Calibrating the Energy of the Muon Collider/Neutrino Factory using Spin Precession

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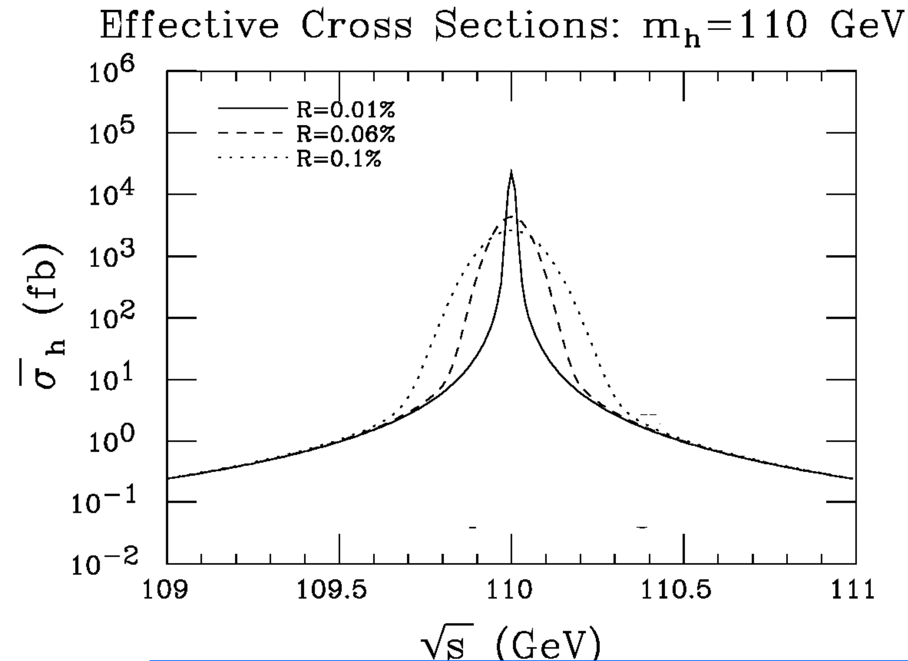
Muon Collider Physics Workshop

Format of talk

- Describe g-2 method.
- R. Raja, A. Tollestrup, **Phys.Rev.D58:013005,1998**
- Planar ideal accelerator
- Explore fractional error in measurement Lorentz factor $\delta\gamma/\gamma$ of muon as a function of number of electrons sampled and polarization.
- Explore variation of $\delta\gamma/\gamma$ as a function of momentum spread $\delta p/p$ of muons
- Examine effects of departures from ideal case.
- Assume that a)Polarized muons can be delivered to the collider ring and b)The polarization can be adequately maintained for 1000 turns.

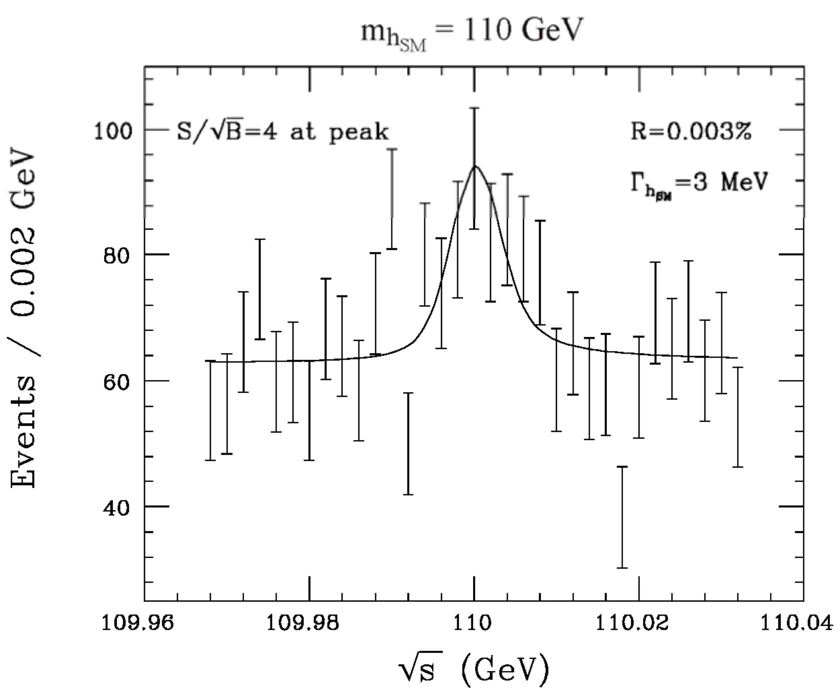
Higgs Factory Scan



$$\sigma^2_{\text{data}} = \sigma^2_{\text{resonance}} + \sigma^2_{\text{beam centroid}}$$

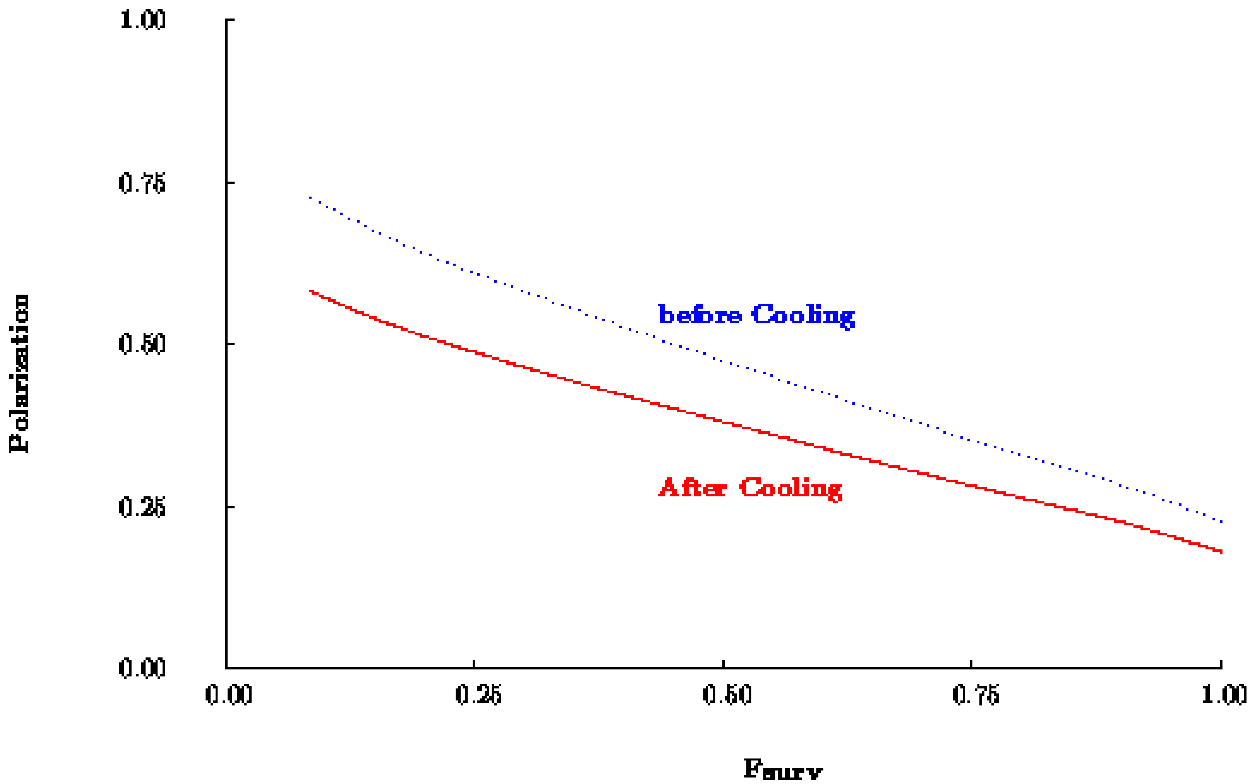
$$\frac{\sigma_{\text{resonance}}}{\sqrt{s}} = \frac{0.003}{110.0} = 2.7\times10^{-5} \approx \frac{\sigma_{\text{beam centroid}}}{\sqrt{s}}$$

Measure $\sigma_{\text{beam centroid}}$ to a part in 10^{-6}



Muon Collider Physics

- Polarization of muons will play a crucial role in many physics areas.
- Both charges polarizable.



Bargmann-Michel-Telegdi equation

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

$$\vec{\Omega} = -\frac{e}{m\gamma}((1+a\gamma)\vec{B}_{\perp} + (1+a)\vec{B}_{\parallel} - (a\gamma + \frac{\gamma}{1+\gamma})\vec{\beta} \times \frac{\vec{E}}{c})$$

$$\vec{\Omega} = \vec{\Omega}_{cyc}(1+a\gamma)$$

$$a = (g-2)/2$$

$B_{\perp}, B_{\parallel}$ are the components of magnetic field perpendicular and parallel particle direction

This equation controls the evolution of the spin vector $\vec{S}$. Polarization is the average of the spin vectors over the muon ensemble. Per revolution spin rotates by $a\gamma 2\pi$ radians more than momentum

Definitions

In the muon rest frame, E is the energy of the electron. Its fractional energy

expressed in terms of the maximum energy ($m_\mu/2$) is x. N is the number of muon decays. θ is the angle of the electron in the muon center of mass

w.r.t muon direction. $\langle E \rangle$ is the average electron energy and $\langle PL \rangle$ is the average longitudinal electron momentum in the muon rest frame.

P is the z component of the muon polarization along the muon direction. $\hat{P}$ is charge*P of the muon.

