The KNT17 approach to calculating the HVP contribution to $(g-2)_\mu$

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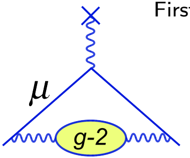
(in collaboration with Daisuke Nomura & Thomas Teubner [KNT17])



First Workshop of the Muon $g-2$ Theory Initiative

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Introduction

Question:

To ensure reliable results with increasing levels of precision, what are *now* the main points of concern when correcting, combining and integrating data to evaluate $a_\mu^{\text{had, VP}}$?

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⇒ When combining data...

→ ...how to best amalgamate large amounts of data from different experiments

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- $\rightarrow$ ...how to best amalgamate large amounts of data from different experiments
- $\rightarrow$ ...the correct implementation of correlated uncertainties (statistical and systematic)

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→ ...finding a solution that is free from bias

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⇒ The reliability of the integral and error estimate

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⇒ The reliability of the integral and error estimate

⇒ The choices when estimating unmeasured hadronic final states

The previous analysis... [HLMNT(11), J. Phys. G**38** (2011), 085003]

$\Rightarrow$ **Back in 2011...**

$\rightarrow$ Cross section measurements from radiative return

$\rightarrow$ Correlated experimental uncertainties* !!

$\rightarrow$ Large radiative correction uncertainties*

$\rightarrow$ Constant cross section clusters*

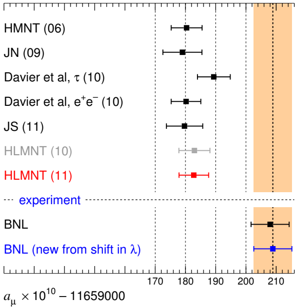
$\rightarrow$ Non-linear χ^2 minimisation fitting nuisance parameters*

$\rightarrow$ Trapezoidal rule integration

$\rightarrow$ Reliance on isospin estimates* !!

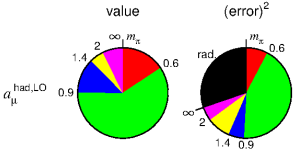
$$a_\mu^{\rm had,LOVP} = 694.9 \pm 3.7_{\rm exp} \pm 2.1_{\rm rad} = 694.9 \pm 4.3_{\rm tot}$$

$$a_\mu^{\rm had,NLOVP} = -9.8 \pm 0.1$$



*** Areas for improvement!!**

$\Rightarrow$ Changes in any of these areas can have drastic effect on mean value and error



!! e.g. - KNT 16/03/17 result - $693.9 \pm 2.6_{\rm tot}$!!

Vacuum polarisation corrections (!!)

⇒ Fully updated, self-consistent VP routine: [vp_knt_v3_0]

→ Cross sections undressed with full photon propagator (must include imaginary part), $\sigma^0_{\rm had}(s) = \sigma_{\rm had}(s)|1-\Pi(s)|^2$

⇒ Applied to all dressed experimental data in all channels

→ Accurate to $\mathcal{O}(1‰)$ precision

⇒ If correcting data, apply corresponding radiative correction uncertainty

→ Take $\frac{1}{3}$ of total correction per channel as conservative extra uncertainty

⇒ Influence/need for VP corrections has changed over time

→ Less prominent in some dominant channels

⇒ Undressing of narrow resonances must be done *excluding* the contribution from the resonance

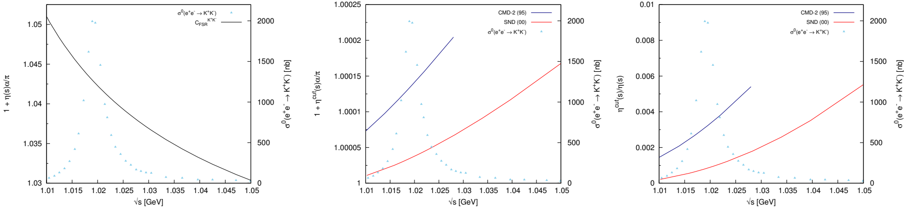
→ ...or would double count contribution

Final state radiation corrections

⇒ For $\pi^+\pi^-$, FSR more frequently included

→ If not, must include through sQED approximation [Eur. Phys. J. C 24 (2002) 51, Eur. Phys. J. C 28 (2003) 261]

⇒ For K^+K^-, is there available phase space for the creation of hard photons?



⇒ Choose to no longer apply FSR correction for K^+K^-

⇒ For higher multiplicity states, difficult to estimate correction

∴ Apply conservative uncertainty

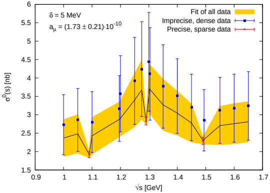
Need new, more developed tools to increase precision here

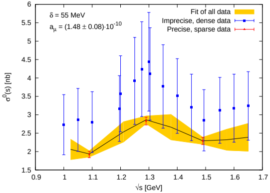
(e.g. - CARLOMAT 3.1 [Eur.Phys.J. C77 (2017) no.4, 254]?)

