Monte Carlo Simulation for Next Generation Source and Channel Coding on OSG Connect

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Outline

- Background and motivation

- Research project 1:
  - Spatially Coupled LDGM (SC-LDGM) codes for lossy source coding and Low-latency Windowed Encoding

- Research project 2:
  - Turbo-like codes for high speed optical communication

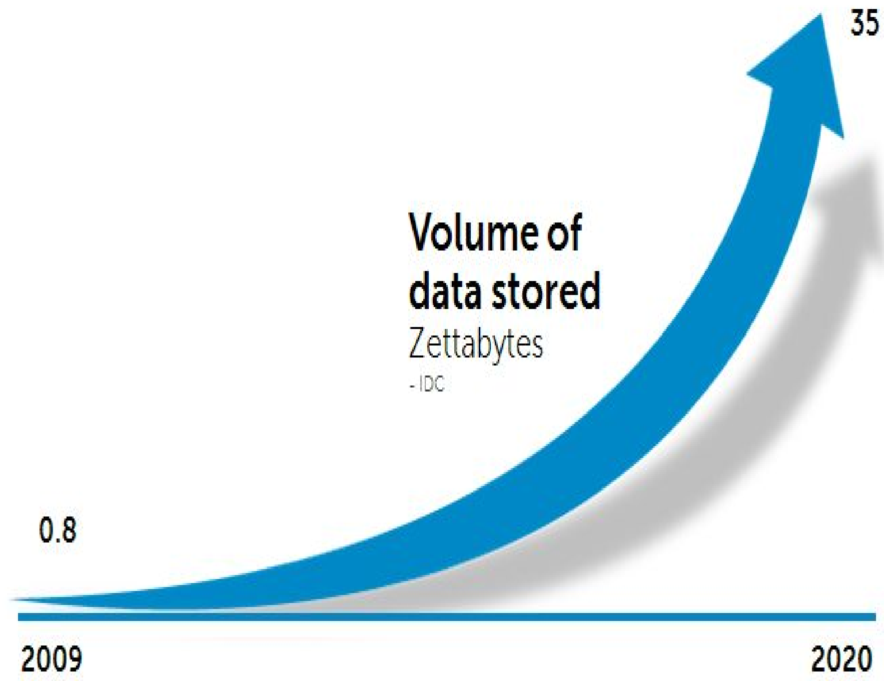
Data Deluge

- Global per capita information usage will grow from 15.5GB in 2014 to 37GB in 2017

- Mobile traffic will increase by factor 10

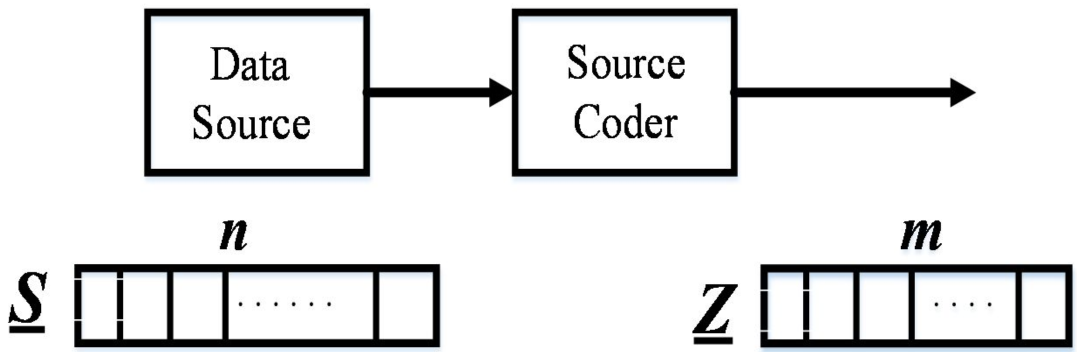
- It is though that the information amount consumed by wireless and mobile devices already exceeds that of wired devices, and will continue to grow.

[Cisco Systems Inc., “The Zettabyte era: trends and analysis”, white paper, May 2015]



Data Compression

- In order to handle the vast amounts of data society will produce, we need efficient and low-complexity algorithms to reliably compress and reconstruct data.



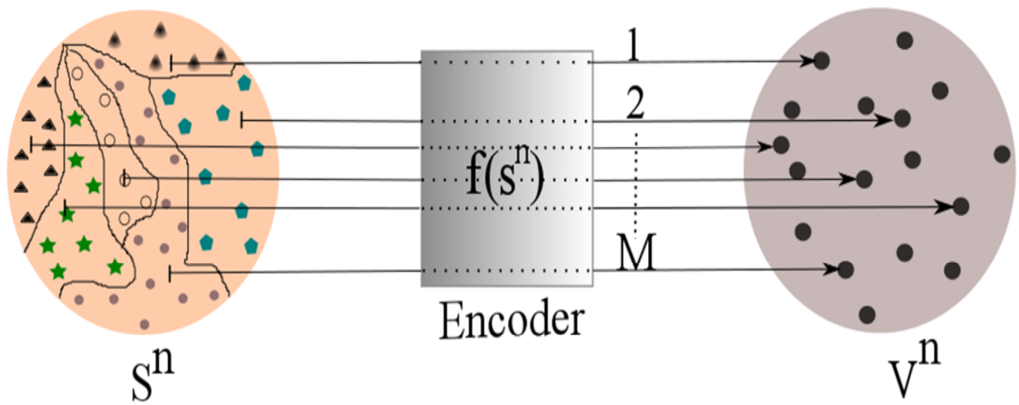
Compression rate

$$R = \frac{m}{n}$$

- We want to reconstruct the sequence with as little distortion as possible with a practical scheme (lossy source coding)

