Next-to-leading order contributions to the pole mass of gluino in the minimal gauge mediation (in progress)

Jae Yong Lee (Korea University)

in collaboration with Yeo Woong Yoon (KIAS)

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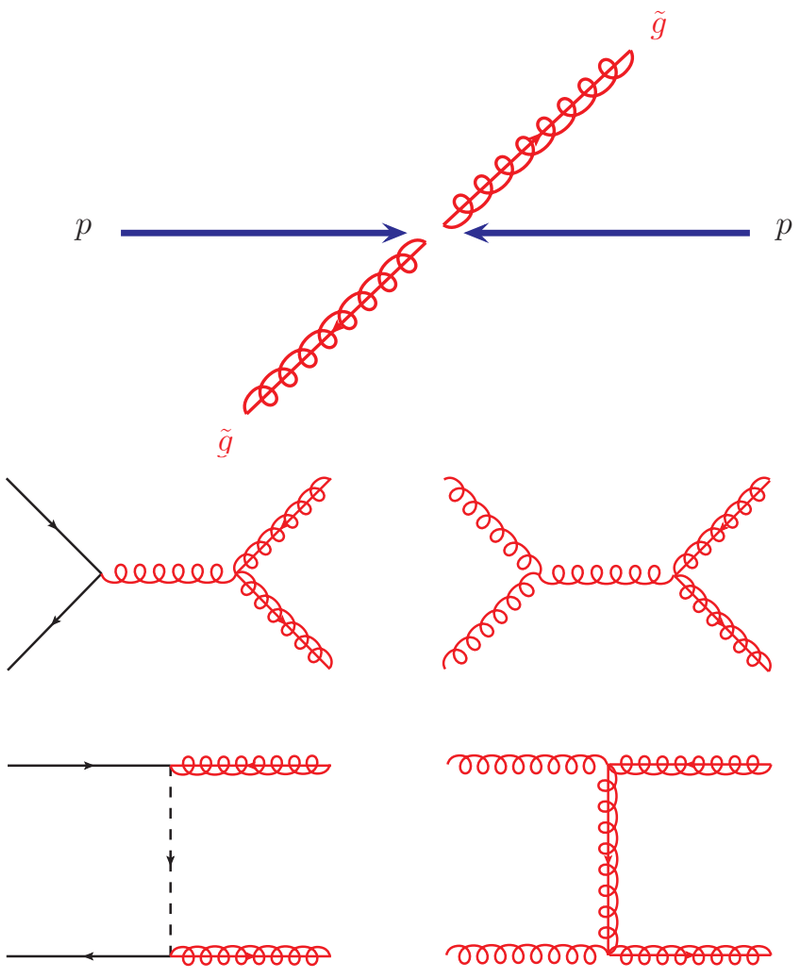
Outline

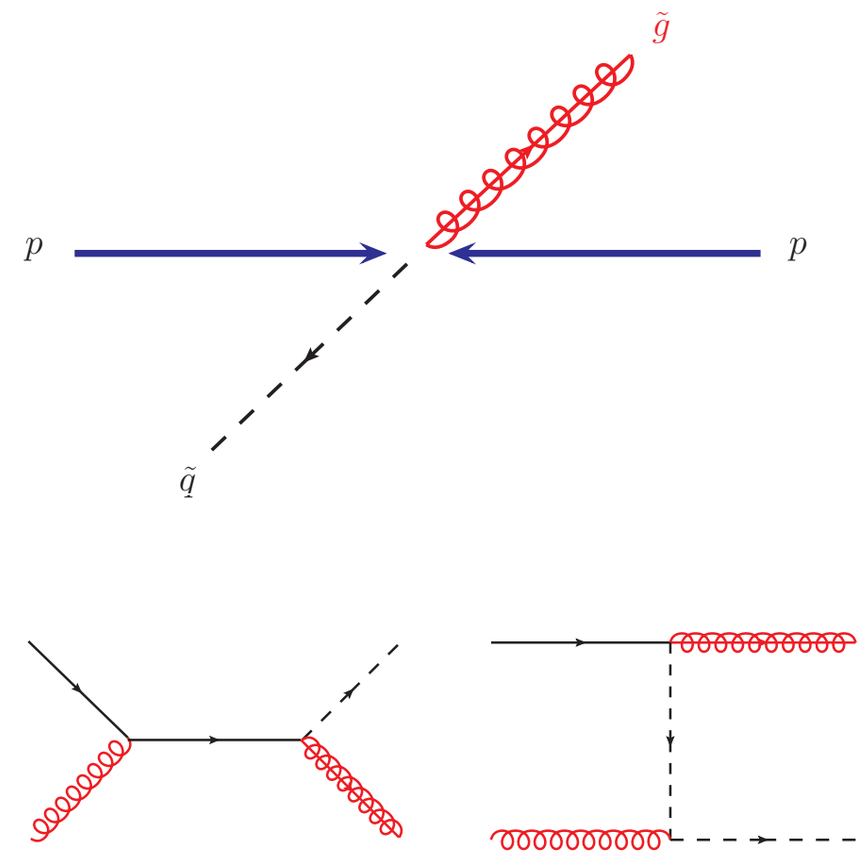
- Gluino in the LHC
- Gluino - Majorana fermion
- Motivation - the study of gluino pole mass in cMSSM
- Gluino in (Minimal) Gauge Mediation
- Gluino Pole Mass at leading order
- Gluino Pole Mass at next-to-leading order
- Summary and Outlook

