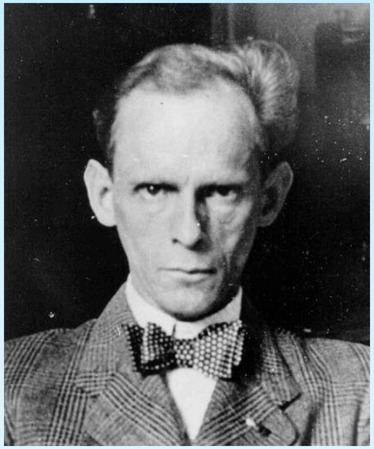
Evolution of Precise Experiments: (A tale of two Leptons)

Thompson

e	μ
ν_e	ν_μ



Kunze

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Anderson
Neddermeyer

Acknowledgements:

I have borrowed heavily from:

Bailey et al, Nuclear Physics B150 (1979)

Miller, de Rafael, Roberts, Rep. Prog. Phys. 70 (2007) 795-881

Farley and Semertzidis, Prog. Nucl. Part. Phys. 52 (2004) 1-83.

Bennett et al., Phys. Rev. D 73, 072003 (2006).

The web, including the AIP Segre archives and the Nobel website for photos.

BNL photo archives

Other talks, including from D. Hertzog.

Outline

- A few words about magnetic moments
- The beginning: $g_e \cong 2$
- The discovery that $g_e > 2$
- The g_e industry
- The discovery that $g_\mu \cong 2$
- The discovery that $g_\mu > 2$
- The CERN $(g_\mu - 2)$ Experiments
- Preparing for the BNL Experiment
- Final remarks

In the preceding two talks we heard:

1. "The goal is to find new physics."
2. "trivial factor of 1"

I would say: "The goal is to understand how nature works."

Remember that quantum mechanics was once "new physics" and everything we know is built on the pioneering work of Schrodinger, Heisenberg, Thomas, Uhlenbeck and Goudsmit,

and especially Dirac, who produced the "trivial factor of 1 (g = 2)"; along with the foundation for modern field theory which assumes the Dirac equation as the beginning.

(remember all those γ matrices on days 1 and 2?)

Magnetic Dipole Moments

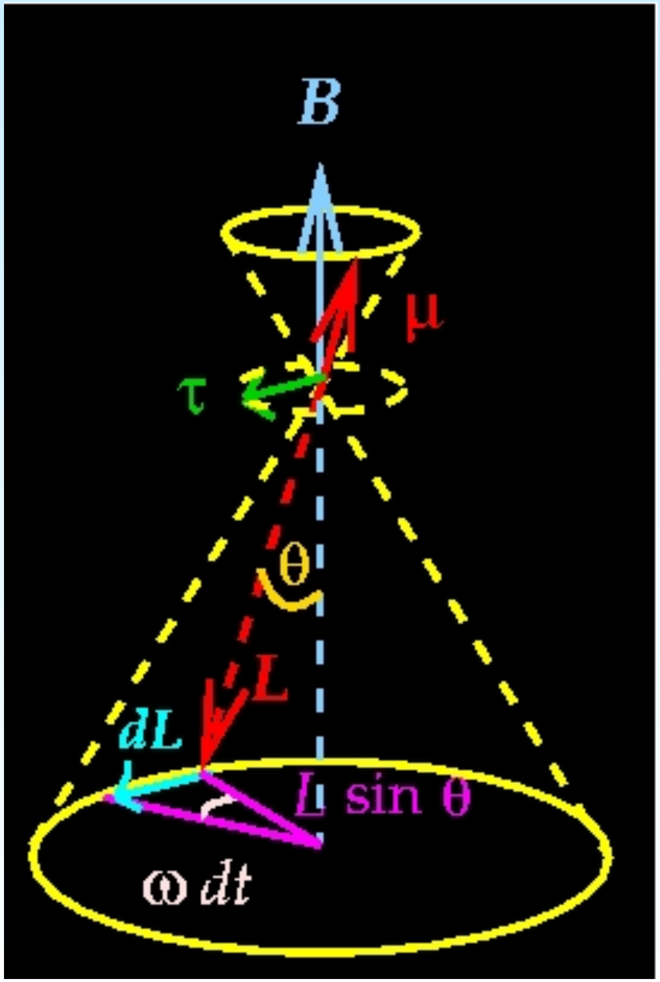
Dipole in a B field:

$$Torque: \quad \vec{N} = \vec{\mu} \times \vec{B}$$

$$Energy: \quad H = -\vec{\mu} \cdot \vec{B}$$

μ_s **for a particle with spin:**

$$\vec{\mu}_s = g_s \left(\frac{Qe\hbar}{2m} \right) \vec{s}; \quad e > 0$$



Larmor precession

Measurements of Magnetic Dipole moments started with a proposal by Otto Stern to study space quantization:

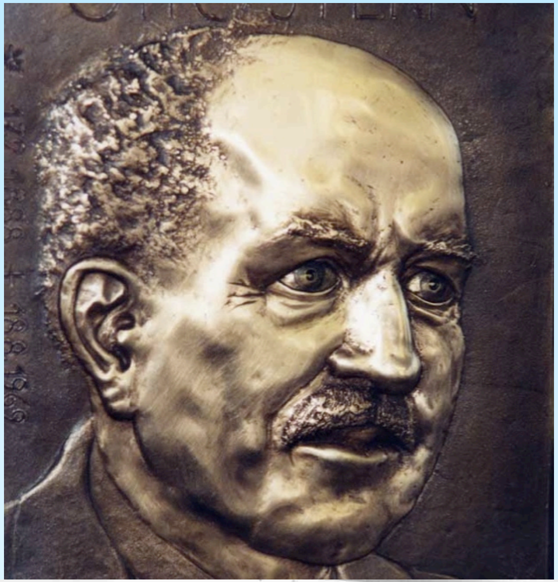
Z. Phys. 7, 249 (1921)

Ein Weg zur experimentellen Prüfung der Richtungsquantelung im Magnetfeld.

Von Otto Stern in Frankfurt a. Main.

Mit zwei Abbildungen. — (Eingegangen am 26. August 1921.)

In der Quantentheorie des Magnetismus und des Zeemaneffektes wird angenommen, daß der Vektor des Impulsmomentes eines Atoms nur ganz bestimmte diskrete Winkel mit der Richtung der magnetischen Feldstärke $\mathfrak{H}$ bilden kann, derart, daß die Komponente des Impulsmomentes in Richtung von $\mathfrak{H}$ ein ganzzahliges Vielfaches von $h/2\pi$ ist[1]. Bringen wir also ein Gas aus Atomen, bei denen das



$$\mathfrak{K} = \mathfrak{m}_x \frac{\partial \mathfrak{H}}{\partial x} + \mathfrak{m}_y \frac{\partial \mathfrak{H}}{\partial y} + \mathfrak{m}_z \frac{\partial \mathfrak{H}}{\partial z}.$$

Nun führt das Atom eine gleichförmige Rotation um die Feldrichtung, d. h. um die z-Achse aus[1]), wobei $\mathfrak{m}_z$ konstant bleibt, während der Mittelwert von $\mathfrak{m}_x$ und $\mathfrak{m}_y$ über einen vollen Umlauf Null wird. Mitteln wir also bei konstantem $\frac{\partial \mathfrak{H}}{\partial x}, \frac{\partial \mathfrak{H}}{\partial y}, \frac{\partial \mathfrak{H}}{\partial z}$ über eine gegen die Umlaufdauer (die z. B. für $\mathfrak{H} = 1000$ Gauß 7.10^{-10} sec ist) große Zeit, so wird die mittlere auf das Atom wirkende Kraft:

$$\overline{\mathfrak{K}} = m_z \frac{\partial \mathfrak{H}}{\partial z}.$$

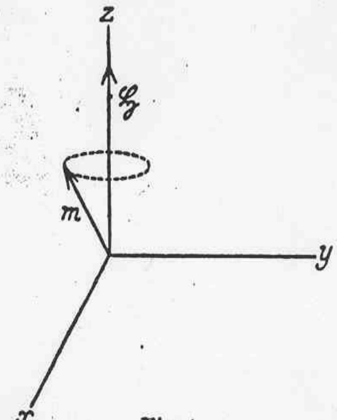


Fig. 1.

Für die auf das Atom wirkende Kraft ist also beim magnetischen Moment

First Result from Walter Gerlach and Otto Stern

Der experimentelle Nachweis der Richtungsquantelung im Magnetfeld.

Von **Walther Gerlach** in Frankfurt a. M. und **Otto Stern** in Rostock.

Mit sieben Abbildungen. (Eingegangen am 1. März 1922.)

Vor kurzem[1]) wurde in dieser Zeitschrift eine Möglichkeit angegeben, die Frage der Richtungsquantelung im Magnetfeld experimentell zu entscheiden. In einer zweiten Mitteilung[2]) wurde gezeigt, daß das normale Silberatom ein magnetisches Moment hat. Durch die Fortsetzung dieser Untersuchungen, über die wir uns im folgenden zu berichten erlauben, wurde die Richtungsquantelung im Magnetfeld als Tatsache erwiesen.

Versuchsanordnung. Methode und Apparatur waren im allgemeinen die gleichen wie bei unseren früheren Versuchen. Im einzelnen wurden jedoch wesentliche Verbesserungen[3]) vorgenommen, welche wir in Ergänzung unserer früheren Angaben hier mitteilen. Der Silberatomstrahl kommt aus einem elektrisch geheizten Öfchen aus Schamotte mit einem Stahleinsatz, in dessen Deckel zum Austritt des Silberstrahls eine 1 mm² große kreisförmige Öffnung sich befand. Der Abstand zwischen Ofenöffnung und erster Strahlenblende wurde auf 2,5 cm vergrößert, wodurch ein Verkleben der Öffnung durch gelegentlich aus dem Öfchen spritzende Silbertröpfchen wie auch ein zu schnelles Zuwachsen durch das Niederschlagen des Atomstrahls verhindert

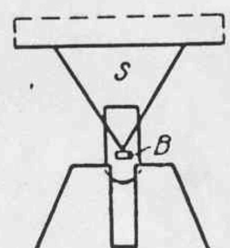
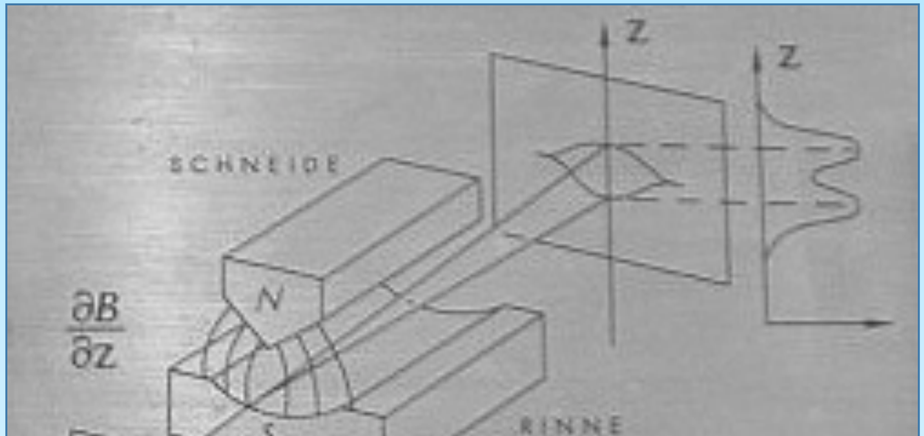
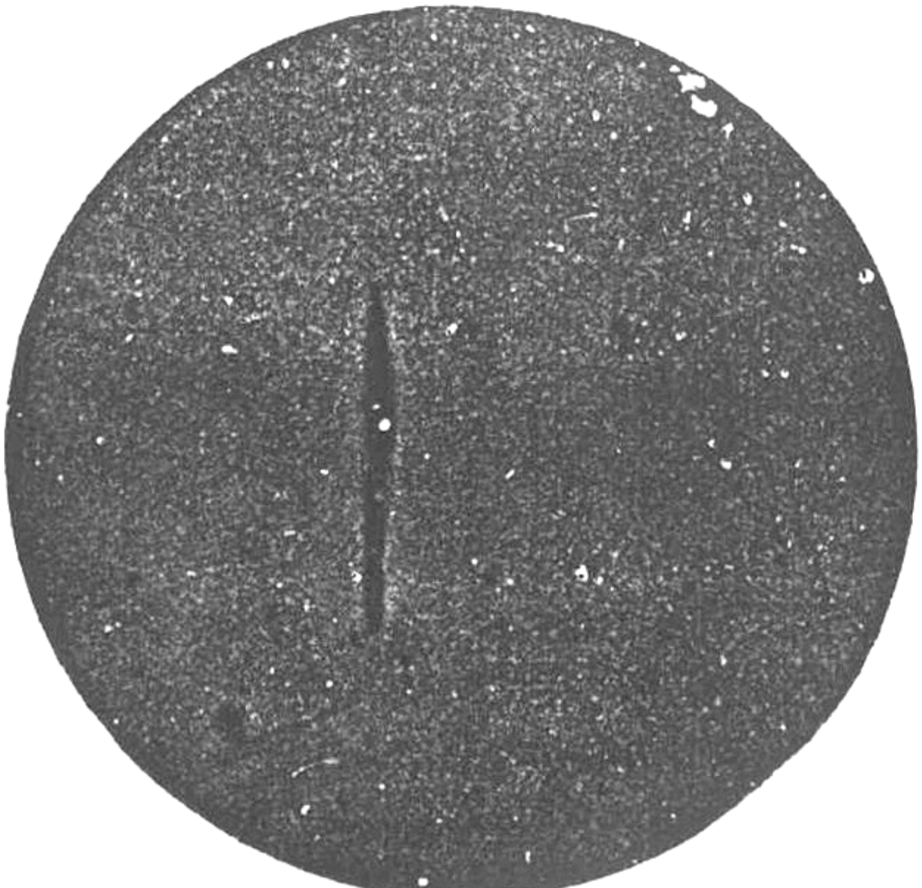


Fig. 1.

Z. Phys. 8, 110 (1922)

W. Gerlach





ohne Magnetfeld

Fig. 2.

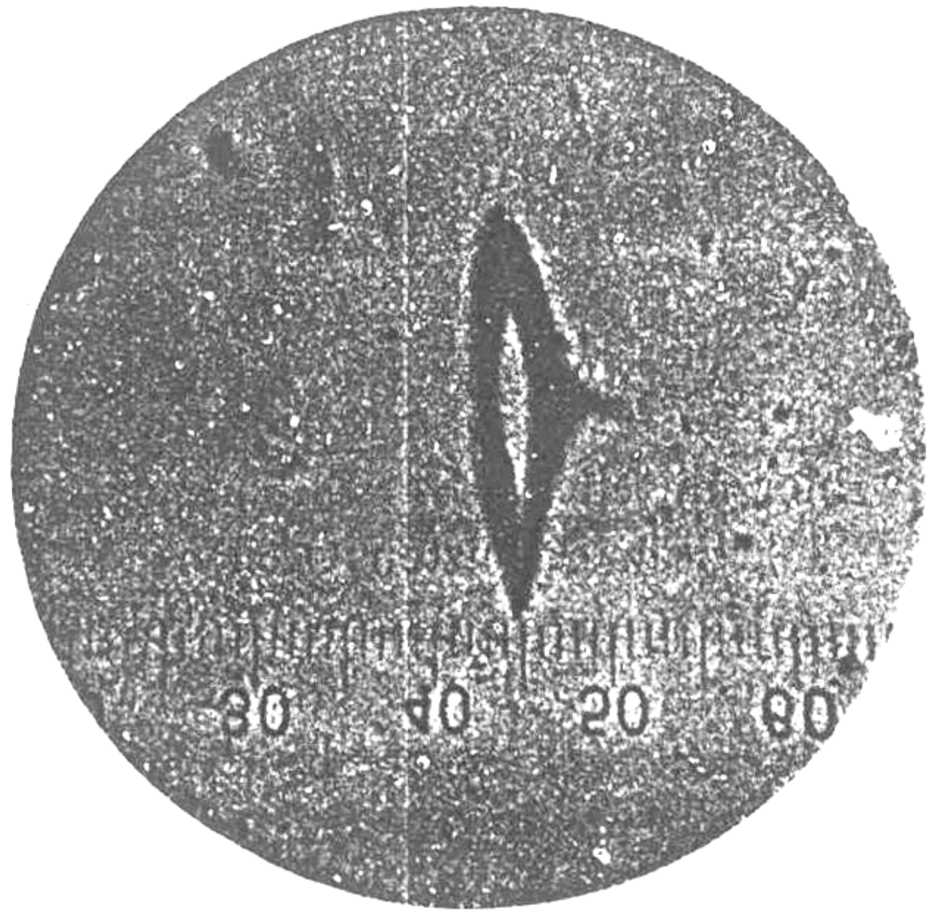


Fig. 3. **mit Magnetfeld**

By 1924 they had published 3 papers plus this review article.

1924. №10.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 74.

1. *Über die Richtungsquantelung im Magnetfeld*[1]; von *Walther Gerlach und Otto Stern.*

(Hierzu Tafel III.)

Nr. der Aufnahme	Entfernung des unabgelenkten Strahles von der Schneide	Mittlere Ablenkung des abgestoßenen Strahles	
		berechnet	beobachtet
15	0,32 mm	$0{,}10_1$ mm	$0{,}10_3$ mm
14	0,21 mm	$0{,}14_6$ mm	0,15 mm

Die Genauigkeit der Messungen schätzen wir auf 10 Proz. Innerhalb dieser Fehlergrenzen zeigen also die Versuche, daß das Silberatom im Normalzustand ein Bohrsches Magneton hat.

10% precision

in modern language

$$\vec{\mu}_s = g_s \left(\frac{Qe}{2m}\right) \vec{s}$$

$$\Rightarrow g_e = 2$$

What had to be understood to conclude $g_e = 2$?

- Quantum Mechanics
- Spin
- Thomas Precession

G.E. Uhlenbeck S. Goudsmit postulated electron spin to explain fine-structure spectra

Klein, U & G

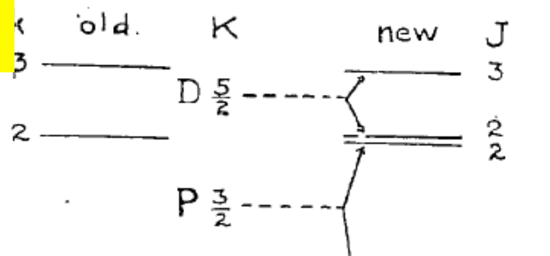
Naturwissenschaften
13, 953 (1925)
Nature
117, 264 (1926)

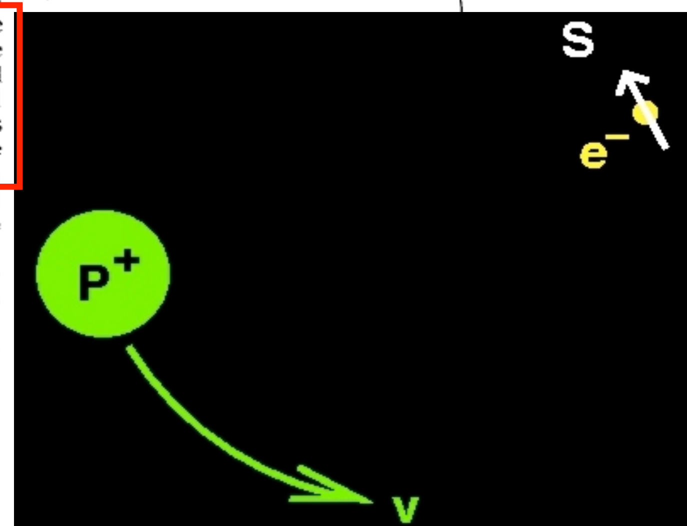
On account of its magnetic moment, the electron will be acted on by a couple just as if it were placed at rest in a magnetic field of magnitude equal to the vector product of the nuclear electric field and the velocity of the electron relative to the nucleus.

general. In this letter we shall try to show how our hypothesis enables us to overcome certain fundamental difficulties which have hitherto hindered the interpretation of the results arrived at by those authors.