- The ultimate limit of compression is known (Shannon 1958)

Lossy Source Encoder



Source Symbols

Code Symbols

- Goal:

Mapping source space of length n to into a subset V^* of V of length n with cardinality $|V^*| = m$

- Problem: the mapping is many-to-one . How can we reconstruct?

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LDGM Codes for Lossy Source Coding

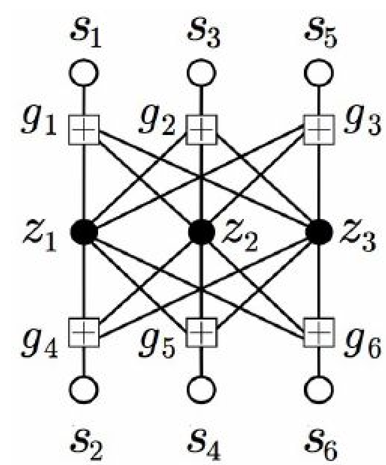
- We construct codes for lossy source coding using a small structured graph (protograph)

- A large graph can be obtained from a protograph by graph lifting with lifting factor M

  - The graph is copied M times and the edges randomly permuted following the graph structure

- Low complexity algorithms based on belief propagation can be defined for the sparse graph

  - Here, messages are passed iteratively forward and backward in the graph until we converge to a codeword $\underline{\boldsymbol{z}}$



(3, 6)- regular

LDGM Codes for Lossy Source Coding

- (J, K) – regular LDGM block codes (LDGM-BCs) achieve the rate-distortion (RD) limit under (high complexity) optimal encoding as the average node degrees increase [Wainwright et al. 2010]

- Unfortunately, directly applying a low-complexity BP algorithm does not work well since there are multiple optimal (or near optimal) solutions
  - An additional decimation step is applied to the most highly reliable nodes. At each iteration, we fix nodes and reduce the size of the graph

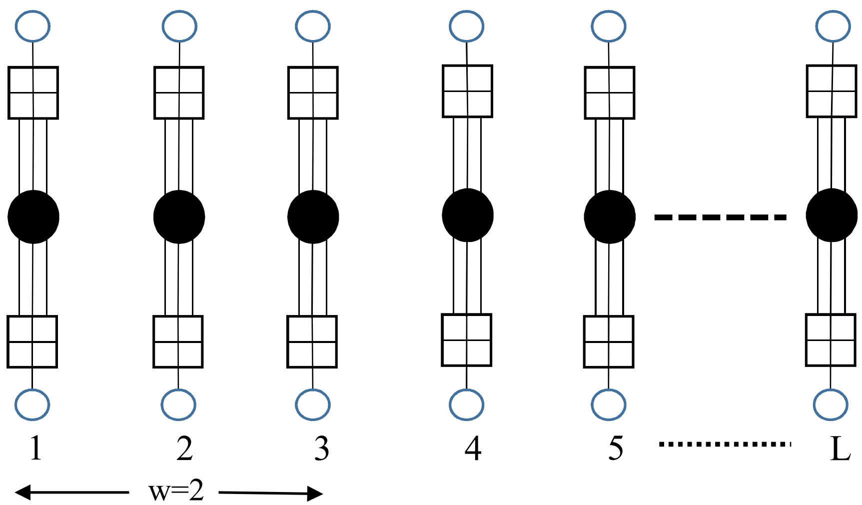
- (J, K) – regular LDGM codes have increasing BPGD distortion with increasing graph density (code length 200000)

(k, 2k)	(3, 6)	(4, 8)	(5, 10)
D_{opt}	0.1139	0.1111	0.1105
D_{BPGD}	0.1357	0.1590	0.1811

[Aref et al., 2012]

SC-LDGM Codes for Lossy Source Coding

(J, K)-regular SC-LDGM codes are constructed by coupling each member of a series of L (J, K)-regular LDGM-BC protographs to w of its neighbors



- Slight irregularity at ends of graph
- Large codes can be lifted from the protograph
- Chain length *L* gives different results
- We denote the (J, K)-regular SC-LDGM ensemble SC(J, K)