Gluino in the LHC

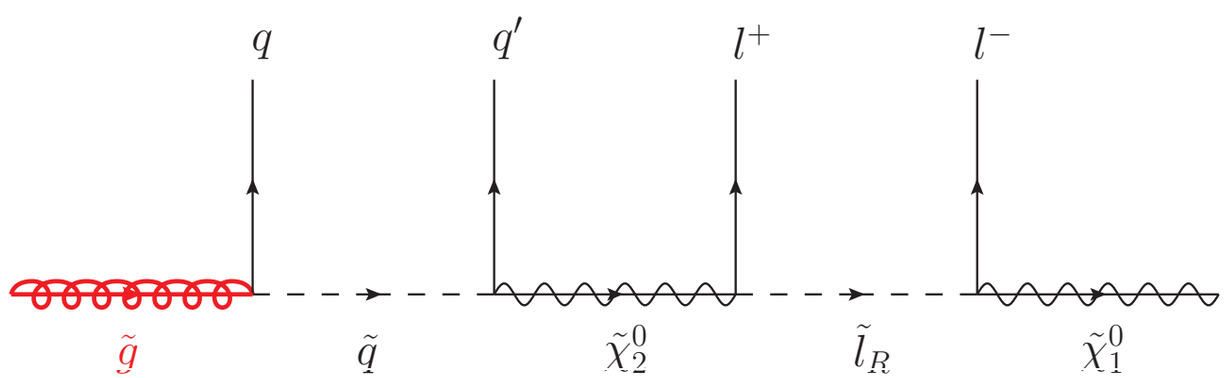
Gluino pair production



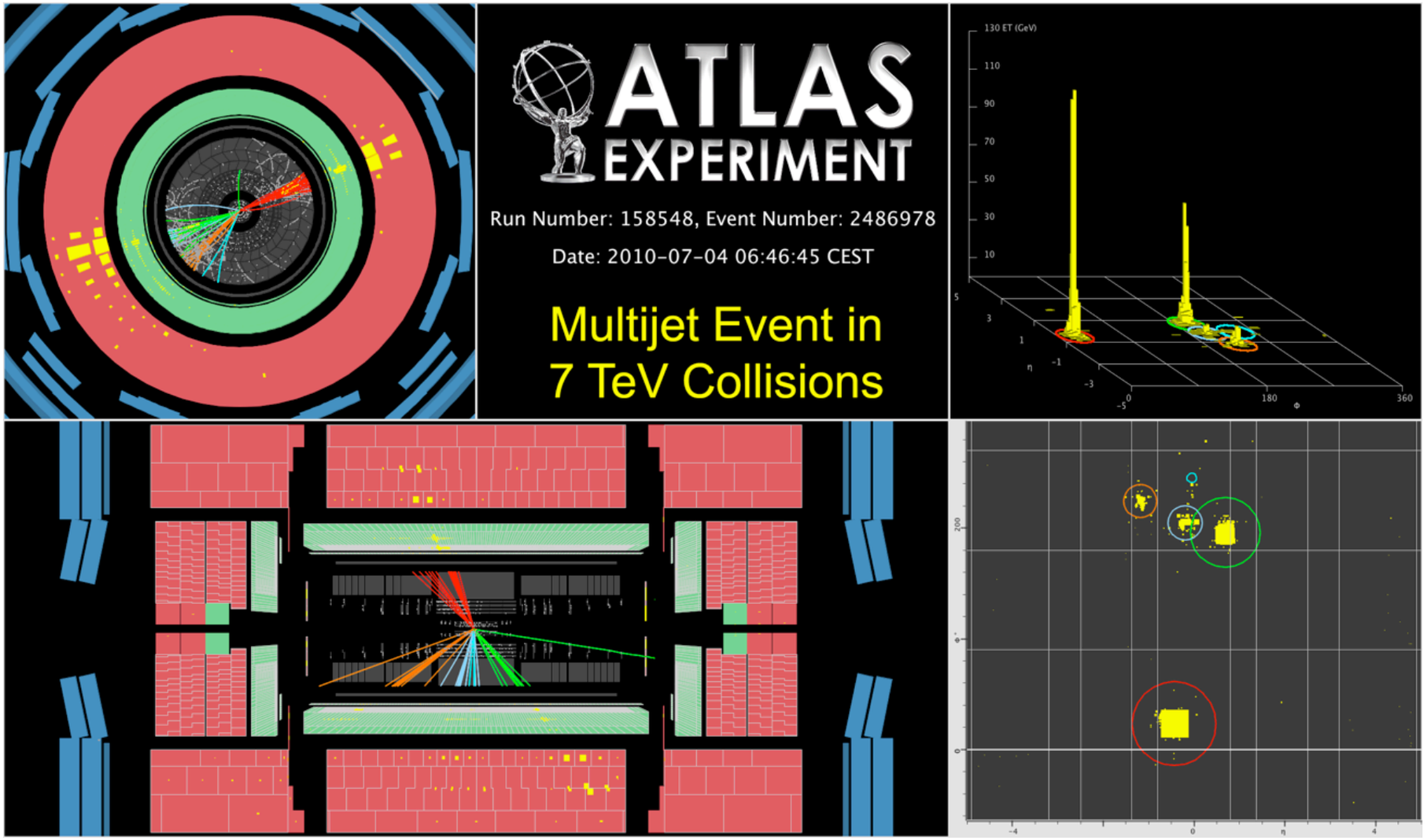
Single gluino production



Gluino decay



What are we looking for at the LHC?



Mass of gluino?

Gluino - Majorana fermion

We adopt two-component notation. H.K. Dreiner, H.E. Haber, S.P. Martin 2010

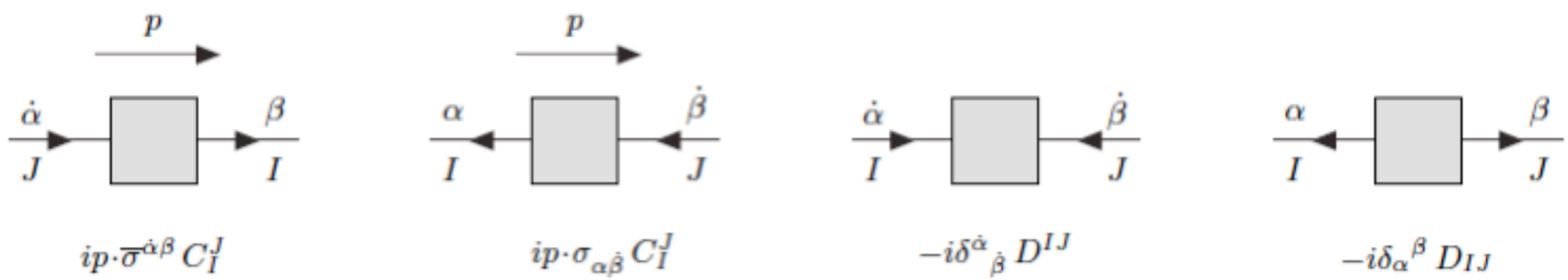


FIG. 1: The full loop-corrected propagators for fermions in two-component notation are associated with functions $C_I^J(s)$, $D^{IJ}(s)$, and $D_{IJ}(s)$, as shown. Here $s = -p^2$ is the external momentum invariant. The shaded boxes represent the sum of all connected Feynman diagrams, and the external legs are included.

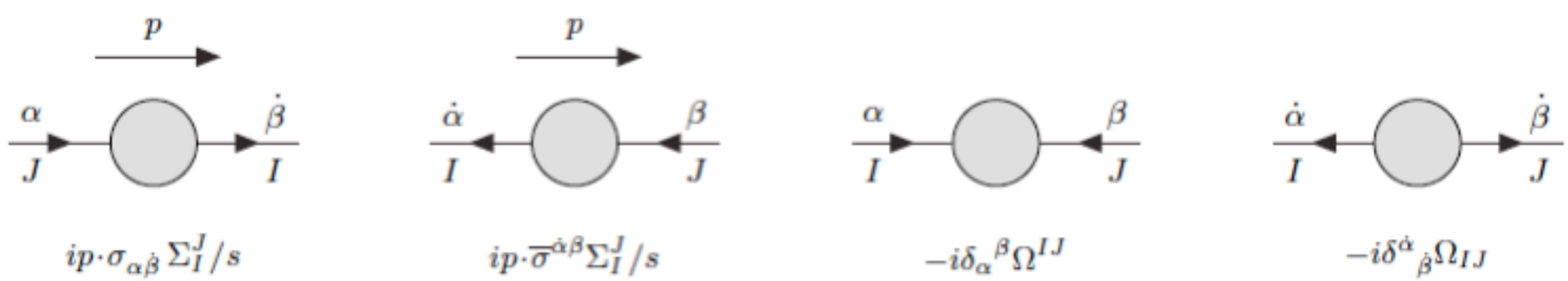


FIG. 2: The self-energies for fermions in two-component notation are associated with functions $\Sigma_J^I(s)$ (for chirality-preserving propagation), and $\Omega^{IJ}(s)$ and $\Omega_{IJ}(s)$ (for chirality-violating propagation), defined as shown. Here $s = -p^2$ is the external momentum invariant. The shaded circles represent the sum of all one-particle irreducible, connected Feynman diagrams, and the external legs are not included.

