Quantum Chromodynamics

& LHC Phenomenology

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Parton shower & Monte Carlo methods

- we will see more explicitly in the next lecture that today one can compute some IR-safe quantities at NLO and very few ones at NNLO. Difficult to expect much more in the coming years.

- we have also seen that sometimes large logs spoil the convergence of PT, NLO etc becomes useless

- here we adopt a different approach: we seek for an approximate result such that enhanced terms are taken into account to all orders

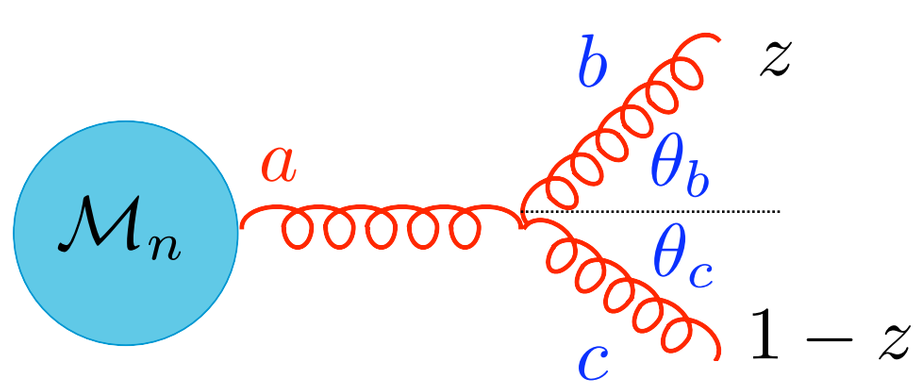
- this will lead to a 'parton shower' picture, which can be implemented in computer simulations, usually called Monte Carlo programs or event generators

Parton branching: the time-like case

Assume: $p_b^2, p_c^2 \ll p_a^2 \equiv t$ (scale of the branching)

$$\begin{aligned} p_a &= (E_a, 0, 0, p_{az}) \\ p_b &= (E_b, 0, E_b \sin\theta_b, E_b \cos\theta_b) \\ p_c &= (E_c, 0, -E_c \sin\theta_c, E_c \cos\theta_c) \end{aligned}$$

Time-like branching: t > 0



Kinematics: $z = \dfrac{E_b}{E_a} = 1 - \dfrac{E_c}{E_a}$

small angle approx.

$$t = (p_b + p_c)^2 = 2E_bE_c(1-\cos\theta) \sim z(1-z)E_a^2\theta^2$$

$$E_b \sin\theta_b = E_c \sin\theta_c \Rightarrow z\theta_b \sim (1-z)\theta_c$$

$$\theta = \theta_b + \theta_c = \frac{\theta_b}{1-z} = \frac{\theta_c}{z}$$

Parton branching: gluon case

Three-gluon vertex:

$$V_{ggg} = i g_s f_{ABC} \epsilon_a^\mu \epsilon_b^\nu \epsilon_c^\rho \left(g_{\mu\nu}(p_a - p_b)_\rho + g_{\nu\rho}(p_b - p_c)_\mu + g_{\rho\mu}(p_c - p_a)_\nu \right)$$

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Use: $\epsilon_i \cdot p_i = 0$ and $p_a + p_b + p_c = 0$

$$V_{ggg} = -2 i g_s f_{ABC} \left[(\epsilon_a \cdot \epsilon_b)(\epsilon_c \cdot p_b) - (\epsilon_b \cdot \epsilon_c)(\epsilon_a \cdot p_b) - (\epsilon_c \cdot \epsilon_a)(\epsilon_b \cdot p_c) \right]$$

Parton branching: gluon case

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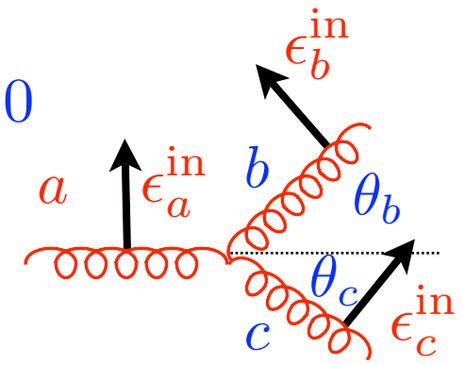
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Branching: in a plane. Natural to split polarization vectors in ϵ_i^{in} and ϵ_i^{out}

Properties: $\epsilon_i^{\text{in}} \cdot \epsilon_j^{\text{in}} = \epsilon_i^{\text{out}} \cdot \epsilon_j^{\text{out}} = -1 \qquad \epsilon_i^{\text{in}} \cdot \epsilon_j^{\text{out}} = \epsilon_i^{\text{out}} \cdot p_j = 0$



Explicitly:

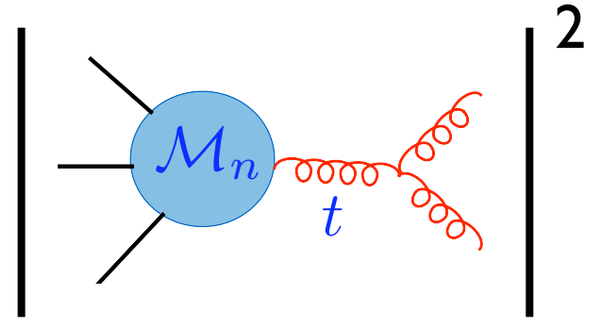
$$\epsilon_a^{\text{in}} = (0, 0, 1, 0)$$
$$\epsilon_b^{\text{in}} = (0, 0, \cos\theta_b, -\sin\theta_b)$$
$$\epsilon_c^{\text{in}} = (0, 0, \cos\theta_c, \sin\theta_c)$$

$$\epsilon_a^{\text{in}} \cdot p_b = -E_b\theta_b = -z(1-z)E_a\theta$$
$$\epsilon_b^{\text{in}} \cdot p_c = E_c\theta = (1-z)E_a\theta$$
$$\epsilon_c^{\text{in}} \cdot p_b = -E_b\theta = -zE_a\theta$$

Parton branching: the gluon case

Squared matrix element for n+1 partons becomes:

$$|\mathcal{M}_{n+1}|^2 = \frac{4g_s^2}{t} C_A F(z;\epsilon_a,\epsilon_b,\epsilon_c)|\mathcal{M}_n|^2$$



NB: one "t" cancels completely

a	b	c	F(z; ε_a, ε_b, ε_c)
in	in	in	(1-z)/z + z/(1-z) + z(1-z)
in	out	out	z(1-z)
out	in	out	(1-z)/z
out	out	in	z/(1-z)

Averaging over incoming and summing over outgoing pol. we get

$$C_A\langle F\rangle = \hat{P}_{gg} = C_a\left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right]$$

The gluon case: remarks

Soft singularities ($z \to 0,1$) are associated to soft gluon in the plane of the branching

Correlation between plane of branching and polarization of incoming gluon: take polarization of gluon at an angle φ to the plane then

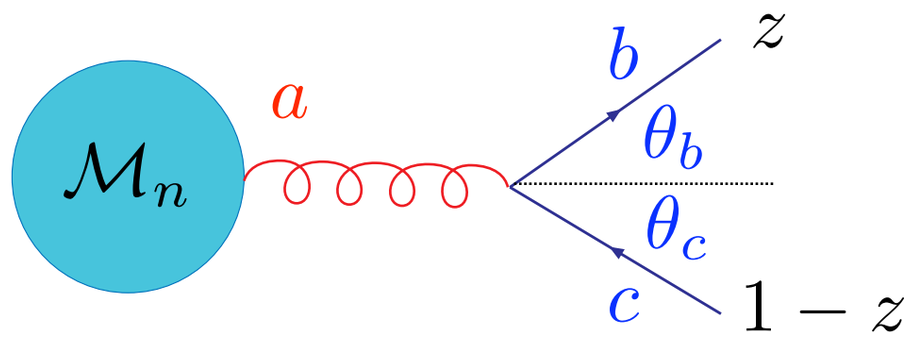
$$
\begin{aligned}
F_\phi &= \sum_{b,c} |\cos\phi\, \mathcal{M}(\epsilon_a^{\text{in}}, \epsilon_c, \epsilon_c) + \sin\phi\, \mathcal{M}(\epsilon_a^{\text{out}}, \epsilon_c, \epsilon_c)|^2 \\
&= \underbrace{\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)}_{\text{unpolarized result}} + \underbrace{z(1-z)\cos 2\phi}_{\text{correction}}
\end{aligned}
$$

Correction favors polarization of branching gluon in the branching plane, but is weak (no soft enhancements)

Gluon splitting to quarks

Similarly start from 3-particle vertex:

$$V_{q\bar{q}g} = -ig_s\, t^A_{bc}\, \bar{u}(p_b)\, \gamma_\mu \epsilon^\mu_a\, v(p_c)$$



Fix a representation of the Dirac algebra (called Dirac rep.):

$$\gamma^0 = \begin{pmatrix} 1_{2\times 2} & 0_{2\times 2} \\ 0_{2\times 2} & -1_{2\times 2} \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0_{2\times 2} & \sigma_i \\ -\sigma_i & 0_{2\times 2} \end{pmatrix}$$

