Quantum Chromodynamics

& LHC Phenomenology

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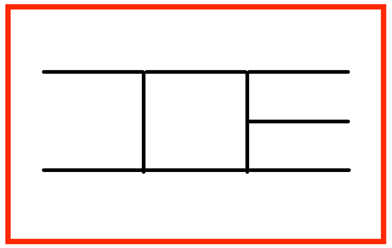
Oxford Theoretical Physics & STFC

2008 CERN-Fermilab Hadron Collider Physics Summer School

Outline

Fourth lecture

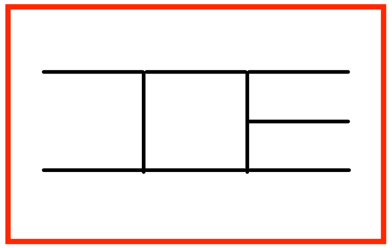
- Fixed (higher) order calculations (NLO & NNLO)
  - bottlenecks, current status
  - sketch of modern techniques for computation of scattering amplitudes
- Jets
  - jet definitions
  - infrared safety
  - applications (jet area, pile-up subtraction, quality measures, jet-substructure)



Benefits of NLO

<u>Last lecture:</u>

leading order only qualitative, very large uncertainties, no precision



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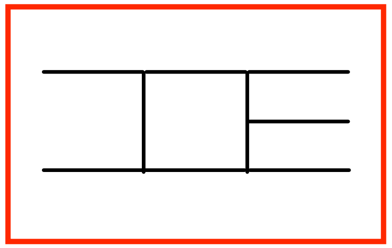

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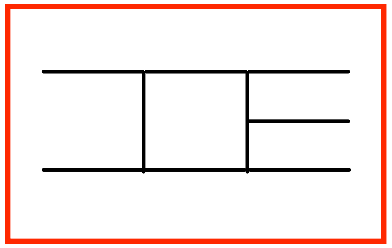
- establish normalization and shape of cross-sections



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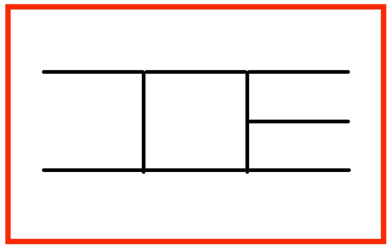

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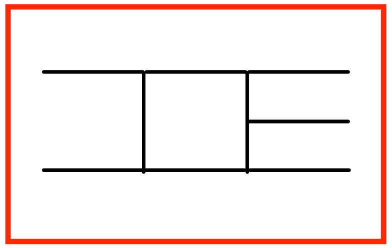

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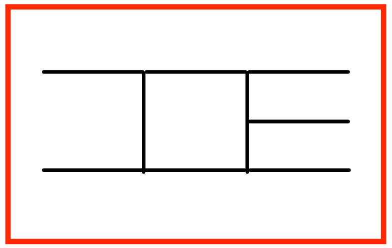
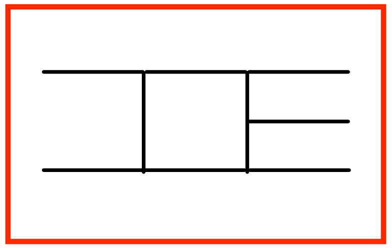
- get indirect information about sectors not directly accessible (through loop effects)



Ingredients at NLO

A full N-particle NLO calculation requires:



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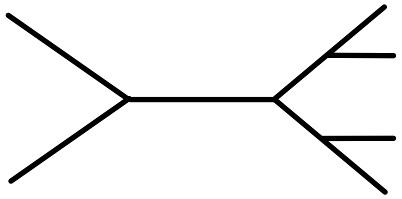
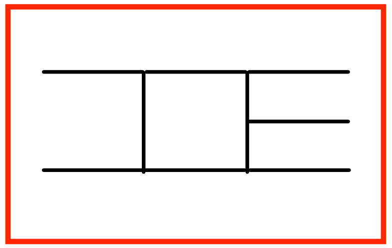
- tree graph rates with N+1 partons
  → soft/collinear divergences





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A full N-particle NLO calculation requires:

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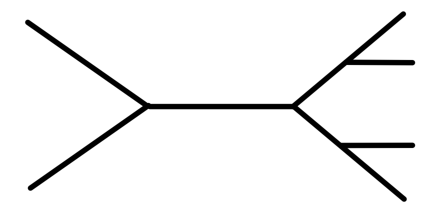
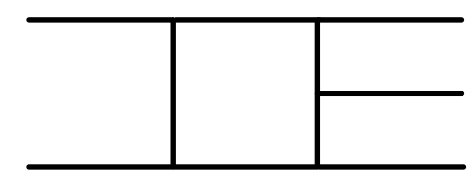
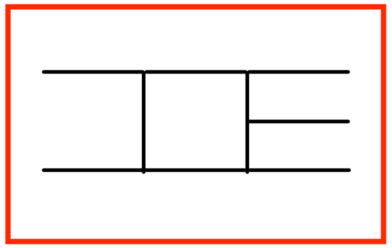
- virtual correction to N-leg process
  → divergence from loop integration,
  use e.g. dimensional regularization







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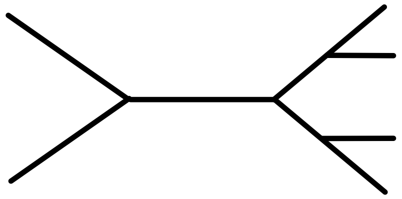
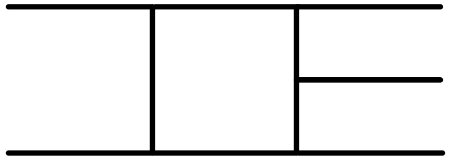
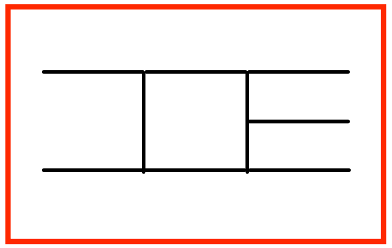
- set of subtraction terms







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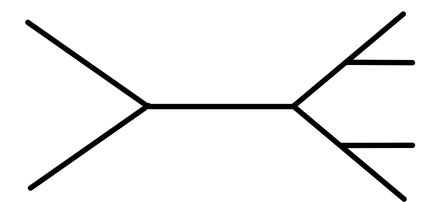

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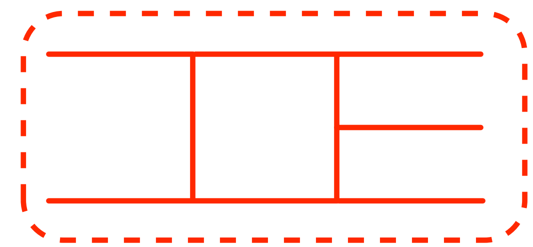
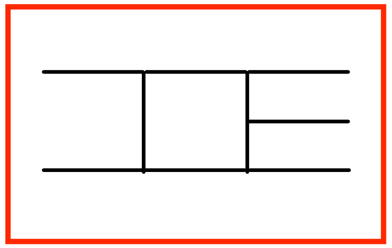


bottleneck

- set of subtraction terms

Gleisberg, Krauss '07
TevJet [public] Seymour Tevlin '08
Hasegawa, Moch, Uwer '08

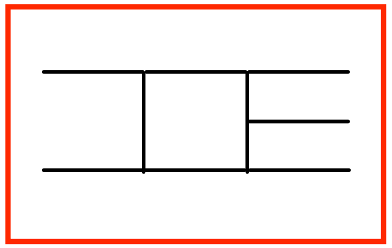


Approaches to NLO

Two complementary approaches:

- Numerical/traditional Feynman diagram methods:
  use robust computational methods [integration by parts, reduction techniques...], then let the computer do the work for you

  Bottleneck:
  factorial growth, $2 \to 4$ barely touched, very difficult to go beyond



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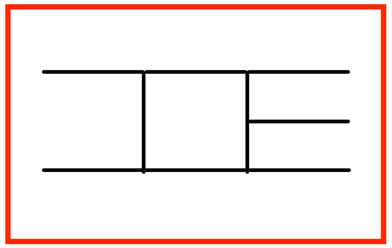
Very recently: unified approaches as a winning strategy ?



Status of NLO

Status of NLO:

☑ 2 → 2: all known (or easy) in SM and beyond

☑ 2 → 3: very few processes left

[but: often do not include decays, newest codes private]

☐ 2 → 4: barely touched ground. Not a single full cross-section calculation for the LHC

The 2005 NLO wish-list

Table 42: The LHC "priority" wishlist for which a NLO computation seems now feasible.

process ($V \in \{Z, W, \gamma\}$)	relevant for
1. $pp \to V\,V$ jet	$t\bar{t}H$, new physics
2. $pp \to t\bar{t}\,b\bar{b}$	$t\bar{t}H$
3. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
4. $pp \to V\,V\,b\bar{b}$	VBF$\to H \to VV$, $t\bar{t}H$, new physics
5. $pp \to V\,V + 2$ jets	VBF$\to H \to VV$
6. $pp \to V + 3$ jets	various new physics signatures
7. $pp \to V\,V\,V$	SUSY trilepton

The QCD, EW & Higgs Working group report hep-ph/0604120

The 2007 update

Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \to VV$jet	WWjet completed by Dittmaier/Kallweit/Uwer [3]; Campbell/Ellis/Zanderighi [4] and Binoth/Karg/Kauer/Sanguinetti (in progress)
2. $pp \to$ Higgs+2jets	NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi [5]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [6,7]
3. $pp \to V\,V\,V$	ZZZ completed by Lazopoulos/Melnikov/Petriello [8] and WWZ by Hankele/Zeppenfeld [9]
Calculations remaining from Les Houches 2005	
4. $pp \to t\bar{t}\,b\bar{b}$	relevant for $t\bar{t}H$
5. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$
6. $pp \to V\,V\,b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
7. $pp \to V\,V$+2jets	relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [10–12]
8. $pp \to V$+3jets	various new physics signatures
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	Higgs and new physics signatures
Calculations beyond NLO added in 2007	
10. $gg \to W^*W^*$ $\mathcal{O}(\alpha^2\alpha_s^3)$	backgrounds to Higgs
11. NNLO $pp \to t\bar{t}$	normalization of a benchmark process
12. NNLO to VBF and Z/γ+jet	Higgs couplings and SM benchmark
Calculations including electroweak effects	
13. NNLO QCD+NLO EW for W/Z	precision calculation of a SM benchmark

Table 1: The updated experimenter's wishlist for LHC processes

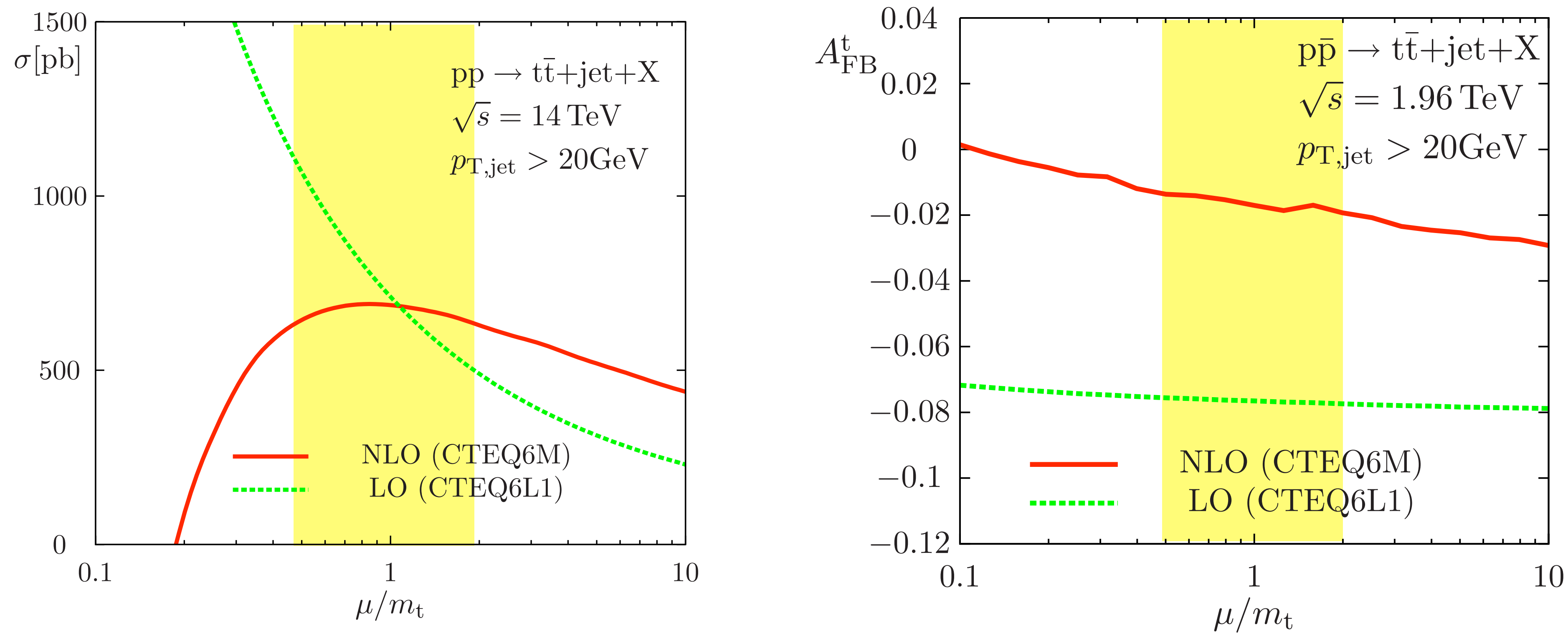
} based on Feynman diagrams; private codes only

The NLO multi-leg Working group report 0803.0494

One NLO example

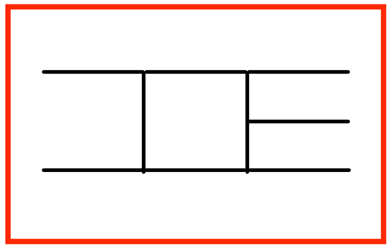
Calculation done with traditional methods

Dittmaier, Kallweit, Uwer ’07-’08



- improved stability of NLO result [but no decays]
- forward-backward asymmetry at the Tevatron compatible with zero
- essential ingredient of NNLO tt production (hot topic)

Also recently computed @ NLO: H+2j, VVV, WWj, ttZ, Wbb & many more

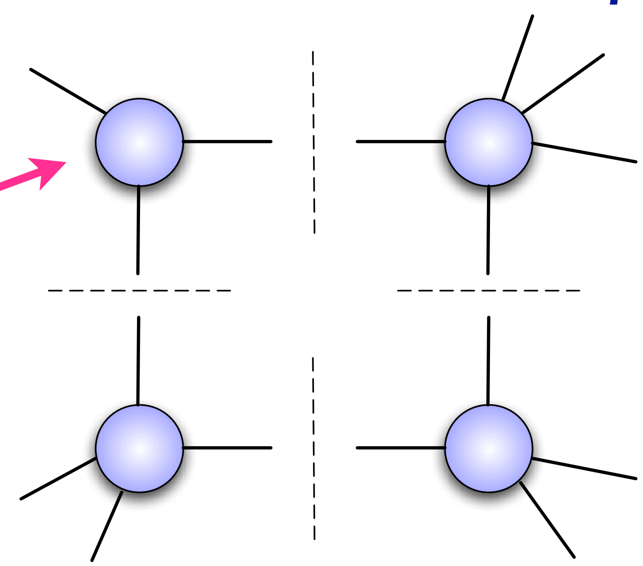


Two breakthrough ideas

Aim: NLO loop integral without doing the integration

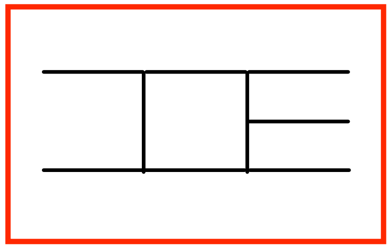
1) "... we show how to use generalized unitarity to read off the (box) coefficients. The generalized cuts we use are quadrupole cuts ..."

NB: non-zero because cut gives complex momenta



Britto, Cachazo, Feng '04

Quadrupole cut, i.e. 4 on-shell conditions on 4 dimensional loop momentum) freezes the integration. But rational part of the amplitude, coming from $D=4-2\varepsilon$ not 4, computed separately

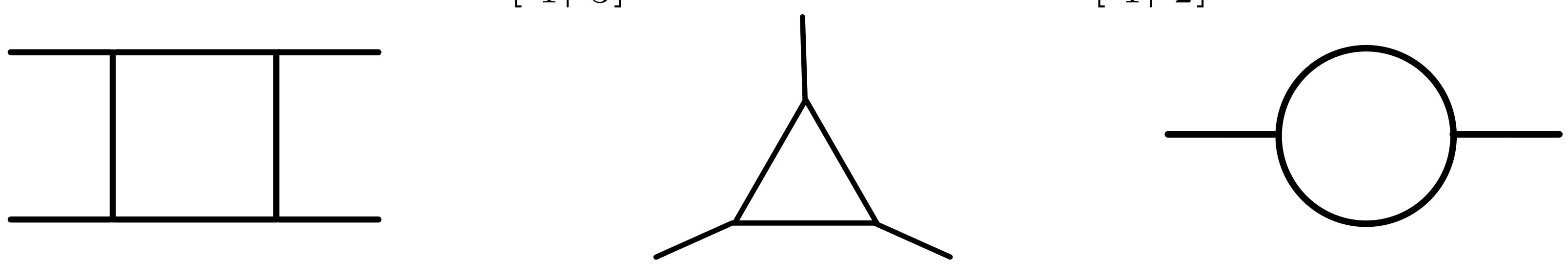


Two breakthrough ideas

Aim: NLO loop integral without doing the integration

2) The OPP method: "We show how to extract the coefficients of 4-, 3-, 2- and 1-point one-loop scalar integrals...."

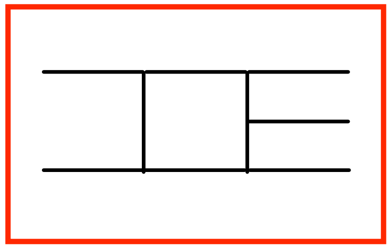
$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3} \; I^{(D)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2} \; I^{(D)}_{i_1 i_2} \right) + \mathcal{R}$$



rational part treated separately

Ossola, Pittau, Papadopolous '06

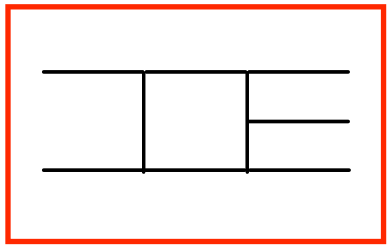
Coefficients can be determined by solving system of equations (no loops, no twistors, just algebra!)



A unified approach ?