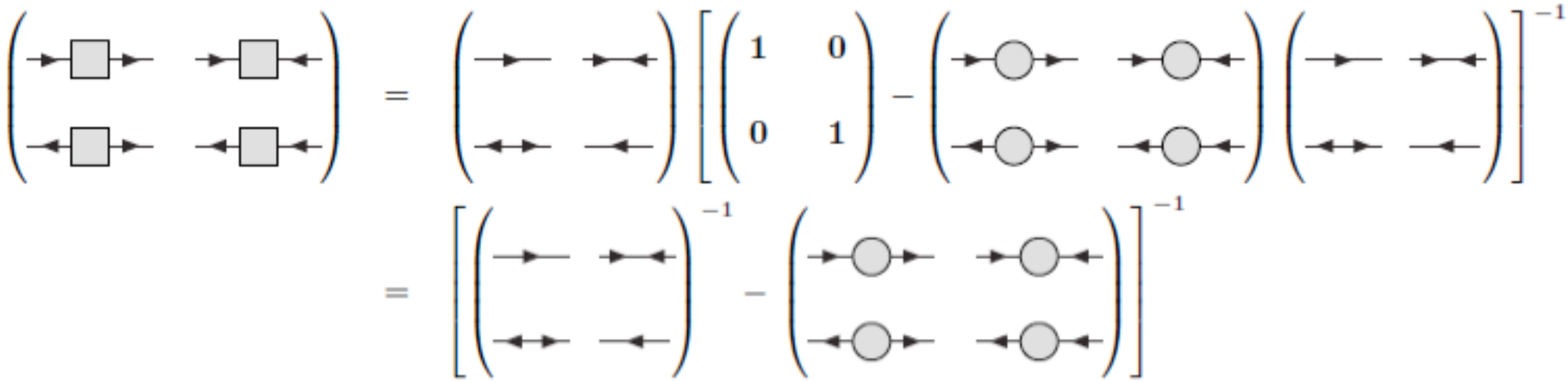


FIG. 3: The diagrammatic version of the fermion self-energy identity in eq. (2.43).

$$\begin{pmatrix} ip\cdot\bar{\sigma}\,\mathbf{C}^T & -i\mathbf{D} \\ -i\overline{\mathbf{D}} & ip\cdot\sigma\,\mathbf{C} \end{pmatrix} = \begin{pmatrix} ip\cdot\sigma\,[\mathbf{1}-\mathbf{\Sigma}^T/s] & i[\mathbf{m}+\mathbf{\Omega}] \\ i[\overline{\mathbf{m}}+\overline{\mathbf{\Omega}}] & ip\cdot\bar{\sigma}\,[\mathbf{1}-\mathbf{\Sigma}/s] \end{pmatrix}^{-1}.$$

$$\begin{aligned}
&\mathbf{C}_I{}^J = (\mathbf{C}^T)^J{}_I \equiv C_I^J, \\
&\mathbf{D}^{IJ} \equiv D^{IJ}, \qquad \overline{\mathbf{D}}_{IJ} \equiv D_{IJ}, \\
&\mathbf{\Sigma}_I{}^J = (\mathbf{\Sigma}^T)^J{}_I \equiv \Sigma_I^J, \\
&\mathbf{\Omega}^{IJ} \equiv \Omega^{IJ}, \qquad \overline{\mathbf{\Omega}}_{IJ} \equiv \Omega_{IJ}, \\
&\mathbf{m}^{IJ} \equiv m^{IJ}, \qquad \overline{\mathbf{m}}_{IJ} \equiv m_{IJ},
\end{aligned}$$

$$\begin{aligned}
\Sigma_I^J &= \frac{1}{16\pi^2}\Sigma_I^{(1)J} + \frac{1}{(16\pi^2)^2}\Sigma_I^{(2)J} + \ldots \\
\Omega^{IJ} &= \frac{1}{16\pi^2}\Omega^{(1)IJ} + \frac{1}{(16\pi^2)^2}\Omega^{(2)IJ} + \ldots \\
\Omega_{IJ} &= \frac{1}{16\pi^2}\Omega_{IJ}^{(1)} + \frac{1}{(16\pi^2)^2}\Omega_{IJ}^{(2)} + \ldots
\end{aligned}$$

$$\mathrm{Det}\big[s\mathbf{1} - (\mathbf{1}-\mathbf{\Sigma}/s)^{-1}(\overline{\mathbf{m}}+\overline{\mathbf{\Omega}})(\mathbf{1}-\mathbf{\Sigma}^T/s)^{-1}(\mathbf{m}+\mathbf{\Omega})\big] = 0.$$

$$s_{\mathrm{pole},I} = M_I^2 - i\Gamma_I M_I$$

Gluino pole mass in Gravity Mediation (cMSSM)

$$\mathcal{L} \supset -\frac{M_3}{2}(\tilde{g}\tilde{g} + \tilde{g}^\dagger\tilde{g}^\dagger)$$

$$m_{\tilde{g}} = M_3 + \delta m_{\tilde{g}}^{(1)} + \delta m_{\tilde{g}}^{(2)} + \cdots$$

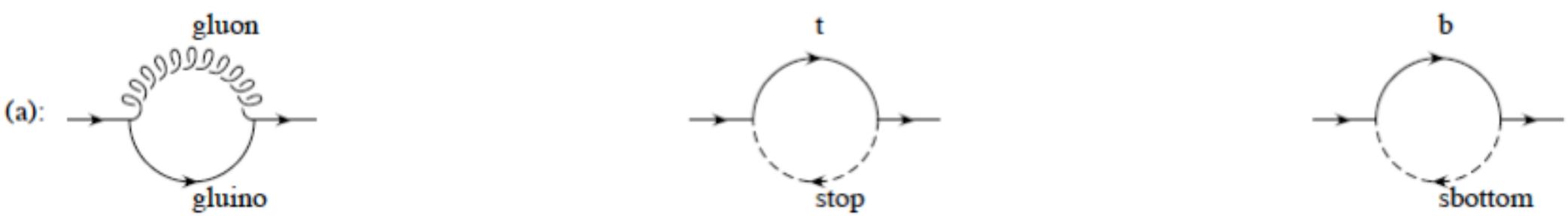
$$\delta m_{\tilde{g}}^{(1)} = \frac{C_V \alpha_s}{4\pi}\left(5 - 6\log\frac{M_3}{Q}\right) + \frac{\alpha_s}{\pi} N_q T_F M_3 B_1(M_3^2, 0, m_{\tilde{g}})$$

D. Pierce, A. Papadopoulos '94

N. V. Krasnikov '95

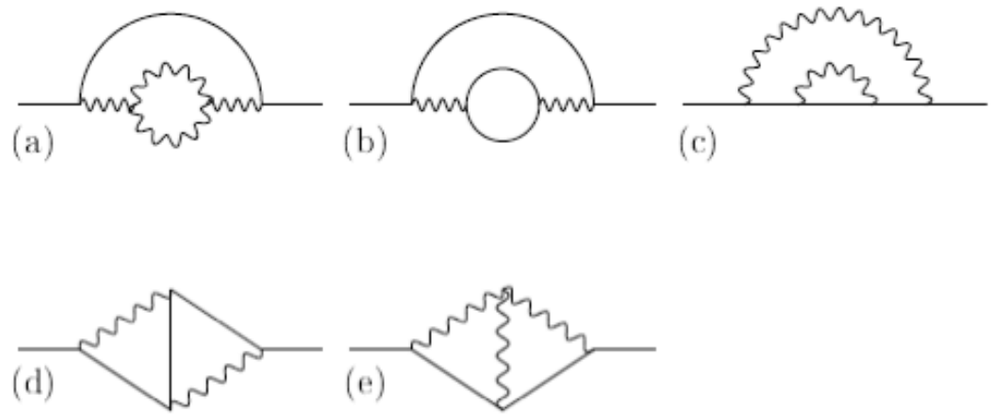
A. Donini '96

D. Pierce, J.A. Bagger, K.T. Matchev, R.J Zhang '97



lots of two-loop diagrams

$\delta m_{\tilde{g}}^{(2)}$



Y. Yamada '05
S. Martin '05

Figure 1: Two-loop $O(\alpha_s^2)$ contributions to the gluino self energy, without quark and squark propagators.