Electron energy distribution

$$x = 2E / m_\mu$$

$$\frac{d^2N}{dx\,d\cos\theta} = N(x^2(3-2x) - \hat{P}x^2(1-2x)\cos\theta)$$

$$< E > = \frac{m_\mu}{2N} \iint x \frac{d^2N}{dx\,d\cos\theta} dx\,d\cos\theta = \frac{7}{10}\frac{m_\mu}{2}$$

$$< P_L > = \frac{m_\mu}{2N} \iint x\cos\theta \frac{d^2N}{dx\,d\cos\theta} dx\,d\cos\theta = \frac{\hat{P}}{10}\frac{m_\mu}{2}$$

Decay distributions

$$< E_{lab} >= \frac{7}{20} E_{\mu} (1 + \frac{\beta}{7} \hat{P})$$

$$E(t) = Ne^{-\alpha t} (\frac{7}{20} E_{\mu} (1 + \frac{\beta}{7} (\hat{P} \cos \omega t + \phi)))$$

$$\omega = \gamma \frac{g-2}{2} 2\pi$$

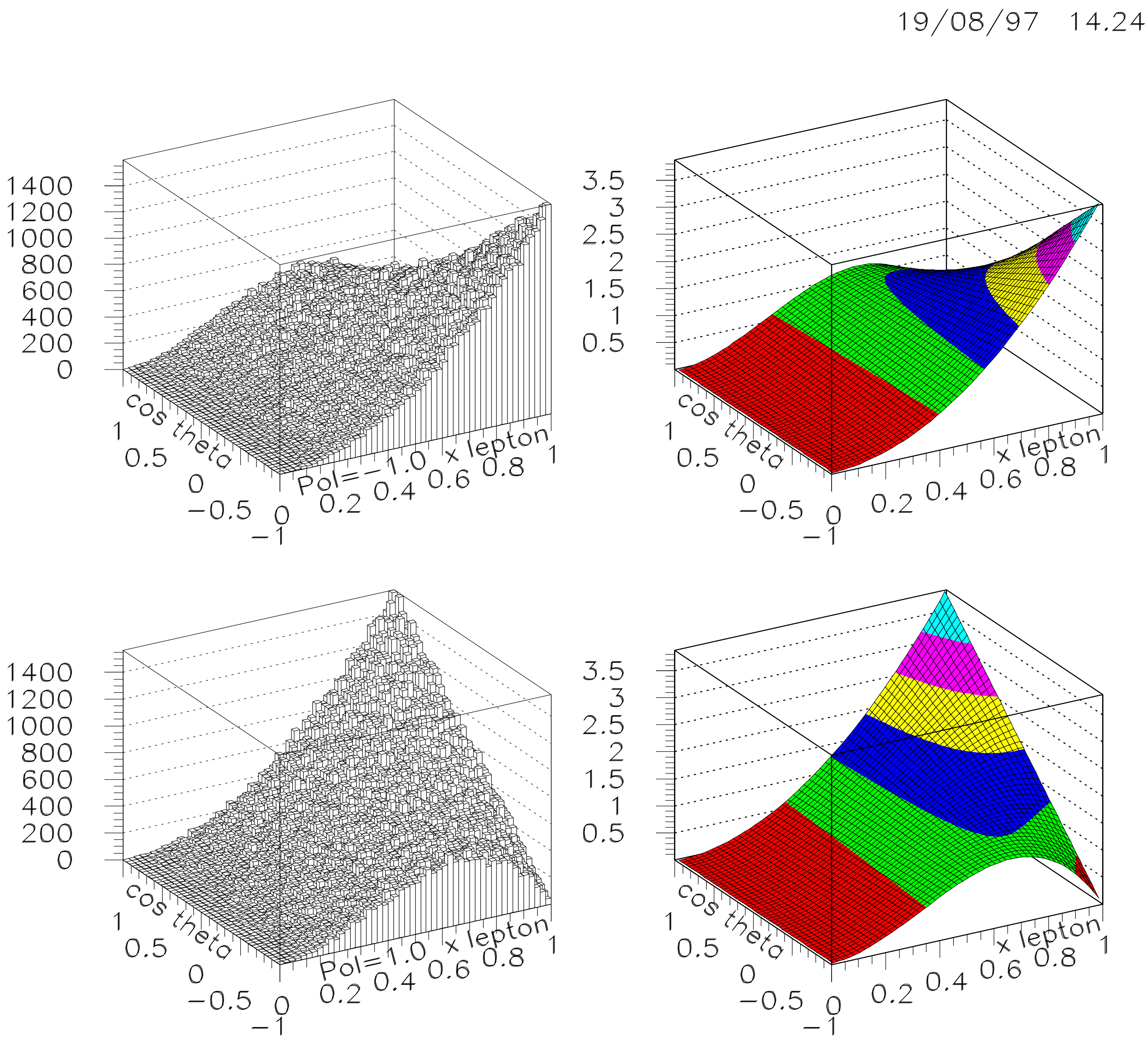
$$\alpha = \frac{t_{circ}}{\gamma t_{life}} = \frac{2\pi m_{\mu}}{0.3Bct_{life}}$$

$$f(t) = Ae^{-Bt} (C \cos(D + Et) + F)$$

$<E_{lab}>$ is the average electron energy in lab. $E(t)$ is the total electron energy during turn t. Determine ω to get γ. γ information also present in α.

$f(t)$ is the fitting function. MINUIT used to fit and extract information.

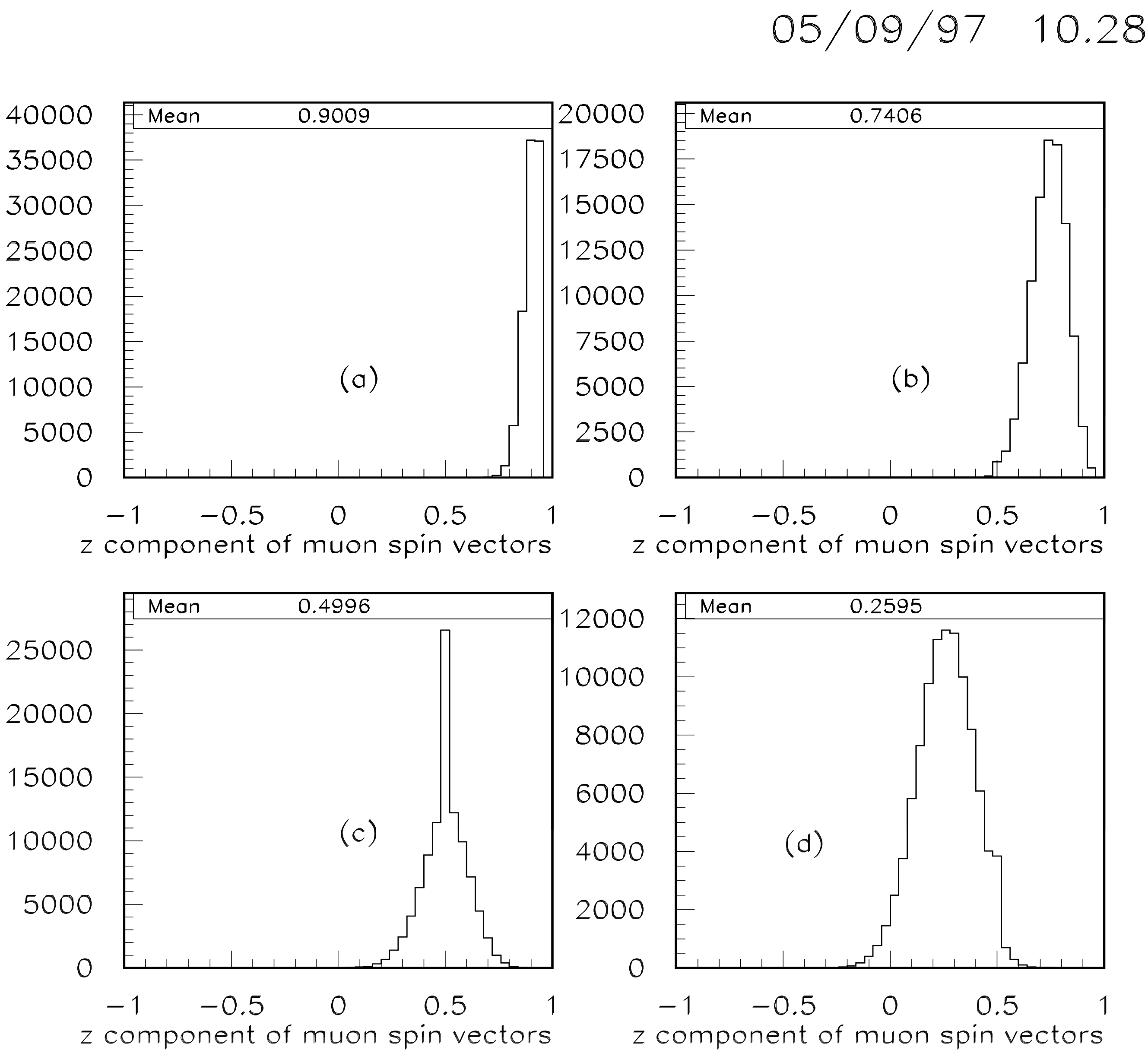
Electron energy and angle distributions in muon rest frame Polarization = -1.0 and 1.0



Variation in spin vectors

The electron energy sampled depends only on $\hat{P}$ which the average of the z component of the individual spin vectors. If all the muons had the same momentum, then the variation of the individual spin vectors is unimportant. Only the average matters. However, $\delta p/p$ is non-zero and the individual spins precess slightly differently turn to turn and a dilution in polarization results. We generate $\hat{P}$ as a binomial distribution about a given average.

Muon Spin vector generation



Idealized collider ring model

In order to simulate the decays, we assume an idealized planar collider ring made up of 4.0 Tesla average bending field. This results in the following parameters.

