Behaviour of the EAS Age parameter in the knee region

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imation B (taking into account only three processe of pair production by the photons, bremsstrahlung b the electrons and collision energy loss suffered by th electrons),

1D Approximation B

$$N_e = \frac{0.31}{\sqrt{ln(E_0/\epsilon_0}} exp[t(1 - 1.5\ ln(s_L)] \qquad (1$$

This relation concerns the total number of electrons o the cascade, whereas a parallel relation for the integra energy spectrum of the electrons is also given, in botl cases with cross sections for bremsstrahlung and pair pro duction in their respective asymptotic form at very higl energy. In this useful synthesis a very simple parametri sation of the longitudinal age s is defined as:

S longitudinal

$$s_L = \frac{3t}{t + 2ln(E/\epsilon_0)} \qquad (2$$

E_0 is the energy of the primary photon generating th cascade, t is expressed here in cascade units (the atmo spheric depth has being divided by the electron radiatio lenth in air taken as 37.1 gcm^{-2}), ϵ_0 is the critical en ergy with a value of $82 MeV$. The developmental stag of a pure electromagnetic cascade is characterized by th

that $\int_0 2\pi x f(x)dx = 1$, are relate
tron density $\Delta_e(r)$ by $\Delta_e(r) = N_e f$
analytical parameterizations of nume
s from the solutions of diffusion e
rom Monte Carlo calculations are co
sified following the earliest forms pro

H. BE

$$= 0.45(1/x + 4)\exp(-4x^{2/3})$$

NKG

$f(x)$ → $$= c(s)x^{s-2}(x + 1)^{s-4.5}$$

$$= g(s)x^{s-a}(x + 1)^{s-b}(1 + dx)^{-c}$$

GAUSSIAN HYPERGEOMET

Kamata [17], to describe the lateral electron distribution in the three-d
is the NKG function:

$$f_{\text{NKG}}(r) = C(s)\left(\frac{r}{r_0}\right)^{s-2}\left(1+\frac{r}{r_0}\right)^{s-4.5}$$

where the coefficient $C(s)$ is given by the normalization condition:

$$C(s) = \frac{1}{2\pi}\frac{\Gamma(4.5-s)}{\Gamma(s)\Gamma(4.5-2s)}$$

and r_0 is the Moliere radius. If N is the size of the shower, the density

$$\Delta_e = \frac{N}{r_0^2} f_{\text{NKG}}(r).$$

The NKG function is based on the Eulerian beta function, $B(u, v)$, ta
cylindrical symmetry, from

$$N = \int_0^\infty 2\pi r\,\Delta_e(r)\,\mathrm{d}r = 2\pi C(s)\int_0^\infty \left(\frac{r}{r_0}\right)^{s-1}\left(\frac{r}{r_0}+\right.$$

where appears the classical form

$$B(u, v) = \int_0^\infty \frac{x^{u+1}}{(1+x)^{u+v}}\,\mathrm{d}x$$

for $x = \frac{r}{r_0}$, $u = s-2$, $v = 6.5-2s$.

NKG analytical representation of 3D solution of cascade transport equations

- **Near axis s→0 r^{s-2}**

Far from axis s→ ∞ $r^{s-4.5}$

NKG is a continuous transition from a steep power law near axis to an asymptotic steeper power law at large distances