To first order in the small angles the spinors are

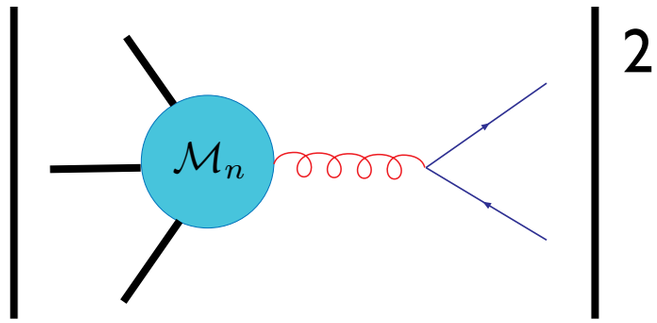
$$\frac{u_+(p_b)}{\sqrt{E_b}} = \begin{pmatrix} 1 \\ \theta_b/2 \\ 1 \\ \theta_b/2 \end{pmatrix} \quad \frac{u_-(p_b)}{\sqrt{E_b}} = \begin{pmatrix} \theta_b/2 \\ -1 \\ \theta_b/2 \\ -1 \end{pmatrix} \quad \frac{v_+(p_c)}{\sqrt{E_c}} = i\begin{pmatrix} -\theta_c/2 \\ -1 \\ \theta_c/2 \\ 1 \end{pmatrix} \quad \frac{v_-(p_c)}{\sqrt{E_c}} = i\begin{pmatrix} -1 \\ \theta_c/2 \\ -1 \\ \theta_c/2 \end{pmatrix}$$

Gluon splitting to quarks

Explicitly we find e.g.

$$-ig_s\bar{u}_+(p_b)\gamma_\mu\epsilon_a^{\text{in},\mu}v_-(p_c) = \sqrt{E_bE_c}(\theta_b-\theta_c) = \sqrt{z(1-z)}(1-2z)E_a\theta$$

Similarly to before define



a	b	c	F(z; ε_a, λ_b, λ_c)
in	$\pm$	$\mp$	$(1-2z)^2$
out	$\pm$	$\mp$	1

$$|\mathcal{M}_{n+1}|^2 = \frac{4g_s^2}{t}T_RF(z;\epsilon_a,\lambda_b,\lambda_c)|\mathcal{M}_n|^2$$

Averaged splitting function: $T_R\langle F\rangle \equiv \hat{P}_{qg}(z) = T_R\left[z^2+(1-z)^2\right]$

Angular correlation: $F_\phi = z^2+(1-z)^2-2z(1-z)\cos 2\phi$ (more important)

Last case: quark emitting gluon

Similarly to the two previous cases one obtains

$$|\mathcal{M}_{n+1}|^2 = \frac{4g_s^2}{t} C_F F(z;\lambda_a,\lambda_b,\epsilon_c)|\mathcal{M}_n|^2$$

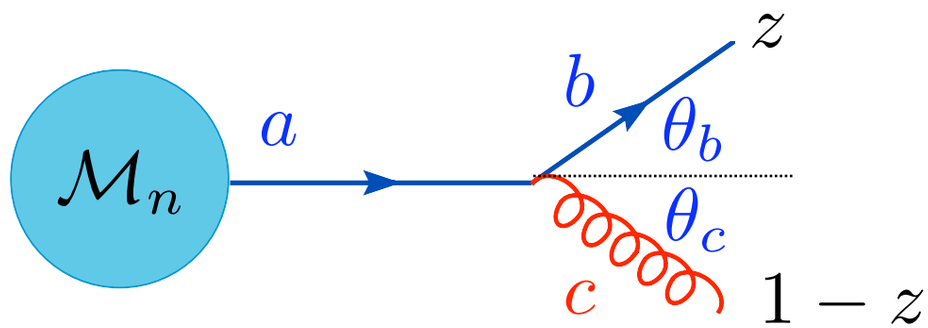
a	b	c	F(z; λ_a, λ_b, ε_c)
$\pm$	$\pm$	in	$(1+z)^2/(1-z)$
$\pm$	$\pm$	out	$1-z$

NB: helicity of the quark does not change during the branching

Averaged splitting function: $C_F\langle F\rangle \equiv \hat{P}_{qq}(z) = C_F \dfrac{1+z^2}{1-z}$

Angular correlation:

$$F_\phi = \frac{1+z^2}{1-z} + \frac{2z}{1-z}\cos 2\phi$$



Phase space

n-particle phase space (without branching): $d\Phi_n = d\Phi_{n-1} \frac{d^3 p_a}{(2\pi)^3 2E_a}$

(n+1)-particle phase space (with branching): $d\Phi_{n+1} = d\Phi_{n-1} \frac{d^3 p_b}{(2\pi)^3 2E_b} \frac{d^3 p_c}{(2\pi)^3 2E_c}$

At fixed p_b: $d^3 p_a = d^3 p_c \quad \Rightarrow \quad d\Phi_{n+1} = d\Phi_n \frac{d^3 p_b}{(2\pi)^3 2E_b} \frac{E_a}{E_c}$

$$\begin{aligned} d^3 p_b &= p_b^2 dp_b \ \sin\theta d\theta \ d\phi \\ &\sim E_b^2 dE_b \ \theta d\theta \ d\phi \\ &= E_a^3 z^2 dz \ \frac{dt}{2z(1-z)E_a^2} \ d\phi \end{aligned}$$

$$\Rightarrow \quad d\Phi_{n+1} = d\Phi_n \frac{1}{4(2\pi)^3} dt \, dz \, d\phi$$

N-particle cross-section: $d\sigma_n = \mathcal{F} \, |\mathcal{M}_n|^2 \, d\Phi_n$ **with** $|\mathcal{M}_{n+1}|^2 = \frac{4g_s^2}{t} C F |\mathcal{M}_n|^2$

$$d\sigma_{n+1} = d\sigma_n \frac{dt}{t} dz \, d\phi \frac{\alpha_s}{2\pi} C \, F$$

Azimuthal averaged result

Averaging over azimuthal angles:

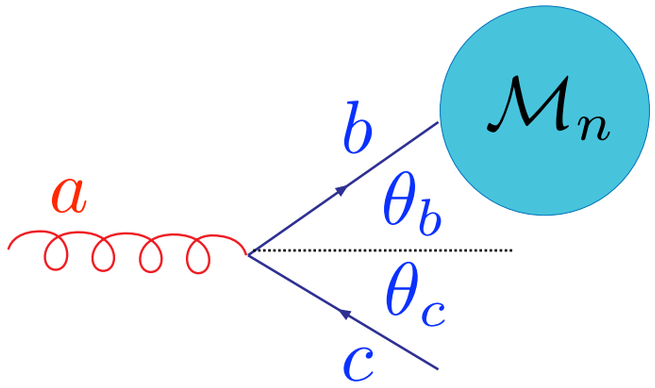
$$\int \frac{d\phi}{2\pi} C\, F = \hat{P}_{ba}(z)$$

The evolution equation becomes:

$$d\sigma_{n+1} = d\sigma_n \frac{dt}{t} dz\, \frac{\alpha_s}{2\pi} \hat{P}_{ba}(z)$$

Space-like branching

What are the modifications needed if an incoming parton splits?



The kinematics changes: $p_a^2, p_c^2 \ll |p_b^2| \equiv t$

Space-like branching: t < 0

Small angle approximation: $t = E_a E_c \theta_c^2$ (verify)

(n+1) particle phase space becomes: $d\Phi_{n+1} = d\Phi_n \frac{1}{4(2\pi)^3} dt \, \frac{dz}{z} \, d\phi$

The additional "z" is compensated by the different flux-factor, we find

Space-like or time-like braching: $d\sigma_{n+1} = d\sigma_n \frac{dt}{t} dz \, \frac{\alpha_s}{2\pi} \hat{P}_{ba}(z)$

Perturbative evolution

In exact analogy with what done for parton densities inside hadrons we want to write an evolution equation for the probability to have partons at the momentum scale Q^2 with momentum fraction z during PT branching

Start from DGLAP equation

$$Q^2 \frac{\partial f(x,Q^2)}{\partial Q^2} = \int_0^1 dz \frac{\alpha_s}{2\pi} \hat{P}(z) \left(\frac{1}{z} f\left(\frac{x}{z}, Q^2\right) - f(x,Q^2) \right)$$

Introduce a cut-off to regulate divergences

$$Q^2 \frac{\partial f(x,Q^2)}{\partial Q^2} = \int_0^{1-\epsilon} \frac{dz}{z} \frac{\alpha_s}{2\pi} \hat{P}(z) f\left(\frac{x}{z}, Q^2\right) - f(x,Q^2) \int_0^{1-\epsilon} dz \frac{\alpha_s}{2\pi} \hat{P}(z)$$

Introduce a Sudakov form factor

$$\Delta(Q^2) = exp\left\{ - \int_{Q_0}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^{1-\epsilon} dz \frac{\alpha_s}{2\pi} \hat{P}(z) \right\}$$