Muon Energy	= 50 GeV
Lorentz factor γ	= 473.22
Spin precession per turn	= 3.4667 radians
Magnetic field	= 4 Tesla
Ring Radius	= 41.667 meters
Circulation time	= 0.873E-6 sec
Dilated muon life time	= 0.104E-2 sec
Turn by turn decay constant α	= 0.8399E-3

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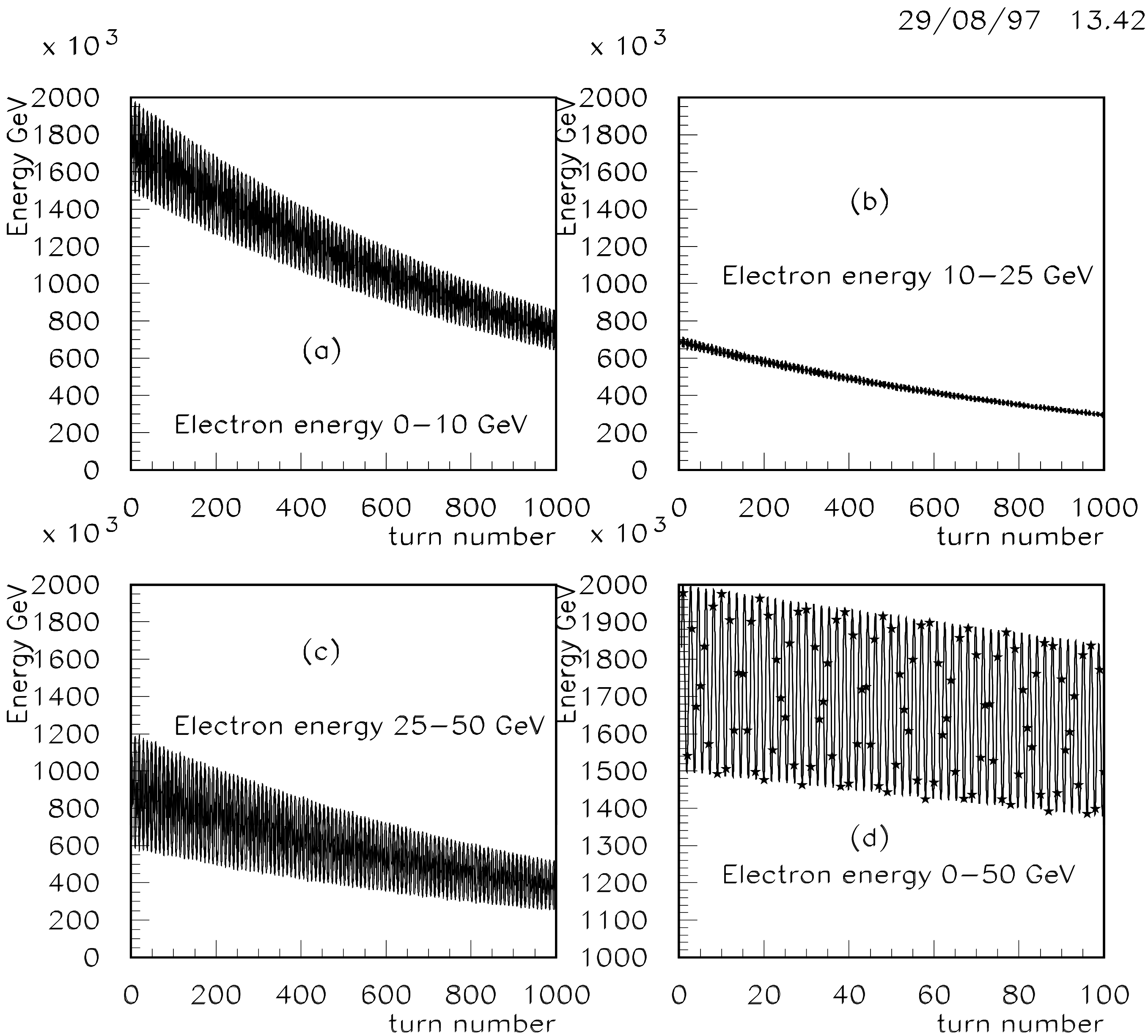
Generation of decays

- We generate an ensemble of 100K muons with appropriate spin and momentum vectors, precess them and decay them for 1000 turns. The measurement errors are added later as appropriate.
- The 100K ensemble of muons represent the beam and are re-used every turn, after precessing. Each individual muon is precessed by the appropriate amount determined by its individual γ. After each turn, the number of muons is reduced by the amount expected by exponential decay. No fluctuations in this are made at this point.
- We will generate muons with $\delta p/p=0.03E-2$ for most of the study and then will explore the dependence of the results on $\delta p/p$.
- We will explore the effect of measurement errors due to sampling fluctuations in the number of electrons.
- We will explore the accuracy obtainable in $\delta\gamma/\gamma$

from measuring α , the turn by turn decay constant.

Electron lab energy spectrum
Pol=1.0, 100K decays



Explanation of previous plot

The variation of the total lab electron energy seen in a calorimeter will depend on the electron energy cut made. Figure (d) has no electron energy cut. For 100 turns, we compare the predictions of the theory with the generated events. Excellent agreement. The amplitude of the oscillations depend on the polarization as well as the average value of $\cos\theta$ sampled. A cut on lab electron energy imposes a selection on $\cos\theta$. Electron energy 10-25GeV for 50 GeV muons samples $\cos\theta = 0$, i .e. 90 degrees in muon rest frame.

Measurement errors

We need an EM calorimeter that measures total energy deposited per bunch from turn to turn. In addition, it would also be useful to have the number of particles going into the calorimeter. This can be done by having a scintillating fiber front end to the calorimeter that measures mips. This can lead to two sets of measurements. Total Electron energy and average electron energy.

We would also like to make cuts on the momenta of elctrons being summed over. This can be done by a spectrometer in front of the calorimeter. With such a device,

we can show that...

Measurement errors

For total energy E, N electrons and average individual electron energy <e>,

for a calorimeter with Constant Sampling and Noise terms (C,S,η)

the fractional resolution in E is given by

$$\frac{\sigma_E^2}{<E>^2} \approx \frac{1}{N}\left(1+\frac{\sigma_e^2}{<e>^2}\right)+C^2+\frac{S^2}{N<e>}+\frac{\eta^2}{N^2<e>^2}$$

For 50 GeV muons, <e>=34.05 GeV,

σ_e =6.046 GeV. For calorimeter,

S=0.15GeV $^{1/2}$ and η may be neglected for large N. If C=0, then

Measurement errors

$$\frac{\sigma_E^2}{<E>^2} \approx \frac{1}{N}(1 + 0.03153 + 0.000661)$$

Neglecting sampling term,

$$PERR^2 \equiv \frac{\sigma_E^2}{<E>^2} \approx \frac{1.03153}{N}$$

PERR=0.5E-2, 1.0E-2, 2.0E-2, 3.0E-2

imply, N=41261, 10315, 2579 and 1146 electrons sampled with e>25 GeV.

For a beam intensity of 10^{12} muons,

there are 3.2E6 decays per meter.

Constant term has to be of order 0.32E-2 for100,000 electrons sampled.

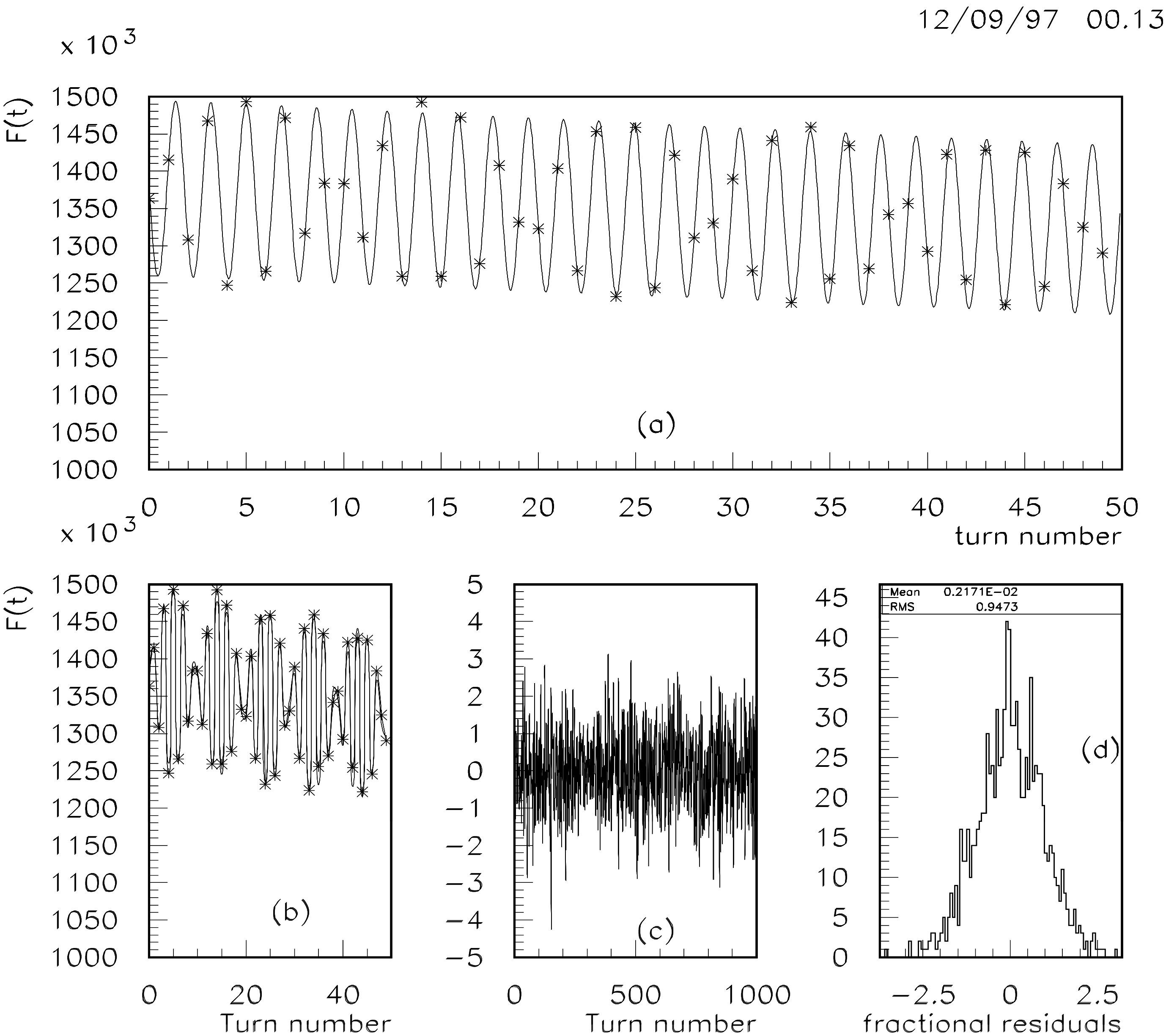
Measurement errors, using average energy

If the number of electrons as well as their total energy is measured, then one can use the average energy. Average energy has lower percentage error, since the error due to the fluctuations in N are removed.

$$\frac{\sigma^2_{Eav}}{<Eav>^2} \approx \frac{1}{<N>}\left(\frac{\sigma^2_e}{<e>^2}\right)\ldots \approx \frac{0.03153}{N}$$

The other advantage of using average energy is that one does not have to model the intensity decay function.

