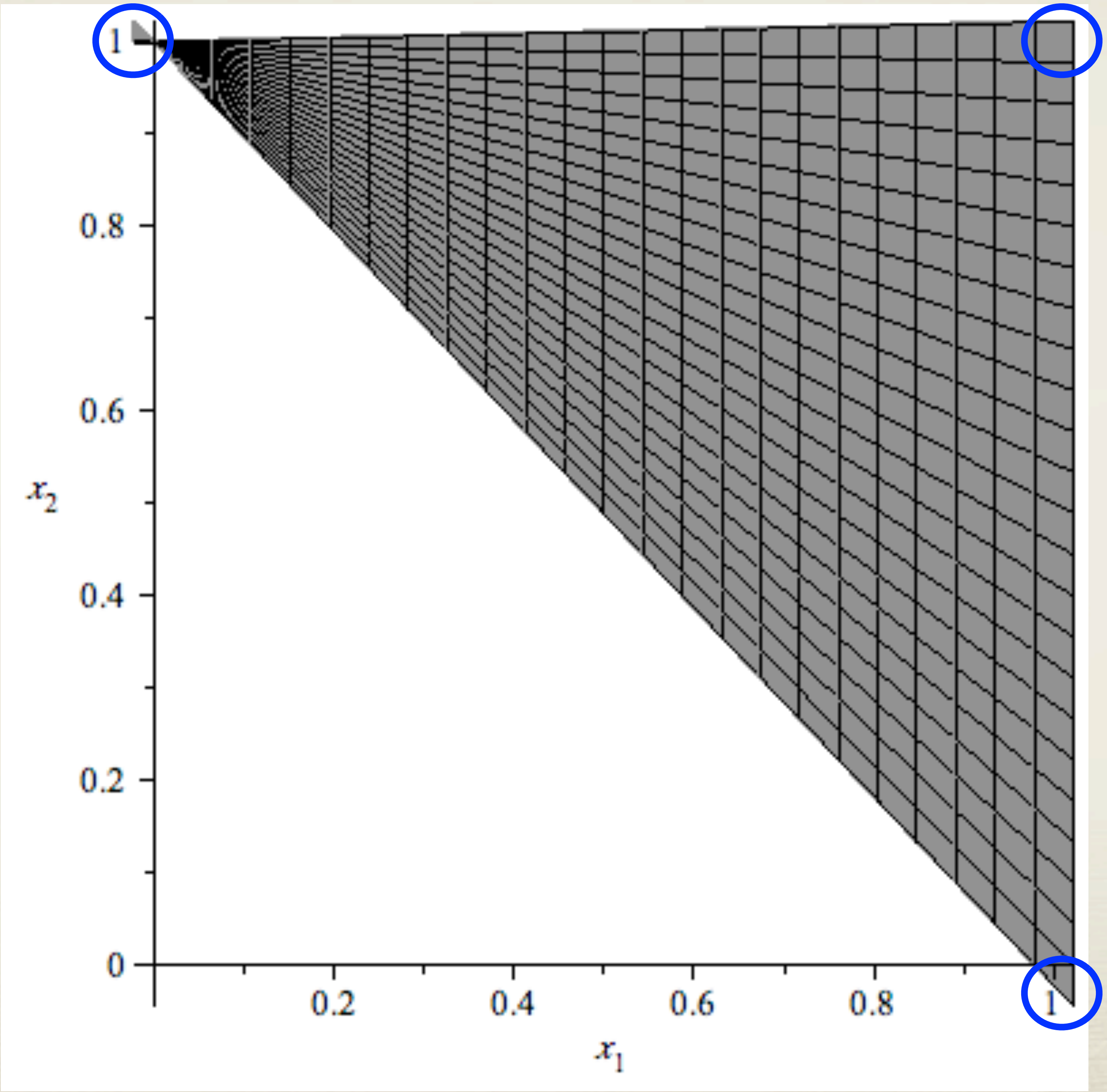
- Introducing $x_1=2E_1/\sqrt{s}$, $x_2=2E_2/\sqrt{s}$, straightforward to derive the d=4 expression:

$$PS_1 = \sqrt{s}\,\frac{\Omega(2)\Omega(3)}{64(2\pi)^5}\int dx_1 dx_2$$
$$= PS_0 \times \frac{s}{16\pi^2}\int dx_1 dx_2$$

Real-emission phase space

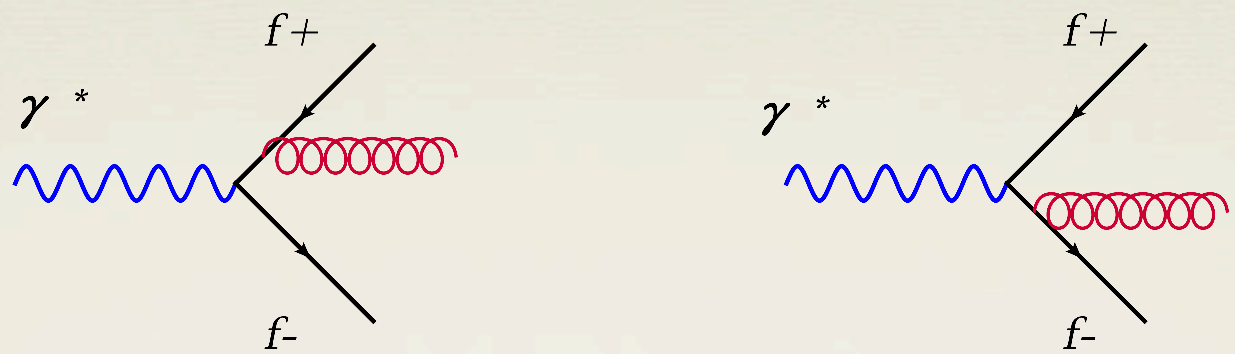
Quark carries no energy

Gluon carries no energy



Anti-quark carries no energy

Real-emission matrix elements



$s_{ij}=(p_i+p_j)^2$

- Work out the matrix elements

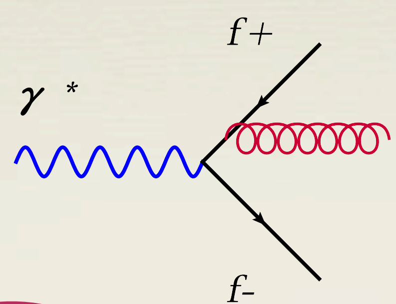
$$
\begin{aligned}
|\bar{\mathcal{M}}_1|^2 &= 2C_F g_s^2 \frac{|\bar{\mathcal{M}}_0|^2}{s} \left\{ \frac{s_{1g}}{s_{2g}} + \frac{s_{2g}}{s_{1g}} + 2\frac{s\, s_{12}}{s_{1g}s_{2g}} \right\} \\
&= 2C_F g_s^2 \frac{|\bar{\mathcal{M}}_0|^2}{s} \frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}
\end{aligned}
$$

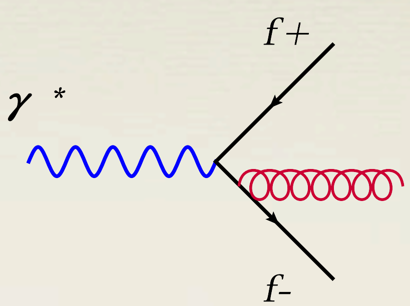
$C_F=4/3$

$$R_1^{q\bar{q}g} = R_0 \times \frac{2g_s^2 C_F}{16\pi^2} \int_0^1 dx_1 \int_{1-x_1}^1 dx_2 \, \frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

⇨ **Singular** for $x_{1,2} \to 1$

Soft and collinear singularities

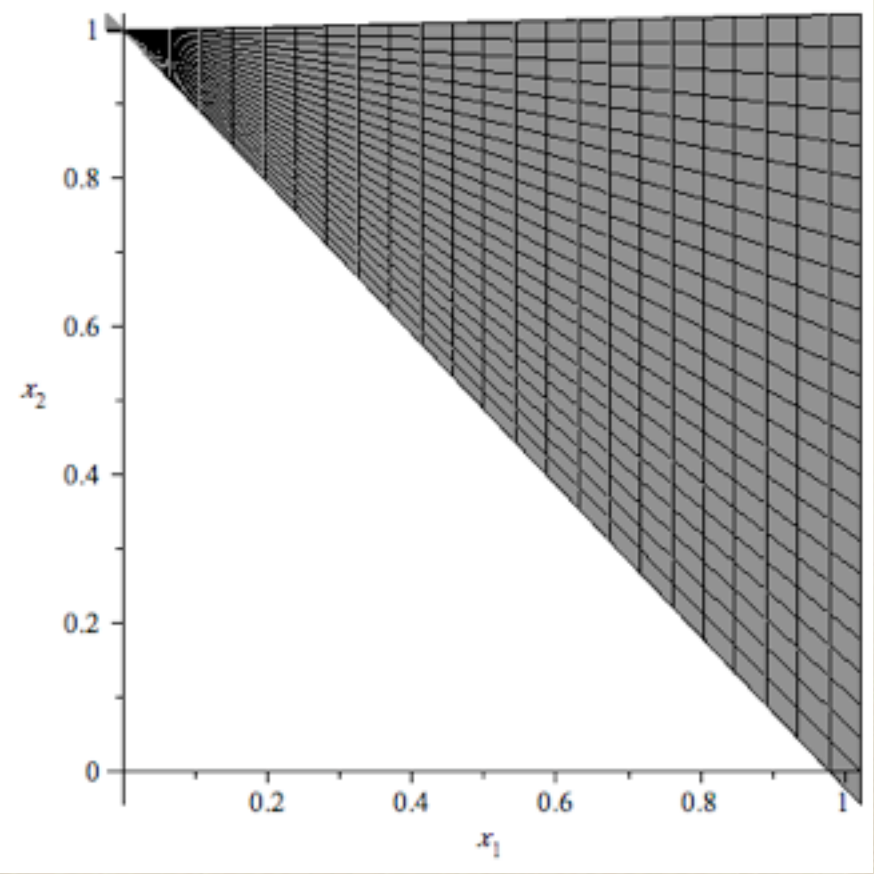




$$
\begin{aligned}
s_{1g} &= 2E_1 E_g (1-\cos\theta_{1g}) \\
s_{2g} &= 2E_2 E_g (1-\cos\theta_{2g})
\end{aligned}
$$

collinear singularities for $p_g \parallel p_1$, $p_g \parallel p_2$

soft singularities when $E_g=(1-x_1-x_2)\sqrt{s}\to 0$



$p_g \parallel p_1$ when $x_2=1$

$p_g \parallel p_2$ when $x_1=1$

KLN theorem

- The cross section for a quark-antiquark pair together with a soft/collinear gluon isn't well-defined in QCD. Experimentally, indistinguishable from just two quarks (in fact, we should be talking about hadrons or jets, not partons; will do later).
- Good question: what is the p_T of the hardest jet
- Bad question: how many gluons are in the final state
- **KLN theorem**: singularities cancel if degenerate energy states summed over $\Rightarrow$ as gluon becomes soft or collinear, indistinguishable from virtual corrections, must add loops.
- First need to *regularize* the real corrections.

Dimensional regularization

- Several ways to regulate soft/collinear divergences: add a gluon mass, take the quarks off-shell
- Method of choice is dimensional regularization: work in $d=4-2\varepsilon$ dimensions. Regulate both UV and IR singularities, introduces no new scales in calculations, maintains gauge symmetry.
- Coupling constant becomes dimensionful: $g_s^2 \to g_s^2 \mu^{2\varepsilon}$
- Useful to know the solid angle in d-dimensions:

$$\Omega(d) = \frac{2\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right)}$$

$$\int d\Omega(d) = \int dc_\theta \, d\phi \, \left[s_\theta^2 s_\phi^2\right]^{-\epsilon}$$

Real emissions corrections, take II

- Recompute the phase space and matrix elements for the real radiation corrections

$$PS_1 \to PS_0 \times \frac{s}{16\pi^2}\frac{1}{\Gamma(1-\epsilon)}\left[\frac{s}{4\pi\mu^2}\right]^{-\epsilon}\int dx_1 dx_2 \left[\underbrace{(1-x_3)}_{x_3=2-x_1-x_2}(1-x_1)(1-x_2)\right]^{-\epsilon}$$

also recomputed in d-dimensions

For ε slightly negative, regulates 1/(1-x₁,₂)

$$|\bar{\mathcal{M}}_1|^2 \quad \to \quad 2C_F g_s^2 \frac{|\bar{\mathcal{M}}_0|^2}{s}\left\{\frac{(1-\epsilon)(x_1^2+x_2^2)+2\epsilon(1-x_3)}{(1-x_1)(1-x_2)} - 2\epsilon\right\}$$

- Combine these to get:

$$\begin{aligned} R_1^{q\bar{q}g} \quad &= \quad R_0 \times \frac{2g_s^2 C_F}{16\pi^2\Gamma(1-\epsilon)}\left[\frac{s}{4\pi\mu^2}\right]^{-\epsilon}\int_0^1 dx_1 \int_{1-x_1}^1 dx_2 \left\{\frac{(1-\epsilon)(x_1^2+x_2^2)+2\epsilon(1-x_3)}{(1-x_1)(1-x_2)} - 2\epsilon\right\} \\ &\times \quad [(1-x_1)(1-x_2)(1-x_3)]^{-\epsilon} \end{aligned}$$

Final result for real emission

- Evaluate integrals (in terms of beta functions) to find:

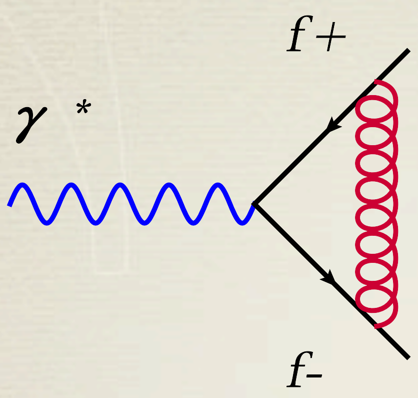
$$R_1^{q\bar{q}g} = R_0 \times \frac{\alpha_s C_F}{2\pi\Gamma(1-\epsilon)}\left[\frac{s}{4\pi\mu^2}\right]^{-\epsilon}\left\{\frac{2}{\epsilon^2}+\frac{3}{\epsilon}+\frac{19}{2}-\pi^2+\mathcal{O}(\epsilon)\right\}$$

double pole: soft+collinear gluon

single pole: soft or collinear gluon

- Regulator dependent! Not a physical observable.
- Add on the virtual corrections next

Virtual corrections and final result

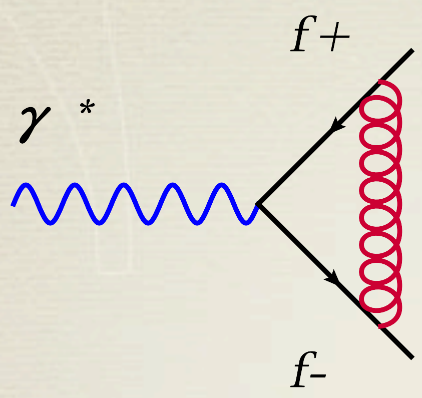


$$R_1^{q\bar{q}} = R_0 \times \frac{\alpha_s C_F \Gamma(1+\epsilon)}{2\pi} \left[\frac{s}{4\pi\mu^2}\right]^{-\epsilon} \left\{ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon) \right\}$$

- As required by the KLN theorem, poles cancel upon addition of real and virtual corrections, leaving:

$$R = R_0 + R_1 + \mathcal{O}(\alpha_s^2) = R_0 \times \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \right\}$$

Virtual corrections and final result



$$R_1^{q\bar{q}} = R_0 \times \frac{\alpha_s C_F \Gamma(1+\epsilon)}{2\pi} \left[\frac{s}{4\pi\mu^2}\right]^{-\epsilon} \left\{ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon) \right\}$$

- As required by the KLN theorem, poles cancel upon addition of real and virtual corrections, leaving:

$$R = R_0 + R_1 + \mathcal{O}(\alpha_s^2) = R_0 \times \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \right\}$$

- (A note about scaleless integrals: $\int d^d k \, \frac{1}{[k^2]^n} \propto \frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} = 0$
- Very useful as long as you don't specifically care about the pole coefficients
- Allows us to neglect the external leg corrections)

Renormalization scale (in)dependence

- The result must be independent of the arbitrary renormalization scale μ. We can derive the following RG equation:

$$\frac{dR}{d\mu} = 0 \to \mu^2 \frac{\partial R}{\partial \mu^2} + \beta_{QCD}(\alpha_s) \frac{\partial R}{\partial \alpha_s} = 0$$

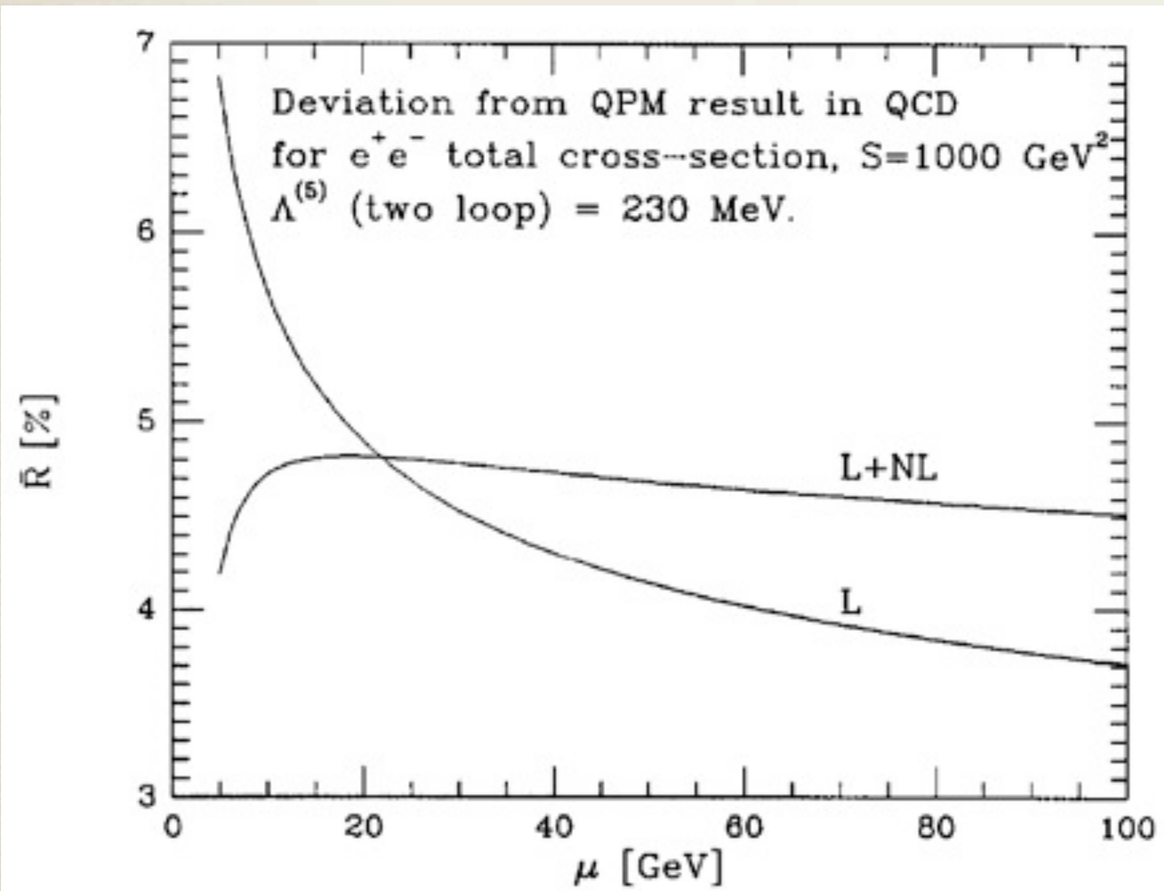
- Can use this to predict the explicit μ dependence at higher orders, by expanding this equation as a perturbative expansion in α_s

$$\mu^2 \frac{\partial R^{(2)}}{\partial \mu^2} = \frac{\beta_0}{4\pi} \alpha_s^2 \frac{\partial R^{(1)}}{d\alpha_s}$$

$$R^{(2)} = \frac{\beta_0}{4} \left(\frac{\alpha_s}{\pi}\right)^2 R^{(0)} \ln \frac{\mu^2}{s} + \underbrace{\ldots}_{\mu \text{ independent}}$$

“Theoretical error”

- Variation of scale in some specified range is often used as an estimate of theoretical uncertainty ⇒ if it was calculated to higher orders, this dependence would vanish

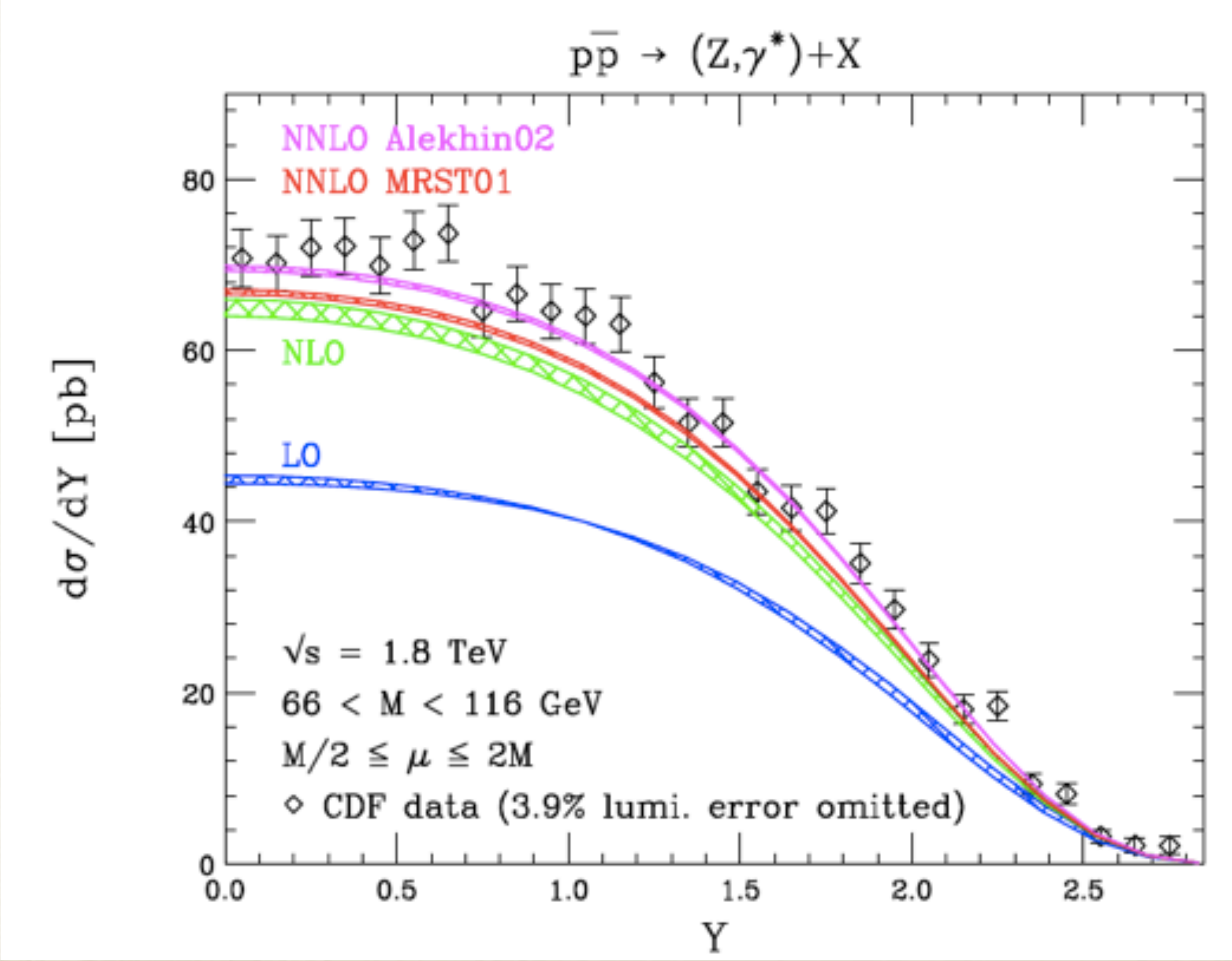


from Ellis, Stirling, Webber
QCD and Collider Physics

- Conventional range: $\sqrt{s}/2 \leq \mu \leq 2\sqrt{s}$
- Often underestimates LO→NLO, especially at hadron colliders where qualitatively new effects can appear at higher orders
- How to pick central value with multiple physical scales?

"Theoretical error"

- Variation of scale in some specified range is often used as an estimate of theoretical uncertainty ⇒ if it was calculated to higher orders, this dependence would vanish

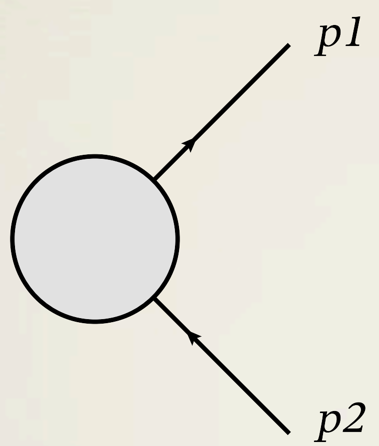


Anastasiou, Dixon, Melnikov, FP 2003

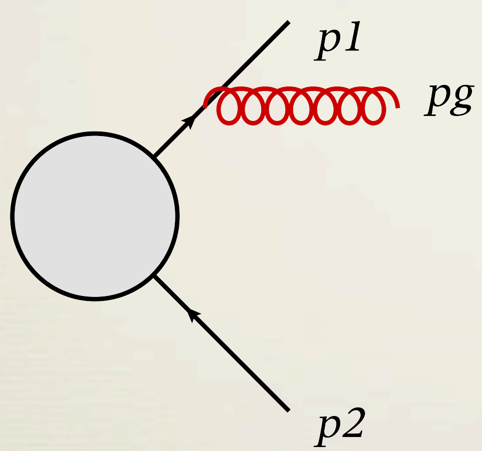
- LO is a qualitative description at best, and the scale variation is not trustable
- If you want to match the data and have any idea about your error, you need higher orders!

