
SM@TLEP : Theoretical Improvements

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Outline

Today I will discuss a few SM topics which may be covered by the TLEP physics program :

The possibilities for measuring $\alpha_s(m_Z)$

Jet physics at lepton colliders

WW pair production

Some comments about extensions to VHE-LHC

(For top physics see Markus' talk)



*Overview : SM Phenomenology @
lepton colliders*



Comparison between hadron and lepton colliders

	Hadron colliders	Lepton colliders
Initial State	PDFs required (with inherent uncertainty)	No PDFs
Higher order corrections	Large QCD corrections, precision difficult	Smaller corrections, precision possible, ee->jets known at high multiplicities
Monte Carlo status	Advanced, many tools and new techniques.	Some dedicated codes, aspects not developed since LEP

The LHC has provided challenging problems to SM phenomenologists, the resulting tools could easily be modified to work for future lepton colliders.



Differences between lepton and hadronic environments

The main difference is of course the ability to study EW processes with much greater precision.

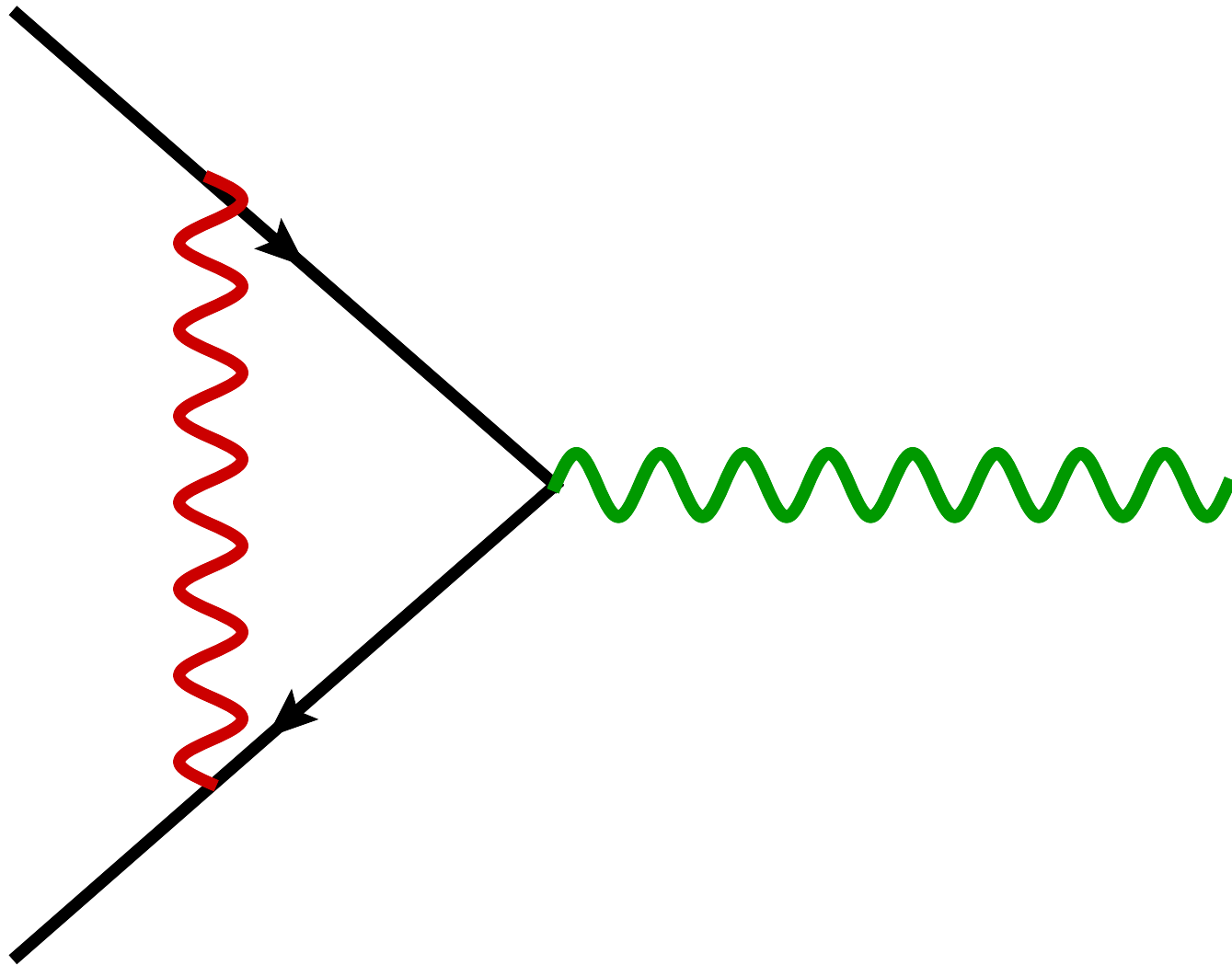
At the LHC NLO EW corrections are of order 5 % (but can be much bigger), around the same order of magnitude as NNLO QCD corrections.

At lepton colliders for EW processes EW corrections are dominant and need to be taken into account to match experimental precision.

So most of the new calculations needed for TLEP will be NLO EW corrections!



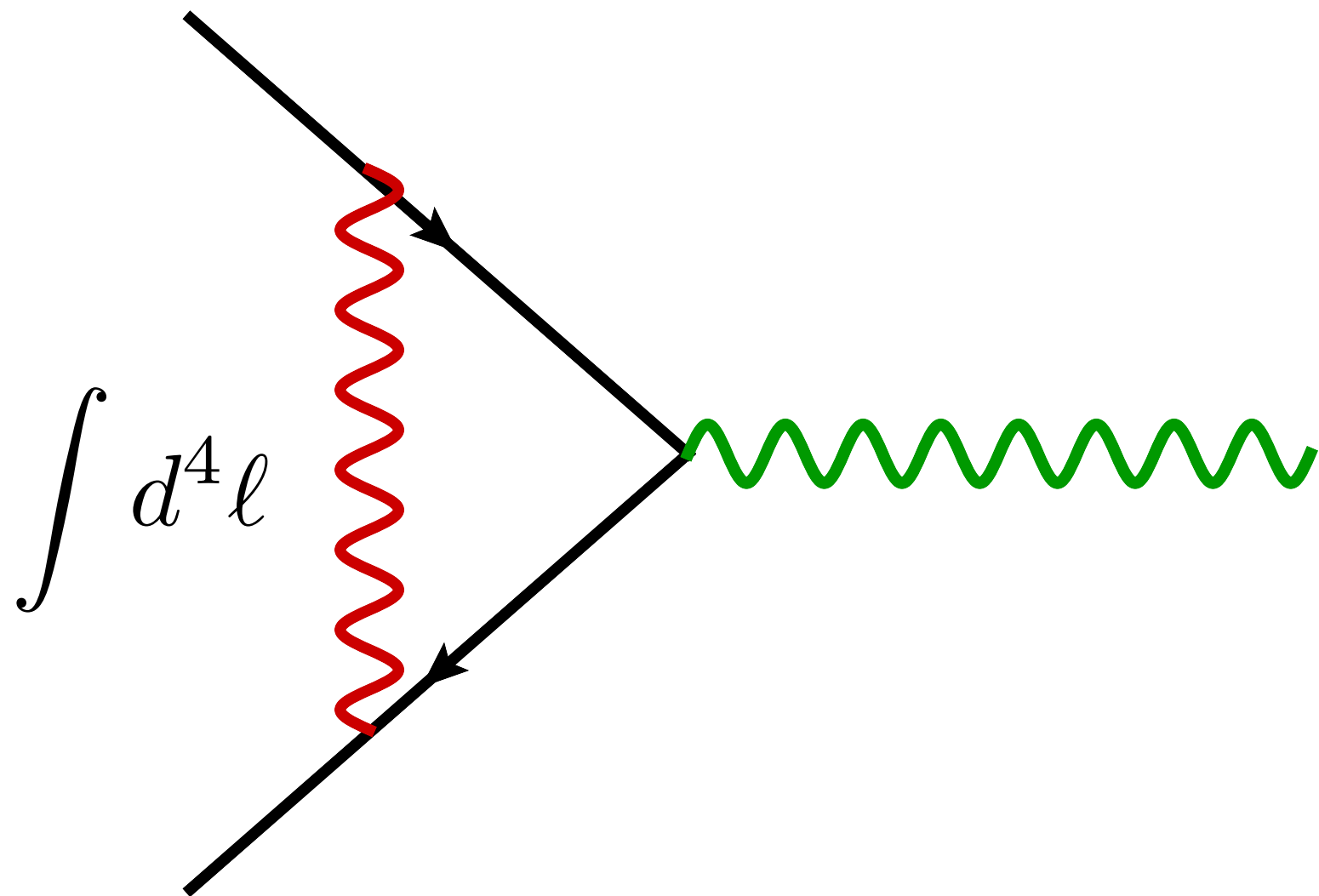
A beginners guide to NLO.



Higher order corrections involve “loop” level Feynman diagrams.



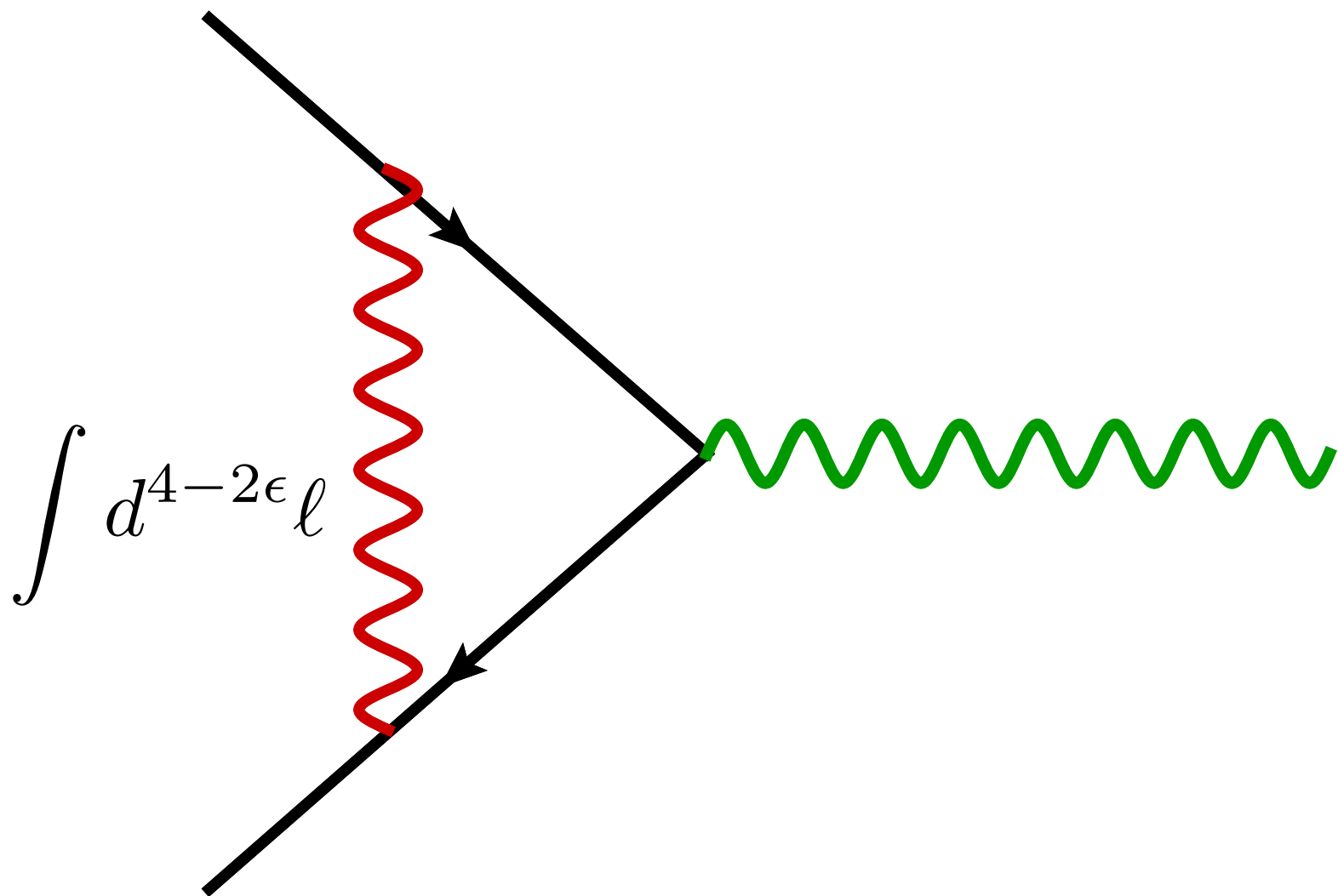
A beginners guide to NLO.