To start with, we shall consider the effect of the spin on the manifold of stationary states which corresponds to motion of an electron round a nucleus. On account of its magnetic moment, the electron will be acted on by a couple just as if it were placed at rest in a magnetic field of magnitude equal to the vector product of the nuclear electric field and the velocity of the electron relative to the nucleus divided by the velocity of light. This couple will cause a slow precession of the spin axis, the conservation of the angular momentum of the atom being ensured by a compensating precession of the orbital plane of the electron. This complexity of the motion requires that, corresponding to each stationary state of an imaginary atom, in which the electron has no spin, there shall in general exist a set of states which differ in the orientation of the spin axis relative to the orbital plane, the other characteristics of the motion remaining unchanged. If the spin corresponds to a one-quantum rotation there will be in general two such states. Further, the energy difference of these states will, as a simple calculation shows, be proportional to the fourth power of the nuclear charge. It will also depend on the quantum numbers which

...oment of momentum is given by $Kh/2\pi$, where ... $\frac{3}{2}$, $\frac{5}{2}$. The total angular momentum of the ...s $Jh/2\pi$, where $J=1, 2, 3$. The symbols K correspond to those used by Landé in his ...cation of the Zeeman effects of the optical ...lets. The letters S, P, D also relate to the ... with the structure of optical spectra which we ...r below. The dotted lines represent the ...i of the energy levels to be expected in the ...e of the spin of the electron. As the arrows in... this spin now splits each level into two, with the ...on of the level $K=\frac{1}{2}$, which is only displaced. ...rder to account for the experimental facts, the ...ng levels must fall in just the same places as ...els given by the older theory. Nevertheless, ...o schemes differ fundamentally. In particular, ...w theory explains at once the occurrence of ... components in the fine structure of the ...en spectrum and of the helium spark spectrum





UNIVERSITY

Only after L.H. Thomas' paper deriving the famous factor of 2, did some people believe in spin.

Letters to the Editor.

[The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, nor to correspond with the writers of, rejected manuscripts intended for this or any other part of NATURE. *No notice is taken of anonymous communications.]*

The Motion of the Spinning Electron.

In a letter published in Nature of February 20, p. 264, Messrs. Uhlenbeck and Goudsmit have shown how great difficulties which atomic theory had met in the attempt to explain spectral structure and Zeeman effects, can be avoided by using the idea of the spinning electron. Although their theory is in complete qualitative agreement with observation, it involved an apparent quantitative discrepancy. The value of the precession of the spin axis in an external magnetic field required to account for Zeeman effects seemed to lead to doublet separations twice those which are observed. This discrepancy, however, disappears when the kinematical problem concerned is examined more closely from the point of view of the theory of relativity.

As usual, letters in heavy type will denote vectors. The anomalous Zeeman effect seems to require that the spin axis of the electron precesses about an external magnetic field **H** with angular velocity

$$\frac{e}{mc}\mathbf{H}, \quad . \quad . \quad . \quad . \quad . \quad (A)$$

To a first approximation

$$\mathbf{f} = -\frac{e}{m}\mathbf{E},$$

so the rate of precession is

$$\frac{e}{2mc^2}[\mathbf{E}\times\mathbf{v}], \quad . \quad . \quad . \quad . \quad (C)$$

just half the expression (B).

The interpretation of the fine structure of the hydrogen lines proposed by Messrs. Uhlenbeck and Goudsmit now no longer involves any discrepancy. In fact, as Dr. Pauli and Dr. Heisenberg have kindly communicated in letters to Prof. Bohr, it seems possible to treat the doublet separation as well as the anomalous Zeeman effect rigorously on the basis of the new quantum mechanics. The result seems to be full agreement with experiment when the calculation is based on formulæ (A) and (C).

I hope in a later paper to develop the above kinematical argument in greater detail.

In conclusion, I wish to express my appreciation of the encouragement and help of Prof. Bohr and Dr. Kramers.

L. H. Thomas.

Universitetets Institut for Teoretisk Fysik, Copenhagen, February 20.

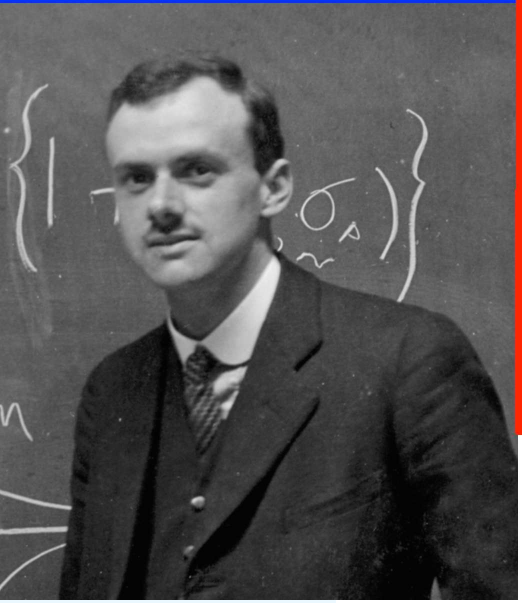
Nature **117**, 514 (1926)
Phil. Mag. **3**, 1 (1927)

Thomas later told Goudsmit that Pauli ridiculed the idea when he first heard the suggestion from Kronig.

L.H. Thomas

I think you and Uhlenbeck have been very lucky to get your spinning electron published and talked about before Pauli heard of it. It appears that more than a year ago Kronig believed in the spinning electron and worked out something; the first person he showed it to was Pauli. Pauli ridiculed the whole thing so much that the first person became also the last and no one else heard anything of it. Which all goes to show that the infallibility of the Deity does not extend to his self-styled vicar on earth.

Theory of Magnetic and Electric Dipole Moments



$$i(\partial_\mu - ieA_\mu(x))\gamma^\mu\psi(x) = m\psi(x)$$

$$\vec{\mu} = g\left(\frac{Qe}{2m}\right)\vec{s}, \quad e > 0 \qquad g \equiv 2$$

The Quantum Theory of the Electron.

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

Proc. R. Soc. (London) **A117**, 610 (1928)

Dirac was surprised when his theory gave the correct magnetic moment of the electron, i.e. g=2

"an unexpected bonus for me, completely unexpected."

quoted by A. Pais in *Paul Dirac: The Man and His Work*, P. Goddard, ed., Cambridge U. Press, New York (1998).

Non-relativistic reduction of the Dirac Equation for an electron in a weak magnetic field.

$$i\hbar\frac{\partial\psi}{\partial t} = [\frac{p^2}{2m} - \frac{e}{2m}(\vec{L}+2\vec{S})\cdot\vec{B}]\psi$$

$$g_L = 1 \quad g_S = 2$$

But … the hydrogen hyperfine structure was not as expected …

Nafe, Nelson, Rabi, Phys. Rev. 71, 914 (1947)

The Hyperfine Structure of Atomic Hydrogen and Deuterium†

J. E. Nafe, E. B. Nelson, and I. I. Rabi
Columbia University, New York, New York
May 19, 1947

THE hyperfine structure separation, ν_H and ν_D, of atomic hydrogen and deuterium were measured directly by means of the atomic beam magnetic resonance method.[1-3] For each atom two resonance lines were measured, each at the same value of the magnetic field, and the ν_H and ν_D were evaluated entirely from differences in the frequencies. Neither the value of the magnetic field nor the g values of the atomic and nuclear systems enter into the final result.

In H, where the value of the nuclear spin $I=1/2$ and the atomic $J=1/2$, the π-transitions $(1,1)\leftrightarrow(0,0)$ and $(1,0)\leftrightarrow(1,-1)$ were measured at the same value of the magnet current. The difference between these two frequencies gives ν_H directly (see Eqs. 9–12 of reference 3). For D, where $I=1$ and $J=1/2$, the line $(3/2,1/2)\leftrightarrow(1/2,-1/2)$, $(3/2,-1/2)\leftrightarrow(1/2,1/2)$, an unresolved doublet, and the line $(3/2,3/2)\leftrightarrow(1/2,1/2)$ were measured in quite weak fields of the order of one gauss. The first line gives ν_D almost directly, and the difference in frequency of the two lines gives a small correction of less than 0.01 percent.

$$\mu_P = 2.7896 \pm 0.0008$$
$$\mu_D = 0.85648 \pm 0.00037,$$

for α^2, R_∞, and C we have the values given by Birge[6]

$$\alpha^2 = (5.3256 \pm 0.0013) \times 10^{-5}$$
$$R_\infty = 109737.303 \pm 0.017\ \text{cm}^{-1}$$
$$C = (2.99776 \pm 0.00004) \times 10^{10}\ \text{cm sec.}^{-1}$$

With these values and the value of the ratio μ_P/μ_D given by Kellogg, Rabi, Ramsey, and Zacharias[2] and by Arnold and Roberts[7] as 3.2571 ± 0.001, we obtain the results given in Table I.

Table I. The hyperfine structure separation of H and D.

	Measured	Computed from Eq. (2)
ν_H	1421.3 ± 0.2 Mc	1416.90 ± 0.54 Mc
ν_D	327.37 ± 0.03 Mc	326.53 ± 0.16 Mc
ν_H/ν_D	4.3416 ± 0.0007	4.3393 ± 0.0014

There is clearly an important difference between the measured and calculated values of ν_H and ν_D of about 0.26 percent compared with the probable error of the calculated value of 0.05 percent. The difference is five times greater than the claimed probable error in the natural constants. Whether the failure of theory and experiment to agree is because of some unknown factor in the theory of the hydrogen atom or simply an error in the estimate of

Discrepancy confirmed by Nagle, Julian and Zacharias

PHYSICAL REVIEW VOLUME 72, NUMBER 10 NOVEMBER 15, 1947

Letters to the Editor

PUBLICATION of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is five weeks prior to the date of issue. No proof will be sent to the authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not exceed 600 words in length.

The Hyperfine Structure of Atomic Hydrogen and Deuterium*

DARRAGH E. NAGLE, RENNE S. JULIAN, AND JERROLD R. ZACHARIAS
Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts

September 29, 1947

IN a recent Letter to the Editor, Nafe, Nelson, and Rabi[1] reported on measurements of the hyperfine structure separations of the ground states of atomic hydrogen and of atomic deuterium. We have performed very similar atomic-beam magnetic-resonance experiments and have obtained similar values of these constants.

Figure 1 shows the variation with magnetic field of the energy levels for atoms with a $^2S_{\frac{1}{2}}$ electronic ground state and with nuclear spins $\frac{1}{2}$ and 1. These are appropriate to

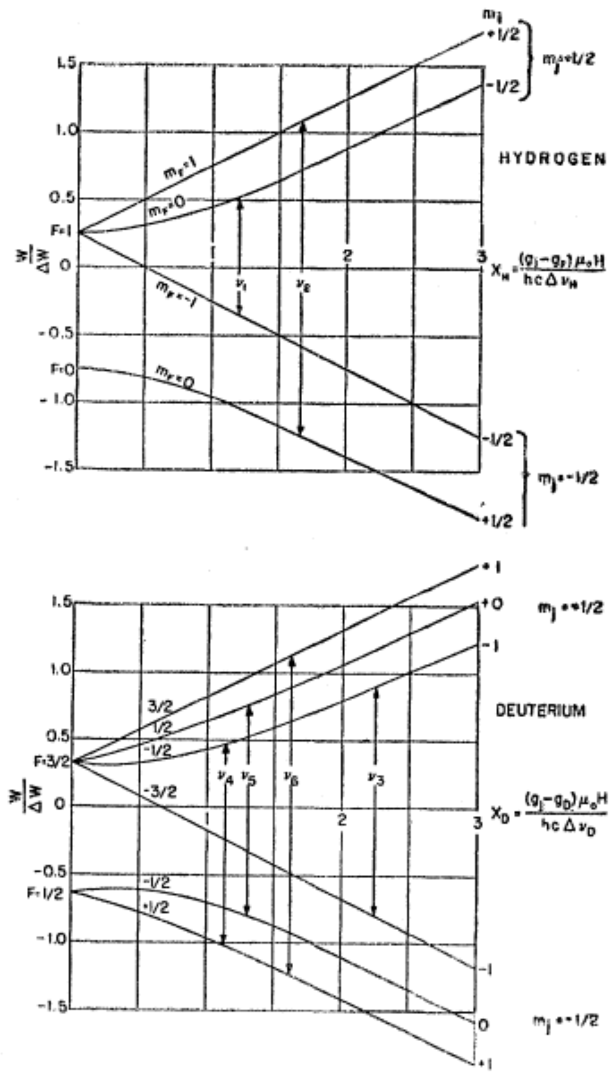


FIG. 1. Hyperfine structure of hydrogen and deuterium; energy levels in a magnetic field.

PHYSICAL REVIEW VOLUME 74, NUMBER 3 AUGUST 1, 1948

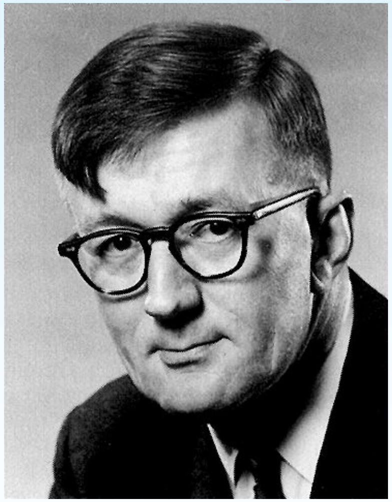
The Magnetic Moment of the Electron†

P. KUSCH AND H. M. FOLEY
Department of Physics, Columbia University, New York, New York

(Received April 19, 1948)

A comparison of the g_J values of Ga in the ${}^2P_{3/2}$ and ${}^2P_{\frac{1}{2}}$ states, In in the ${}^2P_{\frac{1}{2}}$ state, and Na in the ${}^2S_{\frac{1}{2}}$ state has been made by a measurement of the frequencies of lines in the *hfs* spectra in a constant magnetic field. The ratios of the g_J values depart from the values obtained on the basis of the assumption that the electron spin gyromagnetic ratio is 2 and that the orbital electron gyromagnetic ratio is 1. Except for small residual effects, the results can be described by the statement that $\boxed{g_L=1 \text{ and } g_S=2(1.00119\pm0.00005).}$ The possibility that the observed effects may be explained by perturbations is precluded by the consistency of the result as obtained by various comparisons and also on the basis of theoretical considerations.

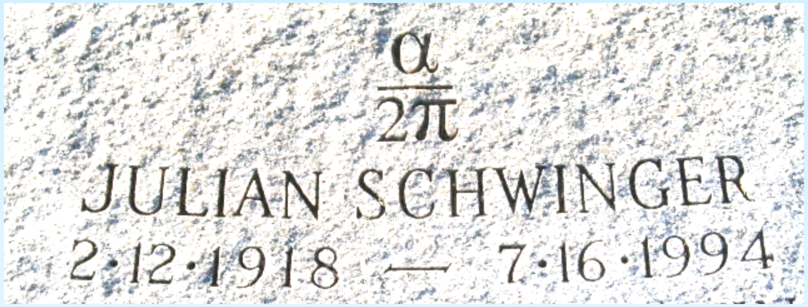
$$g_L = 1 \text{ and } g_S = 2(1.00119 \pm 0.00005)$$

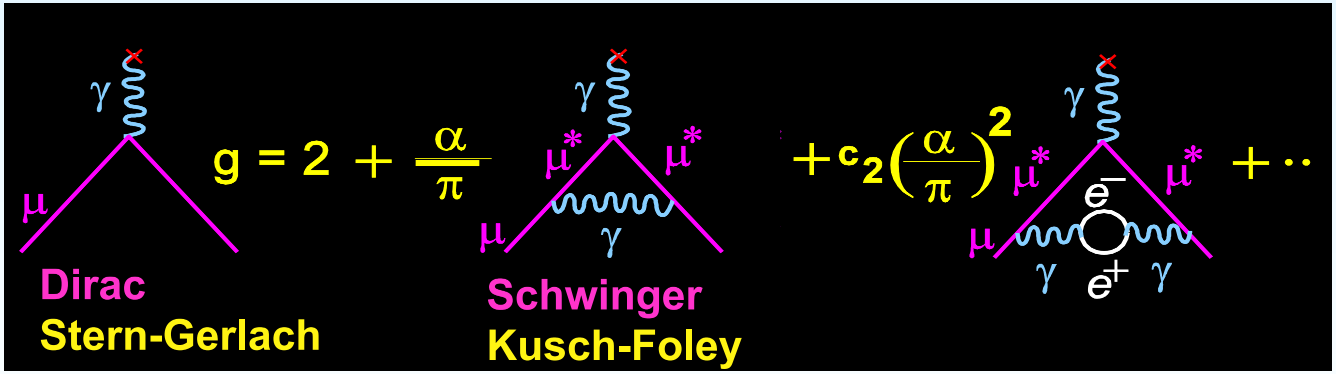


So What?