Eikonal approximation

- Useful to have diagnostic tools to check pieces of a calculation: ‘eikonal’ approximation for soft gluons gets double pole



$$= \bar{u}^i(p_1)\left[i\mathcal{M}_0^{ij}\right] v^j(p_2)$$



$$\begin{aligned} &= \bar{u}^i(p_1)\left\{ i g_s\, \not\epsilon_g^a T^a_{ij} \frac{i(\not p_1 + \not p_g)}{(p_1+p_g)^2}\left[i\mathcal{M}_0^{jk}\right]\right\} v^k(p_2) \\ &\approx -g_s \frac{p_1\cdot\epsilon_g^a}{p_1\cdot p_g}\bar{u}^i(p_1)\left\{T^a_{ij}\left[i\mathcal{M}_0^{jk}\right]\right\} v^k(p_2) \end{aligned}$$

Proportional to the lower-order amplitude, with a color correlation. Emission off the other leg also simplifies

Eikonal approximation

- Phase space also factorizes, into the soft-gluon component times the remainder. Can derive simplified expressions for the cross section in this limit.

sum over the hard gluons

the soft gluon

$$d\sigma_s = \left[\frac{\alpha_s}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu^2}{s_{12}}\right)^{\epsilon}\right]\sum_{f,f'} d\sigma^0_{ff'}\int dS\,\frac{-p_f\cdot p_{f'}}{p_f\cdot p_s p_{f'}\cdot p_s}$$

partonic CM energy squared

$$dS = \frac{1}{\pi}\left(\frac{4}{s_{12}}\right)^{-\epsilon}\int_0^{\delta_s\sqrt{s_{12}}2} dE_s\, dc_\theta\, d\phi\, E_s^{1-2\epsilon} s_\theta^{-2\epsilon} s_\phi^{-2\epsilon}$$

δ_s restricts to the soft region

$$\mathcal{M}^0_{ff'} = \left[\mathcal{M}_{c_1 \ldots b_f \ldots b_{f'} \ldots c_n}\right]^* T^a_{b_f d_f} T^a_{b_{f'} d_{f'}} \mathcal{M}_{c_1 \ldots d_f \ldots d_{f'} \ldots c_n}$$

color-correlated lower-order amplitude

different expressions depending on soft-particle color representation

from Harris & Owens hep-ph/0102128, a useful reference for relevant formulae

Eikonal approximation

- Application to the current process yields:

$$R^{q\bar{q}g}_{1,soft} = R_0 \times \frac{\alpha_s C_F}{\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{s}{4\pi\mu^2}\right)^{-\epsilon} \left\{ \frac{1}{\epsilon^2} - \frac{2}{\epsilon} \ln\delta + 2\ln^2\delta + \text{finite} \right\}$$

agrees with our full calculation

Cutoff dependence must cancel against other regions of gluon phase space

- The 1/ε² poles must cancel against virtual corrections

Collinear approximation

- Another singular region to consider: collinear gluon emission. A simple way of calculating this phase-space region also exists. Study the region $p_1 \parallel p_g$. Sudakov parameterization of momenta:

$$\begin{aligned} p_g^\mu =& z p^\mu + k_\perp^\mu - \frac{k_\perp^2}{z}\frac{n^\mu}{2p\cdot n} \\ p_1^\mu =& (1-z)p^\mu - k_\perp^\mu - \frac{k_\perp^2}{1-z}\frac{n^\mu}{2p\cdot n} \end{aligned}$$

$$z = \frac{E_g}{E_1+E_g}, \quad s_{1g} = -\frac{k_\perp^2}{1-z}$$

- $k_\perp \to 0$ is the singular limit. p, n are light-like vectors satisfying $p.k_\perp = n.k_\perp = 0$. p bisects p_1, p_g. The amplitude simplifues in this limit:

$$|\mathcal{M}_1(p_1,p_2,p_g)|^2 \approx \frac{2}{s_{1g}} g_s^2 \mu^{2\epsilon} P_{qq}(z,\epsilon)\,|\mathcal{M}_0(p_1+p_g,p_2)|^2$$

$$P_{qq}(z,\epsilon) = C_F\left[\frac{1+z^2}{1-z} - \epsilon(1-z)\right]$$

Collinear approximation

- Phase space also simplifies in this limit. We're left with the following contribution to the NLO R ratio from the $p_1 \parallel p_g$ region:

$x_2 = 1 - s_{1g}/s$

$$R^{q\bar{q}g}_{1,1||g} = R_0 \times \frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\left[\frac{s}{4\pi\mu^2}\right]^{-\epsilon}\int_{1-\delta_c}^{1} dx_2\,(1-x_2)^{-1-\epsilon}\int_0^{1-\delta} dz\,[z(1-z)]^{-\epsilon}\,P_{qq}(z,\epsilon)$$

$$= R_0 \times \frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\left[\frac{s}{4\pi\mu^2}\right]^{-\epsilon}\left\{\frac{1}{\epsilon}\left(\frac{3}{2}+2\ln\delta\right) - \ln^2\delta - \frac{3}{2}\ln\delta_c - 2\ln\delta\ln\delta_c + \text{finite}\right\}$$

Together with $p_2 \parallel p_g$ region, agrees with full result

Cancels against soft region (with $p_2 \parallel p_g$ region)

- Remaining cutoff dependence cancels against hard region of phase space, which is finite and can be handled numerically in 4 dimensions

Slicing and subtraction

- The splitting functions and eikonal factors are universal
- What we've done forms the basis of a scheme for handling IR singularities at NLO known as *phase-space slicing*
- Split full=soft+Σ(collinear)+hard; eikonal+collinear approximations to get singularities
- Numerical integration of hard region; dependence on $\ln(\delta)$, $\ln(\delta_c)$ must cancel
- Another scheme known as dipole subtraction, that unifies the soft and collinear limits into 'dipoles' for each pair of emittors

Useful references:
Phase-space slicing, Harris, Owens hep-ph/0102128;
Dipole subtraction, Catani, Seymour hep-ph/9605323;
Singular limits of matrix elements: Campbell, Glover hep-ph/9710255;
Catani, Grazzini hep-ph/9908523

Parton Showers and Jets

Sudakov form factor

- Let's study again our real-emission cross section in the collinear limit, setting d=4.

$$d\sigma^{q\bar{q}g}_{collinear} \to \sigma_0 \frac{\alpha_s}{2\pi}\, dz\, P_{qq}(z) \sum_{t=s_{1g},s_{2g}} \frac{dt}{t}$$

$\Rightarrow$ independent emission of gluon from quark, anti-quark

- Focus on collinear region 1||g. Think of $1/\sigma_0 \times d\sigma^{q\bar{q}g}$ as the probability of emitting gluon in interval dt. Also consider probability of no emission.

$$\begin{aligned} dP &= \frac{dt}{t}\frac{\alpha_s}{2\pi}\int dz\, P_{qq}(z) \\ dP_{no} &= 1-\frac{dt}{t}\frac{\alpha_s}{2\pi}\int dz\, P_{qq}(z) \end{aligned}$$

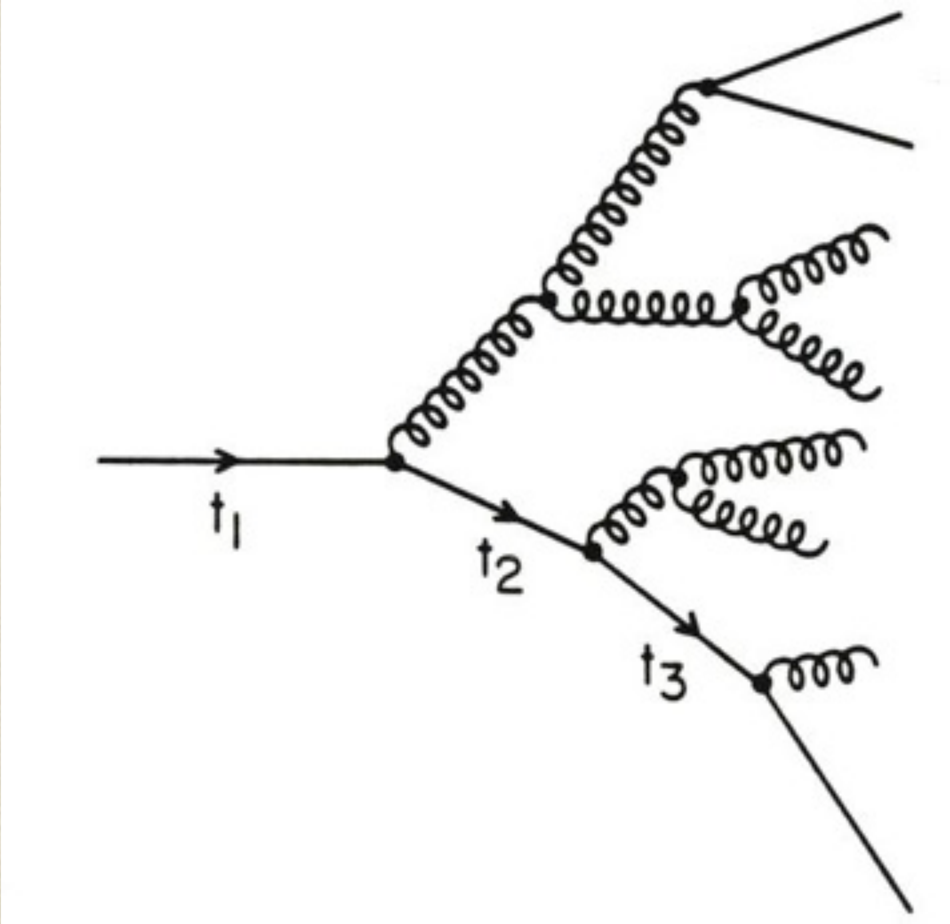
$\Rightarrow$ this *exponentiates*:

$$\Delta(t) = \exp\left\{-\int \frac{dt'}{t'}\frac{\alpha_s}{2\pi}\int dz\, P_{qq}(z)\right\}$$

Sudakov form factor, probability of no emission with invariant mass between upper, lower invariant masses.

The parton shower

- Can use to correctly (within collinear approximation) generate the emission of multiple partons (HERWIG, PYTHIA, SHERPA)
- In our previous example, many partons will be produced as the variable t evolves from high scales to Λ_{QCD}



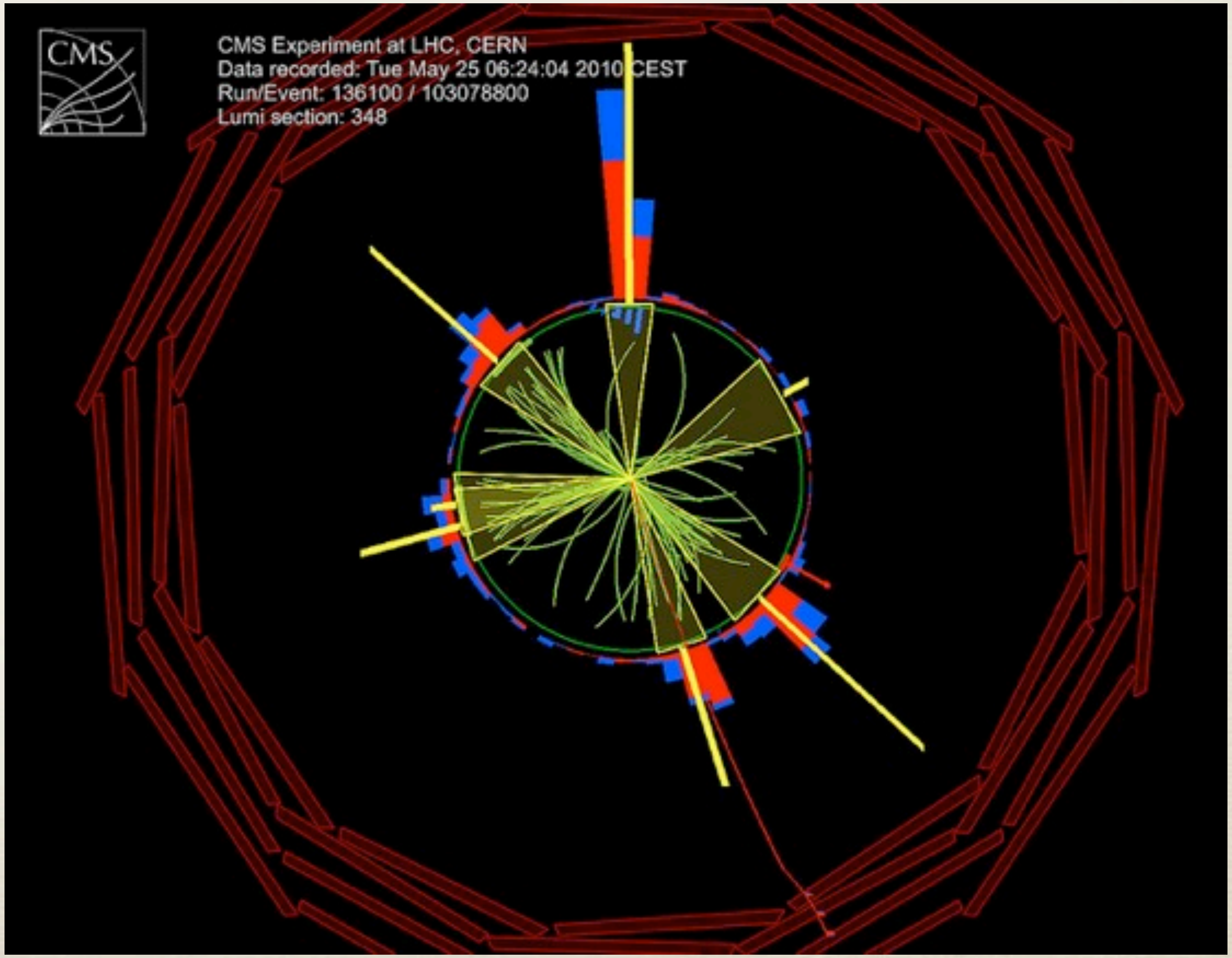
$$\Delta(t) = \exp\left\{ -\int \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int dz\, P_{qq}(z) \right\}$$

- This is the parton shower. In addition to producing high-multiplicity final states, it resums large logarithms that appear in certain regions of phase space

More in John Campbell's lectures

Jets

- When low scales t-Λ_{QCD} are reached, the hadrons will form observed experimentally. Sprays of hadrons form the jets observed experimentally



- Specify a *jet algorithm* for combining the observed particles into jets
- The idea: the jets should reflect the primordial hard partons

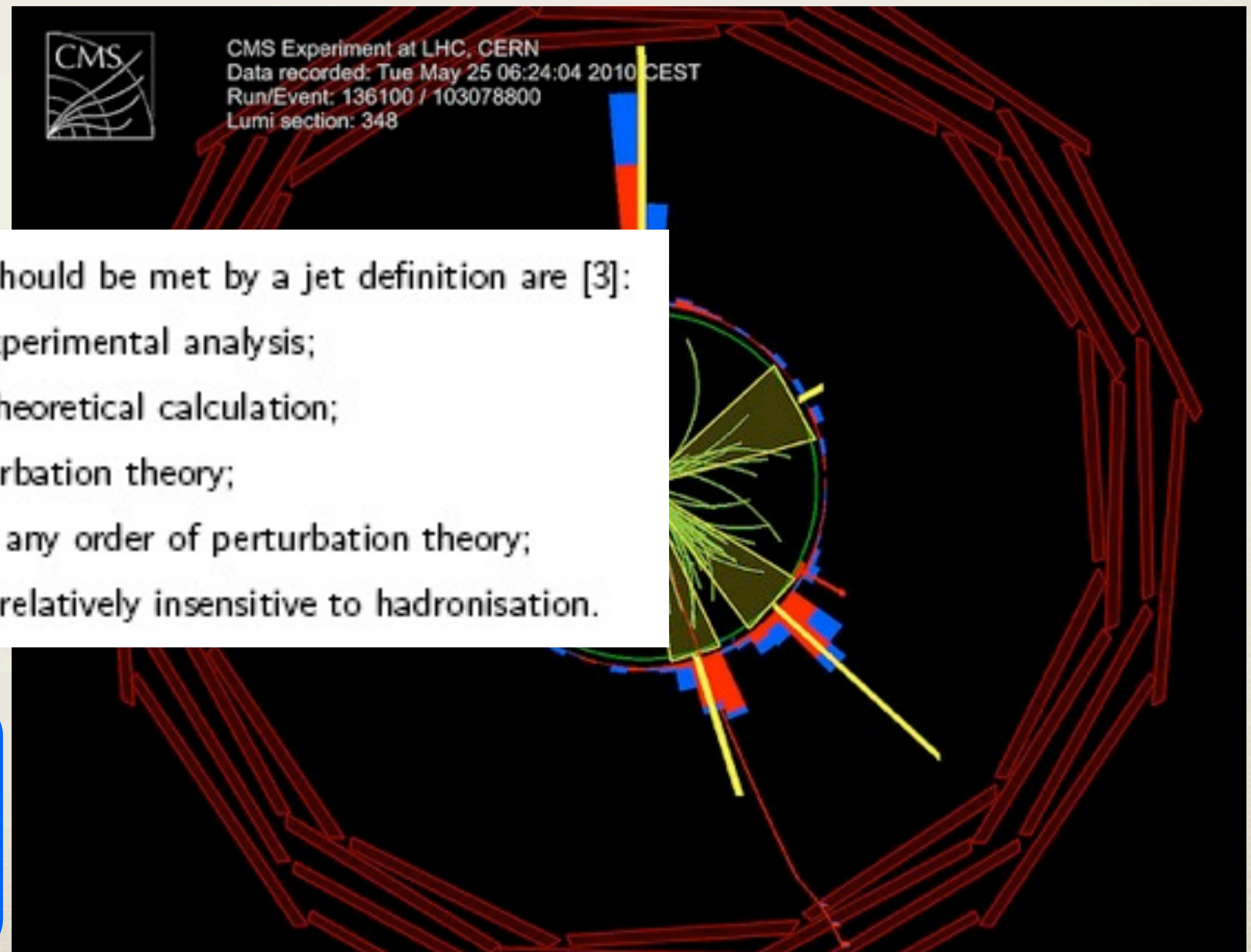
Jets

- When low scales t-Λ_{QCD} are reached, the hadrons will form observed experimentally. Sprays of hadrons form the jets observed experimentally

- Specify a *jet algorithm* for combining the

Several important properties that should be met by a jet definition are [3]:

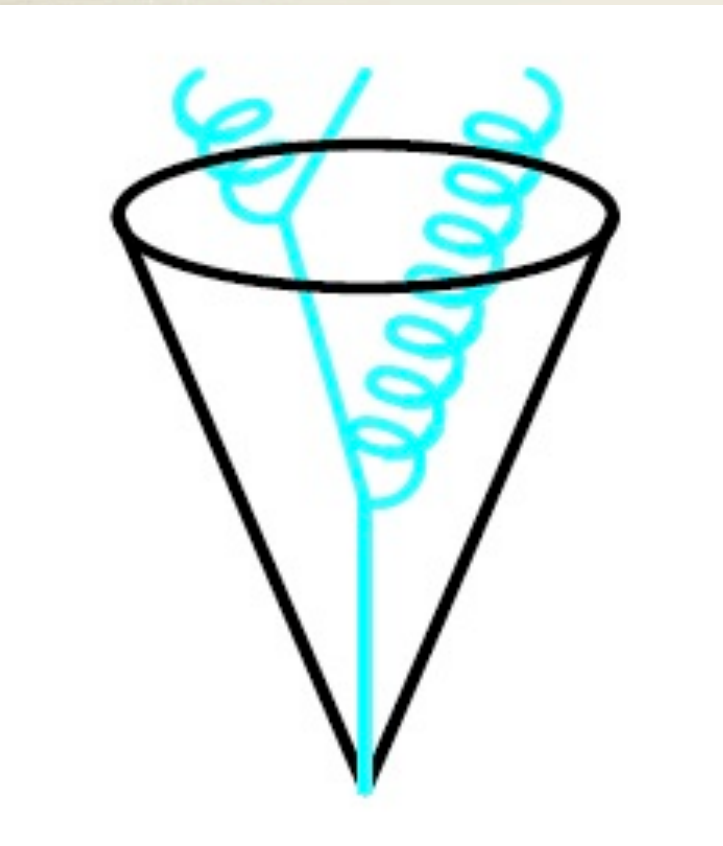
1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross sections at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronisation.



Useful reference: G. Salam, 0906.1833

The cone

- Basic idea: draw a cone around the clusters of energy in the event



Iterated cones:

- Start with seed particle i
- Combine all particles within a cone of radius R

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2 < R^2$$

rapidities — azimuthal angles

- Use the combined 4-momentum as a new seed
- Repeat until stability achieved

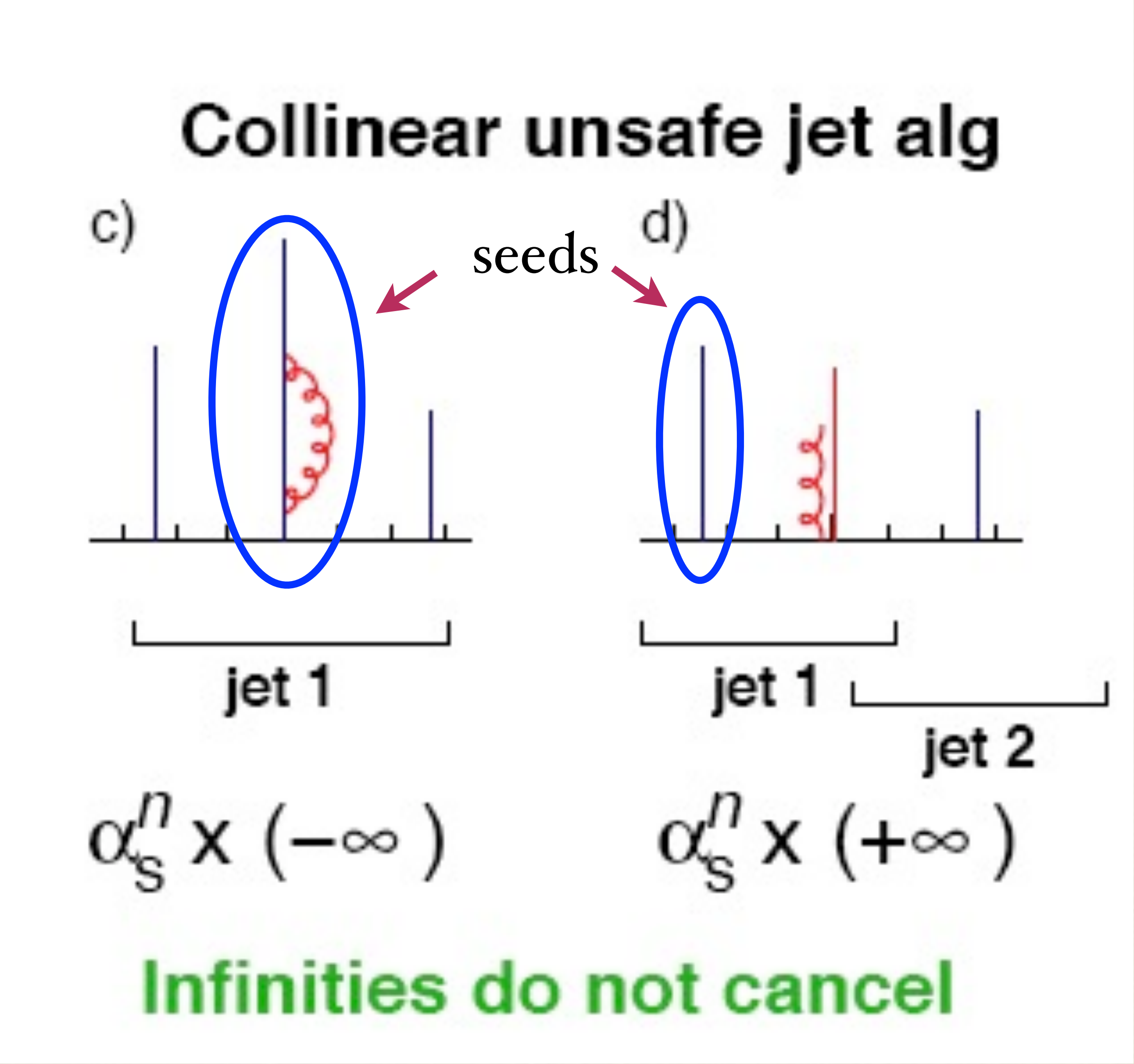
Example:

- Progressive removal (IC-PR): start with largest transverse momentum as seed; after finding stable cone, call it a jet and remove; go to next largest p_T

Infrared safety

- We saw before that IR singularities cancel between real, virtual corrections ⇒ *infrared safety*. The jet algorithm shouldn't spoil this cancellation. The example on the previous slide does.

