Nuclear Matrix Elements for ββ Decay

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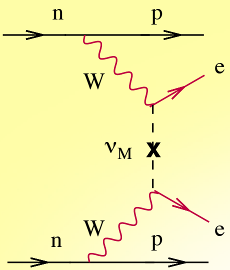
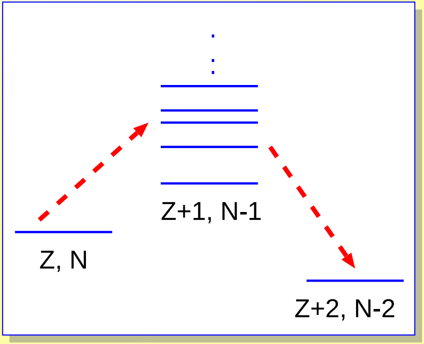
June 6, 2014



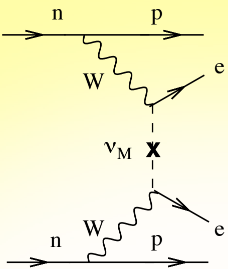
Neutrinoless Double-Beta Decay

If energetics are right (ordinary beta decay forbidden)...



and neutrinos are their own antiparticles...

can observe two neutrons turning into protons, emitting two electrons and nothing else.



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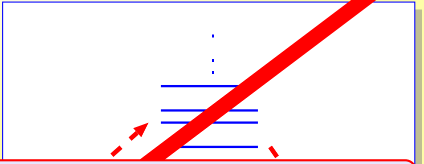

and

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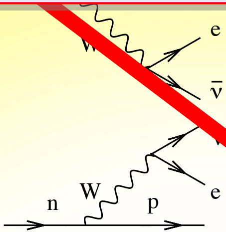
Introductory material: ββ decay is awesome, blah, blah, ...

can observe two neutrons turning into protons, emitting two electrons and nothing else.



Different from already observed two-neutrino process.

Usefulness of Double-Beta Decay

If it's observed, neutrinos are their own antiparticles!

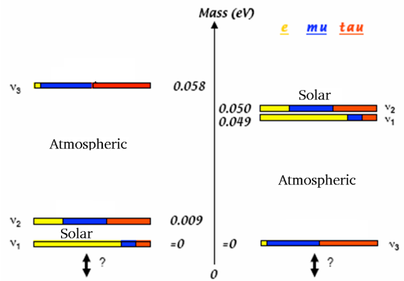
and

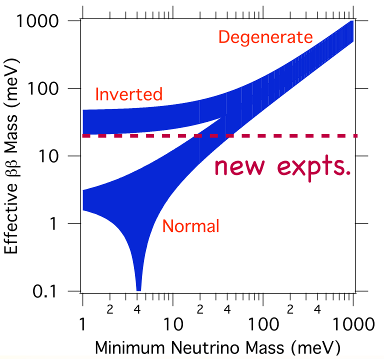
Light-ν-exchange amplitude proportional to "effective mass"

$$m_{eff} \equiv \sum_{i=1}^{3} m_i U_{ei}^2$$

If lightest neutrino is light:

- $m_{eff} \approx \sqrt{\Delta m^2_{sol}} \sin^2 \theta_{sol}$ (normal)
- $m_{eff} \approx \sqrt{\Delta m^2_{atm}} \cos 2\theta_{sol}$ (inverted)





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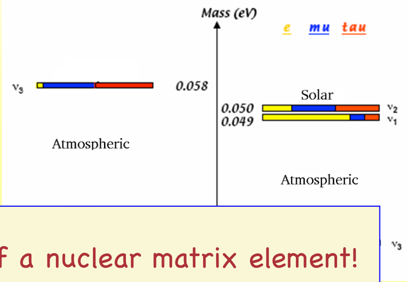

But rate also proportional to square of a nuclear matrix element!