Perturbative evolution

The DGLAP equation becomes

$$Q^2 \frac{\partial}{\partial Q^2}\left(\frac{f(x,Q^2)}{\Delta(Q^2)}\right) = \frac{1}{\Delta(Q^2)} \int_0^{1-\epsilon} \frac{dz}{z} \frac{\alpha_s}{2\pi} \hat{P}(z) f\left(\frac{x}{z}, Q^2\right)$$

Integrating the above equation one gets

$$f(x,Q^2) = f(x,Q_0^2)\frac{\Delta(Q^2)}{\Delta(Q_0^2)} + \int_{Q^0}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \frac{\Delta(Q^2)}{\Delta(k_\perp^2)} \int_0^{1-\epsilon} \frac{dz}{z} \frac{\alpha_s}{2\pi} \hat{P}(z) f\left(\frac{x}{z}, k_\perp^2\right)$$

This equation has a probabilistic interpretation

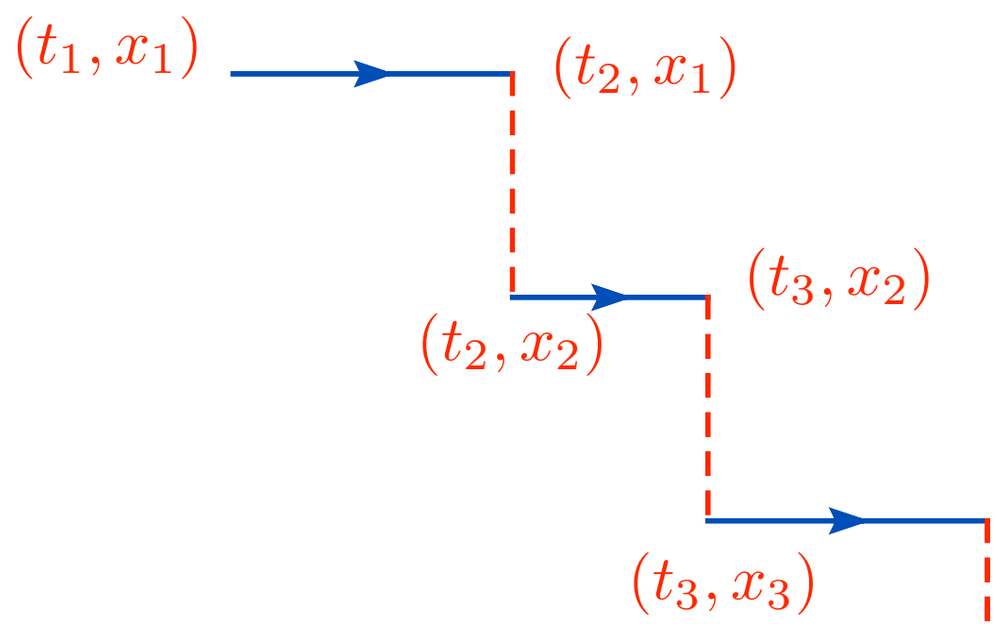
- First term: probability of evolving from Q_0^2 to Q^2 without emissions (ratio of Sudakovs $\Delta(Q^2)/\Delta(Q_0^2)$)
- Second term: emission at scale $k_\perp^2$ and evolution from $k_\perp^2$ to Q^2 without further emissions

Multiple branchings

Multiple branching can now be described using the above probabilistic equation

Denote by t the evolution variable (e.g $t = Q^2$)
Start from one parton at scale t_1 and momentum fraction x_1

The question is how to generate the values of t_2, x_2 and φ_2



Multiple branchings

1. t_2 generated with the correct probability by solving the equation
   (r = random number in [0,1])

$$\Delta(t_1)/\Delta(t_2) = r$$

   If t_2 smaller than cut-off evolution stops (no further branching)

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2. Else, generate momentum fraction $z = x_2/x_1$ with $\text{Prob.} \sim \frac{\alpha_s}{2\pi} P(z)$

$$\int_\epsilon^{x_2/x_1} dz \frac{\alpha_s}{2\pi} P(z) = r' \int_\epsilon^{1-\epsilon} dz \frac{\alpha_s}{2\pi} P(z)$$

   ε: IR cut-off for resolvable branching

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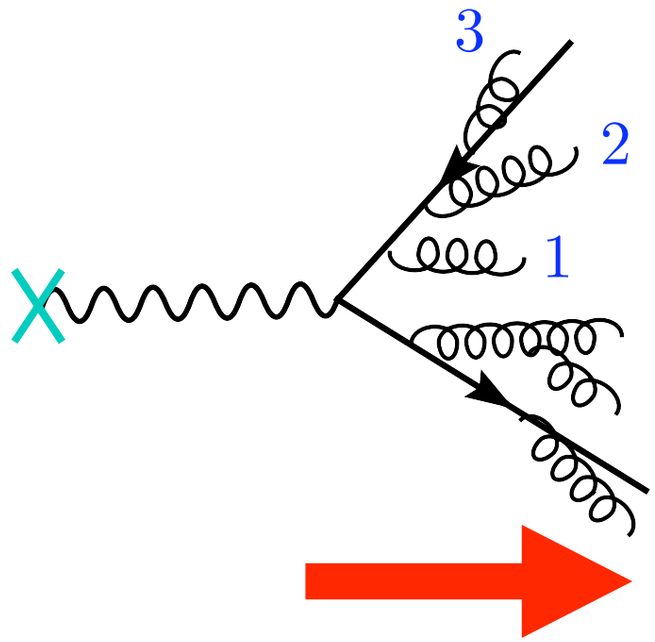
$$\int_\epsilon^{x_2/x_1} dz \frac{\alpha_s}{2\pi} P(z) = r' \int_\epsilon^{1-\epsilon} dz \frac{\alpha_s}{2\pi} P(z)$$

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3. Azimuthal angles: generated uniformly in $(0,2\pi)$ (or taking into account polarization correlations)

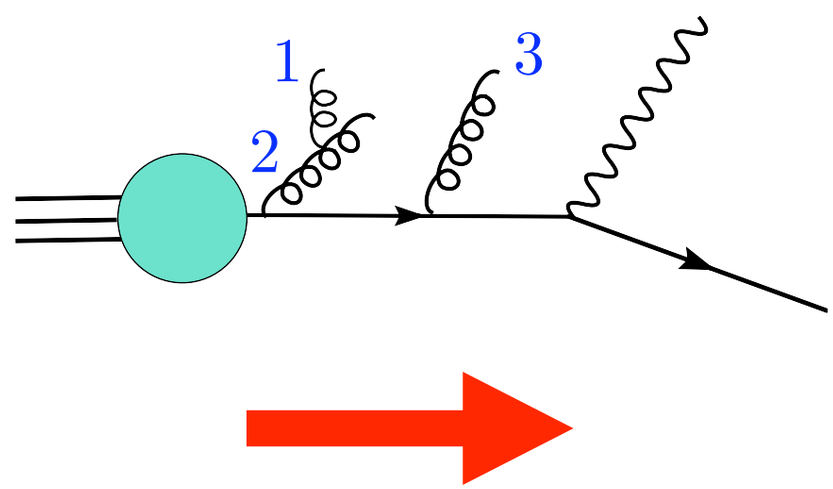
Space-like vs time-like evolution

Time-like: t evolves from a hard-scale downwards to an IR cut-off



$$Q > t_1 > t_2 > \cdots > Q_0$$

Space-like: t increases in the evolution up to the hard scale Q^2



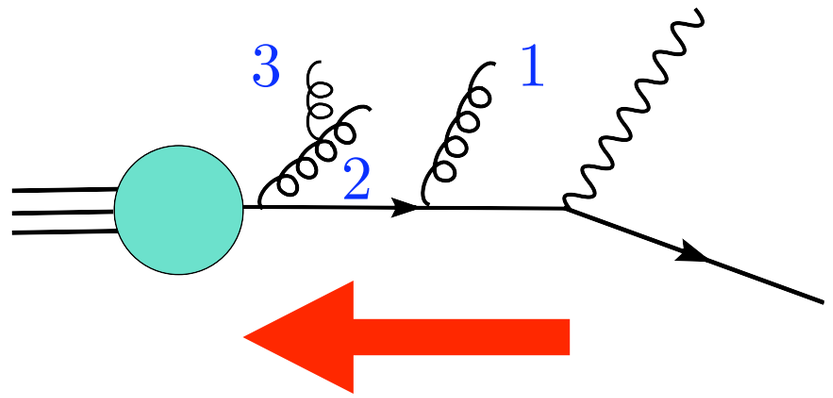
$$Q_0 < t_1 < t_2 < \ldots, Q$$

Each outgoing parton becomes a source of the new branching until the "no-branching" step is met (cut-off essential in parton shower)

$\Rightarrow$ a parton cascade develops, when all branchings are done partons are converted into hadrons via a hadronization model

Backward evolution

In space-like cases it is more convenient to start from the momentum fraction of the outgoing parton x_n and generate $x_{n-1}, .. x_0$ by backward evolution



$$Q > t_1 > t_2 > \cdots > Q_0$$

Essentially, the evolution proceeds as before but with a modified form factor which take the local parton density into account

We will not discuss backward evolution, despite its wide-spread use

Angular ordering

In the branching formalism discussed now we considered collinear enhancements to all orders in PT. But there are also soft enhancements.