✘ IC-PR algorithm starts from different seed after emission of a hard collinear parton



Consequences

- Consequence: $1/\varepsilon \to \ln(p_T/\Lambda_{QCD}) \sim 1/\alpha_S \Rightarrow$ no suppression of higher-order contributions, no expansion possible

$$\underbrace{\alpha_s\alpha_{EW}}_{\text{LO}} + \underbrace{\alpha_s^2\alpha_{EW}}_{\text{NLO}} + \underbrace{\alpha_s^3\alpha_{EW}\ln\frac{p_t}{\Lambda}}_{\text{NNLO}} + \underbrace{\alpha_s^4\alpha_{EW}\ln^2\frac{p_t}{\Lambda}}_{\text{NNNLO}} + \cdots,$$
$$\sim \underbrace{\alpha_s\alpha_{EW}}_{\text{LO}} + \underbrace{\alpha_s^2\alpha_{EW}}_{\text{NLO}} + \underbrace{\alpha_s^2\alpha_{EW}}_{\text{NNLO}} + \underbrace{\alpha_s^2\alpha_{EW}}_{\text{NNNLO}} + \cdots$$

Observable	1st miss cones at	Last meaningful order
Inclusive jet cross section	NNLO	NLO
$W/Z/H + 1$ jet cross section	NNLO	NLO
3 jet cross section	NLO	LO
$W/Z/H + 2$ jet cross section	NLO	LO
jet masses in 3 jets, $W/Z/H + 2$ jets	LO	none

Situation for midpoint cone, from Salam & Soyez 0704.0292

- Can modify algorithms so that addition of soft/collinear particles doesn't modify hard jets in the event: SIScone (seedless infrared safe)

Sequential recombination

- k_t algorithm:

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2)\frac{\Delta R_{ij}^2}{R^2}$$
$$d_{iB} = p_{ti}^2$$

  - Work out all d_{ij}, d_{iB}, find minimum
  - If it is a d_{ij}, combine i and j and restart
  - If it is a d_{iB}, call i a jet and remove it
  - Stop after no particles remain

- Generalizations use a slightly different distance measure

$$d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p})\frac{\Delta R_{ij}^2}{R^2}$$
$$d_{iB} = p_{ti}^{2p}$$

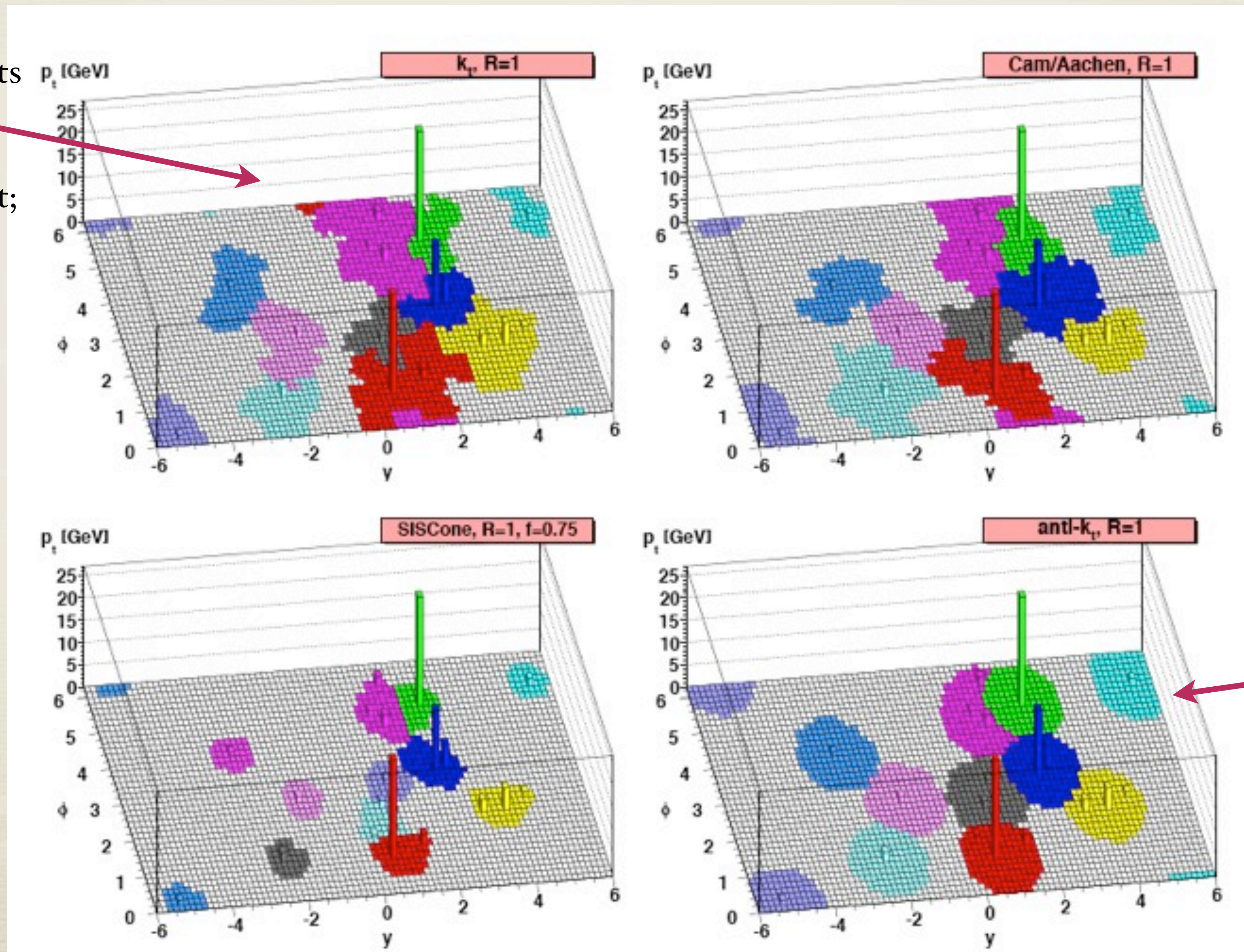
  - p=-1: anti-k_t
  - p=0: Cambridge-Aachen

- Roughly, soft and collinear emissions come with small distance measure and are always recombined ⇒ IR safe

Jets in pictures

- Areas denote where soft radiation would be "soaked up" by jet

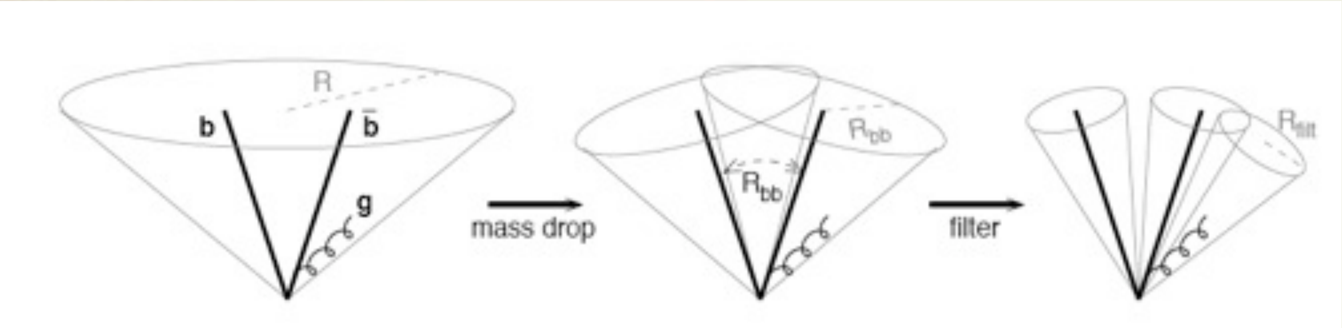
First clusters all sorts of soft particles, which eventually become added to jet; more sensitive to underlying event, pile-up



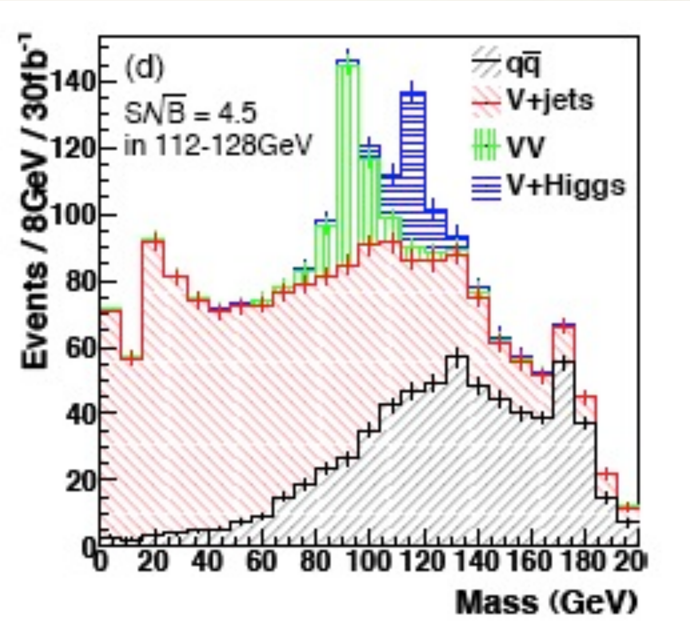
Avoids this with the $1/p_t^2$ in d_{ij}; the preferred choice for LHC studies

Jet substructure

- Recent interest in using substructure of jets to distinguish signal from background. For example, highly-boosted Higgs will produce a "fat jet" with two b subjets inside.



Undo last stage of clustering and look for significant mass drop, consistent with heavy particle decaying to jets



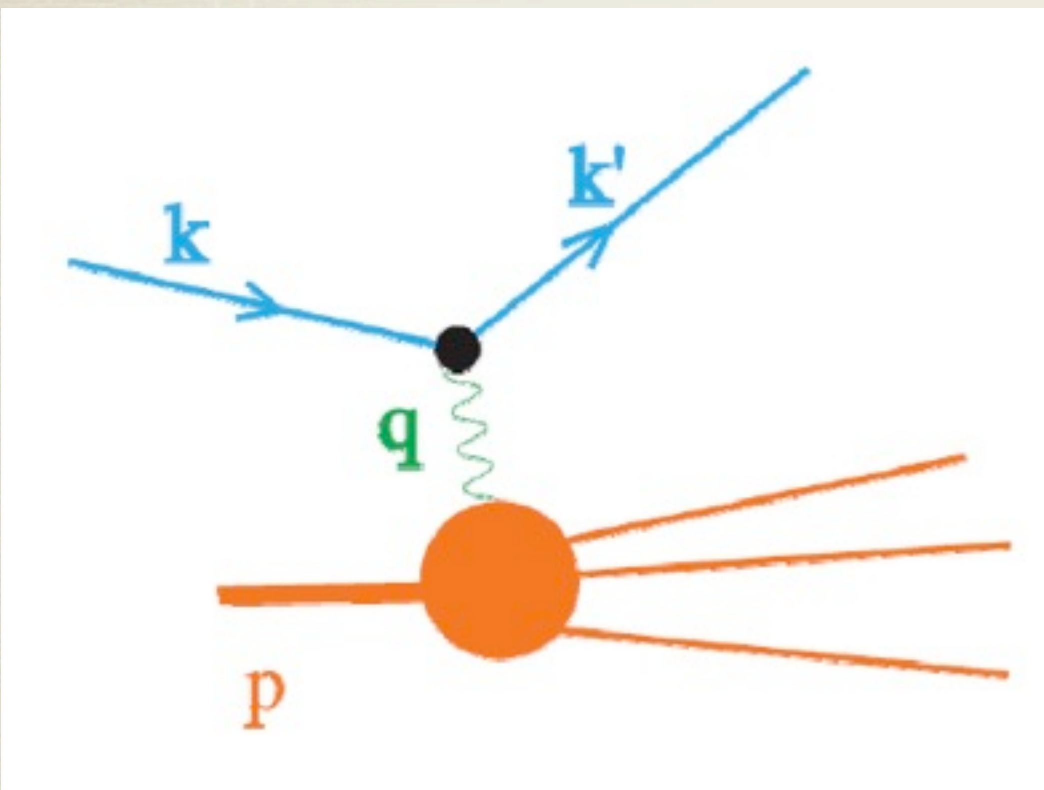
Butterworth et al., 0802.2470

- Boosted tops, W/Z bosons have been studied in various contexts

Example 2: Deep inelastic scattering and PDFs

Deep inelastic scattering

- Putting one hadron in the initial state leads to DIS ⇒ still gives most of our information on PDFs (ep at DESY)



Kinematics:

$$
\begin{aligned}
q^\mu &= k^\mu - k'^\mu \\
Q^2 &= -q^2 \\
x &= \frac{Q^2}{2P\cdot q} \\
y &= \frac{P\cdot q}{P\cdot k} \stackrel{lab}{=} \frac{E-E'}{E}
\end{aligned}
$$

$$
d\sigma = \frac{4\alpha^2}{s}\frac{d^3\vec{k}'}{2|\vec{k}'|}\frac{1}{Q^4}L^{\mu\nu}(k,q)W_{\mu\nu}(p,q)
$$

phase space scat. lepton

photon propagator²

leptonic tensor

hadronic tensor contains information about hadronic structure

Hadronic tensor

- Hermiticity, parity, current conservation allow us to simplify $W_{\mu\nu}$

EM current

$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4z\, e^{iq\cdot z}\langle P|J_\nu^\dagger(z) J_\mu(0)|P\rangle$$
$$= \left\{g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right\} F_1(x,Q^2) + \left\{P_\mu + \frac{q_\mu}{2x}\right\}\left\{P_\nu + \frac{q_\nu}{2x}\right\}\frac{F_2(x,Q^2)}{P\cdot q}$$

Structure functions

$$\frac{d\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left\{\left[1+(1-y)^2\right]\, F_1 + \frac{1-y}{x}\left[F_2 - 2x\, F_1\right]\right\}$$

- *Factorization* tells us that EM probe scatters off partons

PDFs

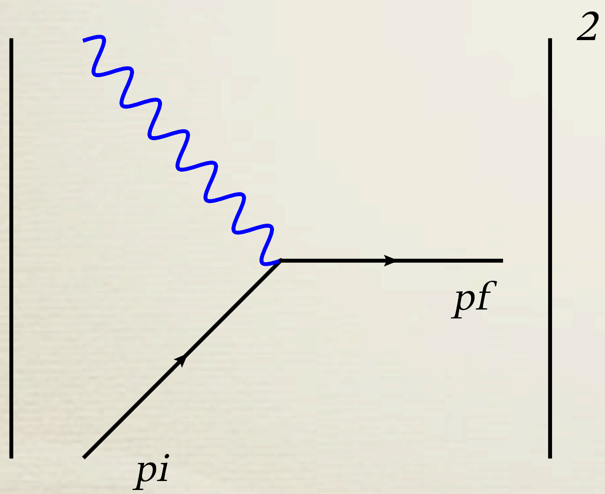
$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4z\, e^{iq\cdot z}\int_0^1 \frac{d\xi}{\xi}\sum_a f_a(\xi)\langle p|J_\nu^\dagger(z) J_\mu(0)|p\rangle_{p=\xi P}$$

Calculating the structure function

- We will calculate the structure function F_2. Note that we can obtain it by applying the following projection operator to W:

$$\begin{aligned} F_2 &= R^{\mu\nu} W_{\mu\nu} \\ R^{\mu\nu} &= \frac{2x}{d-2}\left\{ g^{\mu\nu} - 4\,(d-1)\frac{x^2}{Q^2} P^\mu P^\nu \right\} \end{aligned}$$

- Calculate by inserting a complete set of states between currents; at LO, have a single-quark final state:



- Parameterize momenta as:

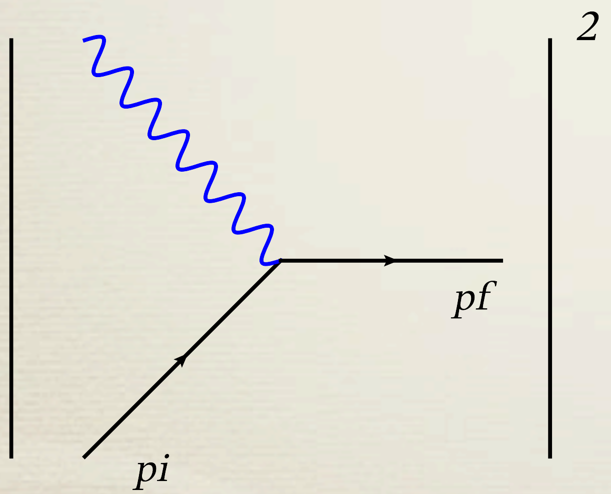
$$\begin{aligned} P^\mu &= \frac{Q}{2x}\left(1, \vec{0}, 1\right) \\ p^\mu &= \frac{\xi Q}{2x}\left(1, \vec{0}, 1\right) \\ q^\mu &= \left(0, \vec{0}, -Q\right) \end{aligned}$$

Calculating the structure function

- We will calculate the structure function F_2. Note that we can obtain it by applying the following projection operator to W:

$$\begin{aligned} F_2 &= R^{\mu\nu} W_{\mu\nu} \\ R^{\mu\nu} &= \frac{2x}{d-2}\left\{ g^{\mu\nu} - 4\,(d-1)\frac{x^2}{Q^2} P^\mu P^\nu \right\} \end{aligned}$$

- Calculate by inserting a complete set of states between currents; at LO, have a single-quark final state:



Derive the following phase space expression:

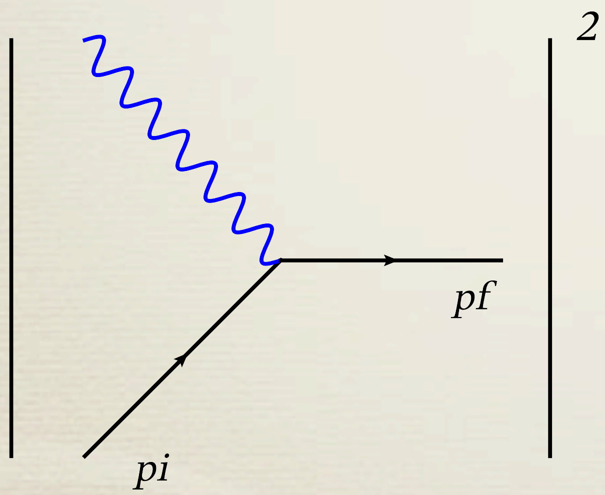
$$\begin{aligned} PS &= \int \frac{d^d p_f}{(2\pi)^{d-1}} \delta(p_f^2)(2\pi)^d \delta^{(d)}(q + p - p_f) \\ &= \frac{2\pi}{Q^2}\delta\left(1 - \frac{x}{\xi}\right) \end{aligned}$$

Calculating the structure function

- We will calculate the structure function F_2. Note that we can obtain it by applying the following projection operator to W:

$$
\begin{aligned}
F_2 &= R^{\mu\nu} W_{\mu\nu} \\
R^{\mu\nu} &= \frac{2x}{d-2}\left\{ g^{\mu\nu} - 4\,(d-1)\frac{x^2}{Q^2} P^\mu P^\nu \right\}
\end{aligned}
$$

- Calculate by inserting a complete set of states between currents; at LO, have a single-quark final state:

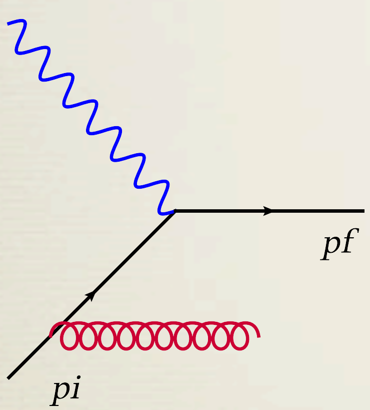


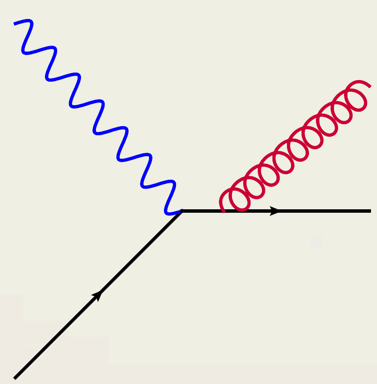
Obtain the structure function:

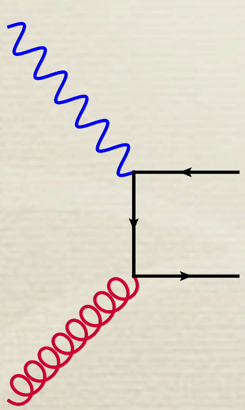
$$
\begin{aligned}
F_2 = \frac{1}{4\pi}\int \frac{d\xi}{\xi} \sum_q f_q(\xi) \times \frac{PS}{2N} \times R^{\mu\nu} \times \\
&= \sum_q e^2 Q_q^2 \int d\xi\, f_q(\xi)\, \xi\, \delta(x-\xi) \\
&= \sum_q e^2 Q_q^2\, x\, f_q(x)
\end{aligned}
$$

Scaling

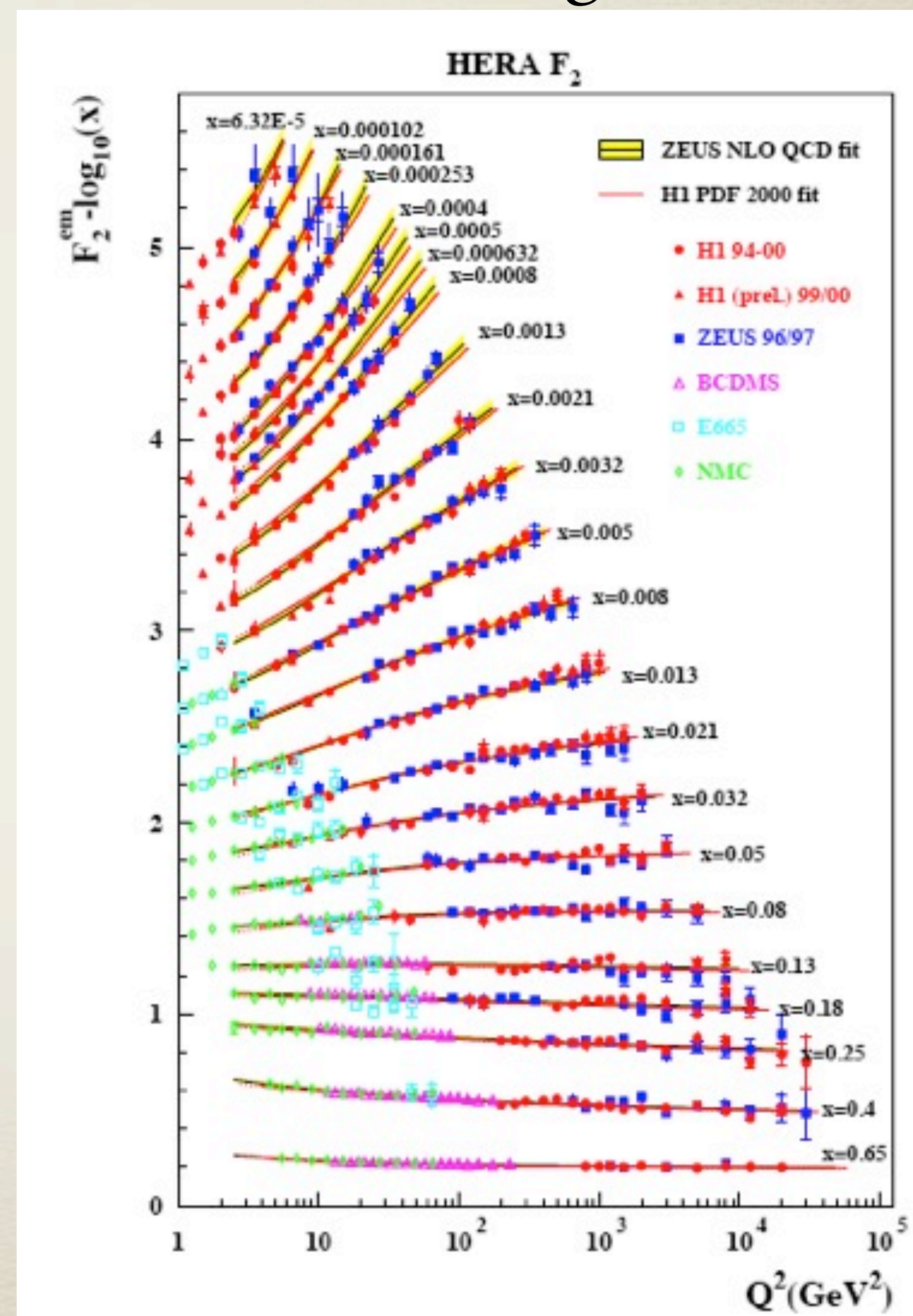
- No Q^2 dependence in $F_2 \Rightarrow$ ***scaling***, comes from scattering off point-like constituents of proton
  - Clearly a good approximation, but also clearly violated
  - Goal: check to see that QCD reproduces the scaling violation
  - Possible NLO real-emission terms:







$\Rightarrow$ we'll do the quark pieces and quote the answer for these



Real-emission phase space

- Focus on new aspects with respect to $e^+e^- \to$ hadrons; first, derive a useful parameterization of the phase space

$$
\begin{aligned}
PS =& \frac{1}{(2\pi)^{d-2}} \int d^d p_f d^d p_g \delta(p_g^2)\delta(p_f^2)\delta^{(d)}(q+p-p_f-p_g) \\
=& \frac{1}{(2\pi)^{d-2}} \int ds_{pg} \int d^d p_f d^d p_g \delta(p_g^2)\delta(p_f^2)\delta(s_{pg}+2p\cdot p_g)\delta^{(d)}(q+p-p_f-p_g)
\end{aligned}
$$

Parameterize p_g as: $p_g = (E, p_T, 0, k)$; use delta functions to remove these integrations. Set $s_{pg}=-Q^2\xi y/x$ to derive:

$$
PS = \frac{\Omega(d-2)}{4(2\pi)^{d-2}} \int_0^1 \left[Q^2 y(1-y)\frac{\xi}{x}\left(1-\frac{x}{\xi}\right)\right]^{-\epsilon}
$$

$$
\begin{aligned}
p \cdot p_g =& \frac{\xi}{2x} Q^2 y \\
p_f \cdot p_g =& \frac{\xi}{2x} Q^2 \left(1-\frac{x}{\xi}\right)
\end{aligned}
$$

Real-emission matrix elements

- Spin, color summed/averaged+projected matrix elements; focus on the potentially divergent terms

$$|\bar{\mathcal{M}}|^2 = 4\,C_F\,e^2 Q_q^2\,g_s^2 \mu^{2\epsilon}\left\{\frac{p_f\cdot p_g}{p\cdot p_g}+\frac{p\cdot p_g}{p_f\cdot p_g}+\frac{Q^2 p\cdot p_f}{p_f\cdot p_g\,p\cdot p_g}+\underbrace{\ldots}_{\text{finite terms}}\right\}$$

- Need to integrate over y, include $\frac{1}{4\pi}\int\frac{d\xi}{\xi}f_q(\xi)$

$$\begin{aligned}F_{2,q}^{(1),real} &= e^2 Q_q^2\,x\,\frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\left[\frac{Q^2}{4\pi\mu^2}\right]^{-\epsilon}\left(\frac{x}{\xi}\right)^{\epsilon}\left(1-\frac{x}{\xi}\right)^{-\epsilon}\\ &\times \int_x^1\frac{d\xi}{\xi}f_q(\xi)\left\{-\frac{C_F}{\epsilon}\frac{1+(x/\xi)^2}{1-x/\xi}-2C_F\frac{x/\xi}{1-x/\xi}+\ldots\right\}\end{aligned}$$

This term is bad news, no way it can cancel against virtual correction, which are δ(x-ξ)

Looks like P_{qq} ⇒ collinear singularity

Notice the singularity when x= ξ ⇒ soft singularity

Factorization of IR singularities

- We are not satisfying the KLN theorem, which tells us to sum over degenerate final **and initial** states. The quark from the proton can emit a collinear gluon. This changes the momentum of the quark that enters the partonic scattering process, but is indistinguishable. The virtuality associated with this splitting is very small, and this is a long-distance effect sensitive to low-energy QCD.
- Solution: must absorb initial-state collinear singularity into PDF. Redo calculation with $f_q \rightarrow f_{q,0}$, a bare PDF. Choose the bare PDF to remove $1/\varepsilon$ pole.
- Must also add virtual corrections, deal with the $x=\xi$ soft singularity of real emission.

