Simulation-Based Circular e+e- Higgs Factory Design

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TLEP Workshop

Fermilab, July 25-26, 2013

Outline

Definition of “Higgs Factory”

Ring Layout

“Saturated Tune Shift” Operation

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Beam Height Equilibrium: Beam-Beam Heating vs. Radiation Cooling

The Parameter Space for Beam Energy E

Unique Reconciliation of Luminosity and Beamstrahlung

Optimized Performance vs Beam Energy E

Definition of “Higgs Factory”

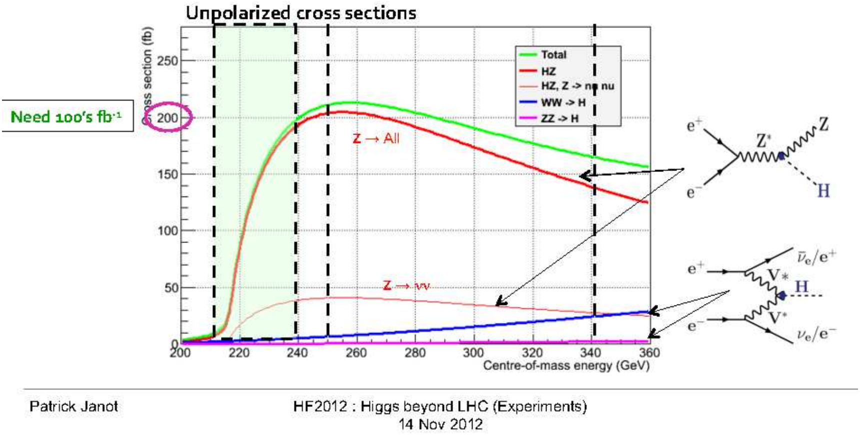
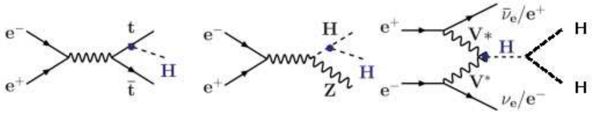


Figure: **Phase I** at e+e- ring; Higgs particle cross sections up to $\sqrt{s} = 0.3\,\text{TeV}$; $\mathcal{L} \geq 2 \times 10^{34}\,/\mathrm{cm}^2/\mathrm{s}$, or $2\,\text{fb}/\text{day}$, will produce 400 Higgs per day in this range.

Other couplings : Htt and HHH



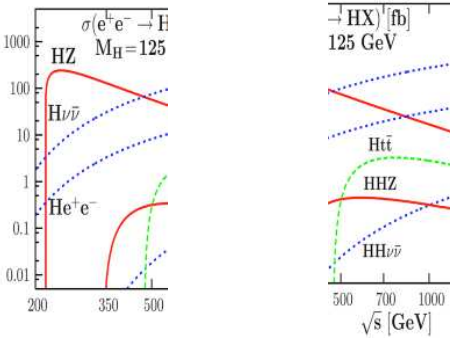
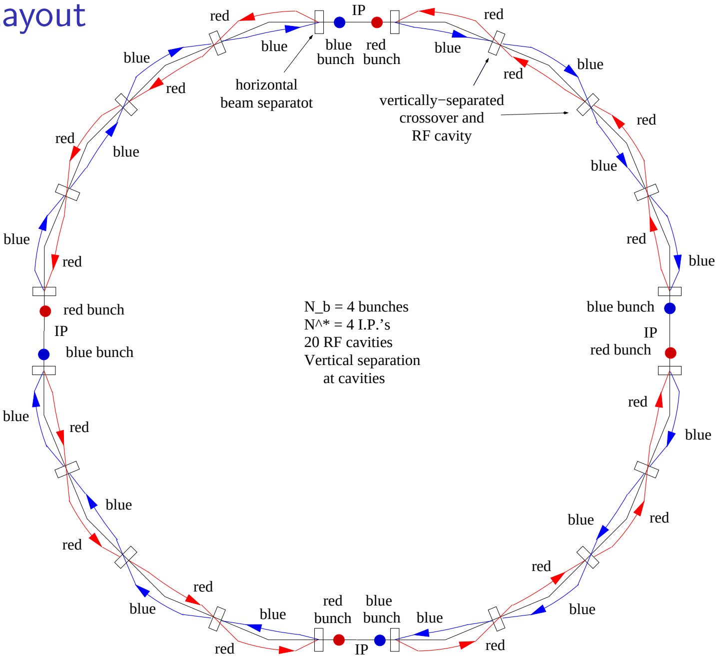


Figure: **Phases II** at e+e- ring; $\mathcal{L} = 0.5 \times 10^{34}\ /\mathrm{cm}^2/\mathrm{s}$ will include fifty $\mathrm{H}\nu\bar{\nu}$, five He+e- and one HHZ or Ht$\bar{t}$ per day at $\sqrt{s} = 500\,\mathrm{GeV}$. **Phase III**, $E > 0.5\,\mathrm{TeV}$, will require linear or $\mu-$collider.

Ring Layout


IP
red
blue
blue bunch
red bunch
horizontal beam separatot
vertically−separated crossover and RF cavity
red bunch
IP
blue bunch
blue bunch
IP
red bunch
N_b = 4 bunches
N^* = 4 I.P.'s
20 RF cavities
Vertical separation at cavities
red bunch
blue bunch
IP

- Especially at high energies the design orbit spirals in significantly; this requires the RF acceleration to be distributed quite uniformly.
- Basically the ring is a “curved linac”.
- The layout shown exploits the spiralling in of counter-circulating orbits and horizontal electric separation to separate the beams in the arcs.
- Beams cross over, vertically separated, at the multiple RF locations.