When a soft gluon is radiated from a ($p_i p_j$) dipole one gets a universal eikonal factor

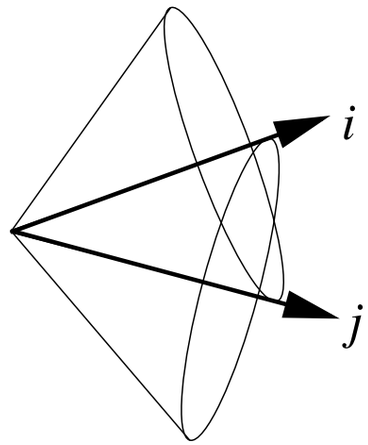
$$\omega_{ij} = \frac{p_i p_j}{p_i k \, p_j k} = \frac{1 - v_i v_j \cos\theta_{ij}}{\omega_k^2 (1 - v_i \cos\theta_{ik})(1 - v_j \cos\theta_{jk})}$$

Massless emitting lines $v_i = v_j = 1$, then

$$\omega_{ij} = \omega_{ij}^{[i]} + \omega_{ij}^{[j]} \qquad \omega_{ij}^{[i]} = \frac{1}{2}\left(\omega_{ij} + \frac{1}{1-\cos\theta_{ik}} - \frac{1}{1-\cos\theta_{jk}}\right)$$

Angular ordering

$$\int_0^{2\pi} \frac{d\phi}{2\pi} \omega_{ij}^{[i]} = \begin{cases} \frac{1}{\omega_k^2 (1-\cos\theta_{ik})} & \theta_{ik} < \theta_{ij} \\ 0 & \theta_{ik} > \theta_{ij} \end{cases}$$



Proof: see e.g. QCD and collider physics, Ellis, Stirling, Webber

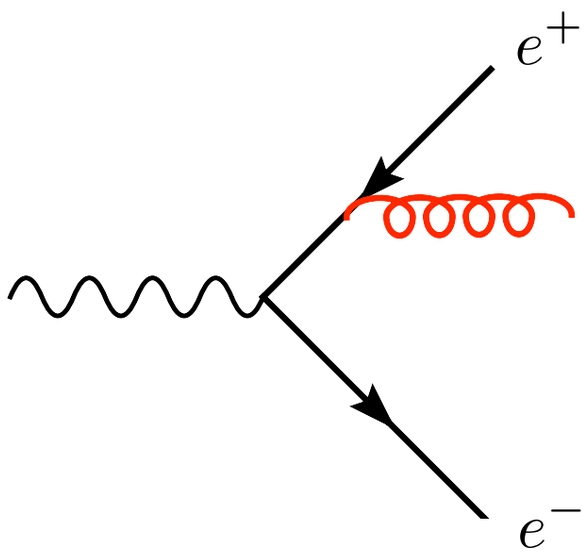
Angular ordering & coherence

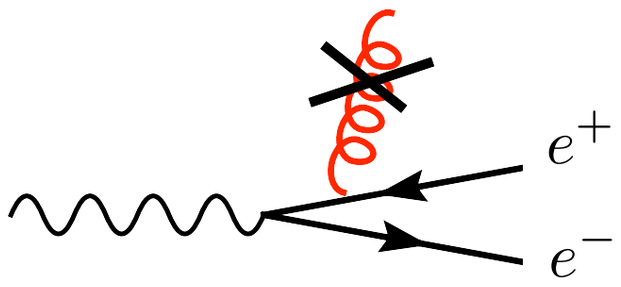
A. O. is a manifestation of coherence of radiation in gauge theories

<u>In QED</u>

suppression of soft bremsstrahlung from an e+e- pair (Chudakov effect)

At large angles the e^+e^- pair is seen coherently as a system without total charge ⇒ radiation is suppressed





Angular ordering & coherence

Coherent a → b + c branching: replace the ordering variable $t = p_a^2$ with

$$\zeta = \frac{p_b p_c}{E_b E_c} \sim 1 - \cos\theta_{bc}$$

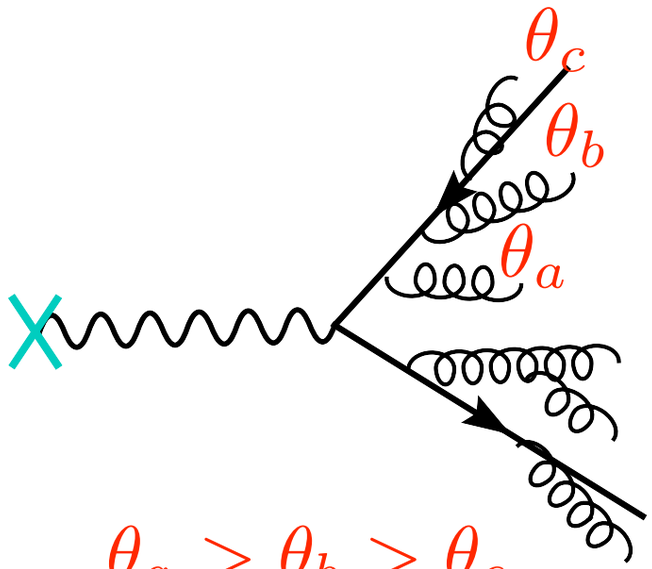
and require $\zeta' < \zeta$ at successive branchings

The basic formula for coherent branching

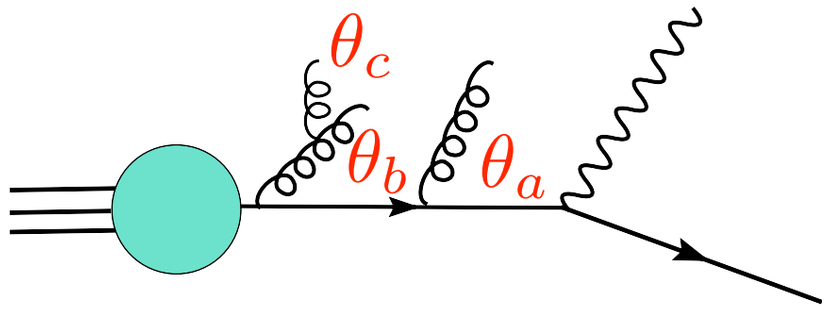
$$d\sigma_{n+1} = d\sigma_n \frac{d\zeta}{\zeta} dz \, \frac{\alpha_s}{2\pi} \hat{P}_{ba}(z)$$

NB: need collinear cut-off. Simplest choice: $\zeta_0 = \dfrac{t_0}{E^2}$

AO: time like vs space-like case



$\theta_a > \theta_b > \theta_c$



$\theta_a > \theta_b > \theta_c$

NB: angles decrease when moving away from the hard vertex, i.e. in the space-like case angles increase during the evolution

Accuracy issue

Formally, Monte Carlos are Leading Logs showers

- because they don't include any higher order corrections to the $1 \to 2$ splitting
- because they don't have any $1 \to 3$ splittings
- ....

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However, they fare better than analytic Leading Log calculations

- because they have energy conservation (NLO effect) implemented
- because they have coherence
- because they have optimized choices for the coupling
- because they provide an exclusive description of the final state

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- because they have coherence
- because they have optimized choices for the coupling
- because they provide an exclusive description of the final state

So, despite not guaranteeing NLL accuracy, they fare usually better than Leading Log analytic calculations

The real issue is that we are not able to estimate the uncertainty

Warning

The above discussion is a simplification

- many details/subtleties not discussed enough, some not at all
- various MC differ in the choice of the ordering variable and in many details, but the basic idea remains the same
- purpose was to give an overall idea of how Monte Carlos and what they can/can't do

Warning

The above discussion is a simplification

- many details/subtleties not discussed enough, some not at all
- various MC differ in the choice of the ordering variable and in many details, but the basic idea remains the same
- purpose was to give an overall idea of how Monte Carlos and what they can/can't do

What I want to discuss next is

- what we have on the market today
- improuvements to parton showers
- hadronization/U.E. minimum bias and all that...

Available parton showers

Standard Monte Carlo:

[Ariadne, Pythia, Herwig, Isajet ...]