L

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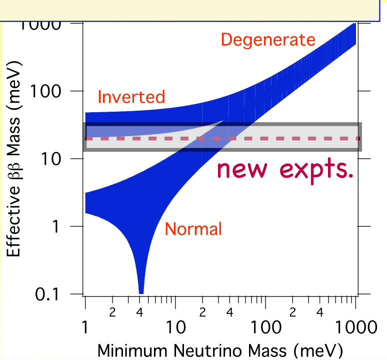
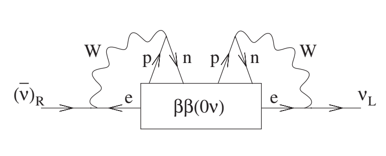
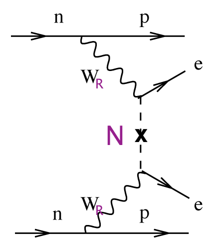


Other Mechanisms

If neutrinoless decay occurs then ν's are Majorana, no matter what:



but light neutrinos may not drive the decay:



Exchange of heavy right-handed neutrino in left-right symmetric model.

For $m_R \approx 1$ TeV, exotic processes can occur with roughly same rate as light-ν exchange. Untangling the two is a long story; focus here on light ν's since we know they exist.

Form of Nuclear Matrix Element

$$M_{0\nu} = M_{0\nu}^{GT} - \frac{g_V^2}{g_A^2} M_{0\nu}^{F} + \ldots$$

with

$$M_{0\nu}^{GT} = \langle f | \sum_{a,b} H(r_{ab}, \overline{E}) \vec{\sigma}_a \cdot \vec{\sigma}_b \tau_a^+ \tau_b^+ | i \rangle + \ldots$$

$$M_{0\nu}^{F} = \langle f | \sum_{a,b} H(r_{ab}, \overline{E}) \tau_a^+ \tau_b^+ | i \rangle + \ldots$$

$$H(r, \overline{E}) \approx \frac{2R}{\pi r} \int_0^\infty dq \frac{\sin qr}{q + \overline{E} - (E_i + E_f)/2} \approx \frac{R}{r}$$

Corrections ("forbidden" terms, weak form factors ...) $\approx 30\%$.

Calculating Matrix Elements

It's hard, because

- Relevant nuclei heavy ($A > 75$) and complicated.
- Never measured; nothing to calibrate to.
- Structure of initial and final nuclear ground states quite different $\Longrightarrow$ matrix element small and sensitive.

$\vdots$

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State of Nuclear-Structure Theory

In light nuclei, theory has made transition from art to science. In heavy nuclei, it's now somewhere in between.

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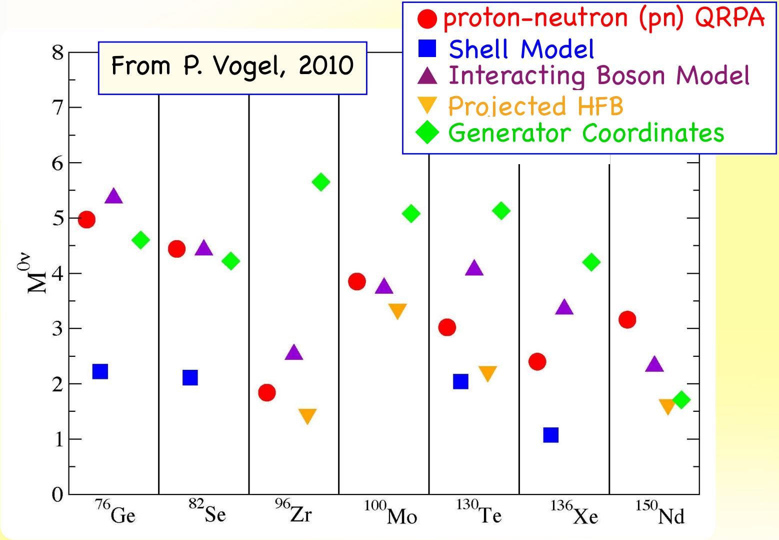
Q: Is it enough of a science yet to get accurate double-beta matrix elements?