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- Horizontal separation electrode gaps are large enough to be masked from synchrotron radiation.
- Beam is separated vertically at cross-over points. These are the only intentional vertical deflections in the ring.

“Saturated Tune Shift” Operation

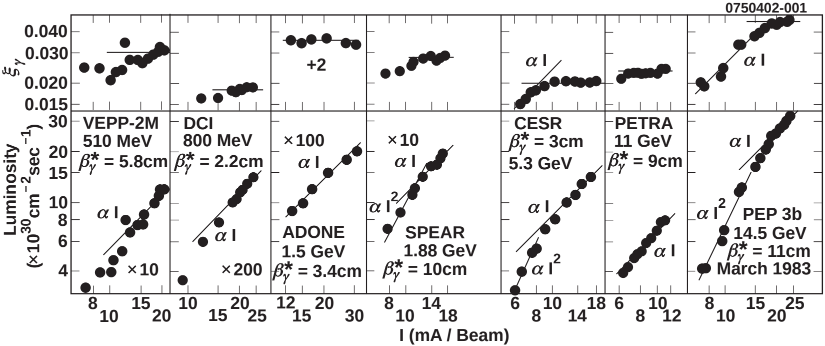
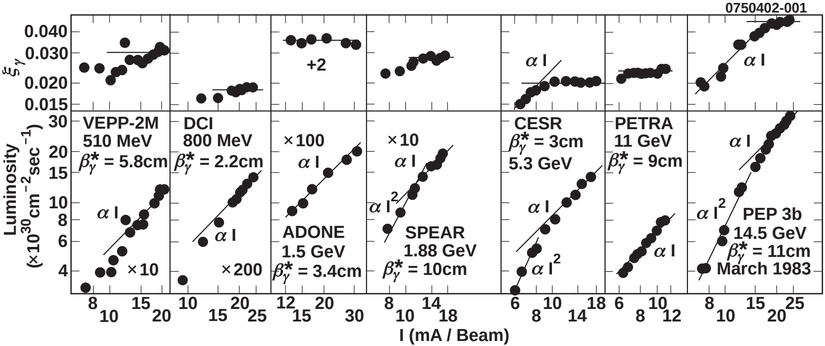


Figure: John Seeman plots of luminosity performance.



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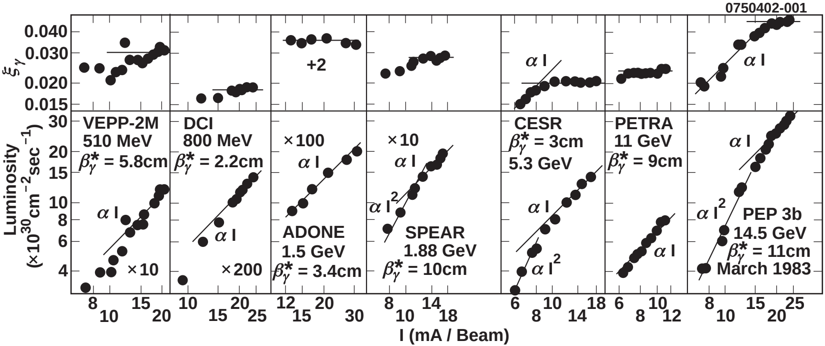

Figure: John Seeman plots of luminosity performance.

- "Tune shift saturation" marks transition from quadratic to linear dependence of luminosity on beam current.
- Above saturation "specific luminosity" (luminosity/current) is constant.

Simulation Results

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- Saturation Principle: the beam height adjusts itself to the smallest value for which the least stable particle (of probable amplitude) is barely stable.
- There is no beam loss though; amplitude detuning causes a particle to lose lock and decay back toward zero.

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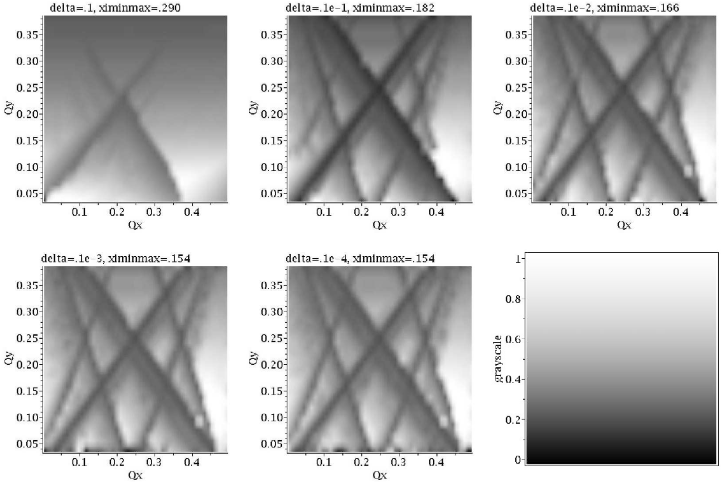
Table: Parameters of some circular, flat beam, e+e- colliding rings, and the saturation tune shift values predicted by the simulation, which has *no adjustable parameters*.