- hard (2→2) scattering
- parton shower
- hadronization model + U.E. model

❖ Different Monte Carlos differ in the ordering variable of the shower (e.g. angle Herwig, transverse momentum Ariadne, virtuality Pythia), in U.E. model, in the hadronization model

❖ Comparison between different MC is often the only way to estimate uncertainties

All fail to describe high multiplicity final states

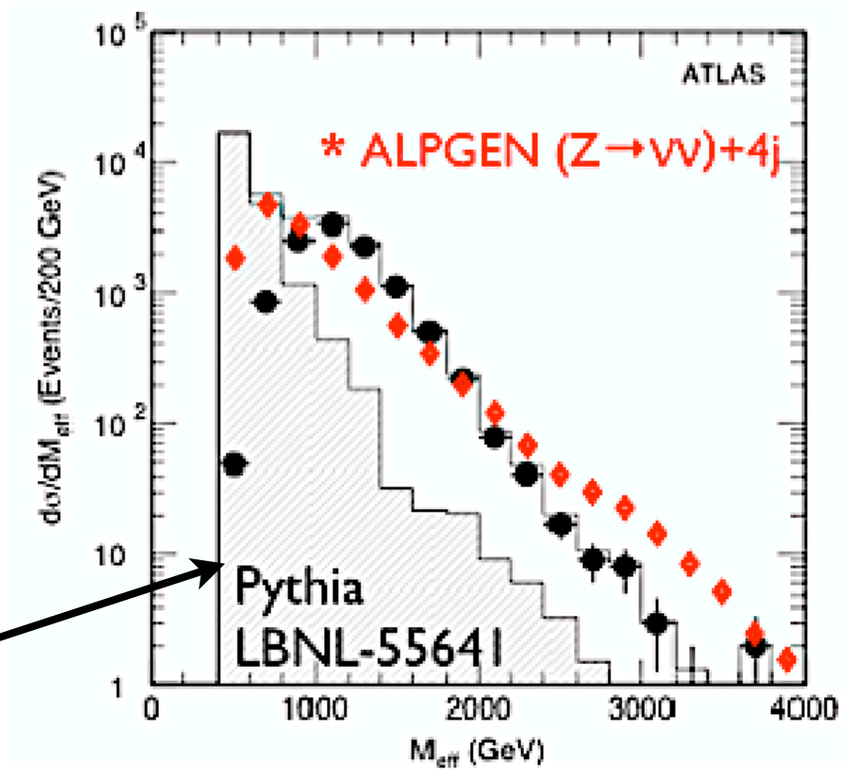
Failure with multiple hard radiation

Soft and collinear emissions ⇒ logarithmically enhanced terms

Monte Carlos take a hard (2→2) scattering and add a soft/collinear parton shower

They are by construction unable to describe multiple hard radiation

This seem trivial but look ... !



Atlas TDR '99!

SUSY search

$$M_{\mathrm{eff}} = E_{\mathrm{T,Mis}} + \sum_{j=1}^{4} E_{\mathrm{T,j}}$$

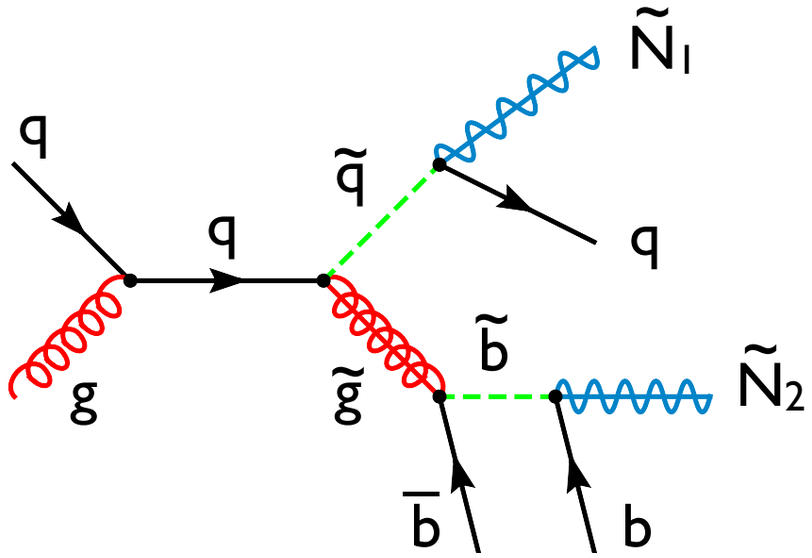
High multiplicity final states

The LHC will operate in a new regime: highest energy & luminosity

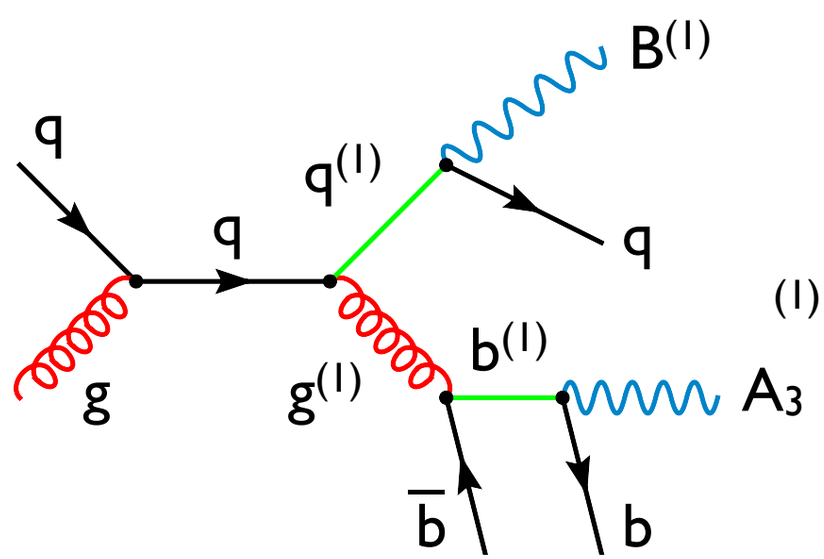
Very large number of high-multiplicity events

- typical SM process is accompanied by radiation multi-jet events
- most signals involve pair-production and subsequent chain decays

SUSY:



UED:



More important than ever to describe high-multiplicity final states

Alternative to parton shower

Matrix element calculation

exact leading order matrix element for a $2 \to n$ scattering (n depends on the process, current edge n=8)

[Alpgen, Madgraph, Sherpa ...]

- valid when partons are well-separated (no virtual!)
- fixed order calculation lower multiplicity compared to parton showers

Description remains leading order

$\Rightarrow$ large uncertainties on shapes and normalizations

Leading order: Feynman diagrams

Get *any* LO cross-section from the Lagrangian

1. draw all Feynman diagrams
2. put in the explicit Feynman rules and get the amplitude
3. do some algebra, simplifications
4. square the amplitude
5. integrate over phase space + flux factor + sum/average over outgoing/ incoming states

Automated tools for (1-3): FeynArts/Qgraf, Mathematica/Form etc.

Bottlenecks

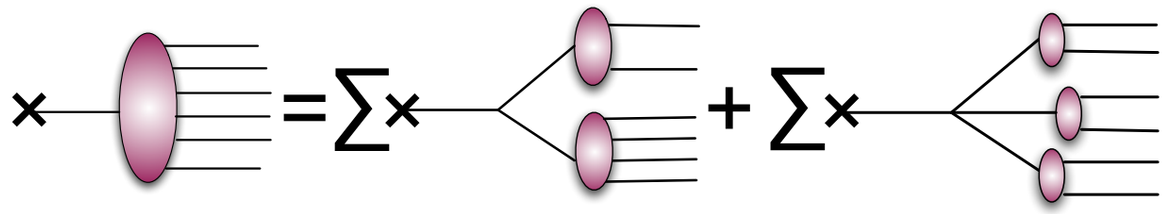
a) number of Feynman diagrams diverges factorially

b) algebra becomes more cumbersome with more particles

But given enough computer power everything can be computed at LO!

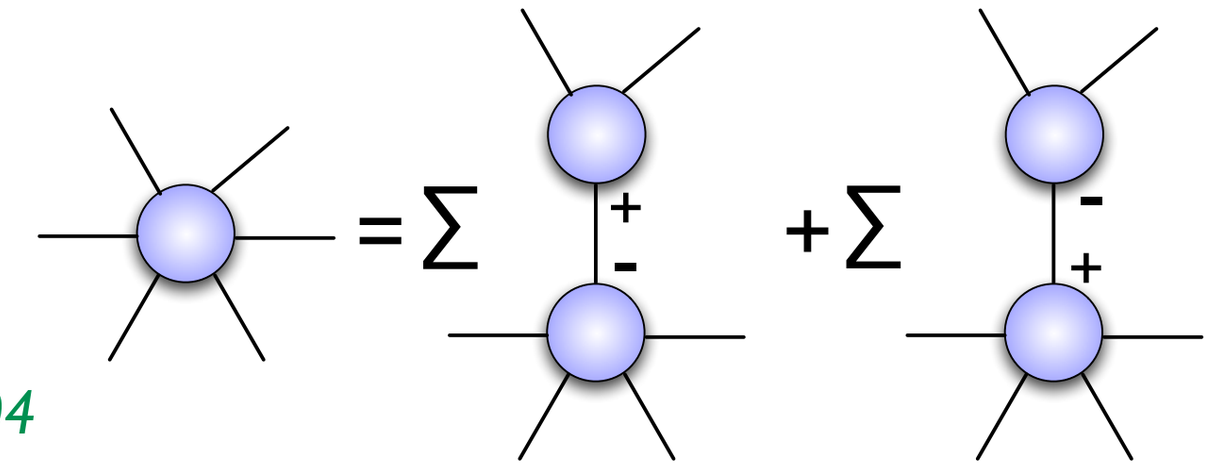
LO: 3 methods beyond Feynman diagrams

✓ Berends-Giele relations: compute helicity amplitudes recursively using off-shell currents



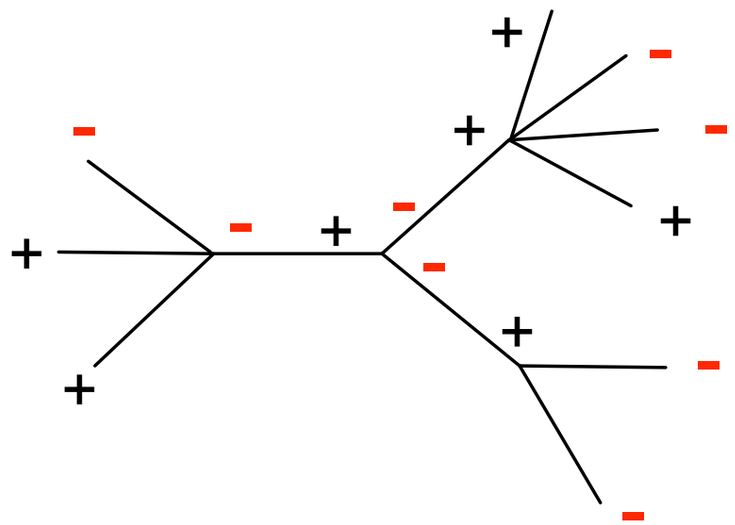
Berends, Giele '88

✓ BCF relations: compute helicity amplitudes via on-shell recursions (use complex momentum shifts)



Britto, Cachazo, Feng '04

✓ CSW relations: compute helicity amplitudes by sewing together MHV amplitudes [- - + + ... +]



Cachazo, Svrcek, Witten '04

Who is faster?