The first loop calculation:

$$a = \left(\frac{\alpha}{2\pi}\right)$$





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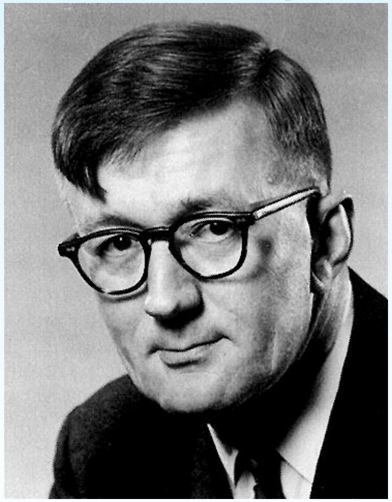
$$g_L = 1 \text{ and } g_S = 2(1.00119 \pm 0.00005)$$

$$a = \frac{\alpha}{2\pi} = 0.001161$$



g_S measured to 50 × 10^{-6} (50 ppm)

a_e to 42 × 10^{-3} (42 parts per mil)

Summary of magnetic moments

- Since g > 2, need to add a "Pauli" (anomalous) moment a

$$\vec{\mu} = g\left(\frac{Qe}{2m}\right)\vec{s}, \quad e > 0$$

$$\mu = (1 + a)\frac{Qe}{2m}$$ **PDG value**

$$g = 2(1 + a) \rightarrow a = \frac{g - 2}{2}$$

Spin Motion in a Magnetic Field

Particle: $q = Qe$ moving in a magnetic field:
momentum turns with cyclotron frequency ω_C,

$$\omega_C = -\frac{QeB}{m\gamma}$$

spin turns with ω_S

$$\omega_S = -g\frac{QeB}{2m} - (1-\gamma)\frac{QeB}{\gamma m}$$

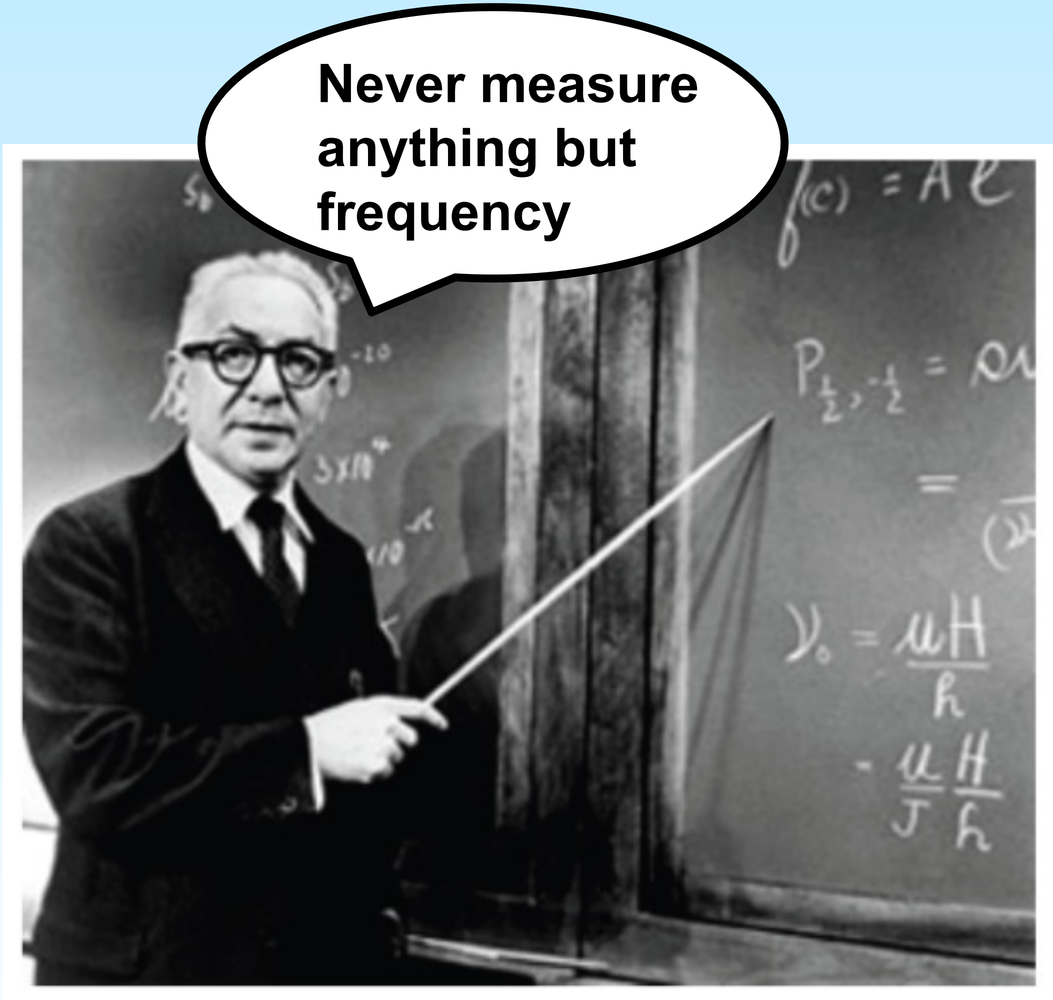
Spin turns relative to the momentum with ω_{diff}

$$\omega_{diff} = \omega_S - \omega_C = -\left(\frac{g-2}{2}\right)\frac{qB}{m} = -a\frac{QeB}{m}$$

which we call ω_a

$$\omega_a = -a\frac{QeB}{m}$$

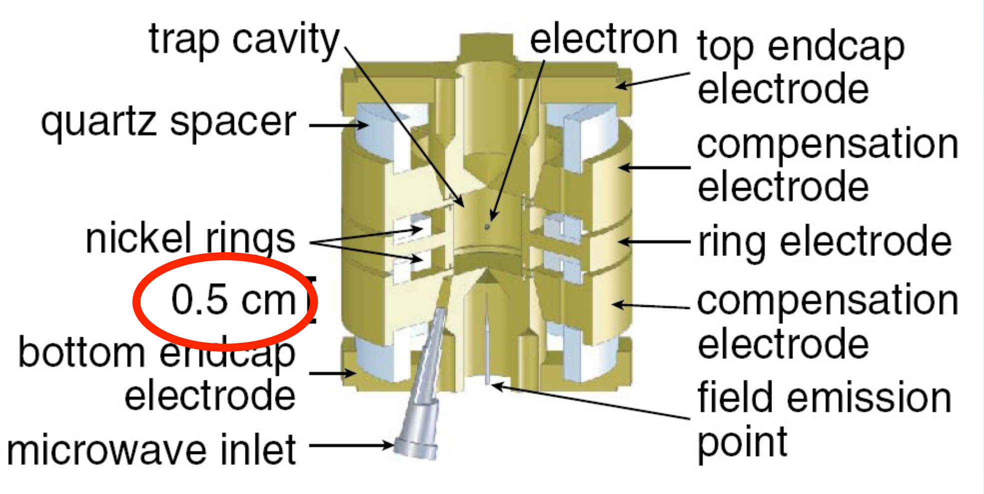
I.I. Rabi

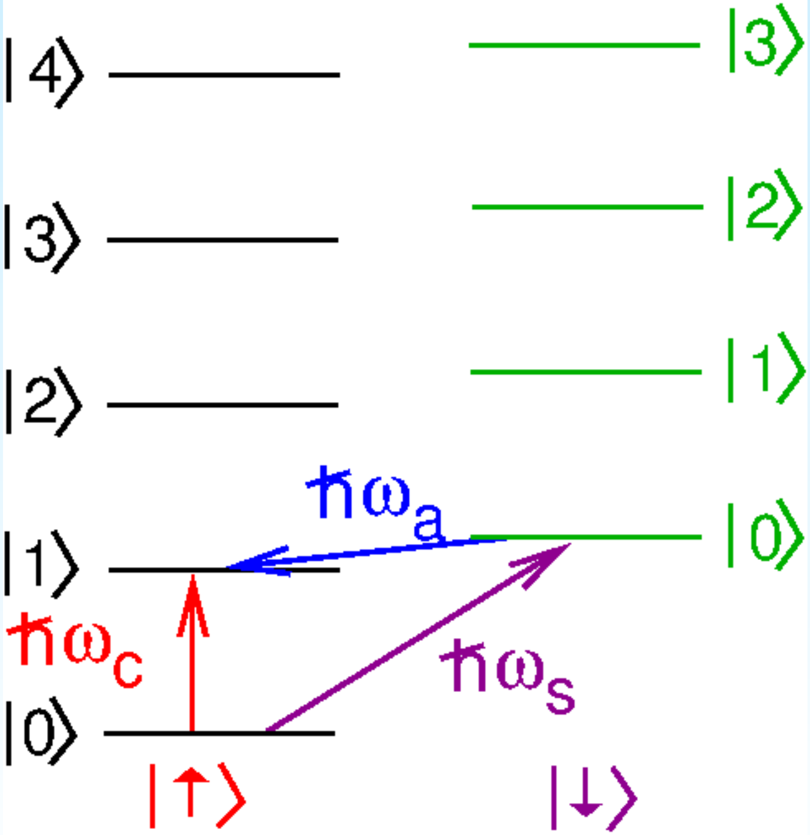


The quantum cyclotron

A single electron in a penning trap, cooled to mK temperatures

Spontaneous emission in the cylindrical trap inhibited by ~140





$$\frac{\omega_a}{\omega_c} = \frac{g-2}{2} = a$$

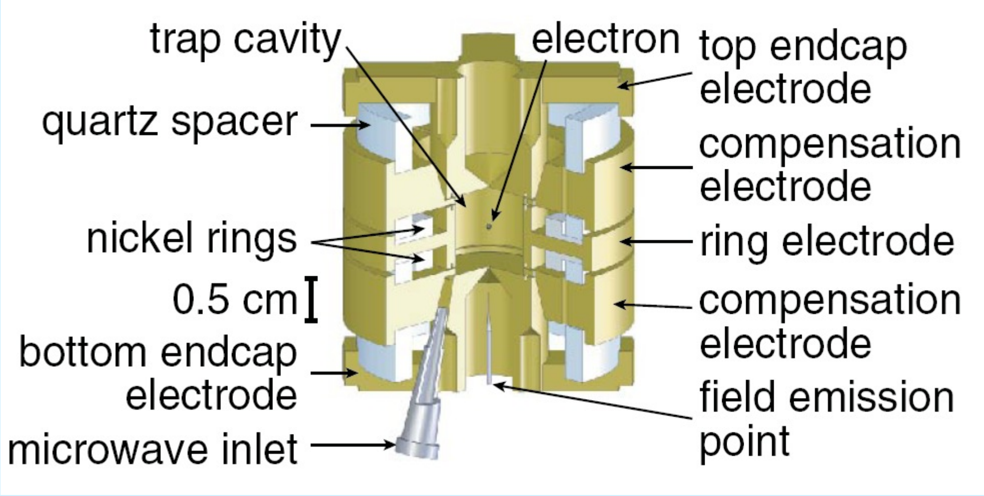
No fundamental constants needed!

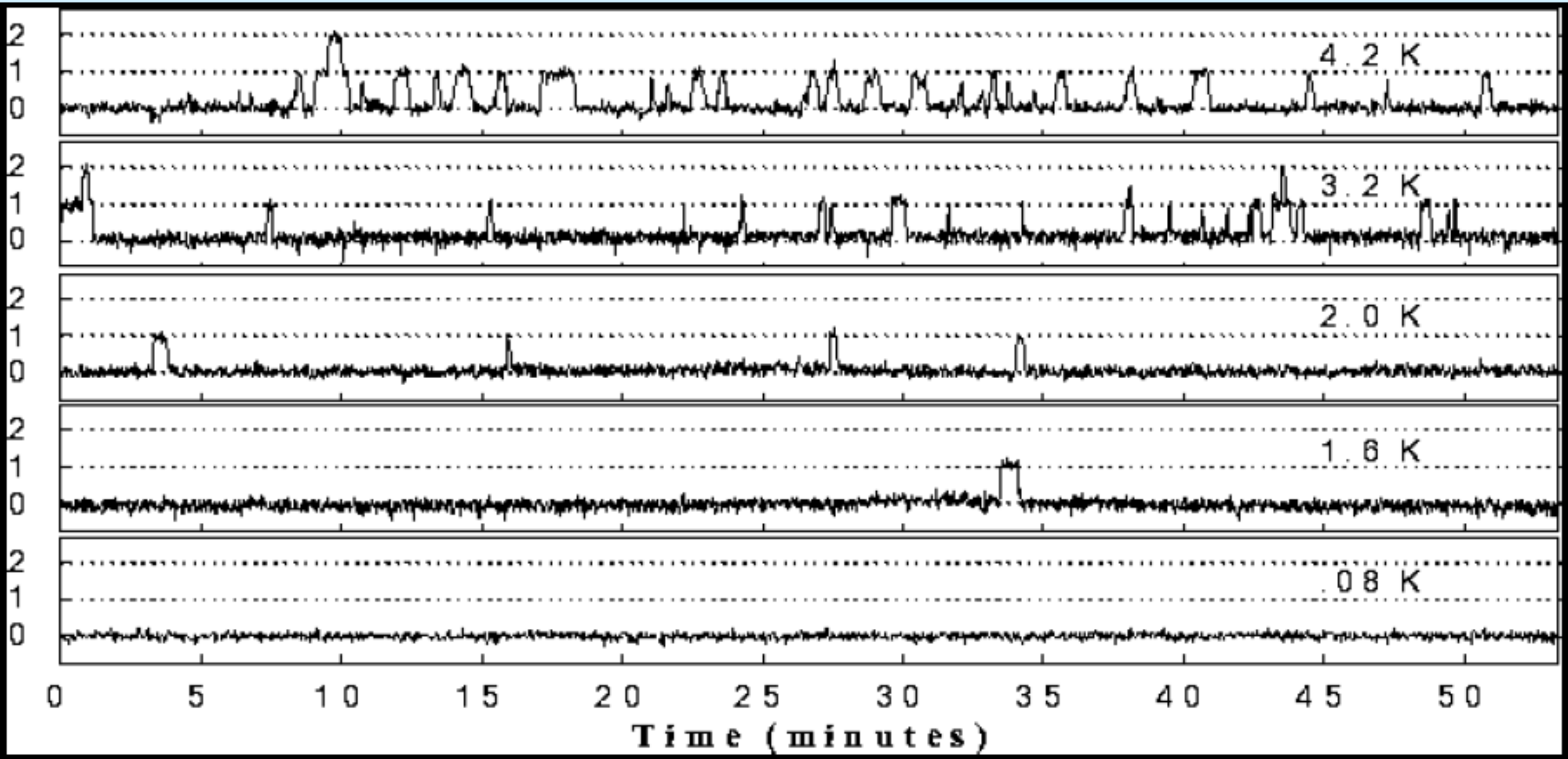
$$\omega_a \simeq 170 \text{ MHz}$$

$$\omega_{s,c} \simeq 160 \text{ GHz}$$

The quantum cyclotron

At 80 mK all ground state





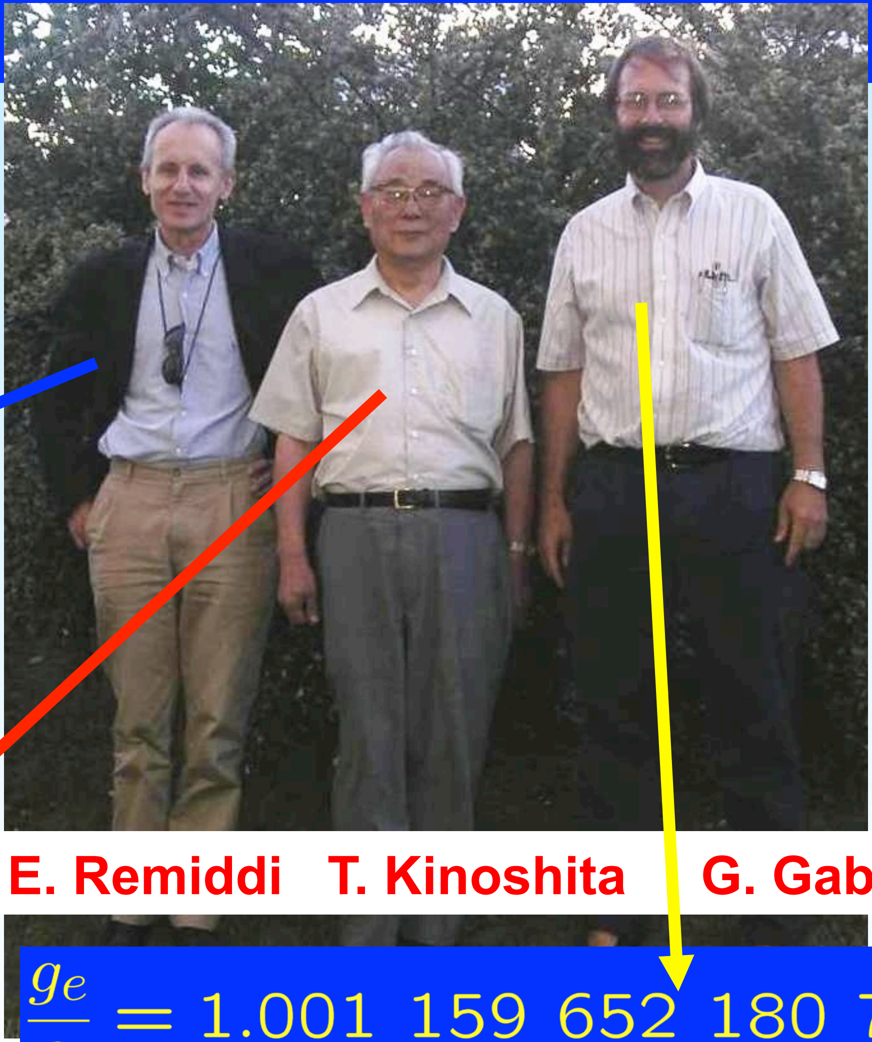
Theory and Experiment

$$a_e = \left(\frac{g_e - 2}{2}\right)$$

$$\frac{g}{2} = 1 + C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + C_5\left(\frac{\alpha}{\pi}\right)^5 + \delta a$$

S. Laporta, E. Remiddi

12,672 diagrams



E. Remiddi T. Kinoshita G. Gabrielse

$$\frac{g_e}{2} = 1.001\ 159\ 652\ 180\ 73(28)$$

BOSTON UNIVERSITY

8^{th} and 10^{th} order QED calculations

Towards analytic $(g-2)_\mu$ @ 4 loops

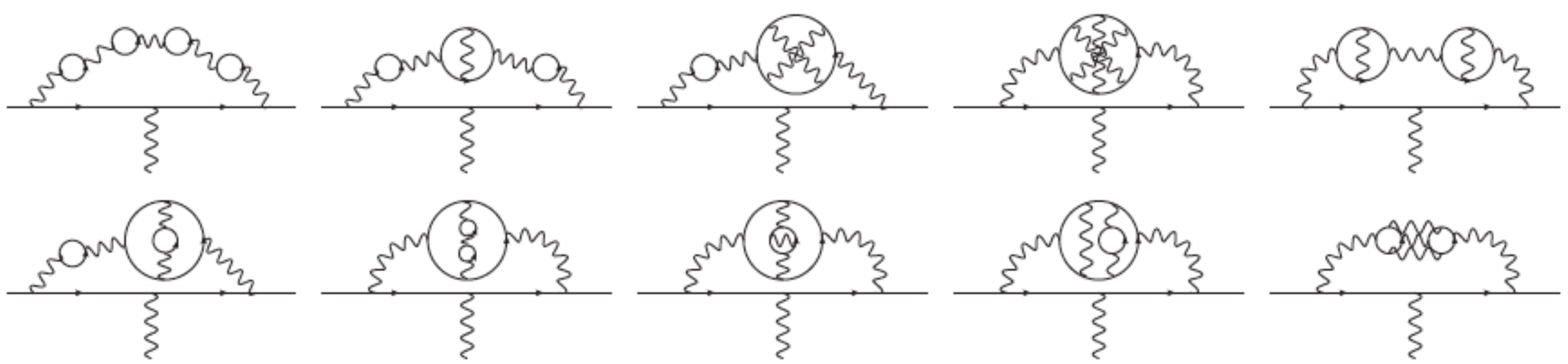
PhiPsi13, Rome, September 9-12, 2013

Matthias Steinhauser | TTP Karlsruhe

Polarization function insertions

4 loops: [Baikov,Broadhurst'95]

5 loops: [Baikov,Chetyrkin,Kühn,Sturm'13; Baikov,Marquard,Maier'13] (see also [Aguilar,Greynat,De Rafael'08])



Experiment and Theory for a_e

PRL 100, 120801 (2008) PHYSICAL REVIEW LETTERS week ending 28 MARCH 2008

New Measurement of the Electron Magnetic Moment and the Fine Structure Constant

D. Hanneke, S. Fogwell, and G. Gabrielse*
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA
(Received 4 January 2008; published 26 March 2008)

$$a_e^{exp} = 1\,159\,652\,180.73\,(28) \times 10^{-12}$$

PRL 109, 111807 (2012) PHYSICAL REVIEW LETTERS week ending 14 SEPTEMBER 2012

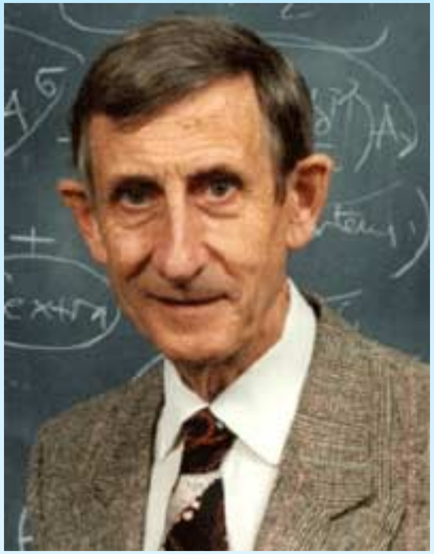
Tenth-Order QED Contribution to the Electron $g - 2$ and an Improved Value of the Fine Structure Constant

Tatsumi Aoyama,[1,2] Masashi Hayakawa,[3,2] Toichiro Kinoshita,[4,2] and Makiko Nio[2]
[1]*Kobayashi-Maskawa Institute for the Origin of Particles and the Universe (KMI), Nagoya University, Nagoya, 464-8602, Japan*
[2]*Nishina Center, RIKEN, Wako, Japan 351-0198*
[3]*Department of Physics, Nagoya University, Nagoya, Japan 464-8602*
[4]*Laboratory for Elementary Particle Physics, Cornell University, Ithaca, New York, 14853, USA*
(Received 24 May 2012; published 13 September 2012)

$$a_e^{theory} = 1\,159\,652\,181.78\,(77) \times 10^{-12}$$

$$\Delta a_e = (1.05 \pm 0.82) \times 10^{-12}$$

What would the inventors of QED think about the astoninshing ageement between experiment and theory for a_e?