Ring IP's	Q_x/IP	Q_y/IP	Q_s/IP	σ_z	β_y^*	$10^4\delta_y$	$\xi_{\rm th.}$	$\Delta Q_{y,{\rm exp.}}$	th/exp
VEPP4 1	8.55	9.57	0.024	0.06	0.12	1.68	0.028	0.046	0.61
PEP-1IP 1	21.296	18.205	0.024	0.021	0.05	6.86	0.076	0.049	1.55
PEP-2IP 2	5.303	9.1065	0.0175	0.020	0.14	4.08	0.050	0.054	0.93
CESR-4.7 2	4.697	4.682	0.049	0.020	0.03	0.38	0.037	0.018	2.06
CESR-5.0 2	4.697	4.682	0.049	0.021	0.03	0.46	0.034	0.022	1.55
CESR-5.3 2	4.697	4.682	0.049	0.023	0.03	0.55	0.029	0.025	1.16
CESR-5.5 2	4.697	4.682	0.049	0.024	0.03	0.61	0.027	0.027	1.00
CESR-2000 1	10.52	9.57	0.055	0.019	0.02	1.113	0.028	0.043	0.65
KEK-1IP 1	10.13	10.27	0.037	0.014	0.03	2.84	0.046	0.047	0.98
KEK-2IP 2	4.565	4.60	0.021	0.015	0.03	1.42	0.048	0.027	1.78
PEP-LER 1	38.65	36.58	0.027	0.0123	0.0125	1.17	0.044	0.044	1.00
KEK-LER 1	45.518	44.096	0.021	0.0057	0.007	2.34	0.042	0.032	1.31
BEPC 1	5.80	6.70	0.020	0.05	0.05	0.16	0.068	0.039	1.74

$$\frac{\text{theory}}{\text{experiment}} = 1.26 \pm 0.45 \qquad (1)$$

Saturated Tune Shift $\xi^{\text{sat.}}$ in (Q_x, Q_y) Plane, for 5 Orders of Magnitude Range of Damping Decrement δ



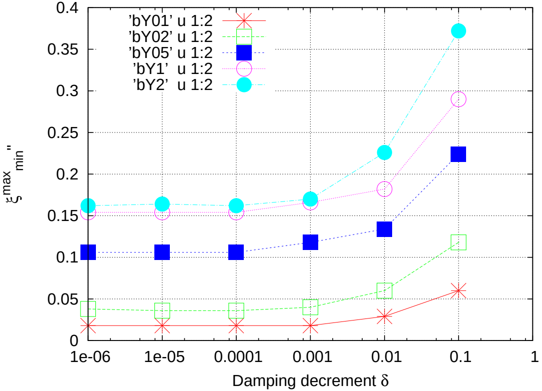


Figure: Plot of saturation tune shift, $\xi^{\mathrm{sat.}}$ versus damping decrement δ, for $\beta_y = 1,2,5,10$, and 20 mm. In all cases $\sigma_z = 0.01$ m, $Q_s = 0.03$.

- **Note:** As well as depending on damping decrement δ, the saturation tune shift depends strongly on other parameters, especially vertical beta function β_y and bunch length σ_z.

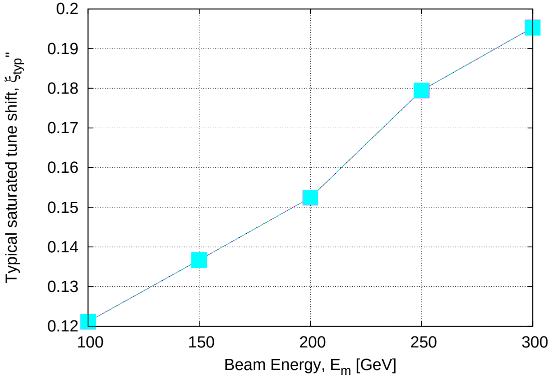


Figure: Plot of "typical" saturated tune shift $\xi_{\rm typ}$ as a function of maximum beam energy E_m for ring radius R scaling as $E_m^{1.25}$. $\beta_y = \sigma_z = 5\,\mathrm{mm}$.

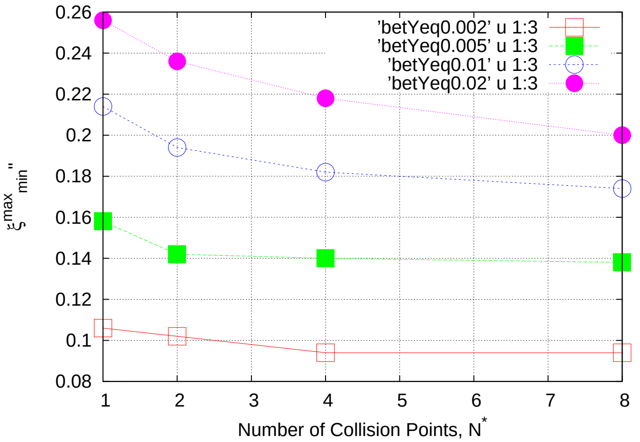


Figure: Plot of saturation tune shift value $\xi^{\text{sat.}}$ versus number of collision points N^*, for $\beta_y = 2{,}5{,}10$, and 20 mm. $Q_s = 0.03/N^*$.

Beam Height Equilibrium: Beam-Beam Heating vs. Betatron Cooling

- Under ideal single beam conditions beam height $\sigma_y \approx 0$.
- This would give infinite luminosity which is unphysical. Nature "abhors" both zero and infinity.
- In fact beam-beam forces cause the beam height to grow into a new equilibrium with normal radiation damping.
- The parametric modulation provides a force with resonance driving strength proportional to $1/\sigma_y$, which is guaranteed to countermand the miniscule single beam height.
- Amplitude dependent detuning limits the growth, so there is no particle loss.
- The simulation automatically accounts for whatever resonances are nearby.

- For Higgs factory design, scan the tune plane, for various vertical beta function values (as well as other, less influential, parameters.)
- Read the ratio $\xi^{\mathrm{sat.}}/\beta_y$ from the figure.