Fit to 50 GeV μ^-, P=0.26
$\delta p/p$=0.03E-2



$\delta\gamma/\gamma$ vs measurement error and Polarization $\delta p/p$=0.03E-2

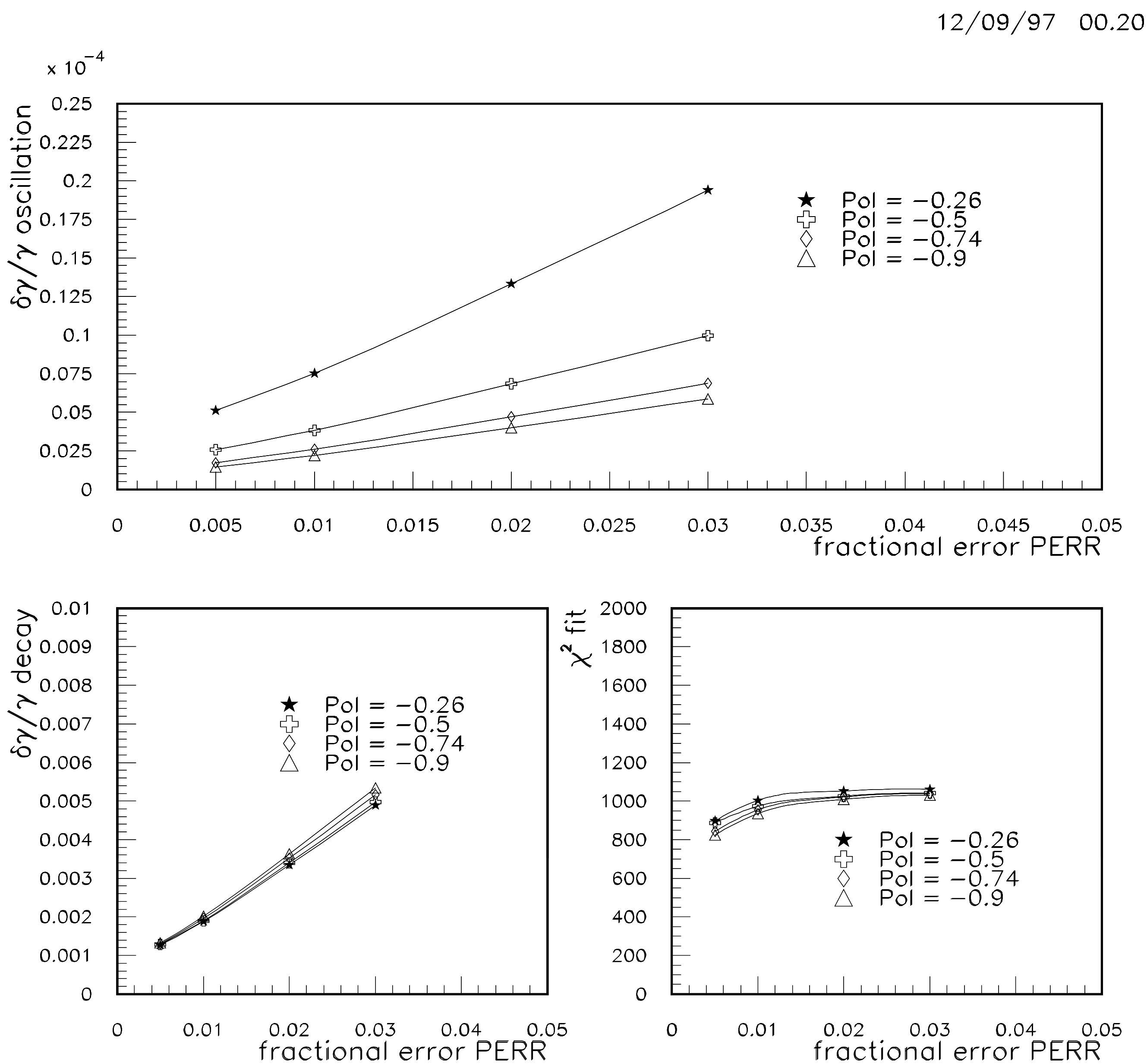
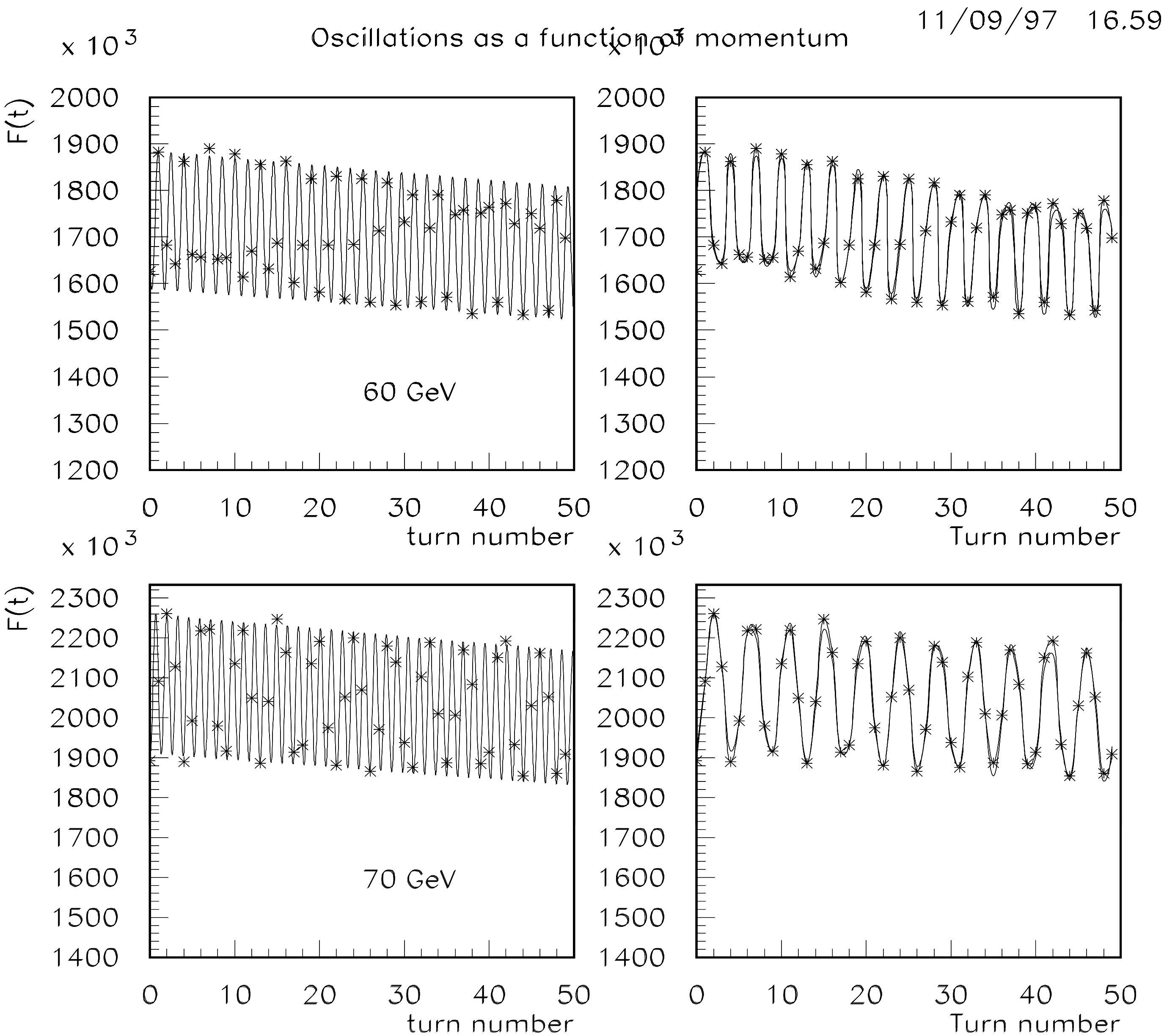


Table of fit parameters

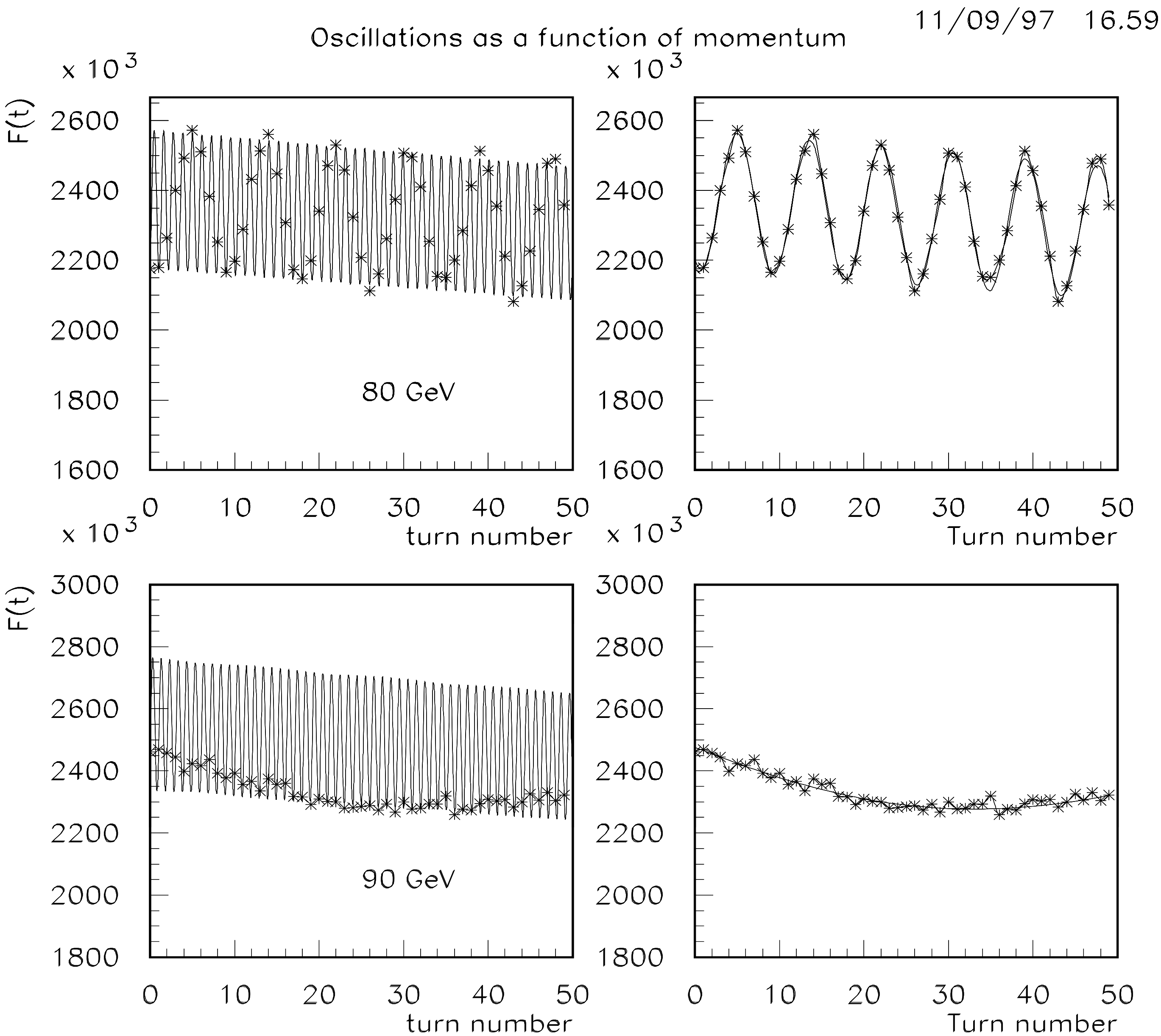
$\hat{P}$	PERR	Number of electrons sampled	$\delta\gamma/\gamma oscillations$	$\delta\gamma/\gamma decay$	χ^2 for NDF=1000
-0.90	0.50E-02	41261	0.14568E-05	0.13227E-02	824.
-0.90	0.10E-01	10315	0.22147E-05	0.20124E-02	936.
-0.90	0.20E-01	2579	0.39999E-05	0.36398E-02	1009.
-0.90	0.30E-01	1146	0.58659E-05	0.53457E-02	1030.
-0.74	0.50E-02	41261	0.17418E-05	0.13019E-02	843.
-0.74	0.10E-01	10315	0.26183E-05	0.19591E-02	954.
-0.74	0.20E-01	2579	0.46981E-05	0.35229E-02	1021.
-0.74	0.30E-01	1146	0.68765E-05	0.51672E-02	1039.
-0.50	0.50E-02	41261	0.25903E-05	0.12813E-02	888.
-0.50	0.10E-01	10315	0.38407E-05	0.19029E-02	973.
-0.50	0.20E-01	2579	0.68338E-05	0.33972E-02	1026.
-0.50	0.30E-01	1146	0.99744E-05	0.49749E-02	1041.
-0.26	0.50E-02	41261	0.51242E-05	0.12688E-02	898.
-0.26	0.10E-01	10315	0.75317E-05	0.18791E-02	1004.
-0.26	0.20E-01	2579	0.13324E-04	0.33447E-02	1053.
-0.26	0.30E-01	1146	0.19380E-04	0.48950E-02	1061.