"The main point was that all of us who put QED together, including especially Feynman, considered it a jerry-built and provisional structure which would either collapse or be replaced by something more permanent within a few years. So I find it amazing that it has lasted for fifty years and still agrees with experiments to twelve significant figures. It seems that Nature is telling us something. Perhaps she is telling us that she loves sloppiness."

Freeman Dyson (private communication) – December 2006

An aside: The g-value of the proton

- **In 1933, against the advice of Pauli who believed that the proton was a pure Dirac particle, Stern and his collaborators showed that:**

$$g_p \simeq 5.5$$

which has something to do with its internal structure.

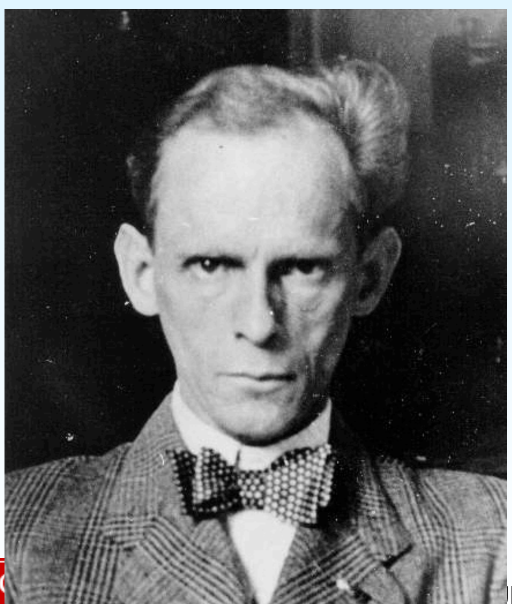
- **Internal structure can also change g from 2.**

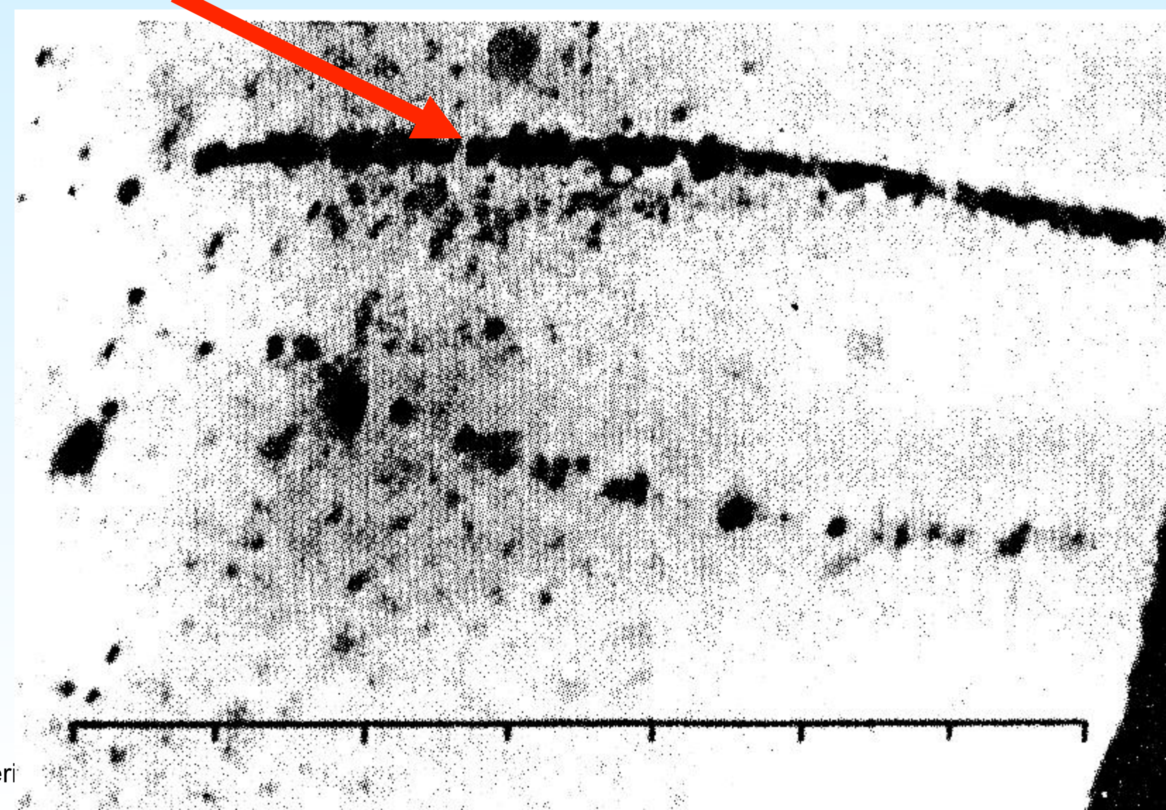
The Muon

First observed in cosmic rays in 1933

"a particle of uncertain nature"

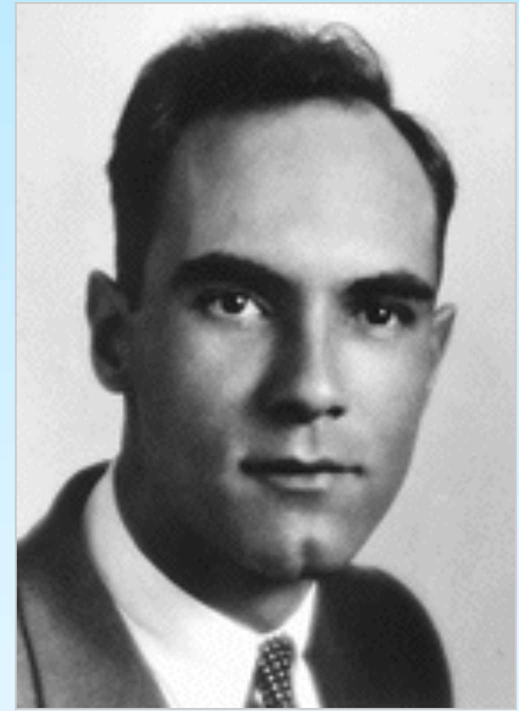
Paul Kunze,
Z. Phys. 83, 1 (1933)





Identified in 1936

Study of cosmic rays by Seth Neddermeyer and Carl Anderson



MAY 15, 1937 PHYSICAL REVIEW VOLUME 51

Note on the Nature of Cosmic-Ray Particles

SETH H. NEDDERMEYER AND CARL D. ANDERSON
California Institute of Technology, Pasadena, California

(Received March 30, 1937)

MEASUREMENTS[1] of the energy loss of particles occurring in the cosmic-ray showers have shown that this loss is proportional massive than protons but more penetrating than electrons obeying the Bethe-Heitler theory, we have taken about 6000 counter-tripped photo-

Confirmed by Street and Stevenson

NOVEMBER 1, 1937 PHYSICAL REVIEW VOLUME 52

LETTERS TO THE EDITOR

Prompt publication of brief reports of important discoveries in physics may be secured by addressing them to this department. Closing dates for this department are, for the first issue of the month, the eighteenth of the preceding month, for the second issue, the third of the month. Because of the late closing dates for the section no proof can be shown to authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

Communications should not in general exceed 600 words in length.

New Evidence for the Existence of a Particle of Mass Intermediate Between the Proton and Electron

Anderson and Neddermyer[1] have shown that, for energies

between those of the proton and electron. If this is true, it should be possible to distinguish clearly such a particle from an electron or proton by observing its track density

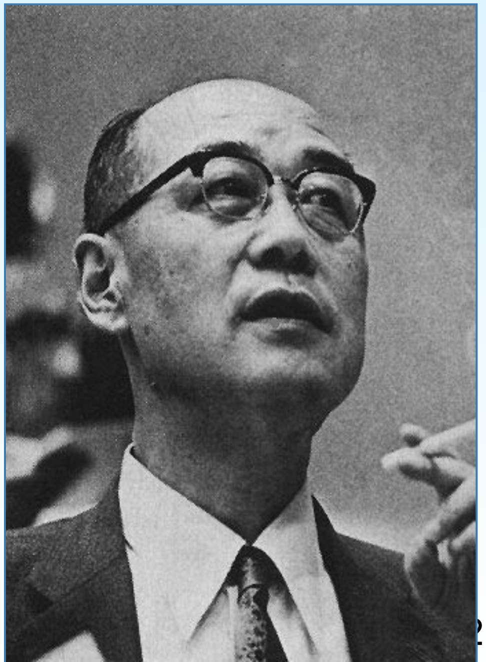
tracks of high energy particles.

J. C. Street
E. C. Stevenson

Research Laboratory of Physics,
Harvard University,
Cambridge, Massachusetts,
October 6, 1937.

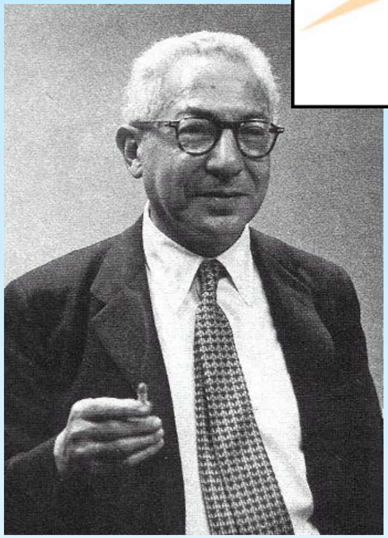
[1] Anderson and Neddermeyer, Phys. Rev. **50**, 263 (1936).
[2] Street and Stevenson, Phys. Rev. **51**, 1005 (1937).

It took 10 years to conclude that the muon interacted too weakly with matter to be the "Yukawa" particle which was postulated to carry the nuclear force



So what is the muon?

- An excited electron?
  - no, $\mu \rightarrow e\,\gamma$ was not observed.
- Was it related to the electron?
  - well it's decay was 3-body, and only an electron was observed



I.I. Rabi

- Did it behave like an electron?
  - conservation of angular momentum could be used to guess that it was spin ½

Fitch and Rainwater looked for fine-structure splitting

PHYSICAL REVIEW VOLUME 92, NUMBER 3 NOVEMBER 1, 1953

Studies of X-Rays from Mu-Mesonic Atoms*

VAL L. FITCH AND JAMES RAINWATER
Department of Physics, Columbia University, New York, New York
(Received July 23, 1953)

A new technique of x-ray spectroscopy of μ-mesonic atoms has been developed. The x-rays are produced when a μ^-meson undergoes transitions between Bohr orbits about nuclei of various Z. The mesons are produced by the Columbia University 164-in. Nevis cyclotron. The x-rays are detected, and their energies are measured to better than 1 percent accuracy (for $Z \geq 22$) using a NaI crystal scintillation spectrometer. The $2p \rightarrow 1s$ transition energies were measured to be 0.35, 0.41, 0.955, 1.55, 1.60, 3.50, 5.80, 6.02, and 6.02 Mev for $Z=13$, 14, 22, 29, 30, 51, 80, 82, and 83. Special attention was paid to the Pb spectrum, and it is believed that an 0.2-Mev fine structure splitting has been observed. This is the expected splitting if the μ^- meson is a spin $\frac{1}{2}$ Dirac "heavy electron" of 210 electron masses, having the expected Dirac magnetic moment and having no strong nonelectromagnetic interaction with nuclear matter.

Since the μ^-meson Bohr orbits are 210 times closer to the nucleus than the equivalent electron orbits, the x-ray energies are quite sensitive to nuclear size for medium and large Z. In the case of Pb, a 1 percent change in nuclear radius gives a 1 percent change in the calculated x-ray energy. Assuming constant proton density inside a spherical nucleus of radius $R_0 = r_0 A^{\frac{1}{3}}$ and the above properties for the μ meson, we obtain $r_0 = 1.17$, 1.21, 1.22, and 1.17×10^{-13} cm for $Z=22$, 29, 51, and 82. The significance of these results in relation to other nuclear size measurements is discussed.

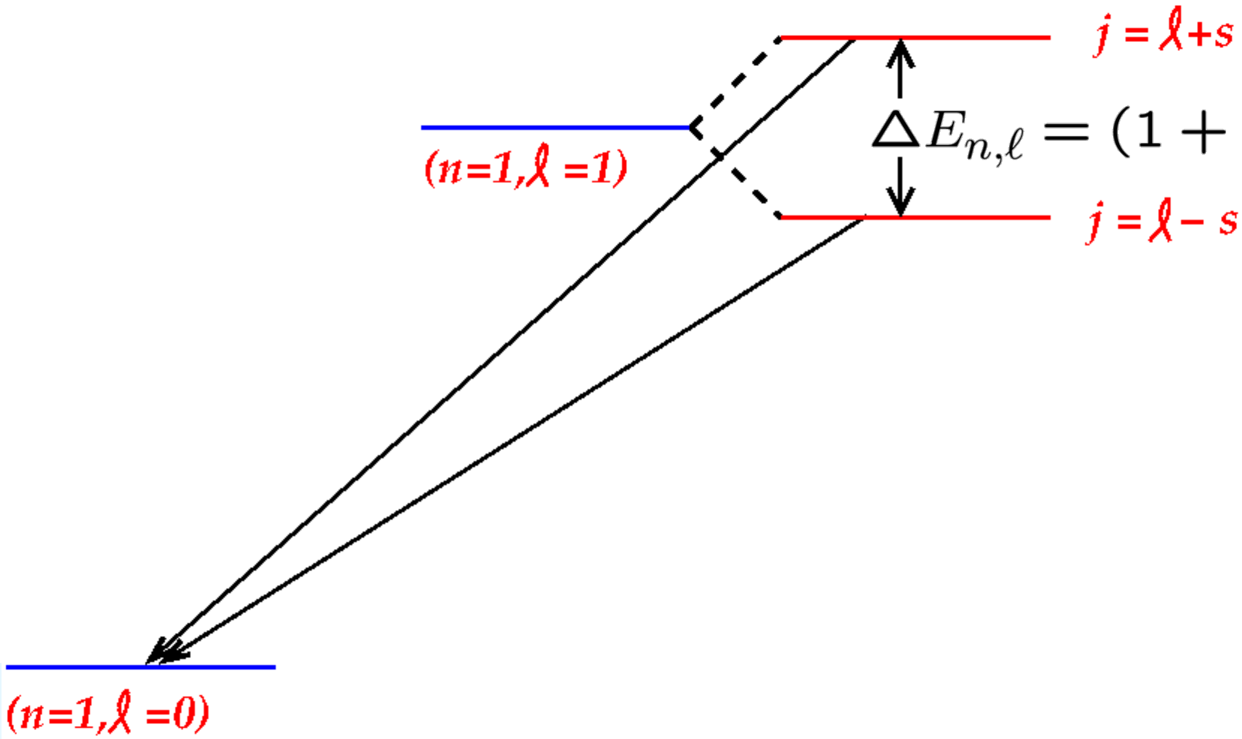
Stop μ^- in matter, forms a muonic Bohr-like atom

$$E_n = -\frac{(Z\alpha)^2 mc^2}{2n^2} \qquad r_n = \frac{n^2 \hbar c}{mc^2 (Z\alpha)}$$

Fitch and Rainwater looked for fine-structure splitting

PHYSICAL REVIEW VOLUME 92, NUMBER 3 NOVEMBER 1, 1953

Studies of X-Rays from Mu-Mesonic Atoms*



$$\Delta E_{n,\ell} = (1+2a)\frac{(Z\alpha)^4}{2n^3}\frac{m}{\ell(\ell+1)}$$

$$E_n = -\frac{(Z\alpha)^2 mc^2}{2n^2}$$

$$r_n = \frac{n^2\hbar c}{mc^2(Z\alpha)}$$

- The fine-structure splitting depends on the magnetic moment; the $2p \rightarrow 1s$ transition is split into 2 lines
- The resolution of the NaI detector was not sufficient to see 2 lines, only a broad blob – no nonlinear least square fitting program (no root or PAW then)

Measuring the Muon Magnetic Dipole Moment

$$\vec{\mu} = g\left(\frac{Qe}{2m}\right)\vec{s}$$

$$g = 2(1 + a) \qquad a = \frac{(g-2)}{2}$$

PHYSICAL REVIEW VOLUME 104, NUMBER 1 OCTOBER 1, 1956

Question of Parity Conservation in Weak Interactions*

T. D. LEE, *Columbia University, New York, New York*

AND

C. N. YANG,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)

The question of parity conservation in β decays and in hyperon and meson decays is examined. Possible experiments are suggested which might test parity conservation in these interactions.