A: It's getting there!

But at Present...



Same level of agreement in 2014. Not so great. And they may all be missing something.

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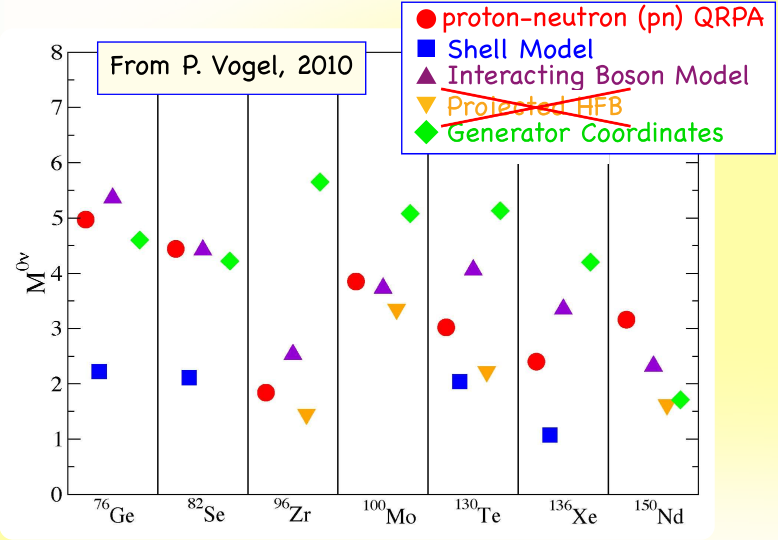

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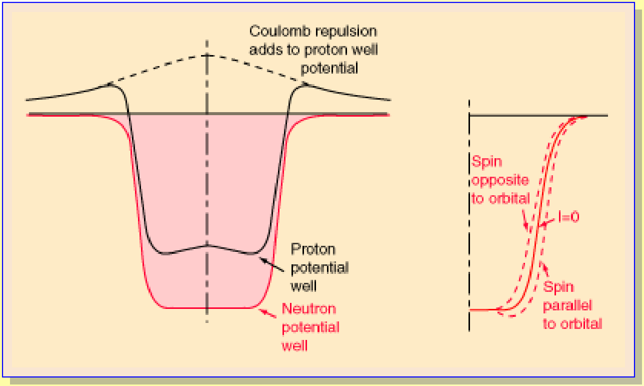
What are these models?

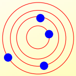
All These Models Start with Mean-Field Potential



In GCM & QRPA mean-field wave functions can include "correlations" by deforming or violating particle-number conservation.

Contrasting the Approaches

Building on Mean Field



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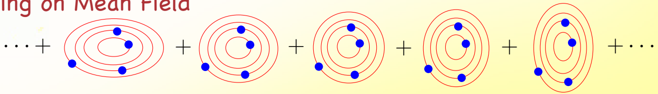

Generator-Coordinate Method (GCM) mixes many such with different collective properties.

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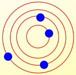
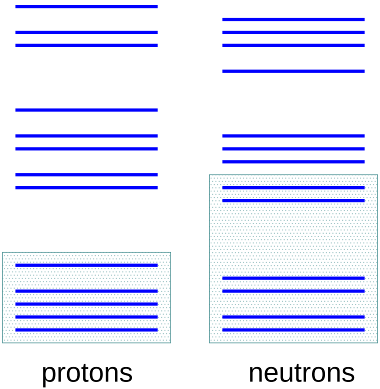


Other methods build on single independent-particle state



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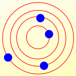

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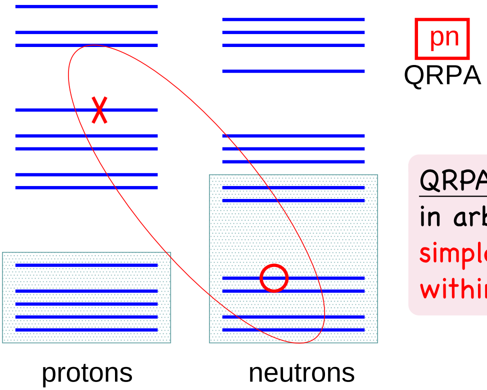
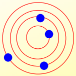


<u>QRPA:</u> Large single-particle spaces in arbitrary single mean field; simple correlations and excitations within the space.