Partial fractioning via OPP + generalized unitarity + BG recursion for tree amplitudes + unitarity in integer higher dimension
⇒ *full one-loop from tree level*

Ellis et al. '07, '08



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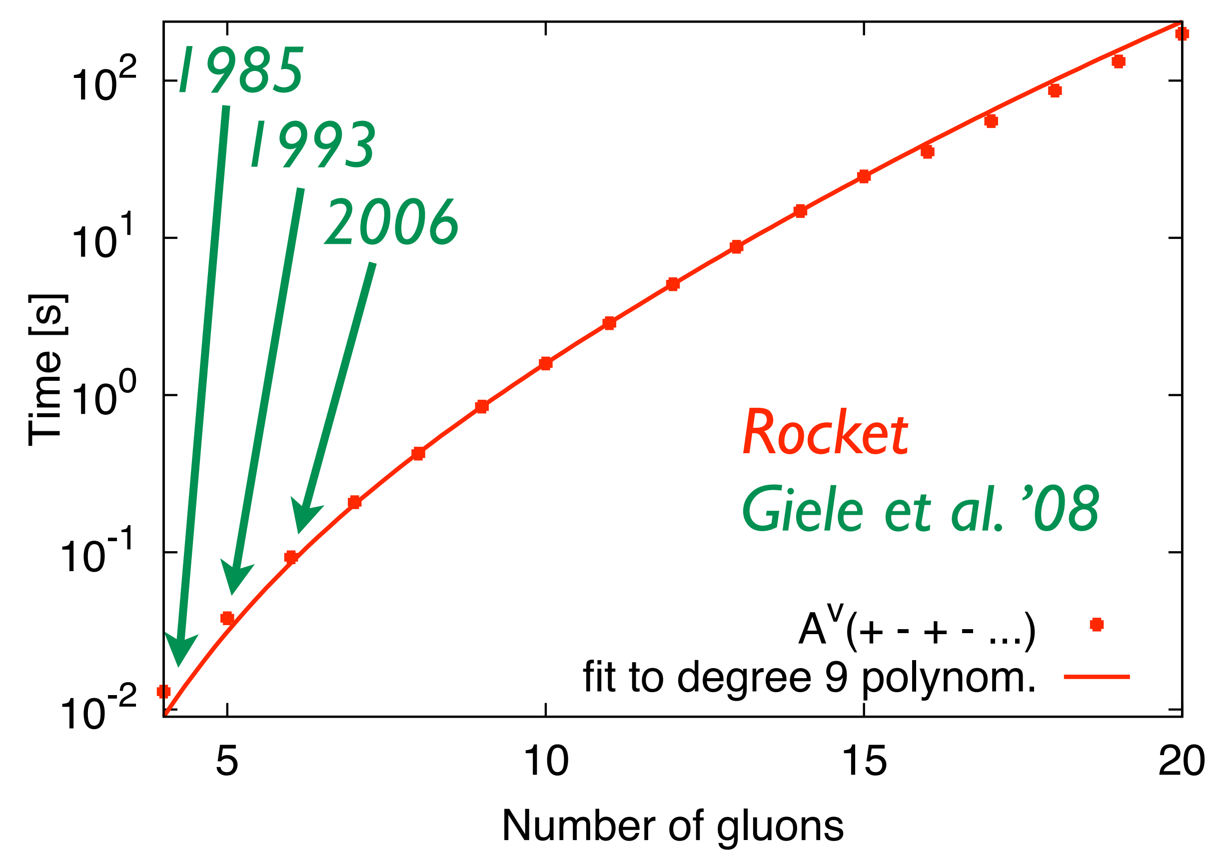
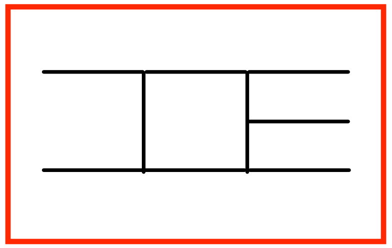
- Practicality? speed, stability

  Excellent performance of the method demonstrated for gluons



Also: Blackhat up to N=6 [7,8 MHV], *Berger et al. '08*



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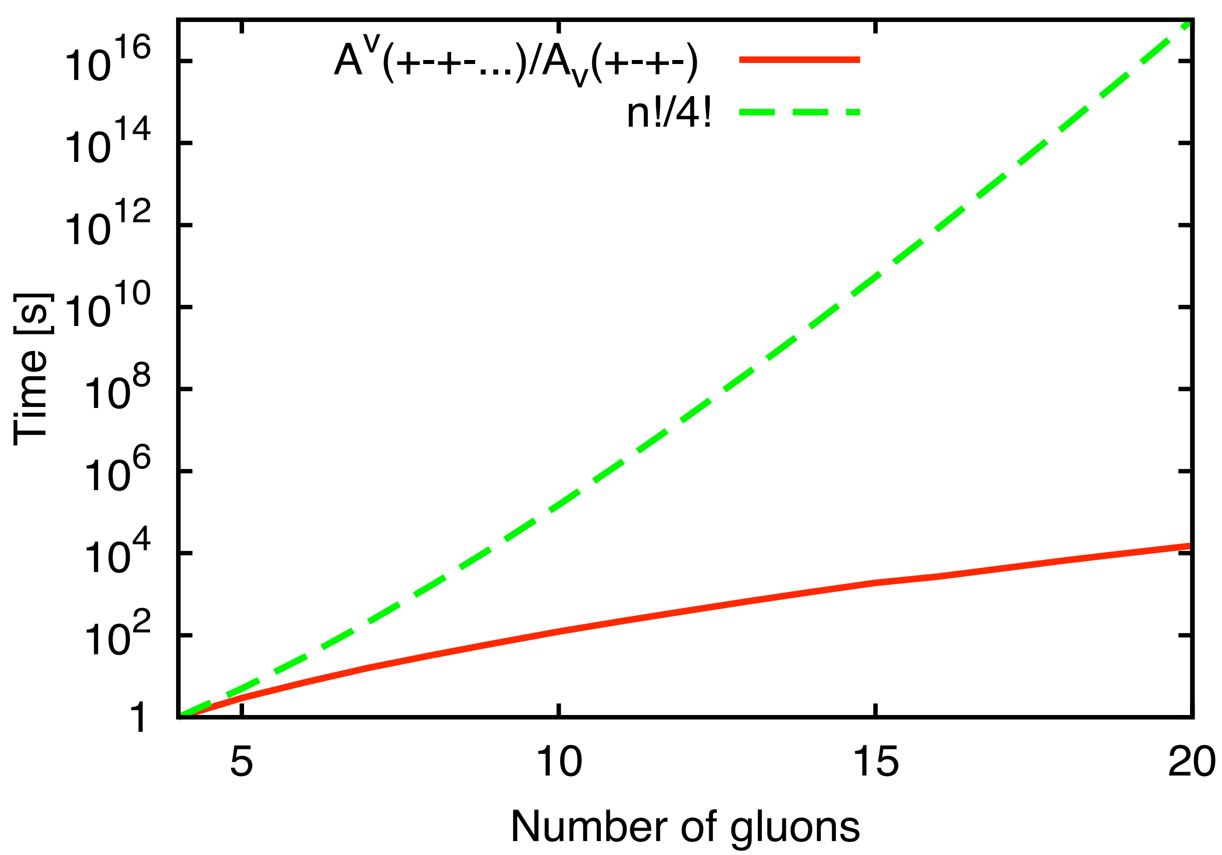

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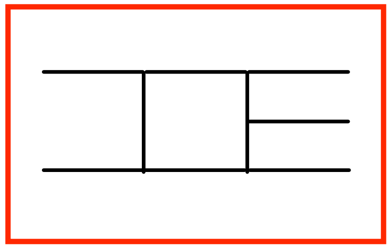

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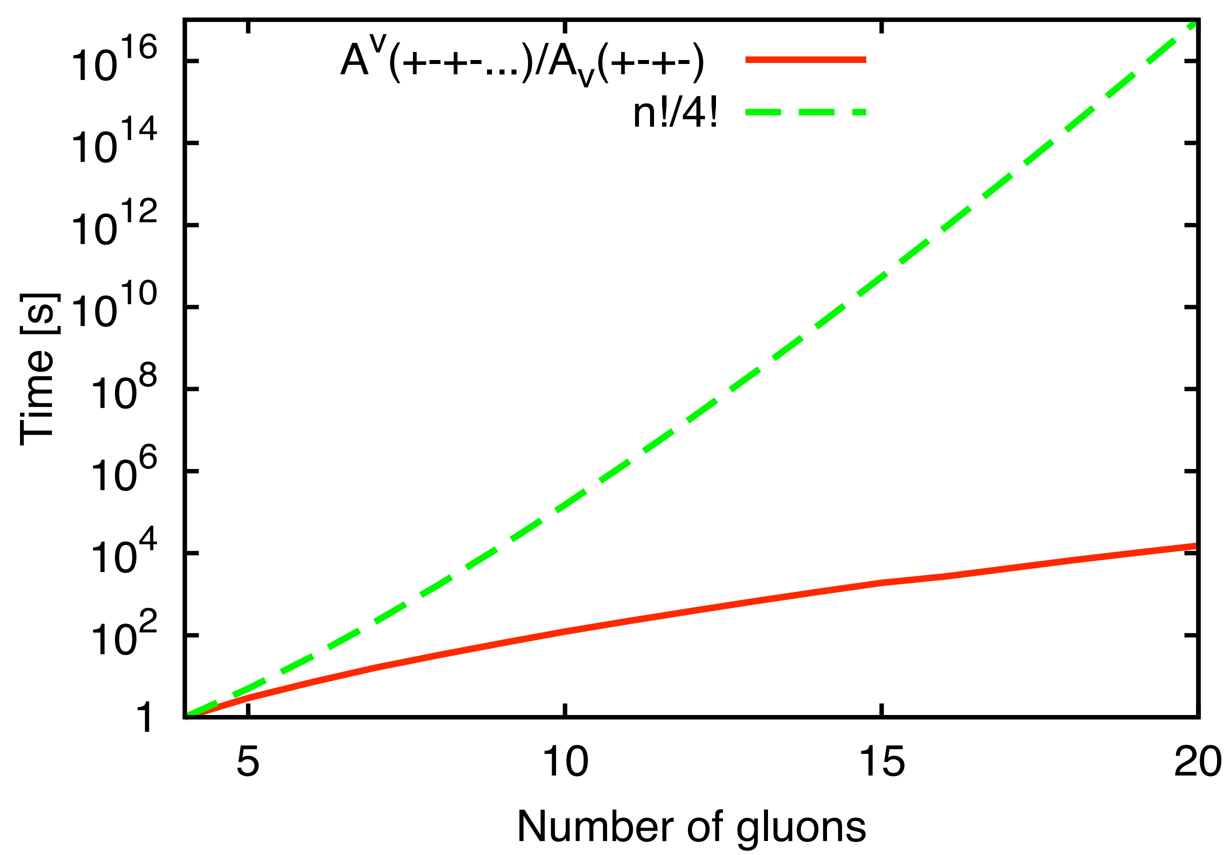


- Generality? what about realistic LHC processes?

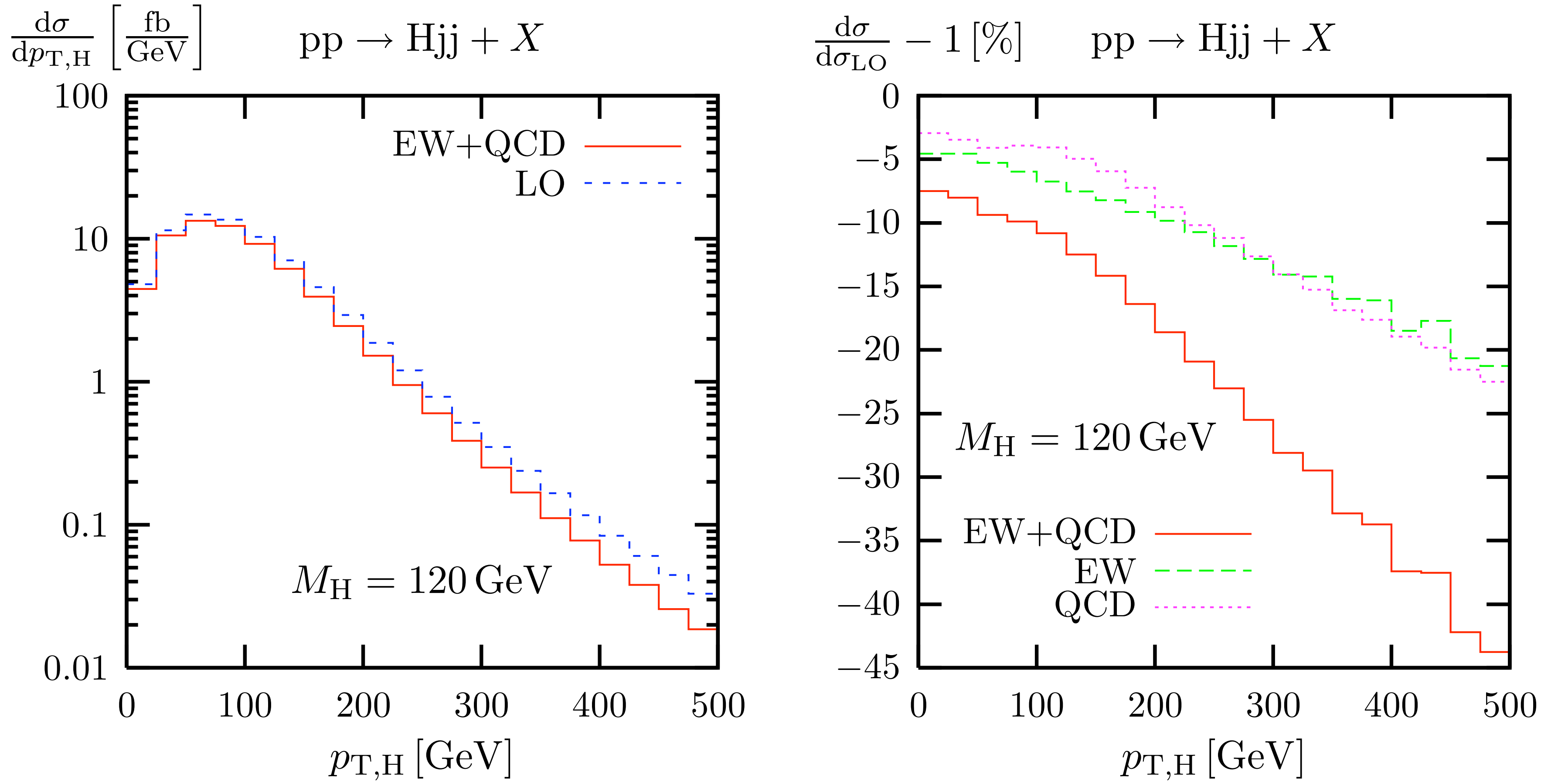
  First case studied:

  $gg \to ttg$, $qq \to Vgg(g)$

Also: Blackhat up to N=6 [7,8 MHV], *Berger et al. '08*

The "not so weak" EW

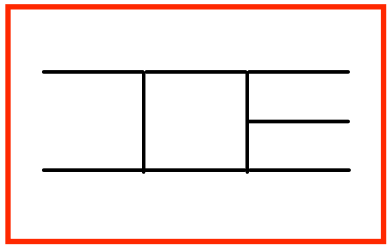
<u>One example:</u> Vector boson fusion Higgs production



Ciccolini, Denner, Dittmaier '07

☛ EW and QCD of the same size, both distort shapes!
Be aware of EW corrections for precision studies [peaks] and in tails of distributions [large EW logarithms]

Also: mixed EW & QCD corrections to Higgs + dijets: Bredenstein, Hagiwara, Jager '08



NLO + parton shower

Combine best features:

Get correct rates (NLO) and hadron-level description of events (PS)
Difficult because need exact NLO subtraction and remove it from PS

Working LHC examples:

- MC@NLO
  - W/Z boson production
  - WW, WZ, ZZ production
  - inclusive Higgs production
  - heavy quark production
  - single-top

Frixione&Webber '02 and later refs.

- POWHEG
  - ZZ production
  - heavy quark production
  - W/Z production
  - Higgs, single top ... in progress

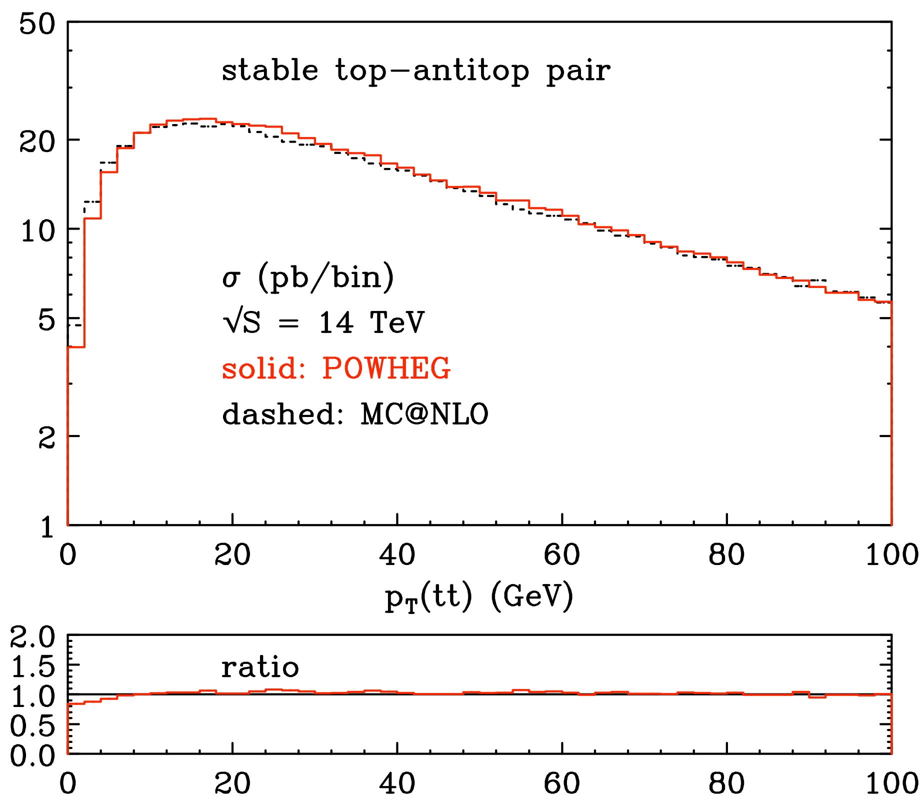
Nason '04 and later refs.

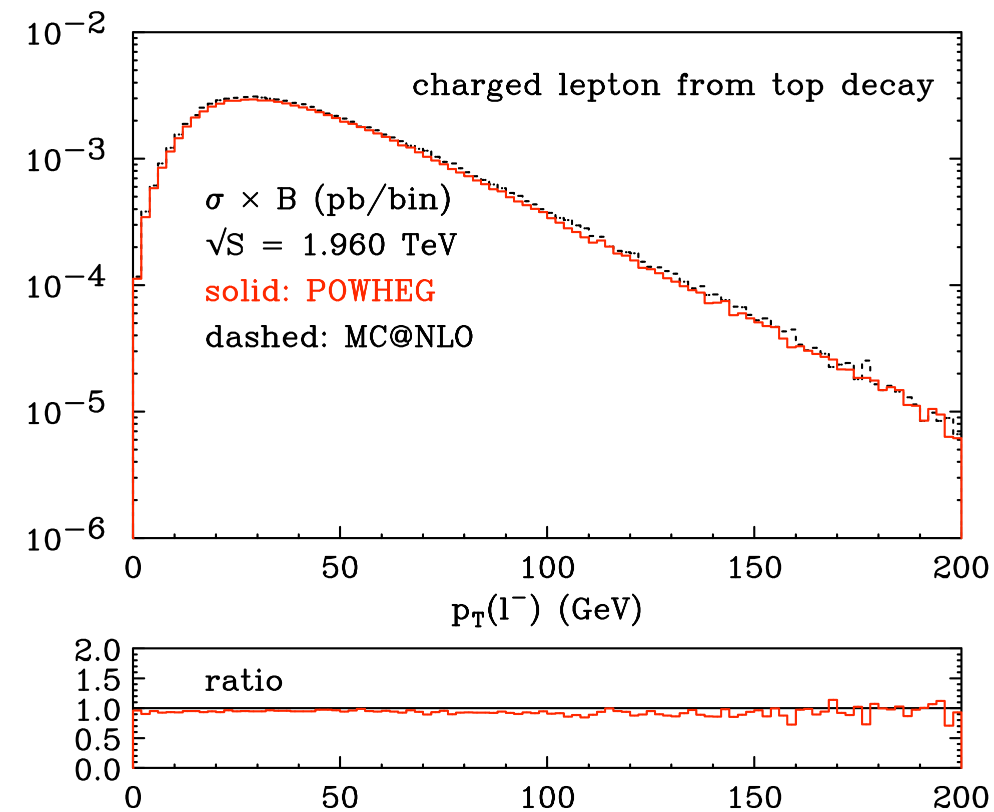
Other recent progress:

Shower with quantum inteference [Nagy, Soper], Geneve (SCET) [Bauer, Schwartz, Tackmann], Vincia (antenna factorization) [Giele et al.], Dipole factorization [Schumann]

MC@NLO vs PowHeg

Top pair production:





Frixione, Nason, Ridolfi '07

⇒ excellent agreement for all observables considered

(difference = different treatment of higher order terms)

NNLO: when is NLO not good enough?