TABLE I. Results of fits for $\delta\gamma/\gamma$ as a function of polarization $\hat{P}$ and noise PERR. Also shown is the χ^2 of the fit for 1000 turns.

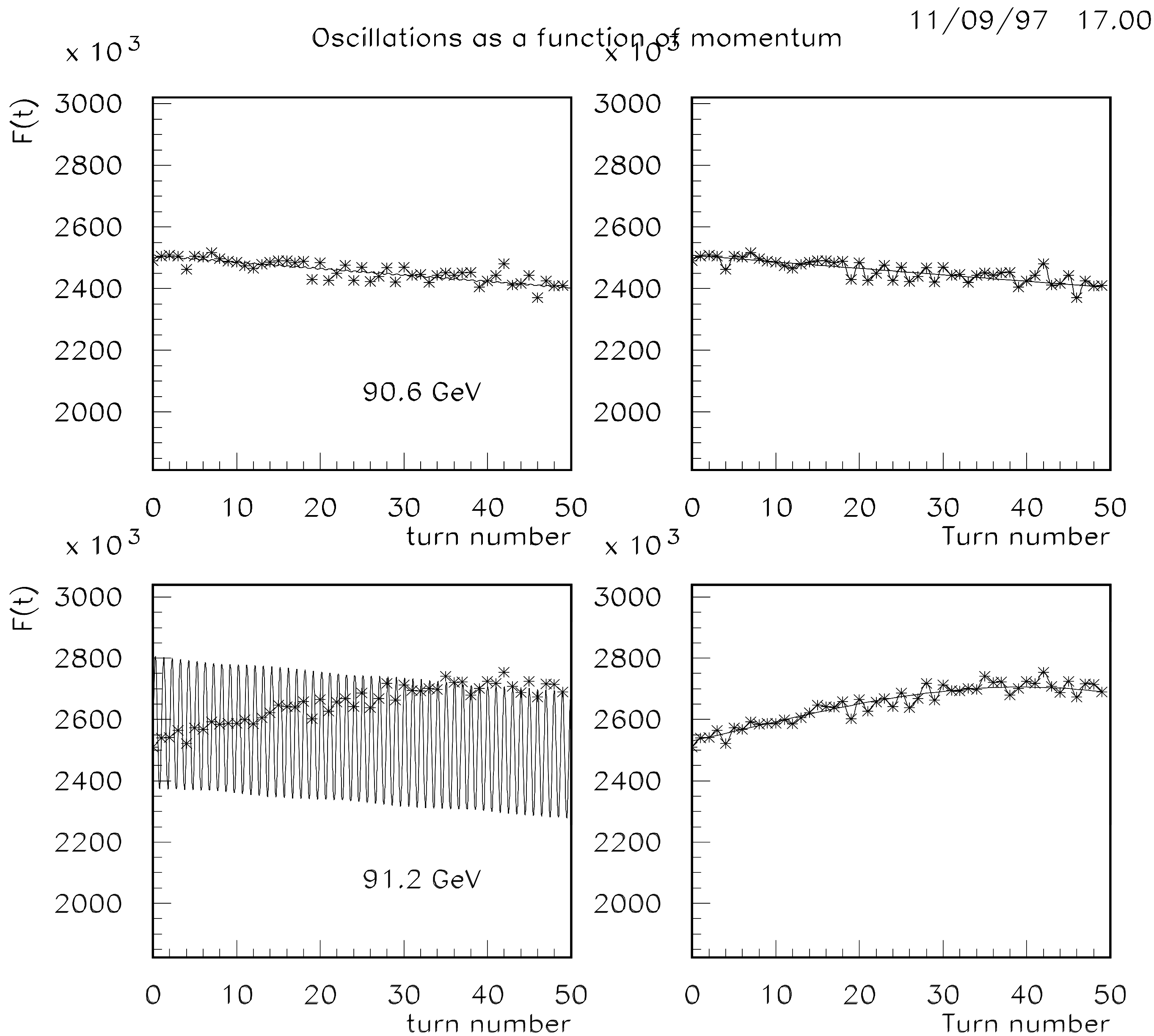
Fits for 60Gev/c and 70 Gev/c muon momenta



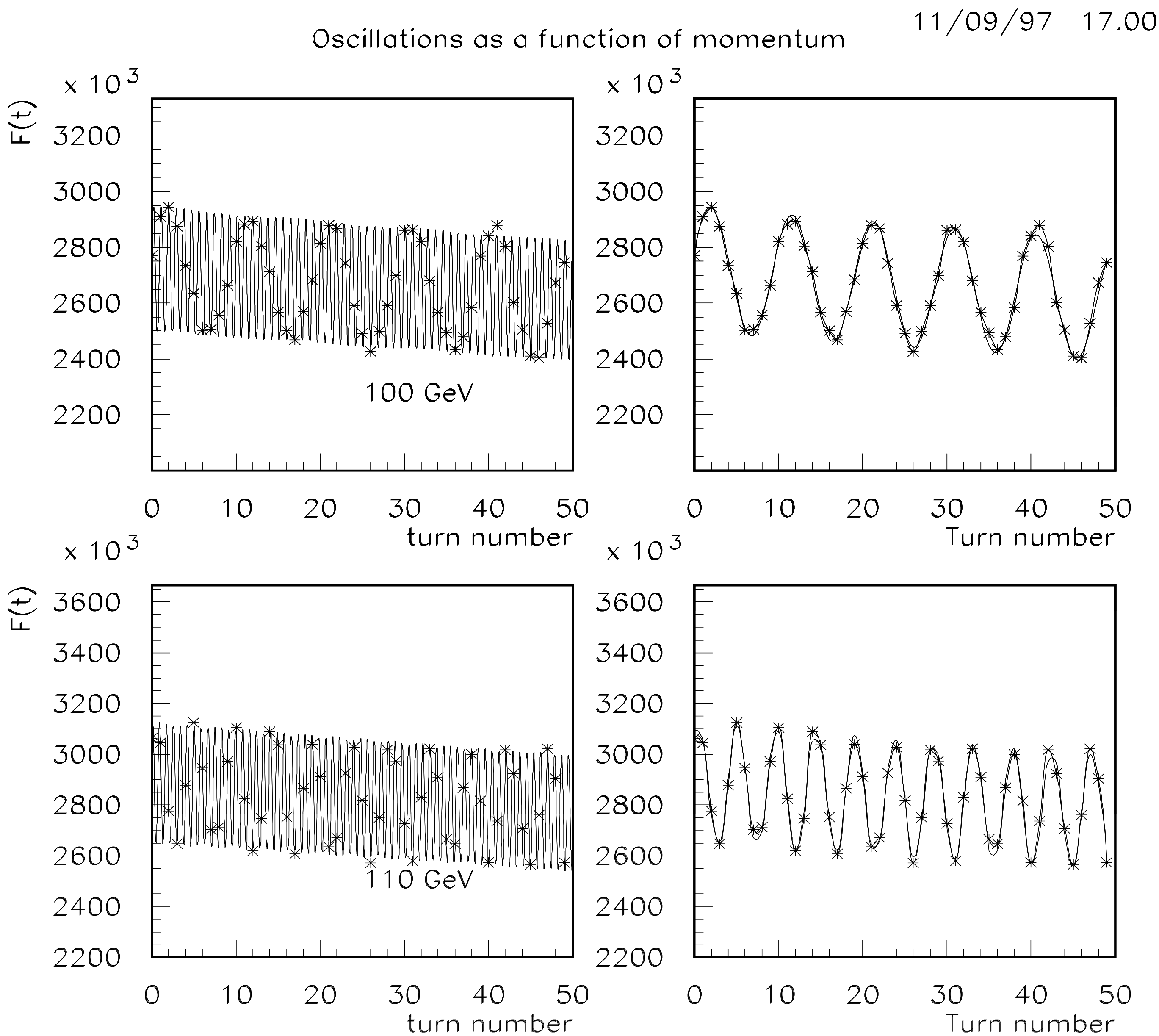
Fits for 80 GeV/c and 90 GeV/c muon momenta