Clustering data

$\Rightarrow$ Re-bin data into *clusters*

Better representation of data combination through adaptive clustering algorithm





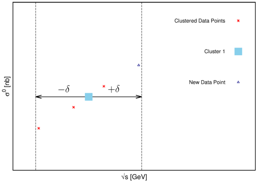
$\rightarrow$ More and more data $\Rightarrow$ risk of over clustering

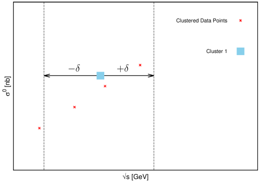
$\Rightarrow$ loss of information on resonance

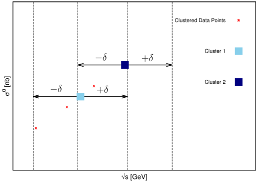
$\rightarrow$ Scan cluster sizes for optimum solution (error, χ^2, check by sight...)

$\Rightarrow$ Scanning/sampling by varying bin widths

$\rightarrow$ Clustering algorithm now adaptive to points at cluster boundaries







Correlation and covariance matrices

⇒ Correlated data beginning to dominate full data compilation...

→ Non-trivial, energy dependent influence on both mean value and error estimate

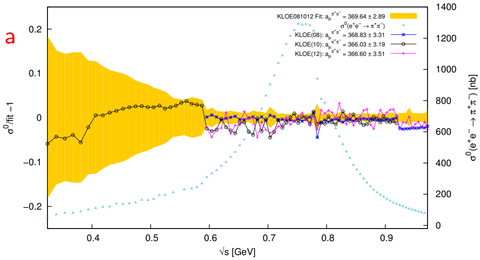
KNT17 prescription

- Construct full covariance matrices for each channel & entire compilation
  ⇒ Framework available for inclusion of any and all inter-experimental correlations
- If experiment does not provide matrices...
  → Statistics occupy diagonal elements only
  → Systematics are 100% correlated
- If experiment does provide matrices...
  → Matrices **must** satisfy properties of a covariance matrix

e.g. - KLOE $\pi^+\pi^-(\gamma)$ combination covariance matrices update

$\Longrightarrow$ Originally, NOT a positive semi-definite matrix:

(This is not an example of bias)



KLOE as an example: Constructing the KLOE $\pi^+\pi^-(\gamma)$ combination covariance matrices (!!) [preliminary]

See talk tomorrow by Stefan Müller

⇒ Three measurements of $\sigma^0_{\pi\pi(\gamma)}$ by KLOE

→ KLOE08, KLOE10 and KLOE12

⇒ They are, in part, highly correlated → **must** be incorporated

→ e.g. - KLOE08 and KLOE12 share the same $\pi\pi(\gamma)$ data, with KLOE12 normalised by the measured $\mu\mu(\gamma)$ cross section

⇒ Must ensure construction satisfies required properties of covariance matrices

e.g. - KLOE0810

→ Correlated statistic and systematics

→ Correlations must cover entire data range

→ KLOE08 is more precise than KLOE10

⇒ **Expected influence on non-overlapping data region**

$$\left(\begin{array}{c|c|c} \begin{matrix}\cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\\ & \mathrm{KLOE08} & \\ & 60\times 60 & \\ \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\end{matrix} & \begin{matrix}\cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\\ \cdots & \mathrm{KLOE0810} & \cdots\\ \cdots & 60\times 75 & \cdots\\ \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\end{matrix} & \begin{matrix}\cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\\ & \mathrm{KLOE0812} & \\ & 60\times 60 & \\ \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\end{matrix} \\ \hline \begin{matrix}\cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\\ & \mathrm{KLOE1008} & \\ & 75\times 60 & \\ \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\end{matrix} & \begin{matrix}\cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\\ \cdots & \mathrm{KLOE10} & \cdots\\ \cdots & 75\times 75 & \cdots\\ \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\end{matrix} & \begin{matrix}\cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\\ & \mathrm{KLOE1012} & \\ & 75\times 60 & \\ \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\end{matrix} \\ \hline \begin{matrix}\cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\\ & \mathrm{KLOE1208} & \\ & 60\times 60 & \\ \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\end{matrix} & \begin{matrix}\cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\\ \cdots & \mathrm{KLOE1210} & \cdots\\ \cdots & 60\times 75 & \cdots\\ \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\end{matrix} & \begin{matrix}\cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\\ & \mathrm{KLOE12} & \\ & 60\times 60 & \\ \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots\end{matrix} \end{array}\right)$$

Prospect of bias

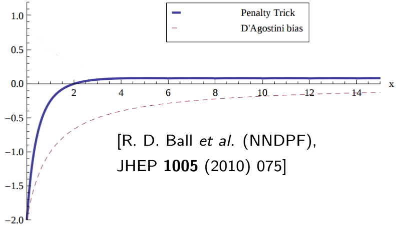
'Statistical bias is a feature of a statistical technique or of its results whereby the expected value of the results differs from the true underlying quantitative parameter being estimated.'