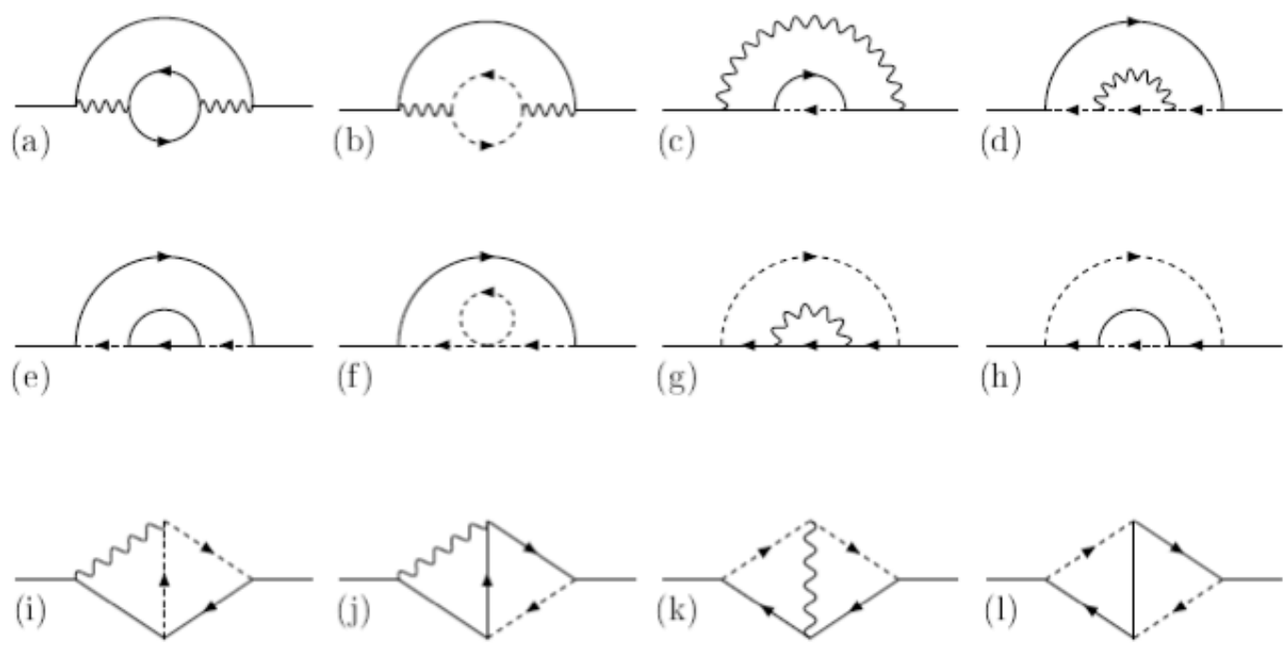


Figure 2: Two-loop $O(\alpha_s^2)$ contributions to the gluino self energy, with quark and squark propagators. Other diagrams obtained by charge conjugation are not shown.

Y. Yamada '05

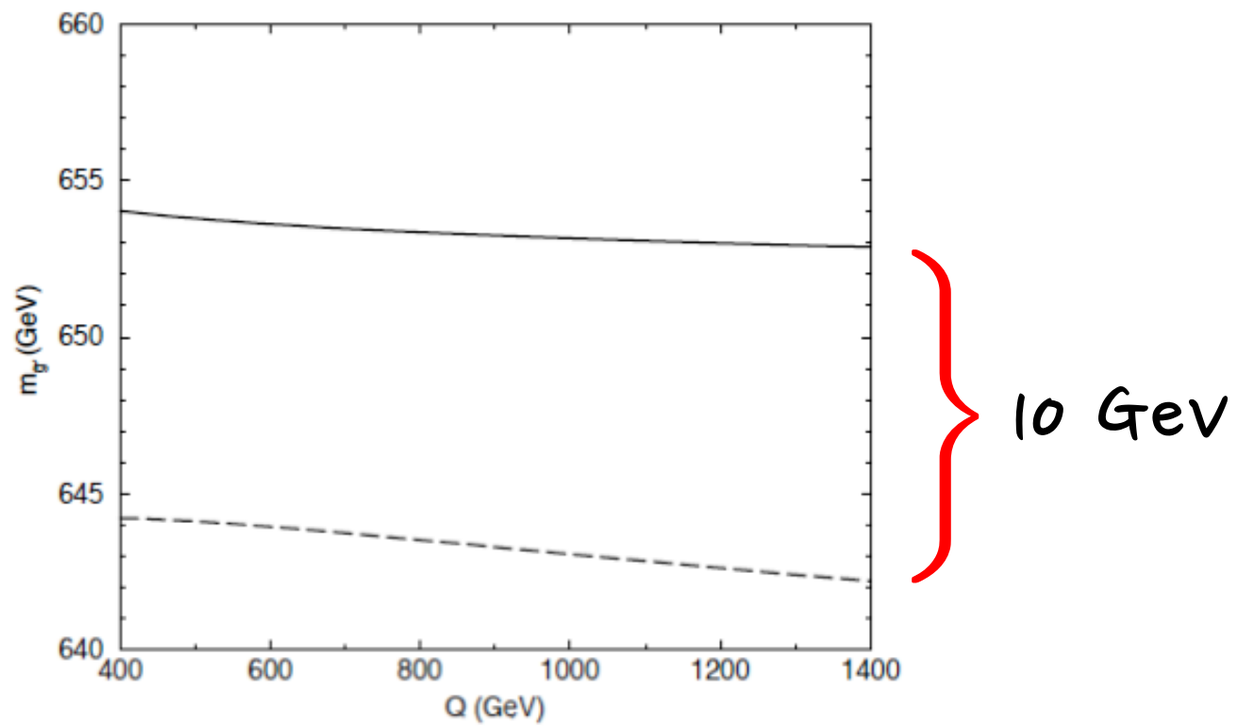


Figure 3: Dependence of the one-loop (dashed) and two-loop (solid) pole masses of the gluino on the renormalization scale Q. Mass parameters are $(M_3, m_{\tilde{q}}) = (580, 800)$ GeV at $Q = 580$ GeV.

$$M_3 = 580 \text{ GeV}, \quad \delta m_{\tilde{g}}^{(1)} \approx 60 \text{ GeV}, \quad \delta m_{\tilde{g}}^{(2)} \approx 10 \text{ GeV}$$

10 % 2 %

Gluino in (Minimal) Gauge Mediation

Lagrangian of Minimal Gauge Mediation

$$
\begin{aligned}
\mathcal{L} \supset & -\frac{1}{4} v^a_{\mu\nu} v^{a\mu\nu} + i\lambda^{\dagger a}\bar{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2}\mathcal{D}^a\mathcal{D}^a \\
& + (D_\mu\phi)^\dagger (D^\mu\phi) + i\psi^\dagger\bar{\sigma}^\mu(D_\mu\psi) + \mathcal{F}^\dagger\mathcal{F} \\
& + (D_\mu\tilde{\phi})(D^\mu\tilde{\phi})^\dagger + i\tilde{\psi}\sigma^\mu(D_\mu\tilde{\psi}^\dagger) + \tilde{\mathcal{F}}\tilde{\mathcal{F}}^\dagger \\
& - \sqrt{2}g(\phi^\dagger \boldsymbol{T}^a\psi - \tilde{\psi}\boldsymbol{T}^a\tilde{\phi}^\dagger)\lambda^a - \sqrt{2}g\lambda^{a\dagger}(\psi^\dagger\boldsymbol{T}^a\phi - \tilde{\phi}\boldsymbol{T}^a\tilde{\psi}^\dagger) \\
& + g\mathcal{D}^a(\phi^\dagger\boldsymbol{T}^a\phi - \tilde{\phi}\boldsymbol{T}^a\tilde{\phi}^\dagger),
\end{aligned}
$$

$$
\begin{aligned}
(D_\mu\phi)_i &= \partial_\mu\phi_i + igv^a_\mu(\boldsymbol{T}^a)_i{}^j\phi_j, \\
(D_\mu\tilde{\phi})^i &= \partial_\mu\tilde{\phi}^i - igv^a_\mu\tilde{\phi}^j(\boldsymbol{T}^a)_j{}^i \\
(D_\mu\tilde{\phi})^\dagger_i &= \partial_\mu\tilde{\phi}^\dagger_i + igv^a_\mu(\boldsymbol{T}^a)_i{}^j\tilde{\phi}^\dagger_j, \\
(D_\mu\psi)_i &= \partial_\mu\psi_i + igv^a_\mu(\boldsymbol{T}^a)_i{}^j\psi_j, \\
(D_\mu\tilde{\psi}^\dagger)_i &= \partial_\mu(\tilde{\psi}^\dagger)_i + igv^a_\mu(\boldsymbol{T}^a)_i{}^j(\tilde{\psi}^\dagger)_j, \\
D_\mu\lambda^a &= \partial_\mu\lambda^a - gf^{abc}v^b_\mu\lambda^c, \\
v^a_{\mu\nu} &= \partial_\mu v^a_\nu - \partial_\nu v^a_\mu - gf^{abc}v^b_\mu v^c_\nu
\end{aligned}
$$