- when NLO corrections are large (NLO correction ~ LO)

  This may happens when

  - process involve very different scales → large logarithms of ratio of scales appear
  - new channels open up at NLO (at NLO they are effectively LO)
  - master example: Higgs production

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Bottleneck: cancel divergences before numerical evaluation

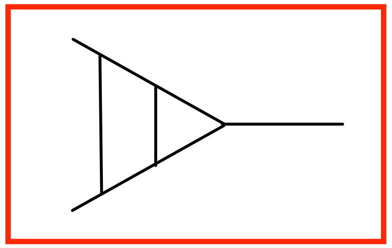
Collider processes known at NNLO

Collider processes known at NNLO today:

(a) Drell-Yan (Z,W)

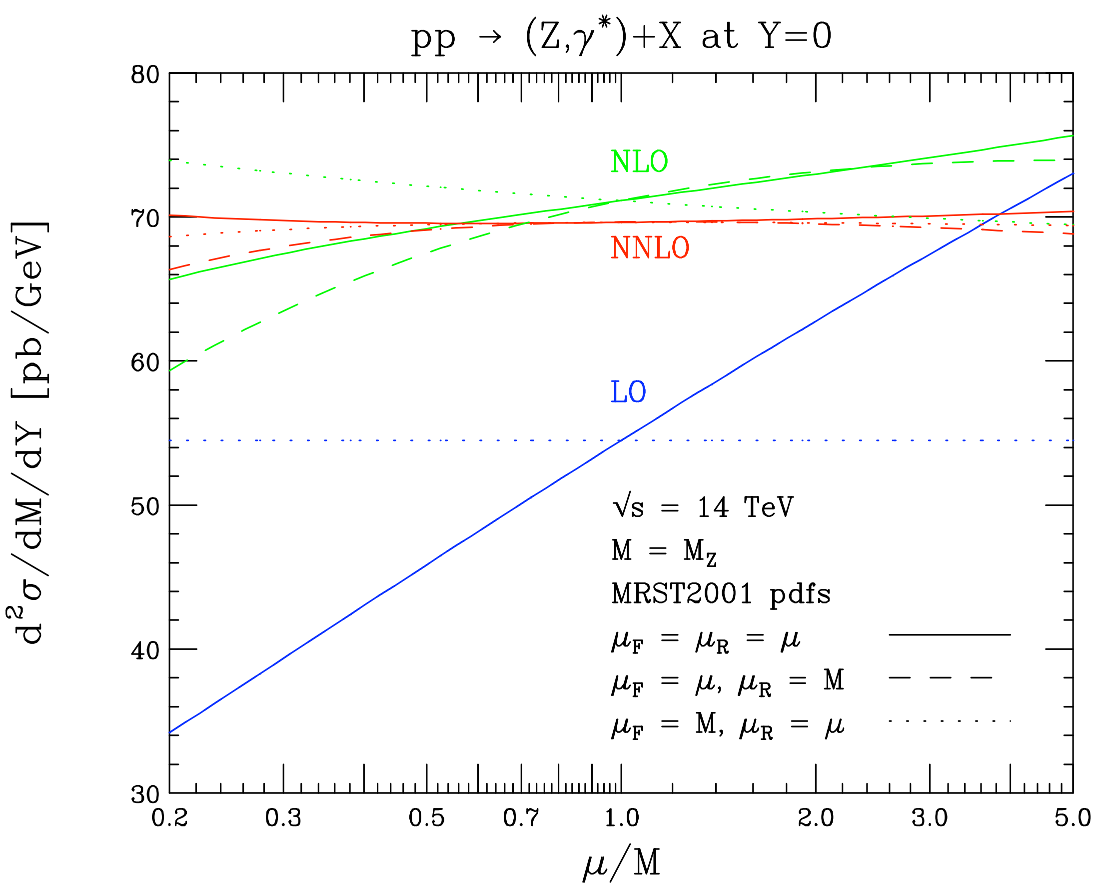
(b) Higgs

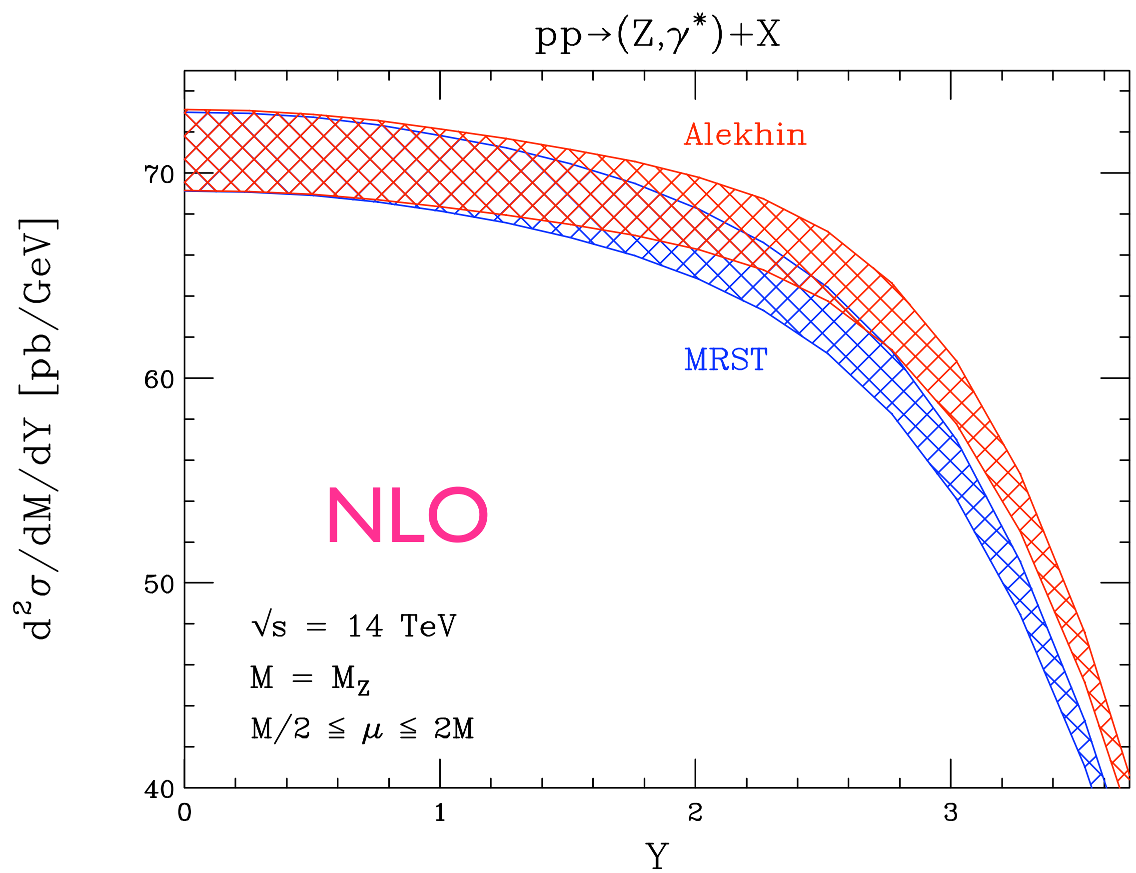
(c) 3-jets in e+e- ('07)



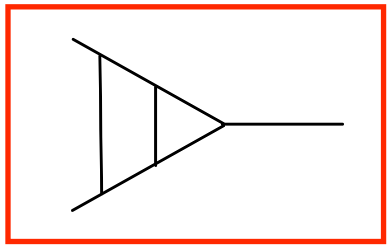
Best known at NNLO: Drell-Yan

- most important and most precise test of the SM at the LHC
- best known process at the LHC: spin-correlations, finite-width effects, γ-Z interference, fully differential in lepton momenta
- sample NNLO results: scale stability & sensitivity to PDFs



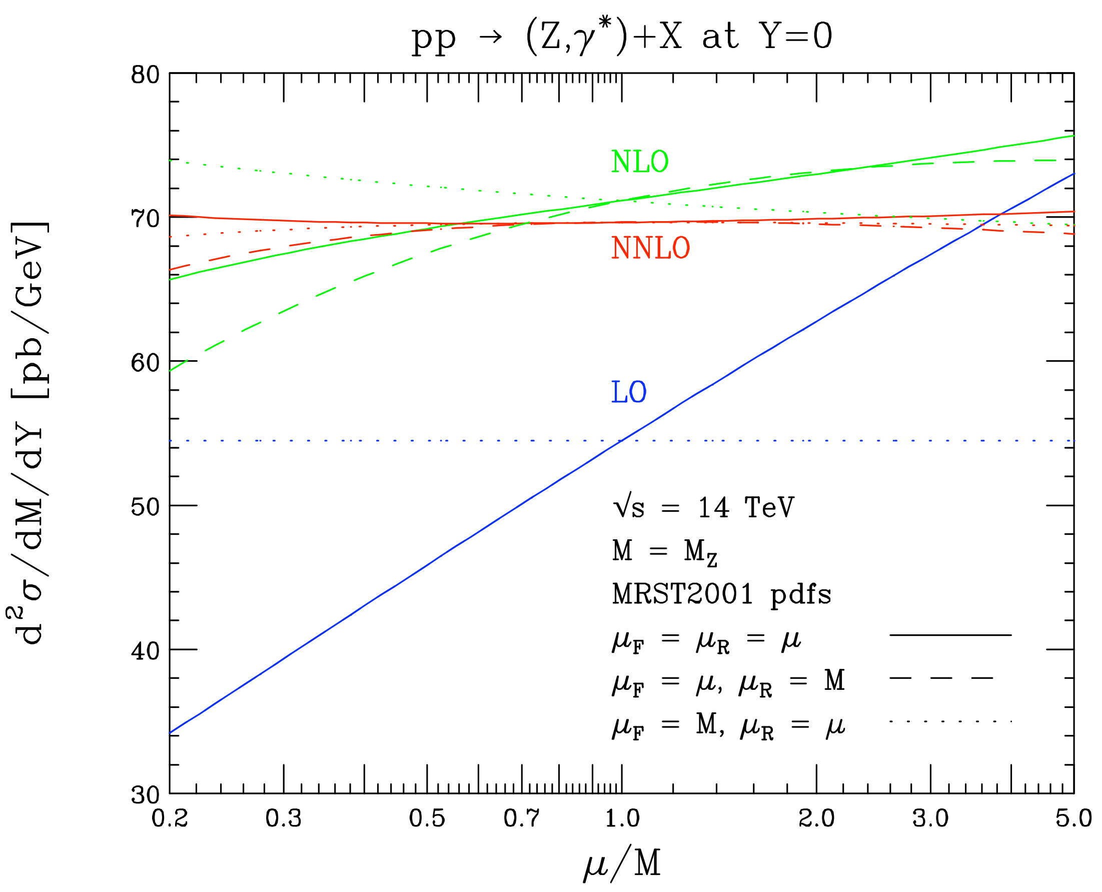


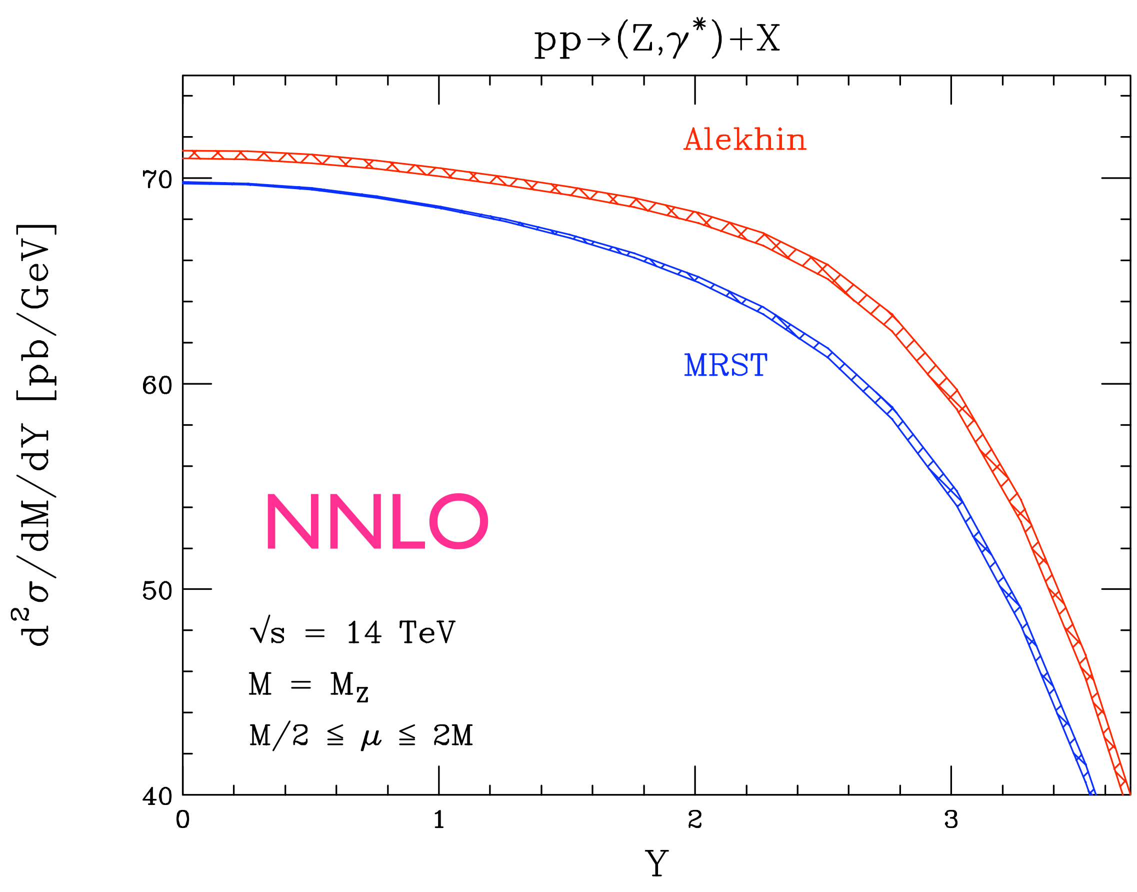
Anastasiou, Dixon, Melnikov, Petriello '03, '05; Melnikov, Petriello '06



Best known at NNLO: Drell-Yan

- Most important and most precise test of the SM at the LHC
- Best known process at the LHC: spin-correlations, finite-width effects, γ-Z interference, fully differential in lepton momenta
- Sample NNLO results: scale stability & sensitivity to PDFs





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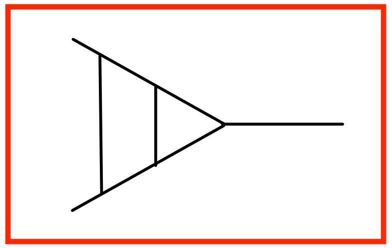
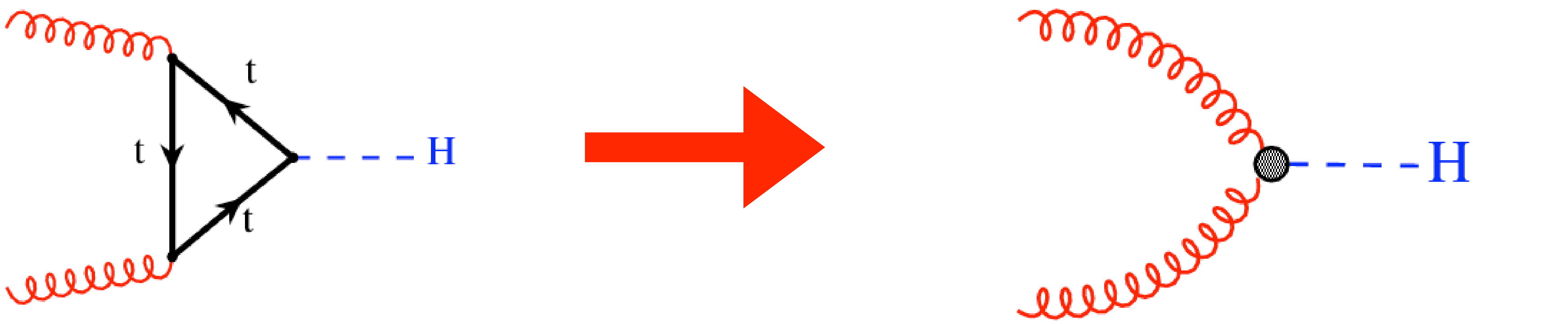
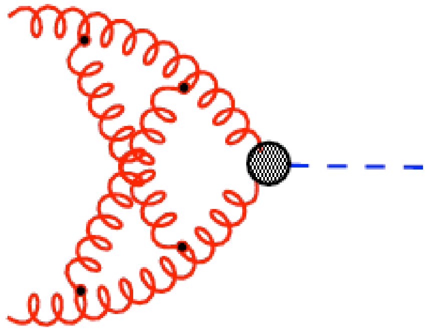


Inclusive NNLO Higgs production

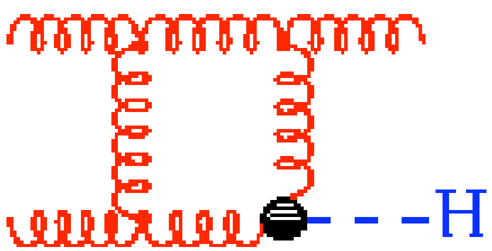
Inclusive Higgs production via gluon-gluon fusion in the large m_t-limit:



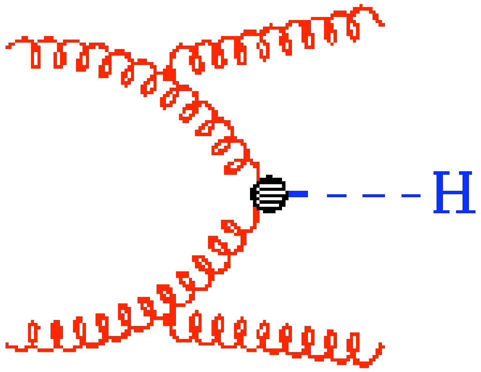
NNLO corrections knows since few years now:



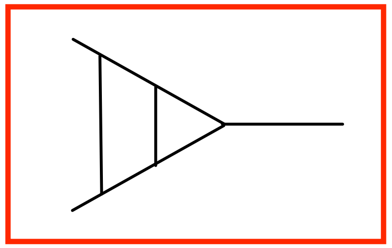
virtual-virtual



real-virtual



real-real



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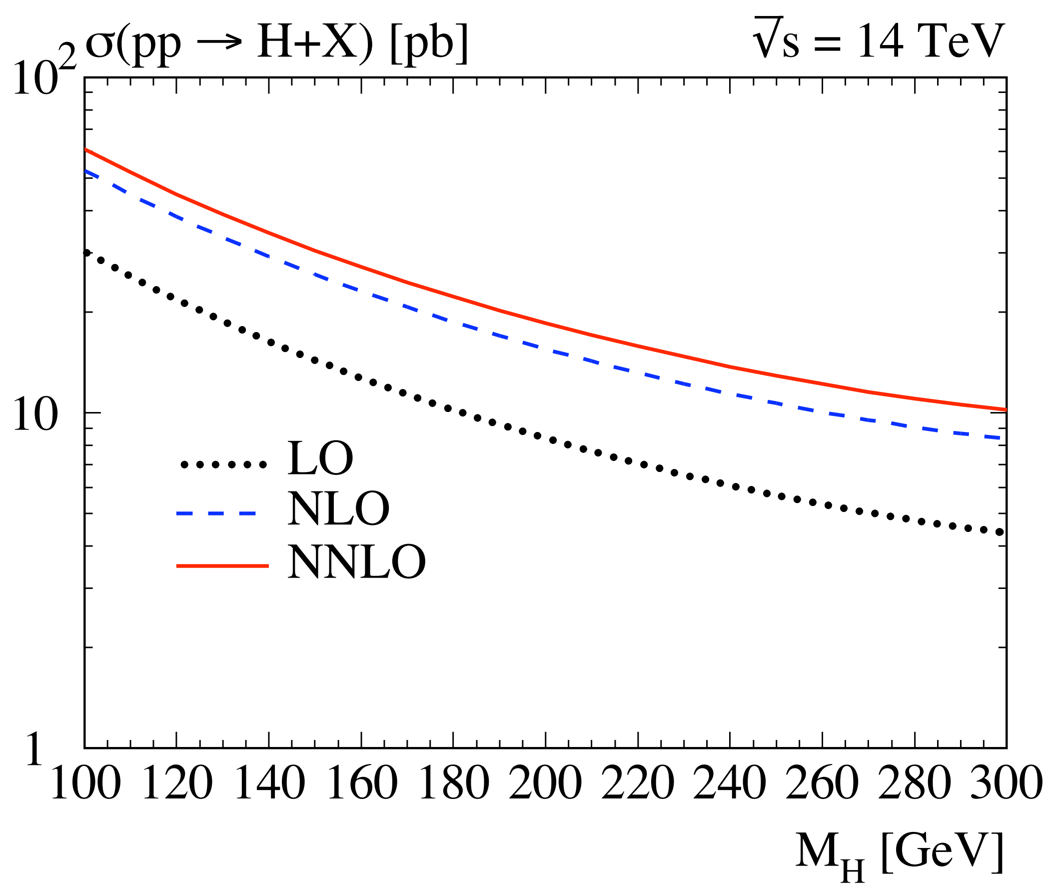
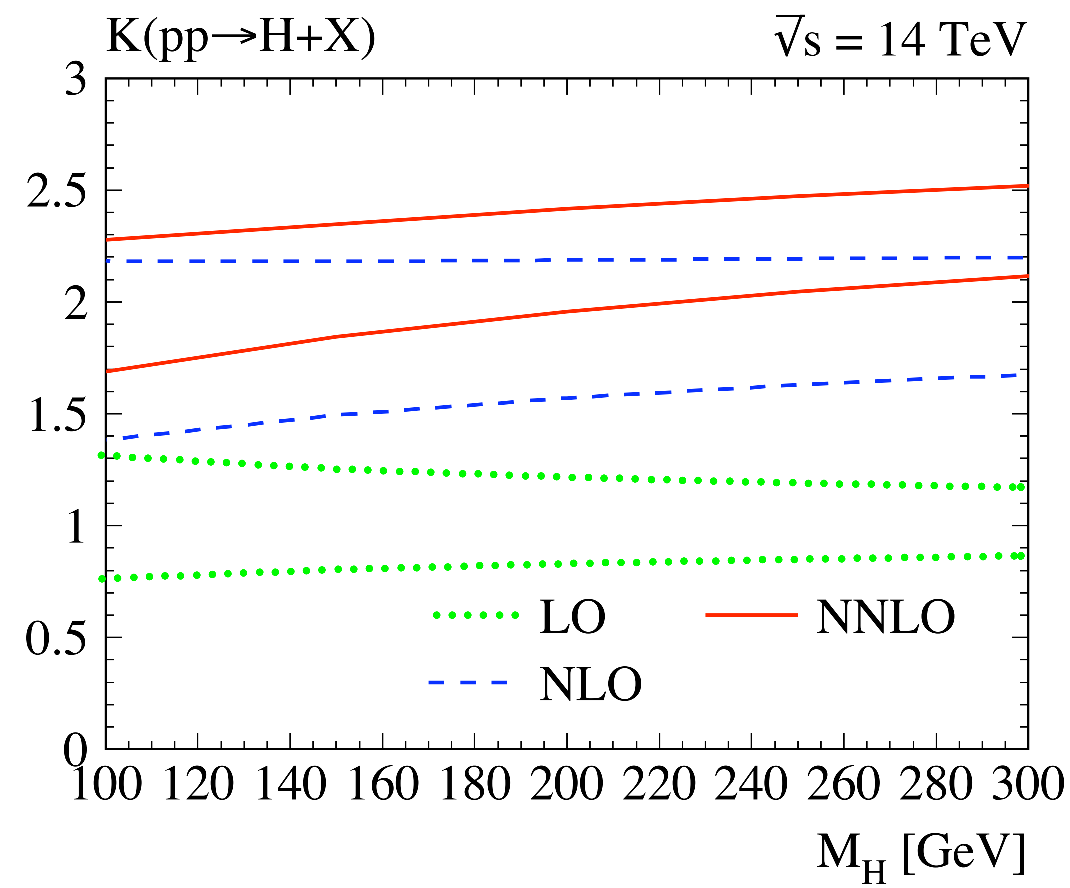
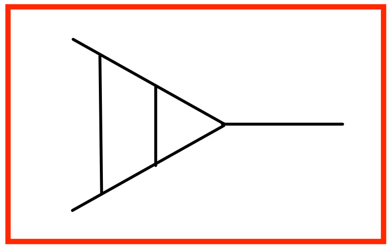