Factorization of IR singularities

- We will perform this 'mass factorization; step-by-step. First define a *plus distribution*:

$$\int_0^1 dx\, f(x)\,[g(x)]_+ = \int_0^1 dx\, g(x)\,[f(x)-f(0)]$$ ⇒ if g ~1/x, removes singularity at x=0

  - After adding virtual corrections and rearranging, our result for the divergent part of F_2 is:

$$F_{2,q} = e^2 Q_q^2\, x \int_x^1 \frac{d\xi}{\xi} f_{q,0}(\xi) \left\{ \delta(1-x/\xi) + \frac{\alpha_s}{2\pi\Gamma(1-\epsilon)} \left[\frac{Q^2}{4\pi\mu^2}\right]^{-\epsilon} \left[-\frac{1}{\epsilon} P_{qq}(x/\xi) + \text{finite} \right] \right\}$$

$$P_{qq}(x) = C_F \left[\frac{1+x^2}{[1-x]_+} + \frac{3}{2}\delta(1-x) \right] \left(\Rightarrow \int_0^1 P_{qq}(x) = 0 \right)$$ ⟵ quark-number conservation

Factorization of IR singularities

- We will perform this 'mass factorization'; step-by-step. First define a *plus distribution*:

$$\int_0^1 dx\, f(x)\, [g(x)]_+ = \int_0^1 dx\, g(x)\, [f(x) - f(0)]$$ ⇒ if g ~1/x, removes singularity at x=0

  - Redefine the PDF according to:

$$f_q(x,\mu^2) = f_{q,0}(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi} f_{q,0}(\xi)\left\{-\frac{1}{\epsilon}P_{qq}(x/\xi) + C(x/\xi)\right\}$$

$\overline{\text{MS}}$: C chosen to remove ln(4π)-γ_E

  - Arrive at the structure function:

$$F_{2,q} = e^2 Q_q^2\, x \int_x^1 \frac{d\xi}{\xi} f_q(\xi,\mu^2)\left\{\delta(1-x/\xi) + \frac{\alpha_s}{2\pi}\left[P_{qq}(x/\xi)\ln\frac{Q^2}{\mu^2} + \text{finite}\right]\right\}$$

Scale variation and DGLAP

- Pole turns into a ln(μ^2) dependence ⇒ F_2 must be independent of this arbitrary *factorization scale*, which leads to an evolution equation for the PDF. Renormalization⇒Evolution.

$$\frac{d\, f_q(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi} f_q(\xi,\mu^2) P_{qq}(x/\xi) \qquad \Rightarrow \textit{\textbf{DGLAP equation}}$$

☑ Leads to a ln(Q^2) dependence of F_2 ⇒ explains the observed scaling violation

- Inclusion of the gluon-initiated partonic processes:

$$\begin{aligned} F_{2,q} &= e^2 Q_q^2\, x \int_x^1 \frac{d\xi}{\xi} f_q(\xi,\mu^2)\left\{\delta(1-x/\xi) + \frac{\alpha_s}{2\pi}\left[P_{qq}(x/\xi)\ln\frac{Q^2}{\mu^2} + \text{finite}\right]\right\} \\ &+ e^2 Q_q^2\, x \int_x^1 \frac{d\xi}{\xi} f_g(\xi,\mu^2)\left\{\frac{\alpha_s}{2\pi}\left[P_{qg}(x/\xi)\ln\frac{Q^2}{\mu^2} + \text{finite}\right]\right\} \end{aligned}$$

$$\frac{d}{d\ln\mu^2}\begin{pmatrix} f_q(x,\mu^2) \\ f_g(x,\mu^2)\end{pmatrix} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\begin{pmatrix} P_{qq}(x/\xi) & P_{qg}(x/\xi) \\ P_{gq}(x/\xi) & P_{gg}(x/\xi)\end{pmatrix}\begin{pmatrix} f_q(x,\mu^2) \\ f_g(x,\mu^2)\end{pmatrix}$$

PDFs

- Get much of our knowledge of PDFs from the DIS process
- PDFS enter every hadron collider prediction, so we'd better know them well. Non-perturbative objects with perturbative evolution. $f(x,Q^2)$: DGLAP governs Q^2 dependence, so we need to extract the x dependence from data.
- On the market today: CTEQ, MSTW, NNPDF (*global fits*) ABM, HERAPDF, JR (non-global)
- Basic idea:

$$\underbrace{\text{hadronic cross section}}_{\text{measure}} = \underbrace{\text{PDFs}}_{\text{extract}} \otimes \underbrace{\text{partonic cross section}}_{\text{calculate}}$$

Determining PDFs

- In more detail (from the *Handbook of Perturbative QCD*):

1. Develop a program to numerically solve the evolution equations — a set of coupled integro-differential equations;
2. Make a choice on experimental data sets, such that the data can give the best constraints on the parton distributions;
3. Select the factorization scheme — the "DIS" or the "$\overline{\text{MS}}$" scheme, and make a consistent set of choices on factorization scale for all the processes;
4. Choose the parametric form for the input parton distributions at μ_0, and then evolve the distributions to any other values of μ_f;
5. Use the evolved distributions to calculate χ^2 between theory and data, and choose an algorithm to minimize the χ^2 by adjusting the parameterizations of the input distributions;
6. Parameterize the final parton distributions at discrete values of x and μ_f by some analytical functions.

1. Develop a program to numerically solve the evolution equations — a set of coupled integro-differential equations;
2. Make a choice on experimental data sets, such that the data can give the best constraints on the parton distributions;
3. Select the factorization scheme — the "DIS" or the "$\overline{\text{MS}}$" scheme, and make a consistent set of choices on factorization scale for all the processes;
4. Choose the parametric form for the input parton distributions at μ_0, and then evolve the distributions to any other values of μ_f;
5. Use the evolved distributions to calculate χ^2 between theory and data, and choose an algorithm to minimize the χ^2 by adjusting the parameterizations of the input distributions;
6. Parameterize the final parton distributions at discrete values of x and μ_f by some analytical functions.

- Global fits typically use HERA charged and neutral current data; fixed-target Drell-Yan and DIS; jet production from the Tevatron/LHC; W/Z data from the Tevatron/LHC
- Non-global fits typically remove one or more of these for various reasons; for example, ABM neglects jet production, since it's not known at NNLO in pQCD

from MSTW:

Process	Subprocess	Partons	x range
$\ell^\pm\{p,n\} \to \ell^\pm X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
$\ell^\pm n/p \to \ell^\pm X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
$pp \to \mu^+\mu^- X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \to \mu^+\mu^- X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu}) N \to \mu^-(\mu^+) X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
$\nu N \to \mu^-\mu^+ X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu} N \to \mu^+\mu^- X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
$e^\pm p \to e^\pm X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
$e^+ p \to \bar{\nu} X$	$W^+\{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
$e^\pm p \to e^\pm c\bar{c} X$	$\gamma^* c \to c, \gamma^* g \to c\bar{c}$	c, g	$0.0001 \lesssim x \lesssim 0.01$
$e^\pm p \to \text{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p} \to \text{jet} + X$	$gg, qg, qq \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
$p\bar{p} \to (W^\pm \to \ell^\pm\nu) X$	$ud \to W, \bar{u}\bar{d} \to W$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
$p\bar{p} \to (Z \to \ell^+\ell^-) X$	$uu, dd \to Z$	d	$x \gtrsim 0.05$

fixed-target DY and DIS

HERA

Tevatron

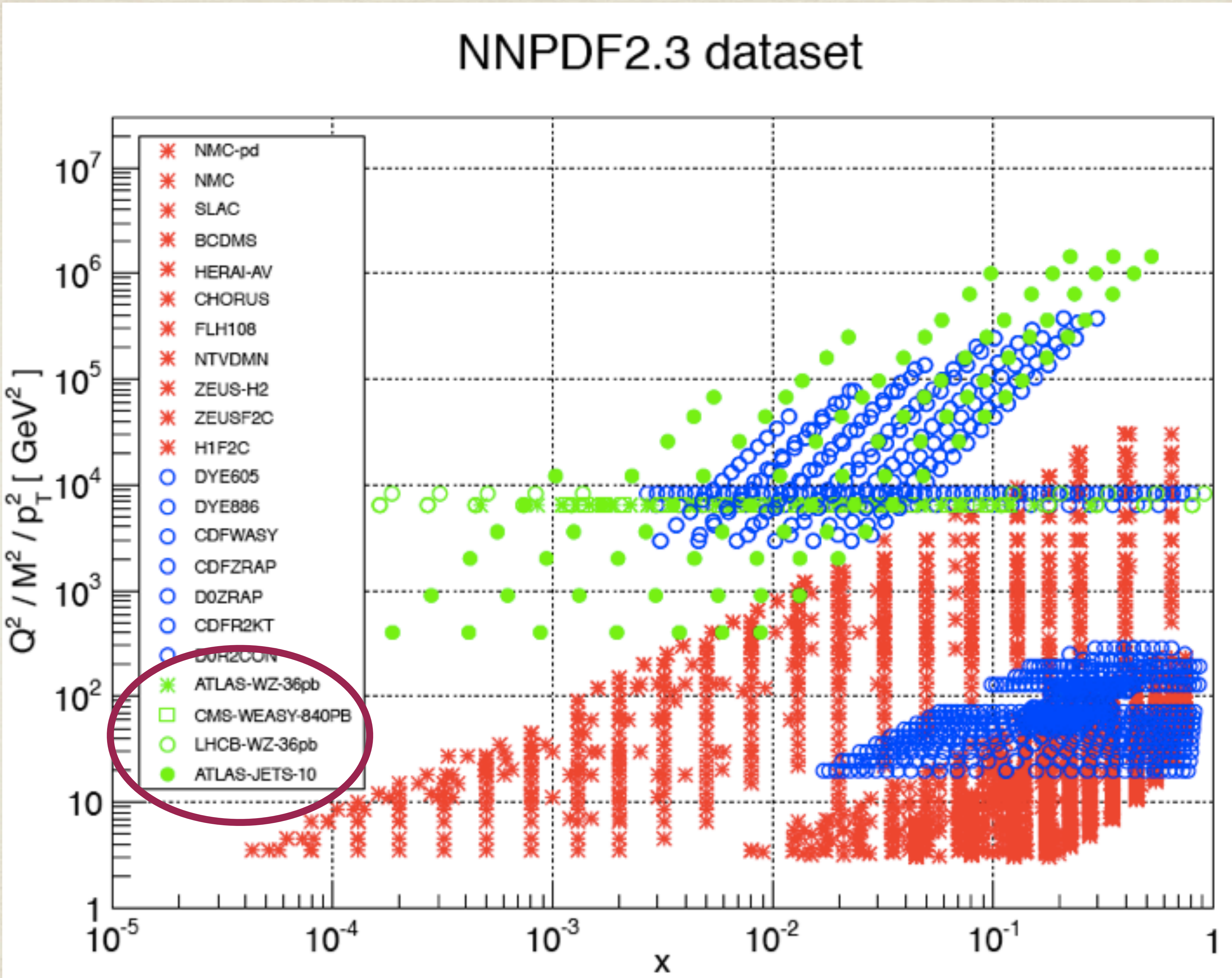
1. Develop a program to numerically solve the evolution equations — a set of coupled integro-differential equations;
2. Make a choice on experimental data sets, such that the data can give the best constraints on the parton distributions;
3. Select the factorization scheme — the "DIS" or the "$\overline{\text{MS}}$" scheme, and make a consistent set of choices on factorization scale for all the processes;
4. Choose the parametric form for the input parton distributions at μ_0, and then evolve the distributions to any other values of μ_f;
5. Use the evolved distributions to calculate χ^2 between theory and data, and choose an algorithm to minimize the χ^2 by adjusting the parameterizations of the input distributions;
6. Parameterize the final parton distributions at discrete values of x and μ_f by some analytical functions.

- Global fits typically use HERA charged and neutral current data; fixed-target Drell-Yan and DIS; jet production from the Tevatron/LHC; W/Z data from the Tevatron/LHC
- Non-global fits typically remove one or more of these for various reasons; for example, ABM neglects jet production, since it's not known at NNLO in pQCD

from MSTW:

Process	Subprocess	Partons	x range
$\ell^\pm \{p,n\} \to \ell^\pm X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
$\ell^\pm n/p \to \ell^\pm X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
$pp \to \mu^+\mu^- X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \to \mu^+\mu^- X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu}) N \to \mu^-(\mu^+) X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
$\nu N \to \mu^-\mu^+ X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu} N \to \mu^+\mu^- X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
$e^\pm p \to e^\pm X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
$e^+ p \to \bar{\nu} X$	$W^+ \{d, s\} \to \{u, c\}$	d, s	$x \gtrsim 0.01$
$e^\pm p \to e^\pm c\bar{c} X$	$\gamma^* c \to c, \gamma^* g \to c\bar{c}$	c, g	$0.0001 \lesssim x \lesssim 0.01$
$e^\pm p \to \text{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p} \to \text{jet} + X$	$gg, qg, qq \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
$p\bar{p} \to (W^\pm \to \ell^\pm \nu) X$	$ud \to W, \bar{u}\bar{d} \to W$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
$p\bar{p} \to (Z \to \ell^+\ell^-) X$	$uu, dd \to Z$	d	$x \gtrsim 0.05$

Need this large multiplicity to get all partons across the needed range of x



LHC data making an important appearance!

1. Develop a program to numerically solve the evolution equations — a set of coupled integro-differential equations;
2. Make a choice on experimental data sets, such that the data can give the best constraints on the parton distributions;
3. Select the factorization scheme — the "DIS" or the "$\overline{\text{MS}}$" scheme, and make a consistent set of choices on factorization scale for all the processes;
4. Choose the parametric form for the input parton distributions at μ_0, and then evolve the distributions to any other values of μ_f;
5. Use the evolved distributions to calculate χ^2 between theory and data, and choose an algorithm to minimize the χ^2 by adjusting the parameterizations of the input distributions;
6. Parameterize the final parton distributions at discrete values of x and μ_f by some analytical functions.

- $\overline{\text{MS}}$ scheme most commonly chosen these days
- Another issue that should appear here: to what order in pQCD are the partonic cross sections calculated?
- All the ones referenced previously (CTEQ, MSTW, NNPDF; ABM, HERAPDF, JR) provide both NLO and NNLO fits
- Note that the NNLO fits of CTEQ, MSTW, NNPDF use NLO QCD predictions for jet production

1. Develop a program to numerically solve the evolution equations — a set of coupled integro-differential equations;
2. Make a choice on experimental data sets, such that the data can give the best constraints on the parton distributions;
3. Select the factorization scheme — the "DIS" or the "$\overline{\text{MS}}$" scheme, and make a consistent set of choices on factorization scale for all the processes;
4. Choose the parametric form for the input parton distributions at μ_0, and then evolve the distributions to any other values of μ_f;
5. Use the evolved distributions to calculate χ^2 between theory and data, and choose an algorithm to minimize the χ^2 by adjusting the parameterizations of the input distributions;
6. Parameterize the final parton distributions at discrete values of x and μ_f by some analytical functions.

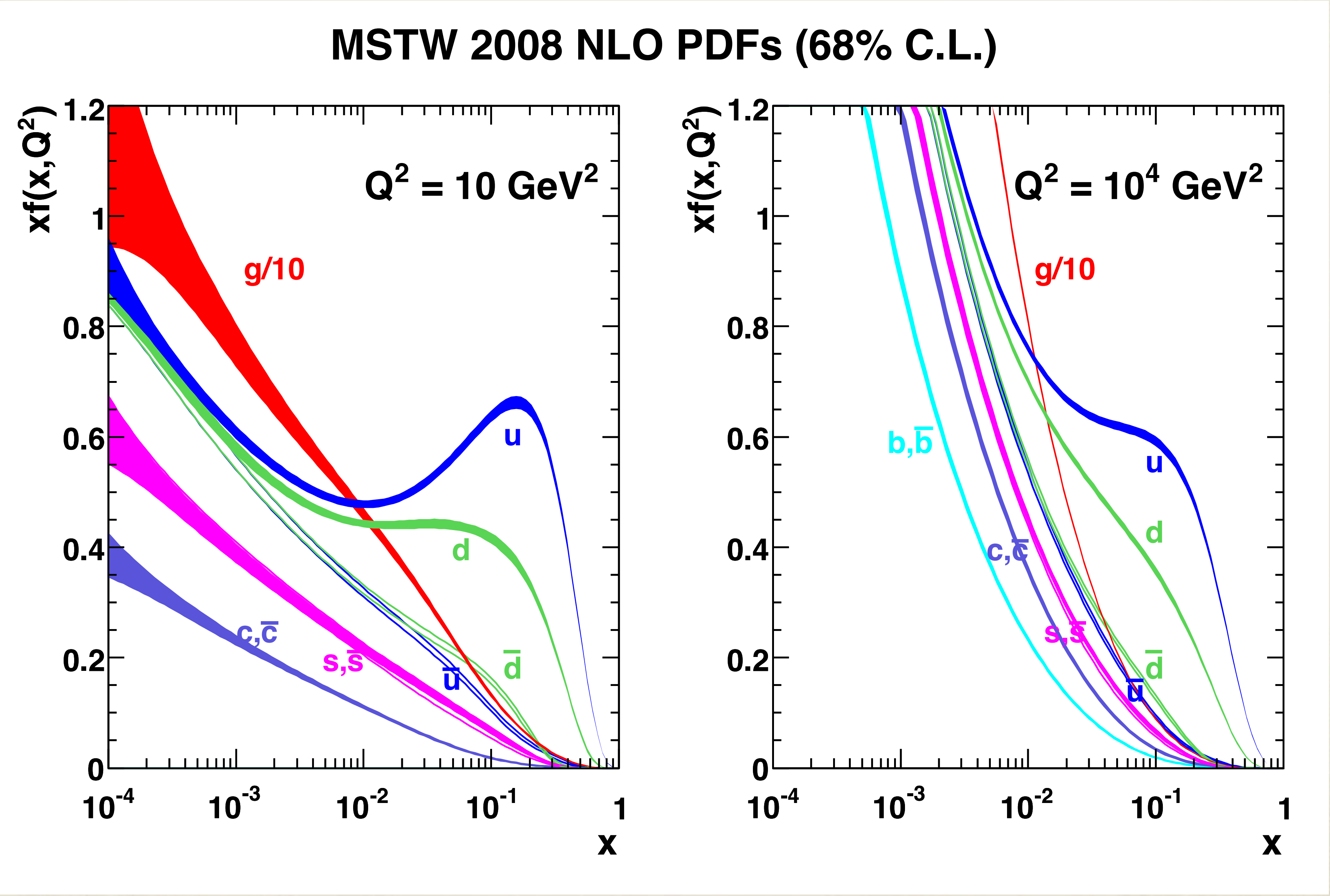
- Traditional choice of CTEQ and MSTW: $f(x,\mu_0) = A_0 x^{A_1}(1-x)^{A_2}P(x)$

from CTEQ: $$q_v(x,\mu_0) = q(x,\mu_0) - \bar{q}(x,\mu_0) = a_0\, x^{a_1}\,(1-x)^{a_2}\,\exp(a_3 x + a_4 x^2 + a_5\sqrt{x})$$

- NNPDF uses instead a neural network parameterization to remove bias: $f(x,\mu_0) = c(x)\times NN(x)$

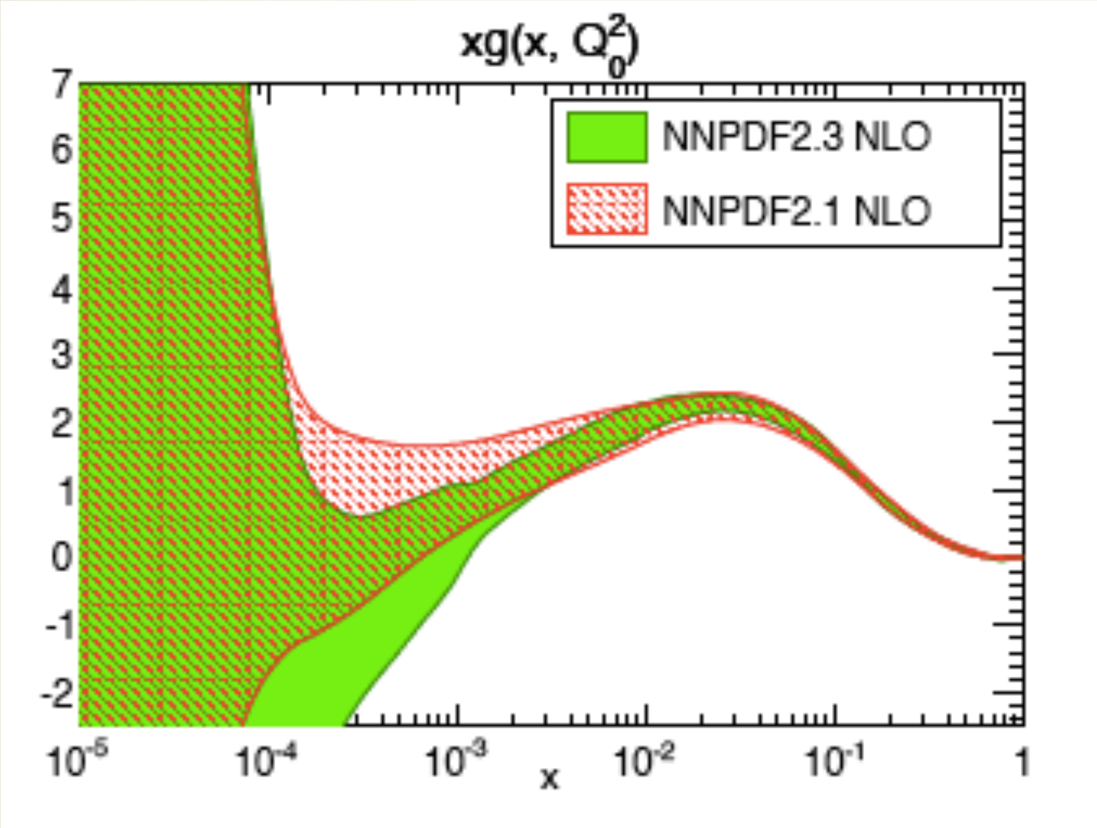
LHC PDFs

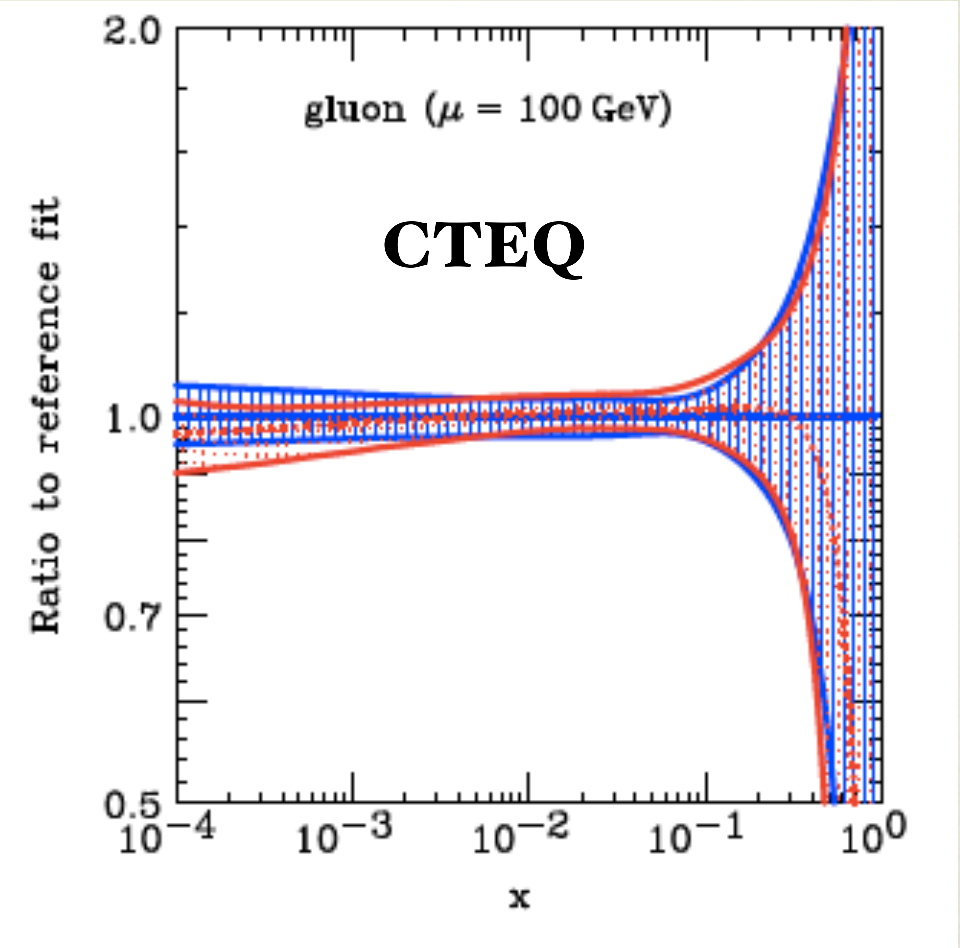
- Lots of gluons!



PDF errors

- Published sets come with errors… what do they mean?





For technical details on how to propagate these errors through to obtain the error on a cross section, see 1101.0536

PDF errors

- Published sets come with errors… what do they mean?

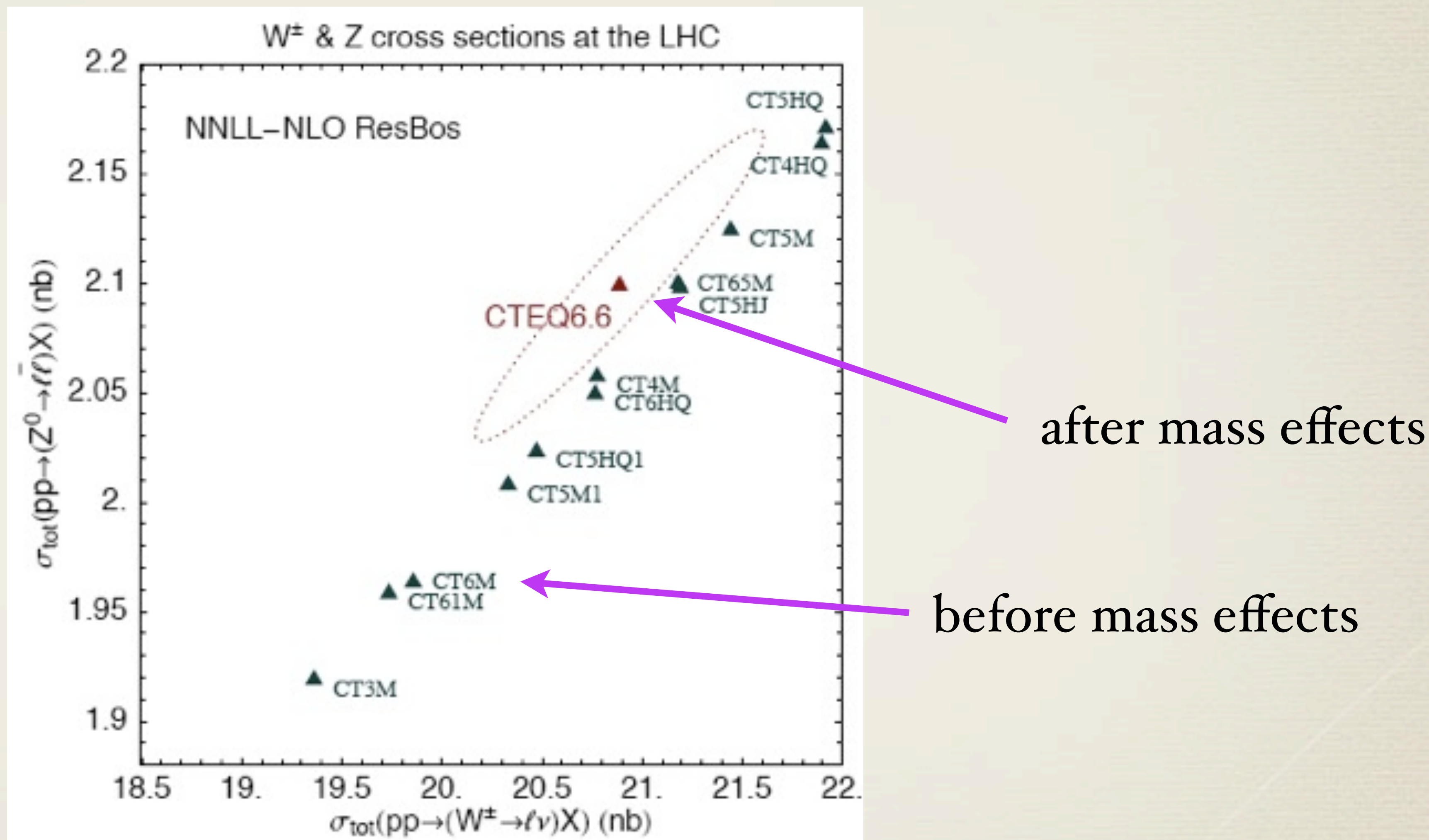
  - There are many sources of uncertainty in the PDFs, some of which we've touched on
    - Data set choice
    - Kinematic cuts
    - Parametrization choices
    - Treatment of heavy quarks, target mass corrections, and higher twist terms
    - Order of perturbation theory
    - Errors on the data ➔ ***Only* error included!**
  - Techniques have been developed to handle the last one
  - The others require judgement and experience, but *are not* included in what are generally referred to as PDF errors.