(ϕ, ψ)	messenger, fundamental
$(\tilde{\phi}, \tilde{\psi})$	messenger, anti-fundamental
λ	gaugino, adjoint

Superpotential

$W \supset X\tilde{\Phi}\Phi,$

singlet $\langle X \rangle = M_0 + \theta\theta F_X,$

(anti-)fundamental messenger: $\tilde{\Phi} = (\tilde{\phi}, \tilde{\psi}),\ \Phi = (\phi, \psi)$

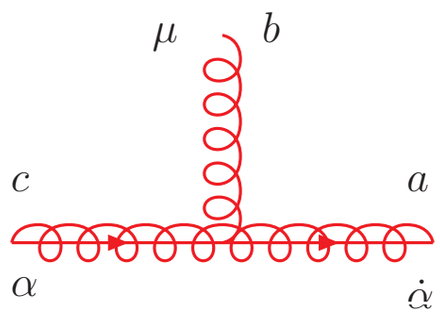
gauge eigenstates: $(\tilde{\phi}, \phi) \quad \Rightarrow \quad$ mass eigenstates: (ϕ_+, ϕ_-)

$$\mathcal{L}_{\text{mass}} \supset -\frac{M_0}{2}(\tilde{\psi}\psi + \tilde{\psi}^\dagger\psi^\dagger) - M_+^2\phi_+^\dagger\phi_+ - M_-^2\phi_-^\dagger\phi_-$$

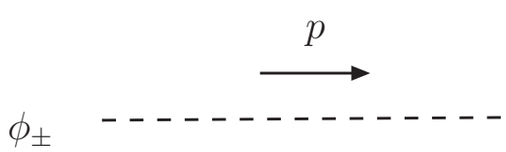
$$\boxed{M_\pm^2 = M_0^2 \pm F_X}$$

Feynman rule 1

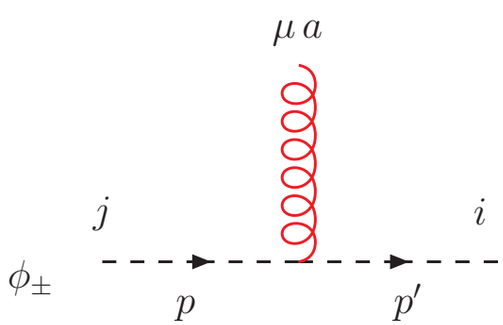
$$-\frac{ip\cdot\bar\sigma^{\dot\alpha\alpha}}{p^2} \quad \text{or} \quad \frac{ip\cdot\sigma_{\alpha\dot\alpha}}{p^2}$$



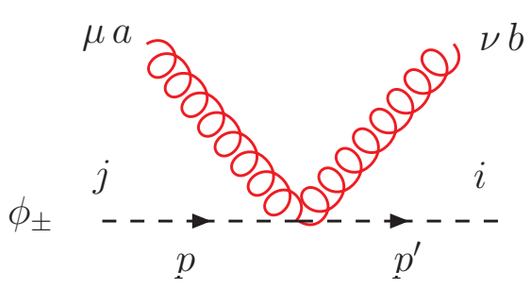
$$g f^{abc}\bar\sigma_\mu^{\dot\alpha\alpha} \quad \text{or} \quad -g f^{abc}\sigma_{\mu\alpha\dot\alpha}$$



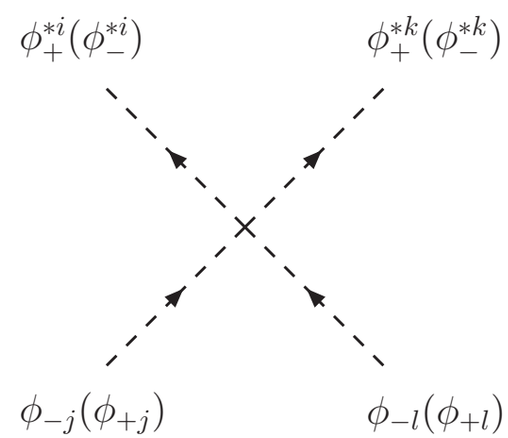
$$\frac{i}{p^2-M_\pm^2}$$



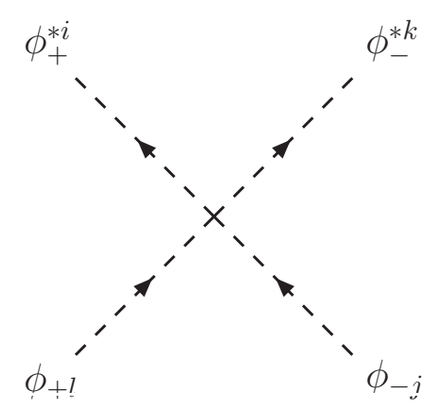
$$-ig(T^a)_i^{\ j}(p+p')_\mu$$



$$ig^2 g^{\mu\nu}\{T^a,T^b\}\delta_i^{\ j}$$

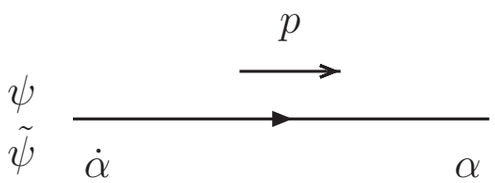


$$-2ig^2\textstyle\sum_a (T^a)_i^{\ j}(T^a)_k^{\ l}$$

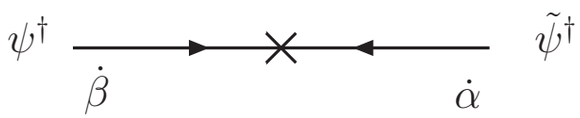


$$-ig^2\textstyle\sum_a (T^a)_i^{\ j}(T^a)_k^{\ l}$$

Feynman rule II

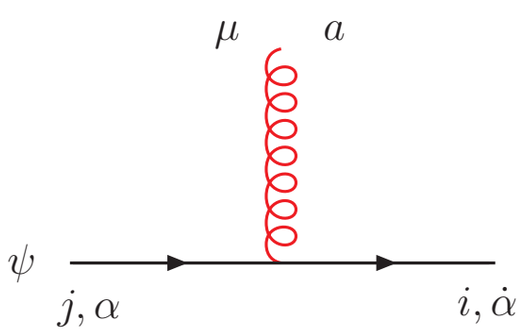


$$-\frac{ip\cdot\bar{\sigma}^{\dot{\alpha}\alpha}}{p^2-M_0^2} \quad \text{or} \quad \frac{ip\cdot\sigma_{\alpha\dot{\alpha}}}{p^2-M_0^2}$$