The momentum in these loops is unconstrained so we have to integrate over them.



A beginners guide to NLO.

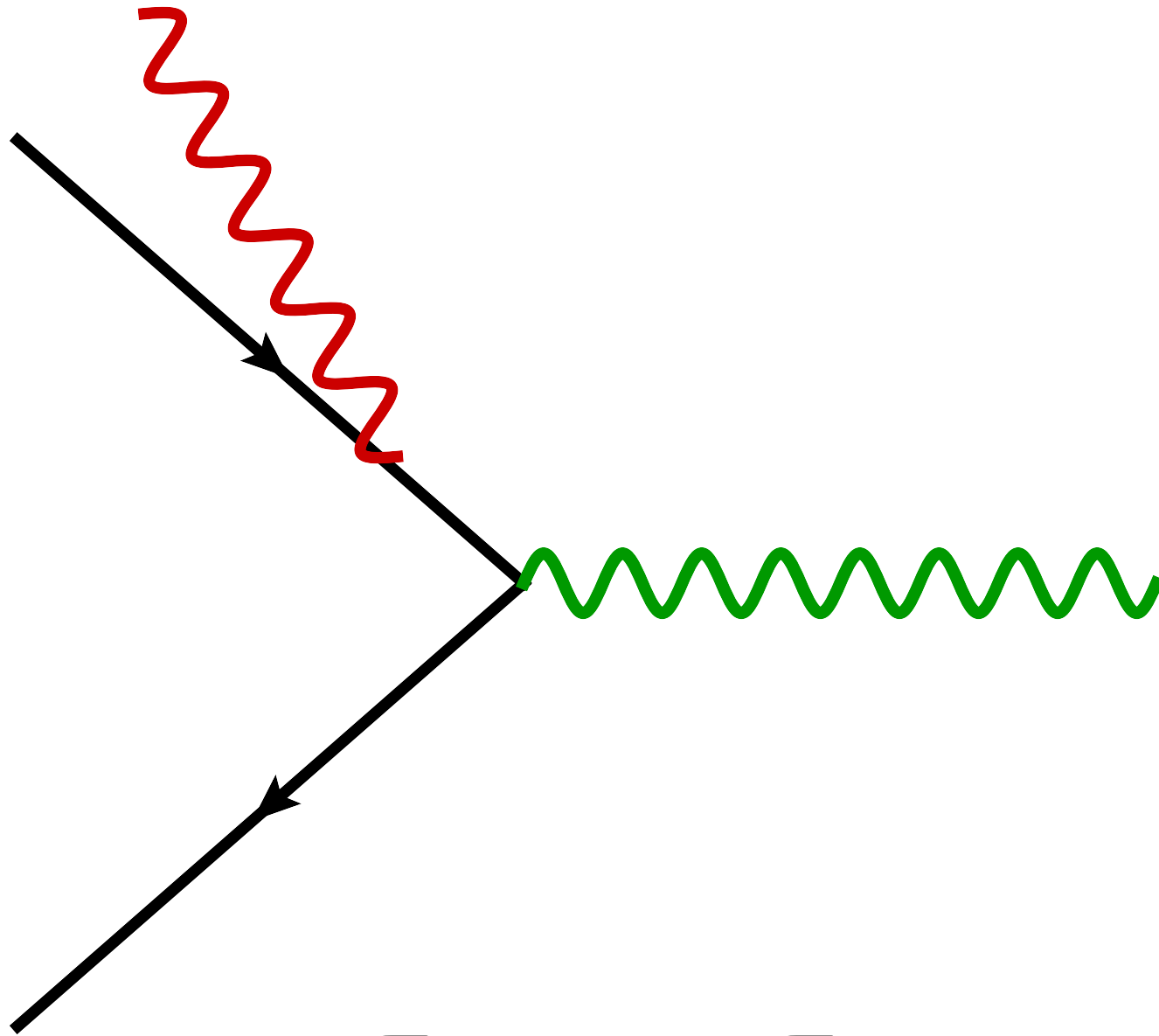


However these integrals diverge in 4 dimensions so we have to regulate the integrand.

$$\frac{\mathcal{I}_{-2}}{\epsilon^2} + \frac{\mathcal{I}_{-1}}{\epsilon} + \mathcal{I}_0 + \mathcal{O}(\epsilon)$$



A beginners guide to NLO.



After UV renormalization the remaining IR singularities are removed when the loop diagrams are combined with the real emissions.

$$-\frac{\mathcal{I}_{-2}}{\epsilon^2} - \frac{\mathcal{I}_{-1}}{\epsilon} + \mathcal{R}_0 + \mathcal{O}(\epsilon)$$



$$e^+e^- \rightarrow \textit{jets}$$



$$e^+e^- \rightarrow jets$$

Jet production at lepton colliders provides a fantastic testing ground for perturbation theory, allowing :

Measurements of α_s

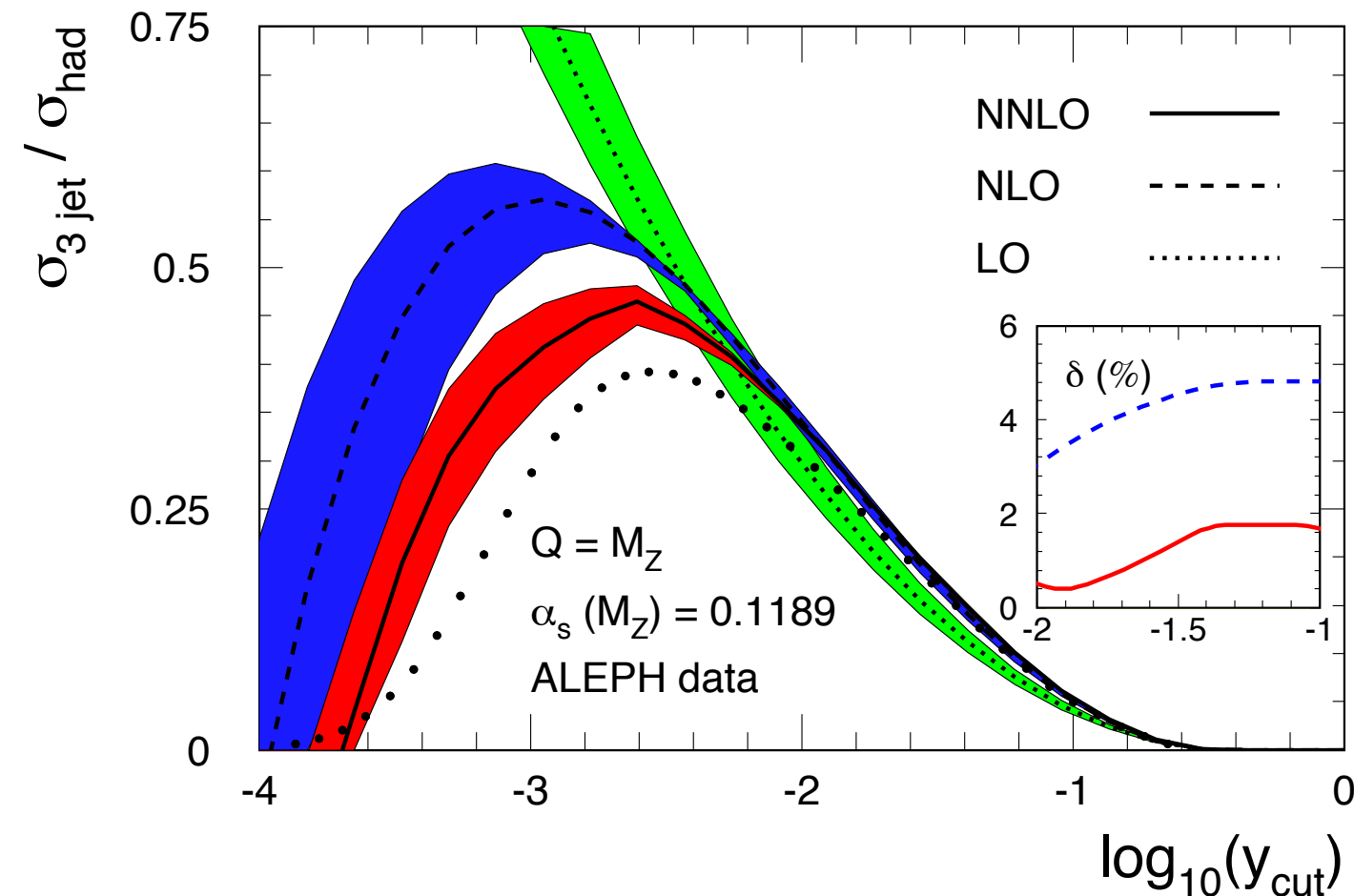
Studies of event shapes, testing perturbation theory, resummation.

Represents a proving ground for high loop computations



$e^+e^- \rightarrow jets$

(Gerhmann-De Ridder, Gehrmann, Glover and Heinrich 08')



Existing LEP data already highlights need for treatment of large logs in regions of phase space which are sensitive to soft/collinear radiation.