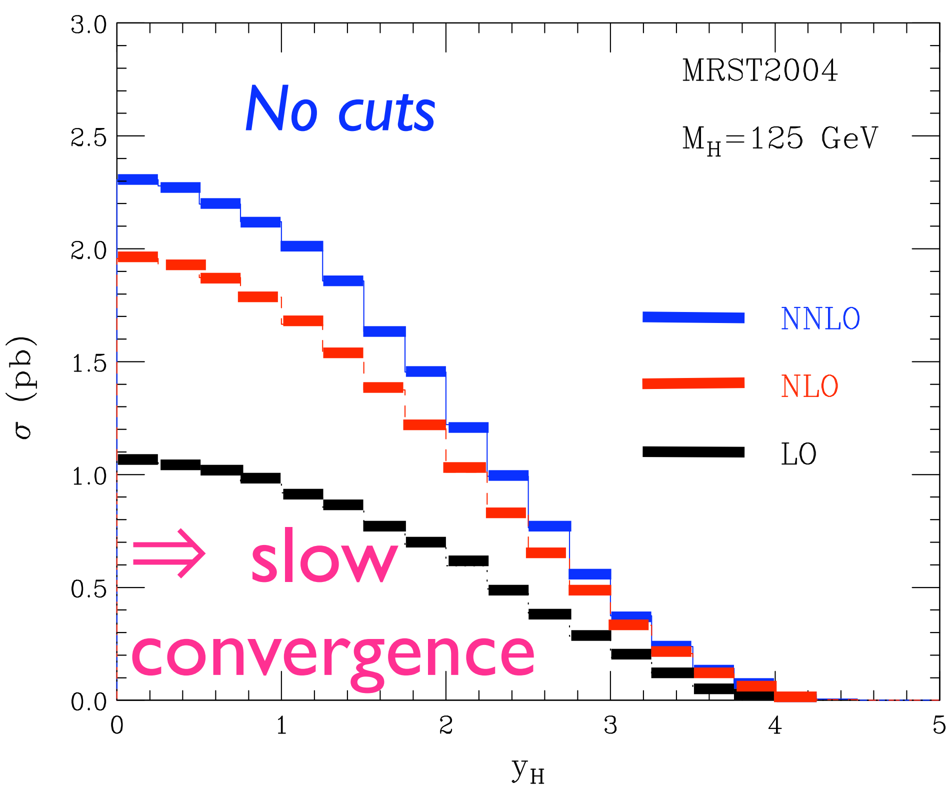


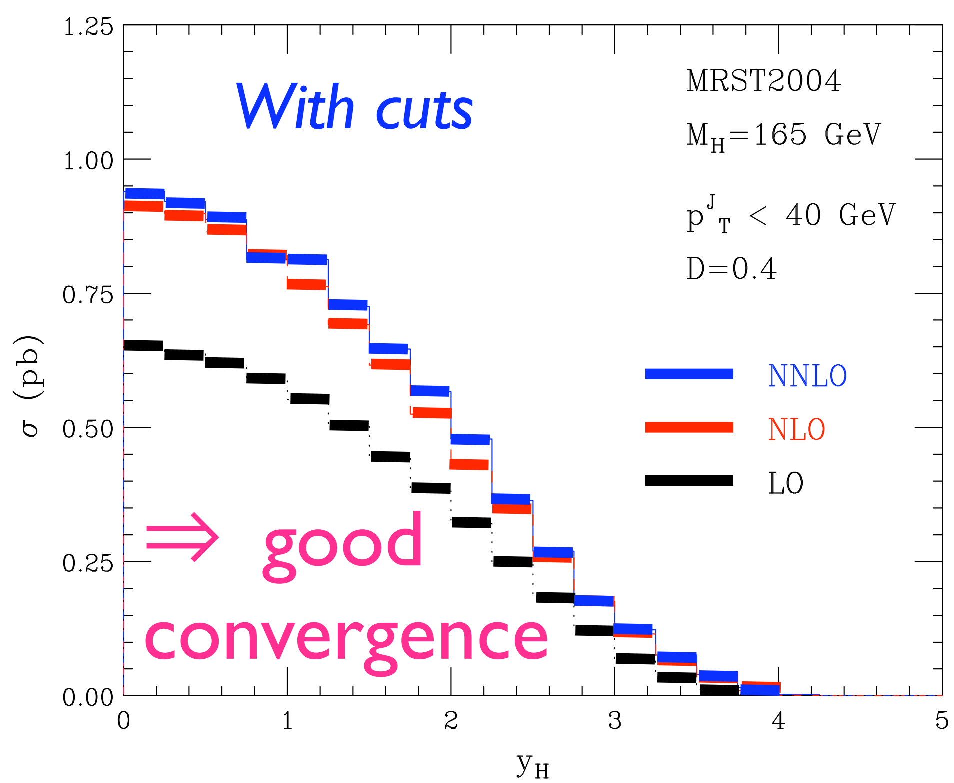
Kilgore, Harlander '02
Anastasiou , Melnikov '02



NNLO Higgs with H→2l 2ν, H→4l

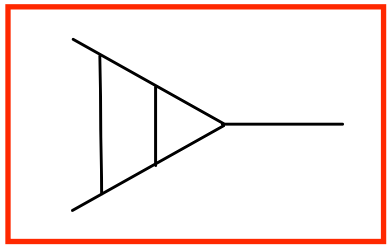
FEHIP, Anastasiou, Dissertori, Stoeckli '07
also: HNNLO Catani, Grazzini '08





⇒ impact of NNLO dramatically reduced by cuts!

Very important to include cuts and decays in realistic studies!



tt cross-section at the LHC

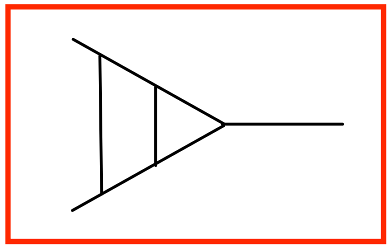
- ☑ tt cross-section at NLO
  - scale uncertainty O(11%)
  - PDF & m_t uncertainty O(2-3%)

    Dawson, Ellis, Nason ’88; Beenakker ‘89

- ☐ experimental goal O(5%)

  Need NNLO!

  Similar aim:
  WW cross section at NNLO



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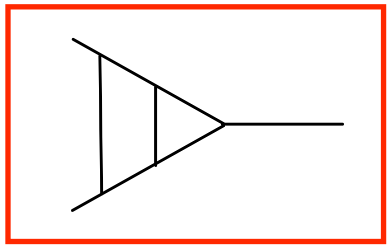
- ☐ experimental goal O(5%)

  Need NNLO!

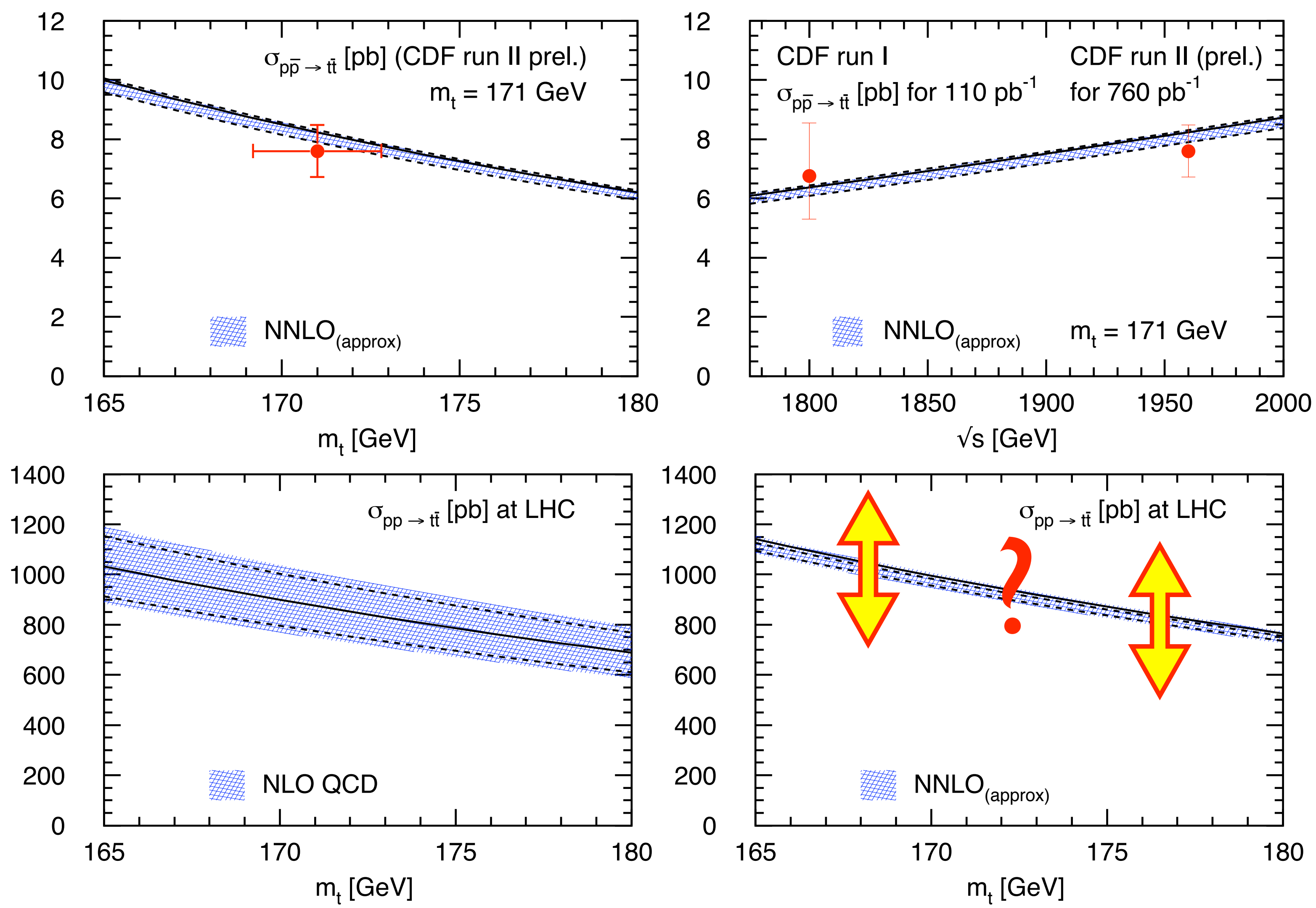
  Similar aim:
  WW cross section at NNLO

- ☐ Pinning down the tt cross-section
  - ✓ NLO + NLL resummed threshold corrections
    Cacciari, Frixione, Mangano, Nason, Ridolfi '08
  - ✓ two-loop virtual qq → tt and gg → tt at $O(m_t^2/s)$
    Czakon, Mitov, Moch '07, '08
  - ✓ full mass dependence at two-loops for qq → tt
    Czakon '08
  - ✓ $\mathrm{NNLO}_{\mathrm{approx}}$ (threshold logs + Coulomb + scale variation)
    Moch, Uwer '08; also Kidonakis, R.Vogt '08
  - ✓ analytic two-loop fermionic corrections for qq → tt
    Bonciani, Ferroglia, Gehrmann, Maitre, Studerus '08
  - ✓ one-loop squared
    Koerner, Merebashvili, Rogal '08



Towards NNLO tt

Moch, Uwer '08

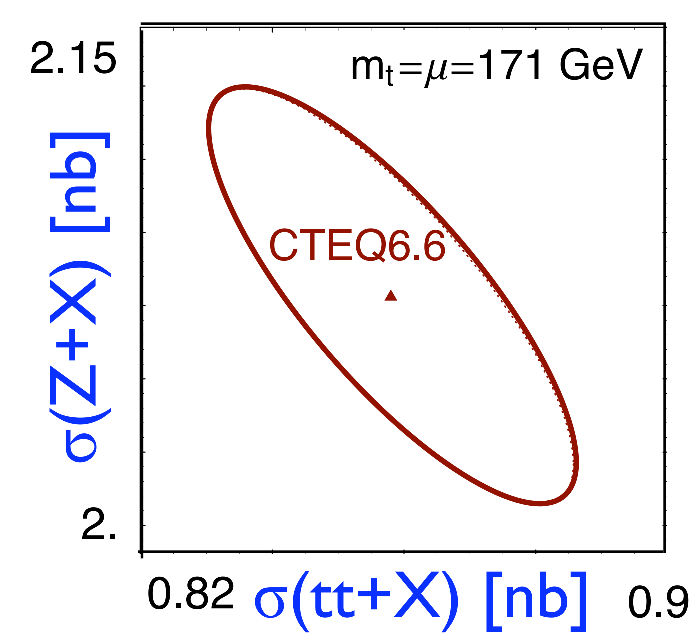


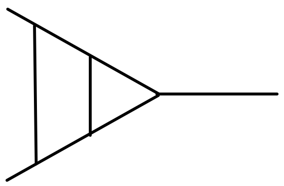
NB: band = simultaneous variation of μ_F and μ_R

- m_t from tt cross-section?
- tt promoted to luminosity monitor?

NB: PDFs in tt anti-correlated to W/Z

Nadolsky et al. '08





NNLO 3-jets in e^+e^-

Motivation: error on α_s from jet-observables

$$\alpha_s(M_Z) = 0.121 \pm 0.001\,(\mathrm{exp.}) \pm 0.005\,(\mathrm{th.})$$

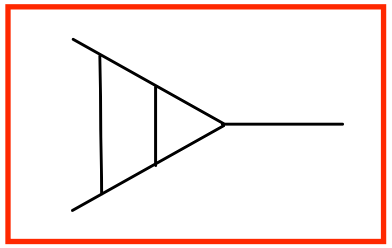
Bethke '06

➥ dominated by theoretical uncertainty

After several years, NNLO 3-jet calculation in e^+e^- completed in 2007

Method: developed antenna subtraction at NNLO

First application: NNLO fit of α_s from event-shapes

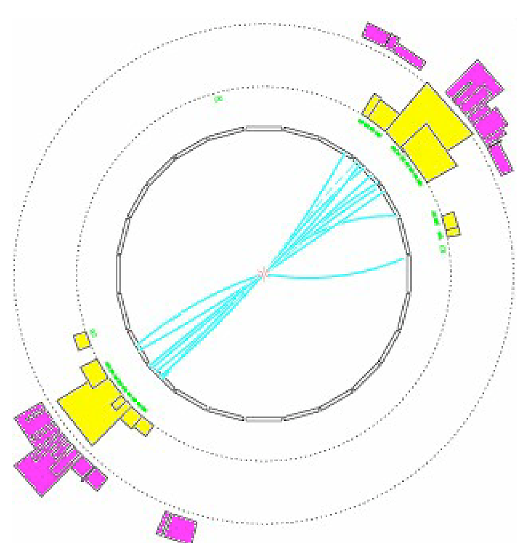


Event shapes

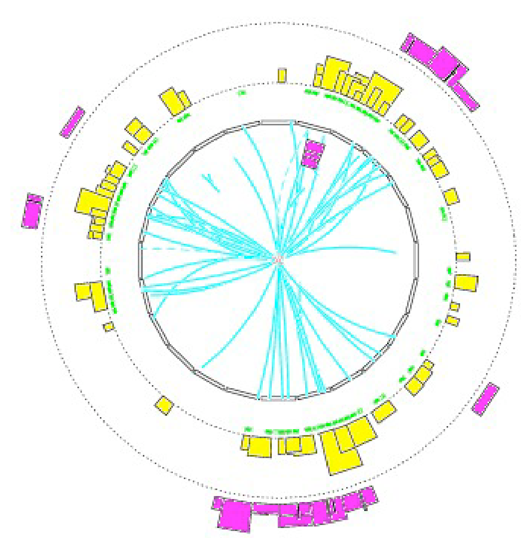
Event-shapes and jet-rates: infrared safe observables describing the energy and momentum flow of the final state.

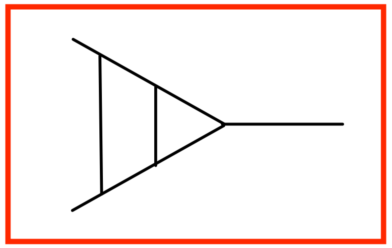
Candle example in e^+e^-: The thrust $T = \max_{\vec{n}} \frac{\sum_i \vec{p}_i \cdot \vec{n}}{\sum_i |\vec{p}_i|}$

Pencil-like event: $1 - T \ll 1$



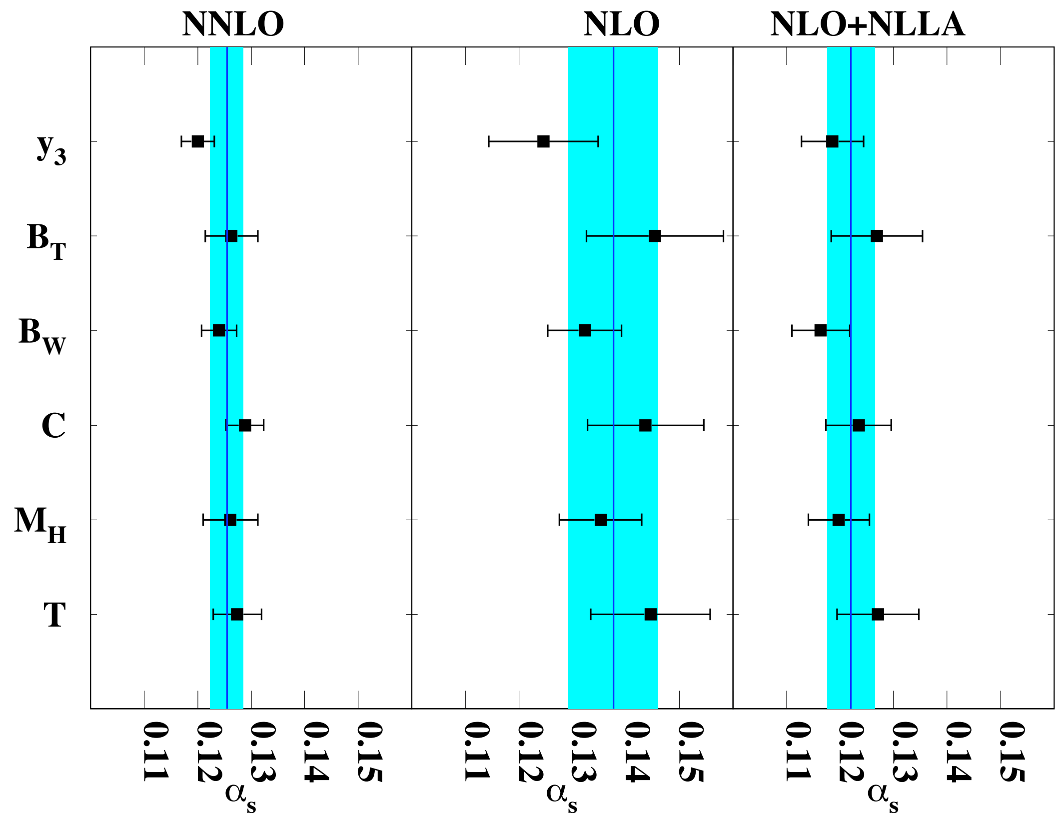
Planar event: $1 - T \sim 1$





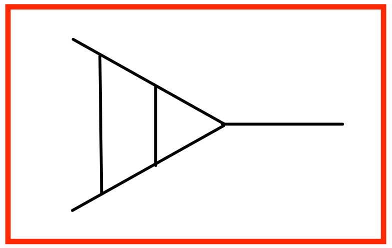
α_s from event shapes at NNLO

- scale variation reduced by a factor 2
- scatter between α_s from different event-shapes reduced
- better χ^2, central value closer to world average



$$\alpha_s(M_Z^2) = 0.1240 \pm 0.0008\,(\text{stat}) \pm 0.0010\,(\text{exp}) \pm 0.0011\,(\text{had}) \pm 0.0029\,(\text{theo})$$

Dissertori, Gehrmann-DeRidder, Gehrmann, Glover, Heinrich, Stenzel '07
Gehrmann, Luisoni, Stenzel '08



α_s from event shapes in e^+e^-

More recently: α_s from N^3LL + NNLO matched thrust distribution (NNLO from above, resummation of soft logs in SCET)

$$\alpha_s(M_Z^2) = 0.1172 \pm 0.0020\,(\mathrm{stat}) \pm 0.0008\,(\mathrm{exp}) \pm 0.0012\,(\mathrm{had}) \pm 0.0012\,(\mathrm{theo})$$

But: SCET resummation points to potential problems in some color structures

Becher, Schwartz '08

Very recently: independent NNLO calculation confirms disagreement in those color structures (α_s unaffected?)

Weinzierl '08

$\Rightarrow$ *Importance of independent calculations!*

Recap on LO, NLO, NNLO

Leading order

- everything can be computed in principle today (practical edge: 8 particles in the final state)
- techniques: standard Feynman diagrams or recursive BG, BCF, CSW ...

Next-to-leading order

- current status 2→3 in the final state. 2→4 very challenging.
- many new, promising techniques, some first "all-N" results

Next-to-next-to-leading order

- few 2→1 processes available (Higgs, Drell-Yan ...)
- recently 3jets in e^+e^-

Remember importance of decays and cuts in realistic studies

Jets: before 2006

Cones are IR unsafe!

The Cone is too rigid!

IR unsafety affects jet cross-sections by less than 1%, so don't need to care!

kt collects too much soft radiation!

Cones have a well-defined circular area!

Jet area not well defined in kt: U.E. and pile-up subtraction too difficult!

What about dark towers??

After all, if D=1.35 R Cone and kt are practically the same thing....

Where do jets enter ?

Essentially everywhere at colliders!