$\Rightarrow$ Iterative fit of covariance matrix as defined by data $\rightarrow$ D'Agostini bias

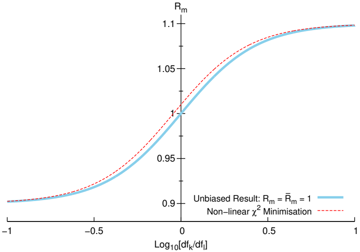
[Nucl.Instrum.Meth. A346 (1994) 306-311]

$\Rightarrow$ HLMNT11 use of non-linear χ^2 minimisation fitting nuisance parameters

$\rightarrow$ Penalty trick bias



[R. D. Ball *et al.* (NNDPF), JHEP **1005** (2010) 075]



$\Rightarrow$ Should we not fit correlated systematics (i.e. - BLUE estimate [Nucl. Instrum. Meth. A 270 (1988) 110])?

$\rightarrow$ Is neglecting the influence of necessary correlations not a bias...?

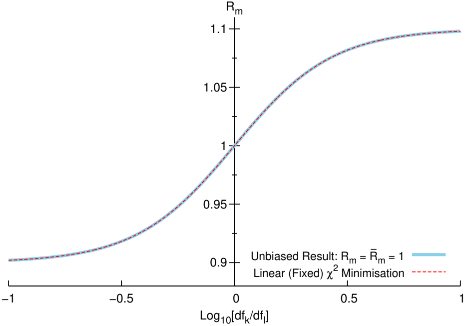
Fixing the covariance matrix [JHEP 1005 (2010) 075, Eur.Phys.J. C75 (2015), 613]

⇒ Apply a procedure to fix the covariance matrix

$$\mathbf{C}_I\left(i^{(m)}, j^{(n)}\right) = \mathsf{C}^{\text{stat}}\left(i^{(m)}, j^{(n)}\right) + \frac{\mathsf{C}^{\text{sys}}\left(i^{(m)}, j^{,n)}\right)}{R_i^{(m)} R_j^{(n)}} R_m R_n \quad ,$$

in an iterative χ^2 minimisation method that, to our best knowledge, is free from bias

- ⇒ Fixing with theory value regulates influence
- ⇒ Can be shown from toy models to be free from bias
- ⇒ Swift convergence
- ⇒ Comparison with past results shows HLMNT11 estimates are largely unaffected



Allows for increased fit flexibility and full use of energy dependent, correlated uncertainties

Linear χ^2 minimisation

$\Rightarrow$ Redefine clusters to have linear cross section

$\rightarrow$ Consistency with trapezoidal rule integration

$\rightarrow$ Fix covariance matrix with linear interpolants at each iteration (extrapolate at boundary)

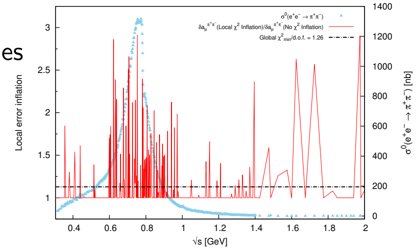
$$\chi^2 = \sum_{i=1}^{N_{\rm tot}} \sum_{j=1}^{N_{\rm tot}} \left(R_i^{(m)} - \mathcal{R}_m^i\right) \mathbf{C}^{-1}\left(i^{(m)}, j^{(n)}\right) \left(R_j^{(n)} - \mathcal{R}_n^j\right)$$

$\Rightarrow$ Through correlations and linearisation, result is the minimised solution of all neighbouring clusters

$\rightarrow$ ...and solution is the product of the influence of all correlated uncertainties

$\Rightarrow$ The flexibly of the fit to vary due to the energy dependent, correlated uncertainties benefits the combination

$\rightarrow$ ...and any data tensions are reflected in a local χ^2 error inflation

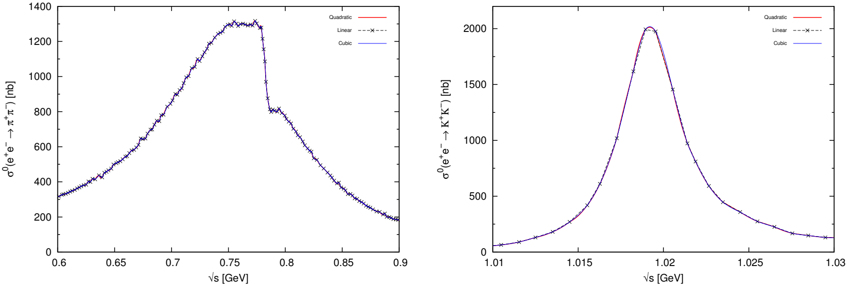


Integration

$\Rightarrow$ Trapezoidal rule integral

$\rightarrow$ Consistency with linear cluster definition

$\rightarrow$ High data population $\therefore$ Accurate estimate from linear integral



