HCPSS-2010, Introduction to the SM: PS1

Question 1: Global symmetries

We talked about the fact that global symmetries are accidental in the SM, that is, that they are broken once non-renormalizable terms are included. Write the lowest dimension terms that break each of the global symmetries of the SM.

Question 2: UV divergence in QM

In order to understand the way we treat UV divergences, let us study a simple QM problem that have a similar characteristics. We consider a particle in an n-dimensional box. That is, the potential in each direction is given by

$$V(x_i) = \begin{cases} 0 & \text{for } |x_i| < L, \\ \infty & \text{for } |x_i| > L. \end{cases} \tag{1}$$

We add a small perturbation

$$V = \lambda L^n \delta^{(n)}(x). \tag{2}$$

Our task is to calculate the corrections to the ground state energy due to this perturbation. Since the perturbation is only at the origin, it is clear that we care only about the states that are finite at the origin. They are given by

$$\phi(x_1, x_2, ...) = \phi_{n_1}(x_1)\phi_{n_2}(x_2)..., \qquad \phi_{n_i}(x_i) = \sqrt{\frac{1}{L}} \cos\left(\frac{n\pi x_i}{2L}\right),$$
$$E_n = C(n_1^2 + n_2^2 + ...), \qquad C = \frac{\hbar^2\pi^2}{8mL^2}, \qquad n_i = 1, 3, 5, ... \tag{3}$$

1. We start with the one dimensional problem where everything is finite. First, write the formula for the second order perturbation and evaluate it. You may like to recall that

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)^2 - 1} = \frac{1}{4}. \tag{4}$$

   Show that the second order perturbation is finite.

2. We now move to two dimensions. Again, write the second order perturbation correction and try to evaluate it. Show that it is logarithmically divergent. To do so, look only at the very high energy modes and approximate the sum by an integral. You have to show that the integral is logarithmically divergent. What can you say about the higher dimensional cases?

3. We are still in $n = 2$. The divergence you found for this case, however, is not physical. Give a physical argument that explain how the sum is cut off in any real physical system. Change the formalism in a way that incorporated the cut-off in it.

4. So far we just regularized the correction. That is, we can make it finite but still the effect depend on the way we do it finite. Yet, as we argued, the final result must be insensitive to the UV physics. In order to see it, we like to ask how do we measure λ. Lets assume that we measured it by looking at the correction to the ground state energy. Then you can determine λ to first order. Express λ to first order based on the measured deviation from the ground state energy. We denote this λ as λ_P. (Note that the $2, 1$ state does not receive corrections so we can really measure the deviation from the zero order result.)

5. Calculate the correction to the $n = (3, 3)$ level only in terms of measured quantities (to the level of perturbation theory we are working at.) To to that, separate the sum into a "low energy" sum that depend on the specific mode, and a "high energy" sum that to a good approximation in universal. Then, show that the final result depends only on the low energy sum and other measured quantities (like C and $\Delta E_{1,1}$.)

6. Now calculate the correction to the $n = (3, 3)$ level and express it in terms of the measured quantities. You can give your result to 10% accuracy. Feel free to use any software you like (like mathematica, matlab or maple) to evaluate any sum or integral you need. Explain how you could get a higher numerical prediction. Also, explain why it does not make sense to get an extremely high numerical precision.