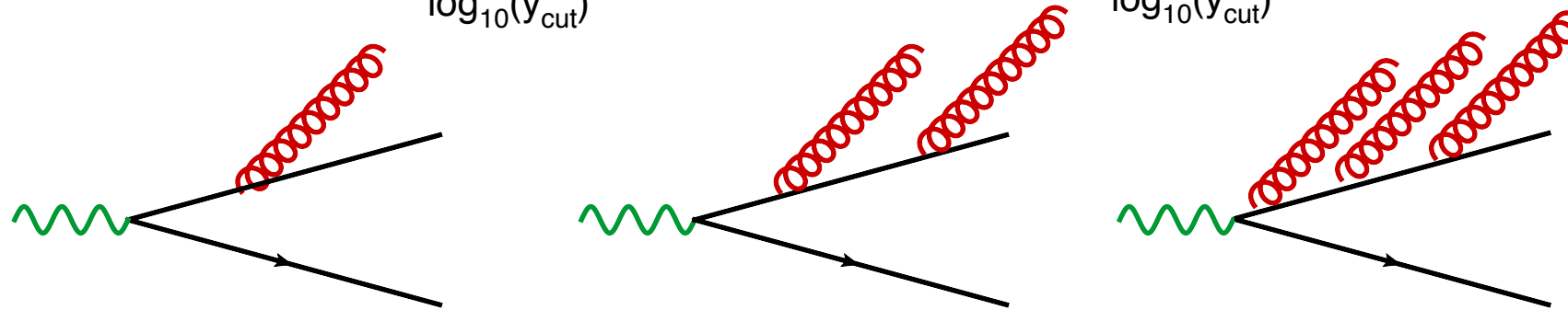
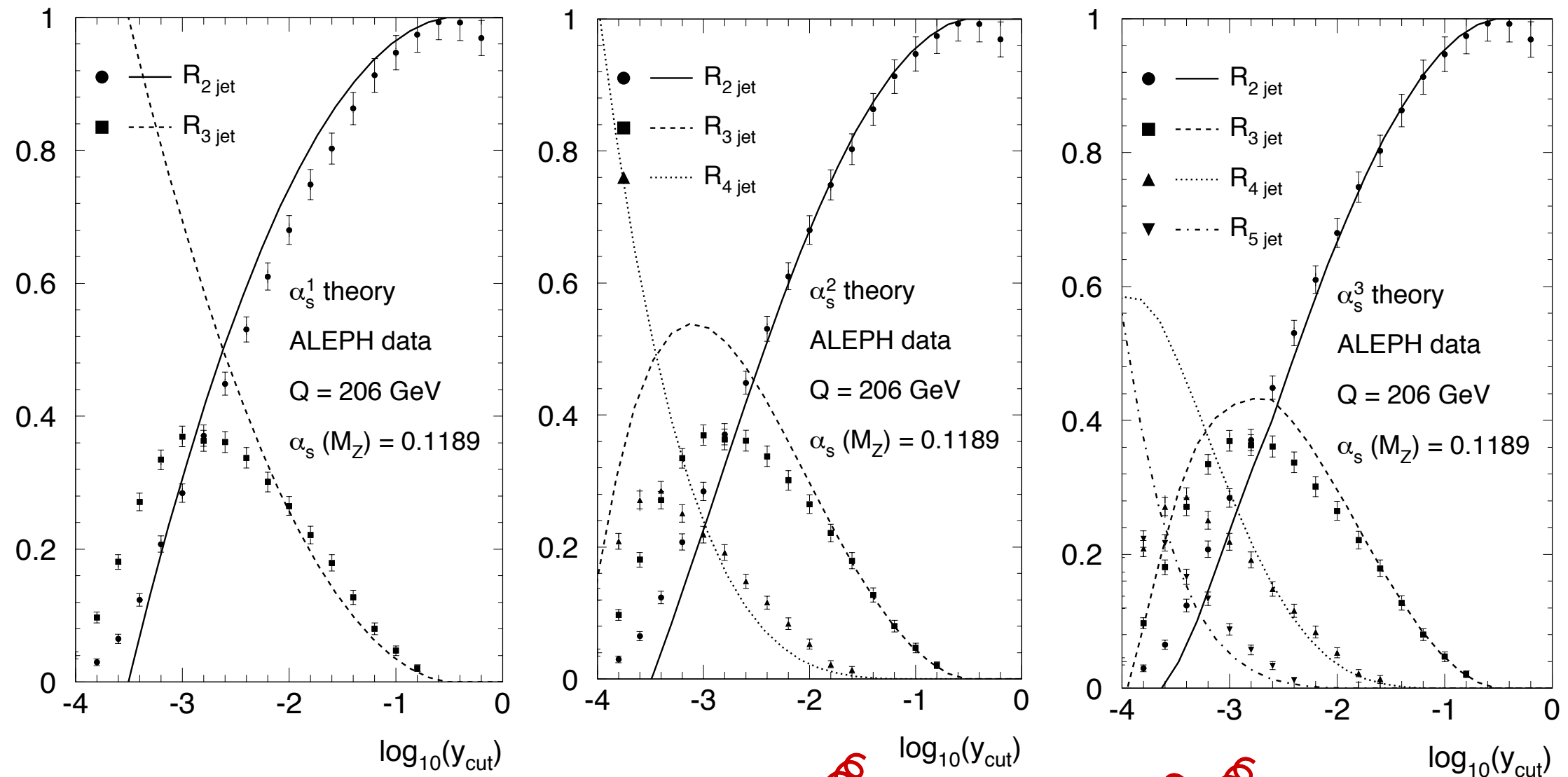
Dedicated NLLA resummations exist, however may need NNLLA and NNNLO to fully exploit future data!

$$y_{ij,D} = 2 \frac{\min(E_i^2, E_j^2)(1 - \cos \theta_{ij})}{E_{vis}^2}$$



Jet fractions at different orders

(Gerhmann-De Ridder,
Gehrmann, Glover and
Heinrich 08')



Each order in perturbation theory results in improved agreement with LEP data. For the relative jet fractions.



$$e^+e^- \rightarrow jets$$

# of jets	Order (and ref)
3	NNLO+NNLA [1-3]
4	NLO [4-5]
5	NLO [6]
6	NLO [7]
7	NLO [7]
8	LO [7]

[1] Gerhmann-De Ridder, Gehrmann, Glover, Heinrich 08'

[2] Weinzierl 08'

[3] Dissertori, Gerhmann-De Ridder, Gehrmann, Glover, Heinrich, Luisoni, and Stenzel 09'

[4] Campbell, Glover and Miller 97'

[5] Weinzierl and Kosower 99'

[6] Becker, Reuschle and Weinzierl 10'

[7] Becker, Goetz, Reuschle, Schwan and Weinzierl 11)

Apologies if I've missed any here!

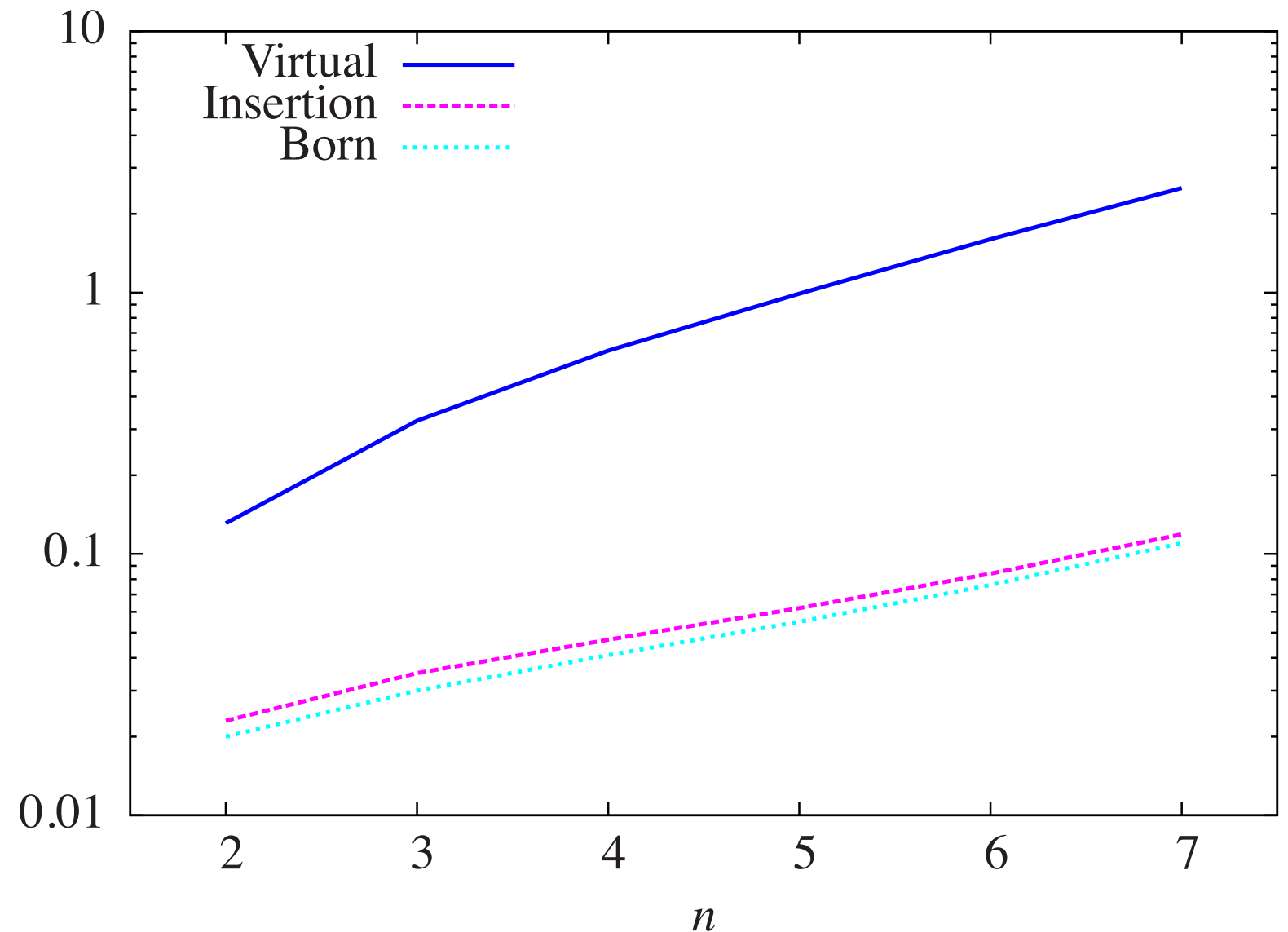
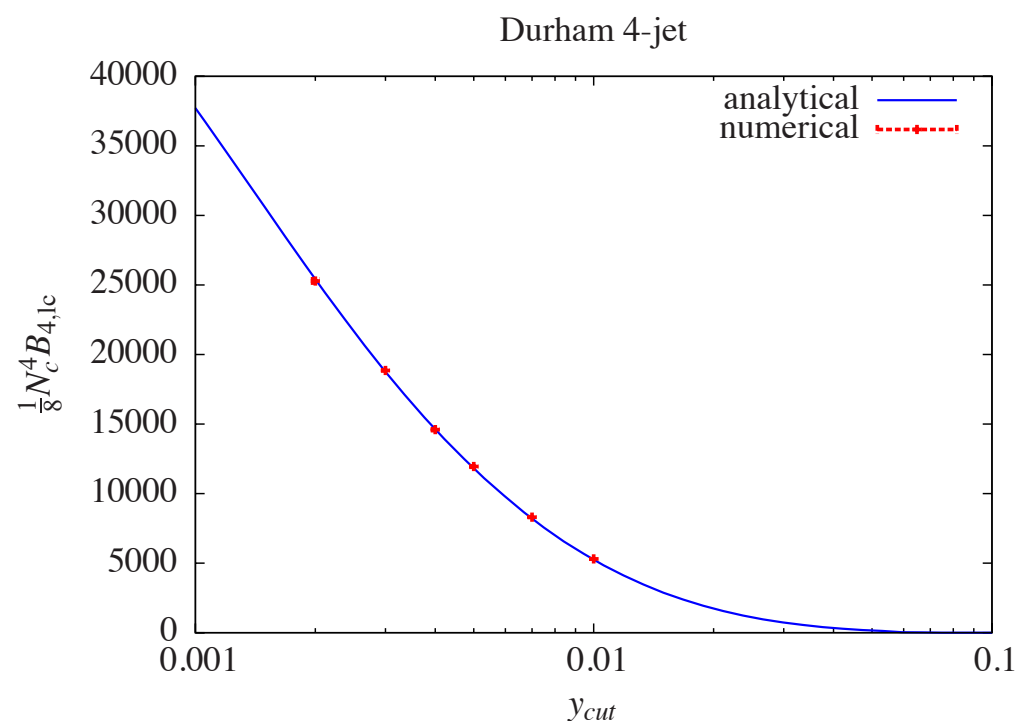


More and more jets.