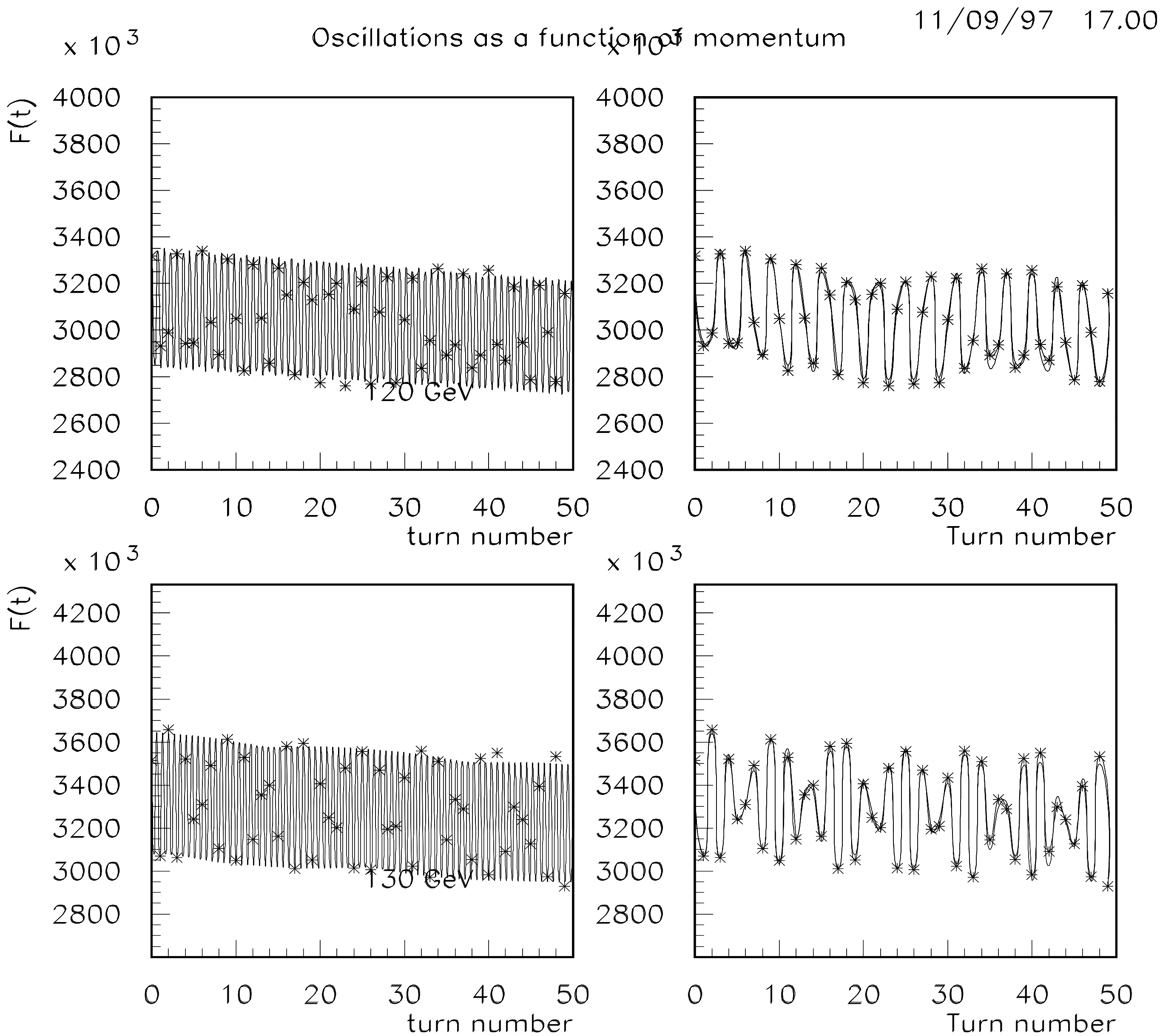
Fits for 90.622 GeV/c and 91.2 GeV/c muon momenta



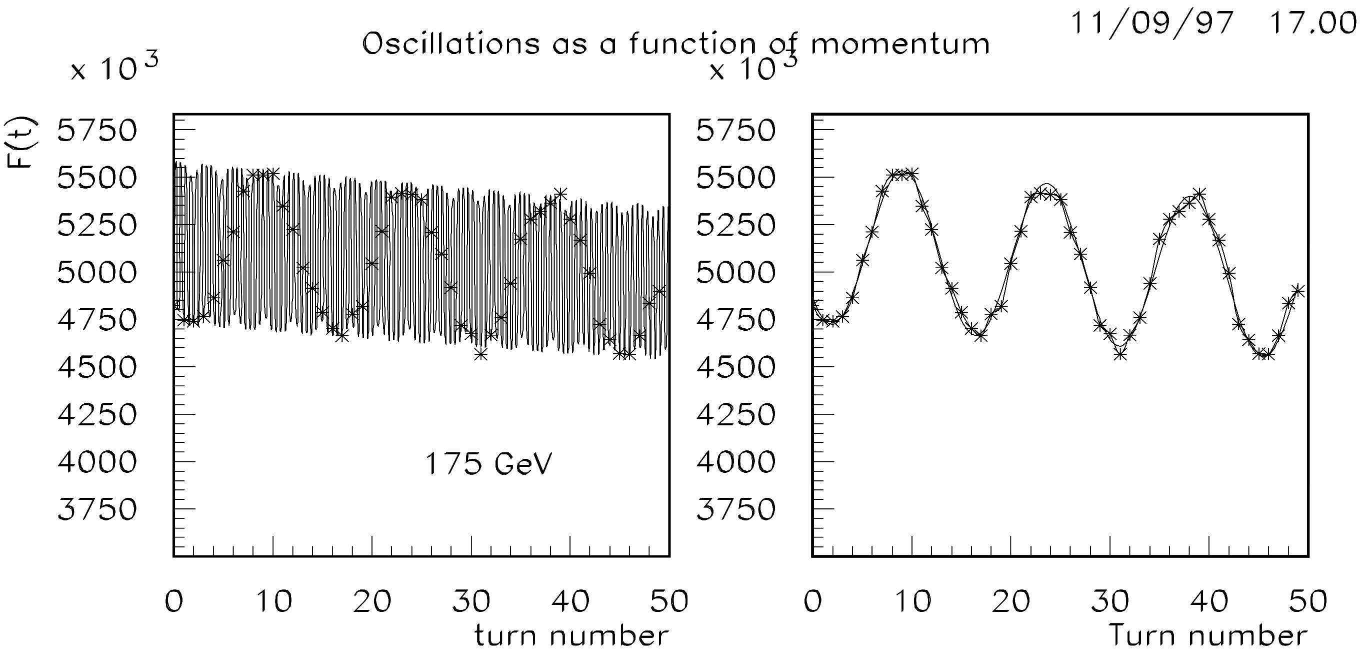
Fits for 100 GeV/c and 110 GeV/c muon momenta



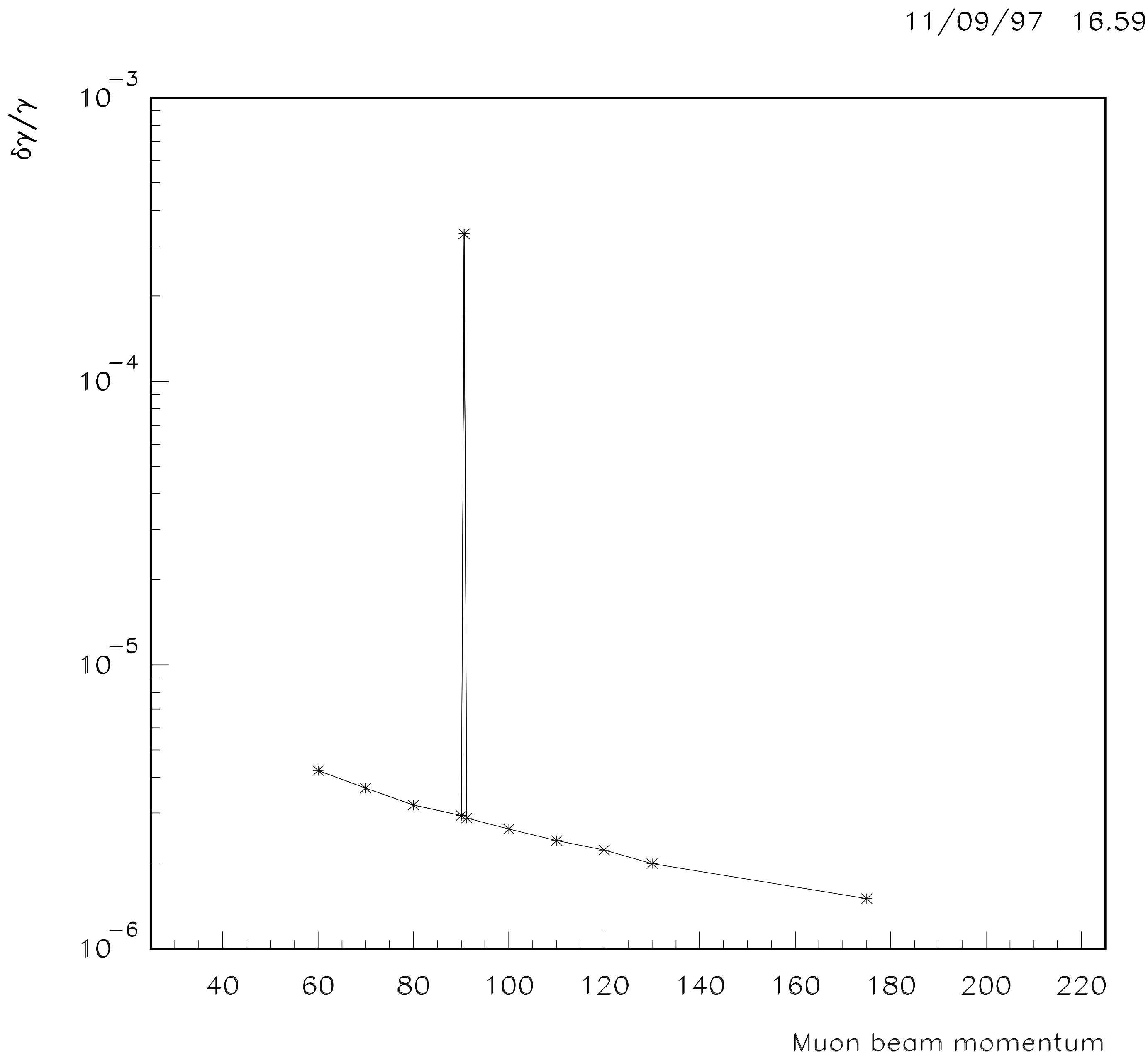
Fits for 120 GeV/c and 130 GeV/c muon momenta