In the decay processes

$$\pi \rightarrow \mu + \nu, \quad (5)$$

$$\mu \rightarrow e + \nu + \nu, \quad (6)$$

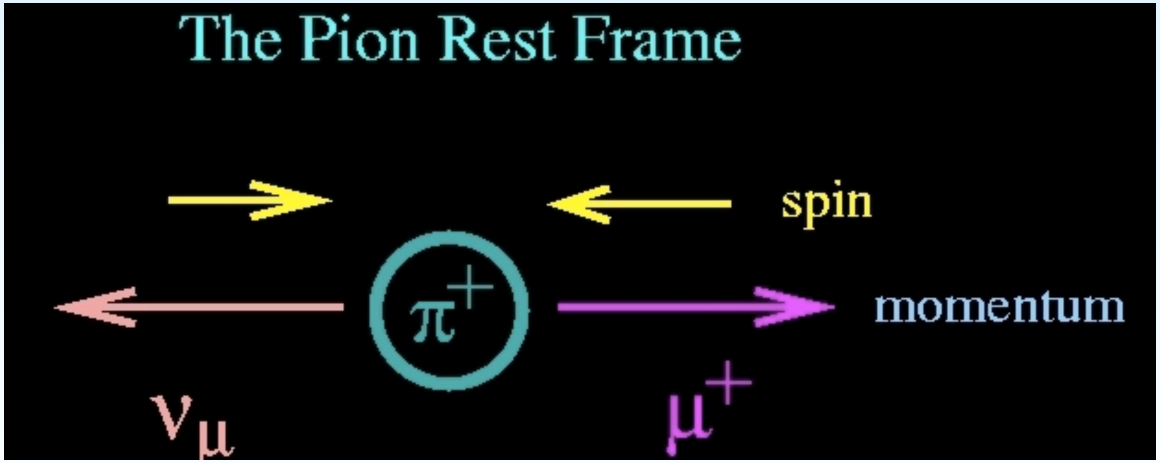
"starting with a π meson at rest, one could study the distribution of the angle θ between the μ-meson momentum and the electron momentum, the latter being in the CM system of the μ meson. If parity is conserved in neither (5) nor (6), the distribution will not in general be identical for θ and $\pi-\theta$. …

If (5) violates parity conservation, the muon would be in general polarized in it's direction of motion…"

subsequent decay (6), the angular distribution problem with respect to θ is therefore closely similar to the angular distribution problem of β rays from oriented nuclei, which we have discussed before. (Entirely

Muon Production: $\pi^+ \rightarrow \mu^+ + \nu_\mu$

- In the rest frame:
  - Initial spin is 0
  - Final spin is 0 – but the neutrino is left-handed so the muon is polarized!
- In the Lab Frame with a pion beam:
  - the very forward muons (highest energy) are highly polarized
  - the very backward muons (lowest energy) are highly polarized



Garwin, Lederman, Weinrich, PR 105,1415 (1957)

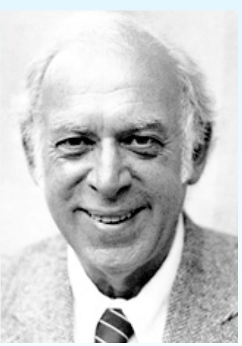

Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon*

RICHARD L. GARWIN,† LEON M. LEDERMAN, AND MARCEL WEINRICH

Physics Department, Nevis Cyclotron Laboratories, Columbia University, Irvington-on-Hudson, New York, New York

(Received January 15, 1957)

Friedman, Telegdi PR105, 1681 (1957)




Nuclear Emulsion Evidence for Parity Nonconservation in the Decay Chain $\pi^+ - \mu^+ - e^{+}$*†

JEROME I. FRIEDMAN AND V. L. TELEGDI

Enrico Fermi Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

(Received January 17, 1957)

Lee and Yang explained that if parity is violated

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

- produces polarized muons along the muon momentum, and the decay

$$\mu^- \rightarrow e^- + \bar{\nu}_e \nu_\mu$$

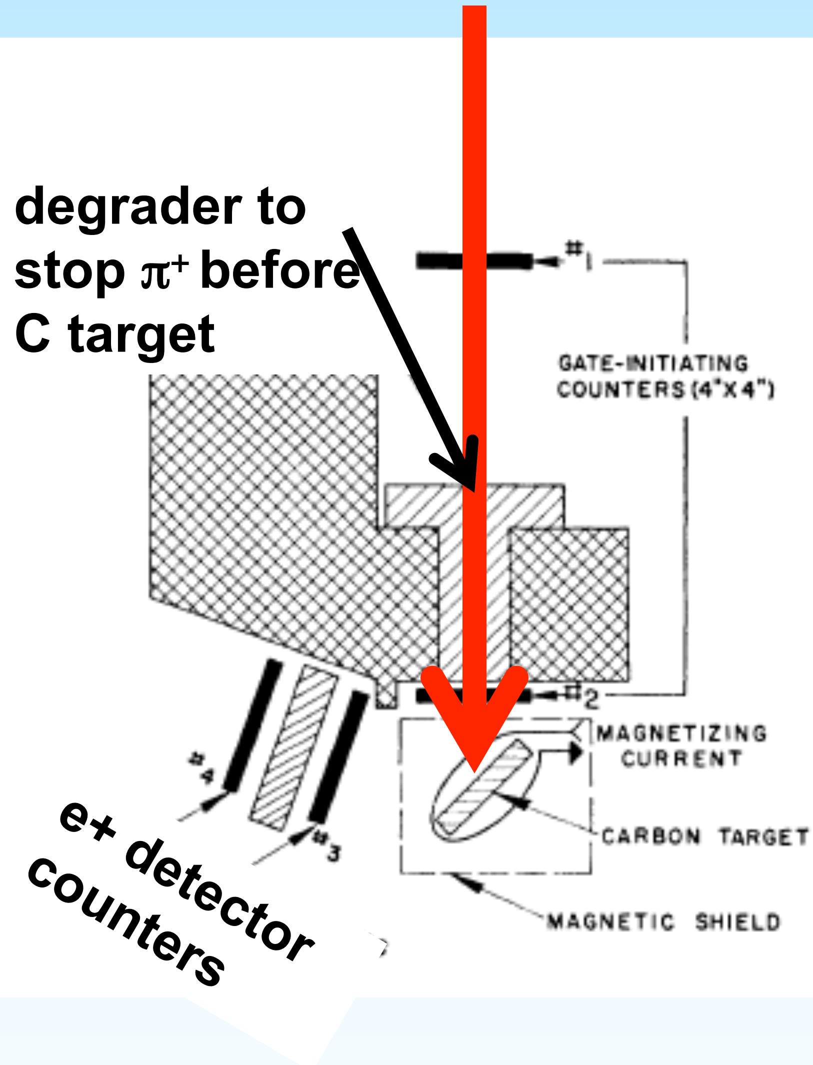
- analyzes the spin orientation at the decay time

"They also point out that the longitudinal polarization of the muon offers a natural way of determining a magnetic moment." – Garwin, Lederman, Weinrich

The Nevis Experiment

85 MeV π^+, μ^+

- Produce a beam of π^+,μ^+
- Stop π^+ in a carbon degrader, permitting muons to stop in a carbon target
- Use a simple telescope to detect e^+ with E_e > 25 MeV
- count e^+ between 0.75 – 2 μs
- look for the angular distribution $(1 + aP\cos\theta)$
- However, the counter only samples e^+ at 100°
- Use a B field to rotate the spin, so <u>for a small time window</u>, the angular distribution is turned into a distribution of counts vs. magnet current



The first muon spin rotation experiment

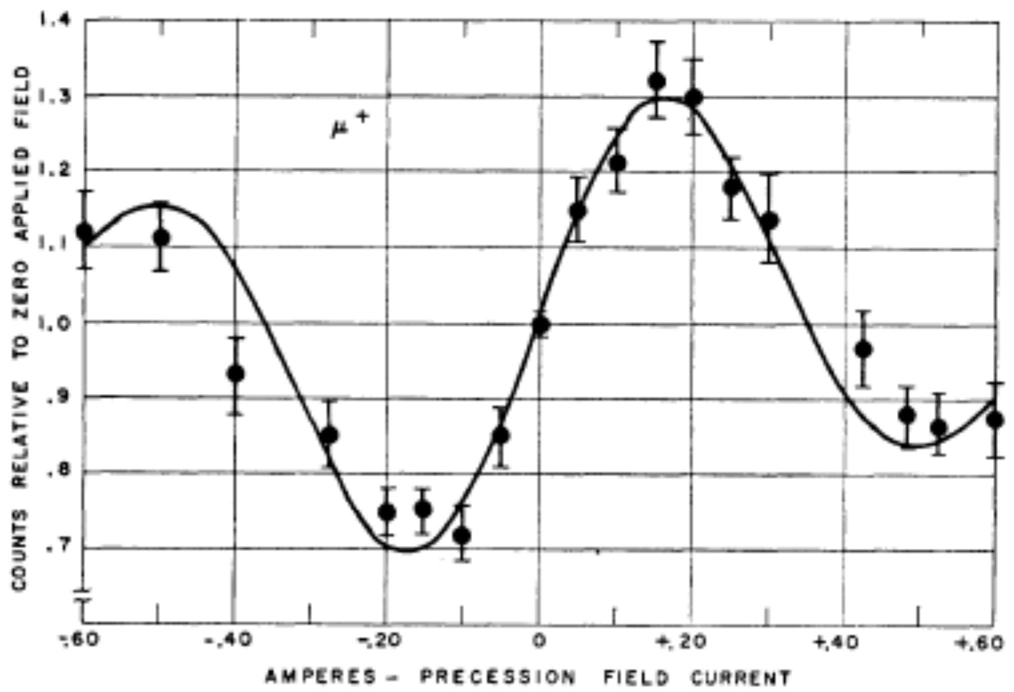


FIG. 2. Variation of gated 3–4 counting rate with magnetizing current. The solid curve is computed from an assumed electron angular distribution $1-\frac{1}{3}\cos\theta$, with counter and gate-width resolution folded in.

Fit to

$$\left(1+\frac{1}{3}\cos\theta\right)$$

$g = 2 \quad \pm 10\%$

n.b. The number of details that must be understood is <u>inversely proportional to the error</u>!

Final Nevis Experiment: PR 118, 271 (1959)

PHYSICAL REVIEW VOLUME 118, NUMBER 1 APRIL 1,

Accurate Determination of the μ^+ Magnetic Moment*

R. L. Garwin,† D. P. Hutchinson, S. Penman,‡ and G. Shapiro§
Columbia University, New York, New York

(Received August 4, 1959)

Using a precession technique, the magnetic moment of the positive mu meson is determined to an accuracy of 0.007%. Muons are brought to rest in a bromoform target situated in a homogeneous magnetic field, oriented at right angles to the initial muon spin direction. The precession of the spin about the field direction, together with the asymmetric decay of the muon, produces a periodic time variation in the probability distribution of electrons emitted in a fixed laboratory direction. The period of this variation is compared with that of a reference oscillator by means of phase measurements of the "beat note" between the two. The magnetic field at which the precession and reference frequencies coincide is measured with reference to a proton nuclear magnetic resonance magnetometer. The ratio of the muon precession frequency to that of the proton in the same magnetic field is thus determined to be 3.1834±0.0002. Using a re-evaluated lower limit to the muon mass, this is shown to yield a lower limit on the muon g factor of $2(1.00122\pm0.00008)$, in agreement with the predictions of quantum electrodynamics.

Note added in proof

$$g_\mu = 2(1.00113^{+0.00016}_{-0.00012})$$

g_μ measured to ~140 × 10^{-6} (140 ppm) **≈ ×3 Kusch and Foley precision**

a_μ to 42 × 10^{-3} (12.4%)

Final Nevis Experiment: PR 118, 271 (1959)

PHYSICAL REVIEW VOLUME 118, NUMBER 1 APRIL 1,

Accurate Determination of the μ^+ Magr… …ment*

R. L. GARWIN,† D. P. HUTCHINSON, S. PENM… …. SHAPIRO§
Columbia University, New Yor'…

(Received August …

Using a precession technique, the magnetic mome… …itive mu meson is determined to an accuracy of 0.007%. Muons are brought to rest in a br… …get situated in a homogeneous magnetic field, oriented at right angles to the initial muon sp… . The precession of the spin about the field direction, together with the asymmetric decay of … produces a periodic time variation in the probability distribution of electrons emitted in a… …atory direction. The period of this variation is compared with that of a reference oscillator … …r phase measurements of the "beat note" between the two. The magnetic field at which th… … and reference frequencies coincide is measured with reference to a proton nuclear magnetic … …agnetometer. The ratio of the muon precession frequency to that of the proton in the same … …d is thus determined to be 3.1834±0.0002. Using a re-evaluated lower limit to the muon m… …own to yield a lower limit on the muon g factor of $2(1.00122\pm0.00008)$, in agreement with t… …s of quantum electrodynamics.

In a magnetic field the muon behaves like a heavy electron!

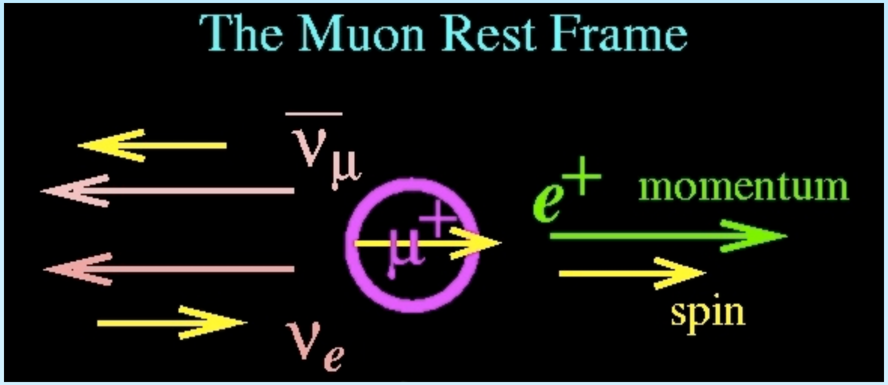
Note added in proof

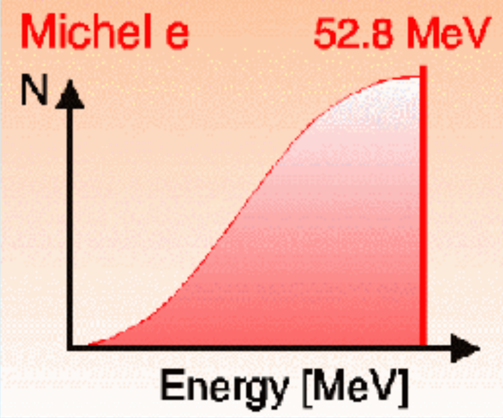
$$g_\mu = 2(1.00113^{+0.00016}_{-0.00012})$$

…easured to ~140 × 10^{-6} (140 ppm) ≈ ×3 Kusch and Foley precision

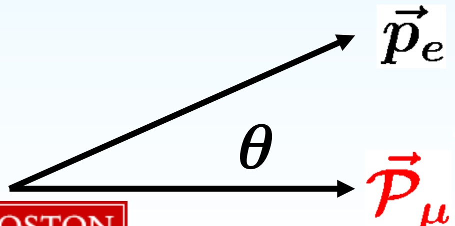
a_μ to 42 × 10^{-3} (12.4%)

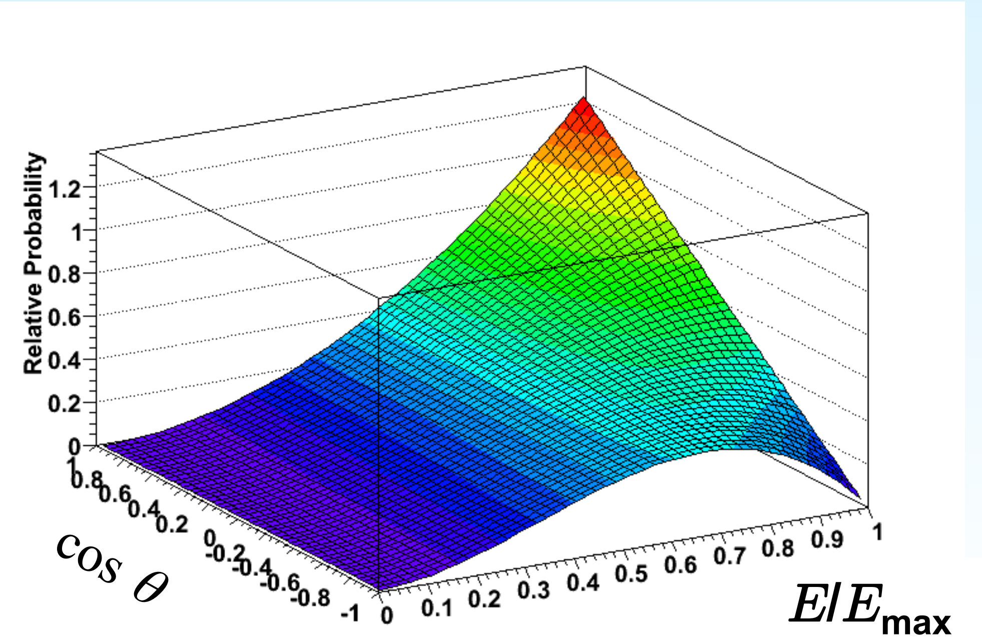
Muon decay

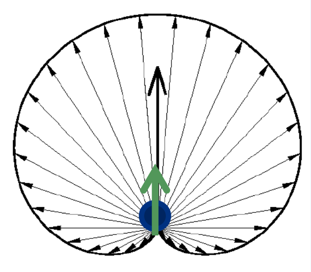




- The highest energy e+, are correlated with the muon spin direction



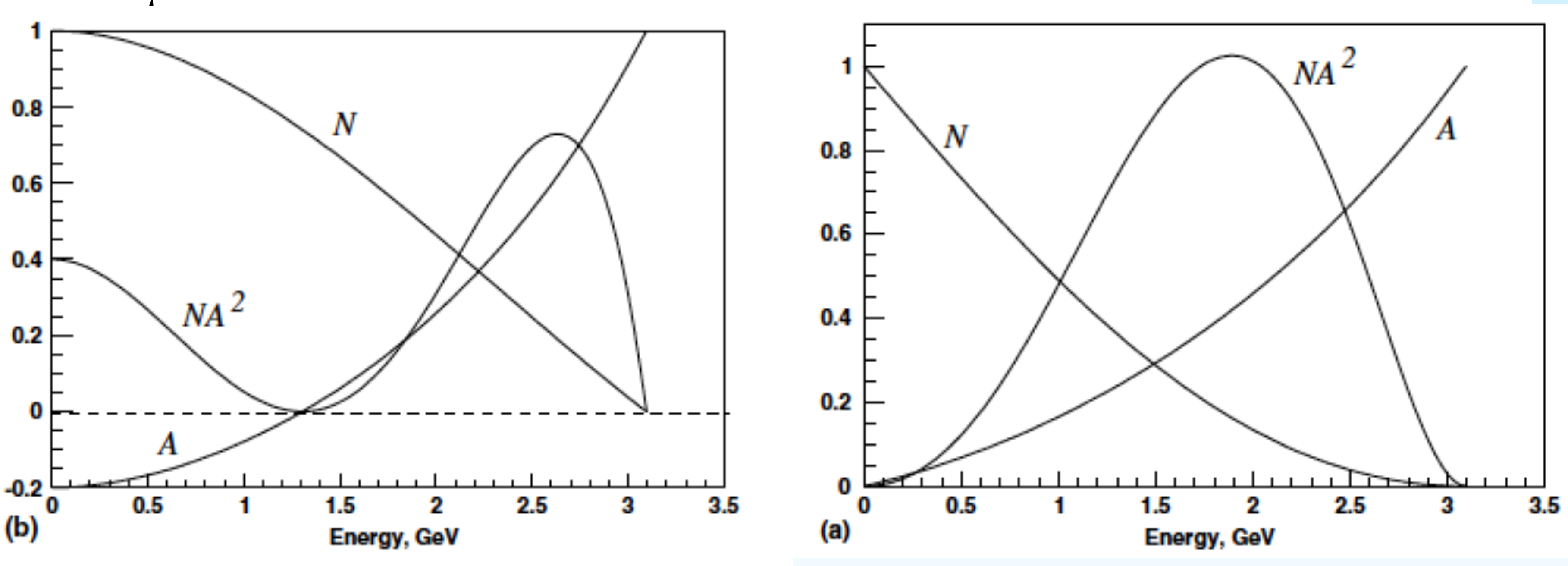




Muon decay in flight

- μ-e decay asymmetry depends on p_e, and the μ beam polarization P.
- For a <u>single energy threshold</u> on the $e^{\pm}$ detectors, the figure of merit, NA^2 peaks at ~0.65 E_μ.