Normalised as a Beta Eulerian function

Options NKG and EGS in CORSIKA

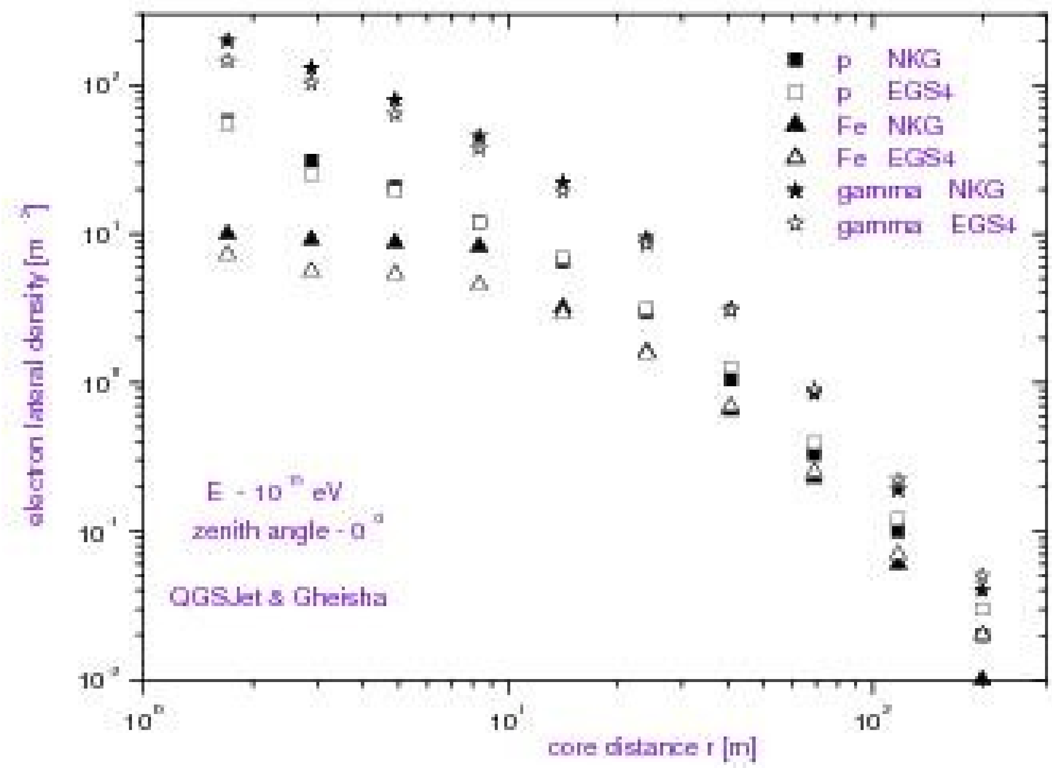
- NKG Approximation B
- Option NKG in CORSIKA is not NKG!
- EGS in CORSIKA contains in addition Annihilation of positron, electrons from muon decay,
- Energy dependant cross section, magnetic field..

NKG option in CORSIKA is based on a Moliere radius reduced by a correlation with longitudinal age parameter, following different solutions of cascade 3D transport equations(from Lagutin et al. 79), see Capdevielle et al. J.Phys G, 6, 901, 1980 , J.Phys. G, 8, 1317,1983

J.Phys.G16, 769, 1990,

CORSIKA reports KFK-4978(1992), FZK 6019(1998)

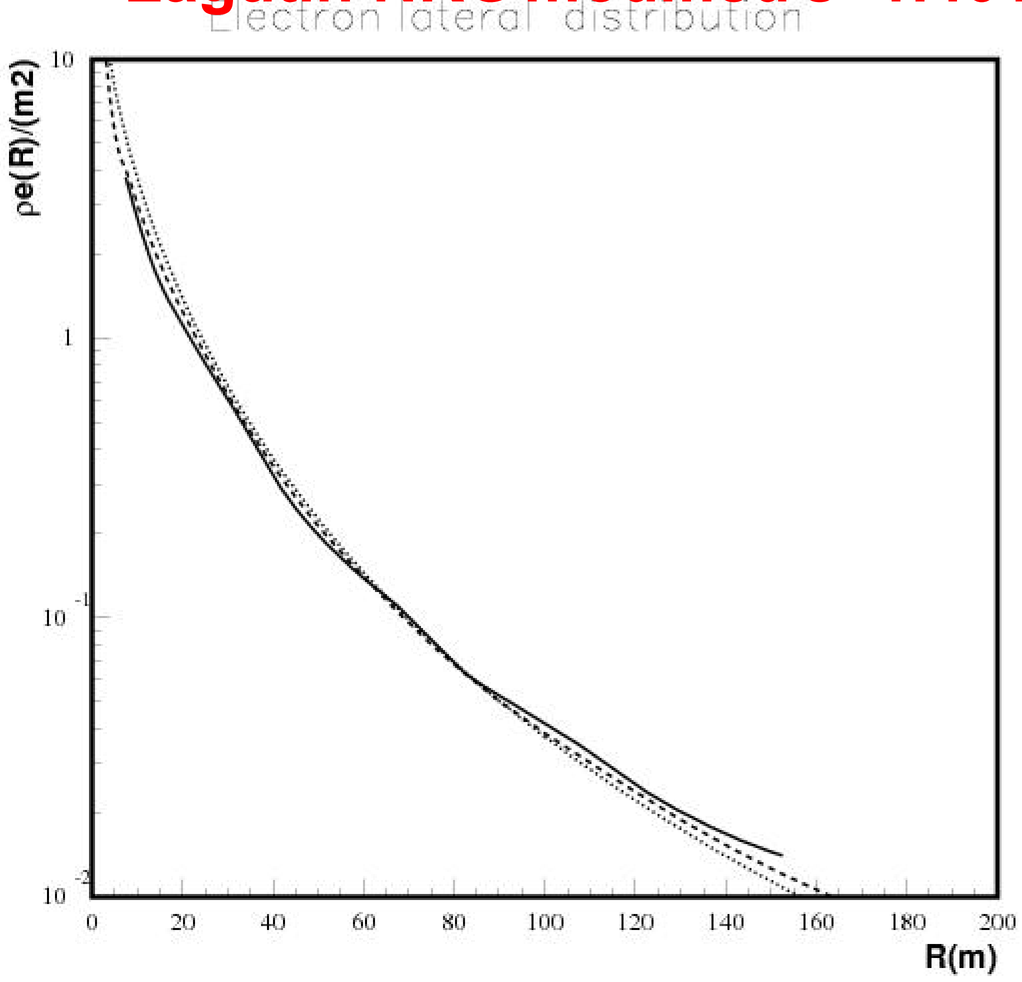
Options NKG and EGS inside CORSIKA



Kascade (solid) logNe [3.9, 4.3]