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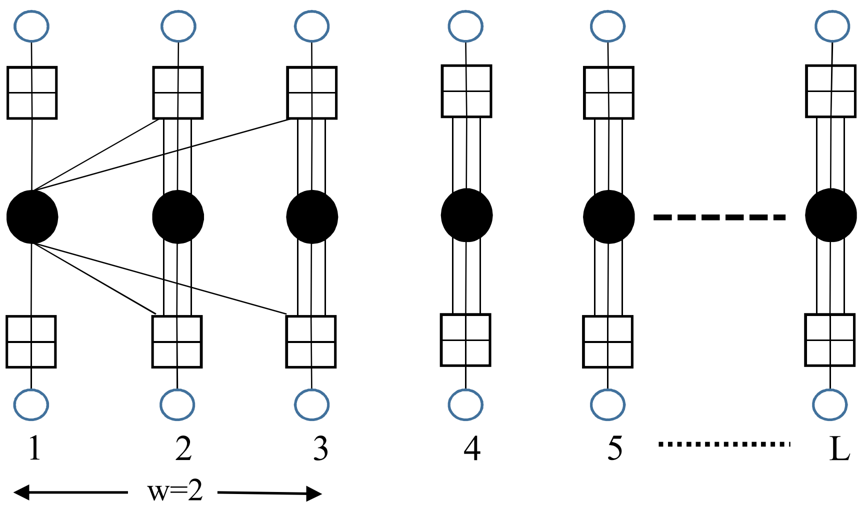

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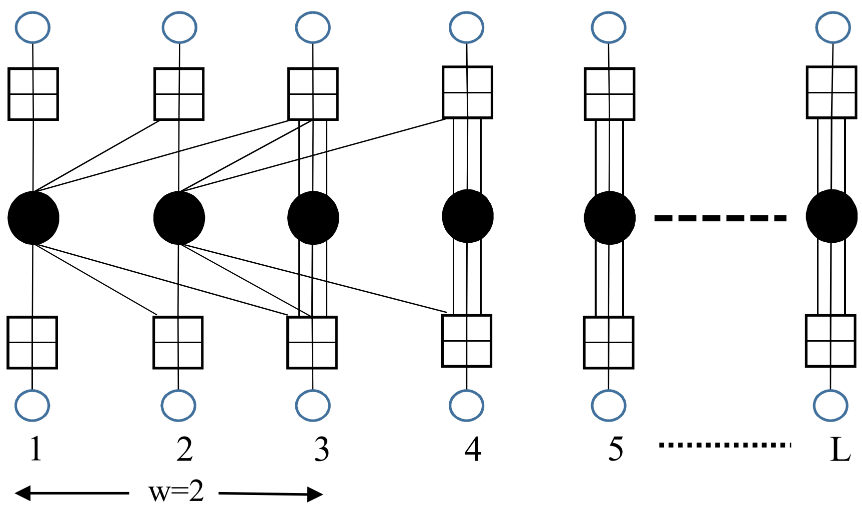

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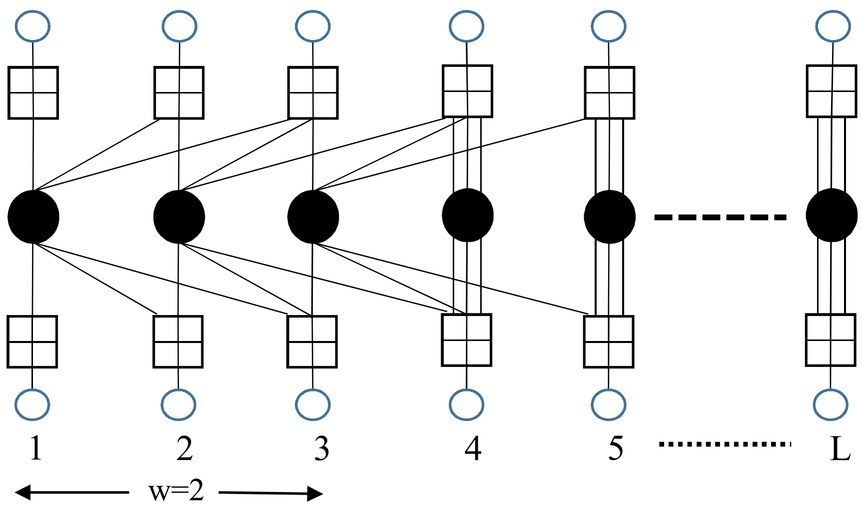

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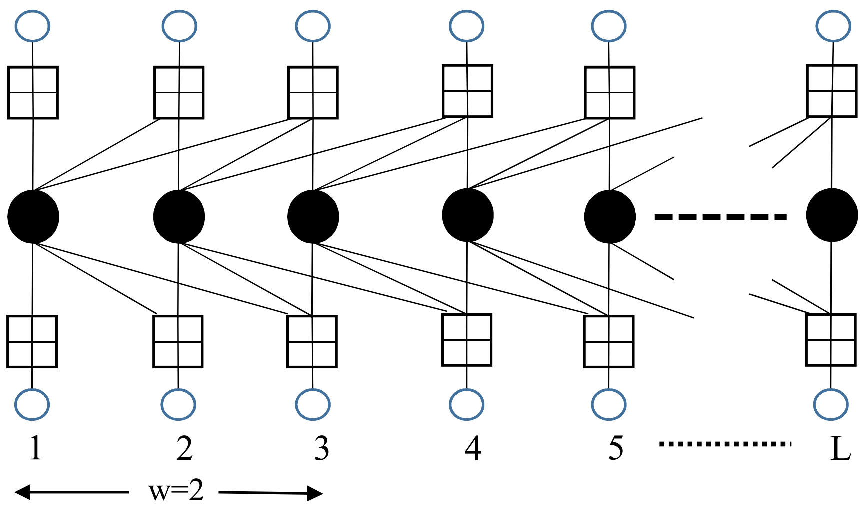