(Becker, Goetz, Reuschle, Schwan and Weinzierl 11)

CPU time

Using innovative advanced numerical techniques this group was able to calculate 6 and 7 jet rates at LEP at NLO!



Extremely efficient in CPU time for such high multiplicities!



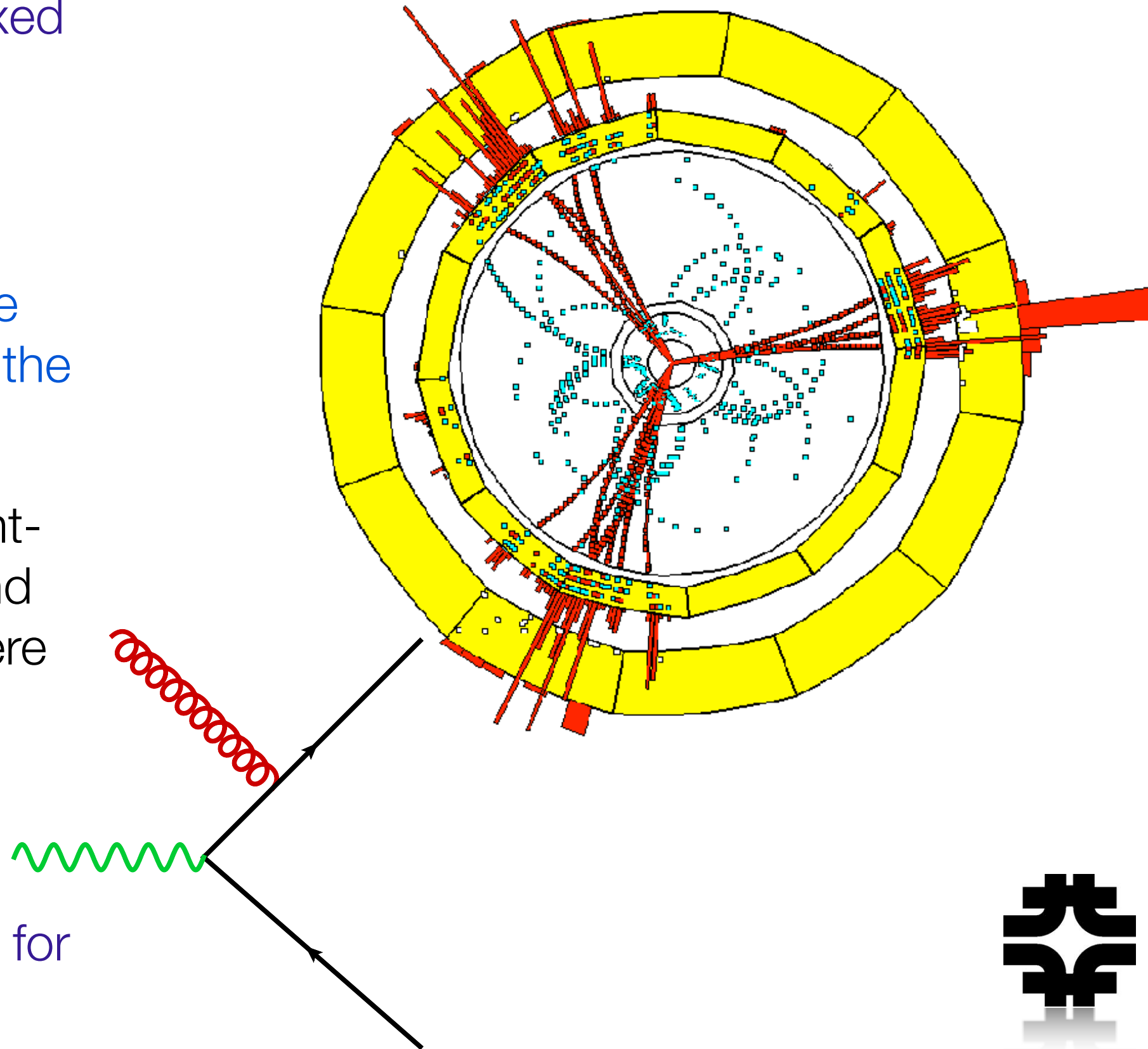
Measurements of α_s

Lepton collisions at a fixed energy provide a great setting to measure the strong coupling.

3 jets is the first time the processes depends on the strong coupling.

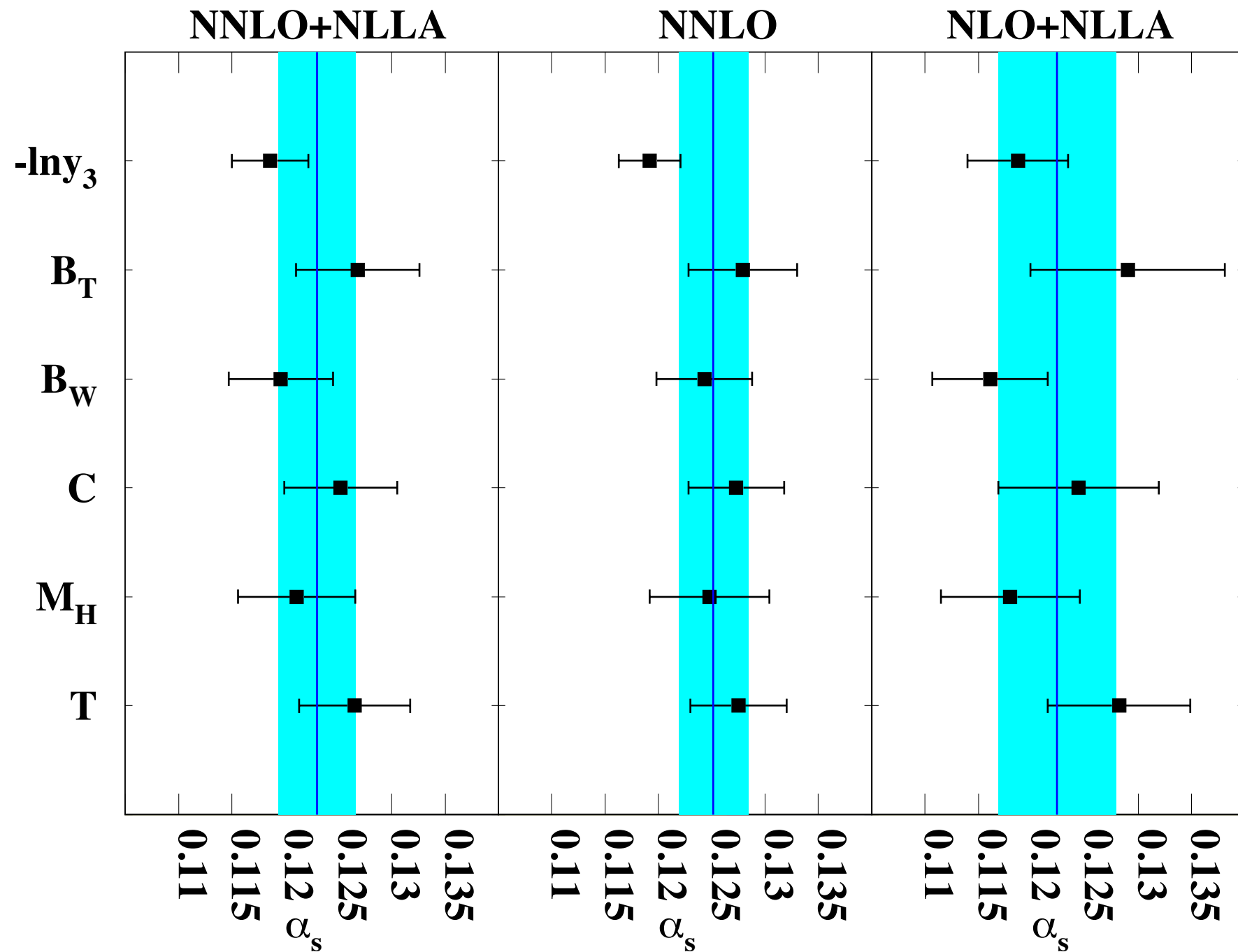
Historically various event-shapes observables (and the overall 3 jet rate) were used.

Whats the theory status for the future?



$e^+e^- \rightarrow jets$

(Dissertori, Gerhmann-De Ridder, Gehrmann, Glover, Heinrich, Luisoni, and Stenzel 09')



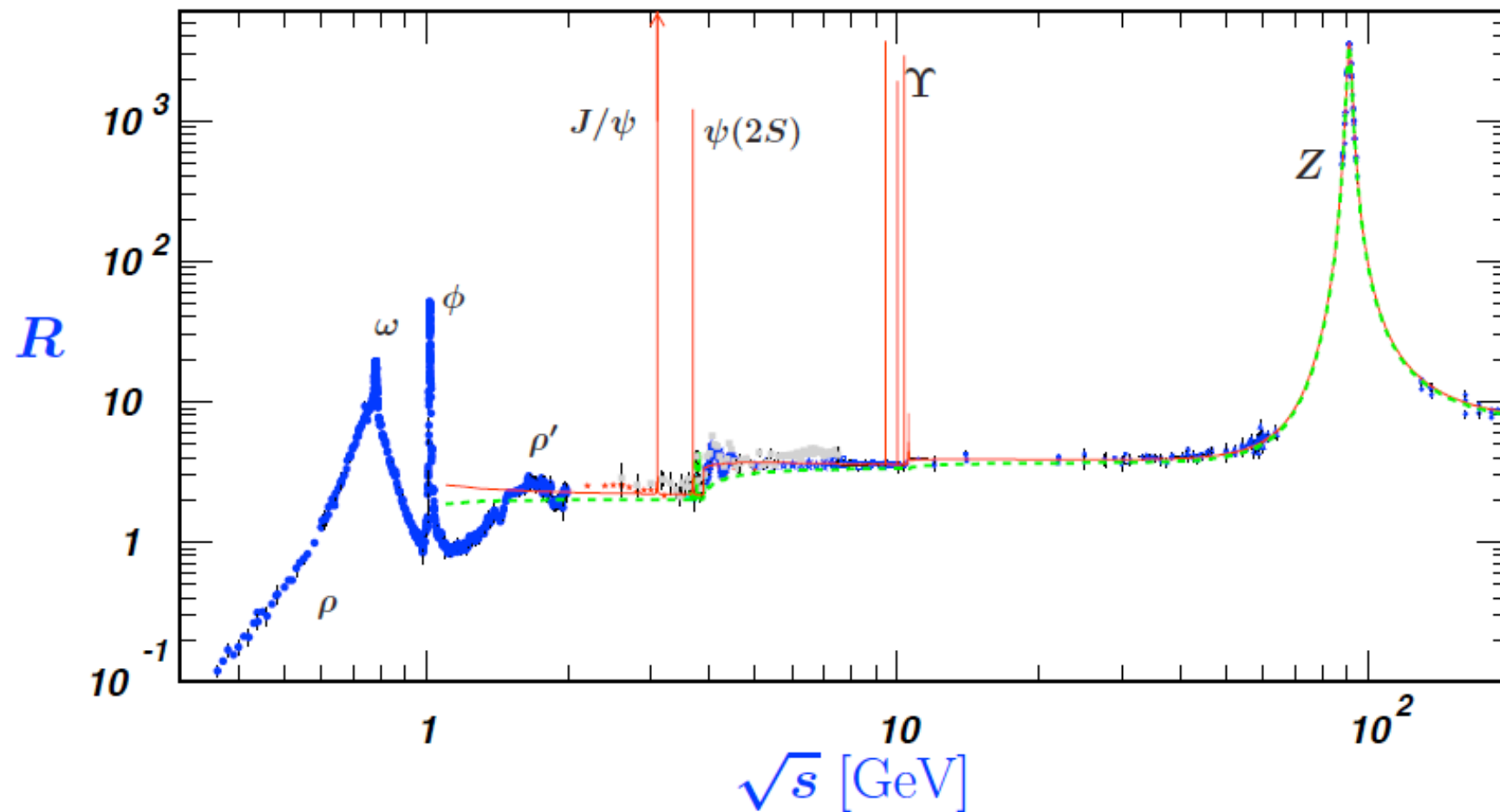
By including the resummation one improves the agreement with data, at the cost of larger theory uncertainty.