CORSIKA NKG dashed

Lagutin NKG modified s=1.404, Ne=11829



The longitudinal age parameter associated with the numerica
EGS cannot be taken from the so-called 'NKG' option of CoRSiKa
differ, according to different energy thresholds and more sophisticat
approximation B and, at very high energy, both cascade behaviours a
LPM effect being implemented only in the Monte Carlo code. We h
EGS program a topological approximation for the longitudinal age par

α = ln(Nmax/Ne)
J.Phys.G 31(2005)507-524

$$s = \exp\left[\frac{2}{3} \times \left\{1 + \frac{\alpha}{t} - \tau\right\}\right] \qquad \text{with} \quad \tau = \frac{t_{\max}}{t}$$

(t is expressed in cascade units). We can observe (figure 4) that at ultra
functions depend on the age parameter for both electrons and muons

$$s_{ij} = \frac{\ln\left(F_{ij} X_{ij}^2 Y_{ij}^{4.5}\right)}{\ln(X_{ij} Y_{ij})}$$

Local Age Parameter

F_{ij} $= f(r_i)/f(r_j)$, $X_{ij} = r_i/r_j$ and $Y_{ij} = (x_i + 1)/(x_j + 1)$. More generally, if
ts a definition of the LAP $s(x)$ (or $s(r)$) at each point:

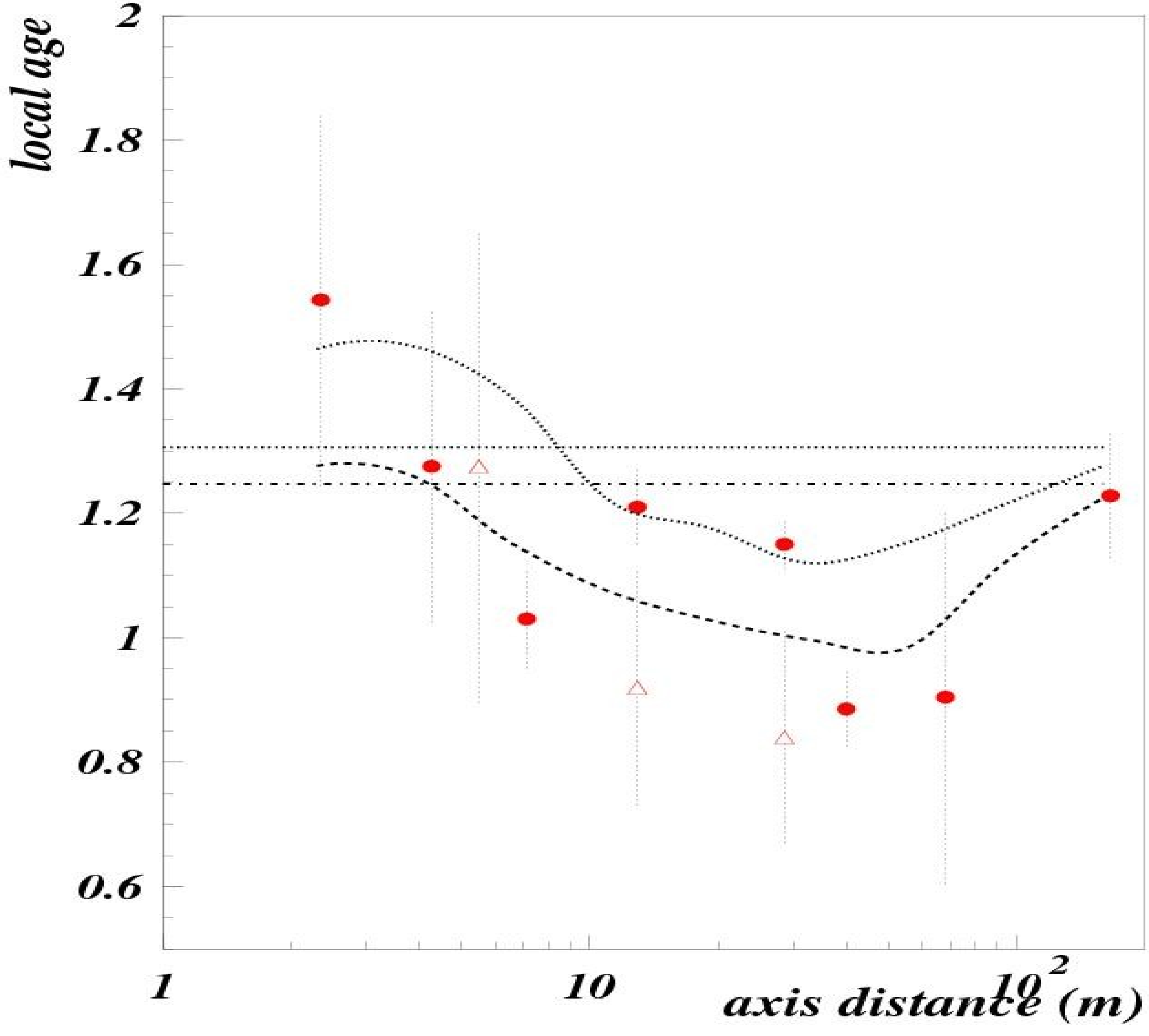
$$s(r) = \frac{1}{2x+1}\left((x+1)\frac{\partial \ln f}{\partial \ln x} + (2 + \beta_0)x + 2\right).$$