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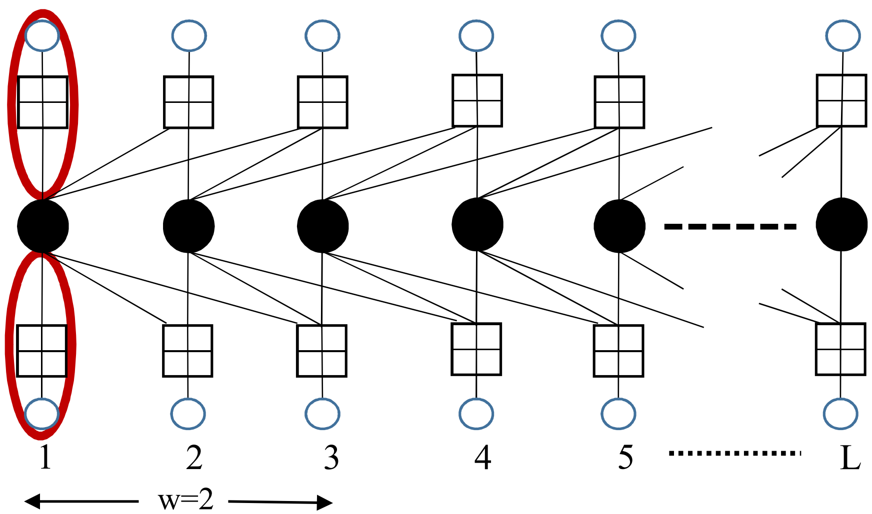

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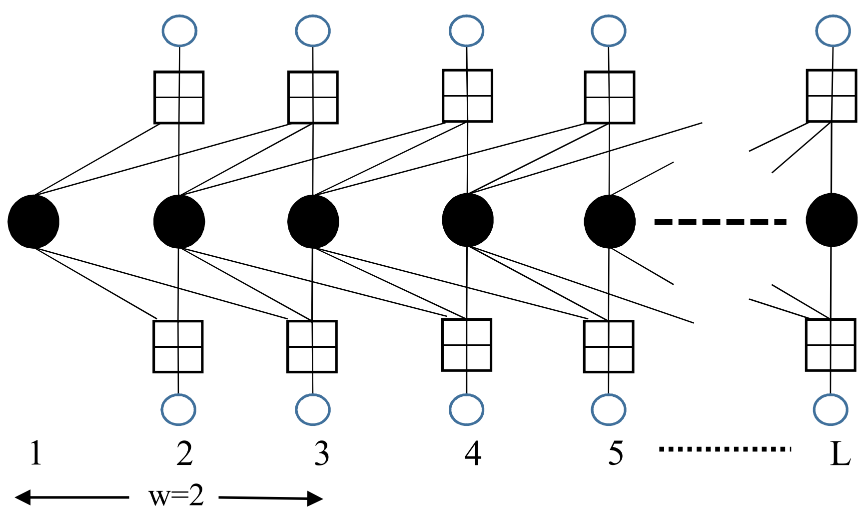

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Monte Carlo Simulation

- Monte Carlo experiments are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. Their essential idea is using randomness to solve problems that might be deterministic in principle