Review by J. Owens at CTEQ 2007 summer school,
http://www.phys.psu.edu/~cteq/schools/summer07/

PDF error examples

- Some examples meant to recommend caution when interpreting quoted errors

CTEQ, P. Nadolsky et al. '08



- Inclusion of m_c, m_b suppresses F_2 at low $Q^2 \Rightarrow$ increase u,d to compensate
- 6-7% increase in LHC W, Z predictions; well outside the quoted error
- Note that the estimated uncertainty from higher-order QCD is 1%

PDF error examples

- Some examples meant to recommend caution when interpreting quoted errors

MSTW 2008 PDF release arXiv:0901.0002

- Run II inclusive jet data
- Quark-mass effects
- Gluon density decreased at x~0.1

M_H=170 GeV Higgs at Tevatron (pb):

MRST 2001	MRST 2004	MRST 2006	MSTW 2008
0.3833	0.3988	0.3943	0.3444

Anastasiou, Boughezal, FP 0811.3458

~15% decrease in predicted cross section !
Previous 90% CL error: ±5%

Importance of global fits

- Error estimates from non-global fits must be carefully scrutinzed

Example: ABM + JR
@ Tevatron

ABM vs MSTW
at 160 GeV

-30% (>5 sigma)

Cross section in picobarns

M_H (GeV)	ABM10 [8]	ABKM09 [9]	JR [10]	MSTW08 [11]	HERAPDF [12]
100	1.438 ± 0.066	1.380 ± 0.076	1.593 ± 0.091	1.682 ± 0.046	1.417
110	1.051 ± 0.052	1.022 ± 0.061	1.209 ± 0.078	1.265 ± 0.038	1.055
115	0.904 ± 0.047	0.885 ± 0.055	1.060 ± 0.072	1.104 ± 0.034	0.917
120	0.781 ± 0.042	0.770 ± 0.050	0.933 ± 0.067	0.968 ± 0.031	0.800
125	0.677 ± 0.038	0.672 ± 0.045	0.823 ± 0.062	0.851 ± 0.029	0.700
130	0.588 ± 0.034	0.589 ± 0.041	0.729 ± 0.058	0.752 ± 0.026	0.615
135	0.513 ± 0.031	0.518 ± 0.037	0.647 ± 0.054	0.666 ± 0.024	0.541
140	0.449 ± 0.028	0.456 ± 0.034	0.576 ± 0.050	0.591 ± 0.022	0.479
145	0.394 ± 0.025	0.403 ± 0.031	0.514 ± 0.047	0.527 ± 0.020	0.424
150	0.347 ± 0.023	0.358 ± 0.028	0.461 ± 0.044	0.471 ± 0.018	0.377
155	0.306 ± 0.020	0.318 ± 0.026	0.413 ± 0.041	0.421 ± 0.017	0.336
160	0.271 ± 0.019	0.283 ± 0.024	0.371 ± 0.039	0.378 ± 0.016	0.300
165	0.240 ± 0.017	0.253 ± 0.022	0.335 ± 0.036	0.341 ± 0.014	0.269
170	0.213 ± 0.015	0.226 ± 0.020	0.302 ± 0.034	0.307 ± 0.013	0.241
175	0.190 ± 0.014	0.203 ± 0.019	0.274 ± 0.032	0.278 ± 0.012	0.217
180	0.169 ± 0.013	0.182 ± 0.017	0.248 ± 0.030	0.251 ± 0.012	0.195
185	0.151 ± 0.012	0.164 ± 0.016	0.225 ± 0.028	0.228 ± 0.011	0.176
190	0.136 ± 0.011	0.148 ± 0.015	0.205 ± 0.027	0.207 ± 0.010	0.159
200	0.109 ± 0.009	0.121 ± 0.013	0.170 ± 0.024	0.172 ± 0.009	0.131

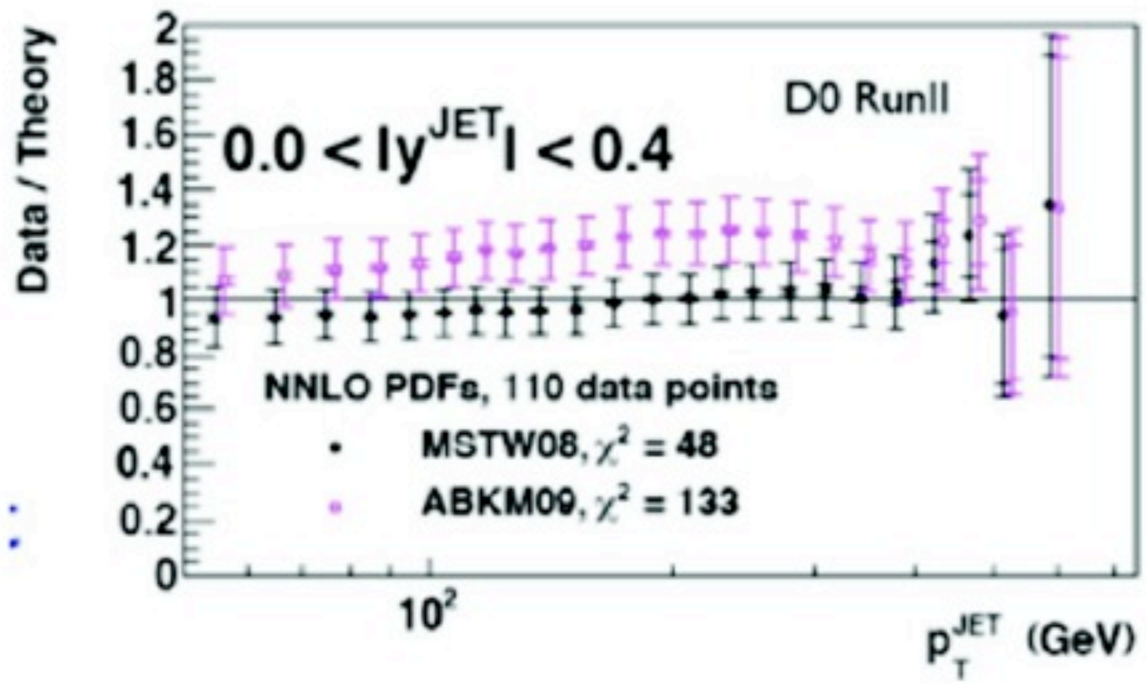
from D. deFlorian

Importance of global fits

- Error estimates from non-global fits must be carefully scrutinzed

• Interesting exercise by Thorne and Watt (2011)

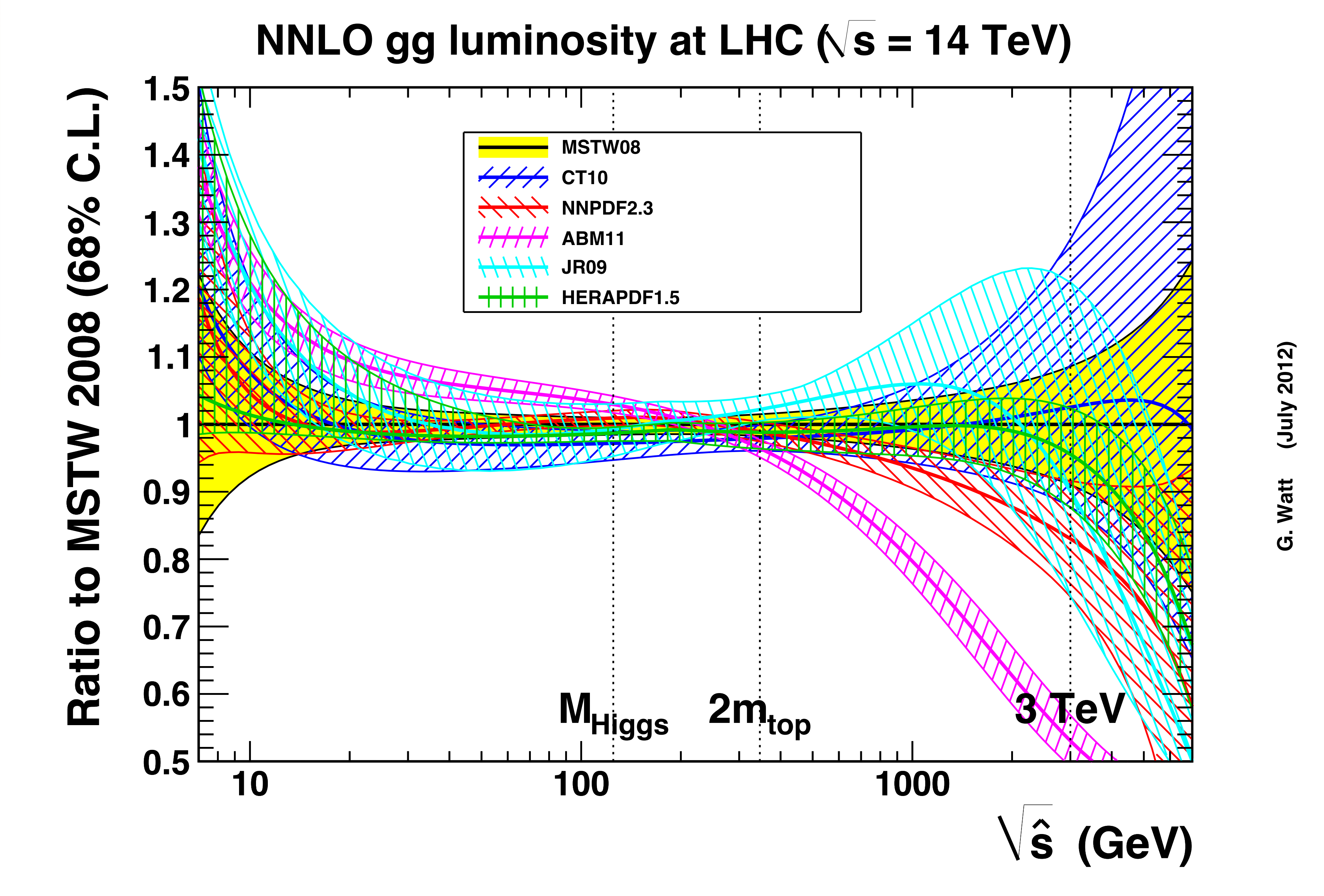
➡ Check how well PDFs reproduce Tevatron jet data



Message from Thorne and Watt: only global analysis provide accurate distributions and uncertainties. No acceptable description of jet data from non-global sets

PDF summary

- Multiple methodologies to cross-check and LHC data gradually increasing robustness of PDF central values and errors
- Global fits in agreement to ~10% over entire kinematic range

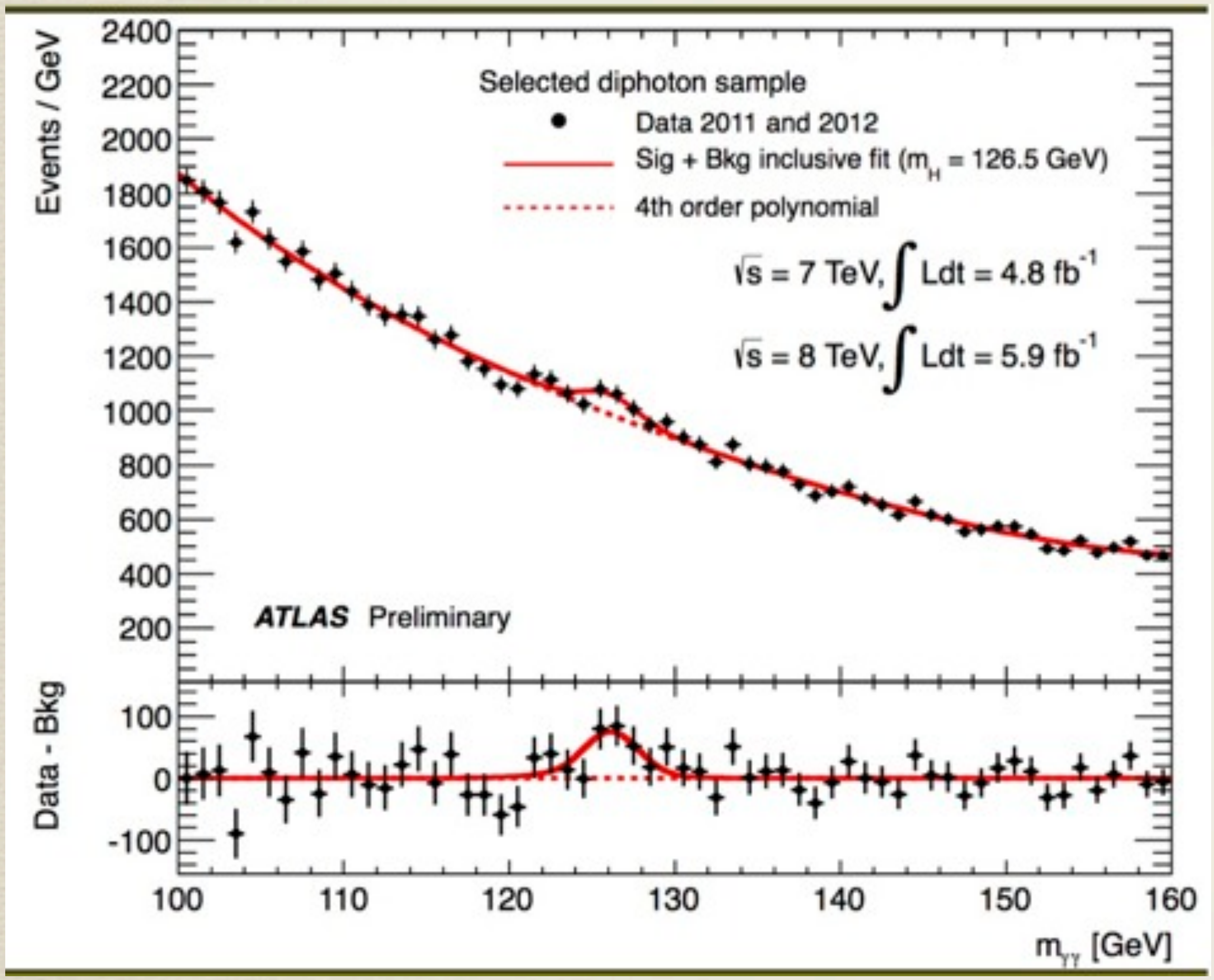


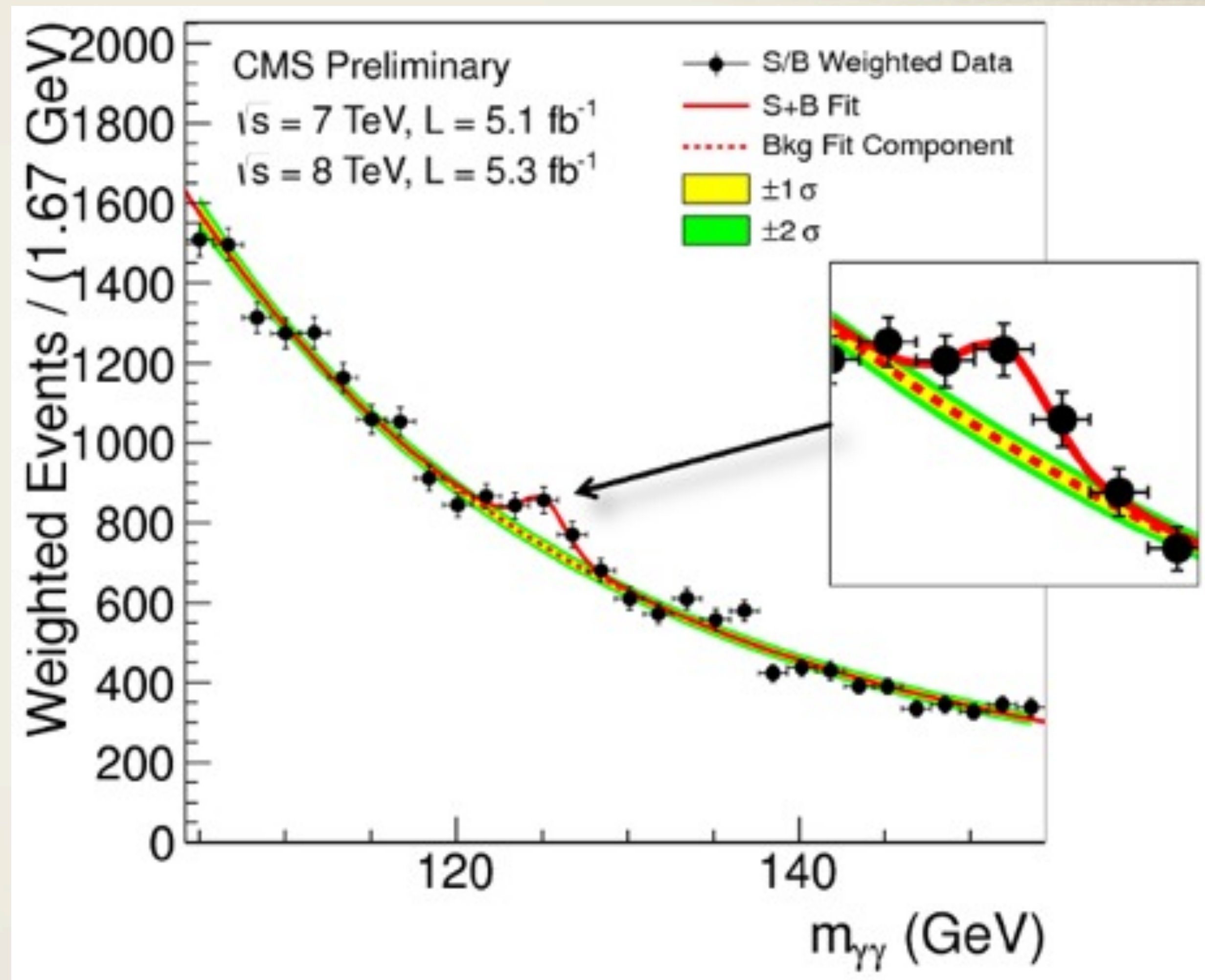
For more details and all references, see 1101.0536

Example 3: Higgs production at NLO

'Higgs' discovery

- You might have heard about the potential discovery of the Higgs recently:

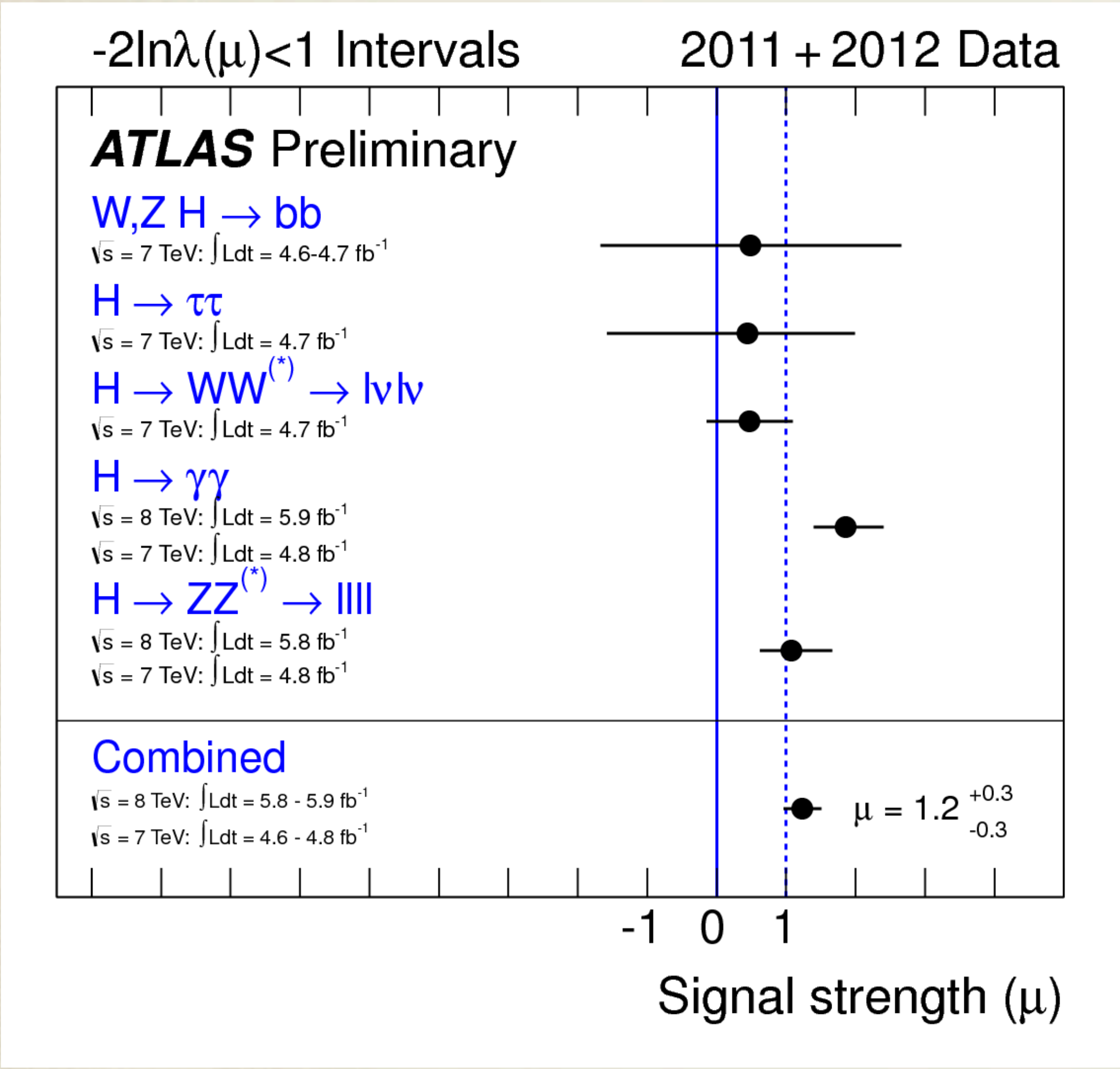


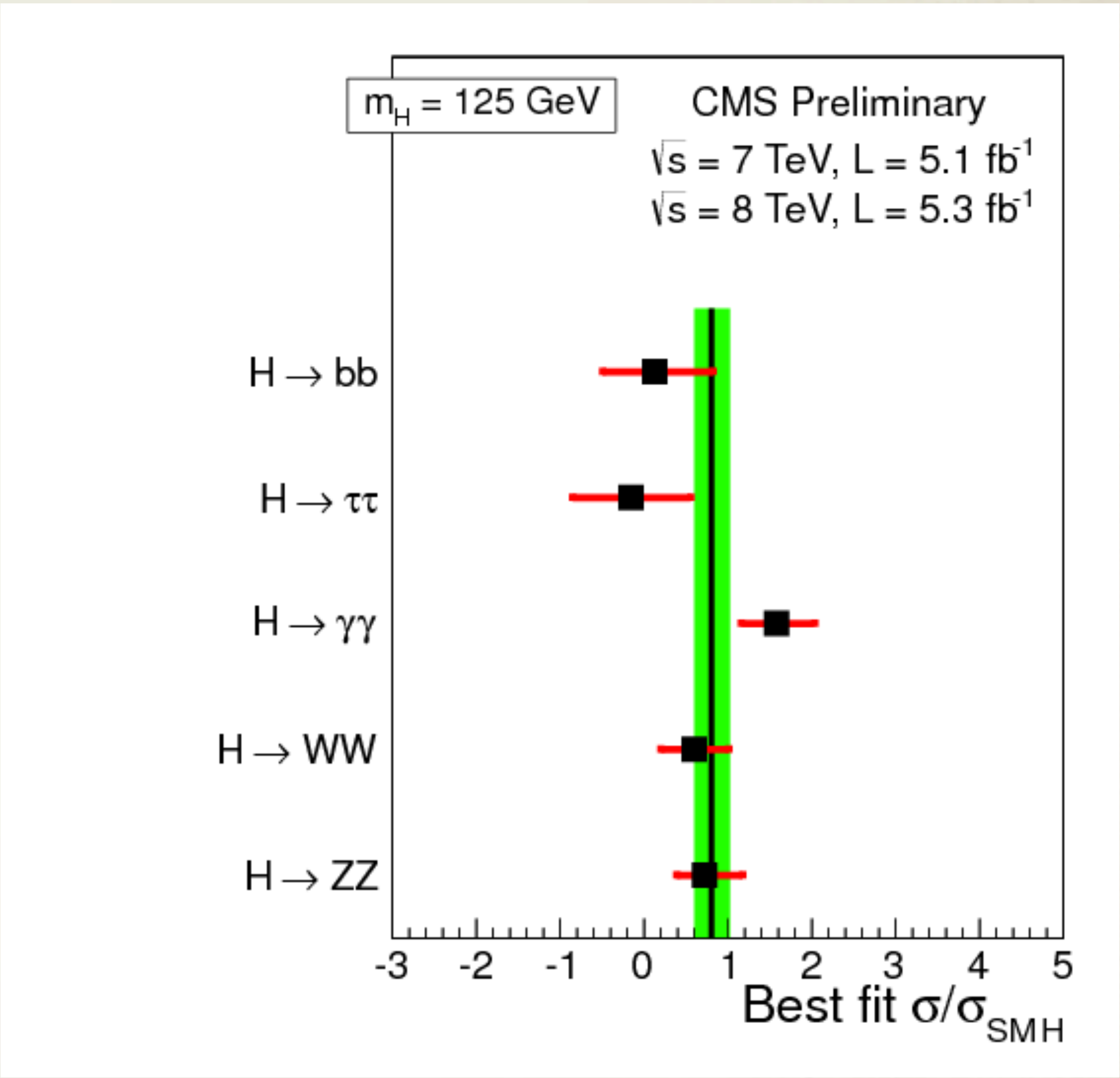


See lectures by Sally Dawson and Tom LeCompte for more on the Higgs boson

What we know so far

- Gross properties of the new state roughly indicate SM-like couplings

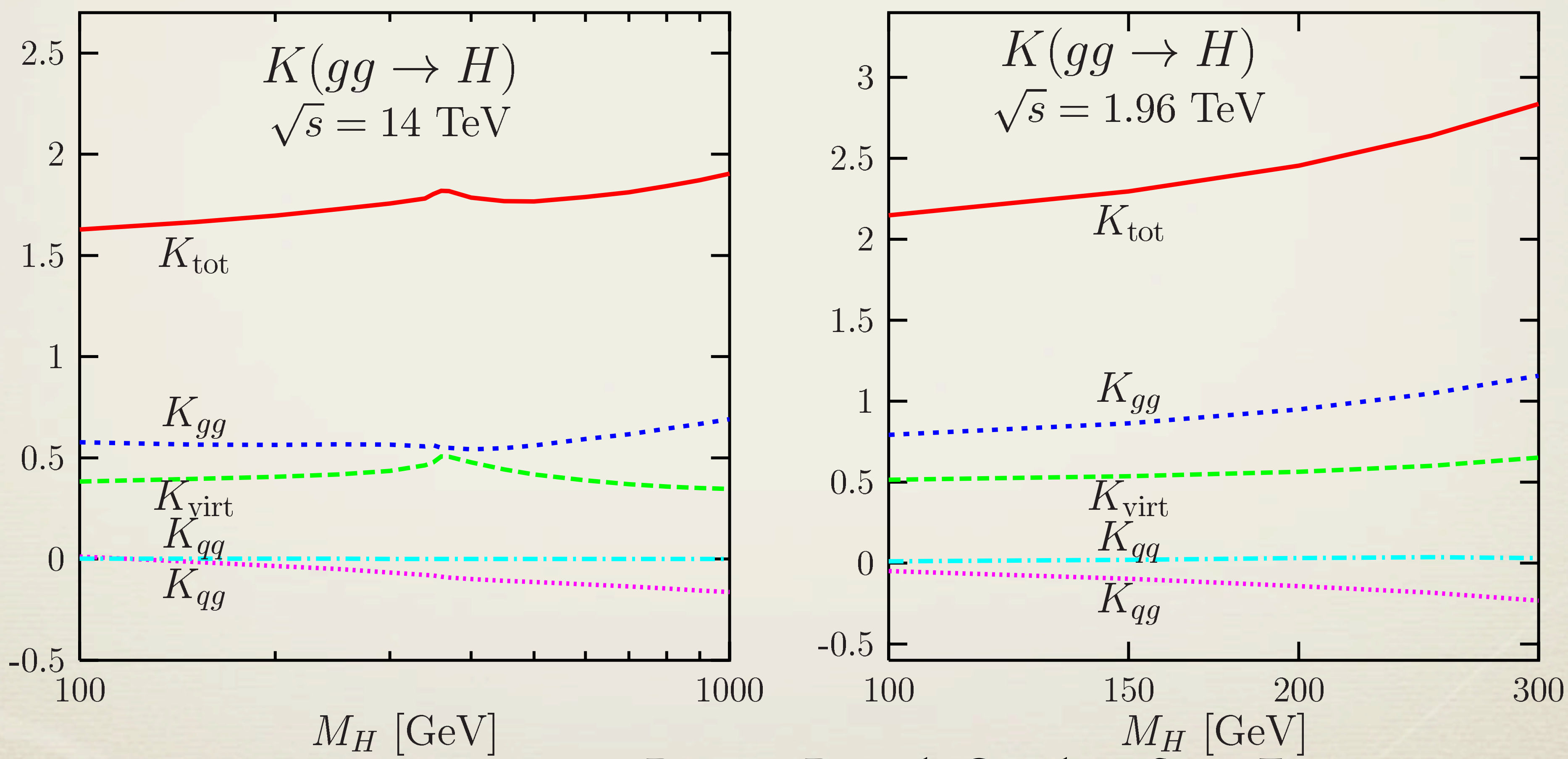




- Biggest signals in γγ and ZZ, which proceed primarily via gg→h

Trouble at NLO

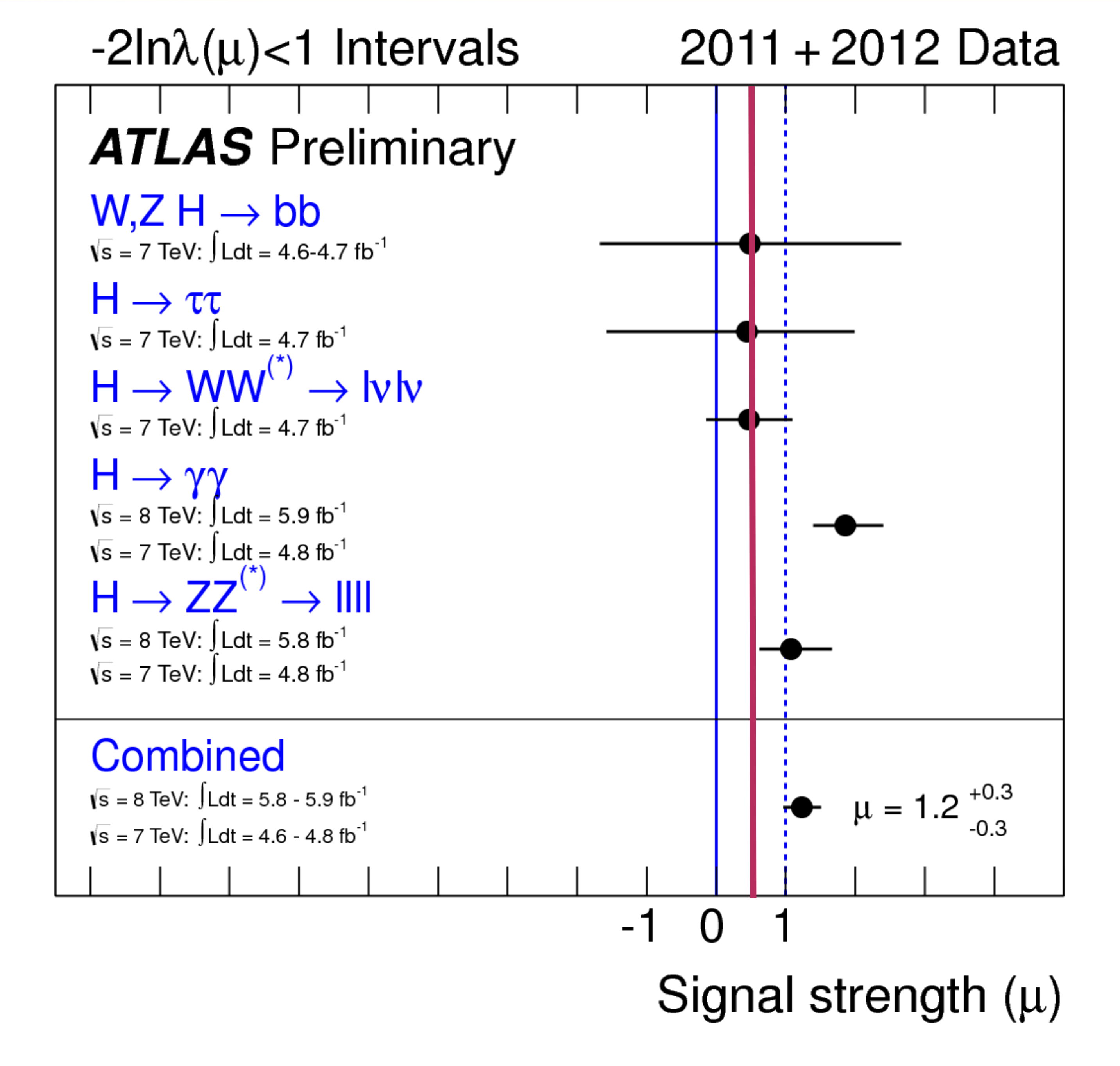
- We showed this plot before indicating that the corrections are large. Our goal now is to compute the NLO cross section for this process and understand why.



Dawson; Djouadi, Graudenz, Spira, Zerwas 1991, 1995

Trouble at NLO

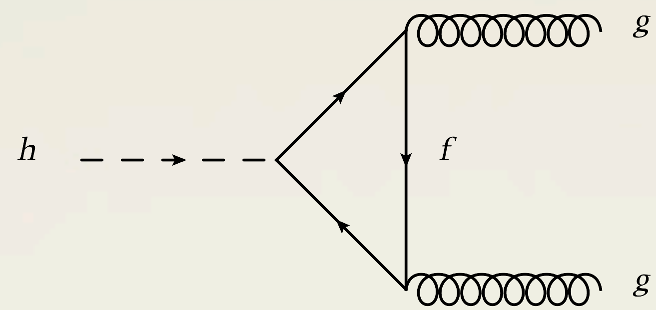
- We showed this plot before indicating that the corrections are large. Our goal now is to compute the NLO cross section for



- Without a detailed understanding of QCD, we would have a factor of 3 excess in the $\gamma\gamma$ channel... and even more theoretical frenzy about beyond the SM physics

Gluon fusion at LO

- Can calculate the LO cross section ⇒ already 1-loop!



$+$ *crossed diagram*

$$\sigma^{LO}_{gg\to h} = \frac{G_F \alpha_s^2}{288\pi\sqrt{2}} \left| \frac{3}{4} \sum_Q \mathcal{F}_{1/2}(\tau_Q) \right|^2 \delta(1-z), \quad \tau_Q = \frac{M_H^2}{4m_Q^2}. \quad z = \frac{M_H^2}{\hat{s}}$$

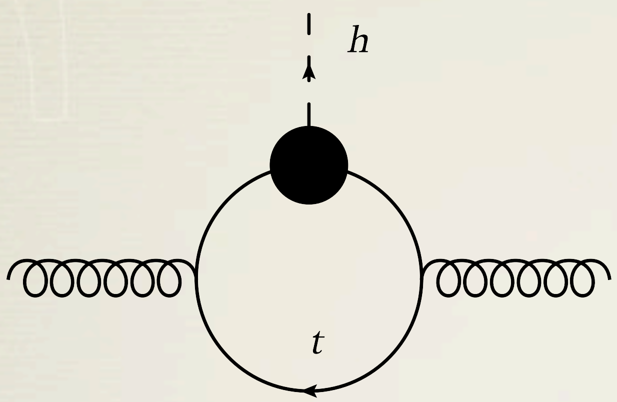
$$\tau \to 0 \quad \Rightarrow \quad \mathcal{F}_{1/2} \to \frac{4}{3}$$

$$\tau \to \infty \quad \Rightarrow \quad \mathcal{F}_{1/2} \to -\frac{2m_Q^2}{M_H^2} \ln \frac{M_H^2}{m_Q^2}$$

- Independent of m_f when $m_f \to \infty$ ⇒ true for *any* heavy fermion that gets its mass entirely from Higgs

Low-energy theorems

- Useful, illuminating alternative approach for $2m_t > M_H$



$$\frac{i}{\not{k} - m_t} \quad \rightarrow \quad \frac{i}{\not{k} - m_t}\frac{-im_t}{v}\frac{i}{\not{k} - m_t} = i\frac{m_t}{v}\left(\frac{1}{\not{k} - m_t}\right)^2$$
$$= \quad \frac{m_t}{v}\frac{\partial}{\partial m_t}\frac{i}{\not{k} - m_t}$$

Generates both diagrams in the $M_H \rightarrow 0$ limit

- Diagrammatically, clear that Higgs interaction comes from derivatives of the top part of the gluon self-energy:

$$\mathcal{M}(hgg) \underbrace{=}_{p_H \rightarrow 0} \frac{m_t}{v}\frac{\partial}{\partial m_t}\mathcal{M}(gg)$$

Effective field theory

- We're going to use an effective field theory to calculate the Higgs production cross section
- EFT: if we are doing experiments at low energies, we shouldn't care about the dynamics of very heavy particles. We should be able to approximate their effects as local, higher-dimension (suppressed by the heavy-particle masses) operators in an effective Lagrangian.
- Well-established in QCD: heavy-quark EFT, soft-collinear EFT
- We will use the separation $2m_t >> M_H$ to form a Higgs EFT

Useful references on EFT:
Manohar and Wise, *Heavy Quark Effective Theory*
Rothstein, hep=ph/0308266

The Higgs effective Lagrangian

- Integrate out the top quark to produce an effective Lagrangian

$$\mathcal{L}_{full} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \mathcal{L}_{top}$$

$$\underbrace{G^{a'}_{\mu}}_{\text{EFT field}} = \underbrace{\sqrt{\zeta_3}}_{\text{decoupling constant}} \underbrace{G^{a}_{\mu}}_{\text{QCD field}}$$

$$\mathcal{L}_{EFT} = -\frac{\zeta_3}{4} G^{a'}_{\mu\nu} G^{\mu\nu'} a$$

(remember to amputate external legs)

- Matching calculation: equate full and EFT propagators

$$-\frac{ig_{\mu\nu}}{p^2}\zeta_3 = -\frac{ig_{\mu\nu}}{p^2} \underbrace{[1+\Pi_t(0)]}_{m_t^2 \gg p^2}$$

top-quark contribution to gluon self-energy

$$\Rightarrow \zeta_3 = 1 + \Pi_t(0)$$

$$\Rightarrow \mathcal{L}_{EFT} = -\frac{[1+\Pi_t(0)]}{4} G^{a\prime}_{\mu\nu} G^{\mu\nu\prime}_a$$

The Higgs effective Lagrangian

- Now apply the low energy theorem to derive HGG operator:

$$\mathcal{L}_{EFT}^{hgg} = -\frac{m_t}{4v}\left(\frac{\partial}{\partial m_t}\Pi_t(0)\right) h\, G_{\mu\nu}^{a\prime} G_a^{\mu\nu\prime}$$

$$\Rightarrow \Pi_t(0) = \frac{\alpha_s}{6\pi}\left[\frac{\bar{\mu}^2}{m_t^2}\right]^{\epsilon}\frac{\Gamma(1+\epsilon)}{\epsilon}$$

$$\Rightarrow \boxed{\mathcal{L}_{EFT}^{hgg} = \frac{\alpha_s}{12\pi}\frac{h}{v} G_{\mu\nu}^{a\prime} G_a^{\mu\nu\prime}}$$

- Numerous nice features of this formulation...

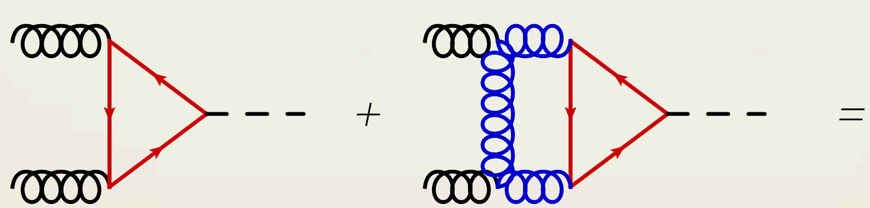
The Higgs effective Lagrangian

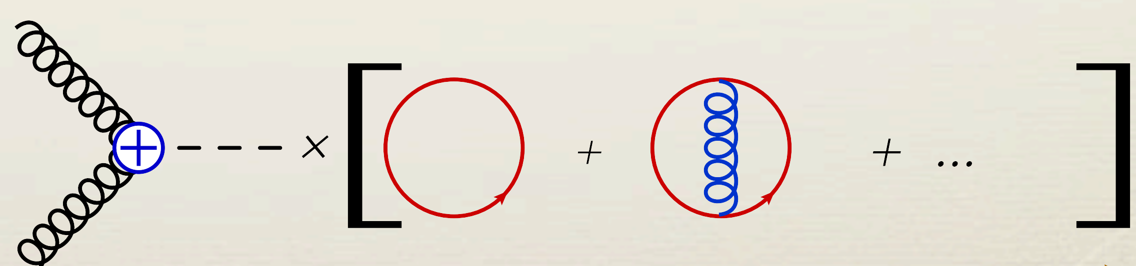
- Systematically, simply extendable to higher orders in QCD

Useful references: Kniehl, Spira hep-ph/9505225; Steinhauser hep-ph/0201075

- Reduces calculations by one loop order; 1-loop becomes tree, etc.; makes a NNLO calculation possible
- Turns a two-scale problem into two one-scale problems

Two scales:
M_{Higgs}, m_{top}





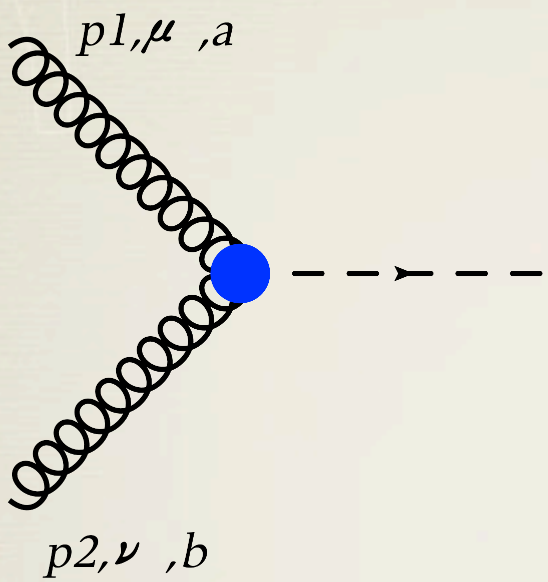
Only M_{Higgs} Only m_{top} $O(M^2_{Higgs}/4m^2_{top})$

The Higgs effective Lagrangian

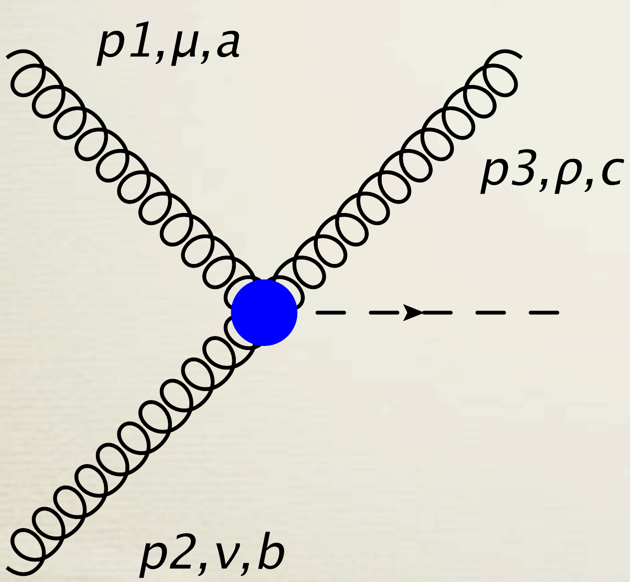
- Factorizes QCD effects (dynamics of gluons, light quarks from L_{EFT}) from new physics (heavy particles into Wilson coefficients)
- Applicable to the $h\gamma\gamma$ coupling also
- Can be used when a particle does not obtain all its mass from the Higgs (for a recent formulation, see Carena et al. 1206.1082)
- Valid much beyond the expected region of validity; forms the basis for much of Tevatron/LHC phenomenology
- Let's try it out, and do a full NLO calculation of a hadron collider cross section

Setup

- Our Feynman rules are 5-flavor QCD plus the EFT vertices:



$$= -i\frac{\alpha_s}{3\pi v}\left\{1+\frac{11}{4}\frac{\alpha_s}{\pi}\right\}\delta^{ab}\left[p_1\cdot p_2 g^{\mu\nu}-p_1^{\nu}p_2^{\mu}\right]$$



$$= g_s\frac{\alpha_s}{3\pi v}f^{abc}\left\{g_{\mu\nu}(p_1-p_2)_\rho\right.$$
$$\left.+g_{\nu\rho}(p_2-p_3)_\mu+(p_3-p_1)_\nu\right\}$$

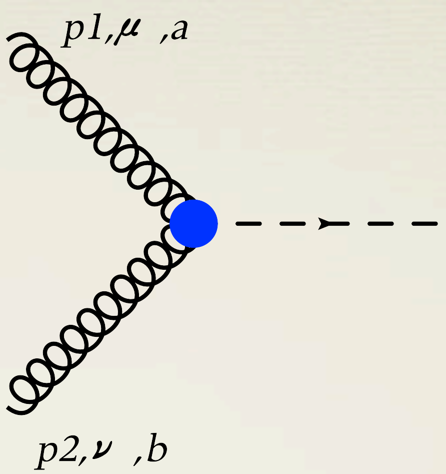
Steps

- Pick a regularization scheme (dimensional regularization for us)
- Get the tree-level result
- Calculate 1-loop diagrams as a Laurent series in ε
- Perform the ultraviolet renormalization
- Calculate the real emission diagrams, extract singularities that appear in soft/collinear regions of phase space
- Absorb initial-state collinear singularities into PDFs
- Get numbers

Tree-level

$$\sigma_{h_1 h_2 \to h} \;=\; \int dx_1\, dx_2 f_g(x_1) f_g(x_2)\, \hat{\sigma}(z)$$
$$+ \;\; \text{smaller partonic channels}$$

$(z = M_H^2/x_1 x_2 s)$



- Calculate the spin-, color-averaged matrix element squared

$$|\bar{\mathcal{M}}|^2 = \underbrace{\frac{1}{256(1-\epsilon)^2}}_{8 \text{ colors, } 2(1-\epsilon) \text{ spins}} \times |\mathcal{M}|^2 = \frac{\hat{s}^2}{576 v^2 (1-\epsilon)} \left(\frac{\alpha_s}{\pi}\right)^2$$

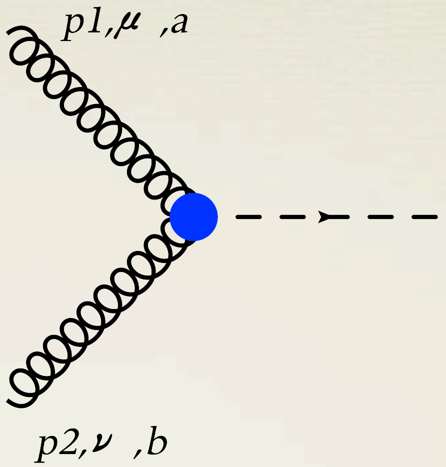
- Get the phase space and flux factor

$$\frac{1}{2\hat{s}} \int \frac{d^d p_h}{(2\pi)^d}\, 2\pi\delta(p_H^2 - M_H^2)\, (2\pi)^d \delta^{(d)}(p_1 + p_2 - p_H) = \frac{\pi}{\hat{s}^2}\delta(1-z)$$

Tree-level

$$\sigma_{h_1 h_2 \to h} = \int dx_1\, dx_2 f_g(x_1) f_g(x_2)\, \hat{\sigma}(z) + \text{smaller partonic channels}$$

$(z = M_H^2/x_1 x_2 s)$



- Combine to get the LO result:

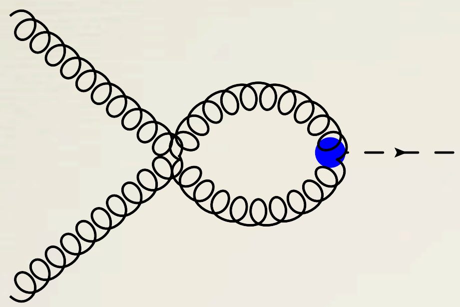
$$\boxed{\hat{\sigma}_0(z) = \sigma_0 \delta(1-z) = \frac{\pi}{576 v^2}\left(\frac{\alpha_s}{\pi}\right)^2 \delta(1-z)}$$

- We will later need the full d-dimensional tree-level result:

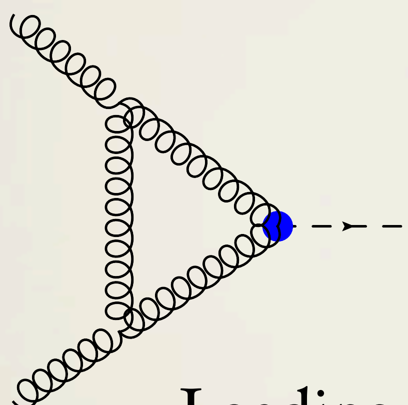
$$\sigma_0^{(d)} = \frac{\sigma_0}{1-\epsilon}$$

Virtual corrections

- Calculate $2\times\mathrm{Re}[(M_0)^*M_1]$, which appears in the cross section



$$= \sigma_0^{(d)}\frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{\hat{s}}{\mu^2}\right)^{-\epsilon}\left\{-\frac{13}{4\epsilon}-\frac{11}{3}\right\}\delta(1-z)$$



$$= \sigma_0^{(d)}\frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{\hat{s}}{\mu^2}\right)^{-\epsilon}\left\{-\frac{3}{\epsilon^2}+\frac{13}{4\epsilon}+\frac{11}{3}+2\pi^2\right\}\delta(1-z)$$

Leading soft+collinear singularity; emitting gluons from gluons gives color factor $C_A=3$

- External leg corrections *scaleless:* $\int d^dk\,(k^2)^n = 0$

UV renormalization

- LO dependence on α_S gives the UV counterterm:

$$\sigma_0^{(d)} \frac{\alpha_s}{\pi} \frac{1}{\epsilon} \frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}} \left\{ -\frac{11}{2} + \frac{N_F}{3} \right\}$$

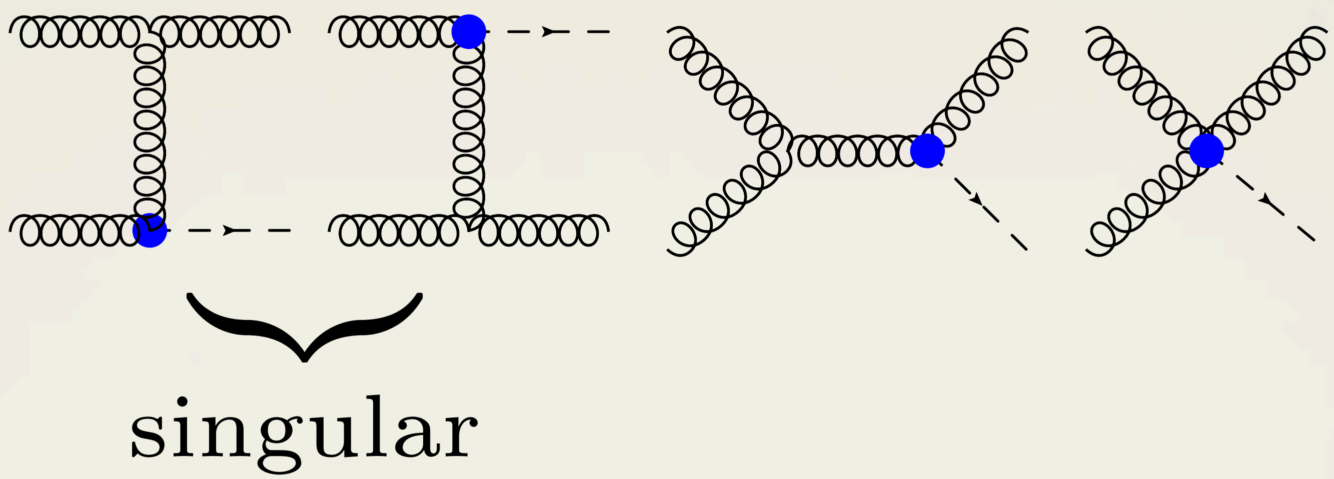
- The remaining singularities are of soft/collinear origin; summing what we have so far yields

$$\sigma_0^{(d)} \frac{\alpha_s}{\pi} \left\{ -\frac{3}{\epsilon^2} + \frac{3}{\epsilon} \ln \frac{\hat{s}}{\mu^2} - \frac{1}{\epsilon} \left(\frac{11}{2} - \frac{N_F}{3} \right) + \text{finite} \right\} \delta(1-z)$$

- The pole structure can be checked to be correct: Catani, hep-ph/9802439

Real radiation corrections

- Get the corrections coming from emission of an additional gluon



$$|\bar{\mathcal{M}}|^2 = 24\,\alpha_s\sigma_0 \left\{ \frac{(1-2\epsilon)}{(1-\epsilon)} \frac{M_H^8 + \hat{s}^4 + \hat{t}^4 + \hat{u}^4}{\hat{s}\hat{t}\hat{u}} + \frac{\epsilon}{2(1-\epsilon)^2} \frac{(M_H^4 + \hat{s}^2 + \hat{t}^2 + \hat{u}^2)^2}{\hat{s}\hat{t}\hat{u}} \right\}$$