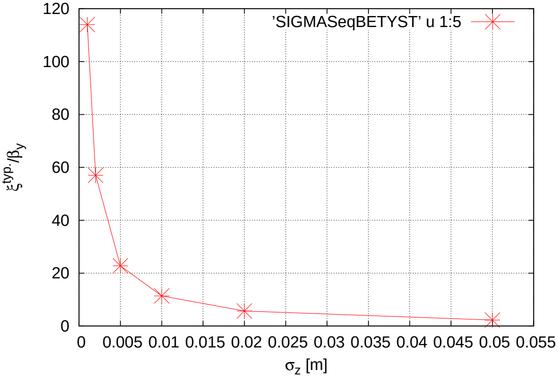


Figure: Plot of $\xi^{\mathrm{typ.}}/\beta_y$ as a function of σ_z, with $\beta_y = \sigma_z$, $\delta = 0.00764$, and synchrotron tune advance between collisions $Q_s = 0.0075$.

- The ratio $\xi^{\rm typ.}/\beta_y$ determines the beam area just sufficient for saturation A_{β_y} according to the formula,

$$A_{\beta_y} = \pi\sigma_x\sigma_y = \frac{N_p r_e}{2\gamma}\frac{1}{(\xi^{\rm sat.}/\beta_y)}. \tag{2}$$

- It is only the product $\sigma_x\sigma_y$ that is fixed but the aspect ratio $a_{xy} = \sigma_x/\sigma_y \approx 15$ is good enough. To within this ambiguity all transverse betatron parameters are then fixed.
- The number of electrons per bunch N_p itself is fixed by the available RF power and the number of bunches N_b. For increasing the luminosity N_b wants to be **reduced**.
- To keep beamstrahlung acceptably small N_b has to be **increased**.
- The maximum achievable luminosity is determined by this compromise.

The Parameter Space for Beam Energy E

$\underline{R}$: bend radius

$\mathcal{C}$: circumference $= 3\pi R$ is good enough

$\underline{N^*}$: number of I.P.′s

$\underline{N_p}$: particles per bunch, $N_{\rm tot.} = N_b N_p$, fixes RF power, $P_{\rm rf}$

β_x : horizontal beta function in arc, fixed by arc design

ϵ_x : horizontal emittance, fixed by arc design

δ : betatron damping decrement, known from R and E

$\underline{\beta_y^*}$: vertical beta function at I.P.

$\underline{\sigma_y^*}$: r.m.s. bunch height at I.P. is to be calculated

ϵ_y : vertical emittance $= {\sigma_y^*}^2/\beta_y^*$ is then known

σ_x^* : r.m.s. bunch width at I.P. $\equiv a_{xy}\sigma_y^* = 15\sigma_y^*$ is good enough

β_x^* : horz. beta function at I.P. $= {\sigma_x^*}^2/\epsilon_x$

σ_z : r.m.s. bunch length $\equiv \beta_y^*/r_{yz} = \beta_y^*/0.6$ is good enough

Q_x, Q_y : transverse tunes (unimportant in simulation)

$\underline{Q_s}$: synchrotron tune (important in simulation)

Reconciling Luminosity and Beamstrahlung

- $\mathcal{L}^{\mathrm{RF}}_{\mathrm{pow}}$ is the RF power limited luminosity
- $\mathcal{L}^{\mathrm{bb}}_{\mathrm{sat}}$ is the beam-beam saturated luminosity
- $\mathcal{L}^{\mathrm{bs}}_{\mathrm{trans}}$ is the beamstrahlung transverse-limited luminosity
- $\mathcal{L}^{\mathrm{bs}}_{\mathrm{longit}}$ is the beamstrahlung longitudinal-limited luminosity

$$\mathcal{L}^{\mathrm{RF}}_{\mathrm{pow}} = \frac{N^*}{N_b} H(r_{yz}) \frac{1}{a_{xy}} \frac{f}{4\pi} \left(\frac{n_1 P_{\mathrm{rf}}[\mathrm{MW}]}{\sigma_y} \right)^2,$$
$$N_{\mathrm{tot}} = n_1 P_{\mathrm{rf}}[\mathrm{MW}]$$

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$$N_{\rm tot} = n_1 P_{\rm rf}[\rm MW]$$

- Single beam dynamics gives $\sigma_y = 0$, $\implies \mathcal{L}^{\rm RF}_{\rm pow} = \infty$? Nonsense. Resonance drive force $\propto 1/\sigma_y$, also.
- Nature "abhors" both zero and infinity. Beam-beam force expands $\sigma_y = 0$ as necessary. **Saturation is automatic**.

$$\mathcal{L}^{\mathrm{RF}}_{\mathrm{pow}} = \frac{N^*}{N_b}\, H(r_{yz})\, \frac{1}{a_{xy}}\, \frac{f}{4\pi} \left(\frac{n_1 P_{\mathrm{rf}}[\mathrm{MW}]}{\sigma_y} \right)^2,$$

$$\mathcal{L}^{\mathrm{bb}}_{\mathrm{sat}} = N^* N_{\mathrm{tot.}} H(r_{yz})\, f\, \frac{\gamma}{2 r_e}\, (\xi^{\mathrm{sat.}}/\beta_y),$$

$$\mathcal{L}^{\mathrm{bs}}_{\mathrm{trans}} = N^* N_b H(r_{yz})\, a_{xy} \sigma_z^2\, f \left(\frac{\sqrt{\pi}\ 1.96 \times 10^5}{28.0\,\mathrm{m}\, \sqrt{2/\pi}} \right)^2 \frac{1}{r_e^2 \widetilde{E}^2} \left(\frac{91\eta}{\ln\left(\frac{1/\tau_{\mathrm{bs}}}{f\, n^*_{\gamma,1}\, \mathcal{R}^{\mathrm{Gauss}}_{\mathrm{unif.}}} \right)} \right.$$