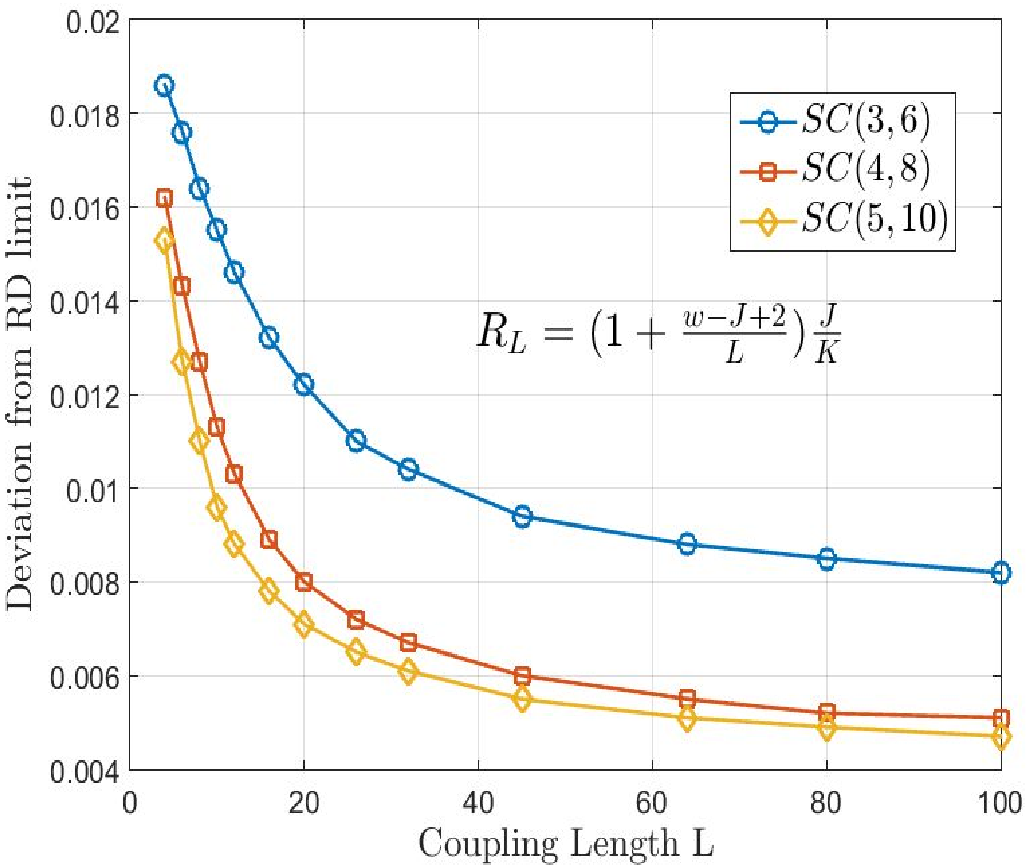
- To verify our approach we use Monte Carlo simulation as follows:
  1. Generate a random source sequence $P(X_i = 0) = P(X_i = 1) = 1/2$
  2. Run Algorithm
  3. Upon termination, measure distortion from original source
  4. Repeat and aggregate the results.

- We typically use 1000 trials per parameter set (~10) to obtain a statistically reliable result

- Each trial can be run independently with different random seeds for source generation, so the simulation is highly parallelizable

Block Encoding

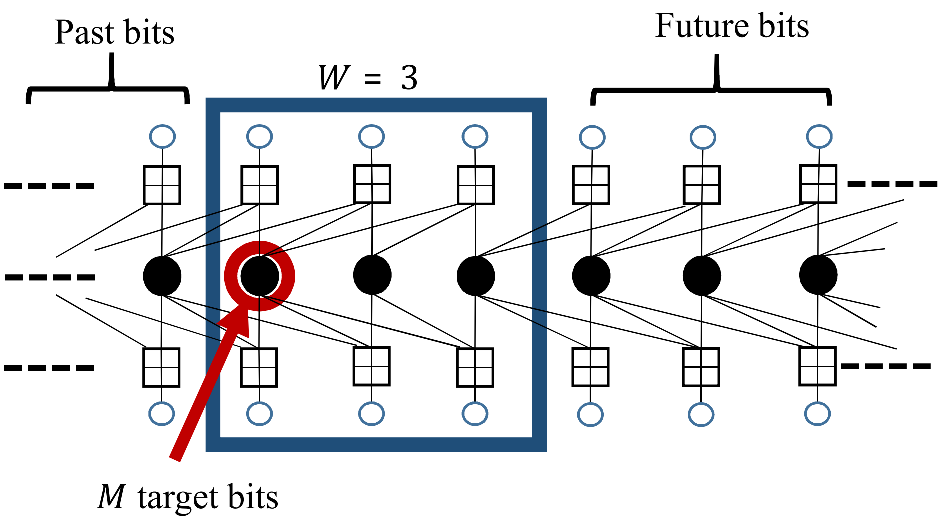
❑ Numerical results for fixed $M = 512$ and increasing L



- As L increases, the rate decreases and the gap to the RD limit decreases exponentially
- Even with low complexity BP encoding, the performance improves with graph density
- We achieve the (optimal) RD limit with low-complexity (suboptimal) message passing decoding
- To get each point, we ran 1000 trials, over different parameter sets
  - 6 hours of computing
  - 2000 parallel jobs

A. Golmohammadi, D. G. M. Mitchell, J. Kliewer, and D. J. Costello, Jr., "Windowed encoding of spatially coupled LDGM codes for lossy source compression." in Proc, IEEE Int. Symp. Inf. Theory, 2016.