Future developments could involve NNLLA resummation and matching to showers to improve hadronization uncertainty.

$$\alpha_s(M_Z) = 0.1224 \pm 0.0009(\text{stat}) \pm 0.0009(\text{exp}) \pm 0.0012(\text{had}) \pm 0.0035(\text{theo})$$



Measurements of α_s



$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

An alternative way of measuring the strong coupling is to use the R ratio. Which is known to $\mathcal{O}(\alpha_s^4)$ in perturbation theory and is experimentally clean.

(Baikov, Chetyrkin and Kühn 08)



Why R? (Dissertori's Energy frontier meeting slide)

α_s from inclusive Z decays



- Advantage of inclusive observables:
 - by now known to NNNLO !
 - non-perturbative effects strongly suppressed

$$R_{\text{exp}} = \frac{\Gamma(Z \rightarrow \text{hadrons})}{\Gamma(Z \rightarrow \text{leptons})} = R_{EW} (1 + \delta_{QCD} + \delta_m + \delta_{np})$$

$$\frac{R_{\text{exp}}}{R_{EW}} = \mathcal{O}(1)$$

$$\sim \mathcal{O}\left(\frac{m_q^2}{M_Z^2}\right) \quad \frac{\Delta\alpha_s}{\alpha_s} \approx \mathcal{O}(\text{few \%}) \cdot \frac{\Delta\delta_m}{\delta_m}$$

Th. Gehrmann:
calculations can be
improved if necessary

$$\delta_{QCD} = \sum_{n=1}^4 c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5)$$

$$c_1 = 1.045 \Rightarrow c_1 \frac{\alpha_s(M_Z)}{\pi} \sim 0.04 = \mathcal{O}(1/25)$$

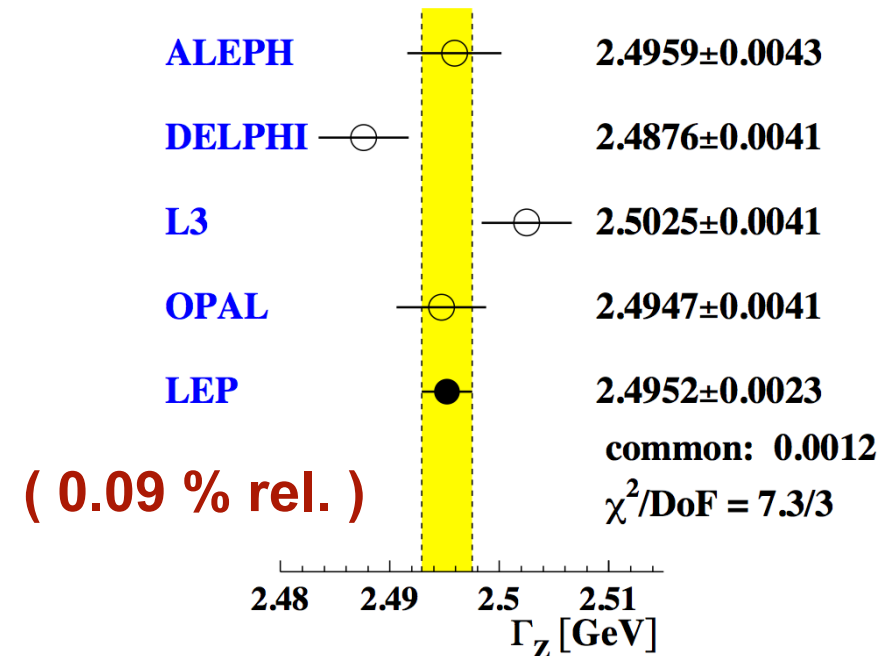
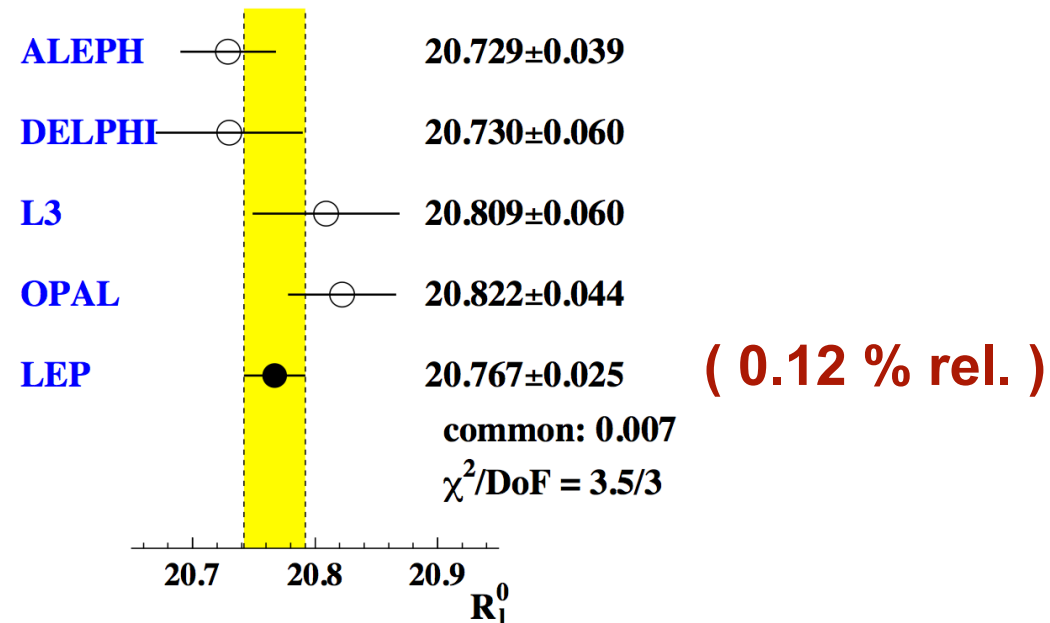
$$\mathcal{O}\left(\frac{\Lambda^4}{M_Z^4}\right)$$

$\ll 0.0001$, no problem



Why R? (Dissertori's Energy frontier meeting slide)

Latest results from LEP EW group



Source	δ	Γ_Z [MeV]	σ_{had}^0 [nb]	R_ℓ^0	R_b^0	ρ_ℓ	$\sin^2 \theta_{\text{eff}}^{\text{lept}}$	m_W [MeV]
$\Delta\alpha_{\text{had}}^{(5)}(m_Z^2)$	0.00035	0.3	0.001	0.002	0.00001	—	0.00012	6
$\alpha_S(m_Z^2)$	0.003	1.6	0.015	0.020	—	—	0.00001	2
m_Z	2.1 MeV	0.2	0.002	—	—	—	0.00002	3
m_t	4.3 GeV	1.0	0.003	0.002	0.00016	0.0004	0.00014	26
$\log_{10}(m_H/\text{GeV})$	0.2	1.3	0.001	0.004	0.00002	0.0003	0.00022	28
Theory		0.1	0.001	0.001	0.00002	—	0.00005	4
Experiment		2.3	0.037	0.025	0.00065	0.0010	0.00016	34

from slide 4:

$$\frac{\Delta\alpha_s}{\alpha_s} \approx 25 \cdot \frac{\Delta R_{\text{exp}}}{R_{\text{exp}}}$$

thus: for a rel. prec.
of ~0.1% on α_s we
need rel. exp. prec.
~25 times better !!