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- If $\mathcal{L}^{\mathrm{bs}}_{\mathrm{trans}} < \mathcal{L}^{\mathrm{bb}}_{\mathrm{sat}}$ we must increase N_b ! $\mathcal{L}^{\mathrm{bs}}_{\mathrm{trans}} \propto N_b$, $\mathcal{L}^{\mathrm{RF}}_{\mathrm{pow}} \propto 1/N_b$,

$$\mathcal{L}^{\mathrm{RF}}_{\mathrm{pow}} = \frac{N^*}{N_b} H(r_{yz}) \frac{1}{a_{xy}} \frac{f}{4\pi} \left(\frac{n_1 P_{\mathrm{rf}}[\mathrm{MW}]}{\sigma_y}\right)^2,$$

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$$N_b = \frac{\mathcal{L}^{\mathrm{bb}}_{\mathrm{sat}}}{\mathcal{L}^{\mathrm{bs}}_{\mathrm{trans}}} \text{ is good enough.}$$

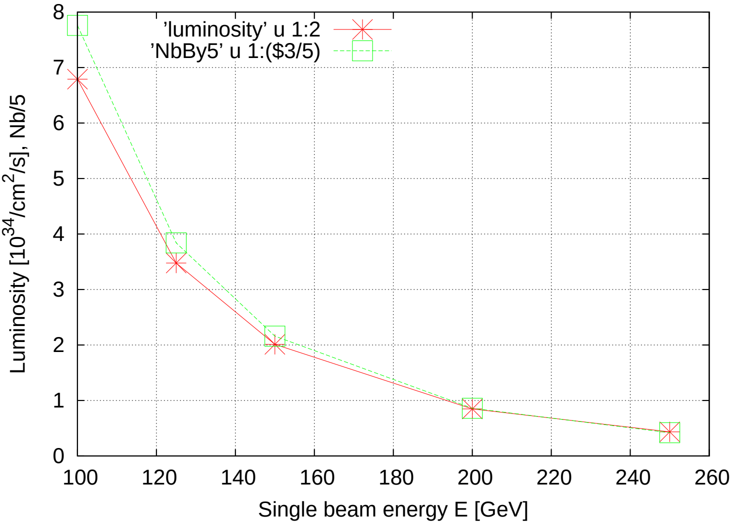

Figure: Dependence of luminosity on single beam energy. (scaled) number of bunches $N_B/5$ is also shown.

Phase II, $E = 250\,\text{GeV}$, $P_{\text{RF}} = 50\,\text{MW}$

E GeV	$\mathcal{C}$ km	R km	f KHz	U_1 GeV	eV_{excess} GeV	n_1 elec./MW	$U_1/(D/2)$ MV/m	$\delta = \alpha_4$	u_c GeV	ϵ_x nm	σ_x^{arc} mm
100	28	3.0	10.60	3.0	62	2.00e+11	0.626	0.0074	0.00074	6.354	0.523
150	28	3.0	10.60	14.9	50	3.94e+10	3.169	0.0249	0.00249	14.297	0.784
200	28	3.0	10.60	47.2	18	1.25e+10	10.016	0.0590	0.00591	25.417	1.05
250	28	3.0	10.60	115.2	-50	5.11e+09	24.453	0.1152	0.01155	39.715	1.31
300	28	3.0	10.60	239.0	-1.7e+02	2.46e+09	50.707	0.1991	0.01995	57.189	1.57
100	57	6.0	5.30	1.5	64	7.98e+11	0.157	0.0037	0.00037	3.177	0.37
150	57	6.0	5.30	7.5	58	1.58e+11	0.792	0.0124	0.00125	7.149	0.554
200	57	6.0	5.30	23.6	41	4.99e+10	2.504	0.0295	0.00296	12.709	0.739
250	57	6.0	5.30	57.6	7.4	2.04e+10	6.113	0.0576	0.00577	19.857	0.924
300	57	6.0	5.30	119.5	-54	9.85e+09	12.677	0.0996	0.00998	28.595	1.11
100	75	8.0	3.98	1.1	64	1.42e+12	0.088	0.0028	0.00028	2.383	0.32
150	75	8.0	3.98	5.6	59	2.80e+11	0.446	0.0093	0.00094	5.361	0.48
200	75	8.0	3.98	17.7	47	8.87e+10	1.409	0.0221	0.00222	9.532	0.64
250	75	8.0	3.98	43.2	22	3.63e+10	3.439	0.0432	0.00433	14.893	0.8
300	75	8.0	3.98	89.6	-25	1.75e+10	7.131	0.0747	0.00748	21.446	0.96
100	94	10.0	3.18	0.9	64	2.22e+12	0.056	0.0022	0.00022	1.906	0.286
150	94	10.0	3.18	4.5	61	4.38e+11	0.285	0.0075	0.00075	4.289	0.429
200	94	10.0	3.18	14.2	51	1.39e+11	0.901	0.0177	0.00177	7.625	0.573
250	94	10.0	3.18	34.6	30	5.68e+10	2.201	0.0346	0.00346	11.914	0.716
300	94	10.0	3.18	71.7	-6.7	2.74e+10	4.564	0.0597	0.00599	17.157	0.859
100	113	12.0	2.65	0.7	64	3.19e+12	0.039	0.0018	0.00018	1.589	0.261
150	113	12.0	2.65	3.7	61	6.31e+11	0.198	0.0062	0.00062	3.574	0.392
200	113	12.0	2.65	11.8	53	2.00e+11	0.626	0.0148	0.00148	6.354	0.523
250	113	12.0	2.65	28.8	36	8.17e+10	1.528	0.0288	0.00289	9.929	0.653
300	113	12.0	2.65	59.7	5.3	3.94e+10	3.169	0.0498	0.00499	14.297	0.784