- This can vanish when either $p_g \to 0$ (soft), or $p_g \parallel p_1$, $p_g \parallel p_2$ (collinear)
- Need a parameterization of phase space to extract these singularities appropriately

$$\hat{s} = (p_1 + p_2)^2$$

$$\hat{t} = (p_1 - p_g)^2$$

$$\hat{u} = (p_2 - p_g)^2$$

Real radiation corrections

$$\frac{1}{2\hat{s}} \int \frac{d^d p_g}{(2\pi)^d} \int \frac{d^d p_H}{(2\pi)^d} (2\pi)\delta(p_g^2)(2\pi)\delta(p_H^2 - M_H^2)(2\pi)^d \delta^{(d)}(p_1 + p_2 - p_g - p_H)$$

- Introduce the following parameterization of p_g:

$$p_g = \frac{\hat{s}(1-z)}{2} \left(1, 2\sqrt{\lambda(1-\lambda)}, 0, 1-2\lambda\right)$$

- Obtain: $$\frac{1}{16\pi\hat{s}} \left(\frac{s}{4\pi}\right)^{-\epsilon} \frac{1}{\Gamma(1-\epsilon)} (1-z)^{1-2\epsilon} \int_0^1 d\lambda \, [\lambda(1-\lambda)]^{-\epsilon}$$

- When we combine matrix elements and phase space, get terms of the following form:

$$(1-z)^{\overbrace{-1}^{\text{singular}}\overbrace{-2\epsilon}^{\text{regulator}}} [\lambda(1-\lambda)]^{-1-\epsilon}$$

λ→0,1: collinear
z→1: soft

Real radiation corrections

- The integrals over λ can be done in terms of Gamma functions, while the soft singularities as z→1 can be extracted using plus distributions:

$$(1-z)^{-1-2\epsilon} = -\frac{1}{2\epsilon}\delta(1-z) + \left[\frac{1}{1-z}\right]_+ - 2\epsilon\left[\frac{\ln(1-z)}{1-z}\right]_+ + \mathcal{O}(\epsilon^2)$$

$$\int_0^1 dz\, f(z)\left[\frac{g(z)}{1-z}\right]_+ = \int_0^1 dz\,\frac{g(z)}{1-z}\left[f(z)-f(1)\right]$$

- Arrive at the following contribution to the cross section:

$$\sigma_0^{(d)}\frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{\hat{s}}{\mu^2}\right)^{-\epsilon}\left\{\overbrace{\frac{3}{\epsilon^2}\delta(1-z)}^{\text{cancels virtual poles}} - \frac{6}{\epsilon}\left[\frac{1}{1-z}\right]_+ + \frac{6z(z^2-z+2)}{\epsilon}\right.$$

$$\left. -\frac{3\pi^2}{2}\delta(1-z) + 12\left[\frac{\ln(1-z)}{1-z}\right]_+ - 12z(z^2-z+2)\ln(1-z) - \frac{11}{2}(1-z)^3\right\}$$

Remaining terms

- Absorb remaining initial-state collinear singularities into PDFs, which amounts to adding the following counterterm:

One for each PDF

$$2 \times \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}}P_{gg}\otimes\hat{\sigma}_0(z) \qquad f\otimes g(z) = \int_0^1 dx\,dy\,f(x)\,g(y)\,\delta(z-xy)$$

Arrive at the contribution:

$$\sigma_0^{(d)}\frac{\alpha_s}{\pi}\frac{1}{\epsilon}\left\{\left(\frac{11}{2}-\frac{N_F}{3}\right)\delta(1-z)+\frac{6}{[1-z]_+}-6z(z^2-z+2)\right\}$$

- This cancels all remaining poles, but we need to add on the NLO correction to the Wilson coefficient in the EFT:

$$\sigma_0^{(d)}\frac{\alpha_s}{\pi}\frac{11}{2}\delta(1-z)$$

Final result

- Arrive at the final NLO result for the inclusive cross section:

$$\Delta\sigma = \sigma_0 \frac{\alpha_s}{\pi}\left\{\left(\frac{11}{2}+\pi^2\right)\delta(1-z) + 12\left[\frac{\ln(1-z)}{1-z}\right]_+ - 12z(-z+z^2+2)\ln(1-z)\right.$$
$$\left. - \frac{11}{2}(1-z)^3 + 6\ln\frac{\hat{s}}{\mu^2}\left[\frac{1}{[1-z]_+} - z(z^2-z+2)\right]\right\} \quad (M^2/s \leq z \leq 1)$$

(integration over PDFs⇒integration over z)

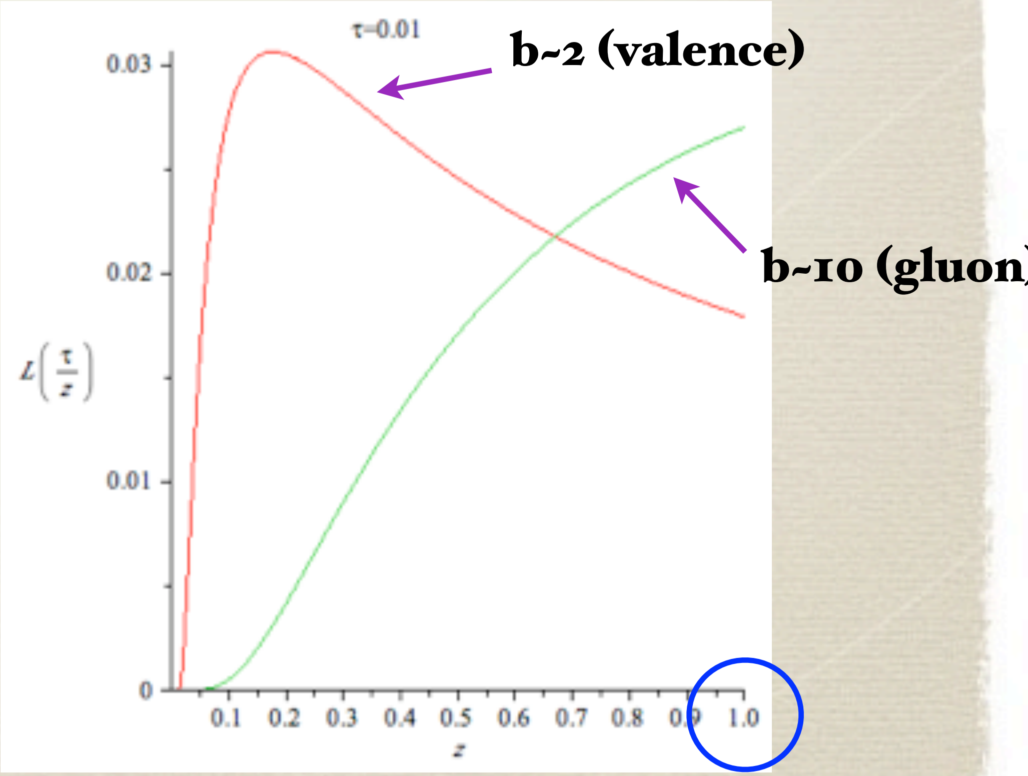
- First source of large correction: $11/2+\pi^2 \Rightarrow 50\%$ increase
- Second source: shape of PDFs enhances *threshold* logarithm

$$\sigma_{had} = \tau \int_\tau^1 dz\, \frac{\sigma(z)}{z}\, \mathcal{L}\left(\frac{\tau}{z}\right)$$

$$\mathcal{L}(y) = \int_y^1 dx\, \frac{y}{x}\, f_1(x) f_2(y/x) \quad \text{(partonic luminosity)}$$

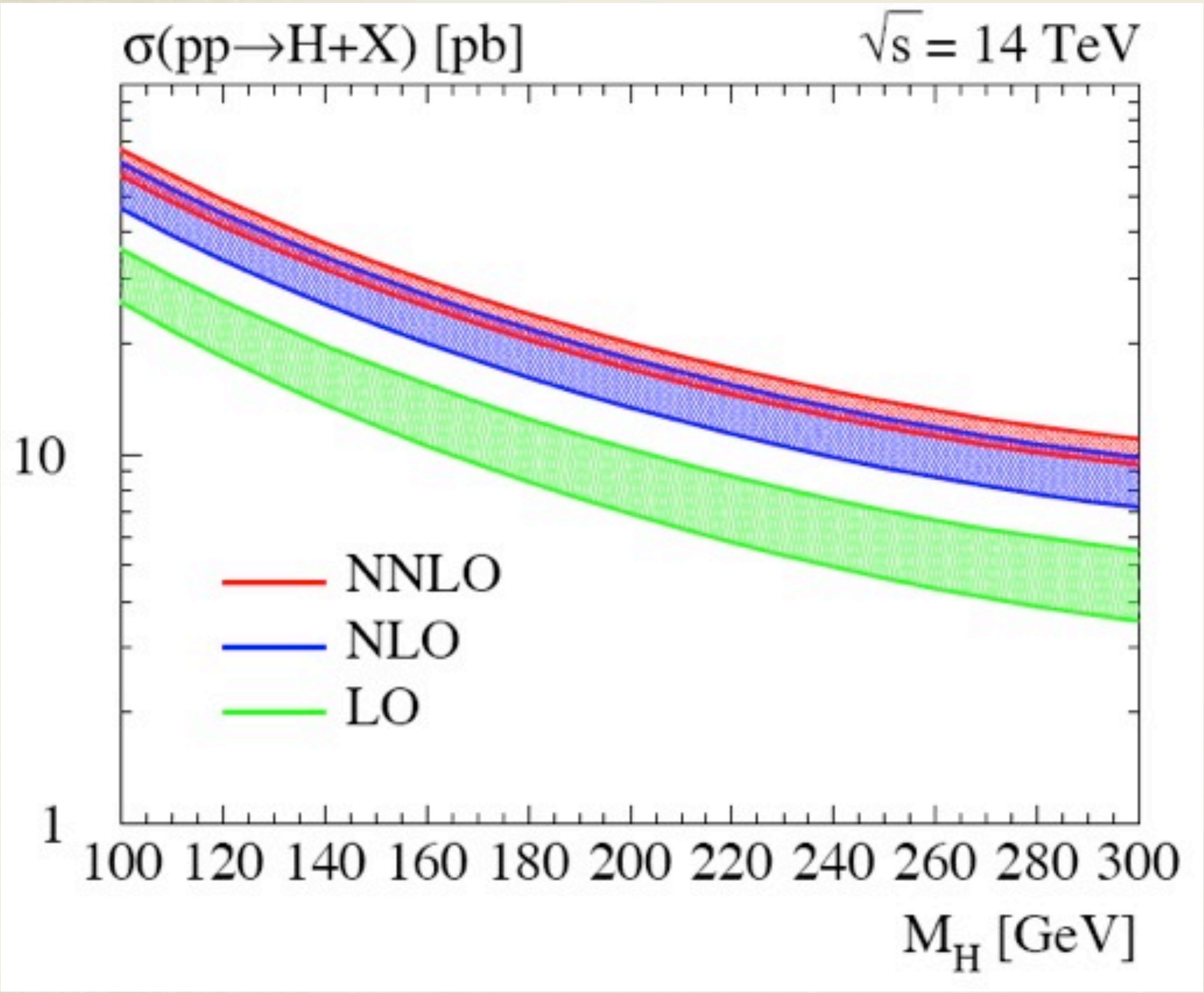
Assume $f_i \sim (1-x)^b$; plot L for various b
Look for peak near $z \approx 1$

⇒Sharp fall-off of gluon PDF enhances correction



NNLO in the EFT

- Use of the EFT allows the NNLO cross section to be obtained



Harlander, Kilgore ‘02; Anastasiou, Melnikov ‘02;
Ravindran, Smith van Neerven ‘03

- Again, scale variation, especially at LO, can badly underestimate error!

Unreasonably effective EFT

- NLO in the EFT:

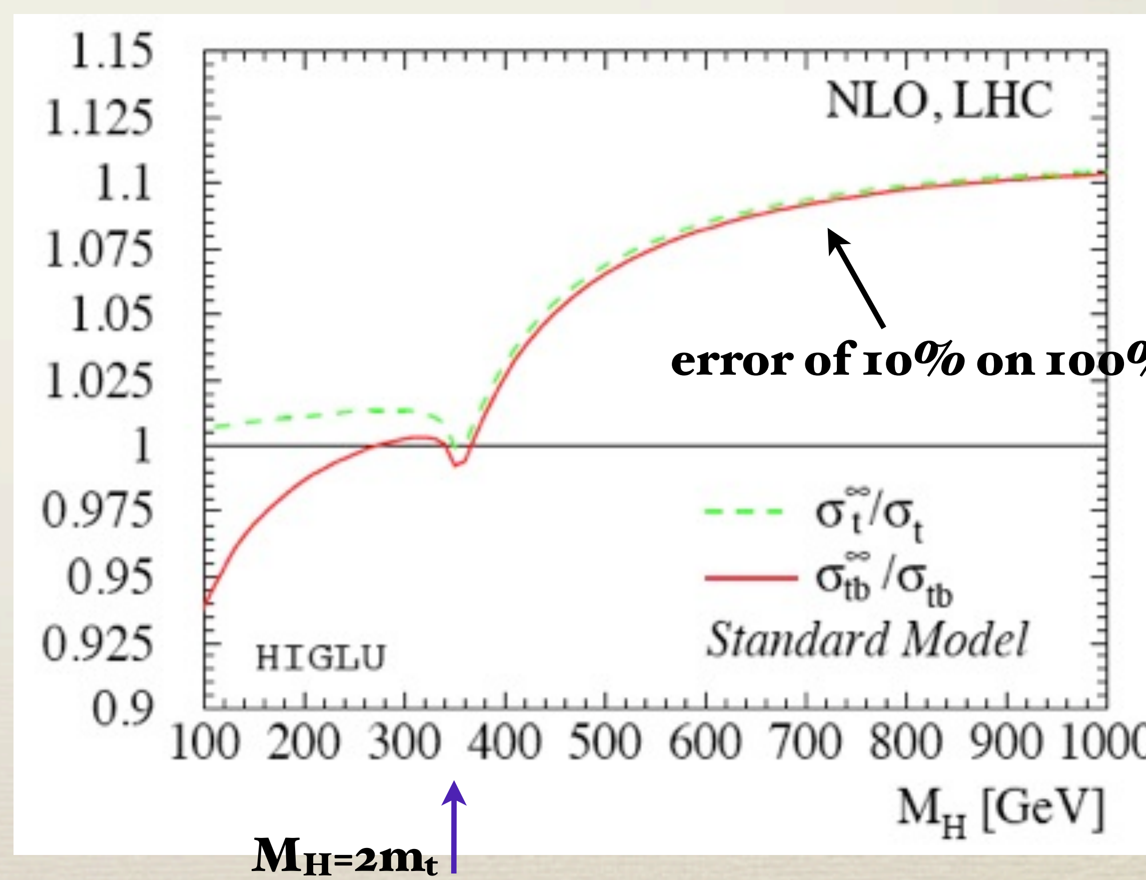
analytic continuation to time-like form factor

$$\Delta\sigma = \sigma_0 \frac{\alpha_s}{\pi}\left\{\left(\frac{11}{2}+\pi^2\right)\delta(1-z)+12\left[\frac{\ln(1-z)}{1-z}\right]_+ - 12z(-z+z^2+2)\ln(1-z) -6\frac{(z^2+1-z)^2}{1-z}\ln(z)-\frac{11}{2}(1-z)^3\right\}$$

eikonal emission of soft gluons

Identical factors in full theory with $\sigma_0 \rightarrow \sigma_{LO,\text{ full theory}}$

$$\sigma^{approx}_{NLO} = \left(\frac{\sigma^{EFT}_{NLO}}{\sigma^{EFT}_{LO}}\right)\sigma^{QCD}_{LO}$$



NNLO study of $1/m_t$ suppressed operators, matched to large s-hat limit, large indicates this persists

Harlander, Mantler, Marzani, Ozeren; Pak, Rogal, Steinhauser 2009

Summary of gluon fusion

- Serves as a very accurate framework for all LHC phenomenology
- Current uncertainty estimates: roughly 10% from uncalculated higher orders, 10% from PDFs, a few percent from other effects (use of EFT, bottom-quark effects, EW effects)

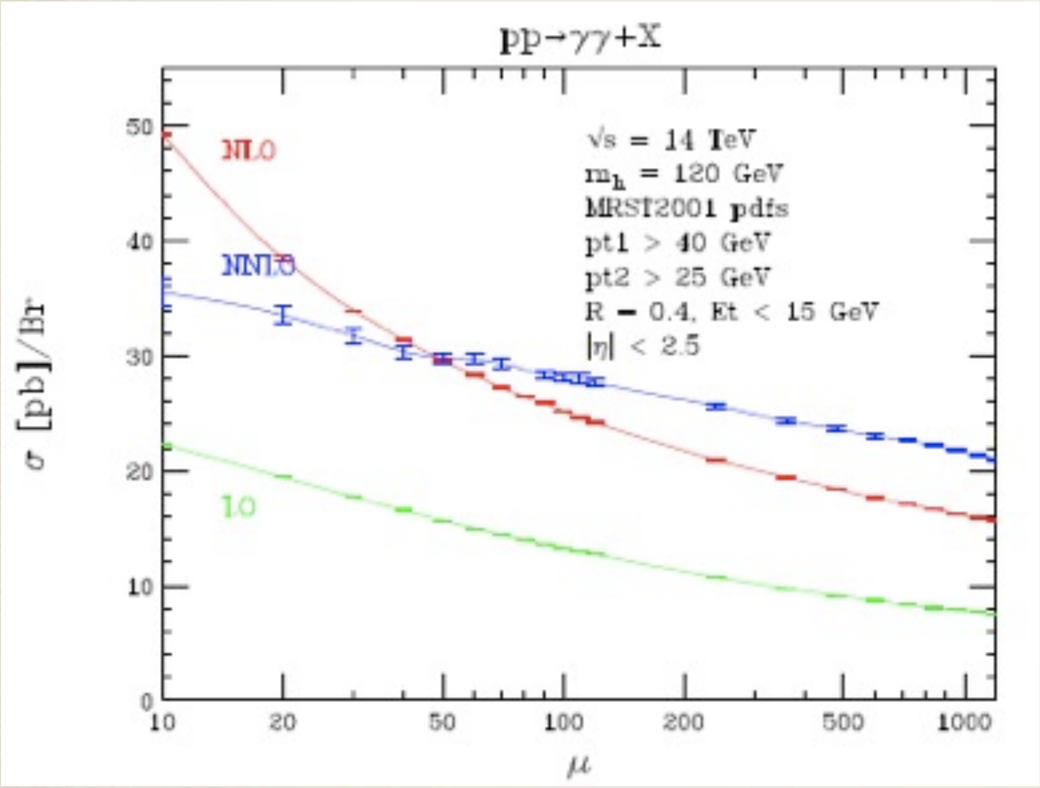
Useful references: S. Dawson, NPB359 (1991) 283-300 and *QCD and Collider Physics* by Ellis, Stirling, Webber (detailed NLO calculation);
1101.0593 (detailed discussion of uncertainties)

Available codes: http://theory.fi.infn.it/grazzini/hcalculators.html
http://www.phys.ethz.ch/~pheno/ihixs/index.html
http://particle.uni-wuppertal.de/harlander/software/ggh@nnlo/
HIGLU: http://people.web.psi.ch/spira/higlu/

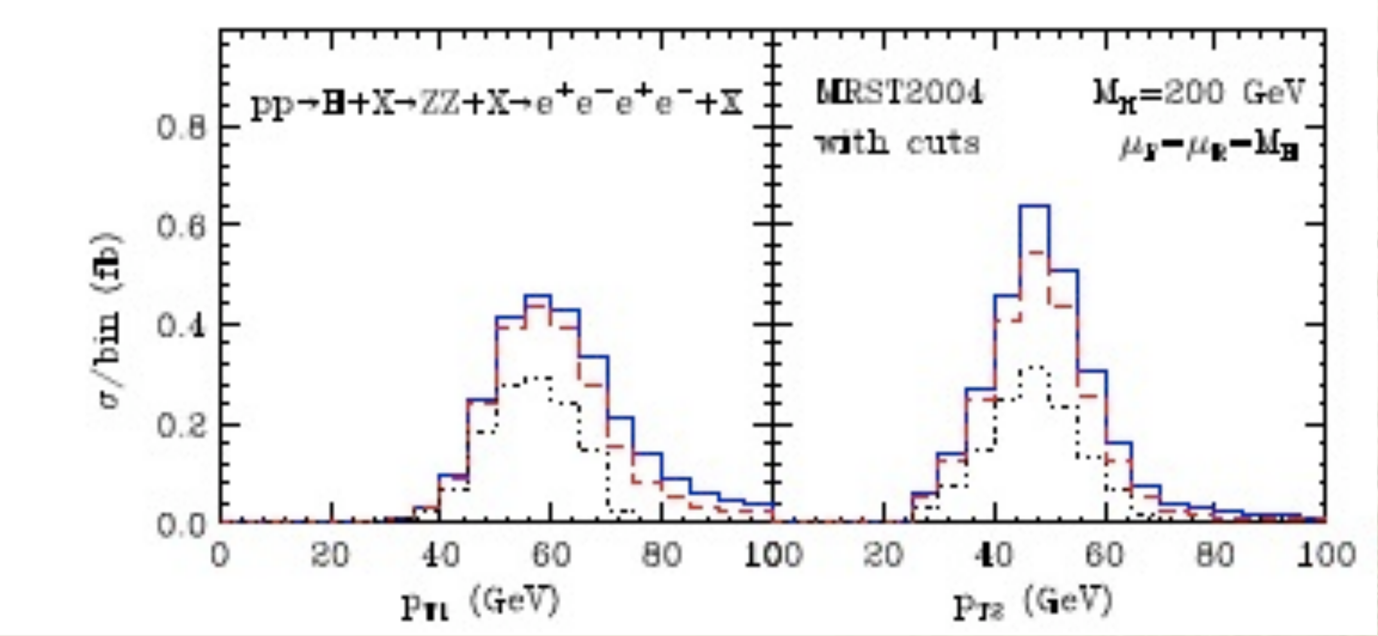
Current topic: jet vetoes in QCD

Confronting reality

- Unfortunately, the overwhelming backgrounds at the LHC require that significant cuts are imposed on the final state.
- For gluon fusion, two NNLO parton-level simulation codes exist



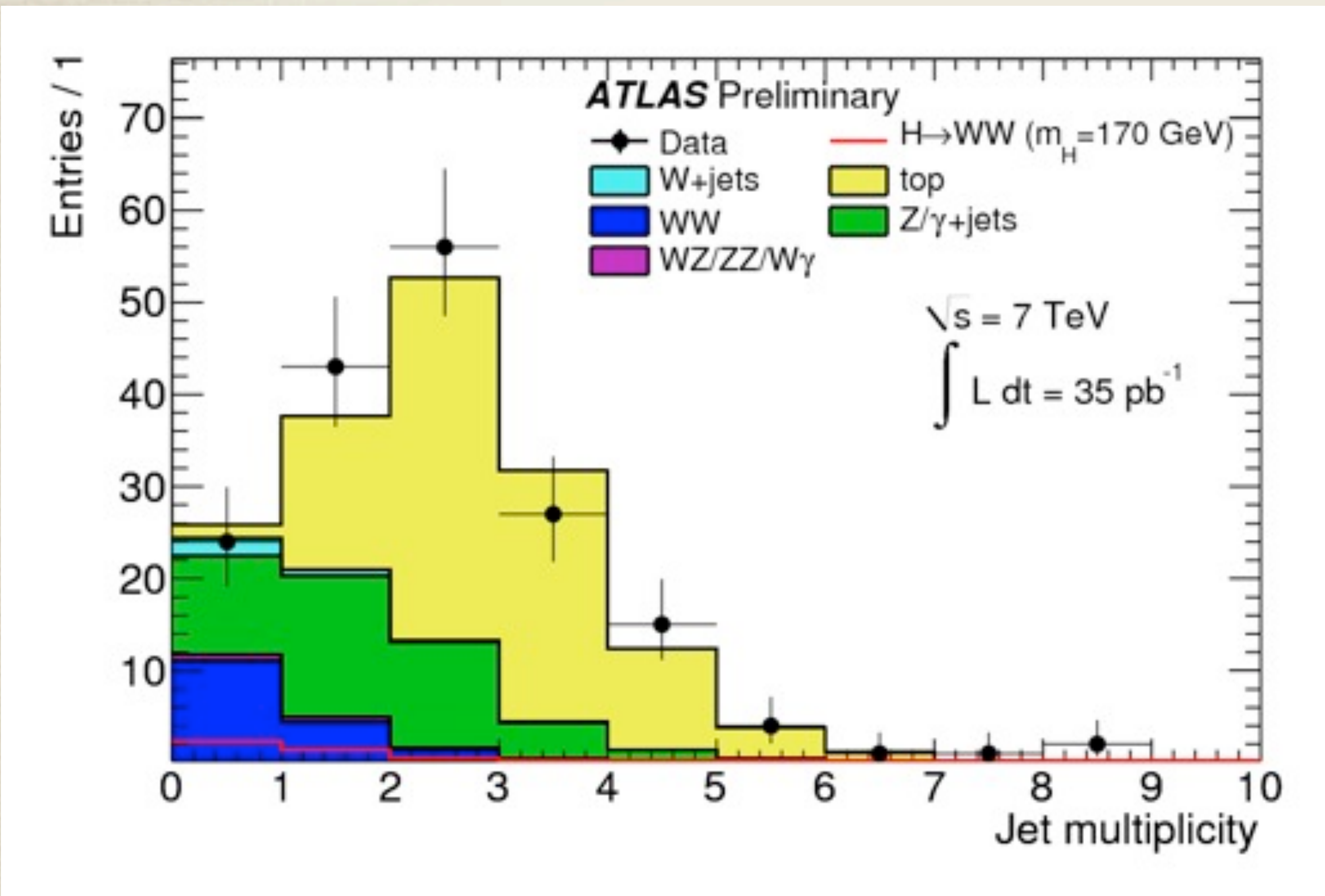
FEHiP: Anastasiou, Melnikov, FP 2005



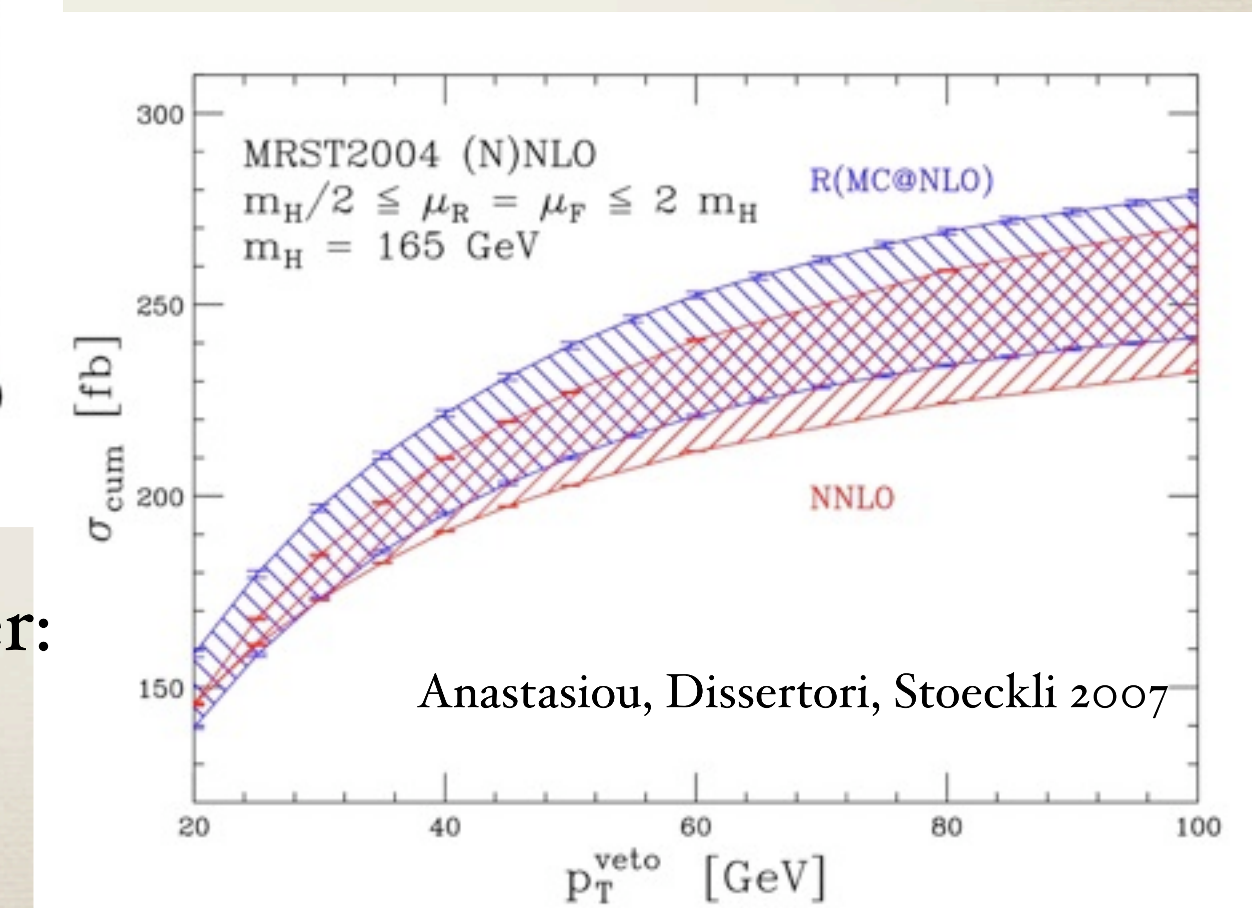
HNNLO: Catani, Grazzini 2007-2008

The jet veto

- A typical cut is to divide the final state into bins of differing jet multiplicity