Time [s] for $2 \to n$ gluon amplitudes for 10^4 points

Duhr et al. '06
also Dinsdale et al. '06

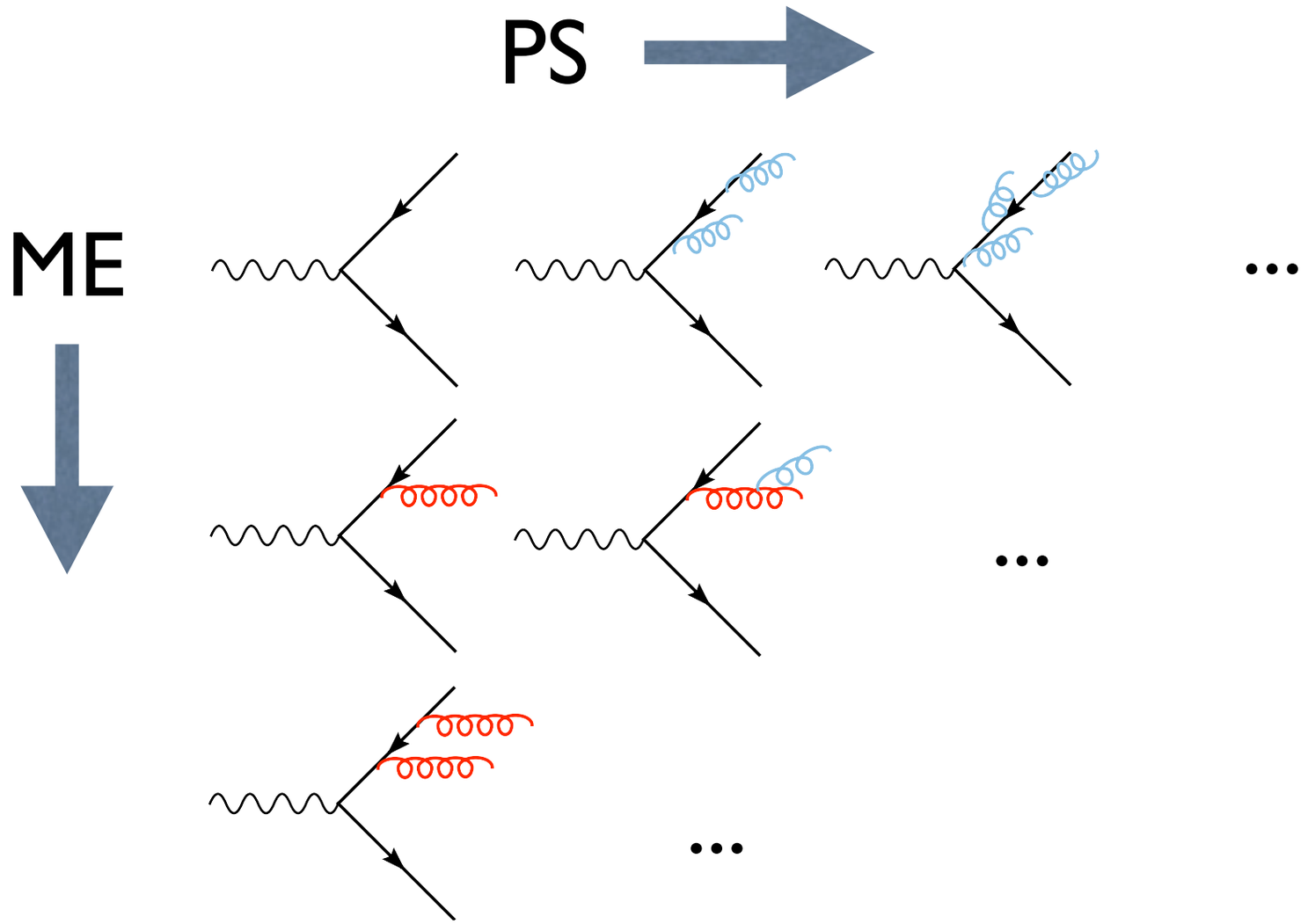
Final state	BG	BCF	CSW
2g	0.28	0.33	0.26
3g	0.48	0.51	0.55
4g	1.04	1.32	1.75
5g	2.69	7.26	5.96
6g	7.19	59.1	30.6
7g	23.7	646	195
8g	82.1	8690	1890
9g	270	127000	29700
10g	864	-	-

☛ undisputed numerical superiority of traditional Berends-Giele compared to twistor inspired methods

Matched matrix elements

Matching:

improve ME in soft-collinear regions (using Sudakov) and parton shower at large angles (using ME)



The issue: avoid double counting

Matched matrix elements

Matching procedures

- CKKW: separate ME&PS domain using a clustering variable [Catani et al. '01]
- MLM: match parton to jet, no modification to the shower (simple) [Mangano '02]
- others (CKKW-L, Pseudo-shower...)

Matched matrix elements

Matching procedures

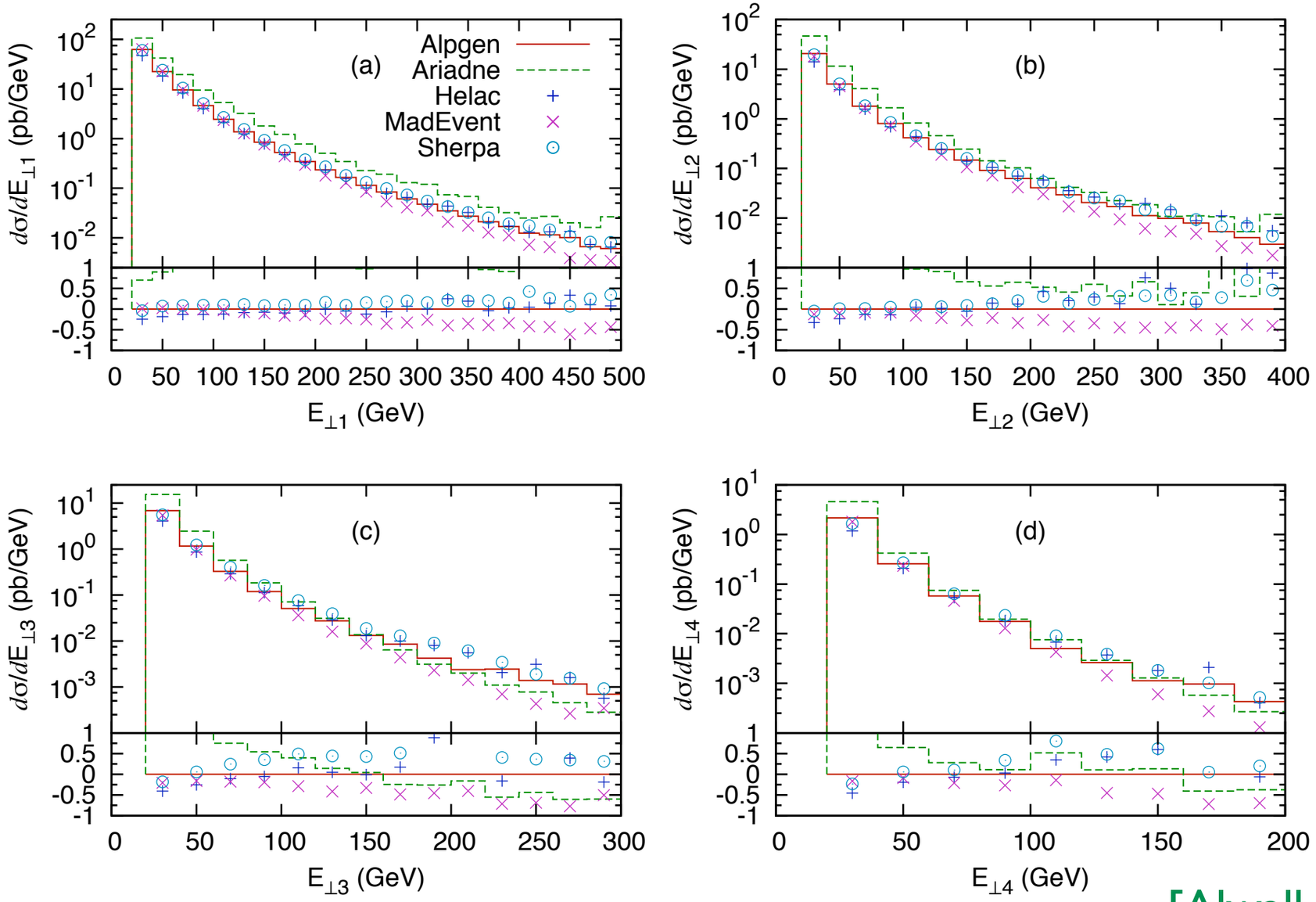
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Altogether

- different matrix element calculation
- different parton shower
- different matching procedure

Agreement?