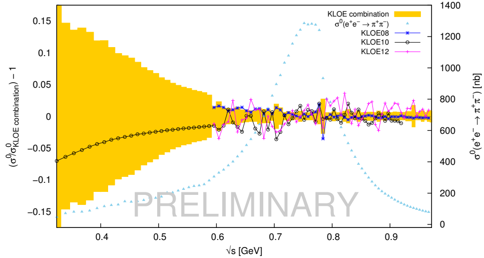
$\rightarrow$ Higher order polynomial integrals give (at maximum) differences of $\sim 10\%$ of error

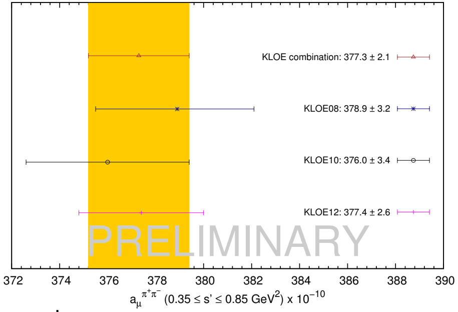
$\Rightarrow$ Estimates of error non-trivial at integral borders

$\longrightarrow$ Extrapolate/interpolate covariance matrices

KLOE as an example: the resulting KLOE $\pi^+\pi^-(\gamma)$ combination (!!) [preliminary]

$\Rightarrow$ Combination of KLOE08, KLOE10 and KLOE12 gives 85 distinct bins between $0.1 \leq s \leq 0.95$ GeV^2





$\rightarrow$ Covariance matrix now correctly constructed

$\Rightarrow$ a positive semi-definite matrix

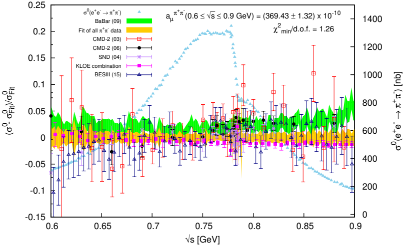
$\rightarrow$ Non-trivial influence of correlated uncertainties on resulting mean value

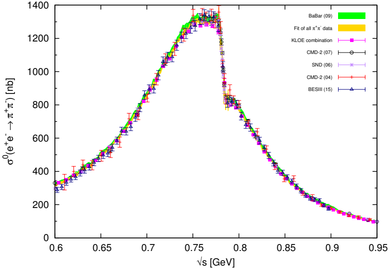
$$a_\mu^{\pi^+\pi^-}(0.1 \leq s' \leq 0.95\ \text{GeV}^2) = (489.9 \pm 2.0_{\text{stat}} \pm 4.3_{\text{sys}}) \times 10^{-10}$$

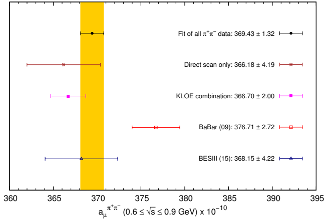
$\pi^+\pi^-$ channel (!!)

$\Rightarrow$ Large improvement for 2π estimate

$\rightarrow$ BESIII [Phys.Lett. B753 (2016) 629-638] and KLOE combination provide downward influence to mean value







$\Rightarrow$ Correlated & experimentally corrected $\sigma^0_{\pi\pi(\gamma)}$ data now entirely dominant

$a_\mu^{\pi^+\pi^-}(0.305 \le \sqrt{s} \le 2.00$ GeV$)$:

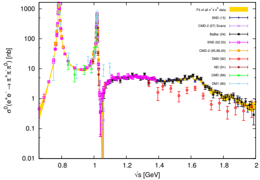
HLMNT11: 505.77 ± 3.09

KNT17: 502.85 ± 1.93 (!!)

(no radiative correction uncertainties)