Contrasting the Approaches

Building on Mean Field



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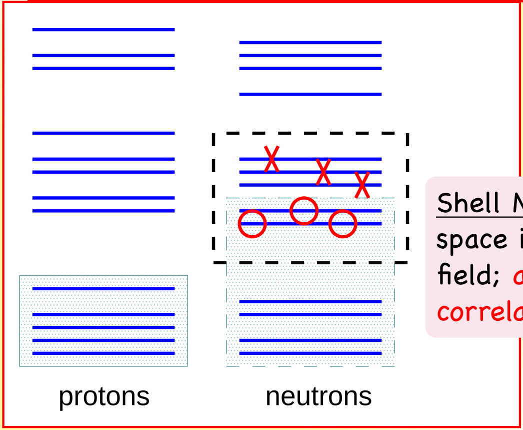
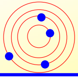


Shell Model: Small single-particle space in simple spherical mean field; arbitrarily complex correlations within the space.

Contrasting the Approaches

Building on Mean Field



Other met

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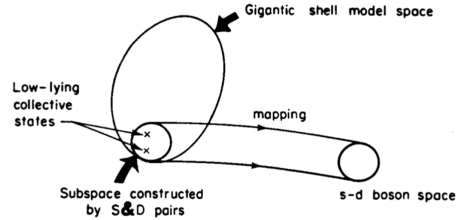
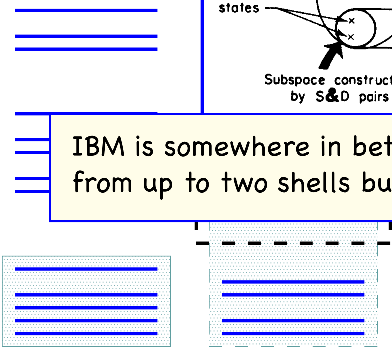


IBM is somewhere in between, mapping matrix elements from up to two shells but truncating to collective pairs.



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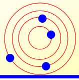

Gigantic shell model space

All these methods fit parameters to data directly in heavy nuclei. Not a bad thing, but makes it hard to estimate accuracy when calculating something different from anything you've ever measured!

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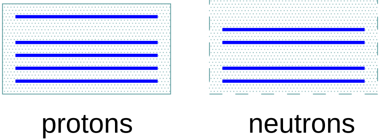


The Way Forward

Two tracks:

- A serious comprehensive statistical analysis of correlation between predictions for matrix element predictions and for other measured observables, across all models.

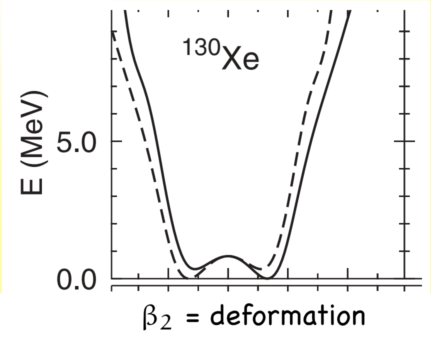
  Can attempt to assign uncertainty; just getting started and I won't talk about it.

- Improving the calculations through
  - incorporating more physics, e.g., combining effects treated by QRPA and GCM.
  - restricting phenomenology to basic level — nucleon-nucleon interaction, etc. — and solving full many-body problem from there.

  These are well underway.

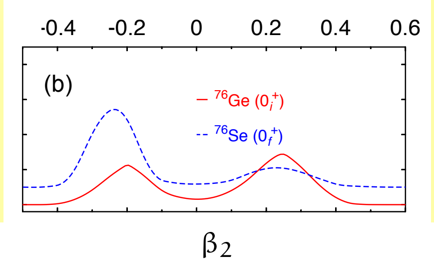
Problems of QRPA I: Single Mean Field

Some of the nuclei in these decays don't have well defined shape, can't be represented by single mean field.