Comparison of ME generators



[Alwall et al. '07]

➡ reasonable good agreement

➡ systematics at Tevatron ~ LHC

☛ tune codes to Tevatron and give consistent predictions for LHC

Leading order

Status: fully automated, edge around outgoing 8 particles

Alpgen, CompHEP, CalcHEP, Helac, Madgraph, Helas, Sherpa, Whizard, ...

Drawbacks of LO: large scale dependences, sensitivity to cuts, poor modeling of jets, ...

Example: W+4 jet cross-section $\propto \alpha_s(Q)^4$

Vary $\alpha_s(Q)$ by ±10% via change of Q ⇒ cross-section varies by ±40%

Why use LO at all?

- fastest option; often the only one
- test quickly new ideas with fully exclusive description
- many working, well-tested approaches
- highly automated, crucial to explore new ground, but no precision

Further improuvements

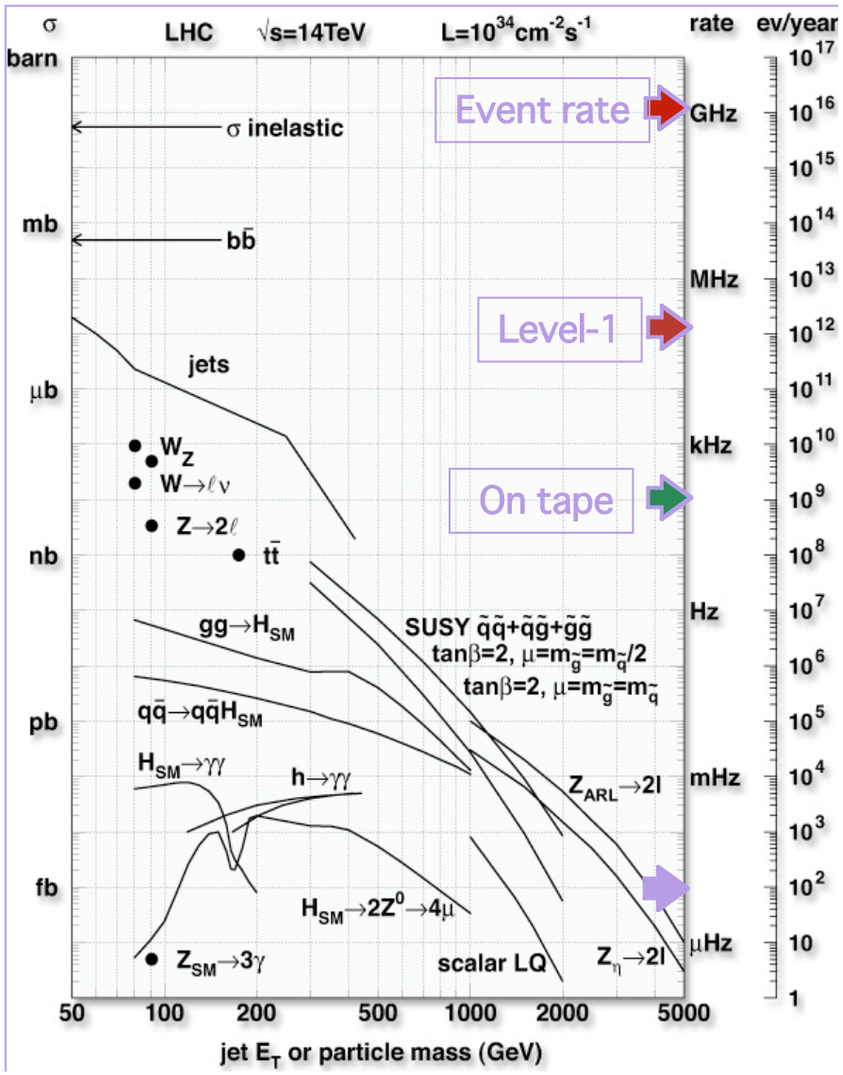
NLO + parton shower:

tomorrow

Few references

- Standard parton showers
  - Herwig++ http://www.hep.phy.cam.ac.uk/theory/Herwig++
  - Pythia 8 http://www.thep.lu.se/~torjorn
- Matrix element generators
  - Sherpa http://www/sherpa-mc.de
  - Isajet http://www.phy.bnl.gov/~isajet
  - Ariadne http://www.thep.lu.se/~leif/ariadne
  - Madgraph http://madgraph.hep.uiuc.edu/
  - Alpgen http://mlm.home.cern.ch/mlm/alpgen/
- Monte Carlo + NLO
  - MC@NLO http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO
  - PowHeg http://moby.mib.infn.it/~nason/POWHEG/

Cross sections



Final state	$\sim \sigma$
Total	100 mb
$W \to e\nu$	20 nb
$E \to e^+e^-$	2 nb
$b\bar{b}$	0.8 mb
$t\bar{t}$	800 pb
$H(m_H = 200\,\mathrm{GeV})$	20 pb

What is the bulk of the total cross-section made of?

Soft interactions

We talked a lot about high-energy scatterings, but what is the most likely thing which can happen when two protons collide at very high energy?

- most of the times, there will be only a low p_t momentum transfer between the partons in the protons

- only occasionally there will be a hard momentum transfer resulting in a hard interaction (outgoing jets at high p_t)

Perturbative QCD can describe hard interactions, but not the soft physics

What we can do is model (parametrize) soft effects, and fit them from data
$\Rightarrow$ Monte Carlo tuning

Some nomenclature

Minimum bias:

- event which one would see with a totally inclusive trigger
- a single inelastic particle-particle (proton-proton) interaction (predominantly dominantly soft)
- on average low transverse momentum, low multiplicity
- many minimum-bias events per bunch crossing at the LHC

Pile-up:

- many additional, generally soft proton-proton interactions

Nomenclature

The underlying event:

- all particle from a single particle collision, except the hard process of interest

  [beam remnant, initial state radiation, multi-parton interactions, minimum bias ...]
- an important area of physics, which will affect all LHC measurements of which we have still a very poor understanding and no first principle calculation

All this soft activity (additional energy) has nothing to do with the hard process $\Rightarrow$ needs to be subtracted

Soft underlying event in standard Herwig

The UA5 model: (herwig default for a long time)

Additional soft hadronic activity generated as a number of clusters distributed flat in rapidity and with exponential transverse momentum distribution

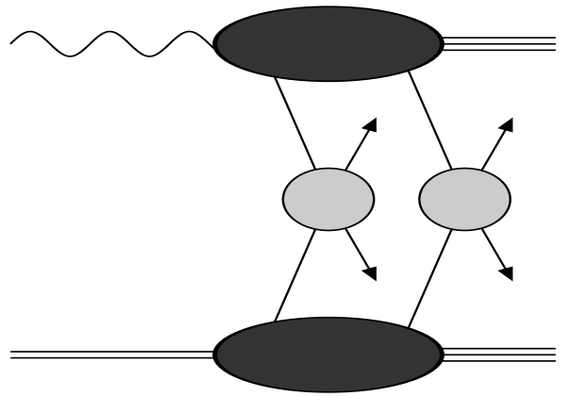
$$\mathrm{Prob}(p_t) \sim p_t e^{-b\sqrt{p_t^2+M^2}}$$

No matrix element, no physical model and practically too soft to fit data

UA5 soft underlying event obsolete: not recommended for serious use

Underlying event in Jimmy

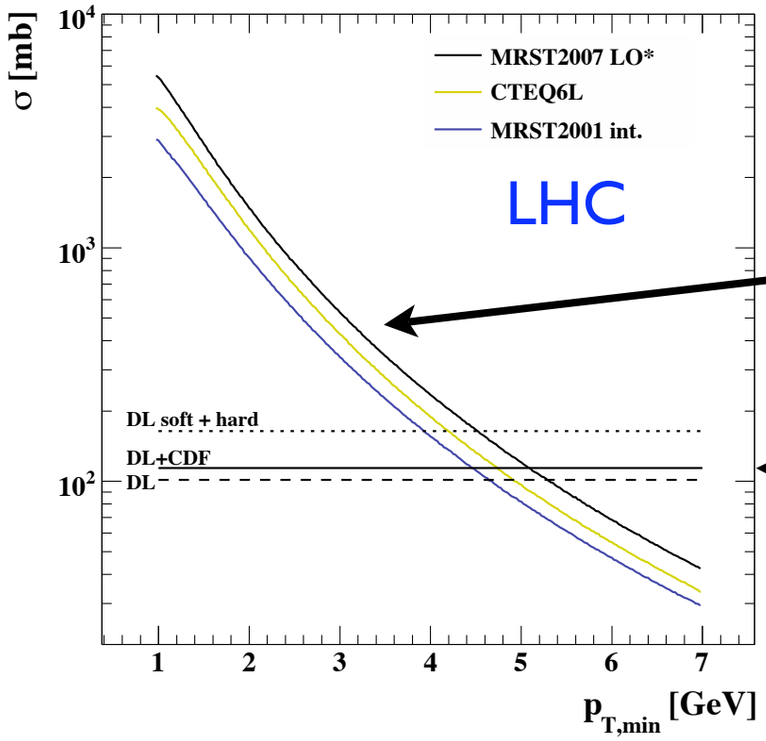
Jimmy is a plug-on to Herwig with a better treatment of the hard part of the U.E.