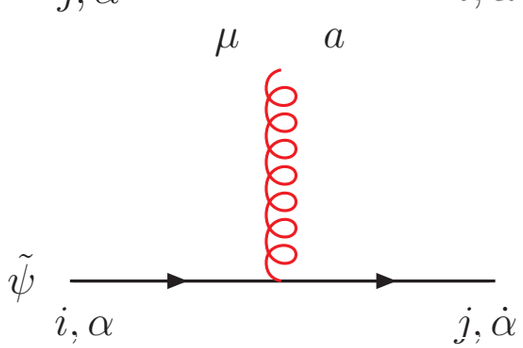


$$\frac{iM_0}{p^2-M_0^2}\delta^{\dot{\alpha}}_{\dot{\beta}}$$

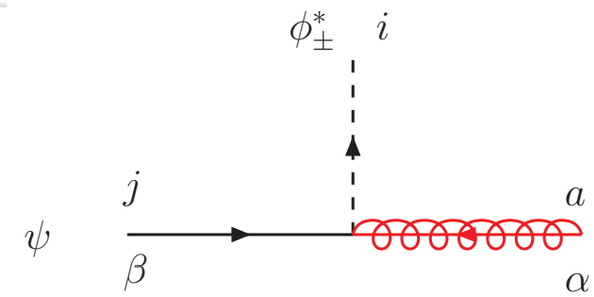
$$\frac{iM_0}{p^2-M_0^2}\delta^{\beta}_{\alpha}$$



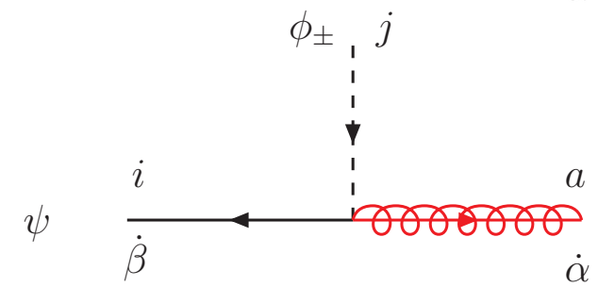
$$-ig(T^a)_i{}^j\bar{\sigma}^{\dot{\alpha}\alpha}_\mu \quad \text{or} \quad ig(T^a)_i{}^j\sigma_{\mu\alpha\dot{\alpha}}$$



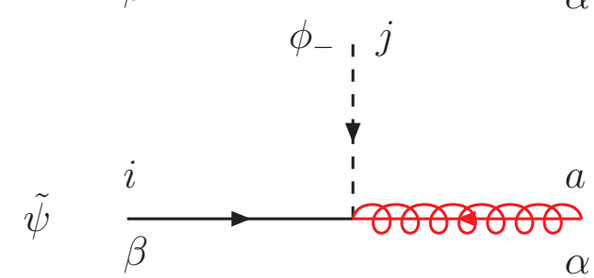
$$ig(T^a)_i{}^j\bar{\sigma}^{\dot{\alpha}\alpha}_\mu \quad \text{or} \quad -ig(T^a)_i{}^j\sigma_{\mu\alpha\dot{\alpha}}$$



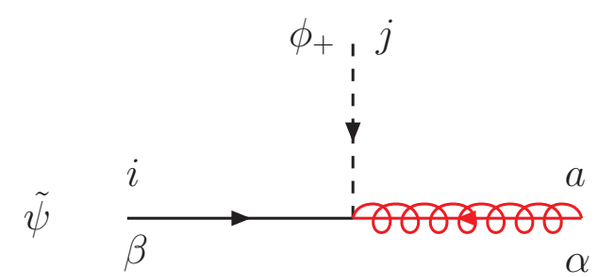
$$-ig(T^a)_i{}^j\delta^{\beta}_{\alpha}$$



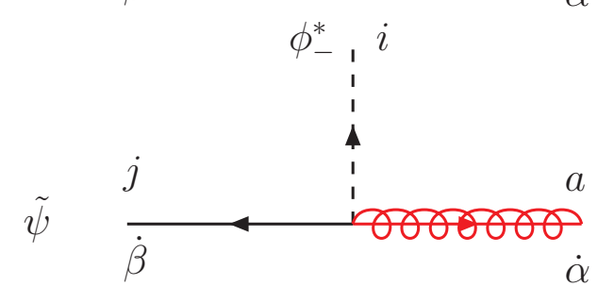
$$-ig(T^a)_i{}^j\delta^{\dot{\beta}}_{\dot{\alpha}}$$



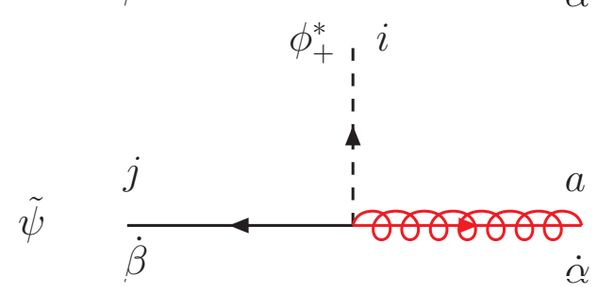
$$ig(T^a)_i{}^j\delta^{\beta}_{\alpha}$$



$$-ig(T^a)_i{}^j\delta^{\beta}_{\alpha}$$



$$ig(T^a)_i{}^j\delta^{\dot{\alpha}}_{\dot{\beta}}$$



$$-ig(T^a)_i{}^j\delta^{\dot{\alpha}}_{\dot{\beta}}$$

Gluino pole mass in the minimal gauge mediation

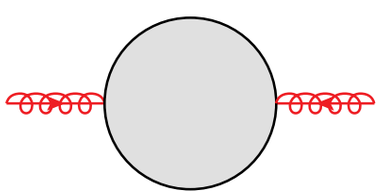
$$s - \frac{\Omega^2}{(1-\Sigma/s)^2} = 0 \quad \Rightarrow \quad s = \frac{1}{2}\left(\sqrt{4\Sigma\Omega^2 + \Omega^4} + 2\Sigma + \Omega^2\right)$$

calculate perturbatively:

$$\Sigma = \Sigma^{(1)} + \Sigma^{(2)} + \cdots ,$$
$$\Omega = \Omega^{(1)} + \Omega^{(2)} + \cdots$$

Gluino Pole mass at leading order

$\Omega^{(1)} =$



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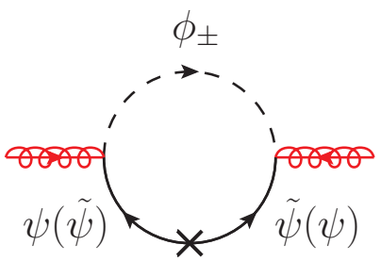
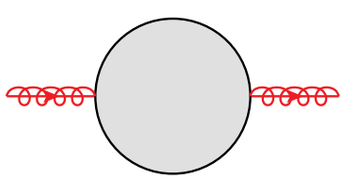


$$\int \frac{d^4 l}{(2\pi)^4}(-ig)\frac{iM_0}{l^2 - M_0^2}(-ig)\frac{i}{(l+p)^2 - M_+^2}$$
$$+ \int \frac{d^4 l}{(2\pi)^4}(-ig)\frac{iM_0}{l^2 - M_0^2}(ig)\frac{i}{(l+p)^2 - M_-^2}$$

No UV divergence!