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300	57	6.0	5.30	119.5	-54	9.85e+09	12.677	0.0996	0.00998	28.595	1.11
100	75	8.0	3.98	1.1	64	1.42e+12	0.088	0.0028	0.00028	2.383	0.32
150	75	8.0	3.98	5.6	59	2.80e+11	0.446	0.0093	0.00094	5.361	0.48
200	75	8.0	3.98	17.7	47	8.87e+10	1.409	0.0221	0.00222	9.532	0.64
250	75	8.0	3.98	43.2	22	3.63e+10	3.439	0.0432	0.00433	14.893	0.8
300	75	8.0	3.98	89.6	-25	1.75e+10	7.131	0.0747	0.00748	21.446	0.96
100	94	10.0	3.18	0.9	64	2.22e+12	0.056	0.0022	0.00022	1.906	0.286
150	94	10.0	3.18	4.5	61	4.38e+11	0.285	0.0075	0.00075	4.289	0.429
200	94	10.0	3.18	14.2	51	1.39e+11	0.901	0.0177	0.00177	7.625	0.573
250	94	10.0	3.18	34.6	30	5.68e+10	2.201	0.0346	0.00346	11.914	0.716
300	94	10.0	3.18	71.7	-6.7	2.74e+10	4.564	0.0597	0.00599	17.157	0.859
100	113	12.0	2.65	0.7	64	3.19e+12	0.039	0.0018	0.00018	1.589	0.261
150	113	12.0	2.65	3.7	61	6.31e+11	0.198	0.0062	0.00062	3.574	0.392
200	113	12.0	2.65	11.8	53	2.00e+11	0.626	0.0148	0.00148	6.354	0.523
250	113	12.0	2.65	28.8	36	8.17e+10	1.528	0.0288	0.00289	9.929	0.653
300	113	12.0	2.65	59.7	5.3	3.94e+10	3.169	0.0498	0.00499	14.297	0.784

E GeV	$\mathcal{C}$ km	R km	f KHz	U_1 GeV	eV_{excess} GeV
250	94	10.0	3.18	34.6	30

Phase II, $E = 250\,\text{GeV}$, $P_{\text{RF}} = 50\,\text{MW}$

E GeV	$\mathcal{C}$ km	R km	f KHz	U_1 GeV	eV_{excess} GeV	n_1 elec./MW	$U_1/(D/2)$ MV/m	$\delta = \alpha_4$	u_c GeV	ϵ_x nm	σ_x^{arc} mm
100	28	3.0	10.60	3.0	62	2.00e+11	0.626	0.0074	0.00074	6.354	0.523
150	28	3.0	10.60	14.9	50	3.94e+10	3.169	0.0249	0.00249	14.297	0.784
200	28	3.0	10.60	47.2	18	1.25e+10	10.016	0.0590	0.00591	25.417	1.05
250	28	3.0	10.60	115.2	-50	5.11e+09	24.453	0.1152	0.01155	39.715	1.31
300	28	3.0	10.60	239.0	-1.7e+02	2.46e+09	50.707	0.1991	0.01995	57.189	1.57
100	57	6.0	5.30	1.5	64	7.98e+11	0.157	0.0037	0.00037	3.177	0.37
150	57	6.0	5.30	7.5	58	1.58e+11	0.792	0.0124	0.00125	7.149	0.554
200	57	6.0	5.30	23.6	41	4.99e+10	2.504	0.0295	0.00296	12.709	0.739
250	57	6.0	5.30	57.6	7.4	2.04e+10	6.113	0.0576	0.00577	19.857	0.924
300	57	6.0	5.30	119.5	-54	9.85e+09	12.677	0.0996	0.00998	28.595	1.11
100	75	8.0	3.98	1.1	64	1.42e+12	0.088	0.0028	0.00028	2.383	0.32
150	75	8.0	3.98	5.6	59	2.80e+11	0.446	0.0093	0.00094	5.361	0.48
200	75	8.0	3.98	17.7	47	8.87e+10	1.409	0.0221	0.00222	9.532	0.64
250	75	8.0	3.98	43.2	22	3.63e+10	3.439	0.0432	0.00433	14.893	0.8
300	75	8.0	3.98	89.6	-25	1.75e+10	7.131	0.0747	0.00748	21.446	0.96
100	94	10.0	3.18	0.9	64	2.22e+12	0.056	0.0022	0.00022	1.906	0.286
150	94	10.0	3.18	4.5	61	4.38e+11	0.285	0.0075	0.00075	4.289	0.429
200	94	10.0	3.18	14.2	51	1.39e+11	0.901	0.0177	0.00177	7.625	0.573
250	94	10.0	3.18	34.6	30	5.68e+10	2.201	0.0346	0.00346	11.914	0.716
300	94	10.0	3.18	71.7	-6.7	2.74e+10	4.564	0.0597	0.00599	17.157	0.859
100	113	12.0	2.65	0.7	64	3.19e+12	0.039	0.0018	0.00018	1.589	0.261
150	113	12.0	2.65	3.7	61	6.31e+11	0.198	0.0062	0.00062	3.574	0.392
200	113	12.0	2.65	11.8	53	2.00e+11	0.626	0.0148	0.00148	6.354	0.523
250	113	12.0	2.65	28.8	36	8.17e+10	1.528	0.0288	0.00289	9.929	0.653
300	113	12.0	2.65	59.7	5.3	3.94e+10	3.169	0.0498	0.00499	14.297	0.784