Other notable exclusive channels

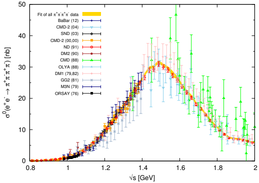
$\pi^+\pi^-\pi^0$



HLMNT11: 47.51 ± 0.99

KNT17: 47.68 ± 0.70

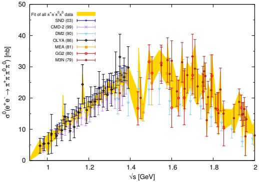
$\pi^+\pi^-\pi^+\pi^-$



HLMNT11: 14.65 ± 0.47

KNT17: 15.18 ± 0.14

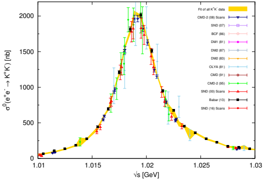
$\pi^+\pi^-\pi^0\pi^0$



HLMNT11: 20.37 ± 1.26

KNT17: 20.07 ± 1.19

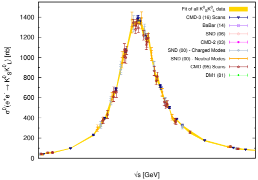
K^+K^-



HLMNT11: 22.15 ± 0.46

KNT17: 22.76 ± 0.22

$K_S^0 K_L^0$



HLMNT11: 13.33 ± 0.16

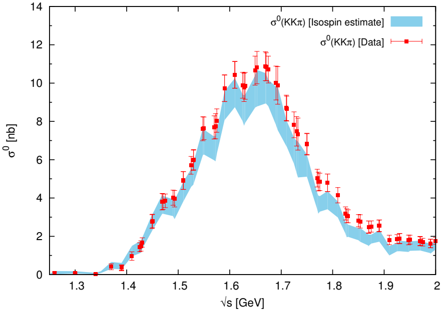
KNT17: 13.09 ± 0.12

$KK\pi$, $KK\pi\pi$ and isospin (!!)

⇒ New BaBar data for $KK\pi$ and $KK\pi\pi$ removes reliance on isospin (only $K^0_S = K^0_L$)

$\underline{KK\pi}$

$K^0_S K^0_L \pi^0$ [Phys.Rev. D95 (2017), 052001]

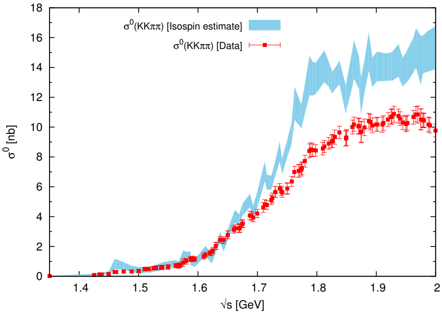


HLMNT11: 2.77 ± 0.15

KNT17: 2.83 ± 0.14

$\underline{KK\pi\pi}$

$K^0_S K^0_L \pi^+\pi^-$ [Phys.Rev. D80 (2014), 092002]
$K^0_S K^0_S \pi^+\pi^-$ [Phys.Rev. D80 (2014), 092002],
$K^0_S K^0_L \pi^0\pi^0$ [Phys.Rev. D95 (2017), 052001]
$K^0_S K^\pm \pi^\mp \pi^0$ [arXiv:1704.05009]



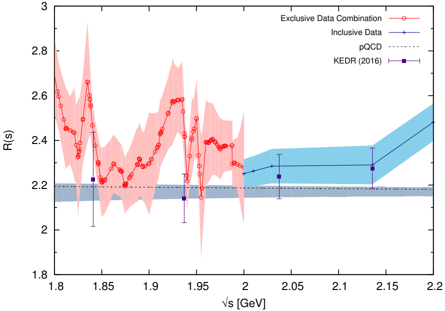
HLMNT11: 3.31 ± 0.58

KNT17: 2.42 ± 0.09

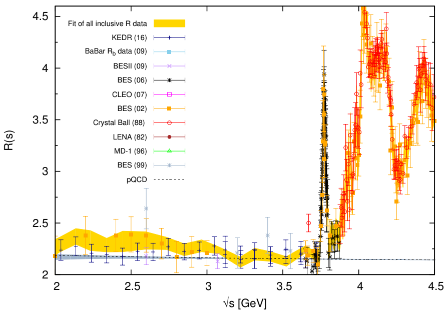
⇒ **But**, still reliant on isospin estimates for $\pi^+\pi^-3\pi^0$, $\pi^+\pi^-4\pi^0$, $KK3\pi$...

Inclusive

$\Rightarrow$ New KEDR inclusive R data ranging $1.84 \leq \sqrt{s} \leq 3.05$ GeV [Phys.Lett. B770 (2017) 174-181] and $3.12 \leq \sqrt{s} \leq 3.72$ GeV [Phys.Lett. B753 (2016) 533-541]



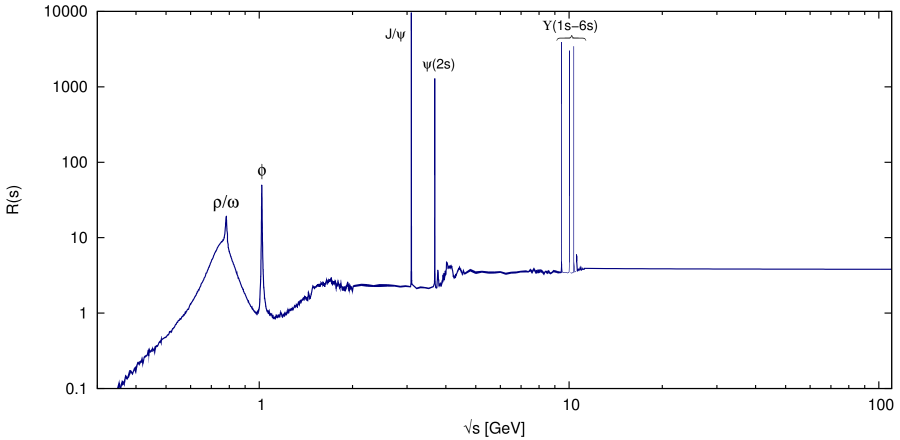
$$a_\mu^{\text{had, LOVP}}(1.84 \leq \sqrt{s} \leq 2.00 \text{ GeV}):$$
$$\text{pQCD}: 6.42 \pm 0.03$$
$$\text{Data}: 6.88 \pm 0.25$$