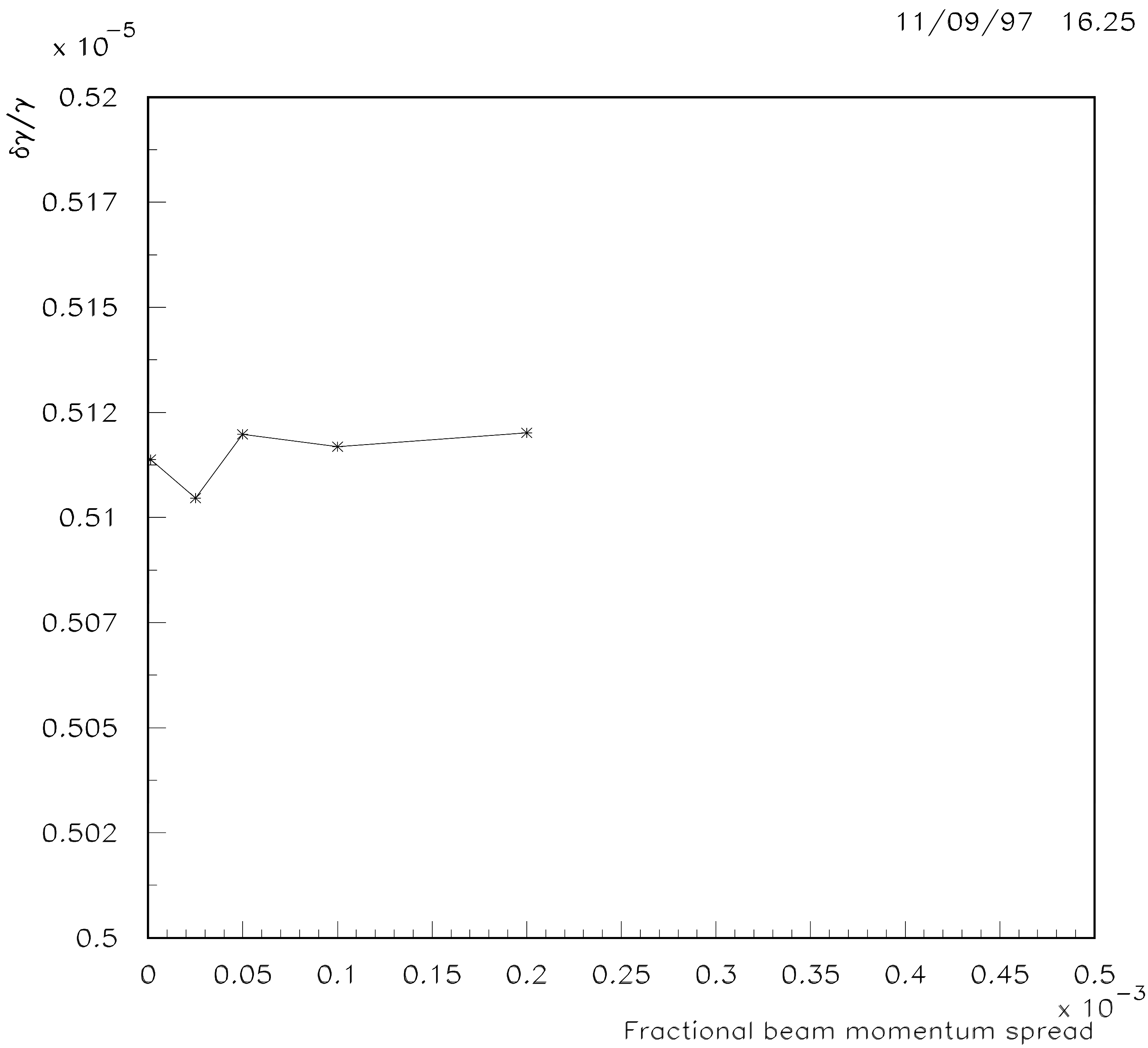
Fit for 175 GeV/c muon momenta



$\delta\gamma/\gamma$ vs muon beam momentum



$\delta\gamma/\gamma$ vs $\delta p/p$ for 50 GeV/c μ^{-}. Perr=.5e-2 P=.26



Departures from ideal case

- Electric fields. Collider ring will have RF but electrostatic separators are not envisaged at present. BMT equation tells that that longitudinal electric fields have no spin precession effect since $\vec{\beta}\times\vec{E}=0$
- Effects of radial magnetic fields
  - » Quadrupole misalignments. FODO
  - » Quad radial magnetic field followed by Horizontal dipole bend followed by reverse quad radial field has a net effect. (Assmann and Koutchouk LEP)

$$\nu_0 = \gamma(g-2)/2$$

$$<\partial\nu> = \frac{\cot \pi\nu}{8\pi}\nu_0^2(n_Q(Kl_Q)^2\sigma_y^2 + n_{cv}\sigma_{\theta cv}^2)$$

$$\sigma_{\partial\nu} = \frac{<\delta\nu>}{\cos \pi\nu_0}$$

Departures from ideal case

- Correction elements may be neglected. These effects are less for the muon collider than for LEP because a)LEP has more quadrupoles b)Muon is 200 times heavier than electron.

Machine	Spin tune ν_0	Quadrupoles	RMS $K\,l_Q$ meters^{-1}	σ_y meters	$\delta\nu$	$\sigma_{\delta\nu}$
46 GeV LEP	100.47	$\approx$ 600	0.032	0.5E-3	5.7E-6 $\equiv$ 3KeV	6.1E-5 $\equiv$ 30KeV
50 GeV Muon Collider	0.5517	70	0.274	0.5E-3	-0.26E-8 $\equiv$ -0.24KeV	1.66E-8 $\equiv$ 1.46KeV

TABLE I. Predictions for spin tune shift $\delta\nu$ and spread in spin tune shift $\sigma_{\delta\nu}$ caused by quadrupoles for LEP compared to the 50 GeV muon collider ring

Effects due to Experimental area solenoid

- Consider a solenoid 1.5 Tesla - 6 meters long = 9Tesla -meters.

$$\theta_s = -\frac{e}{\gamma m_\mu}(1+a)B_s = -(1+a)\frac{B_s l}{B\rho}$$

$$\nu + \delta\nu = \frac{1}{\pi}\arccos(\cos(\pi\nu)\cos(\frac{\theta}{2}))$$

- θ is the angle of rotation due to solenoid. For 9 Tesla-meters, this is 3.09 degrees per turn. This yields δν/ν =-1.72 MeV. LEP will have 200 times less effect, since the tune is 200 times larger. LEP corrects with vertical bumps and horizontal bends. 200 times harder to do for muons. Bucking solenoids optimal.

Preservation of polarization

- Use rf. The effect of RF on polarization in a muon storage ring
  Author(s): Rajendran Raja
  Muon Collider Note Number: MUC//NEUTRINO_SRC//0077

$$\boxed{\begin{array}{l} E = A\cos(\phi_0 + 2\pi Q_s t) \\ \mathrm{E = Energy\ of\ muon} \\ \mathrm{Q_s = fractional\ synchrotron\ tune} \\ \mathrm{t = turn\ number} \end{array}}$$

-

$$\Delta E_{n+1} = \Delta E_n + eV(\sin\phi_n - \sin\phi_s)$$

$$\phi_{n+1} = \phi_n + \frac{2\pi\eta\Delta E_{n+1}}{\beta^2 E_s}$$

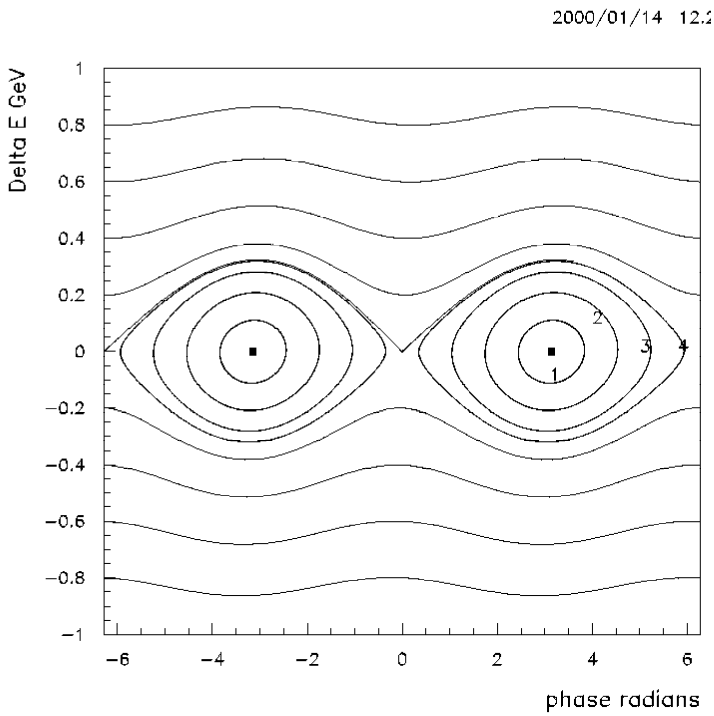
$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}; \text{slip factor}$$