E GeV	$\mathcal{C}$ km	R km	f KHz	U_1 GeV	eV_{excess} GeV
250	94	10.0	3.18	34.6	30

n_1 elec./MW	$U_1/(D/2)$ MV/m	$\delta = \alpha_4$	u_c GeV	ϵ_x nm	σ_x^{arc} mm
5.68e+10	2.201	0.0346	0.00346	11.914	0.716

E GeV	R km	β_y^* m	ϵ_y m	$\xi_{\rm sat}$	$N_{\rm tot}$	σ_y μm	σ_x μm	u_c^* GeV	$n_{\gamma,1}^*$	$\mathcal{L}^{\rm RF}$ 10^{34}	$\mathcal{L}^{\rm bs}_{\rm trans}$ 10^{34}	$\mathcal{L}^{\rm bs}_{\rm longit}$ 10^{34}	$\mathcal{L}^{\rm bb}$ 10^{34}	N_b	β_x^* m
100	3.0	0.006	6.88e-09	0.107	1.0e+13	6.43	96.40	0.014	57.51	2.037	1.21	301	2.037	2.0	1.5
150	3.0	0.006	9.06e-10	0.107	2.0e+12	2.33	34.98	0.018	31.31	0.604	1.29	38.7	0.604	2.0	0.086
200	3.0	0.006	2.15e-10	0.107	6.2e+11	1.14	17.04	0.020	20.33	0.255	1.36	1.55	0.255	2.0	0.011
250	3.0	0.006	7.05e-11	0.107	2.6e+11	0.65	9.75	0.023	14.55	0.000	0	0	0.000	2.0	0.0024
300	3.0	0.006	2.83e-11	0.107	1.2e+11	0.412	6.18	0.025	11.07	0.000	0	0	0.000	2.0	0.00067
100	6.0	0.006	1.47e-08	0.107	4.0e+13	9.4	141.00	0.021	84.12	4.074	1.17	295	4.074	3.7	6.3
150	6.0	0.006	3.63e-09	0.107	7.9e+12	4.66	69.96	0.035	62.61	1.207	0.647	25.6	1.207	2.0	0.68
200	6.0	0.006	8.60e-10	0.107	2.5e+12	2.27	34.08	0.041	40.67	0.509	0.679	4.19	0.509	2.0	0.091
250	6.0	0.006	2.82e-10	0.107	1.0e+12	1.3	19.51	0.045	29.10	0.261	0.706	0.0546	0.261	2.0	0.019
300	6.0	0.006	1.13e-10	0.107	4.9e+11	0.825	12.37	0.050	22.14	0.000	0	0	0.000	2.0	0.0053
100	8.0	0.006	1.96e-08	0.107	7.1e+13	10.9	162.82	0.024	97.14	5.432	1.19	298	5.432	5.0	11
150	8.0	0.006	4.91e-09	0.107	1.4e+13	5.43	81.41	0.041	72.85	1.610	0.647	26.8	1.610	2.6	1.2
200	8.0	0.006	1.53e-09	0.107	4.4e+12	3.03	45.44	0.054	54.22	0.679	0.509	4.1	0.679	2.0	0.22
250	8.0	0.006	5.01e-10	0.107	1.8e+12	1.73	26.01	0.061	38.80	0.348	0.529	0.356	0.348	2.0	0.045
300	8.0	0.006	2.01e-10	0.107	8.8e+11	1.1	16.49	0.066	29.51	0.000	0	0	0.000	2.0	0.013
100	10.0	0.006	2.45e-08	0.107	1.1e+14	12.1	182.04	0.027	108.60	6.790	1.2	301	6.790	6.2	17
150	10.0	0.006	6.14e-09	0.107	2.2e+13	6.07	91.02	0.046	81.45	2.012	0.655	27.9	2.012	3.3	1.9
200	10.0	0.006	2.30e-09	0.107	6.9e+12	3.71	55.68	0.066	66.43	0.849	0.425	3.95	0.849	2.1	0.41
250	10.0	0.006	7.83e-10	0.107	2.8e+12	2.17	32.52	0.076	48.50	0.435	0.423	0.556	0.435	2.0	0.089
300	10.0	0.006	3.15e-10	0.107	1.4e+12	1.37	20.61	0.083	36.89	0.000	0	0	0.000	2.0	0.025
100	12.0	0.006	2.95e-08	0.107	1.6e+14	13.3	199.41	0.030	118.97	8.148	1.22	302	8.148	7.5	25
150	12.0	0.006	7.36e-09	0.107	3.2e+13	6.65	99.70	0.050	89.22	2.414	0.662	28.6	2.414	3.9	2.8
200	12.0	0.006	2.76e-09	0.107	1.0e+13	4.07	60.99	0.073	72.77	1.019	0.429	4.32	1.019	2.5	0.59
250	12.0	0.006	1.13e-09	0.107	4.1e+12	2.6	39.02	0.091	58.20	0.521	0.353	0.656	0.521	2.0	0.15
300	12.0	0.006	4.53e-10	0.107	2.0e+12	1.65	24.74	0.099	44.27	0.302	0.364	0.00669	0.302	2.0	0.043

Nst= 4
BETYST= 0.006 m
XITYPbyBY= 17.800
taubs=600.000 s
RGauUnif= 0.300
Prf= 50.000 MW
eVrf= 65.000 GeV
OVreq= 20.000 GV
axy= 15.000
ryz= 0.600 m
bxarcmax= 43.000 m

E GeV	R km	β_y^* m	ϵ_y m	ξ_{sat}	N_{tot}
250	10.0	0.006	7.83e-10	0.107	2.8e+12

E GeV	R km	β_y^* m	ϵ_y m	$\xi_{\rm sat}$	$N_{\rm tot}$
250	10.0	0.006	7.83e-10	0.107	2.8e+12