$$a_\mu^{\text{had, LOVP}}(2.60 \leq \sqrt{s} \leq 3.73 \text{ GeV}):$$
$$\text{pQCD (inflated errors)}: 10.82 \pm 0.38$$
$$\text{Data}: 11.20 \pm 0.14$$

$\Longrightarrow$ **Choose to adopt entirely data driven estimate from threshold to 11.2 GeV**

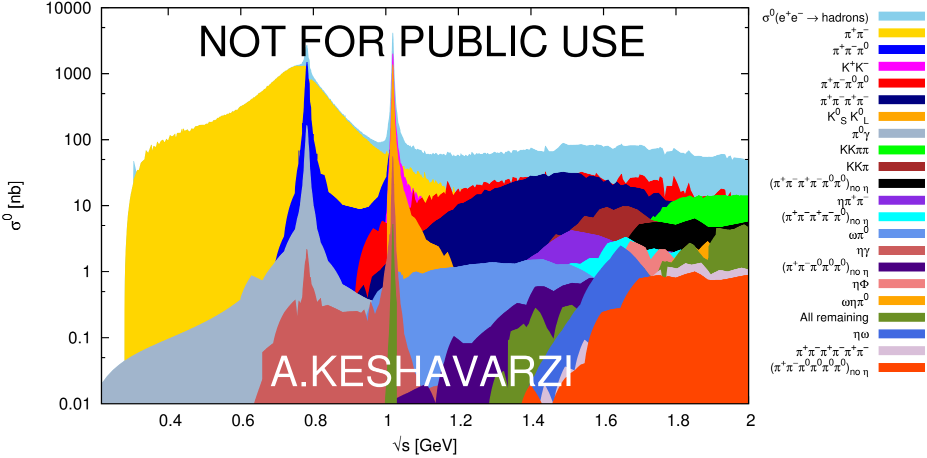
$R(s)$ for $m_\pi \leq \sqrt{s} < \infty$



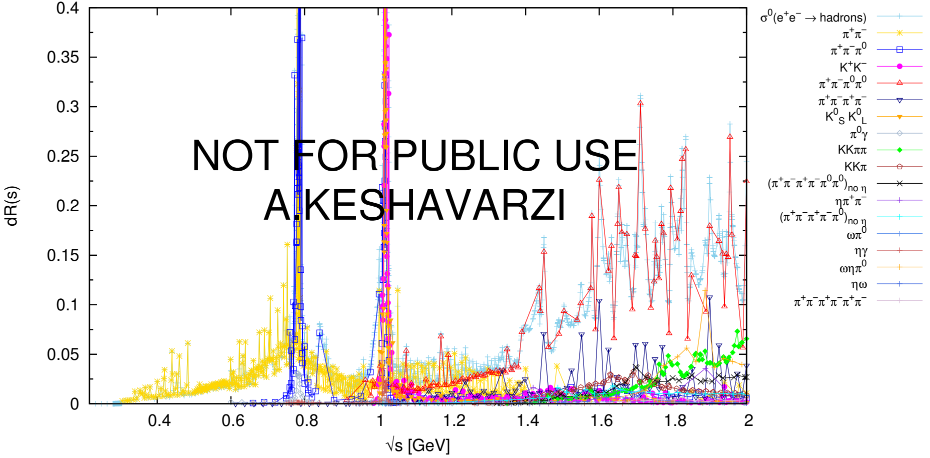
$\Rightarrow$ Full compilation data set for hadronic R-ratio to be made available soon...

$\Longrightarrow$ **...complete with full covariance matrix**

Contributions to mean value below 2GeV



Contributions to uncertainty below 2GeV



KNT17 $a_\mu^{\text{had, VP}}$ update (!!)

HLMNT(11): 694.91 ± 4.27

$\downarrow$

!! KNT 16/03/17 result: $693.9 \pm 1.34_{\text{stat}} \pm 2.15_{\text{sys}} \pm 0.32_{\text{vp}} \pm 0.70_{\text{fsr}}$!!

$\downarrow$

!! Updated KLOE combination covariance matrix construction !!

$\downarrow$

!! $KK\pi\pi$ determination without isospin !!

$\downarrow$

!! New VP iteration !!