Robledo et al.: Energy minima at $\beta_2 \approx \pm .15$

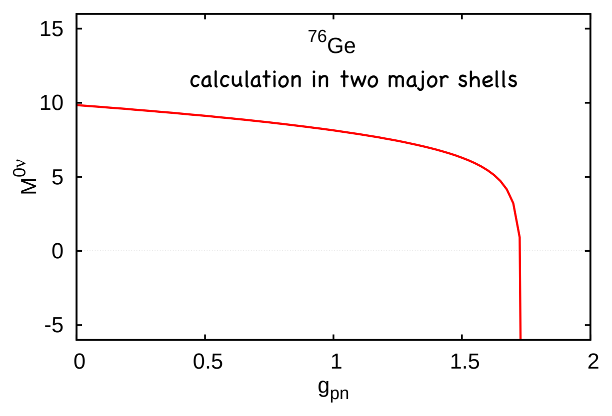
Solid line is actual result; dashed line a symmetric potential for comparison.



Rodríguez and Martinez-Pinedo: Wave functions peaked at $\beta_2 \approx \pm .2$

Problems of QRPA II: Proton-Neutron Pairing

Method treats proton-neutron pairing, an important physical effect, but not ideally:



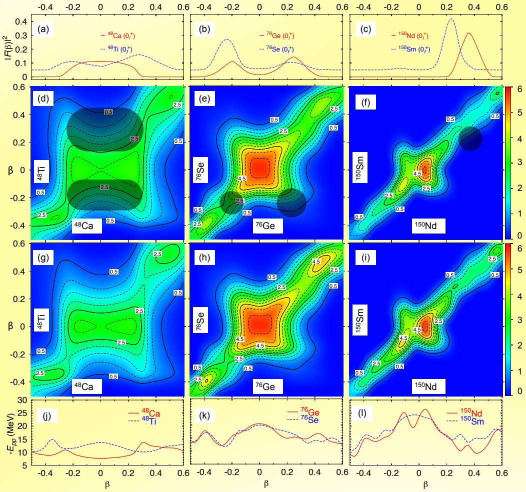
Matrix element blows up when mean-field state changes from like-particle pair condensate to proton-neutron pair condensate.

GCM: Many Mean Fields but No Proton-Neutron Pairing

Basic GCM idea: Construct set of mean fields by constraining coordinate(s), e.g. quadrupole moment $\langle Q_0 \rangle \equiv \langle \sum_i r_i^2 Y_i^{2,0} \rangle$. Minimize

$$\langle H' \rangle = \langle H \rangle - \lambda \langle Q_0 \rangle$$

Then use $\langle Q_0 \rangle$ as a collective coordinate; diagonalize H in space of number- and angular-momentum-projected mean-field states with different values of $\langle Q_0 \rangle$.



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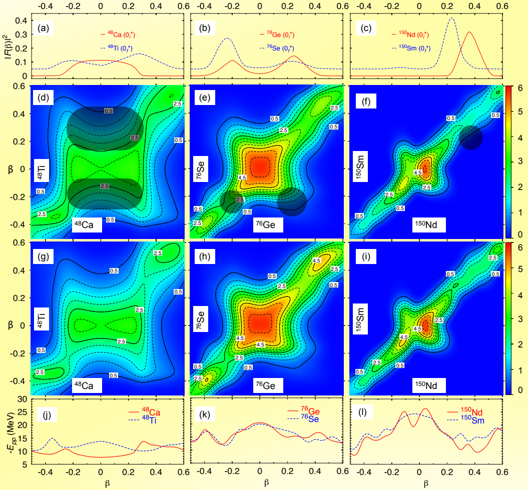


Rodríguez and Martinez-Pinedo

But the states don't contain proton-neutron pairing correlations.