σ_y μm	σ_x μm	u_c^* GeV	$n_{\gamma,1}^*$
2.17	32.52	0.076	48.50

E GeV	R km	β_y^* m	ϵ_y m	$\xi_{\rm sat}$	$N_{\rm tot}$
250	10.0	0.006	7.83e-10	0.107	2.8e+12

σ_y μm	σ_x μm	u_c^* GeV	$n_{\gamma,1}^*$
2.17	32.52	0.076	48.50

$\mathcal{L}^{\rm RF}$ 10^{34}	$\mathcal{L}^{\rm bs}_{\rm trans}$ 10^{34}	$\mathcal{L}^{\rm bs}_{\rm longit}$ 10^{34}	$\mathcal{L}^{\rm bb}$ 10^{34}	N_b	β_x^* m
0.435	0.423	0.556	0.435	2.0	0.089

Optional Stuff

Estimated Cost in $M

System	Phase I detail	Phase I $\sqrt{s} = 0.3$ TeV	Phase II $\sqrt{s} = 0.5$ TeV
construction-below ground	1208		
construction-above ground	177		
construction, total		1065	
main ring magnet	282		
special magnets	64		
installation	131		
vacuum	87		
interaction regions	16		
other accelerator systems	153		
collider, total		2118	
injector chain		1100	
RF, Phase I, 4× LEP2 RF=12 GeV		280	
RF, Phase II,20× LEP2 RF=60 GeV			1400
Detector, Phase I,		300	
Detector, Phase II,			750
totals		4863	2150

- CNA Consulting Engineers, Hatch-Mott-MacDonald, *Estimate of Heavy Civil Underground Construction Costs for a Very Large Hadron Collider in Northern Illinois,* `http://vlhc.org/cna_report.pdf`, 2001
- H.D. Glass, G.W. Foster et al., *Design Study for a Staged Very Large Hadron Collider,* Fermilab-TM-2149, 2001
- CERN, AT-95-37, 1995, RF cost, 19.5/GeV, in million 2013 U.S. dollars
- `http://media.linearcollider.org/ /estimateilcmachine.pdf`, 13.3/GeV, in million 2013 U.S. dollars

A difference equation calculating the vertical displacement on turn $t+1$ (time in units of period between collisions) from the two preceding values at t and $t-1$:

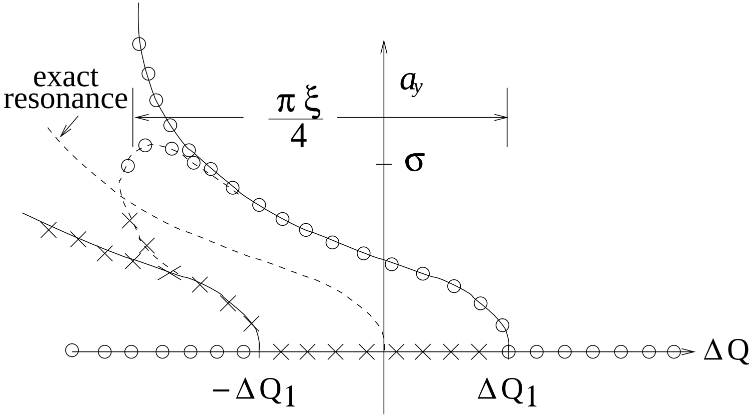
$$y_{t+1} = \frac{1}{1+\delta}\Bigg(2\cos\mu_0 y_t - y_{t-1}(1-\delta) \text{ unperturbed betatron motion}$$

$$-4\pi\xi\sin\mu_0\exp\Big(-a_x^2\cos^2\frac{\mu_x(a_x)(t+t_x)}{2}\Big) \text{ horizontal } \xi\text{-modulation}$$

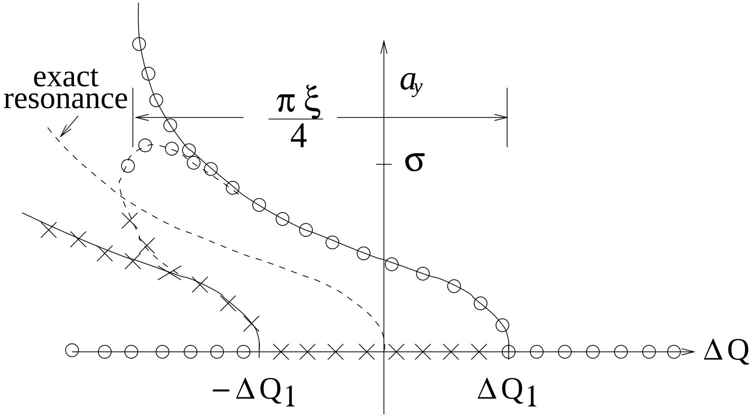
$$\times\sqrt{1+\left(\frac{\sigma_z}{\beta_y^*}\right)^2 a_s^2\cos^2\big(\mu_s(t+t_s)t\big)} \text{ longitudinal } \beta\text{-modulation}$$

$$\times\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\,\frac{y_t}{\sqrt{2}}\Bigg) \text{ vertical force}$$

This is a Mathieu (difference) equation, easily solved analytically.



This is a Mathieu (difference) equation, easily solved analytically.



- There is *always* a nearby resonance or, in fact, more than one.
- Multiple degrees of freedom, continuous amplitude distributions, and tune aliasing dictate numerical treatment.

- I have now applied this code to the design of a Higgs factory. I have not changed the code at all.
- The simulation consists of nothing more than checking (repeatedly and ad nauseum, with gradually increasing amplitude, in an appropriate region of transverse phase space) whether the motion described by the difference equation is "stable" or "unstable", and noting the ξ-value at the transition.