5, $f_{\rm NKG}(r)$ with $s = s(r)$ can be used to fit f in the neighbourhood of r.
... characterized minimum value of

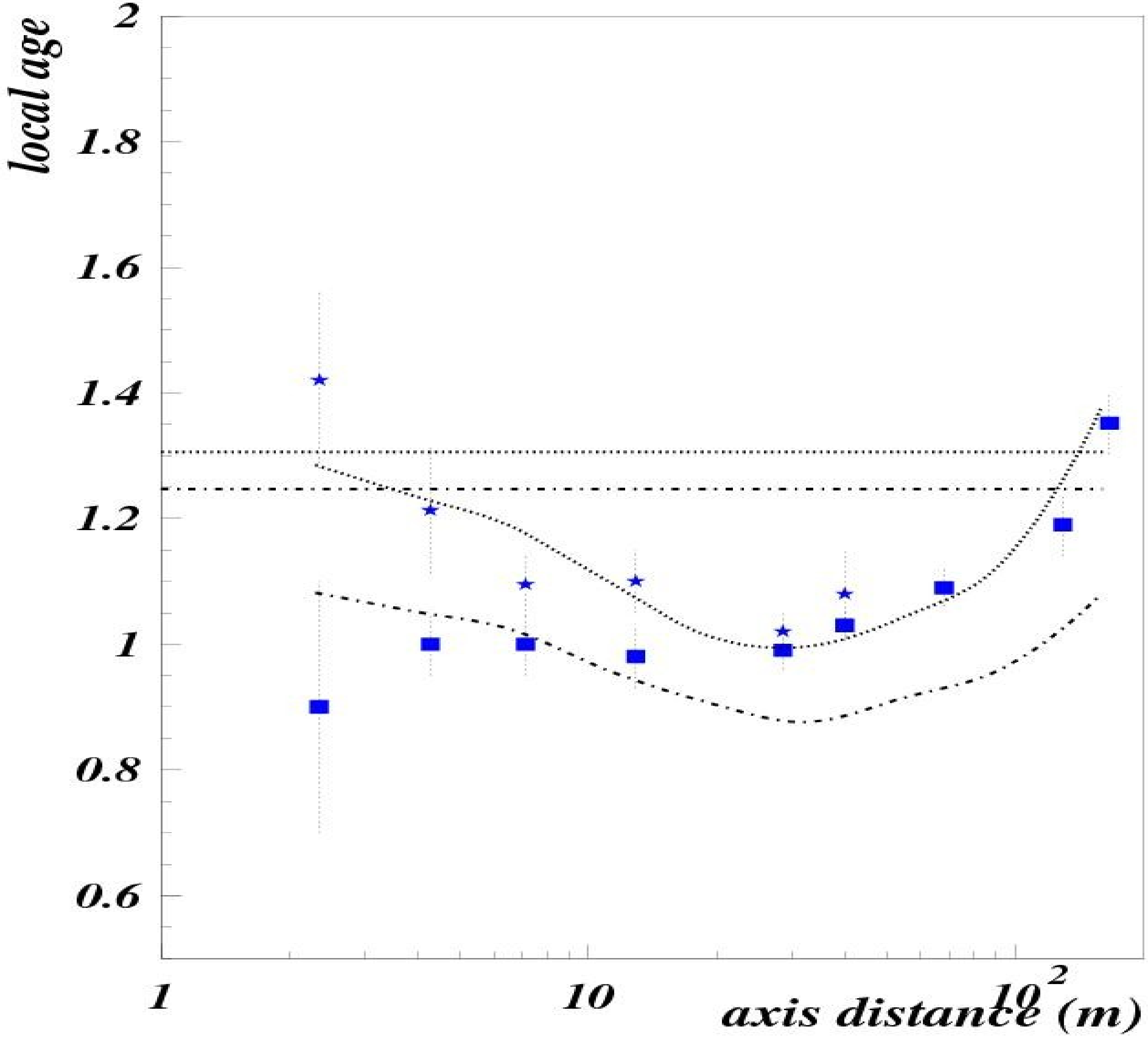
Local Age parameter s(r)