For p_μ = 3 GeV/c; differential and integral N, NA^2 and A



J. Miller, E. de Rafael, BL Roberts, Rep. Prog. Phys. 70 (2007) 795–881

Nevis Experiment was done at rest

What if the particle is moving in the magnetic field?

Particle: $q = Qe$ moving in a magnetic field:
momentum turns with cyclotron frequency ω_C,
spin turns with ω_S

$$\omega_C = -\frac{QeB}{m\gamma}; \qquad \omega_S = -g\frac{QeB}{2m} - (1-\gamma)\frac{QeB}{\gamma m}$$

Spin turns relative to the momentum with ω_a independent of p

$$\omega_a = \omega_S - \omega_C = -\left(\frac{g-2}{2}\right)\frac{QeB}{m} = -a\frac{QeB}{m}$$

BOSTON UNIVERSITY

The featues that make the experiment possible:

- Parity violation
  - produces polarized muons

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

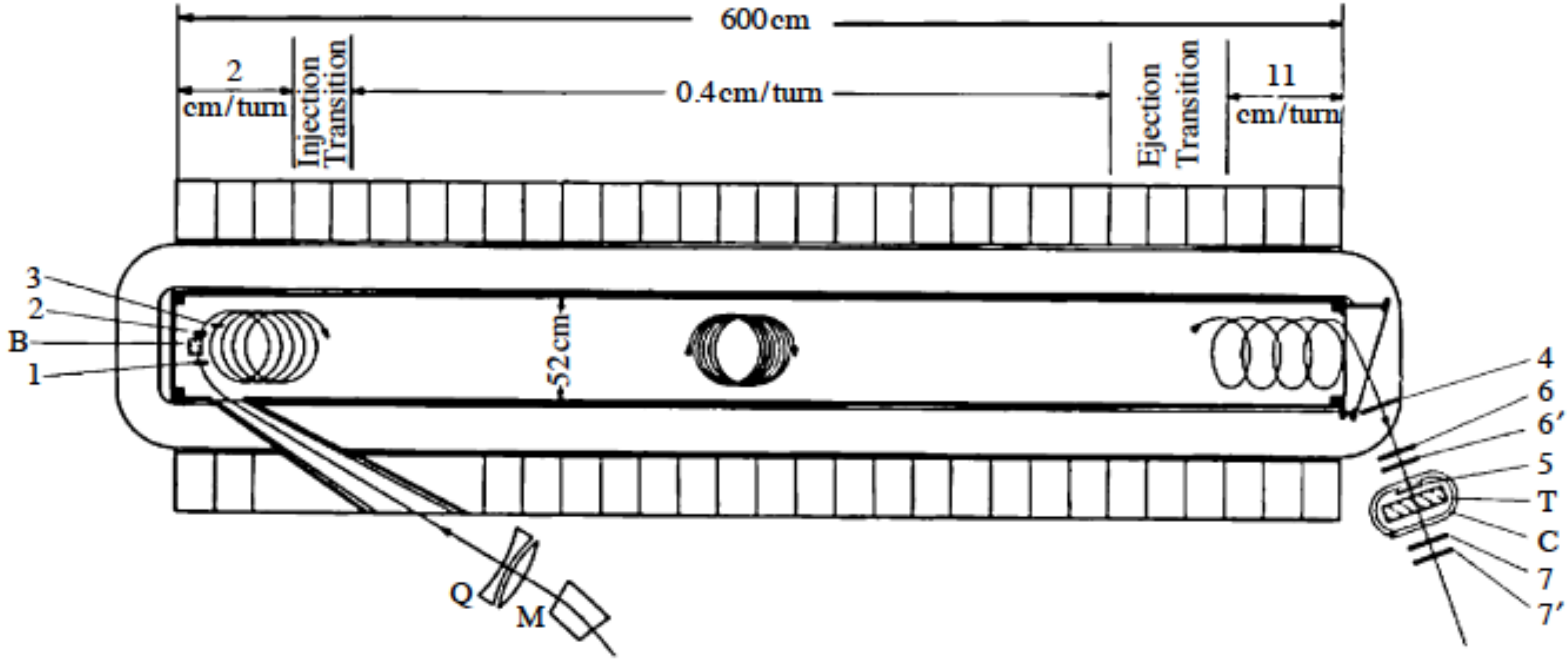
  - analyzes the spin orientation at the decay time

$$\mu^- \rightarrow e^- + \bar{\nu}_e \nu_\mu$$

- The 2.2μs lifetime is practically forever
- All a_μ experiments, except at Nevis, used the rate at which the spin turns relative to the momentum, which only depends on the anomaly and B field.

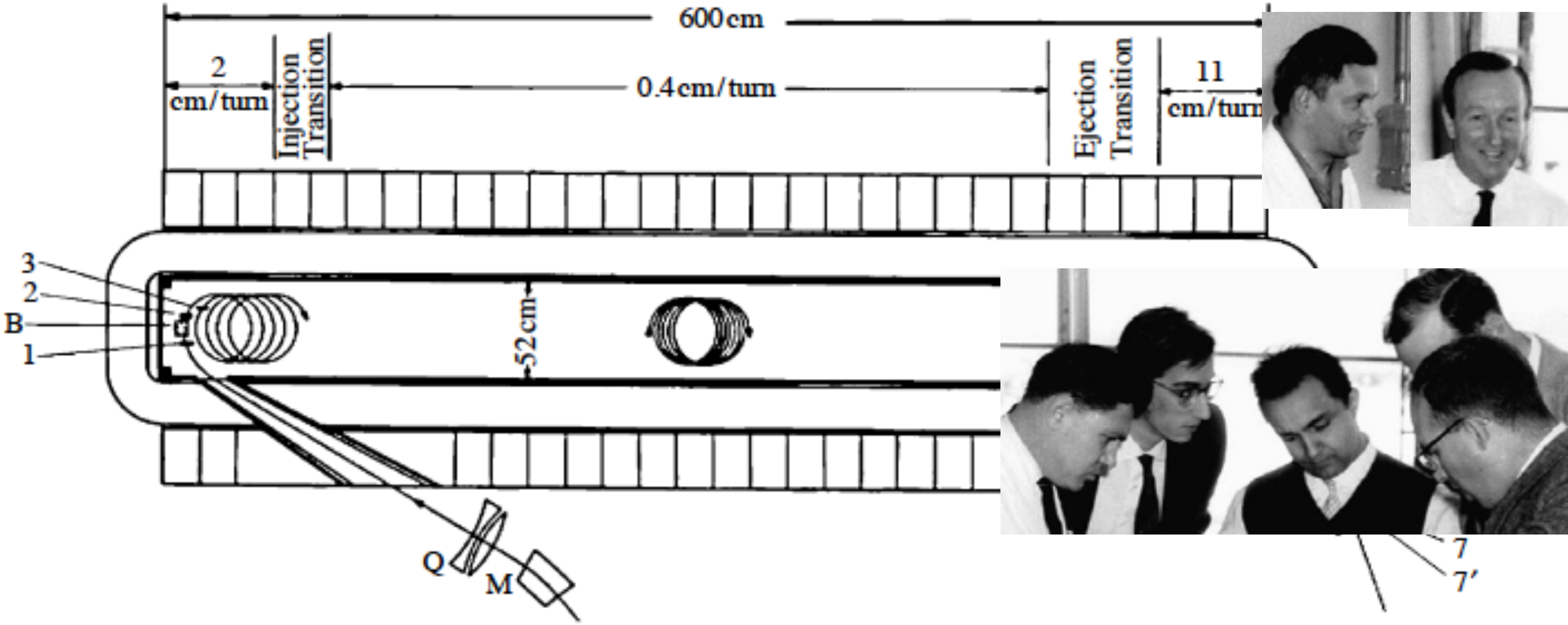
$$\vec{\omega}_a = \omega_S - \omega_C = -\frac{Qe}{m} a_\mu \vec{B}$$

The First CERN Experiment at the Synchrocyclotron



- Inject polarized muon into a long magnet (B ≈ 1.5 T) with a small gradient – particles drift in circular orbits to the other end: 7.5 μs = 1600 turns
- Extract muons with a large gradient into a polarization monitor where they stopped
- Time in the magnetic field was measured by counters
- Use one set of counters and a pulsed field to rotate the spins by π
- Measure the time dependent forward-backward asymmetry

The First CERN Experiment at the Synchrocyclotron



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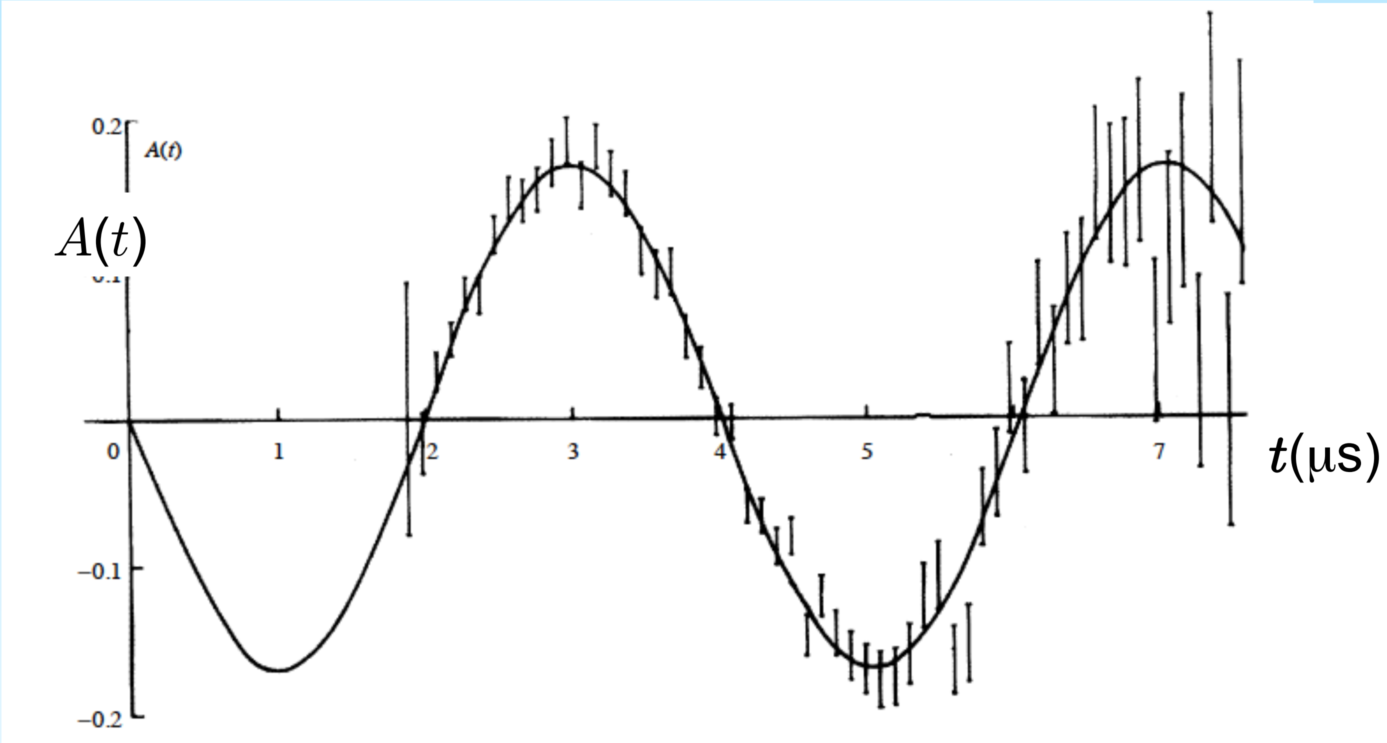
A portion of the CERN data and the final answer



$$a_\mu = 0.001\,162(5)\ (0.43\%)$$ (4300 ppm)

sensitive to $C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2$

- Limitations:
  - not enough data (1 muon/second in analyzer)
  - muon lifetime too short

You need to measure B and ω_a

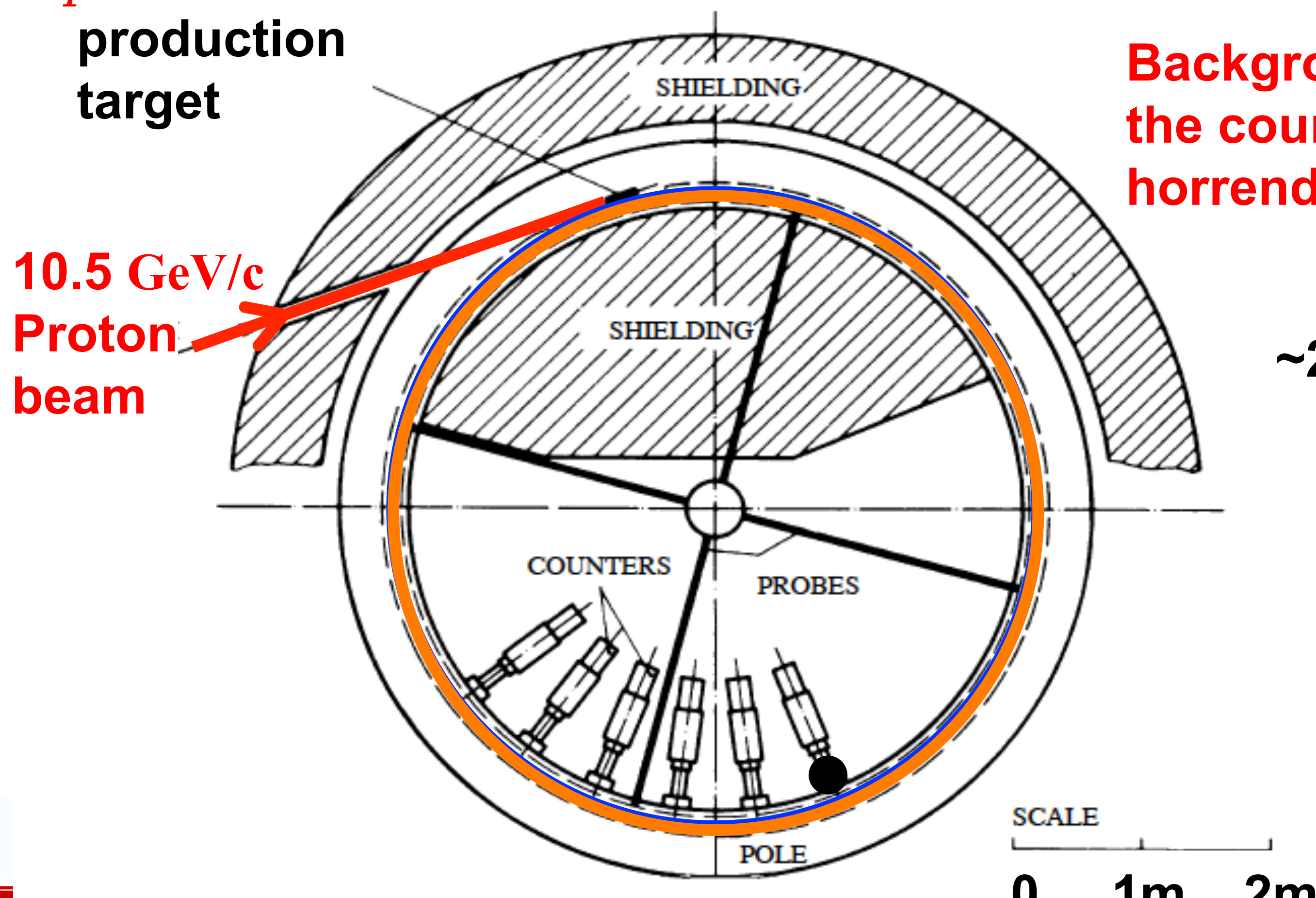
- The magnetic field is normalized to the Larmor frequency of a free proton ω_p.

$$\boxed{a_\mu = \frac{\frac{\omega_a}{\omega_p}}{\frac{\mu_\mu}{\mu_p} - \frac{\omega_a}{\omega_p}}}$$

- Remember what Rabi said; measure frequencies.

The first CERN storage ring; π production target inside

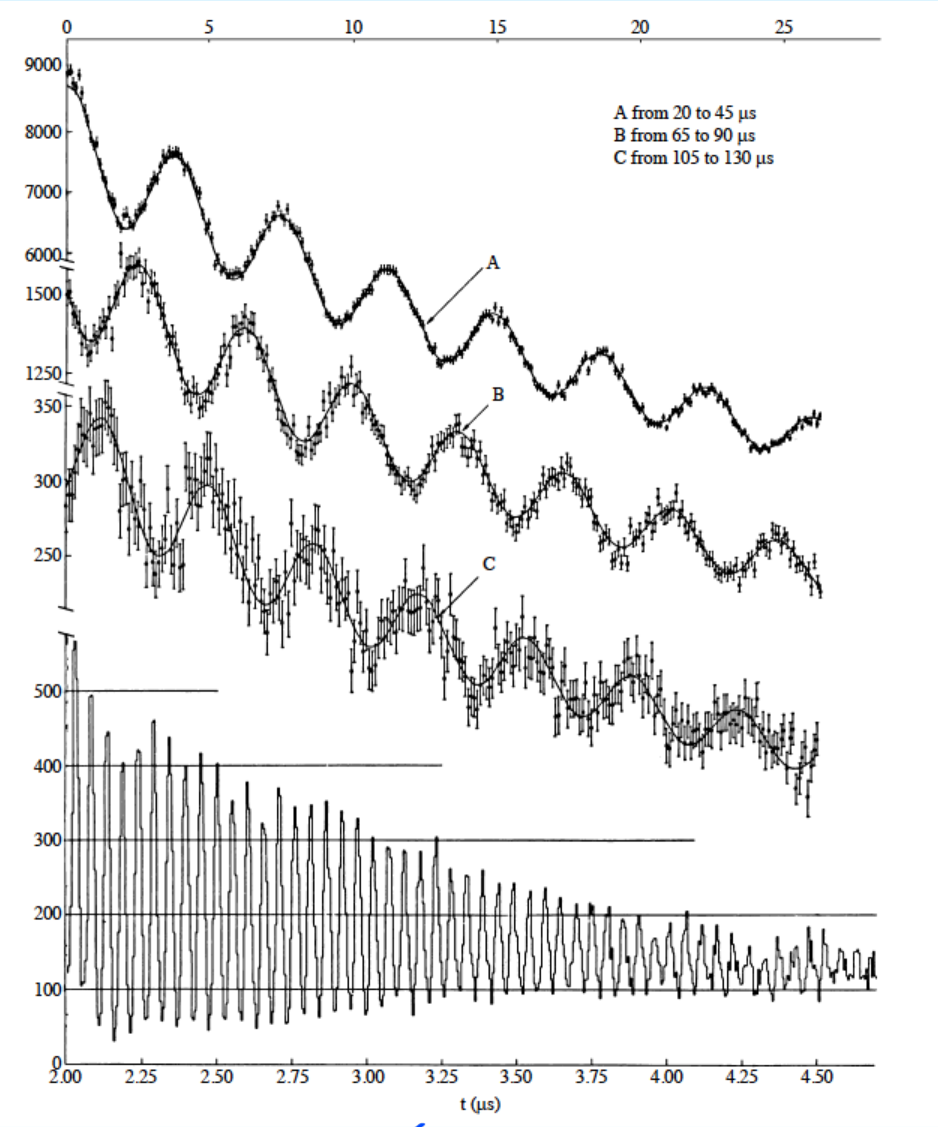
- Go to p_μ = 1.27 GeV/c, γ_μ = 12; $\gamma\tau$ = 27 μs;
- using a weak-focusing magnetic storage ring; B = 1.71 T; n = 0.13; τ_a ≈ 3.7 μs.
- $p + N \rightarrow \pi \rightarrow \mu$ **which are stored** $\rightarrow e$ **which are detected**



Background flash in the counters was horrendous!