Soln: Add Proton-Neutron Correlations to GCM

We generalize GCM in a way that avoids wild QRPA behavior.

Constrain pn pairing as well as deformation, i.e. minimize

$$H' = H - \lambda_Q \langle Q_0 \rangle - \lambda_P \langle P_0^\dagger \rangle$$

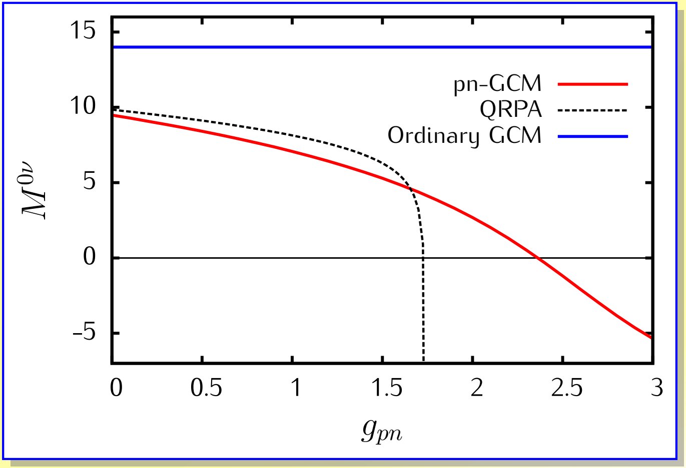
with

$$P_0^\dagger = \sum_\ell \sqrt{2\ell+1} \left[a_\ell^\dagger a_\ell^\dagger \right]^{L=0, S=1, T=0}_{M_S=0}$$

creates spin-1 pn pair

$P_0^\dagger$ has expectation value zero in unconstrained state, but add states that are constrained to have non-zero values, diagonalize in basis of many such states.

Matrix Element in ^{76}Ge

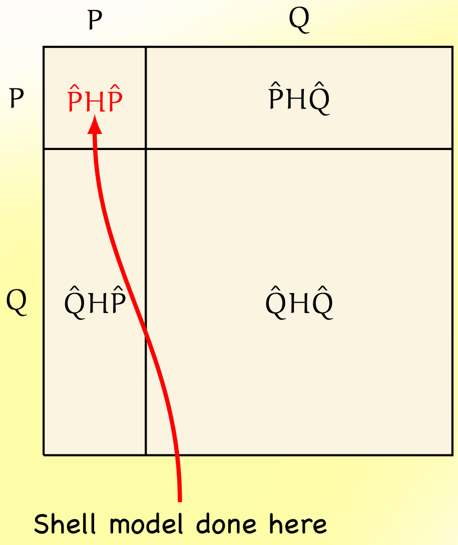


(Realistic value of g_{pn} about 1.5 — 1.6.)

This calculation a prototype; sophisticated version coming soon

Next Idea: Eliminate Nucleus-Level Phenomenology

Ab Initio Shell Model



Partition of Full Hilbert Space

P = valence space
Q = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q, with H_{eff} in P reproducing lowest eigenvalues.

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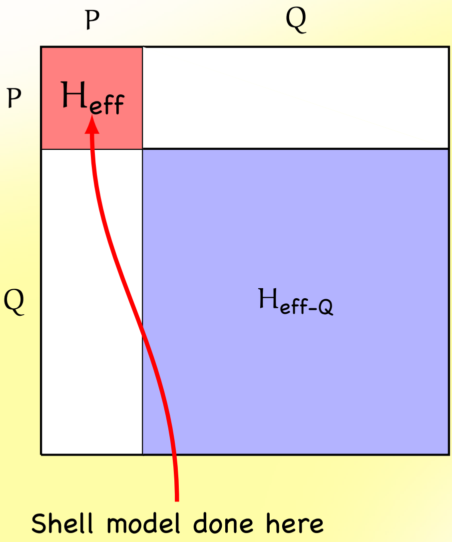