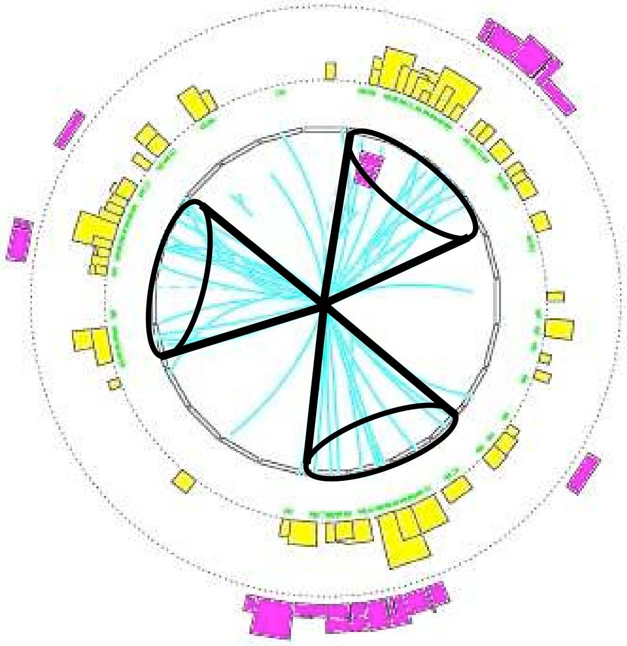
Jets are an essential tool for a variety of studies:

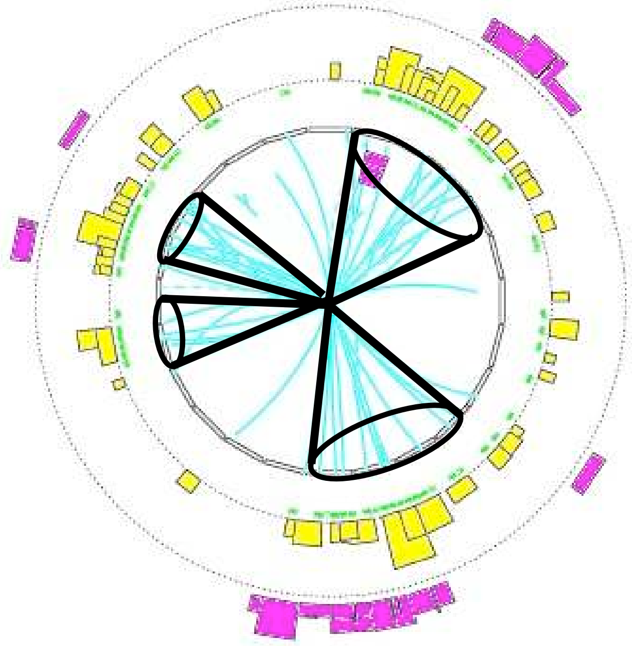
- top reconstruction
- mass measurements
- most Higgs and NP searches
- general tool to attribute structure to an event
- instrumental for QCD studies, e.g. inclusive-jet measurements
  ⇒ important input for PDF determinations

Jets

Jets provide a way of projecting away the multiparticle dynamics of an event ⇒ leave a simple quasi-partonic picture of the hard scattering

The projection is fundamentally ambiguous ⇒ jet physics is a rich subject



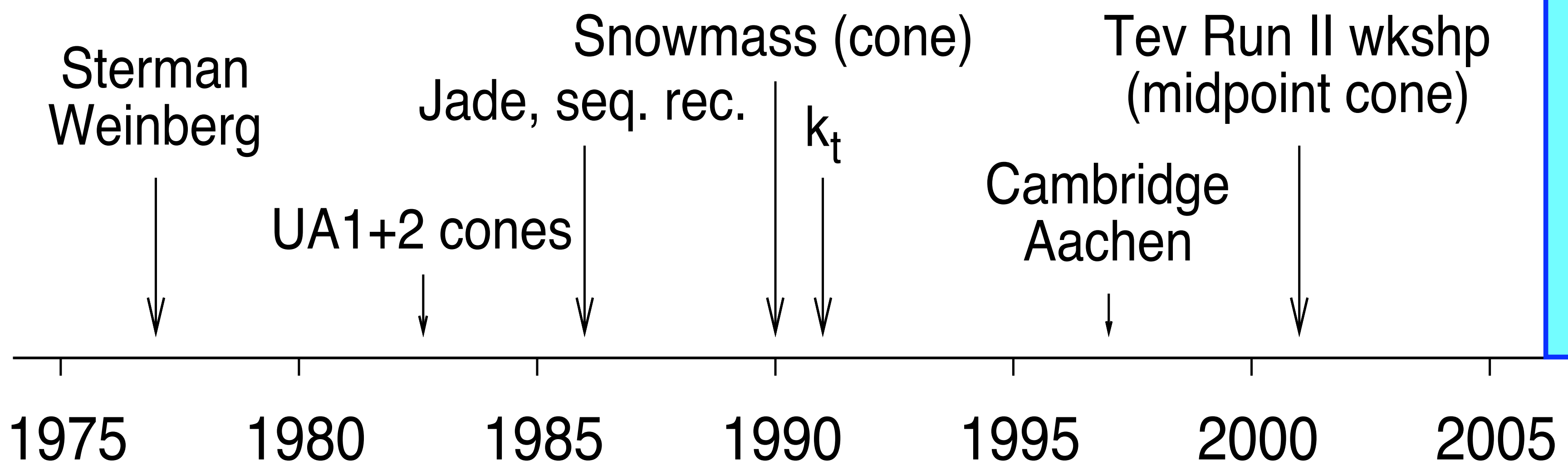


Ambiguities:

1) Which particles should belong to a same jet ?

2) How does recombine the particle momenta to give the jet-momentum?

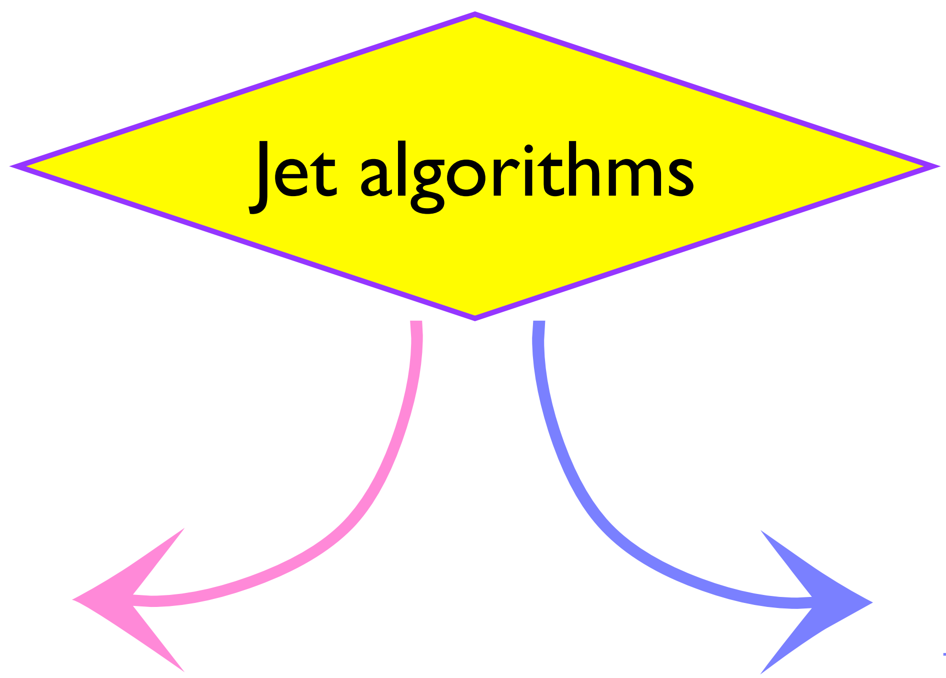
Jet developments



fast-kt, SISCone, anti-kt, jet-areas, jet-flavour, non-perturbative effects, quality measures, jet-substructure as Higgs discovery channel ...

Two broad classes of jet algorithms

Today many extensions of the original Sterman-Weinberg jets.
Modern jet-algorithms divided into two broad classes



Cone type
(UA1,JetCLU, Midpoint, SISCone..)

top down approach: cluster particles according to distance in coordinate-space

Idea: put cones along dominant direction of energy flow

Sequential
(kt-type, Jade, Cambridge/ Aachen...)

bottom up approach: cluster particles according to distance in momentum-space

Idea: undo branchings occurred in the PT evolution

Jet requirements

Snowmass accord

FERMILAB-Conf-90/249-E
[E-741/CDF]

Toward a Standardization of Jet Definitions

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronization.

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Other desirable properties:

- flexibility
- few parameters
- fast algorithms
- transparency
- ...

Inclusive k_t/Durham-algorithm

Catani et. al '92-'93; Ellis&Soper '93

Inclusive algorithm:

1. For any pair of final state particles i,j define the distance

$$d_{ij} = \frac{\Delta y_{ij}^2 + \Delta \phi_{ij}^2}{R^2} \min\{k_{ti}^2, k_{tj}^2\}$$

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2. For each particle i define a distance with respect to the beam

$$d_{iB} = k_{ti}^2$$

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3. Find the smallest distance. If it is a d_{ij} recombine i and j into a new particle ($\Rightarrow$ recombination scheme); if it is d_{iB} declare i to be a jet and remove it from the list of particles

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4. repeat the procedure until no particles are left

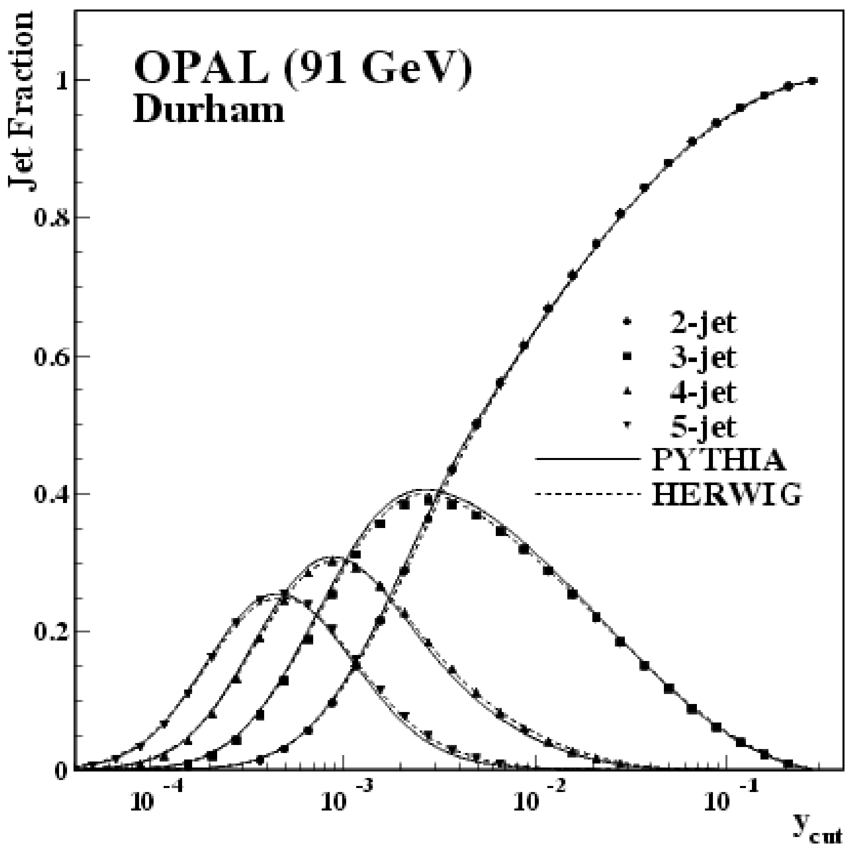
Exclusive version: stop when all d_{ij}, d_{iB} > d_{cut} or when reaching n-jets

k_t/Durham-algorithm in e^+e^-

k_t originaly designed in e^+e^- and most widely used algorithm in e^+e^- (LEP)

$$y_{ij} = 2\min\{E_i^2, E_j^2\}\left(1-\cos\theta_{ij}^2\right)$$

- can specify events using y_{23}, y_{34}, y_{45}, y_{56} ...
- resolution parameter related to minimum transverse momentum between jets



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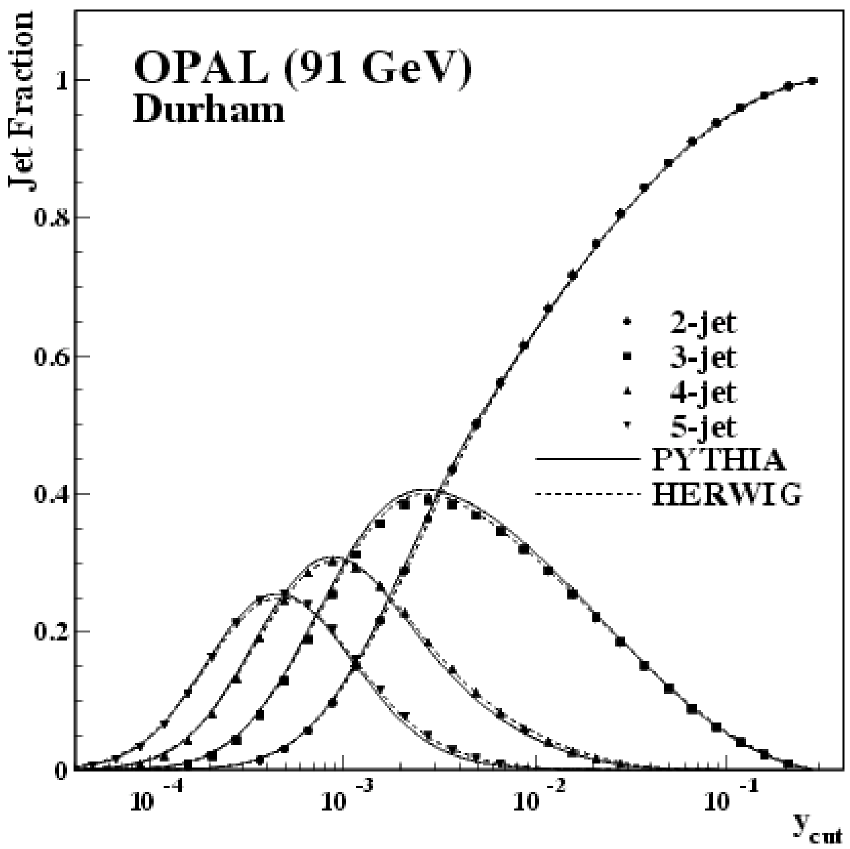

Satisfies fundamental requirements:

1. Collinear safe: collinear particles recombine early on
2. Infrared safe: soft particles do not influence the clustering sequence

$\Rightarrow$ *collinear + infrared safety important: it means that cross-sections can be computed at higher order in pQCD (no divergences)!*

Longitudinally invariant k_t/Durham-algorithm

Questions about k_t-algorithm:

- Is the number of particles in a jet an infrared safe quantity?
- Is the number of jets in e^+e^- an infrared-safe quantity?
- Is the number of jets in pp-collisions an infrared-safe quantity?
- Is the number of jets above some $p_{t,\min}$ an infrared-safe quantity?
- What is bad about the following distance measures in pp collisions?

$$d_{ij} = 2\min\{E_i^2, E_j^2\}\left(1 - \cos\theta_{ij}^2\right)$$

$$d_{iB} = 2E_i^2\left(1 - \cos\theta_i^2\right)$$

The CA and the anti-k_t algorithm

<u>The Cambridge/Aachen</u>: sequential algorithm like k_t, but uses only angular properties to define the distance parameters

$$d_{ij} = \frac{\Delta R_{ij}^2}{R^2} \qquad d_{iB} = 1 \qquad \Delta R_{ij}^2 = (\phi_i - \phi_j)^2 + (y_i - y_j)^2$$

Dotshitzer et. al '97; Wobisch & Wengler '99

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Dotshitzer et. al '97; Wobisch & Wengler '99

<u>The anti-kt algorithm</u>: designed not to recombine soft particles together

$$d_{ij} = \min\{1/k_{ti}^2, 1/k_{tj}^2\}\Delta R_{ij}^2/R^2 \qquad d_{iB} = 1/k_{ti}^2$$

Cacciari, Salam, Soyez '08

Recombination schemes in e^+e^-

Given two massless momenta p_i and p_j how does one recombine them to build p_{ij} ? Several choices are possible.

Most common ones:

1. E-scheme $p_{ij} = p_i + p_j$
2. E_0-scheme $\vec{p}_{ij} = \vec{p}_i + \vec{p}_j$ $\quad E_{ij} = |\vec{p}_{ij}|$
3. P_0-scheme $E_{ij} = E_i + E_j$ $\quad \vec{p}_{ij} = \frac{E_{ij}}{|\vec{p}_i + \vec{p}_j|}(\vec{p}_i + \vec{p}_j)$

E_0/P_0-schemes give massless jets, along with the idea that the hard parton underlying the jet is massless

E-scheme give massive jets. Most used in recent analysis.

Recombination schemes in hh

Most common schemes:

- E-scheme (as in e+e-)
- p_t, p_t^2, E_t, E_t^2 schemes
  - first preprocessing, i.e. make particles massless, rescaling the 3-momentum in the E_t, E_t^2 schemes or the energy in the p_t, p_t^2 schemes
  - then define

$$p_{t,ij} = p_{t,i} + p_{t,j}$$

$$\phi_{ij} = \left(w_i \phi_i + w_j \phi_j\right) / \left(w_i + w_j\right)$$

$$y_{ij} = \left(w_i y_i + w_j y_j\right) / \left(w_i + w_j\right)$$

where the weights w_i are p_{ti} for the p_t, E_t schemes and p_{ti}^2 for the p_t^2 and E_t^2 schemes

NB: *a jet-algorithm is fully specified only once all parameters and its recombination scheme is specified too!*

Cone algorithms

1. A particle i at rapidity and azimuthal angle (yi, φi) ⊂ cone C iff

$$\sqrt{(y_i - y_C)^2 + (\phi_i - \phi_C)^2} \leq R_{\rm cone}$$

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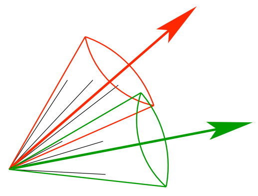

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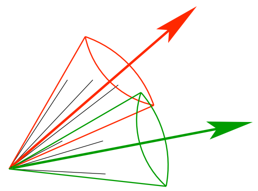

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Practically (JetClu, MidPoint, PxCone..): introduce trial directions (seeds)

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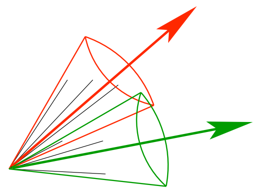

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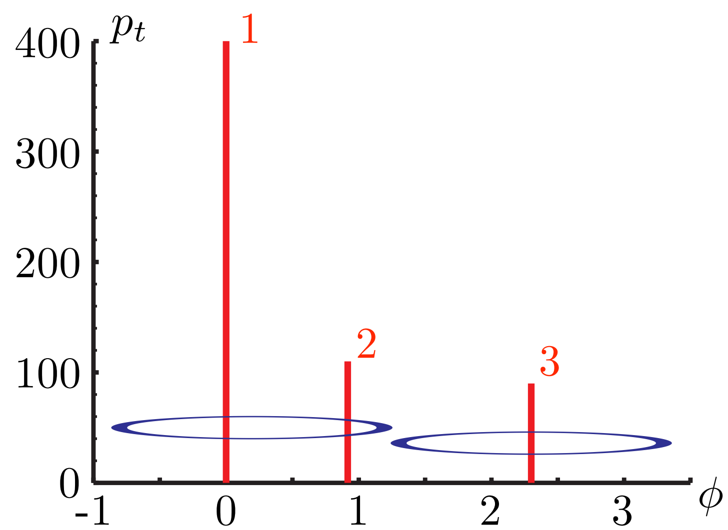
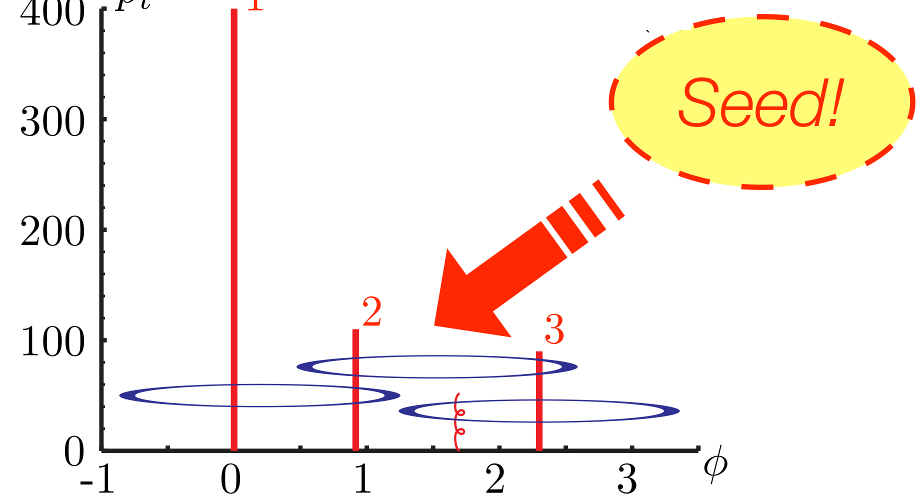
Practically (JetClu, MidPoint, PxCone..): introduce trial directions (seeds)

Seeds make cone algorithms infrared unsafe

Jets: infrared unsafety of cones



3 hard ⇒ 2 stable cones



3 hard + 1 soft ⇒ 3 stable cones

Soft emission changes the hard jets ⇒ algorithm is IR unsafe

Midpoint algo: take as seed position of emissions and midpoint between two emissions (postpones the infrared satefy problem)

Seedless cones

<u>Solution:</u>
use a seedless algorithm, i.e. consider all possible combinations of particles as candidate cones, so find all stable cones [⇒ jets]