~200 μ stored/fill

Arrival time spectrum for E_e > 830 MeV



$$f(t) \simeq N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$

$$\frac{\delta\omega_a}{\omega_a} = \frac{\sqrt{2}}{\omega_a A \gamma \tau \sqrt{N}}$$

270 ppm; Sensitive to

$$C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3$$

$$a_\mu = (116\,616 \pm 31) \times 10^{-8} \quad (266\text{ppm})$$

To get better precision a number of things needed:

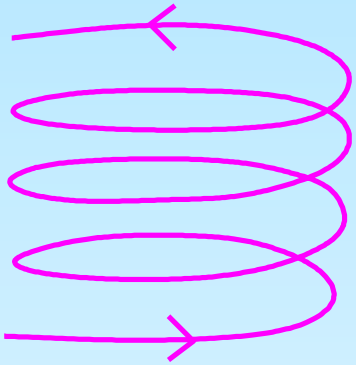
- Longer muon lifetime (more wiggles)
- More muons stored
- To decrease the uncertainty on $\langle B \rangle$, since

$$\omega_a = - - a\frac{Qe}{m}\langle B \rangle_{muon-dist}$$

- With gradients in the field, you have to know the muon trajectories very well to determine $\langle B \rangle$

However, you need vertical focusing:

without vertical focusing



What about an electric quadrupole field for focusing?

- Everybody knows that to a relativistic charged particle, an electric field looks like a combination of E and B

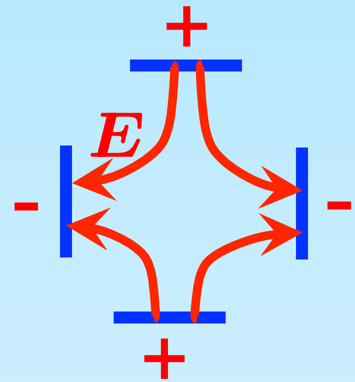
Non-relativistic positive spinning particle

parabolic path
spin unchanged

Relativistic positive spinning particle

parabolic path,
spin leads the momentum

The Magic γ

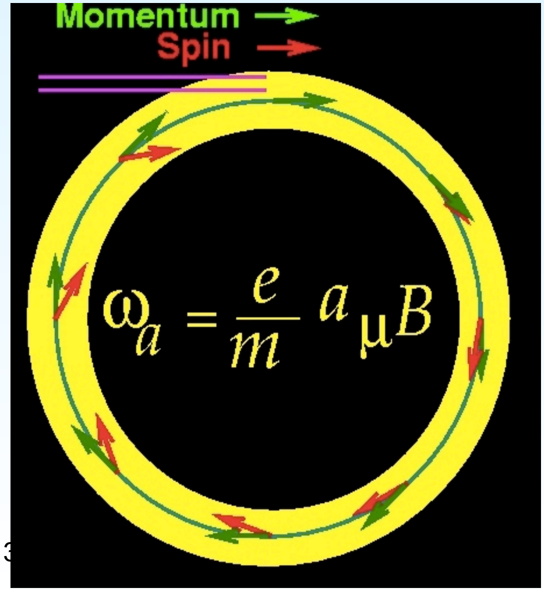


With an electric quadrupole field for vertical focusing

$$\vec{\omega}_a = -\frac{Qe}{m}\left[a_\mu \vec{B} - \cancelto{0}{\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)} \frac{\vec{\beta} \times \vec{E}}{c}\right]$$

$$\gamma_{\mathrm{magic}} = 29.3$$

$$p_{\mathrm{magic}} = 3.09\ \mathrm{GeV/c}$$



Beam Dynamics: Weak Focusing Betatron

$$\text{Field index: } n = \frac{R_0}{\beta B_0}\frac{dE_r}{dr} \simeq 0.135$$

If the quadrupole is uniform, get simple harmonic motion

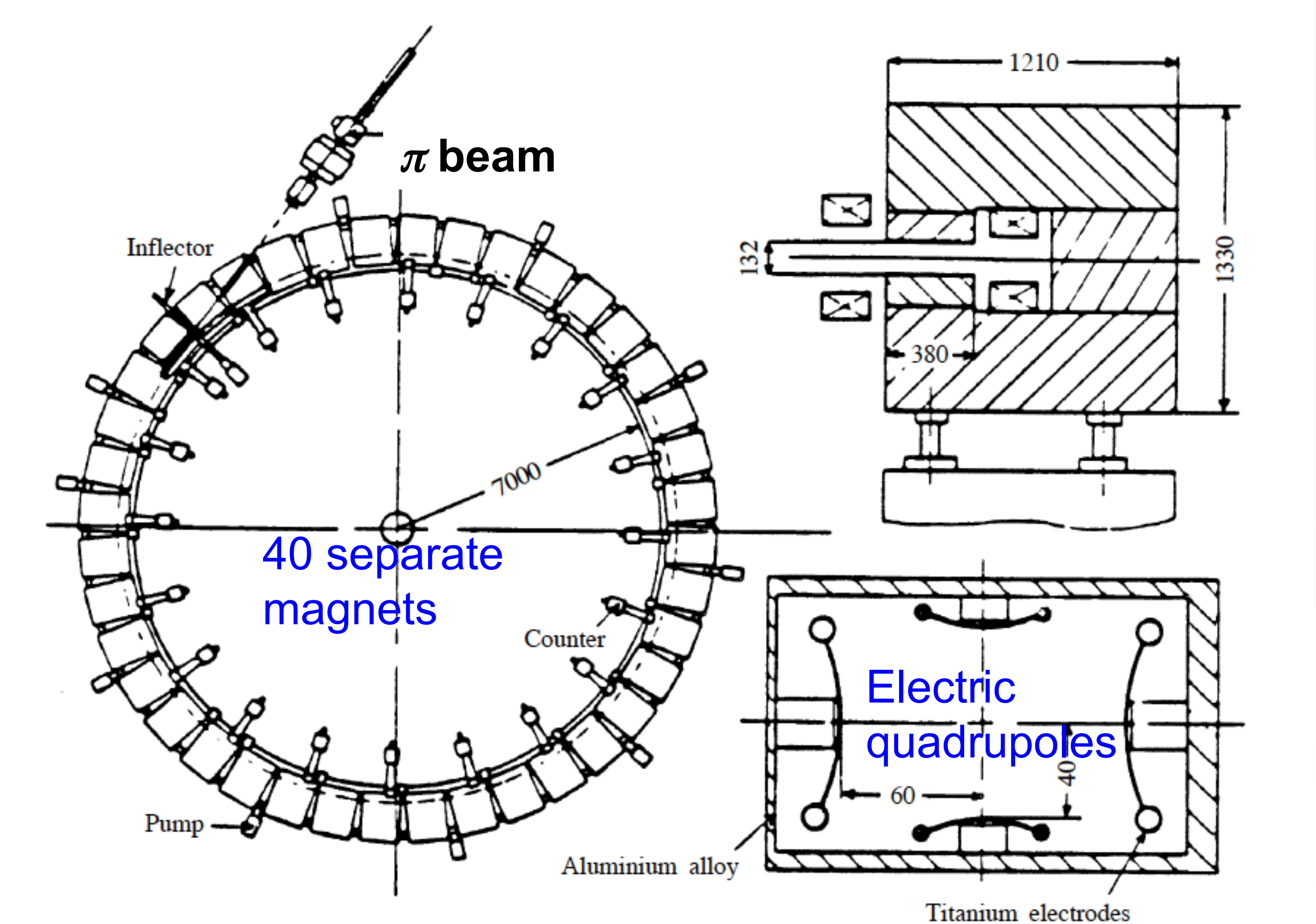
$$f_y = f_C\sqrt{n} \simeq 0.37 f_C; \qquad \lambda_y \simeq 2.7(2\pi R)$$

$$f_x = f_C\sqrt{1-n} \simeq 0.929 f_C \qquad \lambda_x \simeq 1.08(2\pi R)$$

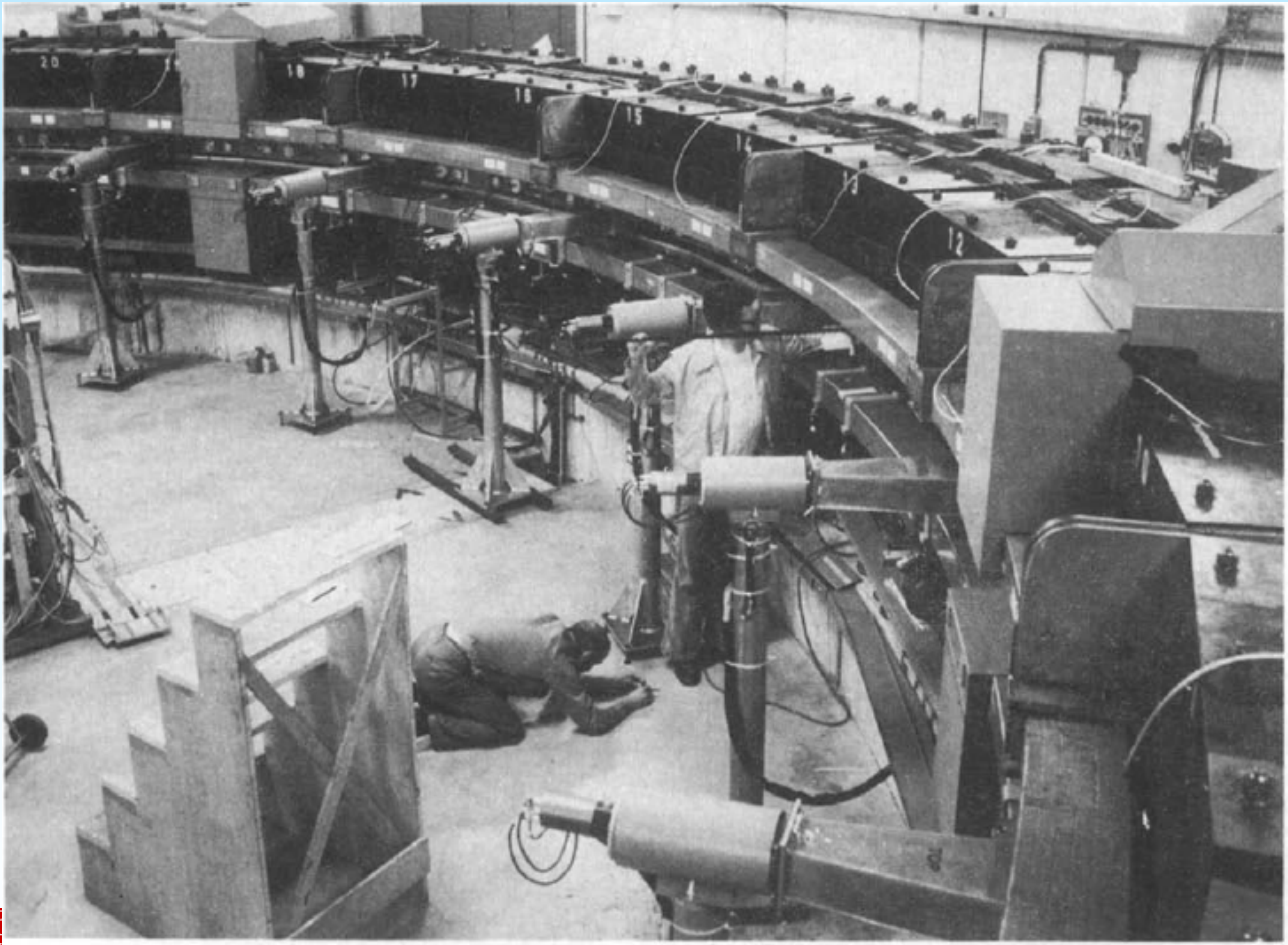
Must adjust the field index to avoid resonances.

The Third CERN Experiment; The magic γ
NPB 150, 1 (1979)

- Inject pions
- Use $\pi \rightarrow \mu$ decay to kick muons onto stable orbits

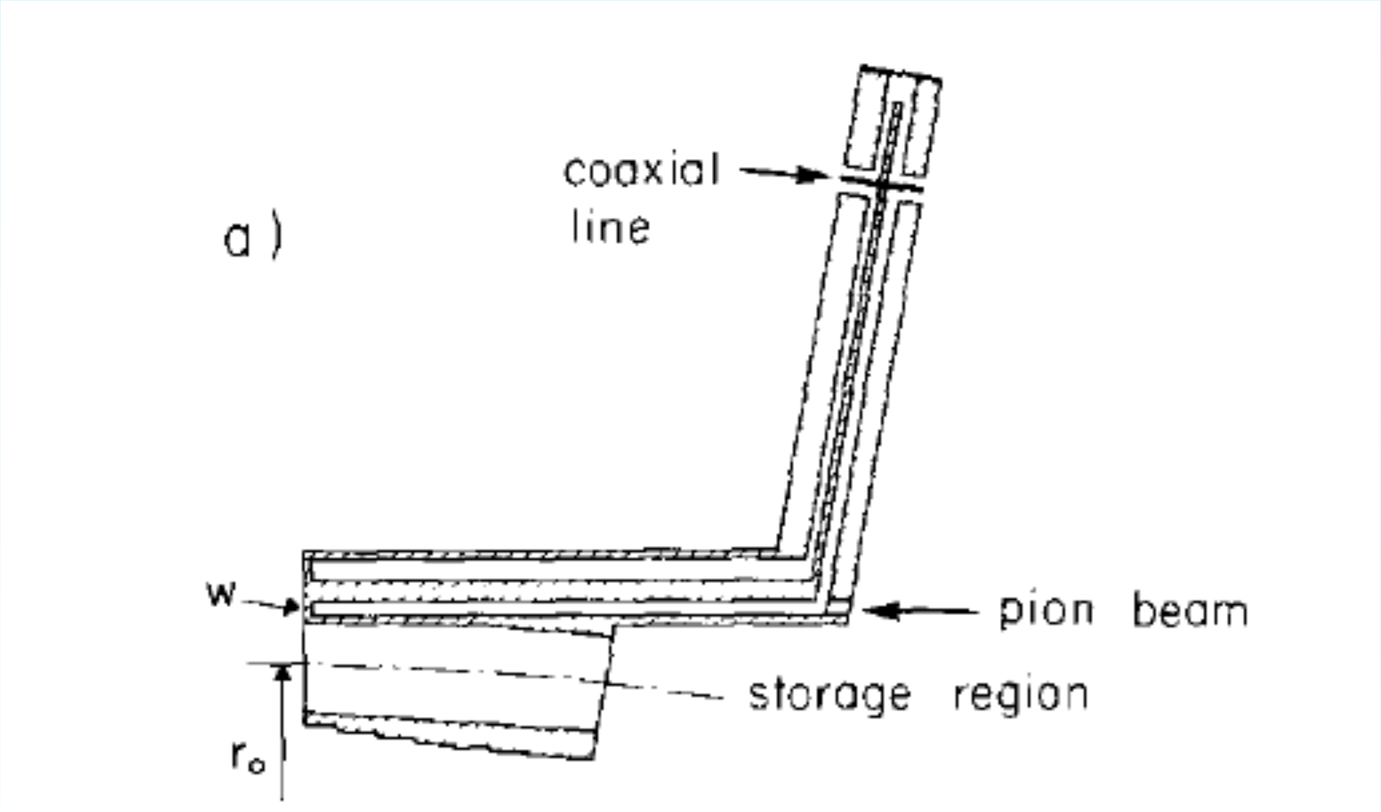


The CERN3 storage ring

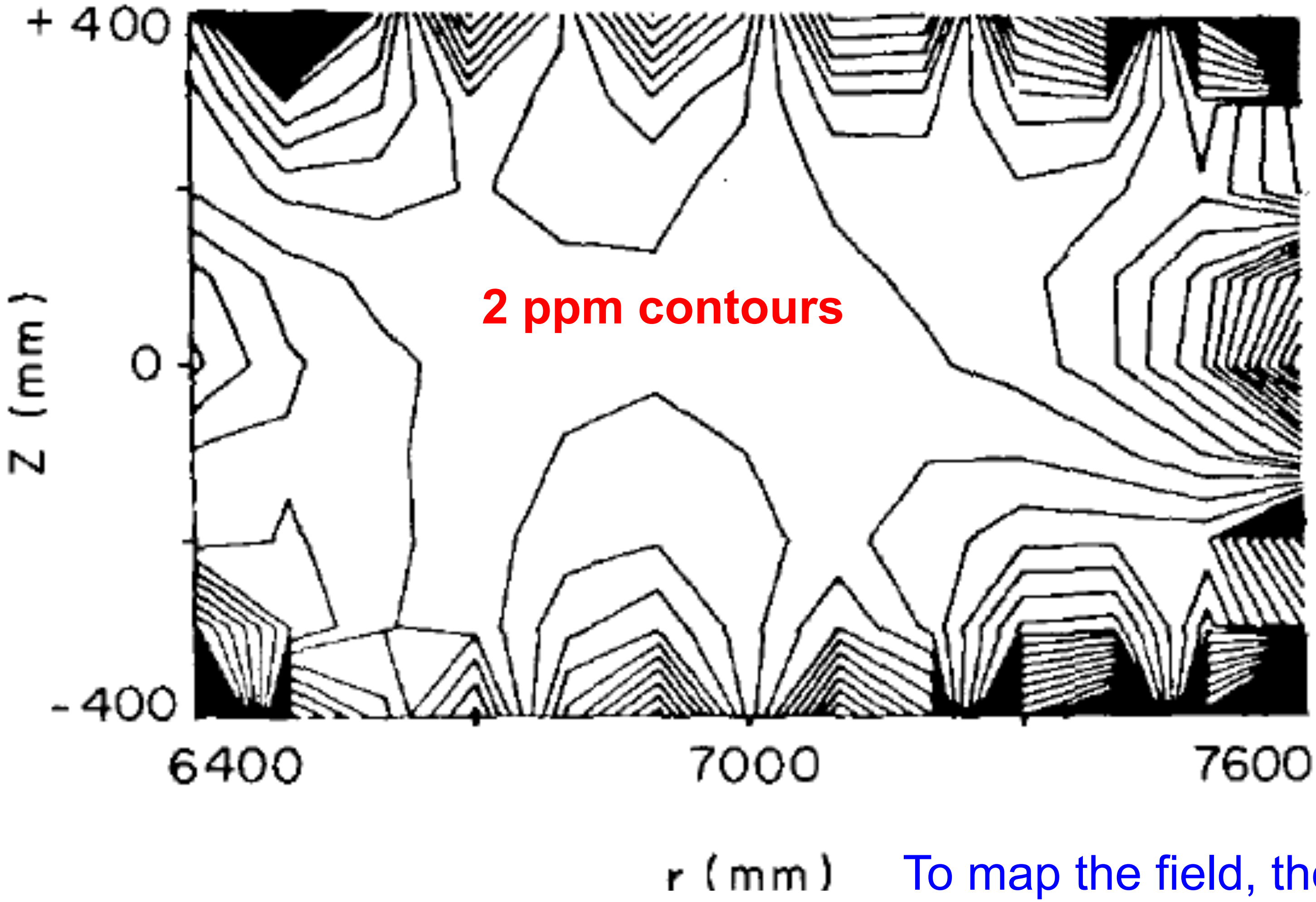


The CERN Inflector Magnet

- To deliver the beam at the edge of the storage region, CERN used a pulsed co-axial line, 300 kA peak current to null the main field.