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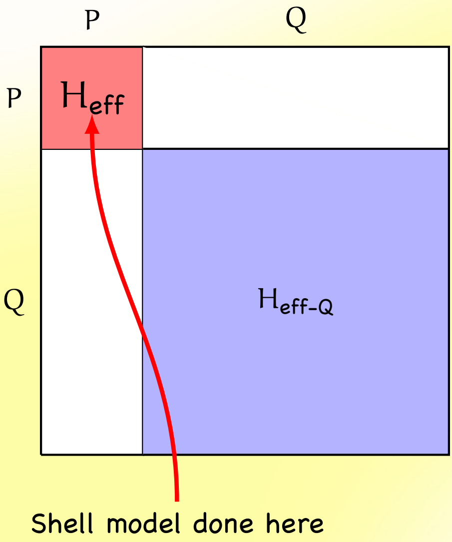

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For transition operator $\hat{M}$, apply same transformation to get $\hat{M}_{eff}$.

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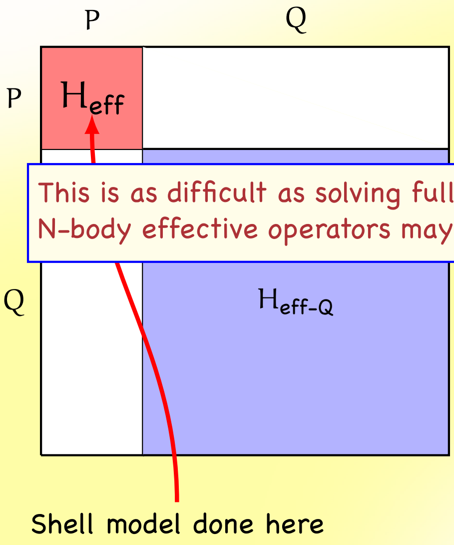

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This is as difficult as solving full problem. But the idea is that N-body effective operators may not be important for N > 2 or 3.

lowest eigenvalues.

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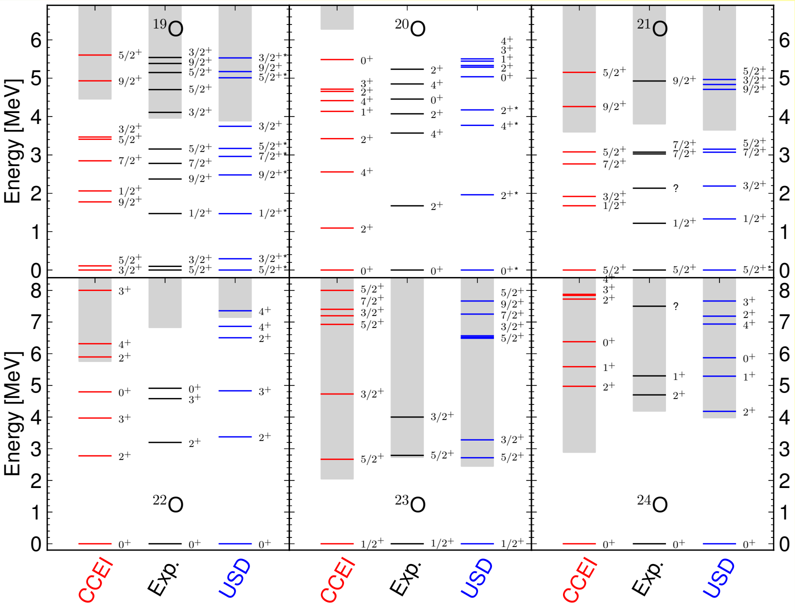
Procedure