Windowed Encoding of SC-LDGM codes



- The chain structure motivates a sliding window encoding for streaming
- After the M target bits are decimated, the window shifts one position and the M associated index symbols are encoded
- We will get lower latency, lower complexity, and reduce memory requirements but how does it perform?
- Similar simulation set-up to verify
- The latency of Windowed Encoding (WE) is $2MW$ compared to the "block encoding" latency $2ML$ ($W \ll L$)

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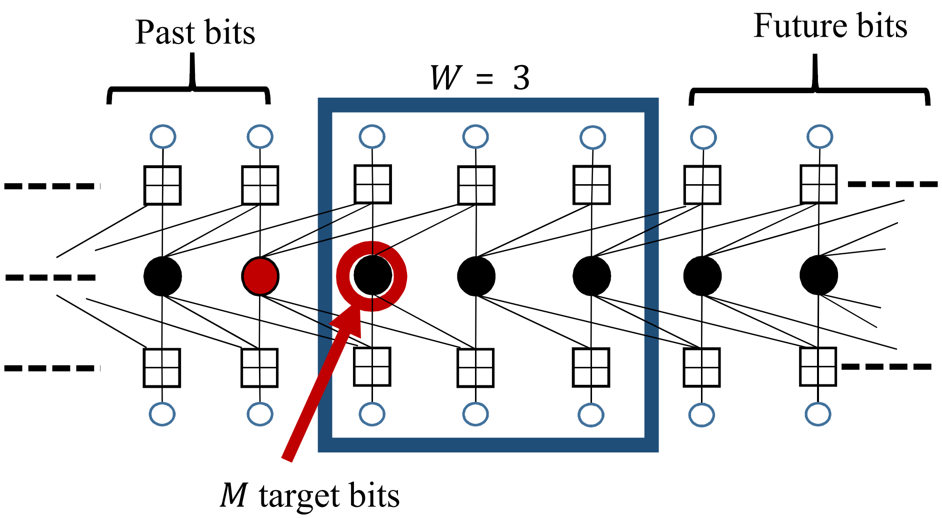

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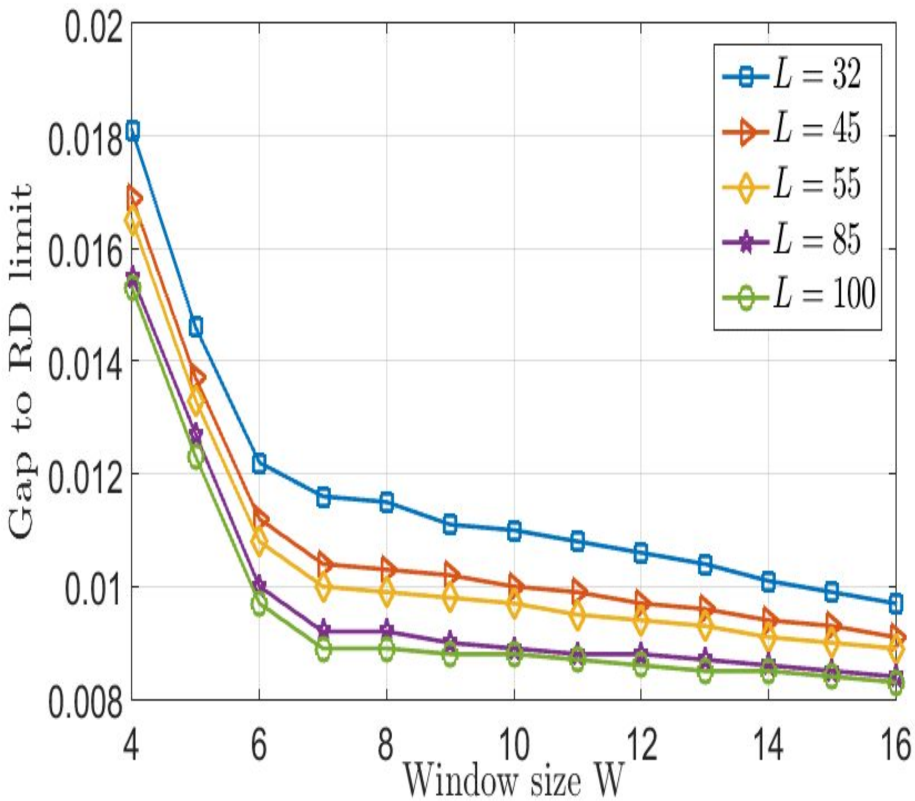
Modified Soft-Decimation



($2MW$ latency)