$\downarrow$

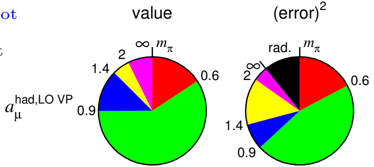
This work: $a_\mu^{\text{had, LOVP}} = 692.23 \pm 1.26_{\text{stat}} \pm 2.02_{\text{sys}} \pm 0.31_{\text{vp}} \pm 0.70_{\text{fsr}}$

$= 692.23 \pm 2.42_{\text{exp}} \pm 0.77_{\text{rad}}$

$= 692.23 \pm 2.54_{\text{tot}}$

$a_\mu^{\text{had, NLOVP}} = -9.83 \pm 0.04_{\text{tot}}$

$\Rightarrow$ Accuracy better then 0.4%
(uncertainties include all available correlations)



KNT17 a_μ^{SM} update

	2011		2017	*to be discussed
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[Phys. Rev. Lett. 109 (2012) 111808]
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]*
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]*
	HLMNT11		**KNT17**	
LO HVP	694.91 (4.27)	$\longrightarrow$	692.23 **(2.54)**	this work*
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.83 (0.04)	this work*
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144] *
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659181.00 (3.62)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	$\longrightarrow$	28.1 (7.3)	this work
Δa_μ	3.3σ	$\longrightarrow$	3.9σ	this work

Conclusions

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✓ Continuously adapt and improve...

Extra Slides

VP corrections of narrow resonances

The undressing of narrow resosnances in the $c\bar{c}$ and $b\bar{b}$ regions requires special attention. Importantly, we must undress the electronic width of an individual resonance, Γ_{ee}, of vacuum polarisation corrections, where the VP correction *excludes* the contribution of that resonance, such that

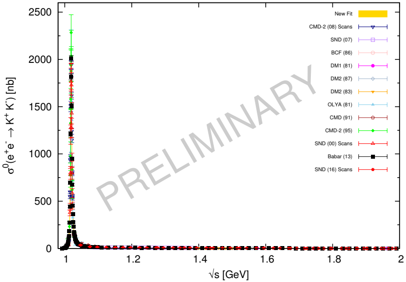
$$\Gamma_{ee}^0 = \frac{\left(\alpha/\alpha_{\textsf{no res}}(M_{\text{res}}^2)\right)^2}{1+3/\alpha(4\pi)}\Gamma_{ee} \ .$$

Here, $\alpha_{\textsf{no res}}$ is the running QED coupling without the contribution of the resonance we are correcting for and is given by

$$\alpha_{\textsf{no res}}(s) \equiv \frac{\alpha}{1-\Delta\alpha_{\textsf{no res}}(s)}$$

where $\Delta\alpha_{\textsf{no res}}(s)$ is determined such that the input $R(s)$ does not include the resonance that we are correcting. To include the resonance would result in a double counting of this contribution.

Kaon FSR study



Inclusive FSR correction was previously applied to K^+K^- cross section

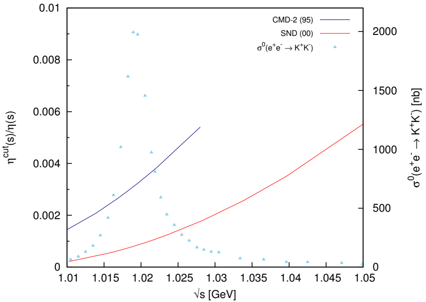
KLN theorem requires all virtual and soft corrections necessarily included in given cross section

$\therefore$ Only hard real radiation is left to be corrected for

BUT K^+K^- cross section is totally dominated by ϕ resonance

$\Rightarrow$ No phase space for creation of hard real photons at ϕ

Inclusive FSR correction is large over-correction $\longrightarrow$



$\therefore$ No longer apply FSR correction

Properties of a covariance matrix

Any covariance matrix, $\mathcal{C}_{ij}$, of dimension $n \times n$ must satisfy the following requirements:

- As the diagonal elements of any covariance matrix are populated by the corresponding variances, all the diagonal elements of the matrix are positive. Therefore, the trace of the covariance matrix must also be positive

$$\text{Trace}(\mathcal{C}_{ij}) = \sum_{i=1}^{n} \sigma_{ii} = \sum_{i=1}^{n} \text{Var}_i > 0$$