$\Sigma^{(1)} =$



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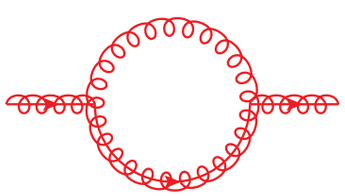

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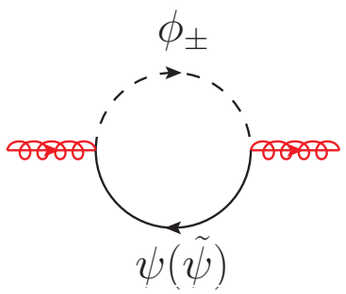


$+$

$$
\begin{aligned}
\Omega^{(1)} &= m_f \frac{2\alpha_s}{4\pi} C_2(R)(4\pi)^{\varepsilon}[I_{22}(\mu^2, q^2, m^2_{\phi^+}, m^2_f) - I_{22}(\mu^2, q^2, m^2_{\phi^-}, m^2_f)],\\
\Sigma^{(1)}_1 &= \frac{\alpha_s}{4\pi} C_2(G)(4\pi)^{\varepsilon} I_{21}(\mu^2, q^2),\\
\Sigma^{(1)}_2 &= -\frac{\alpha_s}{4\pi} C_2(G)(4\pi)^{\varepsilon}\big[\frac{1}{2q^2}\big(I_1(\mu^2, m^2_{\phi^+}) + I_1(\mu^2, m^2_{\phi^-})\big) - \frac{1}{q^2} I_1(\mu^2, m^2_f)\\
&\quad + \frac{q^2 + m^2_f - m^2_{\phi^+}}{2q^2} I_{22}(\mu^2, q^2, m^2_{\phi^+}, m^2_f) + \frac{q^2 + m^2_f - m^2_{\phi^-}}{2q^2} I_{22}(\mu^2, q^2, m^2_{\phi^-}, m^2_f)\big].
\end{aligned}
$$

where

$$
\begin{aligned}
I_1 &= -m_f^2 \Big(\frac{\mu^2}{m_f^2}\Big)^{\epsilon} (e^{\gamma_E \epsilon}) \Gamma(-1+\epsilon)\,.\\
I_{21} &= \Big(\frac{\mu^2}{-q^2}\Big)^{\epsilon} (e^{\gamma_E \epsilon}) \frac{\Gamma(1-\epsilon)^2 \Gamma(\epsilon)}{\Gamma(2-2\epsilon)}\,.\\
\Big(\frac{\mu^2}{m_f^2}\Big)^{-\epsilon} I_{22} &= \frac{1}{\epsilon} + \left(\frac{\eta_1 - \eta_2}{2}\right) \ln\left(\frac{1 + x\eta_2}{1 + x\eta_1}\right) - \frac{\ln r}{2x}(1 + x - r) + 2\\
&\quad + J_{22}(x, r)\,\epsilon + \mathcal{O}(\epsilon^2)\,.
\end{aligned}
$$

with

$$
\begin{aligned}
x &= -\frac{q^2}{m_f^2}, \quad r = \frac{m^2_{\phi_+}}{m_f^2}\,, \quad \tilde{r} = \frac{m^2_{\phi_-}}{m_f^2}\\
\eta_1 &= \frac{r + x - 1}{2x} + \frac{1}{2x}\sqrt{r^2 + 2\,r(x-1) + (x+1)^2}\\
\eta_2 &= \frac{r + x - 1}{2x} - \frac{1}{2x}\sqrt{r^2 + 2\,r(x-1) + (x+1)^2}\,.
\end{aligned}
$$

Gluino Pole Mass at Next-to-Leading Order

(in progress)

Issues for jumping to the next-to-leading order calculation

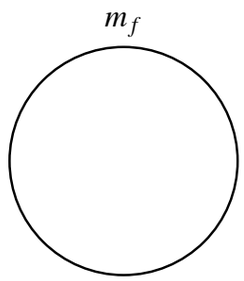
- The number of the 1PI diagrams is large so that the whole calculation is a time-consuming game.
- Propagators with different masses make the calculation a big challenge.
- We use Mathematica to carry out a numerical analysis.

concerns on supersymmetric regularization

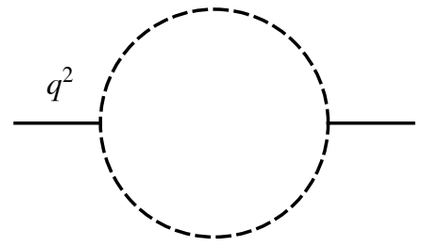
Dimensional Reduction (DR)

- momentum in d-dimensions
- spinor in 4-dimensions
- epsilon-scalar

Master Integrals



I_1



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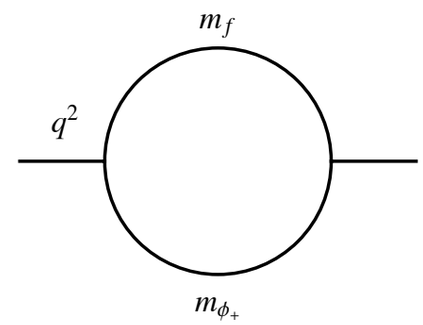

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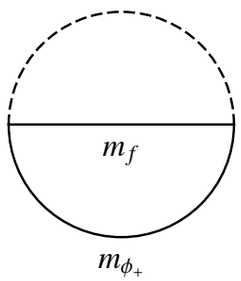

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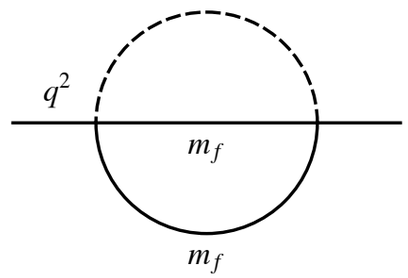
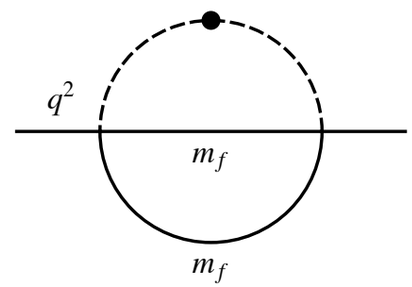
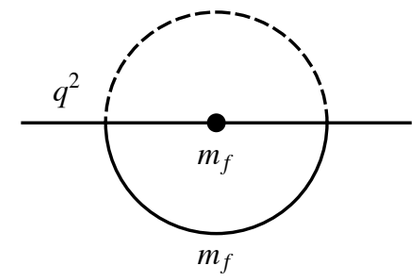


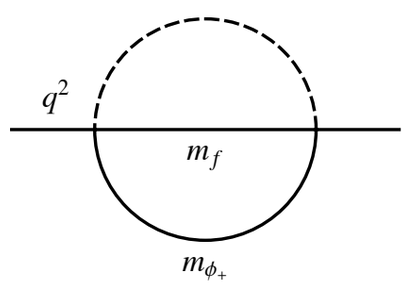
I_{32}



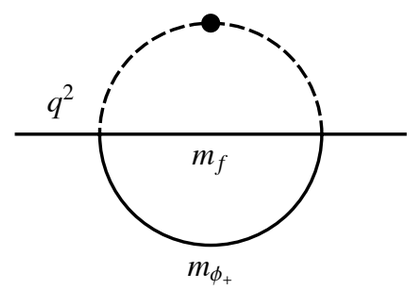
I_{33}



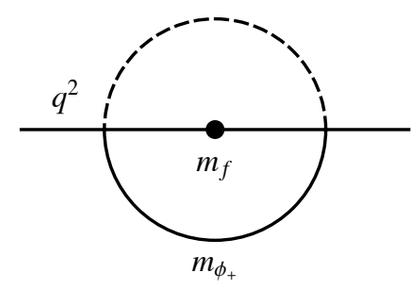
I_{34}



I_{35}



I_{36}



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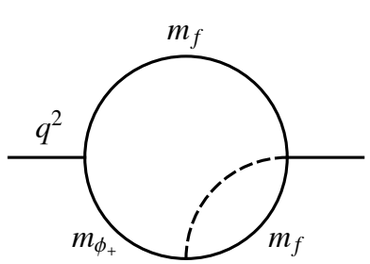