- Increasing W beyond a certain point results in small gains in distortion performance, but increasing latency
- SC(4, 8), $L = 100$, $M = 512$, $R = 0.5050$, $D_{Sh} = 0.1084$, $W = 10$
  - $D_{Win} = 0.1172$
  - $D_{Block} = 0.1135$
- To get each point, we run 1000 trials, over different parameter sets
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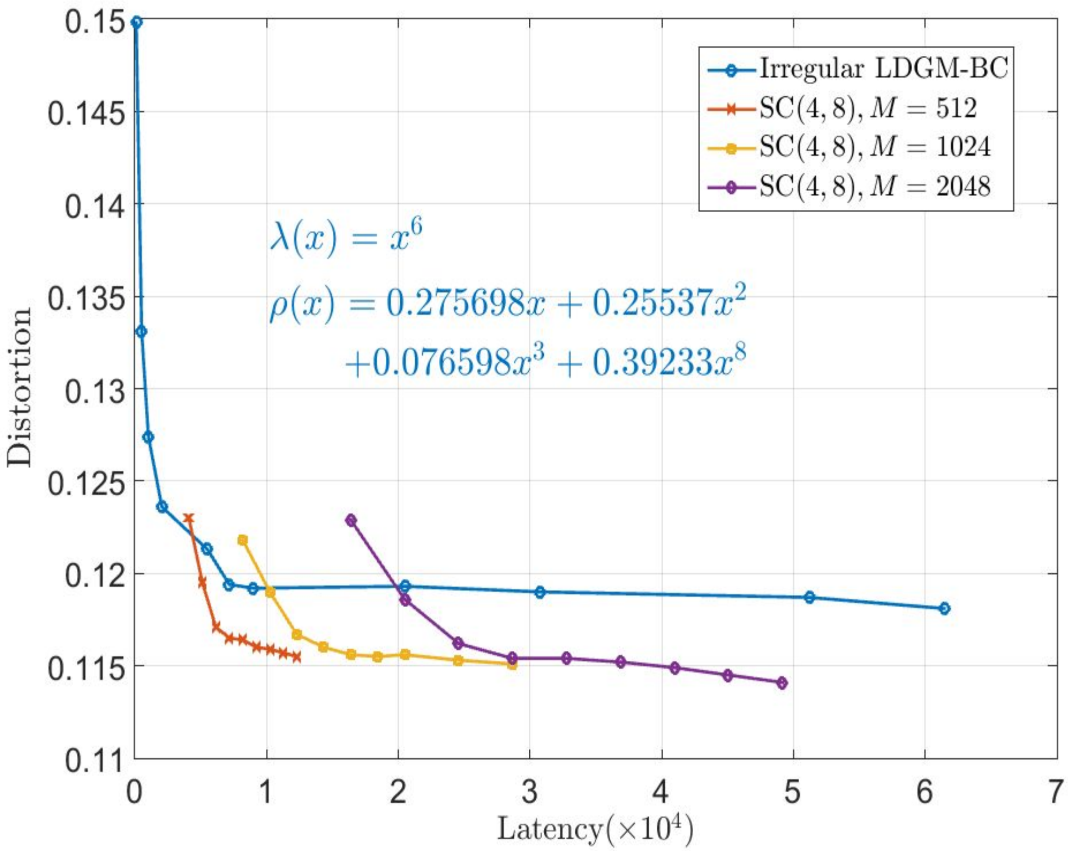
Threshold decimation

Distortion performance for an state-of-the-art irregular optimized LDGM-BC [Castanheira & Gamerio, 2012]



- SC(4, 8) codes ($L = 32$, $W = 4, \ldots, 12$) outperform the irregular codes except for very small window sizes
- The irregular LDGM block code was allowed 202 iterations
- The SC-LDGM codes were encoded with a threshold $\theta = 0.9999$ giving an average number of updates per node of 202

Block/Windowed Encoding Simulation on OSG Connect

- 30000 simulation for each type and code length

Type
SC (2, 4)
SC (3, 6)
SC (4, 8)
SC (5, 10)
SC (6, 12)

SC-LDGM matrix (M=512)	
2k-2L	55k-55L
4k-4L	64k-64L
6k-6L	80k-80L
8k-8L	100k-100L
10k-10L	120k-120L
12k-12L	160k-160L
16k-16L	180k-180L
20k-20L	200k-200L
26k-26L	300k-300L
32k-32L	
45k-45L	

Lifting factor (M)
128
192
256
320
640
832
1024
1760
2048

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Some Problems

- Missing files

1. Some nodes don't generate output files
2. output files are generated, but they are empty
   - Solution: Create scripts to parse and check data- re-run problem simulations

- Execution time

For the same simulation (and expected simulation time), some nodes generates results much faster than the others (one day late in some case)

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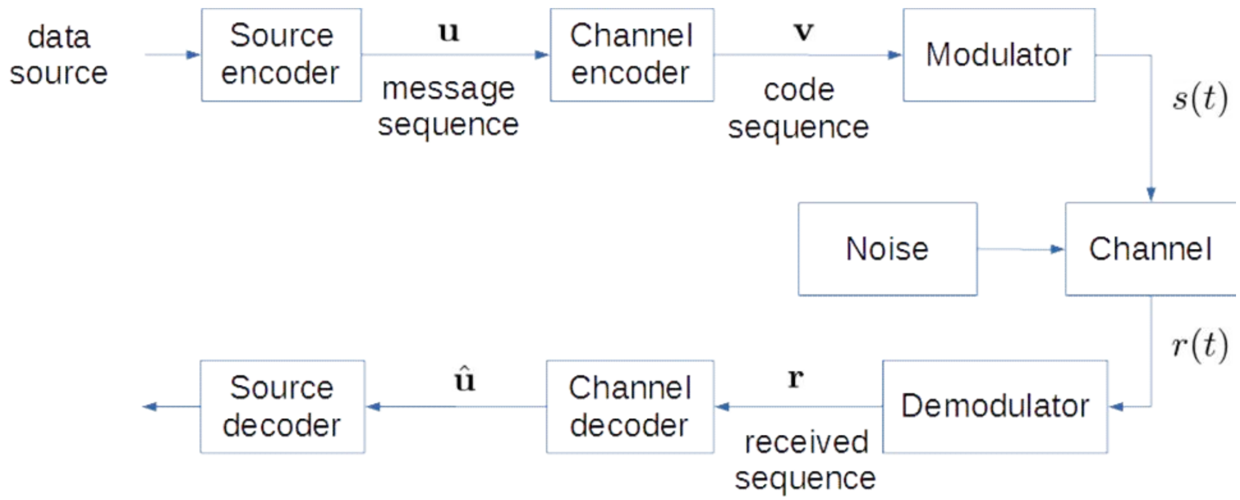
Channel Coding