- It is a symmetric matrix, $\mathcal{C}_{ij} = \mathcal{C}_{ji}$, and is, therefore, equal to its transpose, $\mathcal{C}_{ij} = \mathcal{C}_{ij}^T$
- The covariance matrix is a positive, semi-definite matrix,

$$\mathbf{a}^T \mathcal{C}\ \mathbf{a} \geq 0\ ;\ \mathbf{a} \in \mathbf{R^n},$$

where $\mathbf{a}$ is an eigenvector of the covariance matrix $\mathcal{C}$

- Therefore, the corresponding eigenvalues $\lambda_{\mathbf{a}}$ of the covariance matrix must be real and positive and the distinct eigenvectors are orthogonal

$$\mathbf{b}\ \mathcal{C}\ \mathbf{a} = \lambda_{\mathbf{a}}(\mathbf{b} \cdot \mathbf{a}) = \mathbf{a}\ \mathcal{C}\ \mathbf{b} = \lambda_{\mathbf{b}}(\mathbf{a} \cdot \mathbf{b})$$

$$\therefore \text{if } \lambda_{\mathbf{a}} \neq \lambda_{\mathbf{b}} \Rightarrow (\mathbf{a} \cdot \mathbf{b}) = 0$$

- The determinant of the covariance matrix is positive: $\text{Det}(\mathcal{C}_{ij}) \geq 0$

Tests of reliability of f_k method

Did the f_k method incur a bias?

Compare f_k method and fixed matrix method with only multiplicative normalisation uncertainties.

$\rightarrow$ If we see **differences** in mean value, then bias previously influenced the fit.
$\longrightarrow$ Previous results unreliable

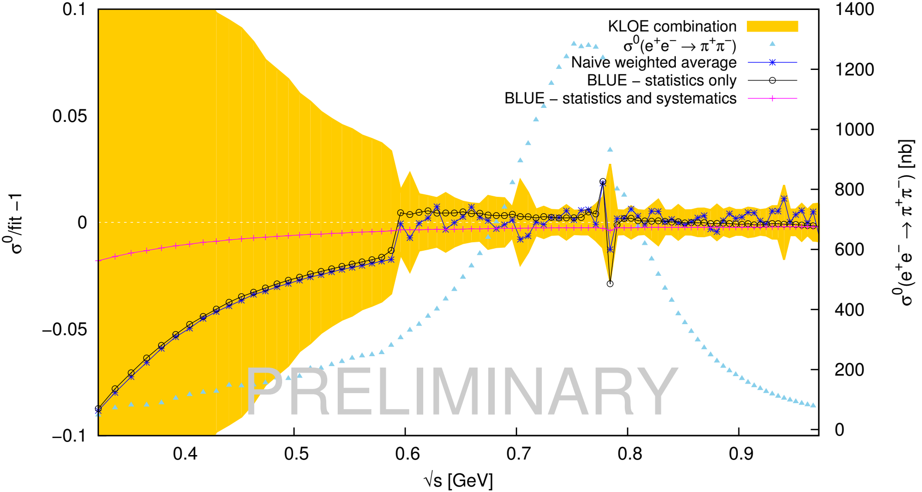
$\rightarrow$ If we see **no differences** in mean value, then bias did not influence fit (any change comes from improved treatment of systematics)
$\longrightarrow$ Previous results reliable

Example - $\pi^+\pi^-$
Set 1 - CMD-2(06) (0.7% Systematic Uncertainty), Set 2 - CMD-2(06) (0.8% Systematic Uncertainty), Set 3 - SND(04) (1.3% Systematic Uncertainty)

From $0.37 \rightarrow 0.97$ GeV

Fit Method:	f_k method		Fixed matrix method		
Channel	a_μ	χ^2_{min}/d.o.f.	a_μ	χ^2_{min}/d.o.f.	Difference
$\pi^+\pi^-$	481.42 ± 4.26	1.10	481.42 ± 4.05	1.02	0.00

Comparison of KLOE combination methods [preliminary]



KNT17 $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2)$ update [preliminary]

Using the same data compilation as for a_μ^{HVP}, we can also determine $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2)$, in order to update our prediction of the value of the QED coupling at the Z boson mass:

HLMNT11: $(276.26 \pm 1.38_{\mathrm{tot}}) \times 10^{-4}$

KNT17: $$\begin{aligned}\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) &= (276.06 \pm 0.39_{\mathrm{stat}} \pm 0.64_{\mathrm{sys}} \pm 0.08_{\mathrm{vp}} \pm 0.82_{\mathrm{fsr}}) \times 10^{-4} \\ &= (276.06 \pm 0.76_{\mathrm{exp}} \pm 0.83_{\mathrm{rad}}) \times 10^{-4} \\ &= (276.06 \pm 1.13_{\mathrm{tot}}) \times 10^{-4}\end{aligned}$$

Analysis comparison for leading channels

Channel	KNT17	DHMZ16	FJ17
$\pi^+\pi^-$	502.73 ± 1.94	506.9 ± 2.55	
$\pi^+\pi^-2\pi^0$	17.80 ± 0.99	18.03 ± 0.56	
$2\pi^+2\pi^-$	14.00 ± 0.19	13.70 ± 0.31	
K^+K^-	22.70 ± 0.25	22.67 ± 0.43	
$K_S^0K_L^0$	13.08 ± 0.14	12.81 ± 0.24	
Total HVP	692.23 ± 2.54	692.6 ± 3.3	688.07 ± 4.14