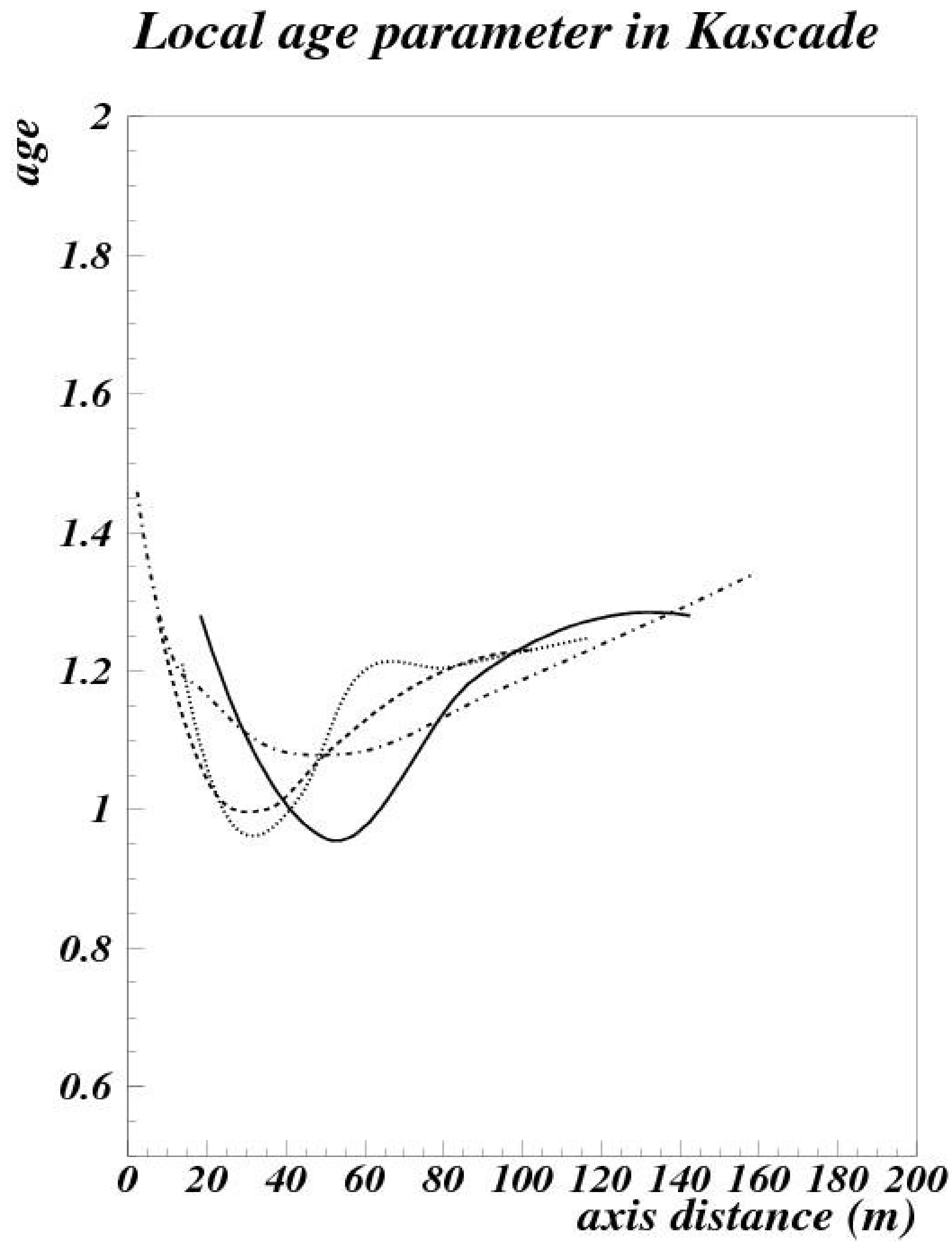
- **Inspired by the topological approach of mathematical functions**
- **Gives the most suitable NKG adjustment in a small interval**
- **See Rapporteur papers of**
- **S.Tonwar 17th ICRC Paris, 13, 330, 1981**
- **M.R. Rao, 18th ICRC Bangalore, 11,1983**
- **R.W. Clay, 19th ICRC, La Jolla, 9, 1985**