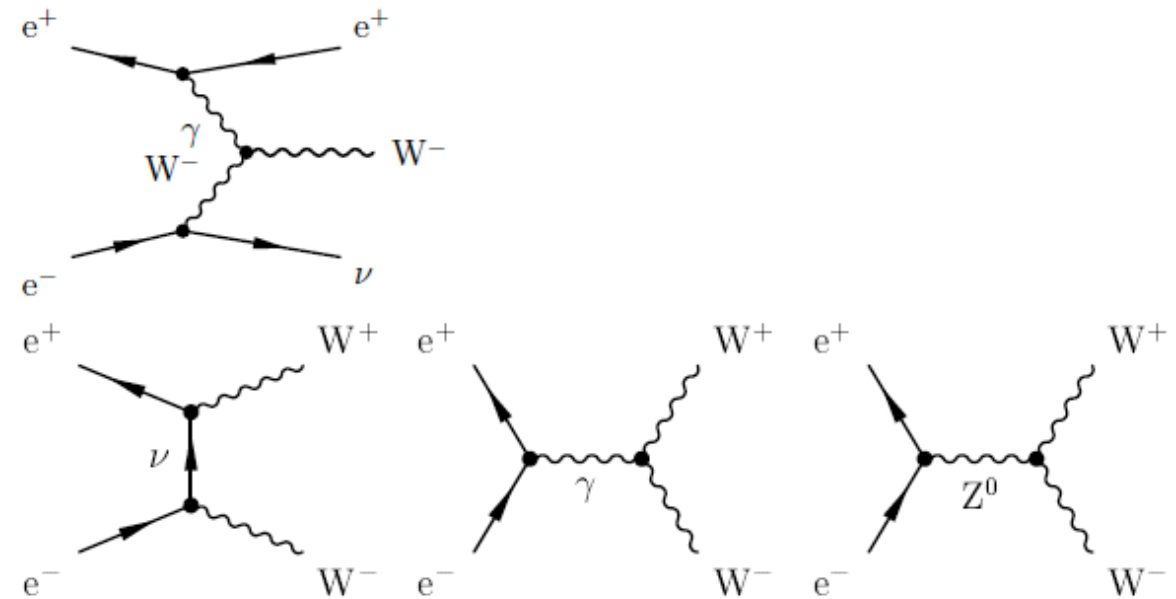
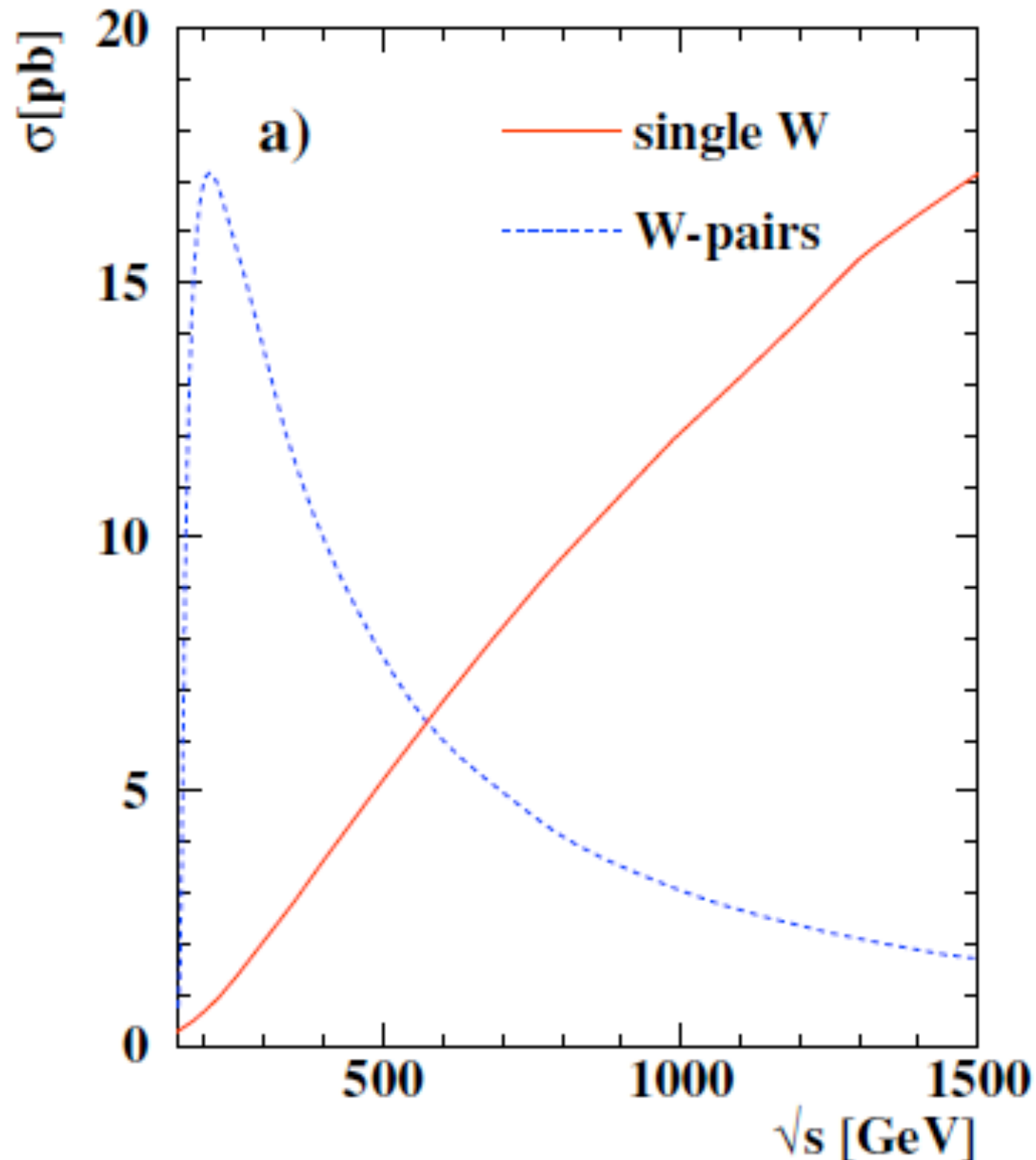


$$e^+e^- \rightarrow 4f(W^+W^-)$$

(for more details see e.g. C. Schwinn's Seattle Snowmass talk)



Production processes at a lepton collider



Counting 'W's yields two topologies (all potential 4f final states)

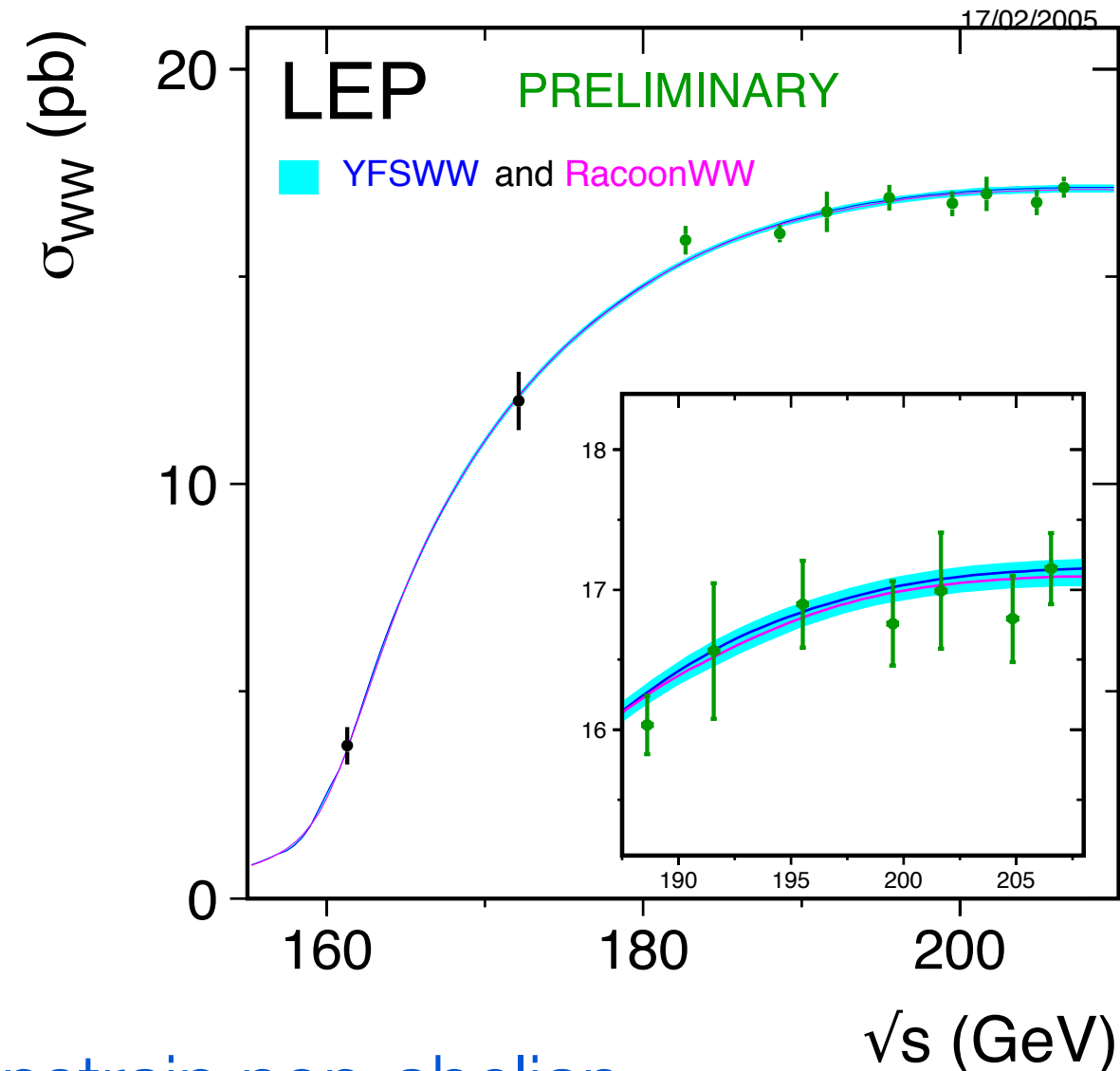
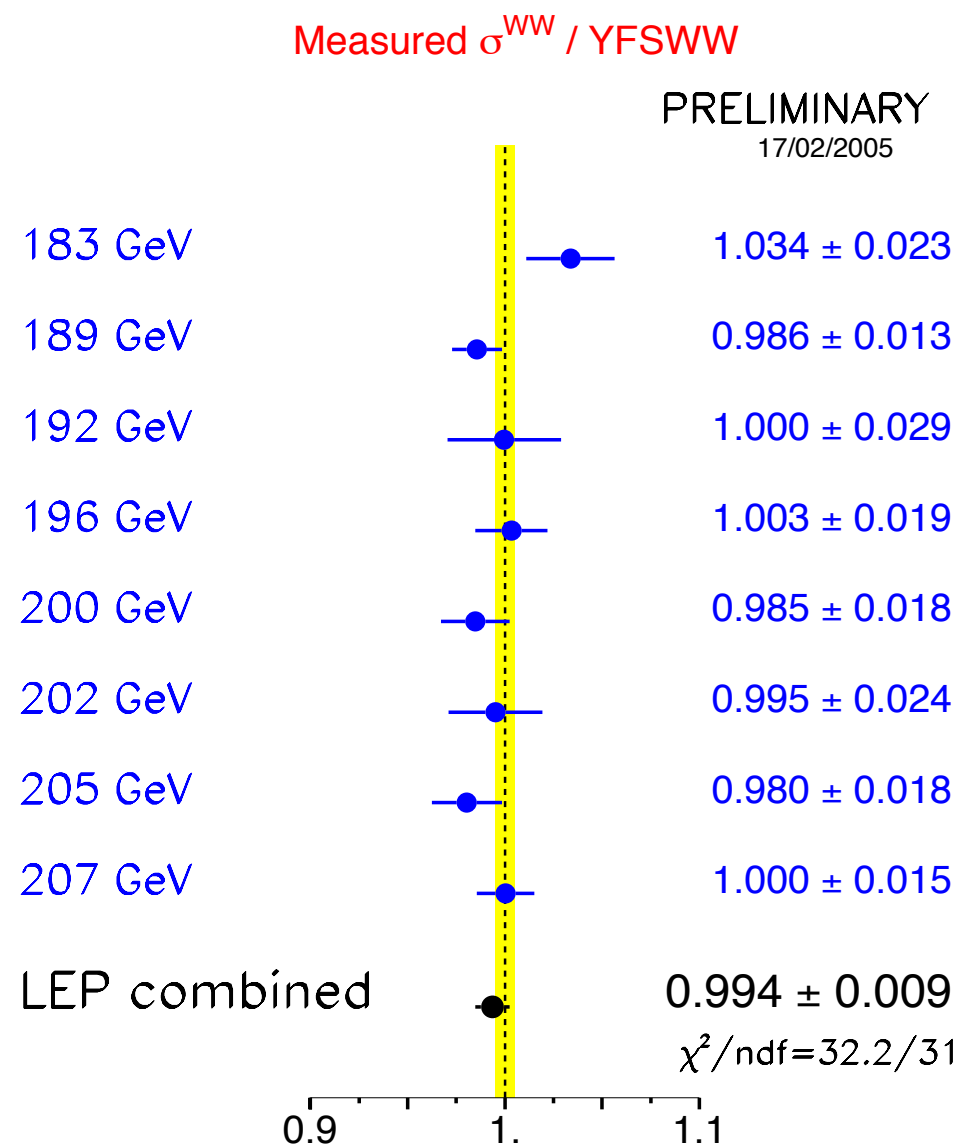
As com energy grows W pair is s-channel suppressed compared to t-channel processes.



WW overview

WW was extremely fruitful for theory and experiment at LEP

1% agreement with SM NLO predictions



Can constrain non-abelian
EW gauge structure through
AGC measurements.

And measure W mass at threshold.