Blazey '00

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The problem:
clustering time growth as $N2^N$. So for an event with 100 particles need 10^{17} ys to cluster the event ⇒ prohibitive beyond PT (N=4,5)

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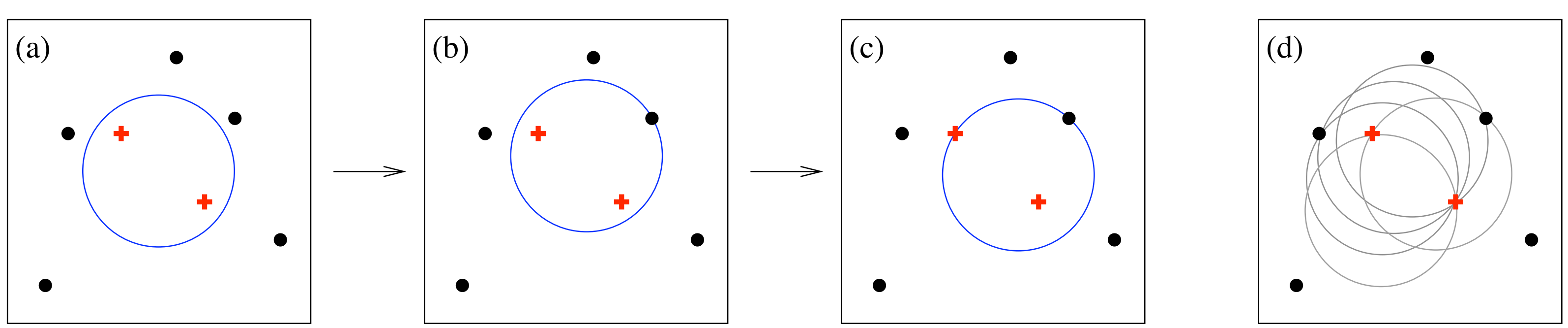
Blazey '00

The problem:

clustering time growth as $N2^N$. So for an event with 100 particles need 10^{17} ys to cluster the event ⇒ prohibitive beyond PT (N=4,5)

Better solution:

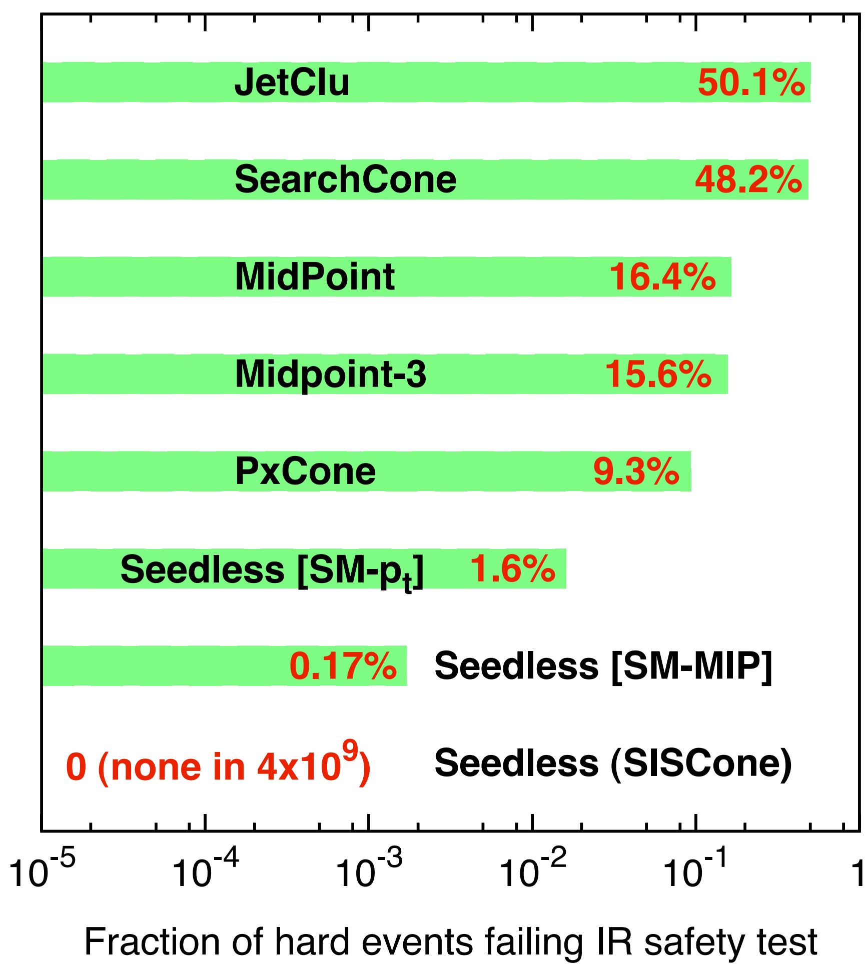
SISCone recasts the problem as a computational geometry problem, the identification of all distinct circular enclosures for points in 2D and finds a solution to that ⇒ $N^2 \ln N$ time IR safe algorithm

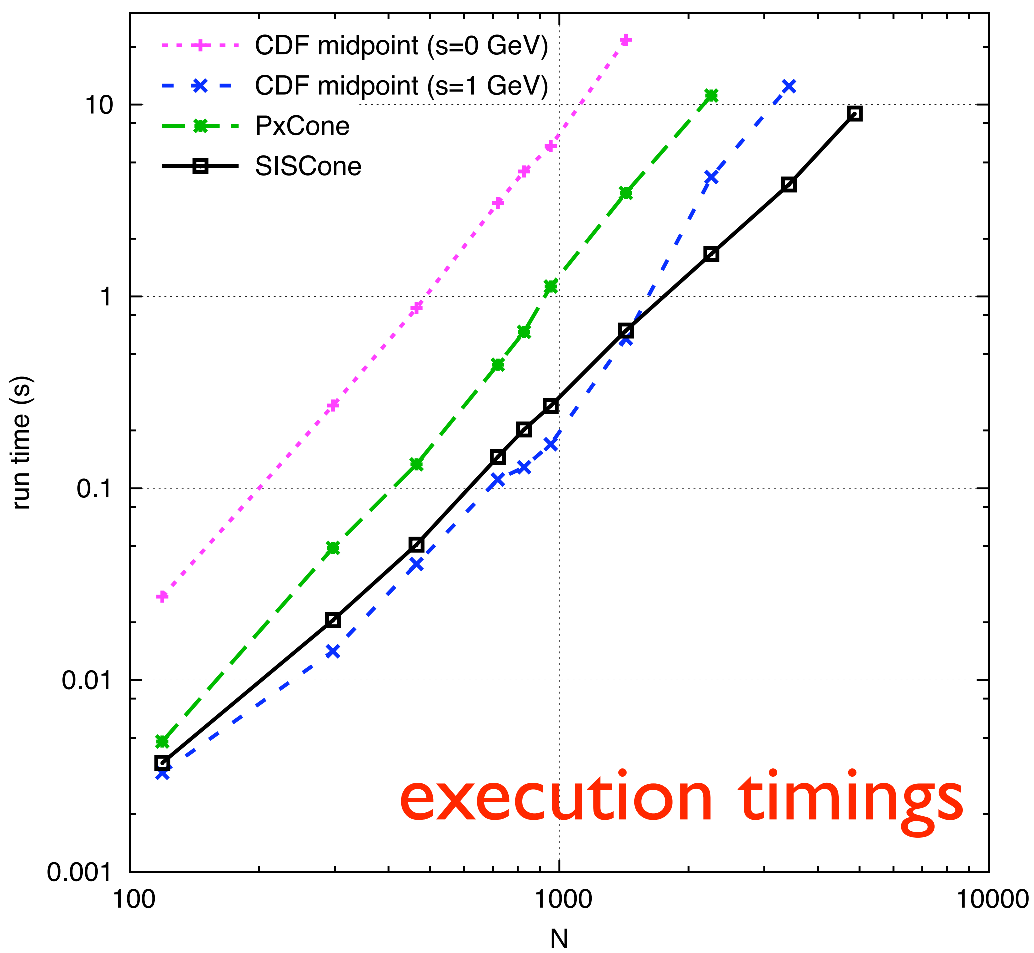


Salam, Soyez '07

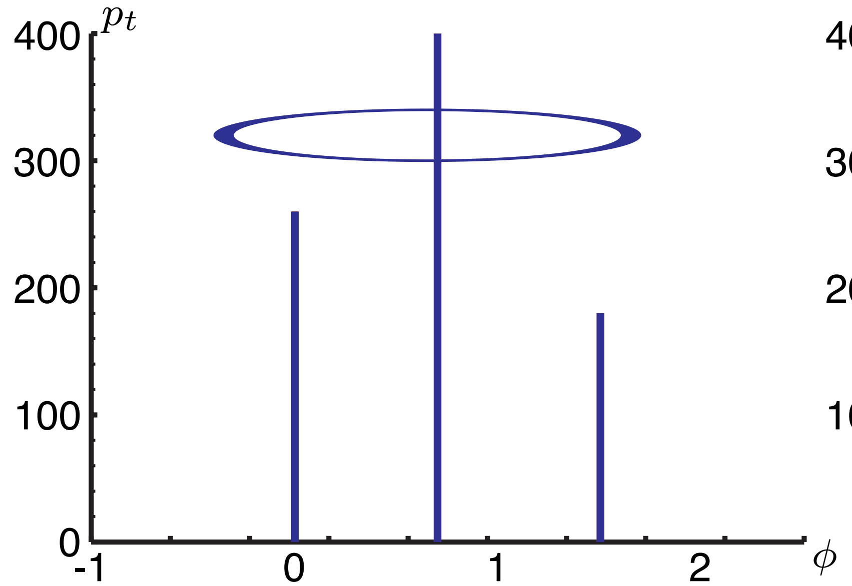
IR safety test & time comparisons

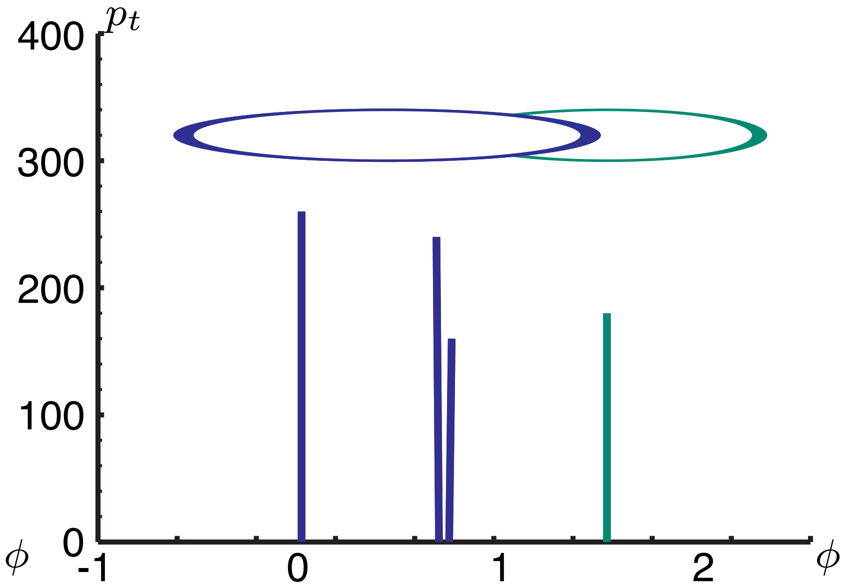
IR safety test: take a random hard event, add very soft emissions, count the number of times the hard jets change due to soft emissions





IR-unsafety of iterative cones





3 hard ⇒ 1 stable cone

collinear splitting ⇒ 2 stable cones

Collinear splitting changes the hard jets ⇒ algorithm is col. unsafe

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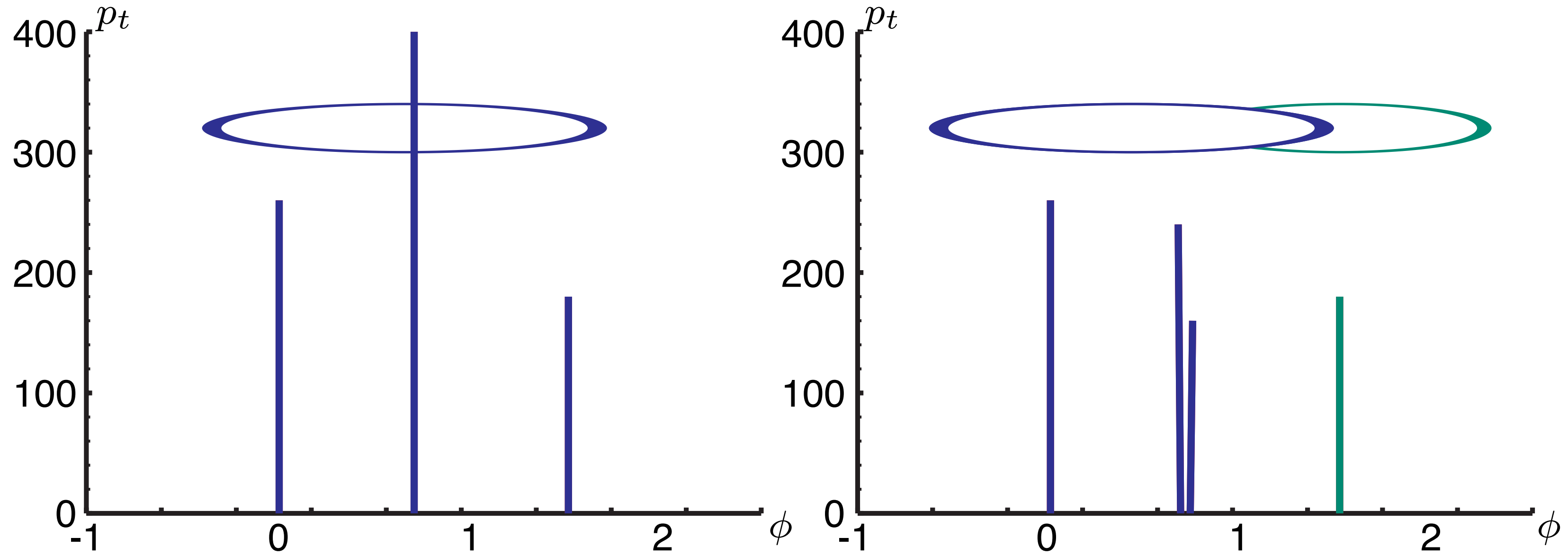


3 hard ⇒ 1 stable cone   collinear splitting ⇒ 2 stable cones

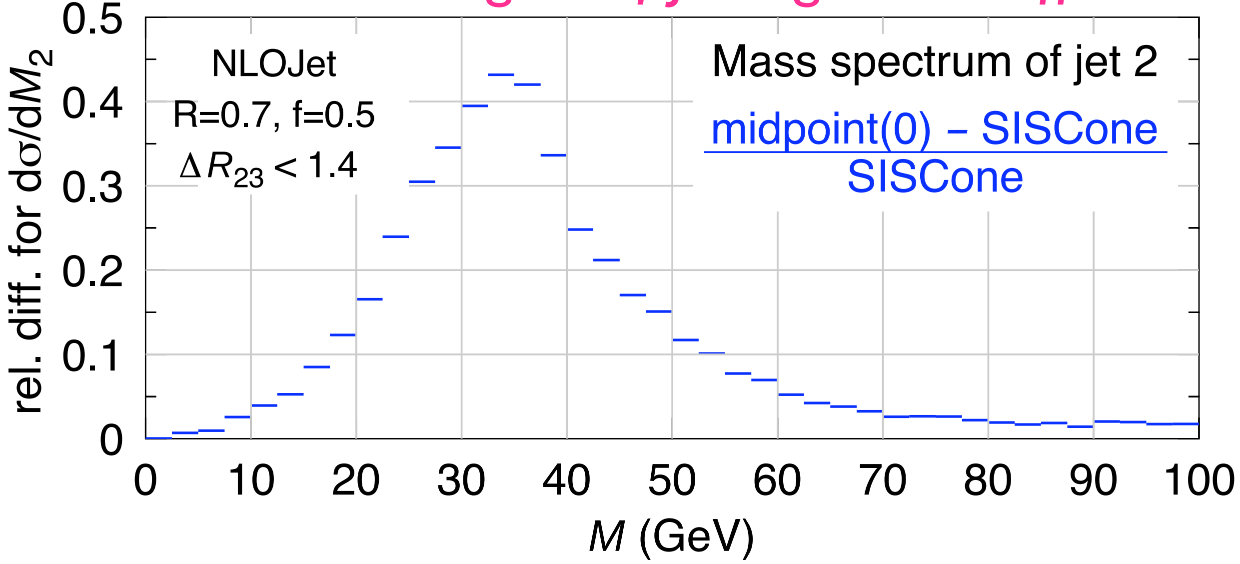
Collinear splitting changes the hard jets ⇒ algorithm is col. unsafe

Solution: anti-kt algorithm $d_{ij} = \min\{1/k_{ti}^2, 1/k_{tj}^2\}\Delta R_{ij}^2/R^2$

large kt ⇔ small distance, so hard partons eat up everything up to a distance R ⇒ circular jets (not modified by soft radiation)

Physical impact of infrared unsafety

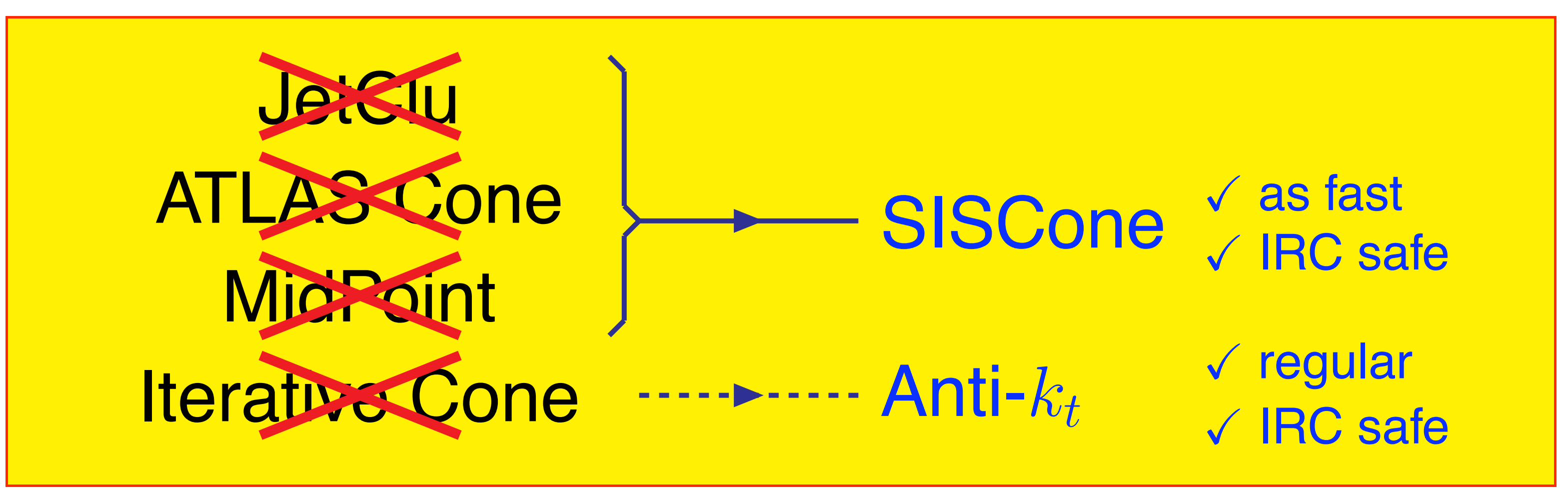
Side remark: comparison data/theory meaningless if jet algos are different



Up to 40% difference in mass spectrum

IR-unsafety is an issue at the LHC

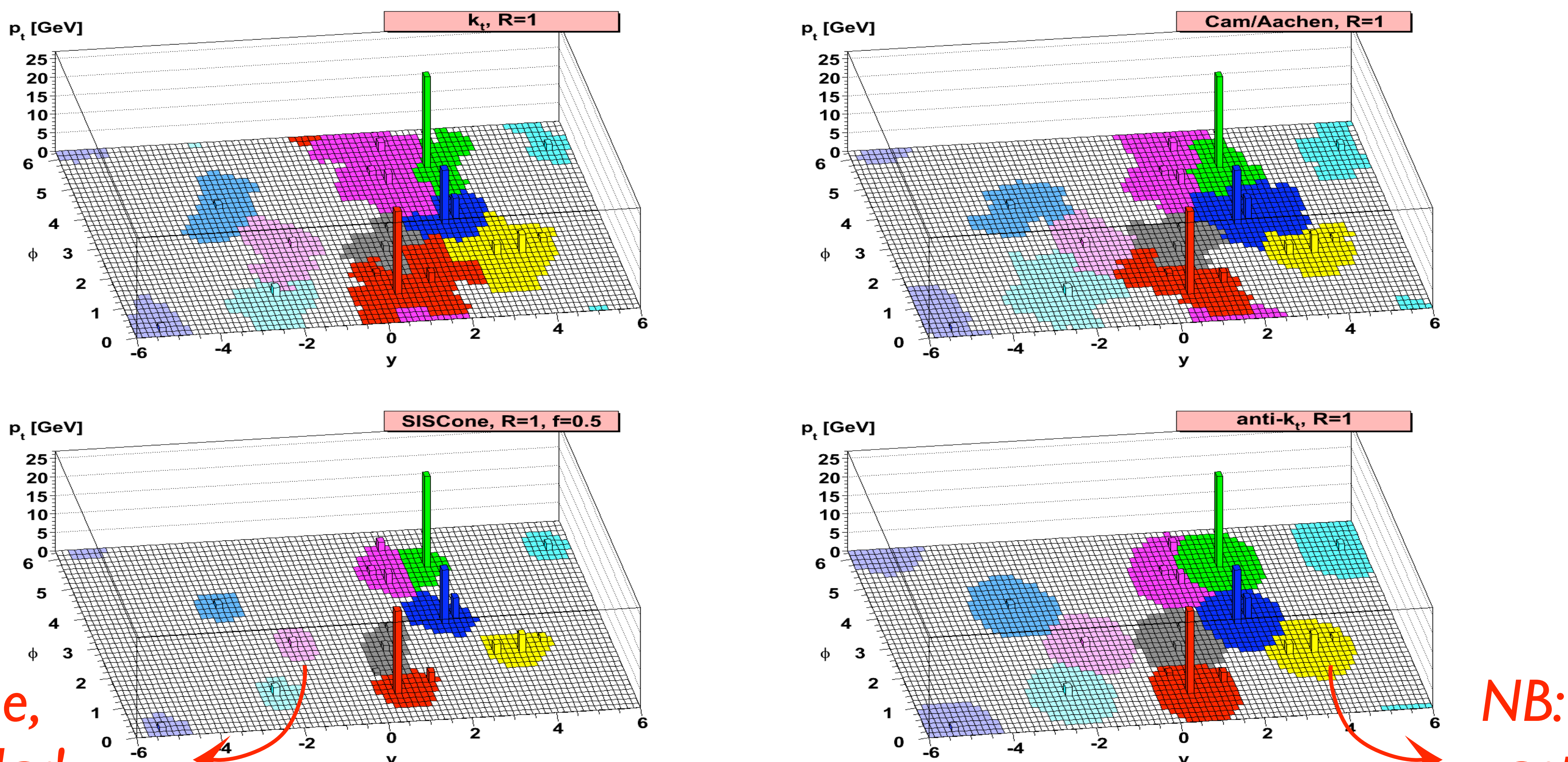
Observable	1st miss cones at	Last meaningful order
Inclusive jet cross section	NNLO	NLO
3 jet cross section	NLO	LO (NLO in NLOJet)
$W/Z/H$ + 2 jet cross sect.	NLO	LO (NLO in MCFM)
jet masses in 3 jets	LO	none (LO in NLOJet)



If you don't want theoretical efforts to be wasted!