Issues:

- at high energies probe low-x PDFs
- the gluon PDF grows at small x
- if the parton density grows, it is reasonable to assume that more than one hard event per collision can take place: multi-particle interactions (MPI)
- this assumption is also necessary to unitarize the cross-section

Problems with unitarity



total "hard" cross-section assuming one parton-parton per pp collision

total cross section (few models)

$\Rightarrow$ assumption of one parton-parton per pp collision leads to inconsistency

- without MPI: cross-section for inclusive jet-production (computed in PT with steep PDFs) exceeds the total (γp, pp) cross-section
- with MPI: inclusive jet-cross section exceeds total cross-section by a factor corresponding to the mean multiplicity of MPI

Standard Jimmy model

Assumption:

- At fixed impact parameter b, scatters are independent and obey Poisson statistics (eikonal model)

$$\sigma_n = \int d^2b \frac{\left(A(b)\sigma_a\right)^n}{n!} e^{-A(b)\sigma_a}$$

normalized matter density

parton-parton cross-section

Inclusive cross-section is

$$\sigma_{\rm inc} = \sum_{n=0}^{\infty} \int d^2b \, n \, \frac{\left(A(b)\sigma_a\right)^n}{n!} e^{-A(b)\sigma_a} = \sigma_a$$

Standard Jimmy model

Total cross-section with at least one scatter of type a is

$$\sigma_{\mathrm{tot,a}} = \sum_{n=1}^{\infty} \int d^2b \frac{(A(b)\sigma_a)^n}{n!} e^{-A(b)\sigma_a} = \int d^2b \left(1 - e^{-A(b)\sigma_a}\right)$$

Probability of n scatters given that there is at least one

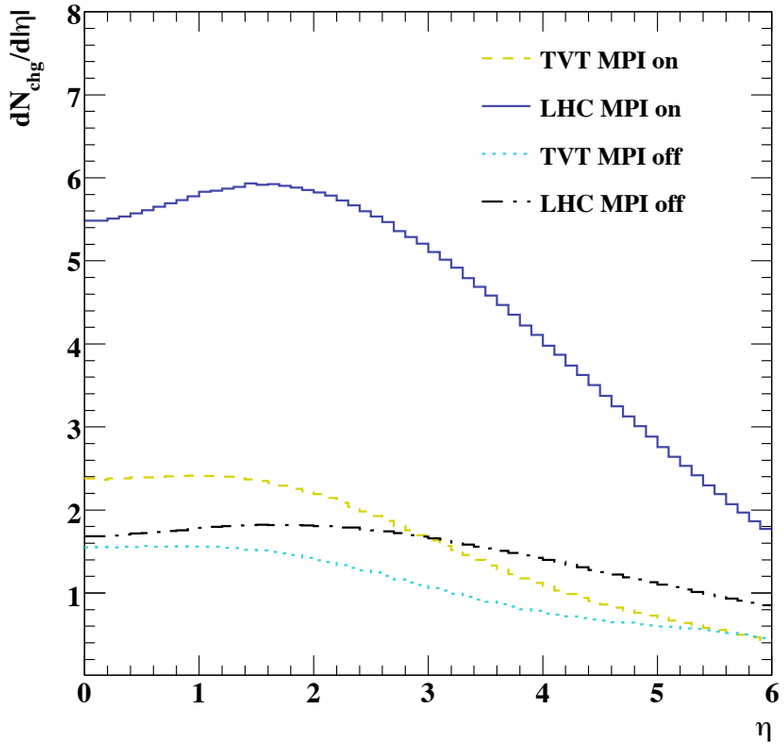
$$P_{n|1} = \frac{\int d^2b \frac{(A(b)\sigma_a)^n}{n!} e^{-A(b)\sigma_a}}{\int d^2b (1 - e^{-A(b)\sigma_a})}$$

Pre-tabulated probability distribution (as a function of s) in Jimmy. Then in a given event n is chosen according to $P_{n|1}$

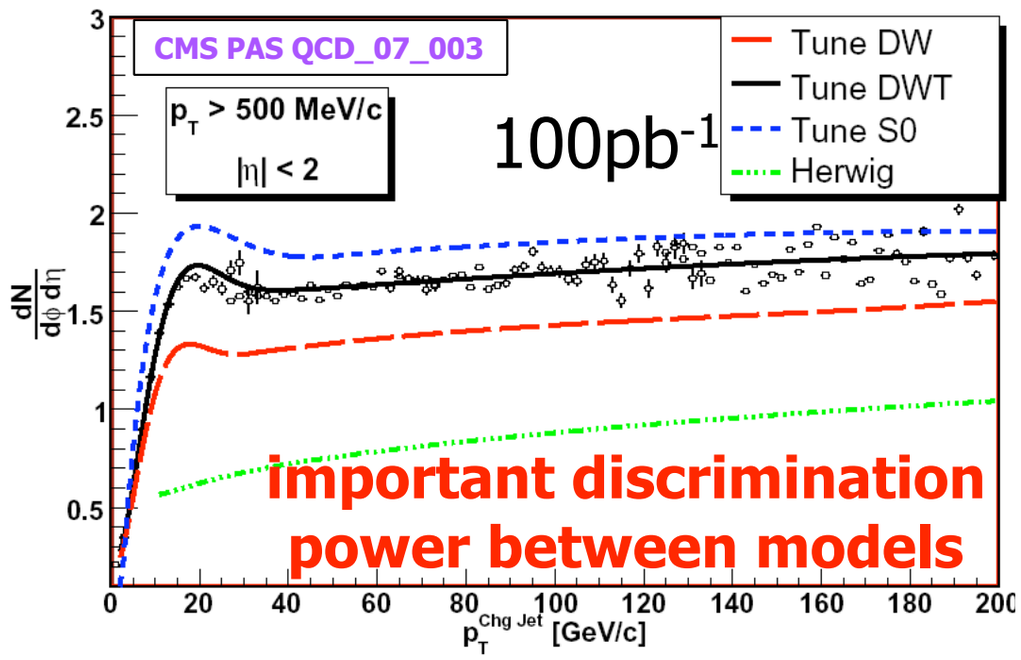
Exercise: show that $\langle n \rangle = \dfrac{\sigma_{\mathrm{inc}}}{\sigma_{\mathrm{tot,a}}}$

N.B. $\sigma_{\mathrm{tot,a}}$ must be less than the total cross-section, but σ_{inc} must not be

MPI: TeV vs LHC



Very large effects at the LHC



Current UE models tuned at the Tevatron give different extrapolations at the LHC

Models based on physical ideas, but a lot of assumptions and extrapolations behind, only data can help constraining from and parameters

Hadronization

Partons produced in a hard scattering loose energy via perturbative radiation, then they will pick the flavour and color form the vacuum so as to create an observable hadron

Simplest example: consider b-hadro production. The inclusive jet spectrum of b-flavoured hadrons is given by

$$\frac{d\sigma_{p_i p_j \to H_b}}{dp_t(H_b)} = \int \frac{dz}{z} D^{b\to H_b}(z) \frac{d\sigma_{p_i p_j \to b}}{dp_t(b)} \qquad p_t(H_b) = z p_t(b)$$

- Fragmentation functions $D^{Q\to H_Q}(z)$ are analogous to PDFs, they can not be computed but are extracted form data (typically in e^+e^-) and are universal
- As for PDFs the functional form is unknown. The parameterization if often a large source of uncertainty which is difficult to estimate

Recap

- higher orders: included only approximately
- parton evolution as branching process from higher to lower x
- parton shower based on Sudakov form factor (Prob. of evolving without branching) with corresponding evolution equation
- branching described by picking randomly 3 numbers (t, x, φ) with the right prob. distributions
- virtuality ordered shower: collinear enhancements
- angular ordering needed for soft enhancements
- parton shower supplemented by hadronization + U.E. (various models $\Rightarrow$ MC tuning) $\Rightarrow$ full event generator
- by construction PS fail to describe multiple hard radiation
- improvements
  - exact matrix elements matched to parton shower (still LO only)
  - NLO + parton shower