Theoretical status of WW calculations

LEP era

NLO corrections split into two parts, Factorizable and Non-Factorizable soft contributions, used in LEP2 MC. RacoonWW and YFSWW

(Denner et al. 99 Jadach et al 99)

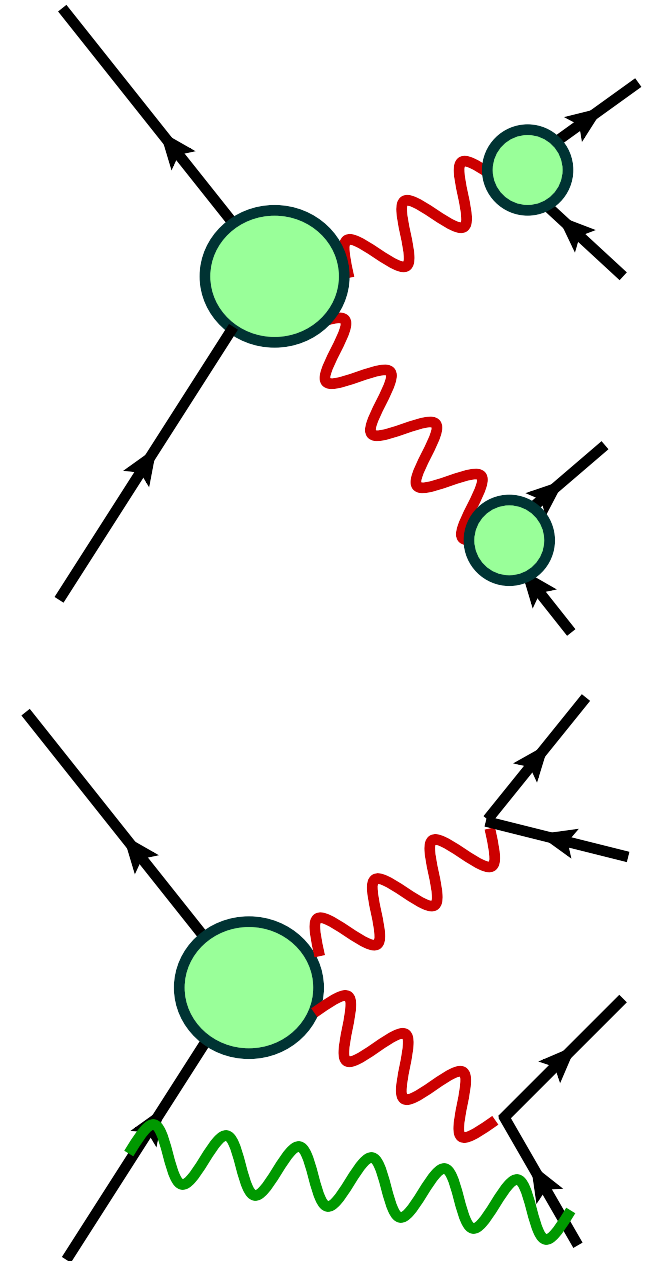
Post LEP era

Complete NLO calculations for the charged currents (Denner et al. 05)

Log-enhanced NNLO corrections for $\sqrt{s} \gg m_W$

(Kühn et al 05)

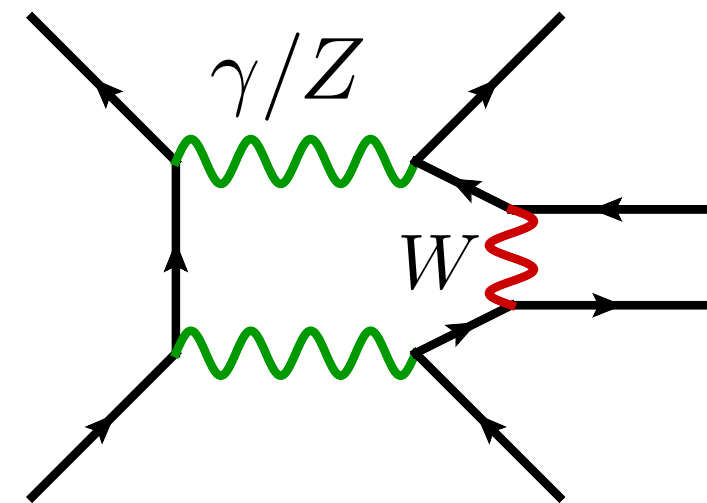
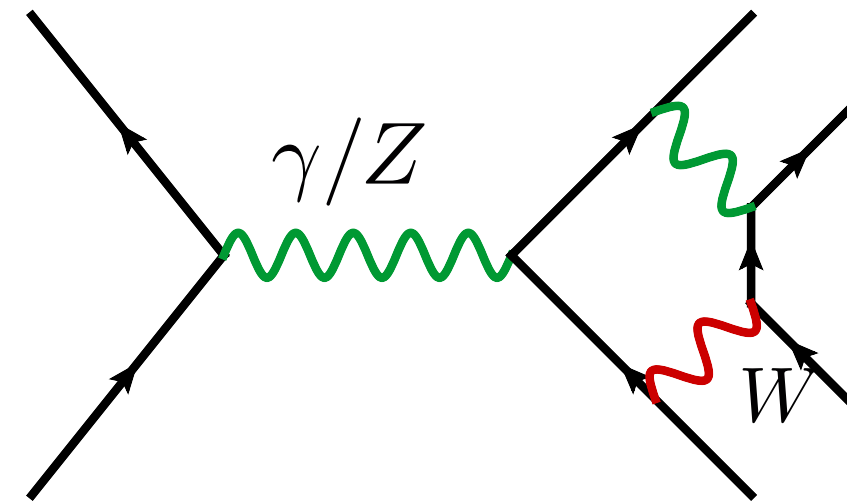
NLO and leading NNLO correction in threshold expansion. (Beneke et al. 08, Actis et al 08)



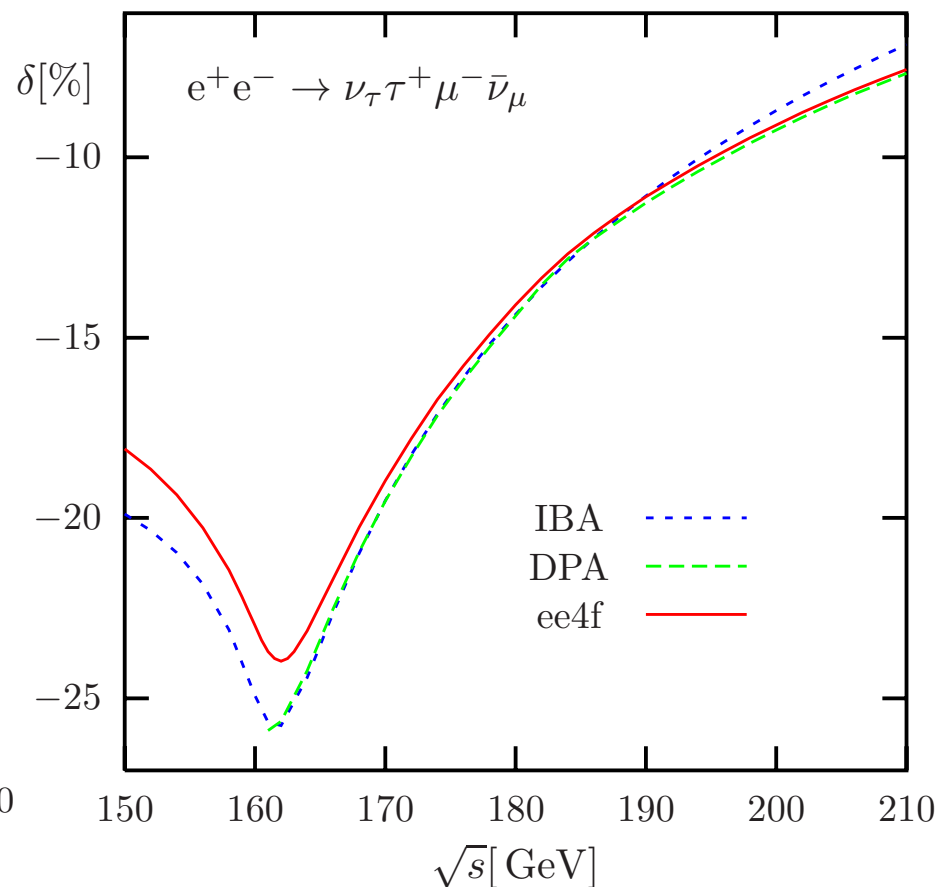
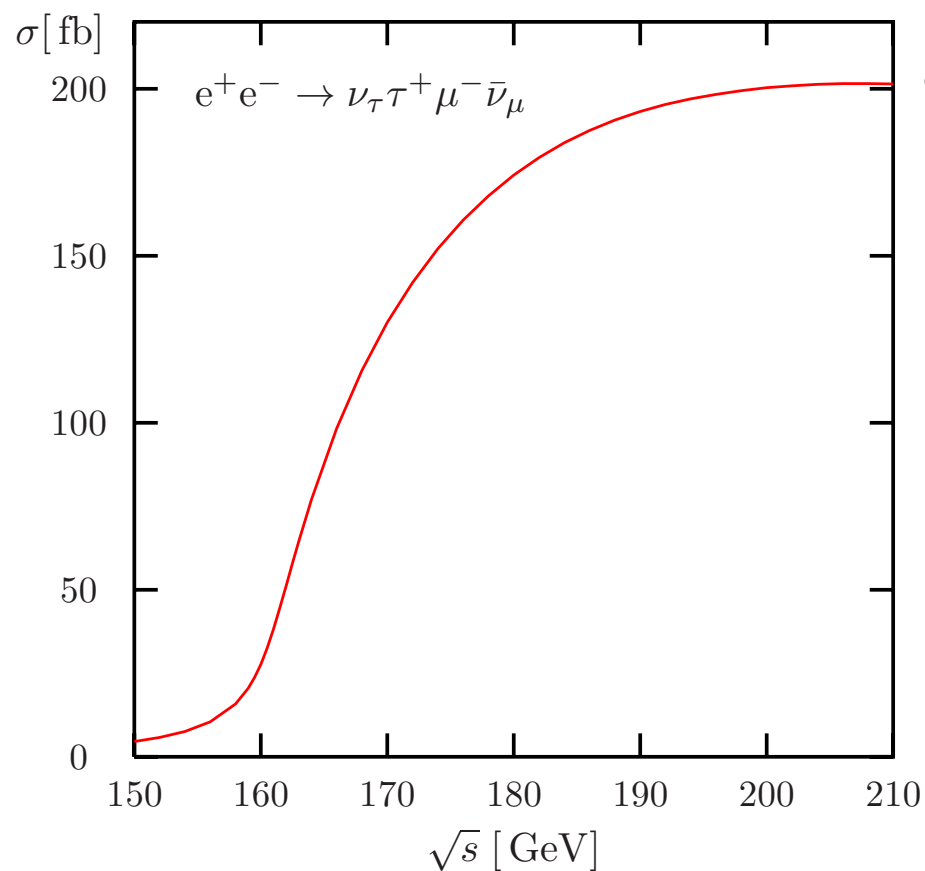
$$e^+e^- \rightarrow 4f$$

The full $e^+e^- \rightarrow 4f$ is now available, and represents a fantastic theoretical achievement, (over 1000 loop diagrams, 6 - point integrals)

(Denner, Dittmaier, Roth and Wieders 05)



Needed to match future exp precision!