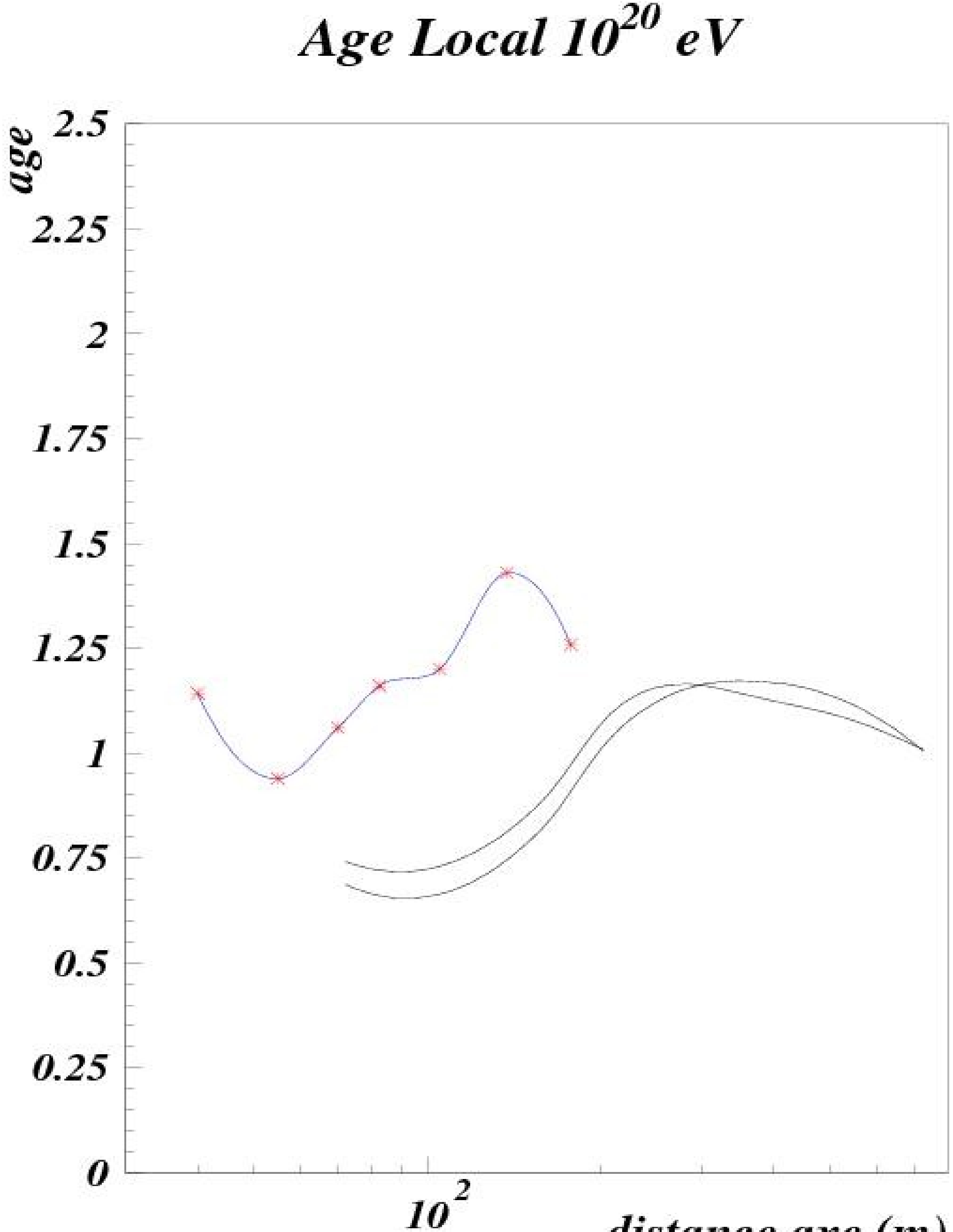
Local age parameter in Akeno



Local age parameter in Akeno




Local age parameter in Kascade
age
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0
20
40
60
80
100
120
140
160
180
200
axis distance (m)



Age Local 10^{20} eV
age
2.5
2.25
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
10^2
distance axe (m)