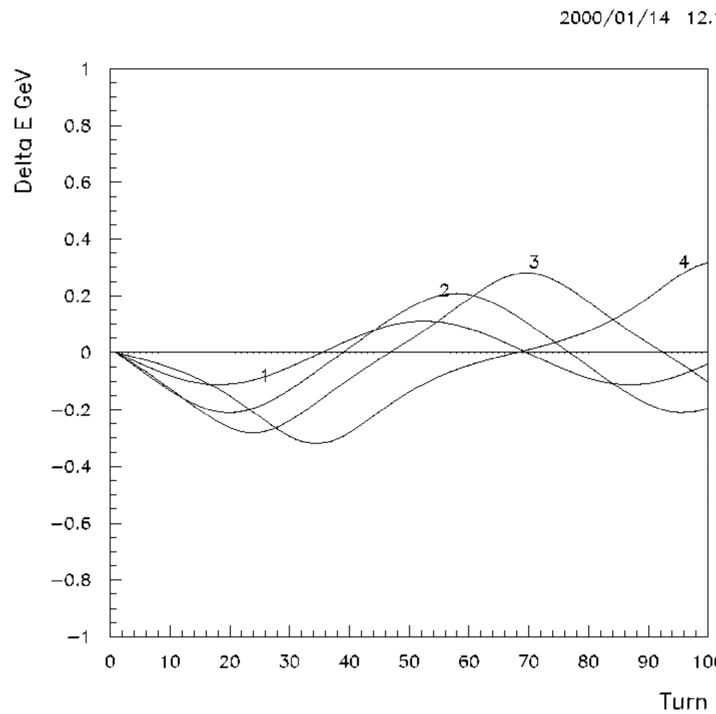
$$\mathrm{Q_s} = \sqrt{\frac{-h\eta eV\cos\phi_s}{2\pi\beta^2 E_s}}; \text{synchronous tune}$$

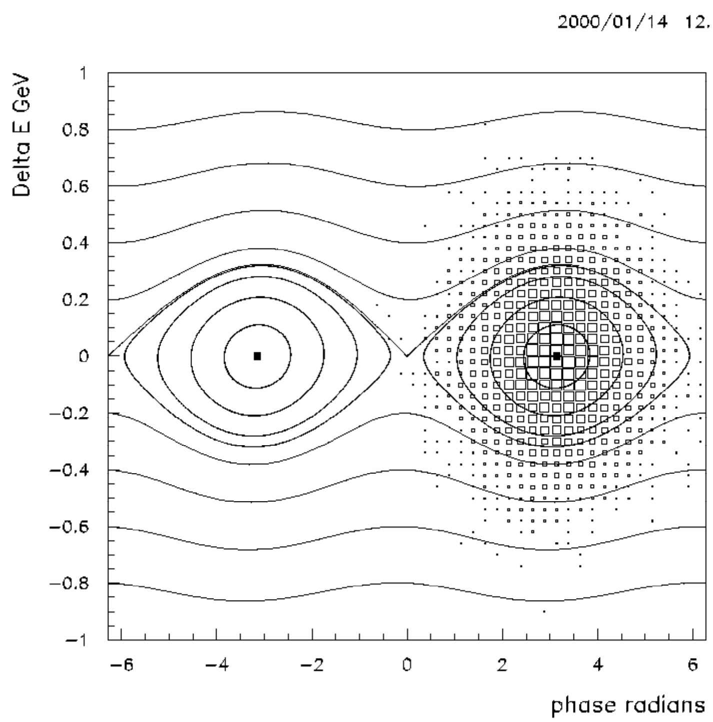
$$\mathrm{eV} = \frac{-2\pi\beta^2 E_s}{h\eta\cos\phi_s} Q_s^2$$

Synchrotron oscillations and polarization preservation

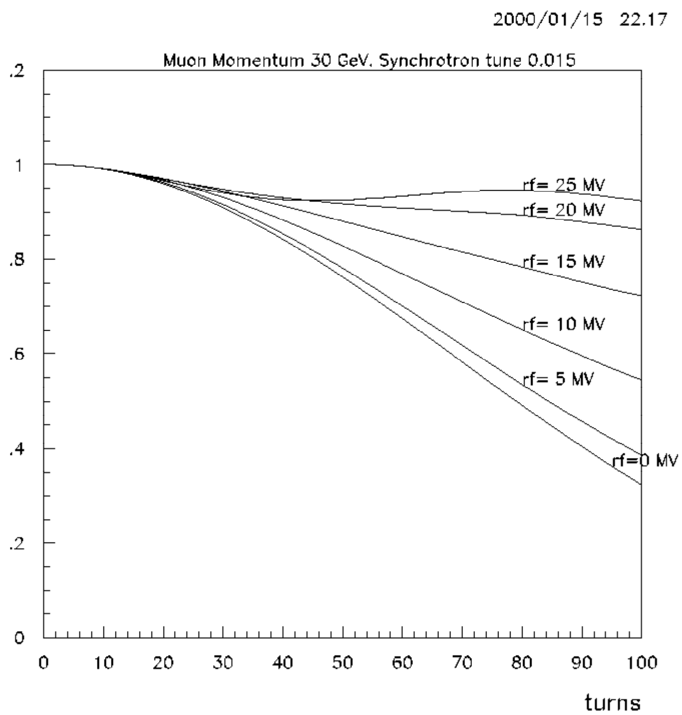
- R.Raja- Mucool note Muc0077

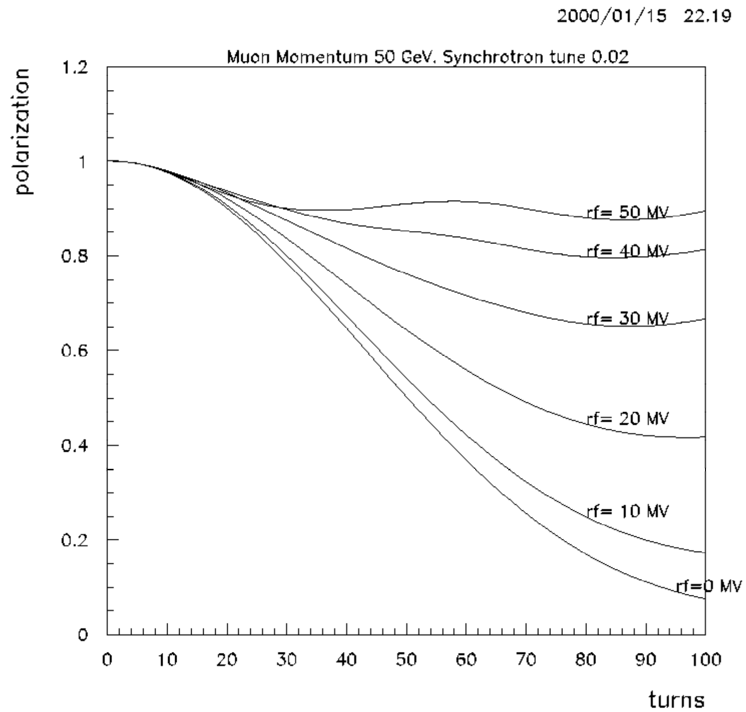






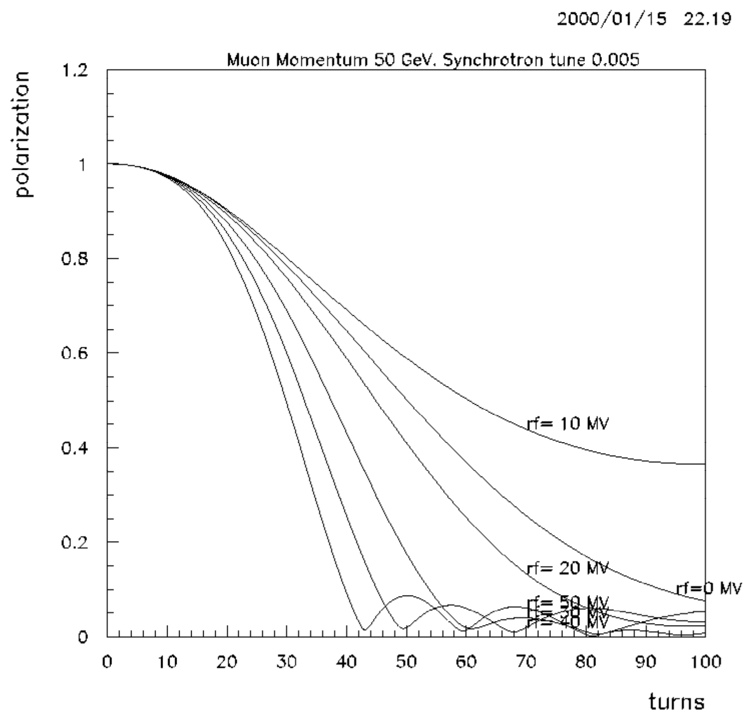
Polarization preservation in ring





- 50 GeV δp/p=.007
- rf=200MHz

Synch. Tune	δp/p 3σ	Rf MeV	<Pol>	<pol>$_{weighted}$
.02	.01	33	0.8	0.826
.03	0.017	76	.936	.94
.04	.021	135	.96	.97



Polarization preservation, measurement and luminosity measurement

- By having an electron calorimeter around the beam pipe, segmented in phi, one can measure
  - » Energy of muon bunch to precisions of a part per million. This can be achieved with a polarization of 0.1 by sampling ~8 million electrons (a bunch of 10^{12} 50 GeV muons produces this in 2.5 meters of decay space!).
  - » Measurement of muon intensity turn by turn. Statistical precision from such a sampling is ~0.03% !
  - » Measurement of $\delta p/p$. Switch off the rf and one can see the polarization decay. This rate is related to $\delta p/p$. So long as this is maintained from store to store, one can measure the beam properties very accurately.
  - » Transverse segmentation measures transverse muon polarization. Paper on this is in the works.
- Measurement of a precessing electron polarization gives information on the parentage of a signal (whether it is from muon neutrino or electron anti-neutrino). Investigating consequences for a near-detector

Conclusions

- We have shown that IF
  - » it is possible to deliver polarized muons to the ring (some polarization is inevitable)
  - » and that it is possible maintain the polarization for 1000 turns (rf with synchronous phase will do the trick)
- g-2 precession can be used to obtain precisions of the order of a few parts per million in the energy of the bunches.
- Method can be used to scan a narrow Higgs, Measure the W mass etc...
- Also valid for preserving polarization for neutrino factory and measuring the energy scale.

Conclusions

- EM Calorimeters needed to select electrons, positrons > 25 GeV (for Higgs factory of 55 GeV/c per beam) and measure total energy and total number of accepted electrons for each turn for 1000 turns.
- rf needed in ring to preserve polarization operating in synchronous mode (no net acceleration) of 25MV or higher.
- Bucking solenoids of half the detector solenoid Integral Bdl needed on either side of detector. These can just hug the beampipe, with shielding. i.e not too much stored energy. These will buck out the cumulative rotation of polarization.