Jet area

Given an infrared safe, fast jet-algorithm, can define the jet area A as follows: fill the event with an infinite number of infinitely soft emissions uniformly distributed in η-φ and make A proportional to the # of emissions clustered in the jet



NB: cone, not circular!

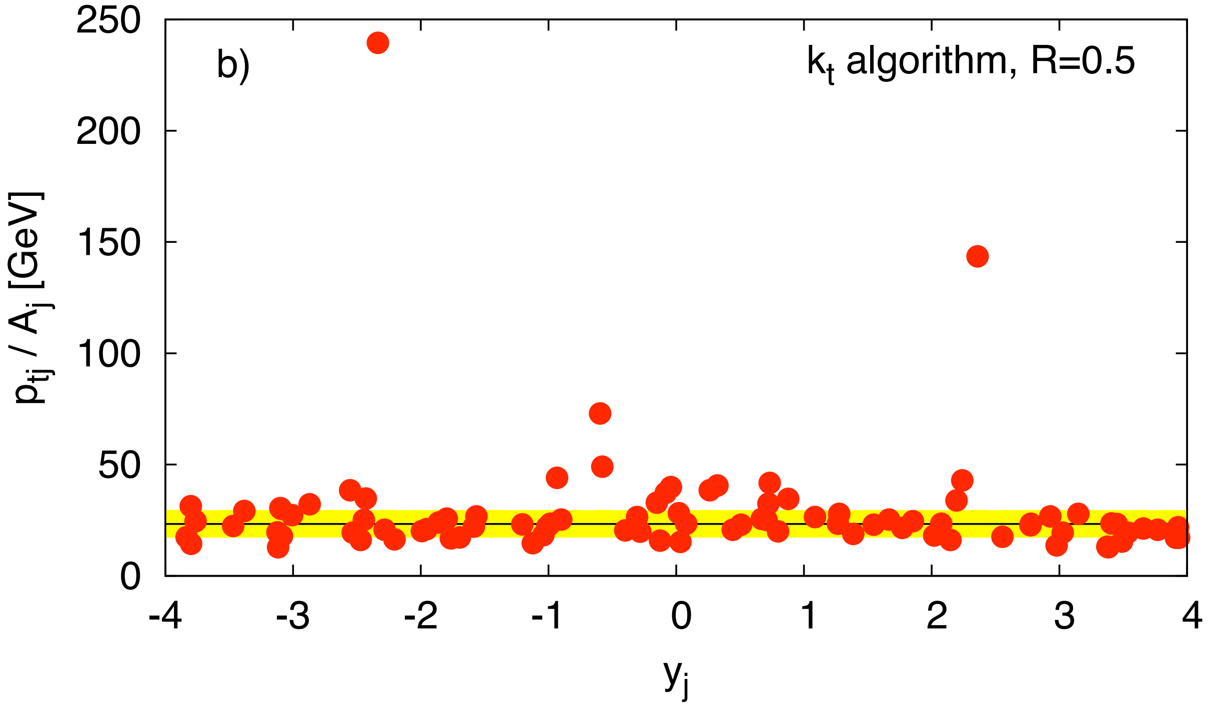
NB: new anti-kt

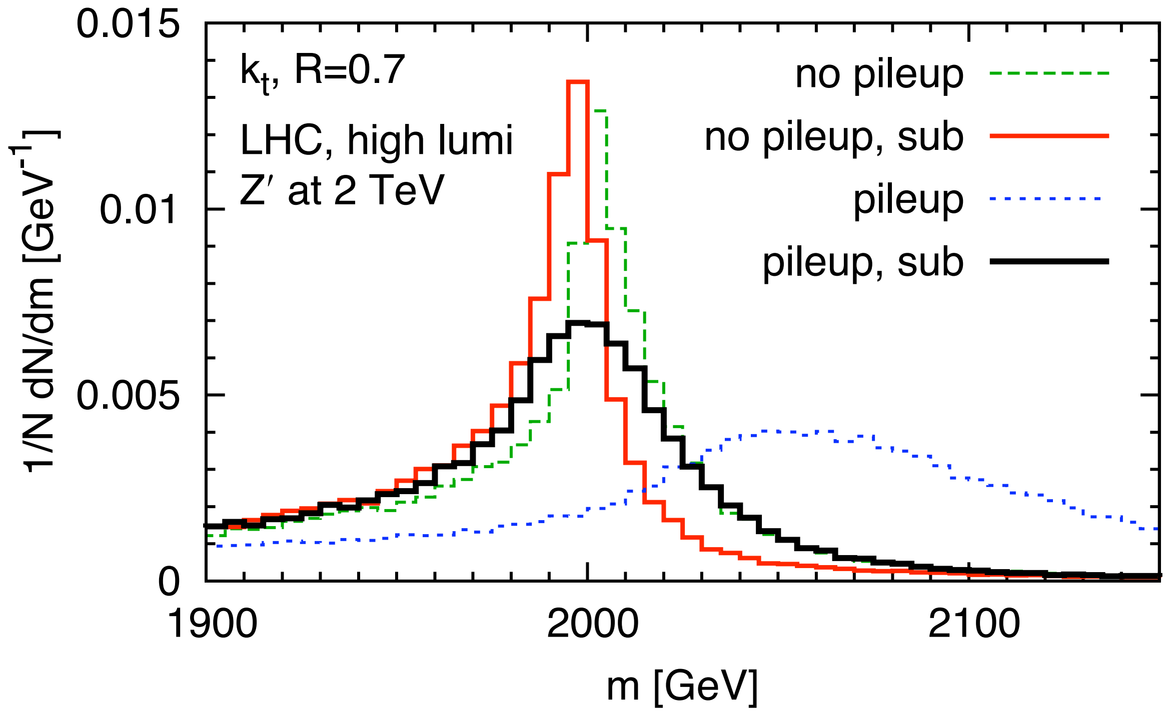
What jet areas are good for

jet-area ≡ catching area of the jet when adding soft emissions

⇒ simple area based subtraction for a variety of algorithms

Get $\rho = \frac{p_{t,j}}{A_j}$ from the majority of (pile-up) jets, define $p_j^{\mathrm{sub}} = p_j - A_j \rho$





Cacciari et al. ’07

Remember: pileup = generic p-p interaction (hard, soft, single-diffractive...) overlapping with hard scattering

Quality measures of jets

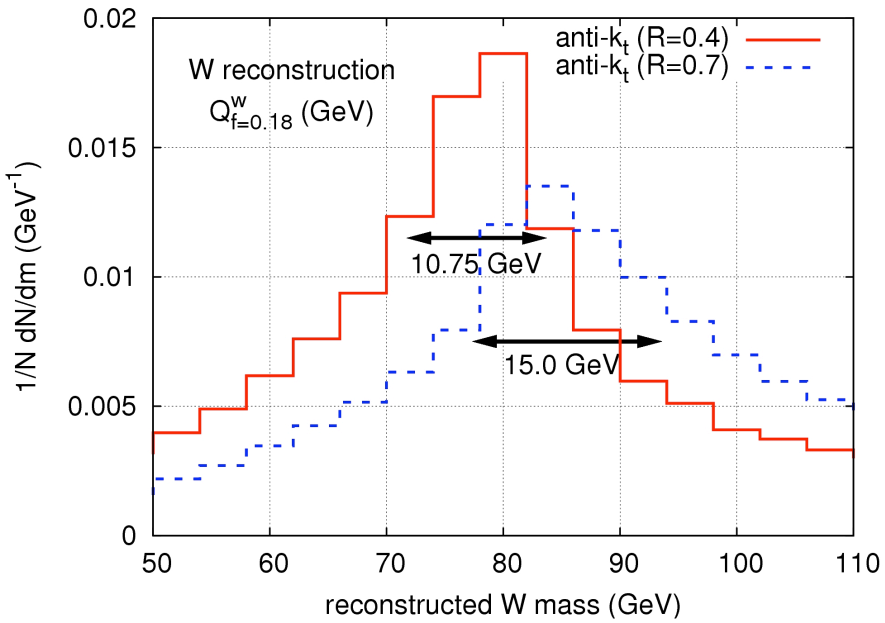
Suppose you are searching for a heavy state (H→gg, Z'→qq, ...)

The object is reconstructed through its decay products
⇒ *Which jet algorithm (JA) is best ? Does the choice of R matter?*

Define: Q^w_f(JA, R) ≡ width of the smallest mass window that contains a fraction f of the generated massive objects

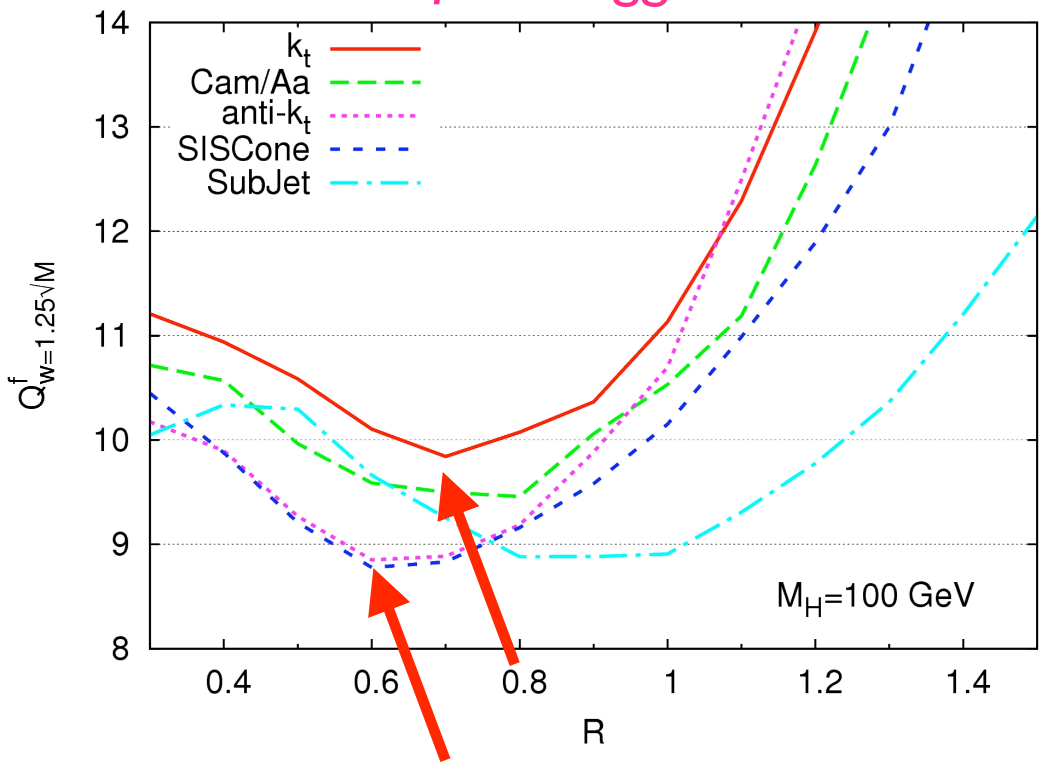
- good algo ⇔ small Q^w_f(JA, R)
- ratios of Q^w_f(JA,R): mapped to ratios of effective luminosity (with same $S/\sqrt{B}$)

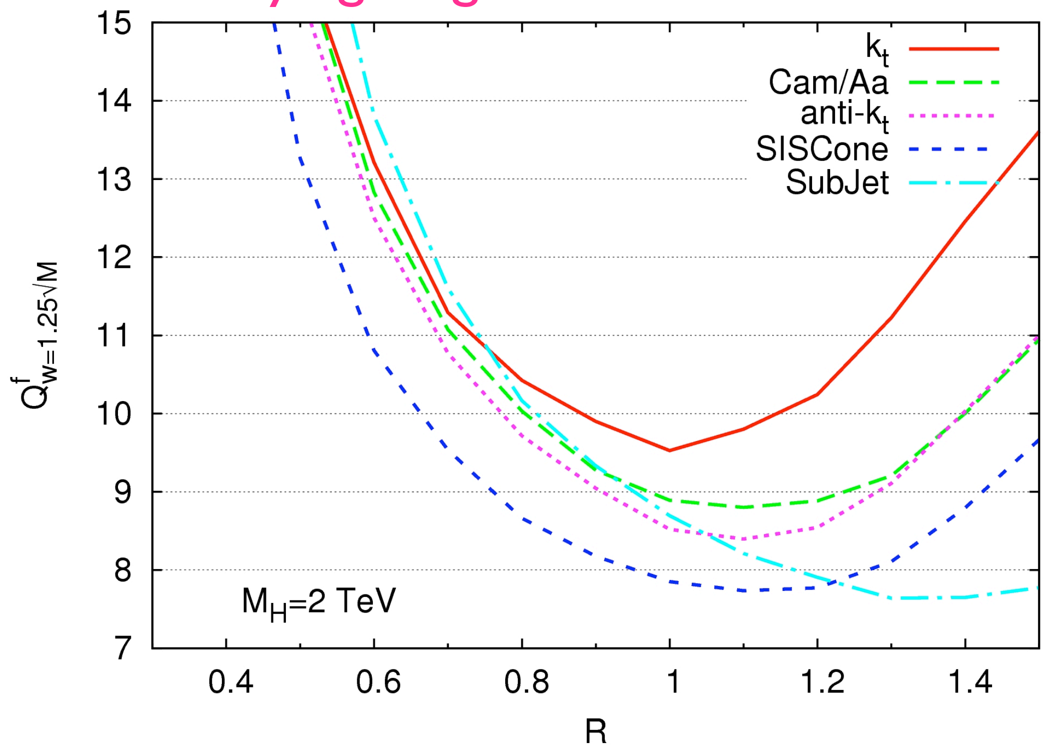
$$\mathcal{L}_2 = \rho_\mathcal{L} \mathcal{L}_1 \qquad \rho_\mathcal{L} = \frac{Q^f_z(JA_2, R_2)}{Q^f_z(JA_1, R_1)}$$



Quality measures: sample results

NB: Here "fake Higgs" = narrow resonance decaying to gluons





- At 100GeV: use a Tevatron standard algo (k_t, R=0.7) instead of best choice (SISCone, R=0.6 $\Rightarrow$ lose $\rho_{\mathcal{L}} = 0.8$ in effective luminosity

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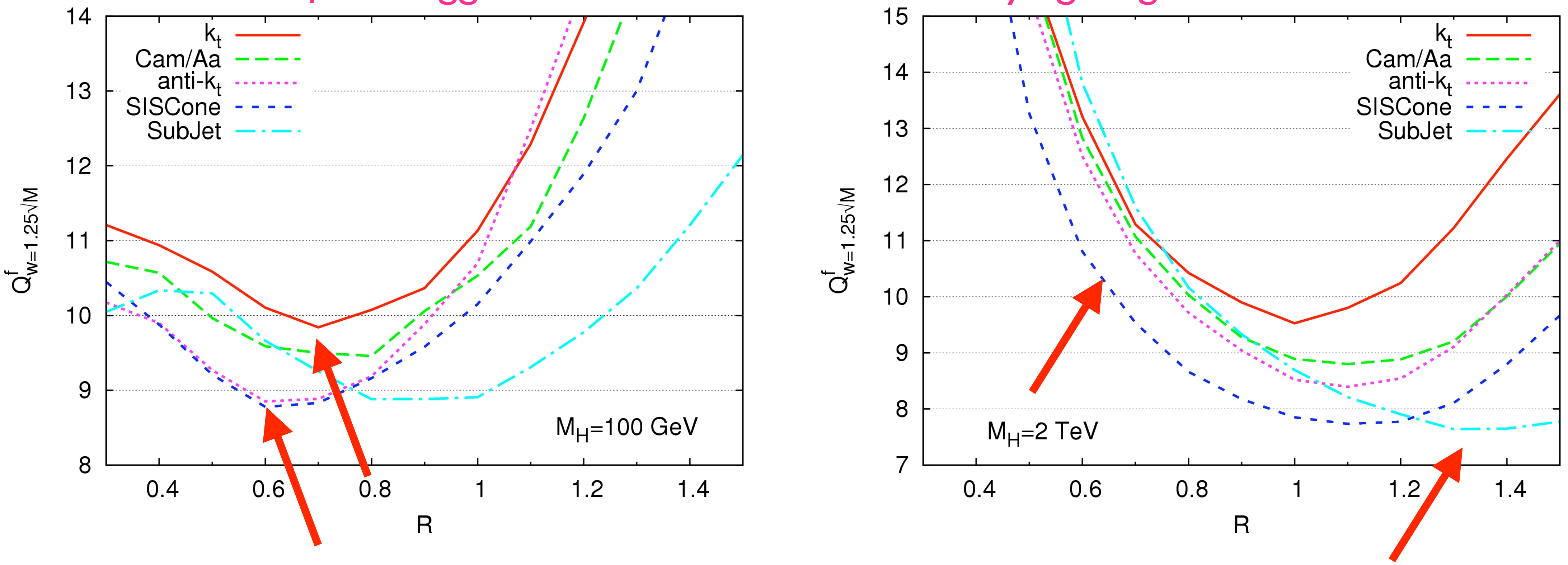

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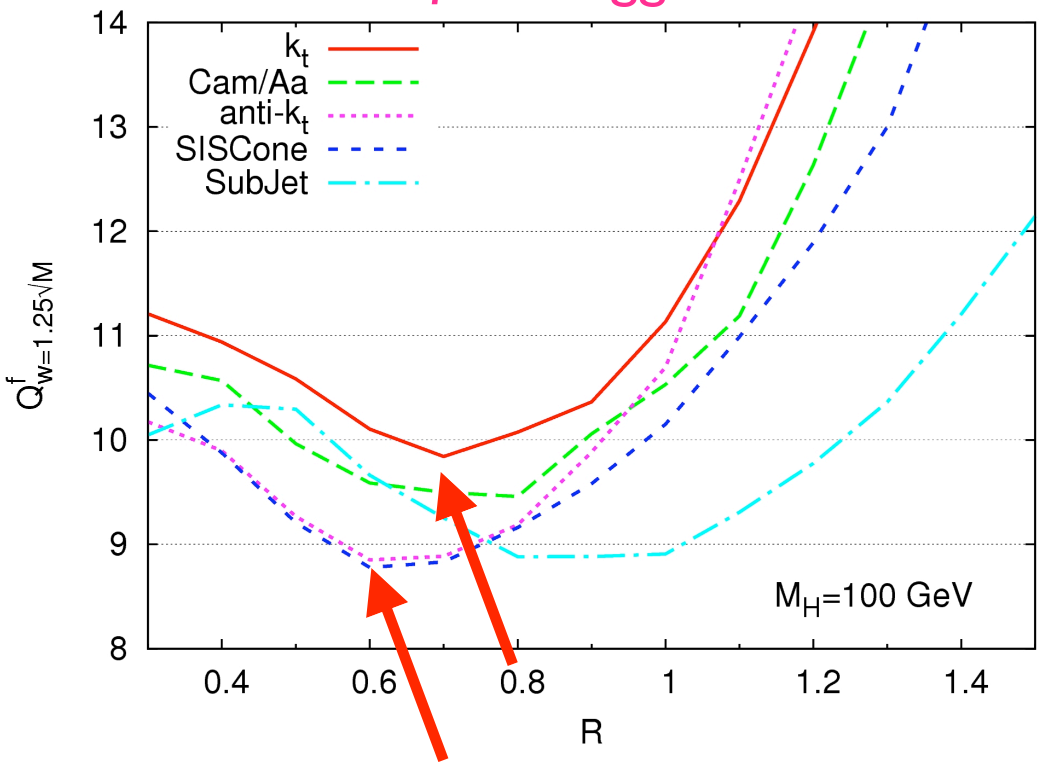