Size fluctuations at different depths

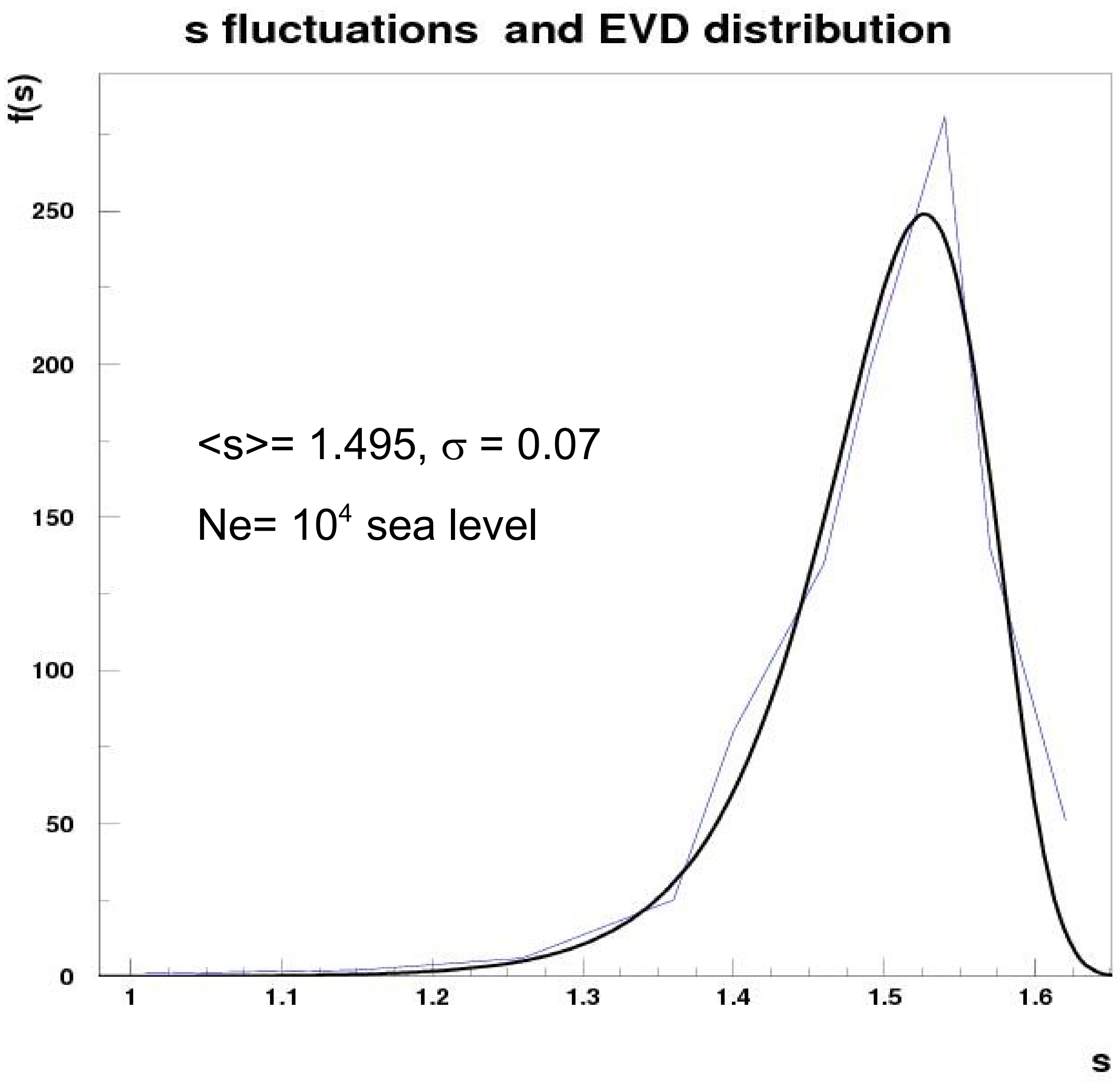
Extreme value distributions (E.V. D.

$$f(N_e) = \frac{1}{\sigma} \exp\left(\pm \frac{\mu - N_e}{\sigma} - e^{\pm \frac{\mu - N_e}{\sigma}} \right)$$