Possible Improvements and future directions.

Treatment of ISR radiation (currently LL resummation)

Threshold resummation, implementation Leading NNLO corrections?

qq->WW @ NNLO should be available soon, could be utilized to improve predictions.

Calculation of Triboson (and ZZ) processes at NLO in EW.



Theory post TLEP, the VHE-LHC

(borrowed from John Campbell's Snowmass Seattle talk)



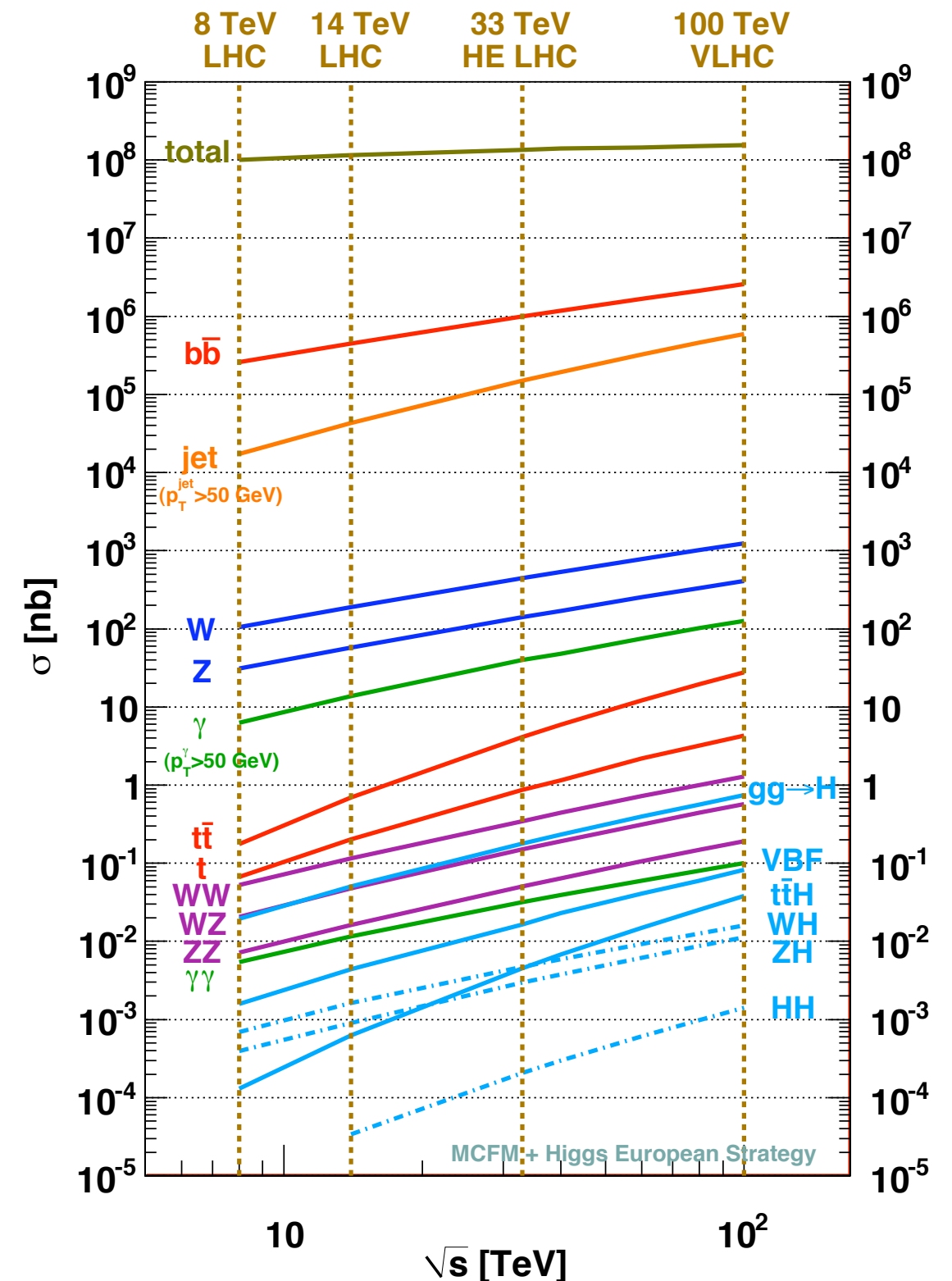
Cross sections at higher energies

- Most important cross sections are reasonably stable under NLO corrections at higher energies.

33 TeV

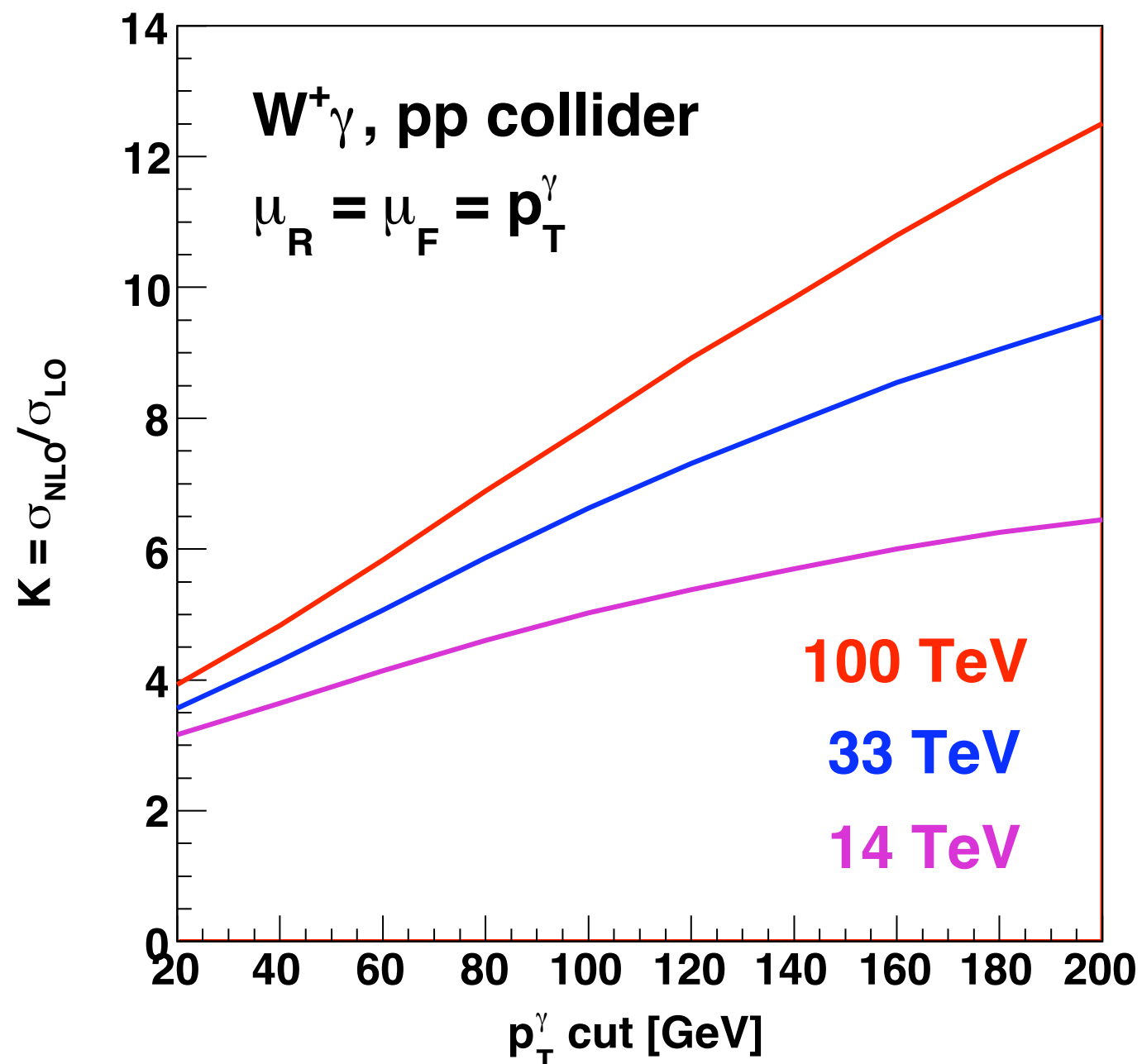
Process	μ_R^2, μ_F^2	σ_{LO} [pb]	σ_{NLO} [pb]	K -factor
$W^+ j$ ($p_T^W > 200$ GeV)	$M_W^2 + p_T^{W^2}$	427	629	1.47
$W^- j$ ($p_T^W > 200$ GeV)	$M_W^2 + p_T^{W^2}$	291	443	1.52
$Z^0 j$ ($p_T^Z > 200$ GeV)	$M_Z^2 + p_T^{Z^2}$	312	460	1.41
γj ($p_T^\gamma > 100$ GeV)	$p_T^{\gamma^2}$	2690	4030	1.47
$W^+ \gamma$ ($p_T^\gamma > 100$ GeV)	$p_T^{\gamma^2}$	1.90	10.0	5.26
$W^- \gamma$ ($p_T^\gamma > 100$ GeV)	$p_T^{\gamma^2}$	1.29	7.50	5.81
$Z^0 \gamma$ ($p_T^\gamma > 100$ GeV)	$p_T^{\gamma^2}$	3.66	7.88	2.15
$\gamma\gamma$ (both $p_T^\gamma > 100$ GeV)	$m_{\gamma\gamma}^2$	2.70	3.65	1.35
$\ell^+ \ell^-$ ($m_{\ell^+ \ell^-} > 150$ GeV)	$m_{\ell^+ \ell^-}^2$	20.9	23.6	1.13

- Some notable exceptions that are understood.
- Scaling of cuts on basic objects assumed (jets, photons).

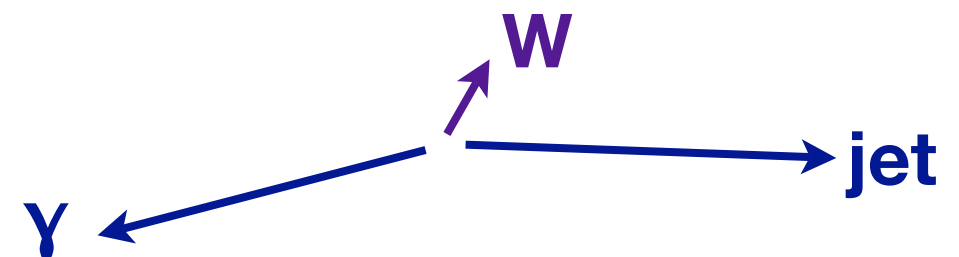


Lessons from NLO

- ◆ Phenomenon of “giant” K-factors well-known - see recent studies by Sapeta & Salam.



- ◆ Caused by new gluon contributions and kinematic configurations.



- ◆ Exacerbated by scaling cuts with energy.
- ◆ Underlines importance of using latest tools (such as matched samples) where these effects are included.

Summary

Many advanced calculations for jet production at lepton colliders exist, extending these will require heroic theoretical effort.

Some “easy” things to do will be to study NLO+PS matching with existing predictions.

Large Luminosity at a future lepton collider will allow access to rarer EW processes, NLO corrections for these will be required.

In general the theoretical SM predictions are in pretty good shape, and can probably be improved as needed (with significant time investment) for the exp results. (e.g. NNLLA resummation, N3LO, NLO EW).

VHE-LHC will require new approaches.....