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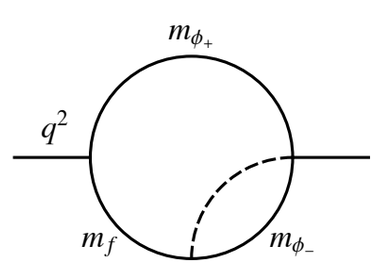
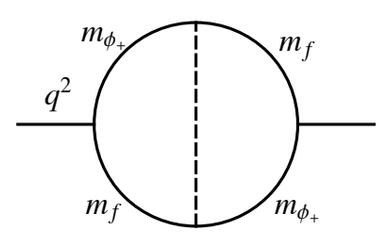
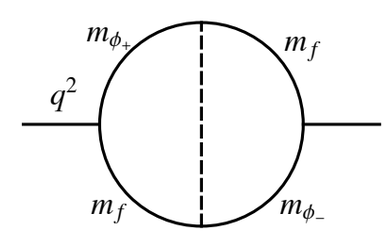


I_{42}

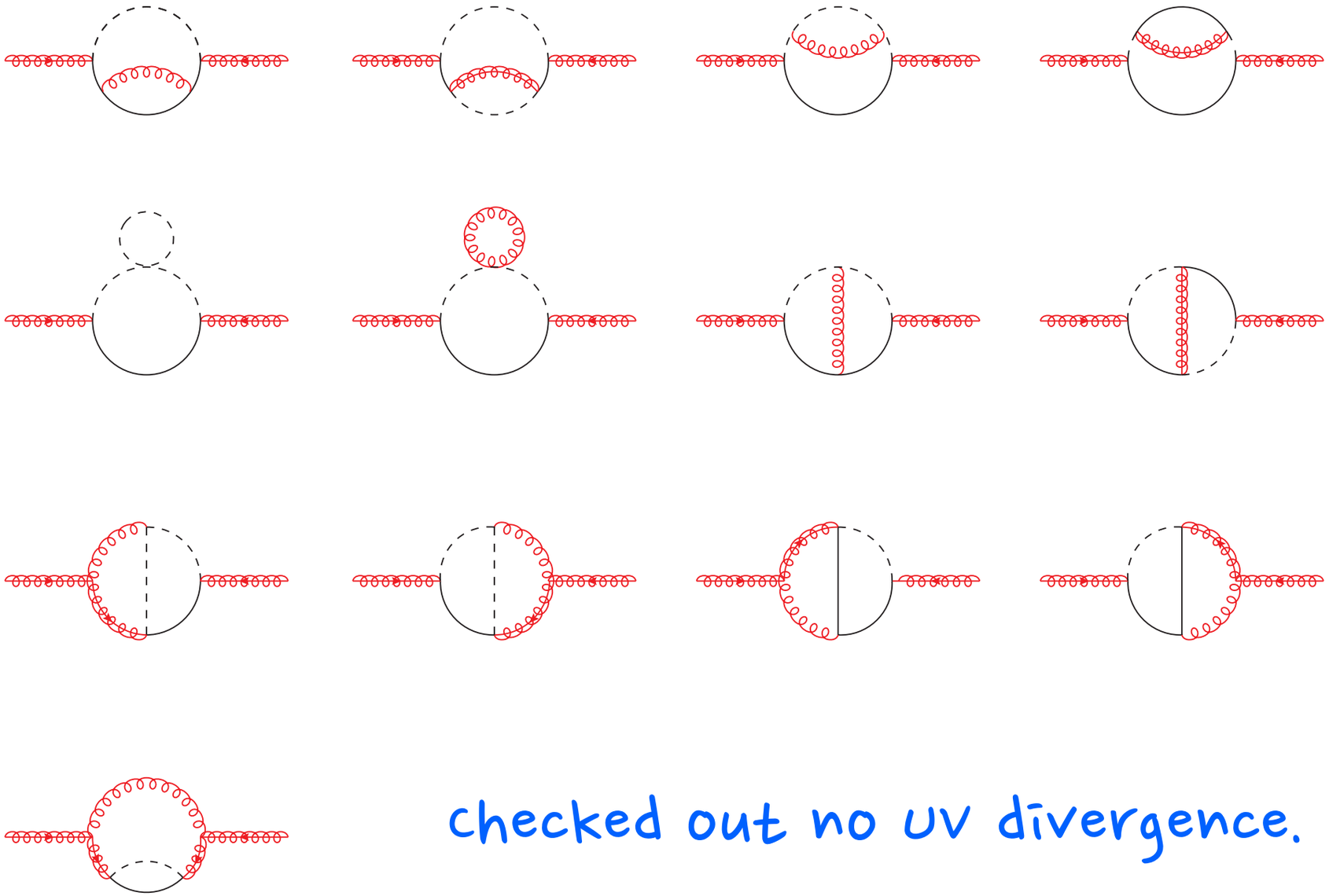


I_{51}



I_{52}

$\Omega^{(2)}$



checked out no UV divergence.

$$
\begin{aligned}
(\text{last diagram}) &= i\left(\frac{\alpha_s}{4\pi}\right)^2 m_f C_2(G) C_2(R) V(m_f^2, x, r) \\
V(m_f^2, x, r) &= \frac{2}{x} J_{35}(x,r) - \frac{(1+r+x)}{x} J_{37}(x,r) - \frac{1}{x}\,\mathrm{Li}_2(1-r) - \frac{r}{x}\,\mathrm{Li}_2\left(\frac{r-1}{r}\right) \\
&\quad + \frac{\left(-2r^2 + rx + 4r - 2x - 2\right)}{2(r-1)x} \ln^2(r) + \left(\frac{5r^2 - 2rx - 4r - x - 1}{(r-1)x} + \frac{r\ln(x)}{r-1}\right)\ln(r) \\
&\quad - \frac{\pi^2 r}{12x} - \frac{13r}{2x} \quad - \left(r \leftrightarrow \tilde{r}\right)
\end{aligned}
$$

We are now in the middle of calculating $\Sigma^{(2)}$.

It has more diagrams compared with $\Omega^{(2)}$.

Stay tuned for the final result in a month or two.

Summary and outlook

- Motivation
  - comparison between gravity mediation and gauge mediation.
  - Review of the pole mass of the gluino in the cMSSM.
- LO gluino pole mass in the MGM.
- NLO contributions to the gluino pole mass in the MGM.
- Future plan
  - NLO contributions to the pole mass of Neutralino in the MGM.
  - NLO gluino pole mass in anomaly mediation.

Thanks for your attention.

Backup : Lagrangian of epsilon-scalar

$$\mathcal{L}_\varepsilon = \frac{1}{2} v^a_{\underline{\mu}} \Box v^{a\underline{\mu}} - g f^{abc} (\partial^\mu v^{a\underline{\nu}}) v^b_\mu v^c_{\underline{\nu}} + \frac{g^2}{2} f^{abc} f^{adf} v^{b\mu} v^{c\underline{\nu}} v^d_\mu v^f_{\underline{\nu}} - \frac{1}{4} \sum_i \lambda_i H_i^{bcdf} v^{b\underline{\mu}} v^{c\underline{\nu}} v^d_{\underline{\mu}} v^f_{\underline{\nu}}$$
$$- i g f^{abc} \lambda^{\dagger a} \bar{\sigma}^{\underline{\mu}} \lambda^c v^b_{\underline{\mu}} - g \psi^\dagger \boldsymbol{T}^a \bar{\sigma}^{\underline{\mu}} \psi v^a_{\underline{\mu}} - g \tilde{\psi} \boldsymbol{T}^a \sigma^{\underline{\mu}} \tilde{\psi}^\dagger v^a_{\underline{\mu}}$$
$$+ g^2 (\phi^* \boldsymbol{T}^a \boldsymbol{T}^b \phi + \tilde{\phi} \boldsymbol{T}^a \boldsymbol{T}^b \tilde{\phi}^\dagger) v^a_{\underline{\mu}} v^{b\underline{\mu}}$$

The epsilon-scalar gives finite contributions when implementing two-loop calculations.

W. Siegel 1979

D.M. Capper, D.R. Jones, P. van Nieuwenhuizen 1980