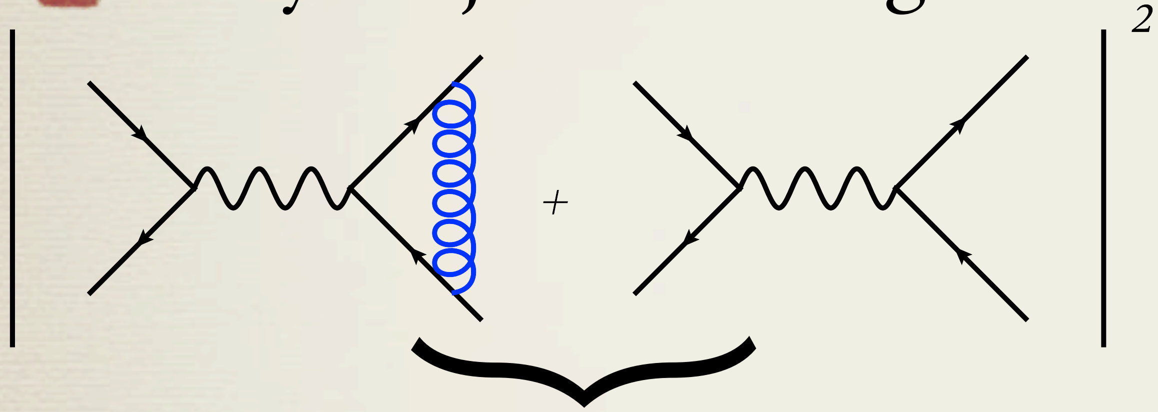
- Required in the WW channel to reduce top-quark background
- 25-30 GeV jet cut used

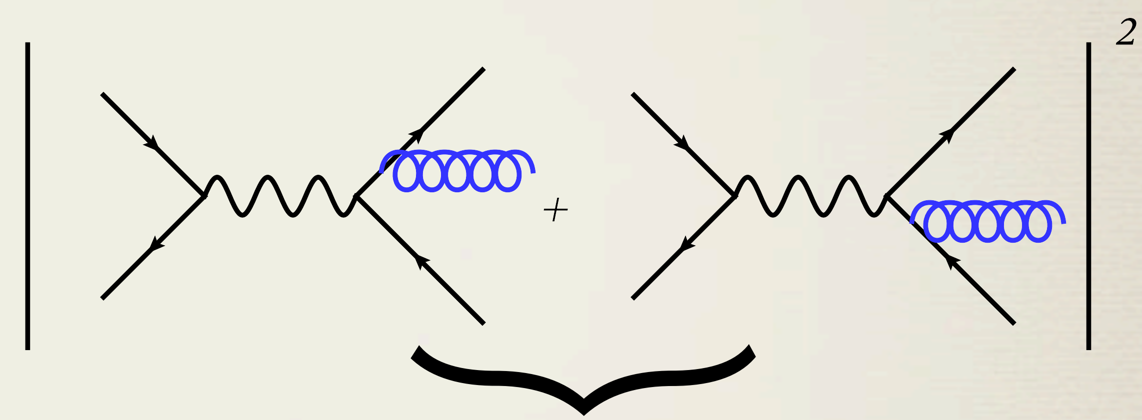


- When we try to compute at fixed order:
- Does the uncertainty really become smaller with a stricter veto?

The jet veto

- Significant interest in trying to understand the impact of jet vetos on Higgs searches Stewart, Tackmann 1107.2117; Banfi, Salam, Zanderighi 1203.5773
- We also saw this in VH, although we'll focus on gluon-fusion here
- Why are jet vetos dangerous?





Virtual corrections: $-1/\varepsilon_{IR}^2$

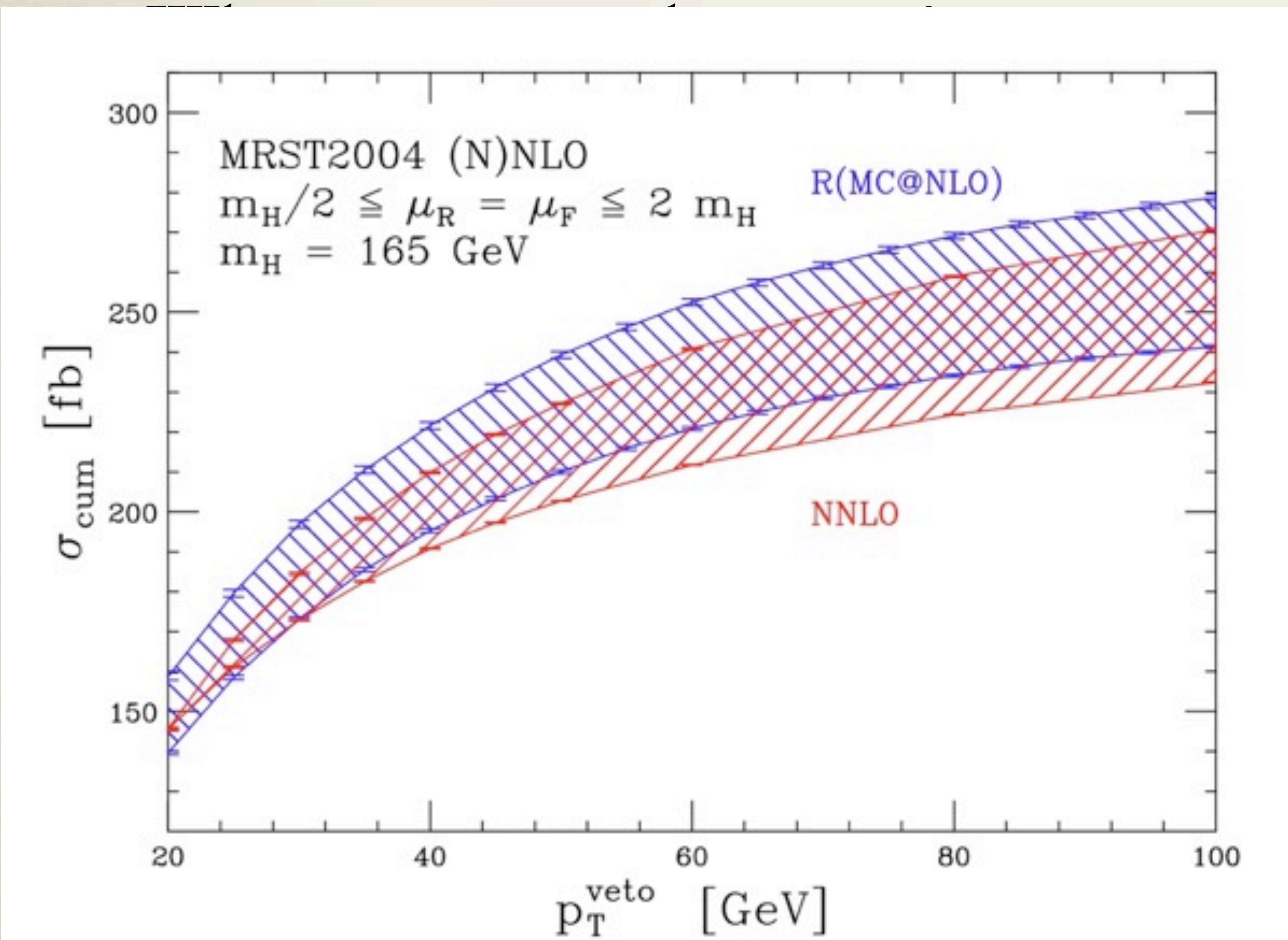
Real corrections: $1/\varepsilon_{IR}^2 - \ln^2(Q/p_{T,cut})$

- Relevant log term for Higgs searches: $6(\alpha_S/\pi)\ln^2(M_H/p_{T,veto}) \sim 1/2$

$\Rightarrow$should be *resummed* to all orders, fixed-order breaks down

The jet veto

- Significant interest in trying to understand the impact of jet vetos on Higgs searches Stewart, Tackmann 1107.2117; Banfi, Salam, Zanderighi 1203.5773
- We also saw this in VH, although we'll focus on gluon-fusion here



- Arises from an accidental cancellation between these logs and the large corrections to the inclusive cross section... no reason to persist at higher orders

The jet veto

- Significant interest in trying to understand the impact of jet vetos on Higgs searches Stewart, Tackmann 1107.2117; Banfi, Salam, Zanderighi 1203.5773

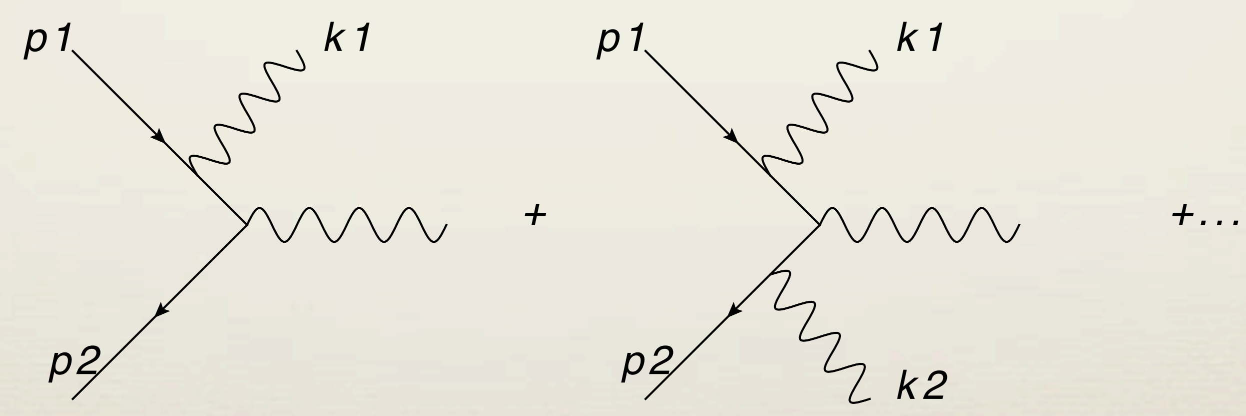
$$\sigma_0(p^{\mathrm{cut}}) = \sigma_{\mathrm{total}} - \sigma_{\geq 1}(p^{\mathrm{cut}})$$
$$\simeq \sigma_B\Big\{\left[1+\alpha_s+\alpha_s^2+\mathcal{O}(\alpha_s^3)\right] - \left[\alpha_s(L^2+L+1)+\alpha_s^2(L^4+L^3+L^2+L+1)+\mathcal{O}(\alpha_s^3L^6)\right]\Big\}$$

$$\sigma_{\mathrm{total}} = (3.32\,\mathrm{pb})\left[1+9.5\,\alpha_s+35\,\alpha_s^2+\mathcal{O}(\alpha_s^3)\right],$$
$$\sigma_{\geq 1}\big(p_T^{\mathrm{jet}} \geq 30\,\mathrm{GeV}, |\eta^{\mathrm{jet}}| \leq 3.0\big) = (3.32\,\mathrm{pb})\left[4.7\,\alpha_s+26\,\alpha_s^2+\mathcal{O}(\alpha_s^3)\right].$$

Arises from an accidental cancellation between these logs and the large corrections to the inclusive cross section... no reason to persist at higher orders

Resumming jet-veto logs

- Option 1: directly resum the logs in the presence of a jet algorithm. This is complicated, and is the subject of 'healthy debate' in the literature Banfi, Monni, Salam, Zanderighi, 1206.4998; Tackmann, Walsh, Zuberi 1206.4312; Becher, Neubert 1205.3806
- Option 2: build intuition from simpler but closely related variables
- Typical choice is p_T of the Higgs; equivalent to a jet veto through $O(\alpha_S)$. Other choices possible Berger et al. 1012.4480
- Toy example of $\ln(p_T)$ resummation: $e^+e^-\rightarrow\gamma^*$, multiple soft-photon effects



Soft emissions in b-space

- Both matrix elements and phase space simplify in this limit

Eikonal approximation for n-photon matrix-elements: $$\mathcal{M}_n \propto g^n \mathcal{M}_0 \left\{ \frac{p_1 \cdot \epsilon_1 \ldots p_1 \cdot \epsilon_n}{p_1 \cdot k_1 \ldots p_1 \cdot k_n} + (-1)^n \frac{p_2 \cdot \epsilon_1 \ldots p_2 \cdot \epsilon_n}{p_2 \cdot k_1 \ldots p_2 \cdot k_n} \right\}$$

Phase-space for n-photon emission: $$d\Pi_n \propto \nu(k_{T1})\, d^2 k_{T1} \ldots \nu(k_{Tn})\, d^2 k_{Tn}\; \overbrace{\delta^{(2)}\left(\vec{p}_T - \sum_i \vec{k}_{Ti}\right)}^{\text{sum to Higgs } p_T}$$

$$\nu(k_T) = k_T^{-2\epsilon} \ln\left(\frac{s}{k_T^2}\right)$$

- Would be independent emissions if not for phase-space constraint
- Fourier transform:

$$\int \frac{d^2 b}{(2\pi)^2}\; \mathrm{e}^{-i\vec{b}\cdot\vec{p}_T} \int d^2 k_{T1}\, f(k_{T1}) \ldots d^2 k_{Tn}\, f(k_{Tn})\, \delta^{(2)}\left(\vec{p}_T - \sum_i \vec{k}_{Ti}\right)$$

$$= \int \frac{d^2 b}{(2\pi)^2} \mathrm{e}^{-i\vec{b}\cdot\vec{p}_T} \left[\tilde{f}(b)\right]^n, \quad \tilde{f}(b) = \int d^2 k_T\, \mathrm{e}^{i\vec{b}\cdot\vec{k}_T} f(k_T)$$

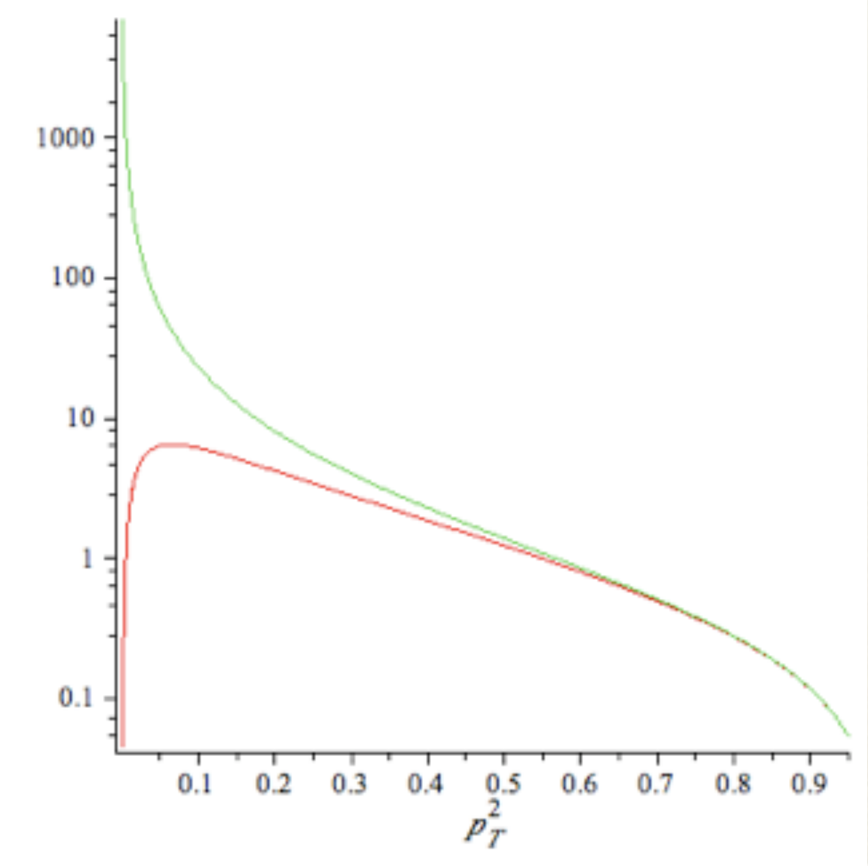
Exponentiation

- Product of matrix elements and phase space now exponentiates

$$\begin{aligned}\frac{d\sigma}{d^2p_T} &= \sigma_0\int\frac{d^2b}{(2\pi)^2}\,\mathrm{e}^{-i\vec{b}\cdot\vec{p}_T}\,\tilde{\sigma}(b)\\ \tilde{\sigma}(b) &= \exp\left\{\frac{g^2}{4\pi^2}\int d^2k_T\,\mathrm{e}^{i\vec{b}\cdot\vec{k}_T}\left[\frac{\ln(s/k_T^2)}{k_T^2}\right]_+\right\}\end{aligned}$$

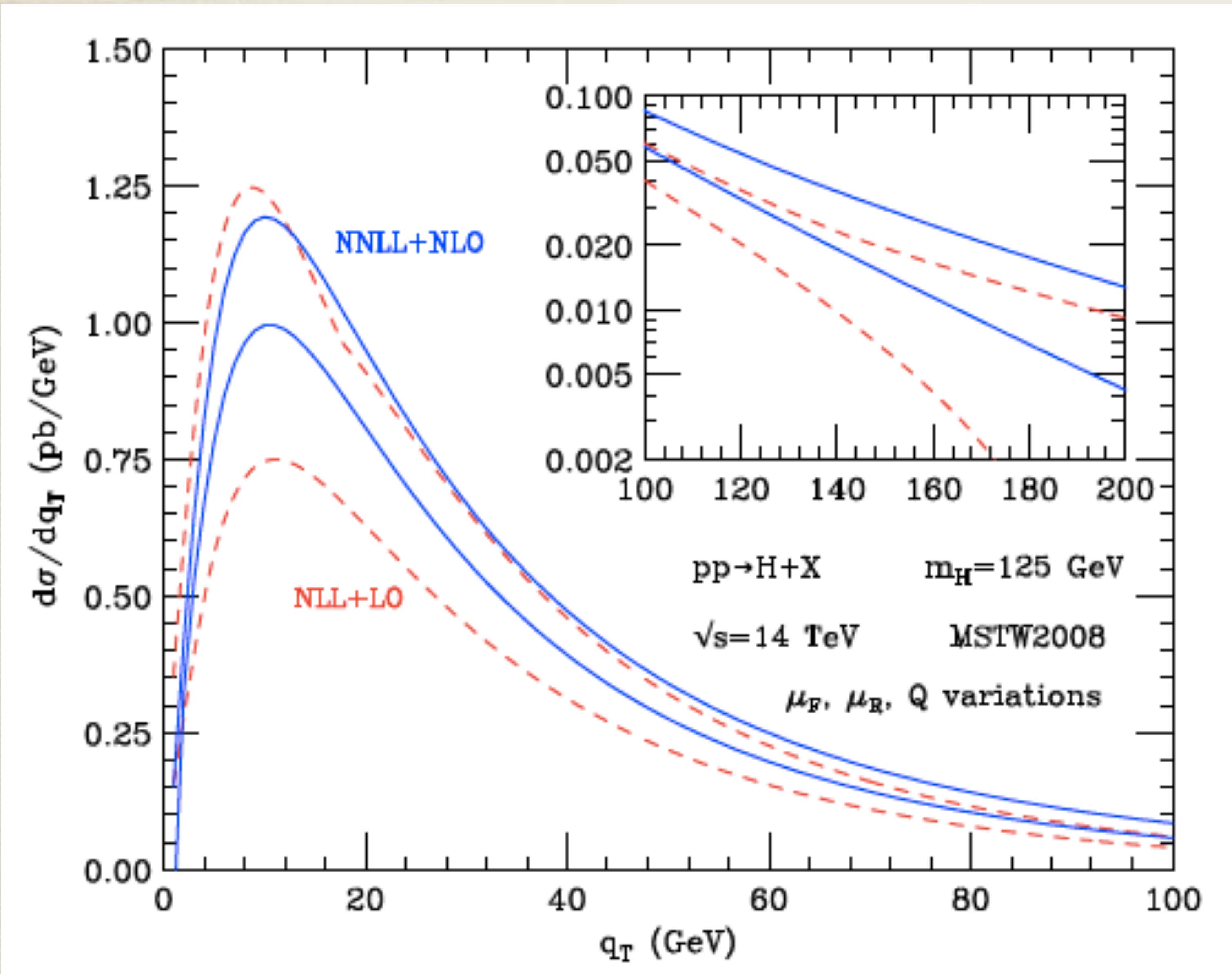
- Large b ⇔ small p_T; inverse transform keeping leading terms

$$\frac{d\sigma}{dp_T^2} = \frac{\alpha}{\pi}\sigma_0\frac{1}{p_T^2}\ln\frac{s}{p_T^2}\exp\left\{-\frac{\alpha}{2\pi}\ln^2\frac{s}{p_T^2}\right\}$$



P_T resummation for Higgs

- Known to the next-to-next-to-leading logarithmic level

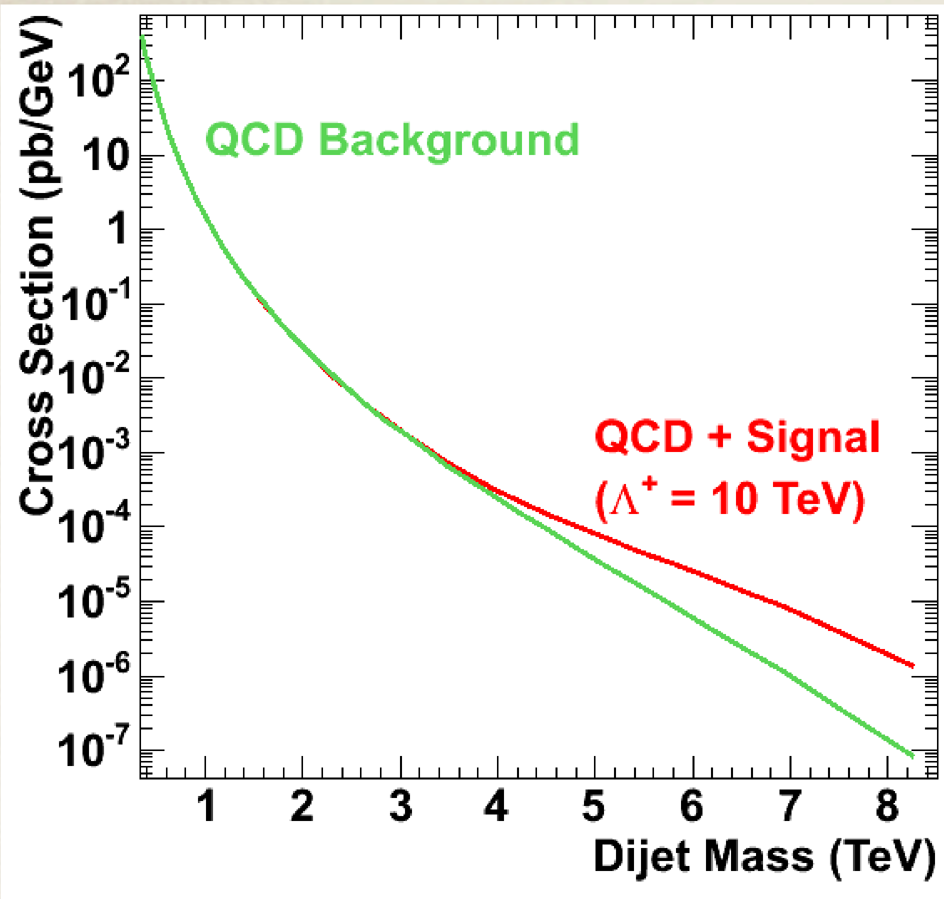


HqT: de Florian, Ferrera, Grazzini, Tommasini 2011

Used to reweight Monte-Carlo simulation programs such as POWHEG, MC@NLO to properly model Higgs kinematics and describe the jet veto

Classic ref for low p_T resummation: Collins, Soper, Sterman NPB250 (1985)
b-space: Parisi, Petronzio NPB154 (1979)

Conclusions



- I hope you learned about the QCD techniques available to avoid confusing the two lines shown on the left
- Serious quantitative predictions at LHC require NLO; this is a very active area!
- Many things can happen at higher orders in QCD, and must be carefully considered in studies: do the cuts enhance corrections? are there large logarithms? are the PDFs well determined?
- Effective field theory methods can simplify calculations with multiple scales
- Enjoy Chicago this weekend!