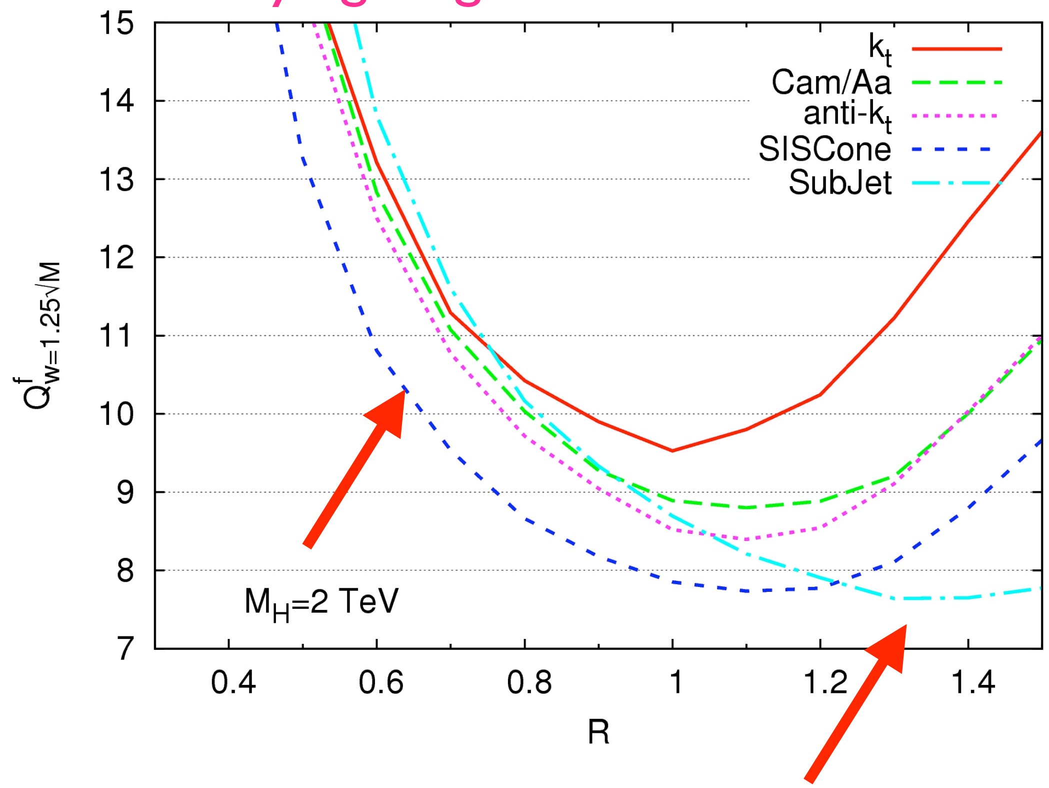

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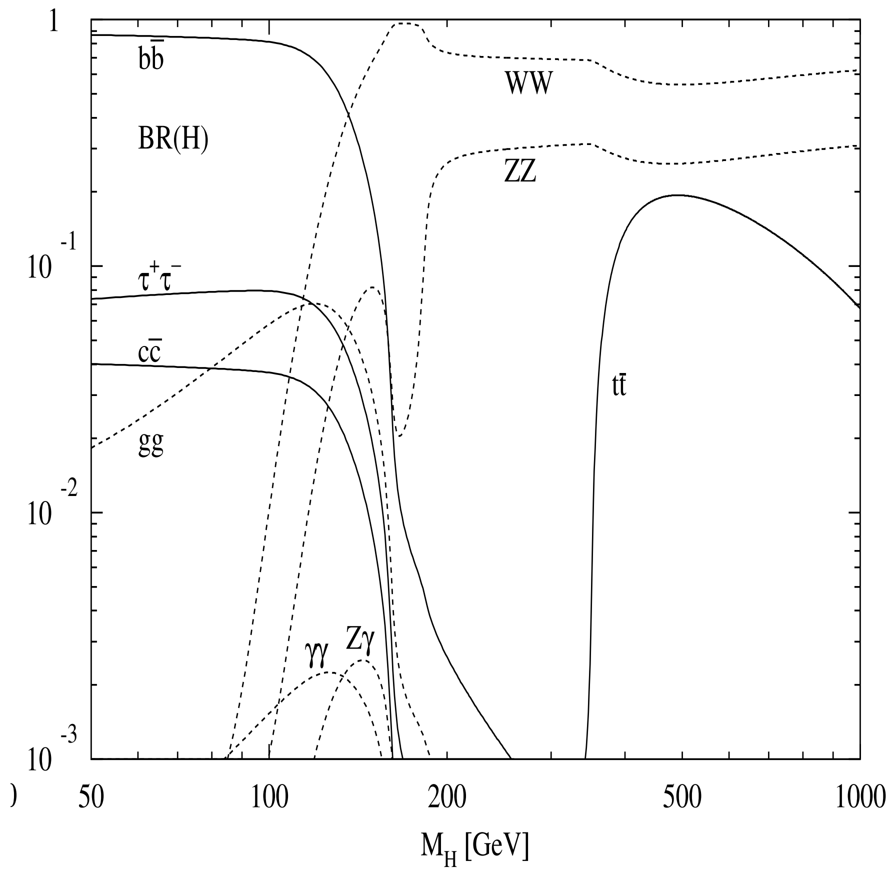
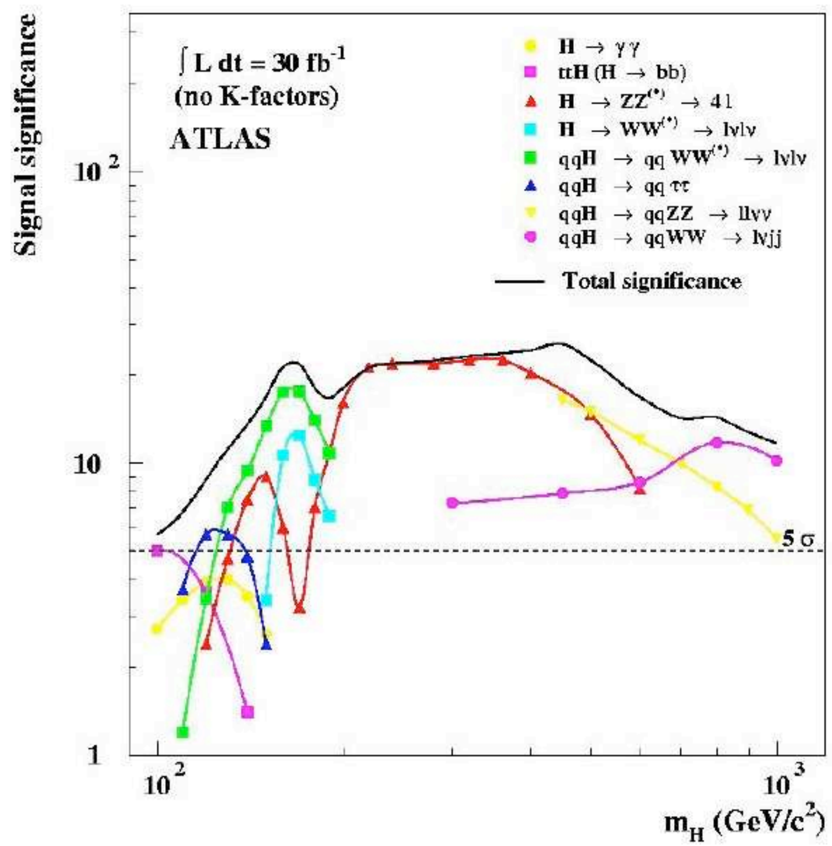
A good choice of jet-algorithm does matter!
Bad choice of algo $\Leftrightarrow$ lost in discrimination power!

Z/W+ H (→bb) rescued ?





⇒ Light Higgs hard: H→bb dominant, but overwhelmed by background

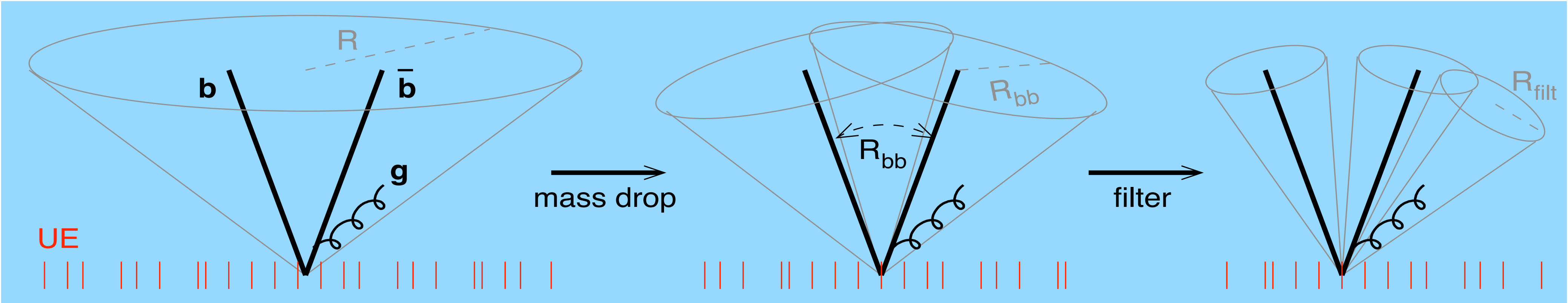
Conclusion [ATLAS TDR]:

The extraction of a signal from H → bb decays in the WH channel will be very difficult at the LHC even under the most optimistic assumptions [...]

Z/W + H (→bb) rescued ?

Boosted Higgs at high p_t: central decay products ⇒ single massive jet

Use jet-finding geared to identify the characteristic structure of fast-moving Higgs that decays into a bb-pair close in angle

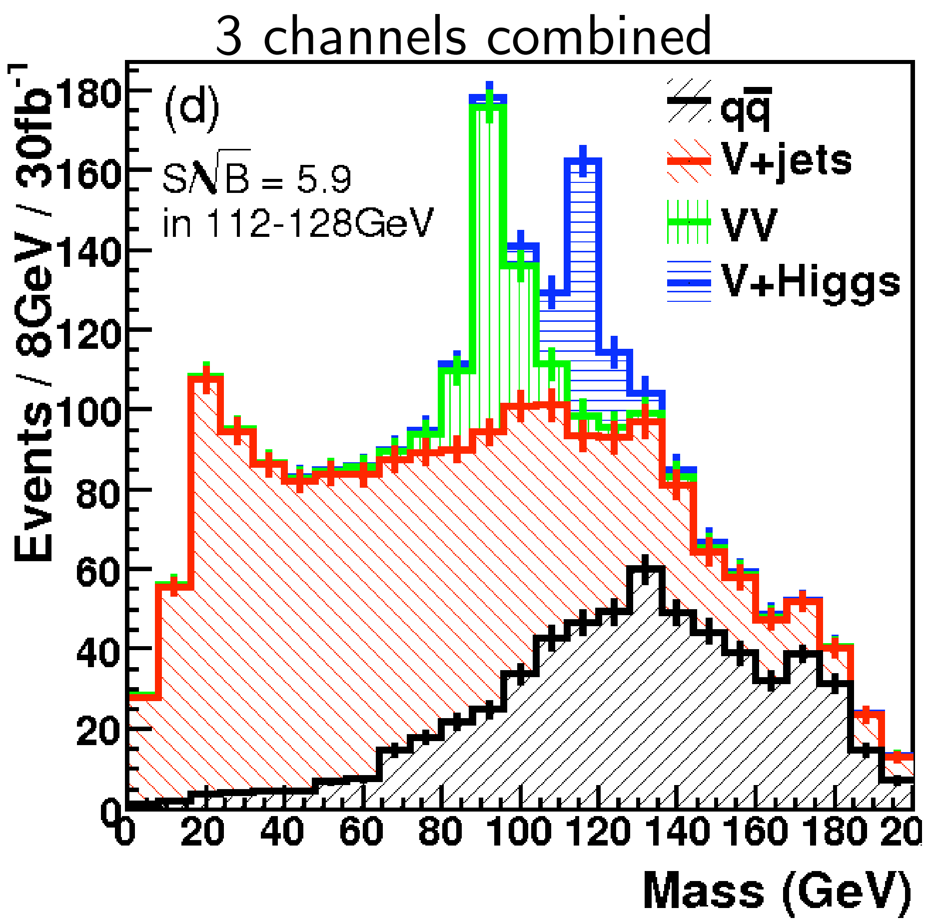


1. cluster the event with e.g. CA algo and large-ish R
2. undo last recomb: large mass drop + symmetric + b tags
3. filter away the UE: take only the 3 hardest sub-jets

<u>Related ideas for 2- and 3-body decays (boosted tops)</u>: Butterworth, Cox & Forshaw; Butterworth, Ellis & Raklev; Skiba & Tucker-Smith; Hodom; Baur; Agashe et al; Lille, Randall & Wang; Contino & Servant; Brooijmans; Thaler & Wang; Kaplan et al.; Almeida et al. [...]

Z/W + H (→bb) rescued ?

Mass of the three hardest sub-jets:



- with common & channel specific cuts: p_{tV}, p_{tH} > 200GeV , ...
- real/fake b-tag rate: 0.7/0.01
- NB: very neat peak for WZ (Z →bb) Important for calibration

Butterworth, Davison, Rubin, Salam '08

5.9σ at 30 fb^{-1}:VH with H → bb recovered as one of the best discovery channels for light Higgs? More (exp) studies to come !

Recap on jets

- Two major jet classes: sequential (k_t, CA, ...) and cones (UA1, midpoint, ...)
- Jet algo is fully specified by: clustering + recombination + split merge or removal procedure + all parameters
- Standard cones based on seeds are IR unsafe
- SISCone is new IR safe cone algo. (no seeds)
- Using IRunsafe algos you might not be able to use available higher order calculations
- Using IRsafe algos: can do sophisticated studies e.g. jet-areas for pile-up subtraction
- Not all algos fare the same for BSM/Higgs searches: quality measures quantify this
- Also remember importance of jet substructure (Higgs example)

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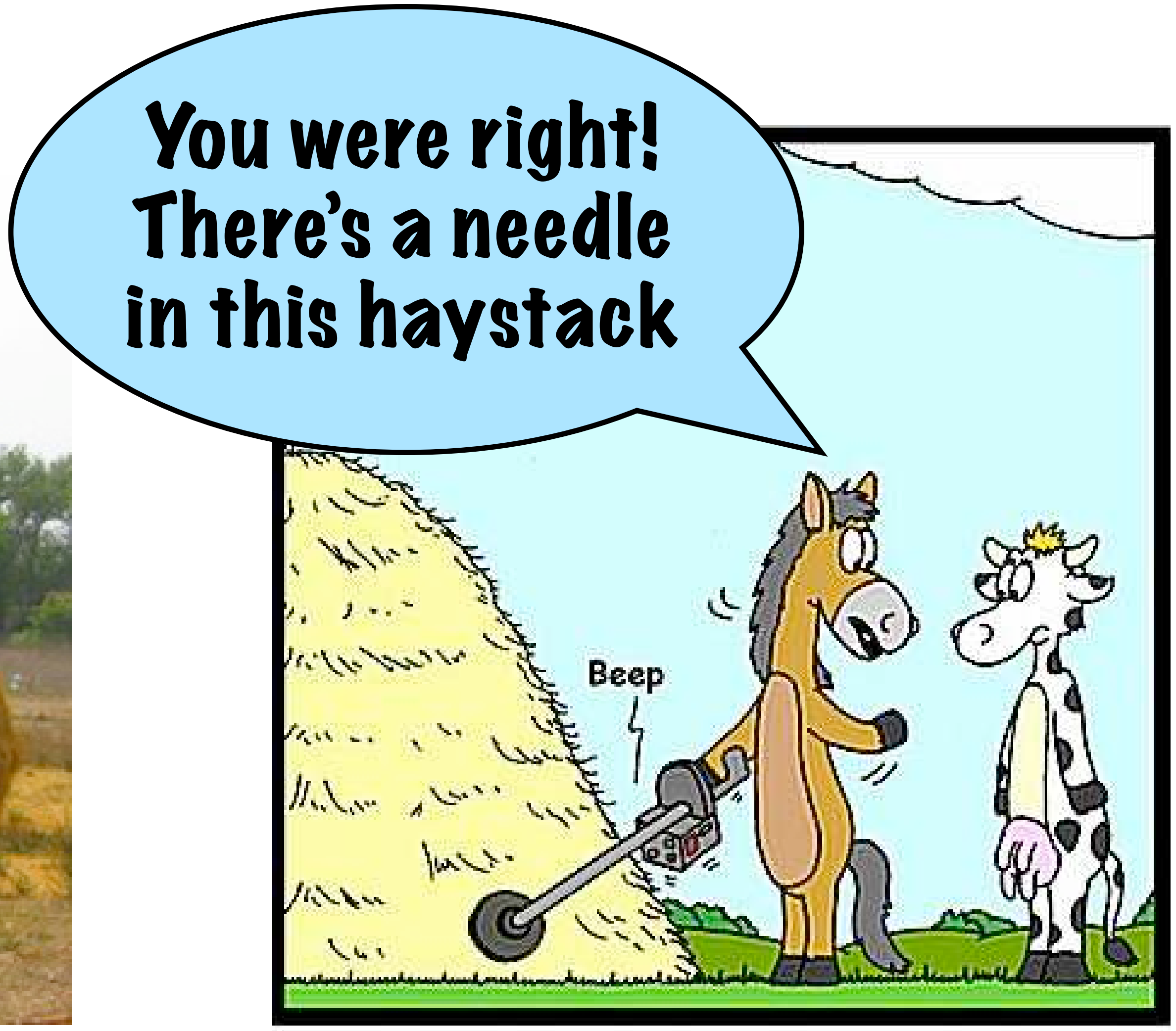

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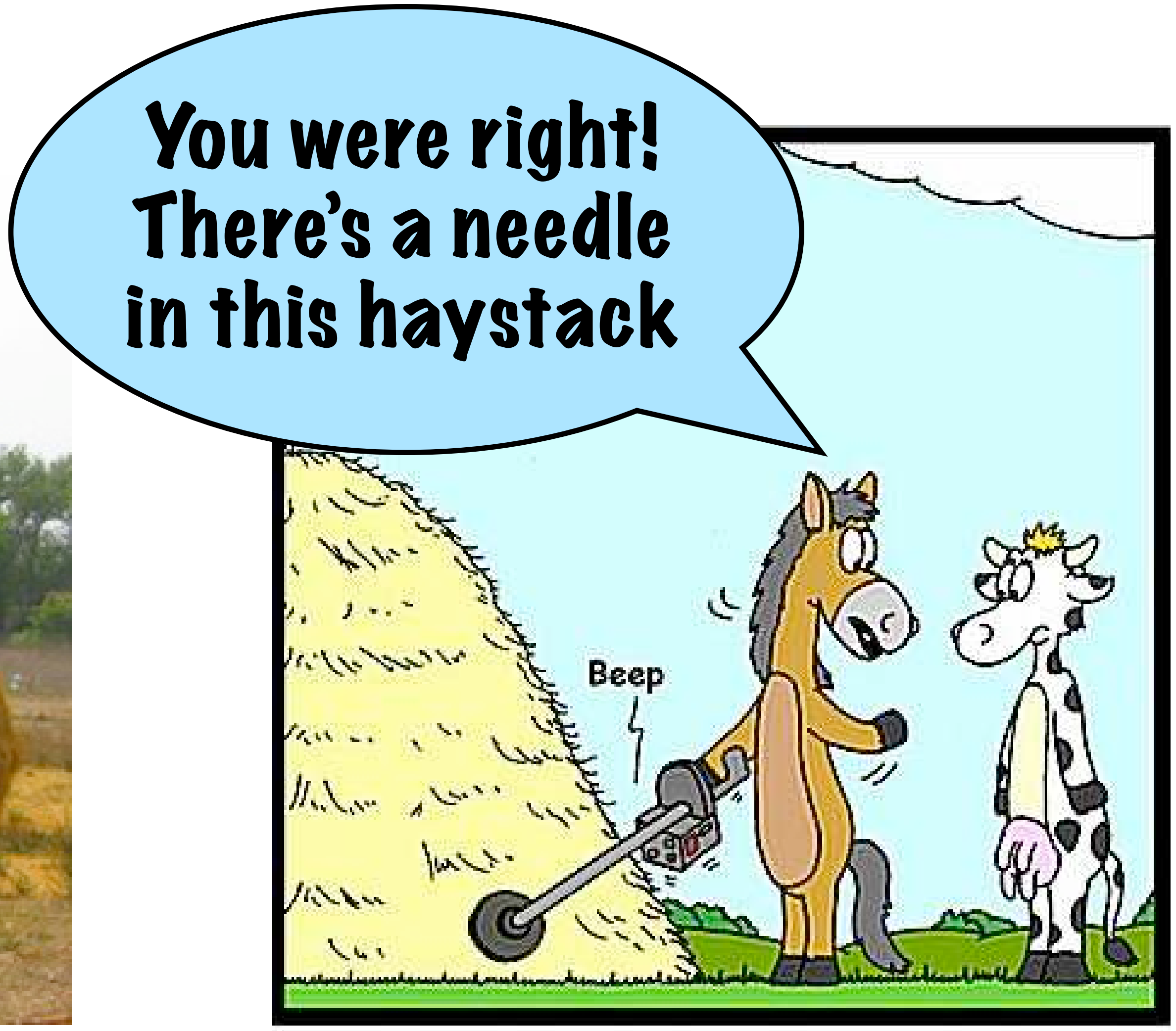

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UNDERSTANDING QCD CRUCIAL TO DEVELOP THE RIGHT TOOLS!