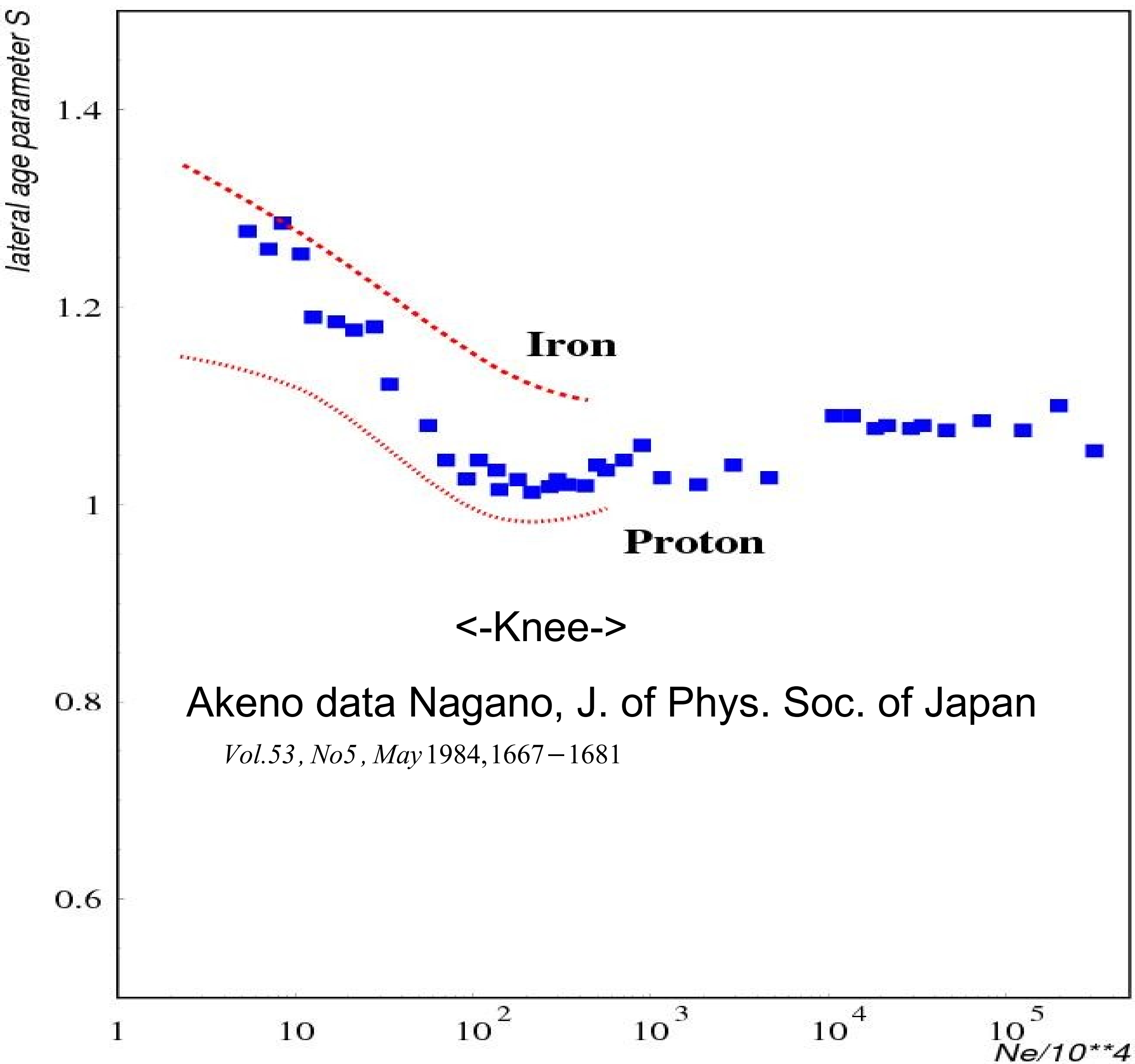
where the parameters μ and σ are relat

$\overline{N_e} = \mu \pm 0.577\sigma$ and $V_{N_e} = 1.645\sigma^2$

S in place of Ne for age fluctuations



s fluctuations and EVD distribution
f(s)
250
200
150
100
50
0
1
1.1
1.2
1.3
1.4
1.5
1.6
s
<s>= 1.495, σ = 0.07
Ne= 10^4 sea level



lateral age parameter S
1.4
1.2
1
0.8
0.6
Iron
Proton
<-Knee->
Akeno data Nagano, J. of Phys. Soc. of Japan
Vol.53, No5, May 1984, 1667−1681
1
10
10²
10³
10⁴
10⁵
Ne/10**4

Conclusion

- **Relation between s(r) , s(long), and s(lat)**
- **s(long)= 1.25 s(lat) in 1st approximation**
-
- **More accurately s(lat) = $\int$s (r)rdr/ $\int$rdr with integration on the shower disk taking into account the grid interval, the size of the detectors and the conditions of trigger.**
-
- **After suitable corrections of bias introduced by the NKG global fitt to the lateral distribution, s behaviour versus size is a good indicator of intrinsic cascading and mass composition.**

CONCLUSION

- **Local age parameter behavior at fixed primary energy (larger near axis, a minimum near 40-50m and a clear increase at large distance)**
- **It can explain the experimental behavior of S(lat) versus energy, with an apparent minimum in knee region and followed by a kind of bump by pure phenomenological effects, remaining in agreement with a mixed composition.**
- **Next simulations with EGS and also above 50 PeV are in progress,as well as interpretation of Kascade Grande data.**