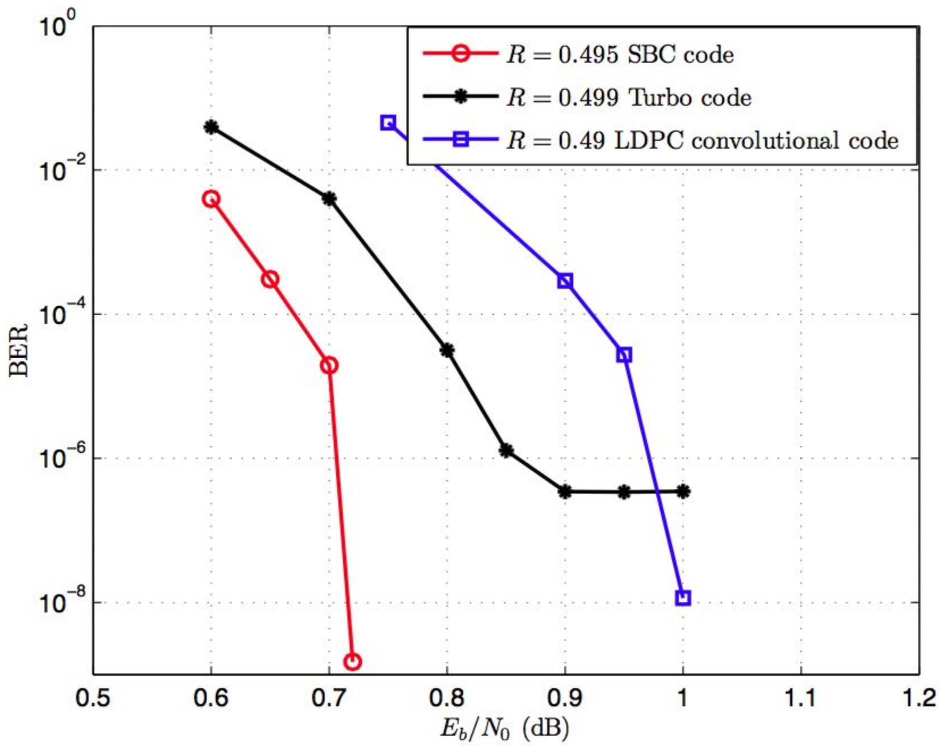
- In channel coding, redundancy is added to protect information for communication over a noisy channel

- Again, Monte Carlo methods can be used, where we generate random noise patterns and repeatedly decode transmitted codewords

- LDPC codes are also defined on sparse graphs, so are suitable for belief propagation decoding

Performance Comparison

- Performance comparison of our designed braided convolutional code (BCC), the CDMA 2000 standard turbo code (4G LTE), and a state-of-the-art (3,6)-regular SC-LDPC code



- All codes have latency 48K bits
- The BCC outperforms the turbo code and the SC-LDPC code
- The BCC simulation shows no indication of an error floor down to a BER of 10^{-8} using 2, 322, 768 hours of a high performance computing cluster.

M. Zhu, D. G. M. Mitchell, M. Lentmaier, D. J. Costello, Jr., and B. Bai, "Window Decoding of Braided Convolutional Codes," Proc. IEEE Information Theory Workshop, pp. 143-147, Jeju Island, Korea, Oct. 2015.*

D. J. Costello, Jr., M. Lentmaier, and D. G. M. Mitchell, "New Perspectives on Braided Convolutional Codes," keynote talk, Proc. International Symposium on Turbo Codes and Iterative Information Processing, pp. 400-405, Brest, France, Sept. 2016.

M. Zhu, D. G. M. Mitchell, M. Lentmaier, D. J. Costello, Jr., and B. Bai, "Braided Convolutional Codes with Low-Latency Sliding Window Decoding," submitted to the IEEE Transactions on Communications.*

Conclusion

- Sparse convolutional graph based source codes can approach the fundamental limit of data compression with a low complexity algorithm

- For the channel coding problem, similar codes can outperform the best known coding schemes under similar system parameters

- To numerically verify these results, we need huge computational hour usage of Monte Carlo methods. This would not be possible without OSG