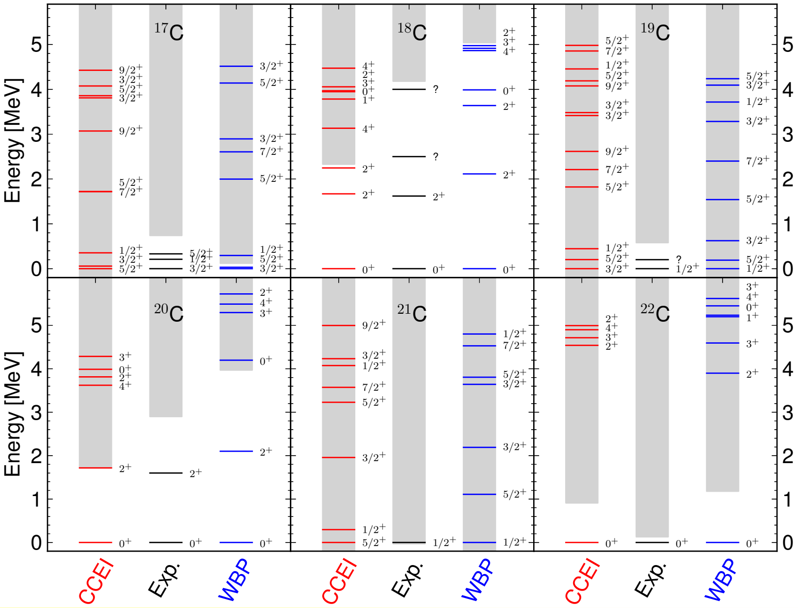
1. Find good NN and NNN interactions by matching to data in NN scattering, He, ..., or QCD. ✓
2. Use **Coupled-Clusters methods** to get good ab initio ground state for closed-shell nucleus ^{56}Ni (28 protons, 28 neutrons). ✓
3. Use extension of same method for low-lying states in nuclei with $A = 57$ and 58. ✓
4. Do "Lee-Suzuki" mapping of lowest eigenstates with $A = 57, 58$ onto $f_{5/2}pg_{9/2}$ shell, determine shell-model Hamiltonian that reproduces energies of these states. ✓
5. Do the same thing for the double-beta-decay operator.
6. Put more nucleons in the valence shell (20 for ^{76}Ge), shut up, and calculate (in the words, allegedly, of Feynman). ✓

✓ = done ✓ = done in lighter nuclei

First Step: Interaction in sd Shell



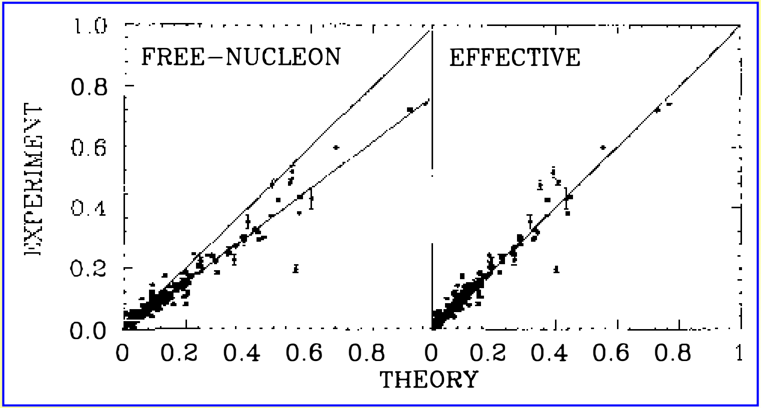
And in p Shell



Finally: "Renormalization of g_A"

Forty(?)-year old problem: Single-beta rates, 2ν double-beta rates, related observables over-predicted.

Brown & Wildenthall: Beta-decay strengths in sd shell



Solution: Not Yet Clear

Typical practice: "Renormalize" g_A to get correct results. But if g_A is renormalized by same amount in 0ν decay as in 2ν decay (a lot in shell model and IBM), experiments are in trouble; rates go as $(g_A)^4$.

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Better practice: Understand reasons for over-prediction. In modern language, must be due to

1. Many-body weak currents (from non-nucleonic degrees of freedom), either modeled explicitly as π, ρ exchange, etc., or treated in effective-field theory.

   Who's right? The old-school practitioners who say meson-exchange effects are small, or the modern effective-field-theory folk, who say they can be large (about 30% in initial studies)?

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People are attacking both sides of this problem.

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That's all.