CERN 3 experiment: NPB 150, 1 (1979)



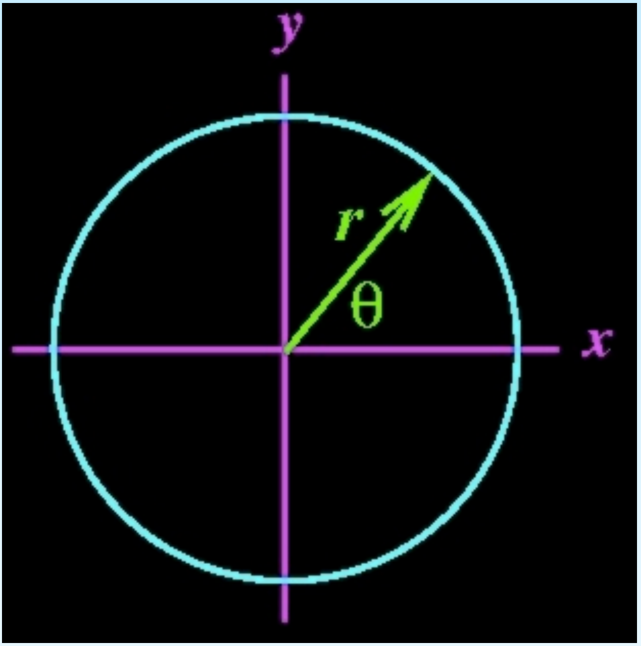
What are these discrete areas?

The width of the grinding wheel

To map the field, they had to take out the vacuum chambers.

The average field

$$\langle B \rangle = \int M(r,\theta) B(r,\theta) r dr d\theta$$

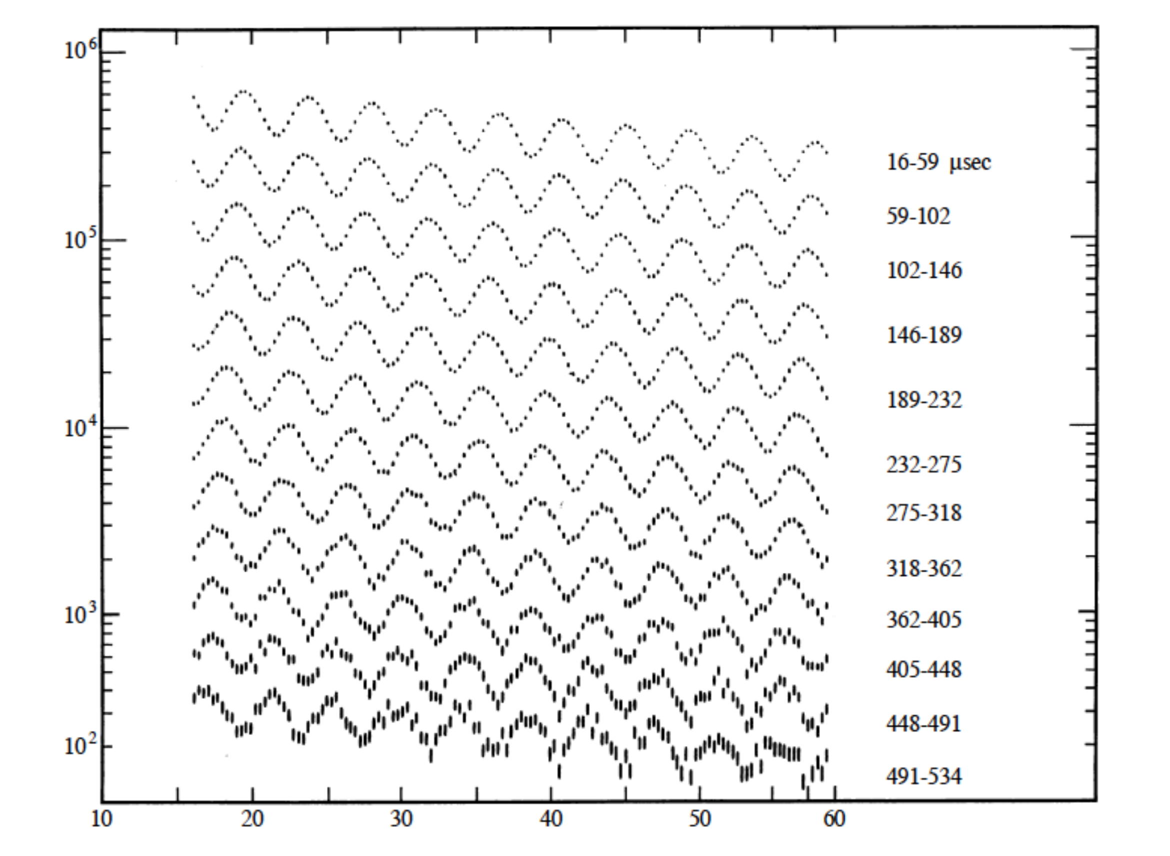


In the average, B couples multipole by multipole with the moments of the muon distribution.

$$B(r,\theta) = \sum_{n=0}^{\infty} r^n \left[c_n \cos n\theta + s_n \sin n\theta \right]$$

$$M(r,\theta) = \sum [\gamma_m(r) \cos m\theta + \sigma_m(r) \sin m\theta]$$

CERN 3 results



A portion of the CERN 3 data

$$a_{\mu^\pm} = (1\,165\,923 \pm 8.5) \times 10^{-9} \quad (7.3 \text{ ppm})$$

Lessons from CERN

- Don't grind the magnet, instead have a shimming kit, including currents
- Make the magnet monolithic
- Don't use a pulsed inflector
- Use a circular aperture

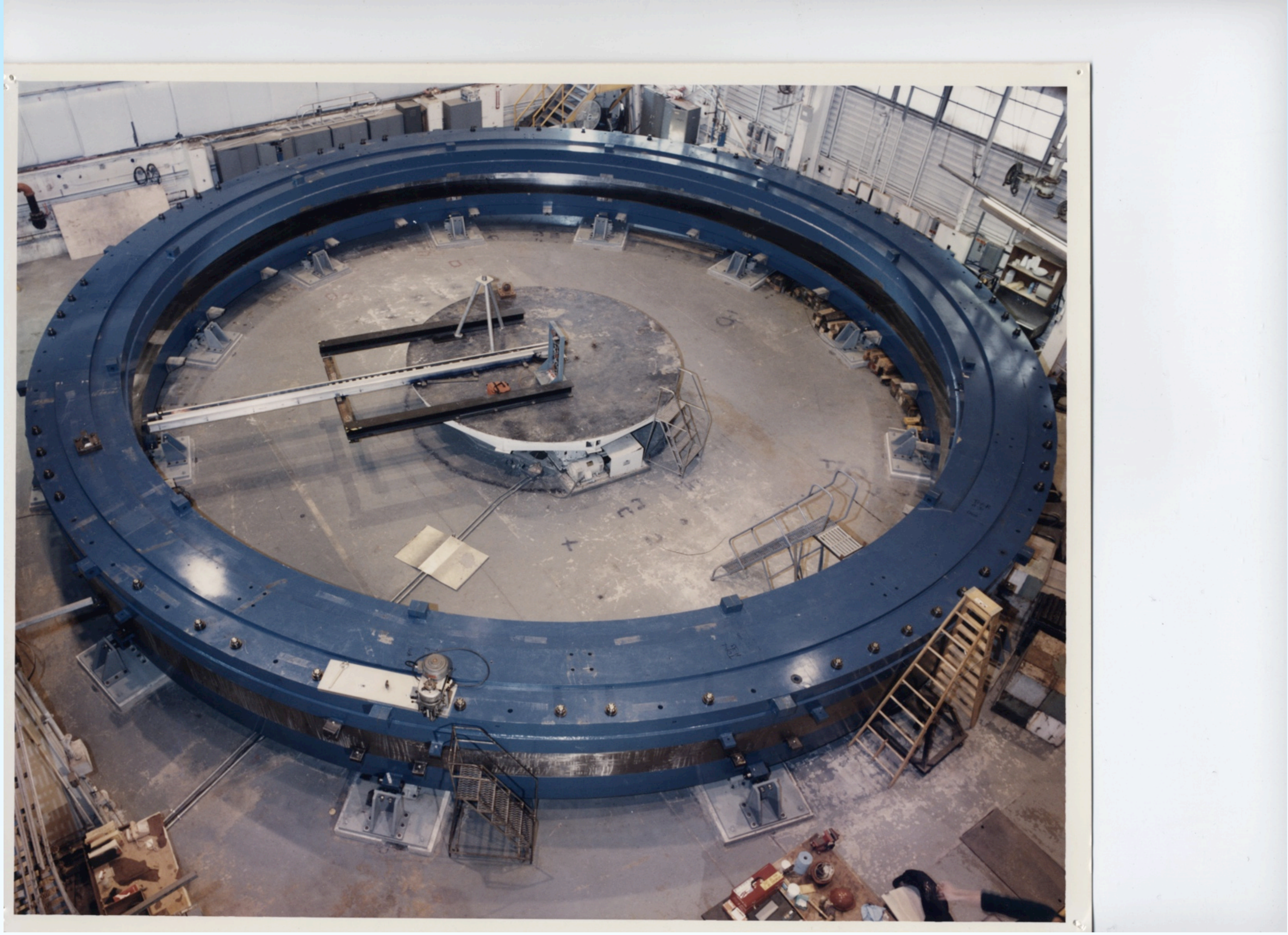
$$\langle B \rangle = \int M(r,\theta) B(r,\theta) r dr d\theta$$

- Use direct muon injection to get rid of the pion flash
- Develop a way to map the B field without powering down and removing the vacuum chambers.
- Monitor the field everywhere during data collection

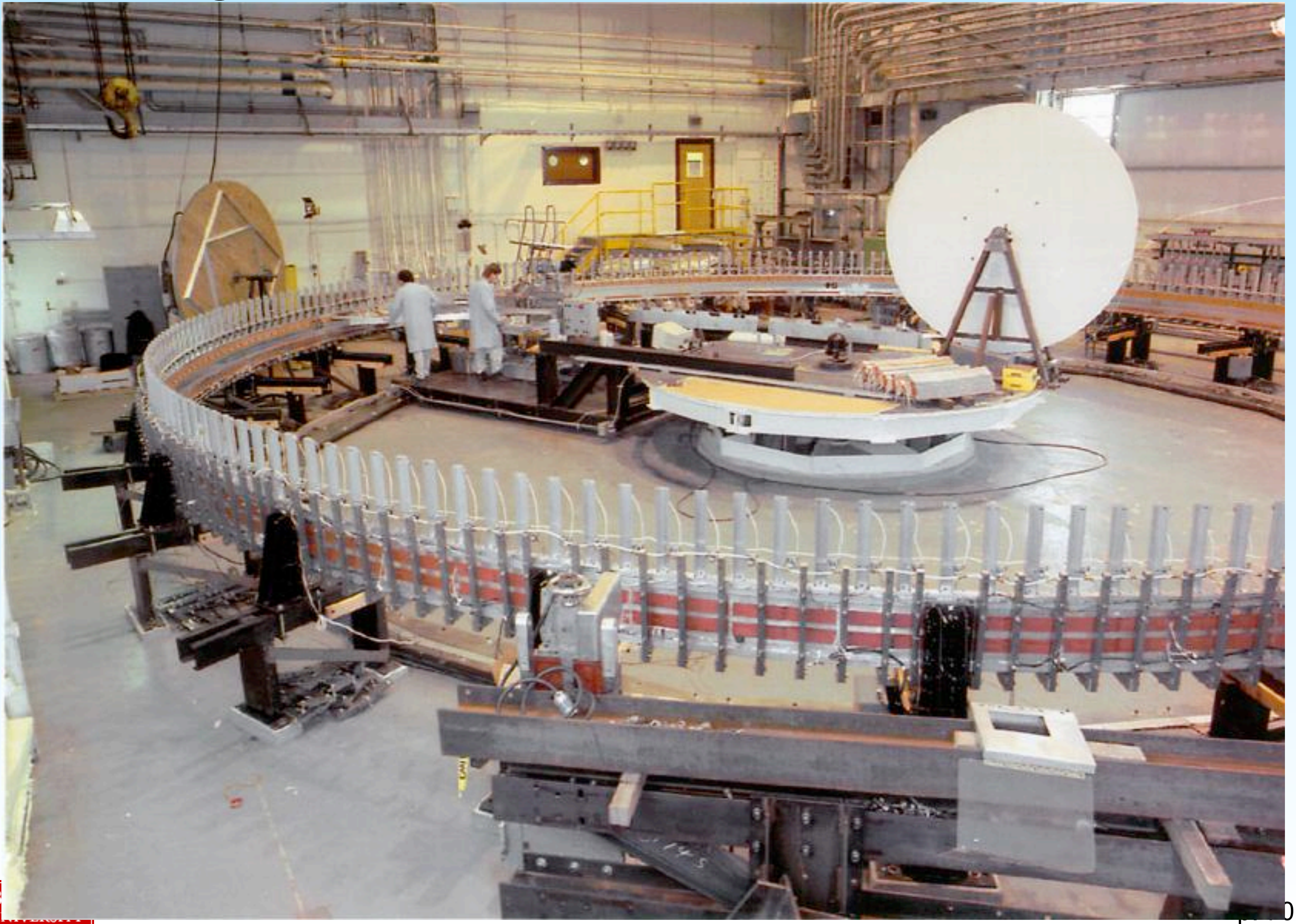
Design Decisions for E821 at BNL

- Make the magnet superferric – i.e. use superconducting coils to excite the magnet, but iron to shape the field.
  - much more stable, can start data collection very soon after power-up (CERN had to wait a week for stability)
- Shim the field to ~ 1 ppm uniformity
- Design a superconducting inflector
  - No transient magnetic fields
- Design a muon kicker that kicks the beam onto orbit, but does not leave any remnant field behind that interferes with the precession measurement
- Use modern electronics and wave-form digitizers

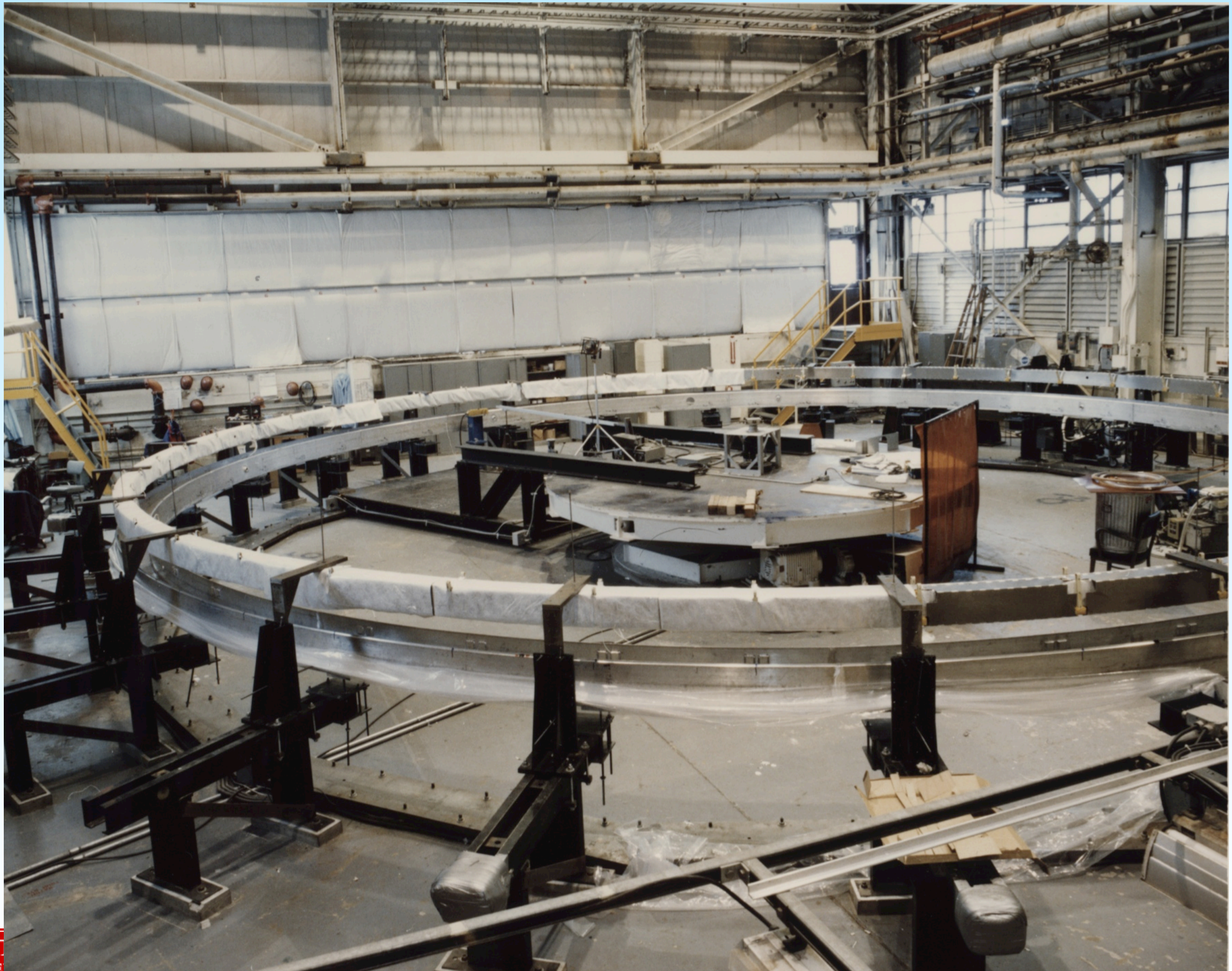
Assembling the yoke for the first time



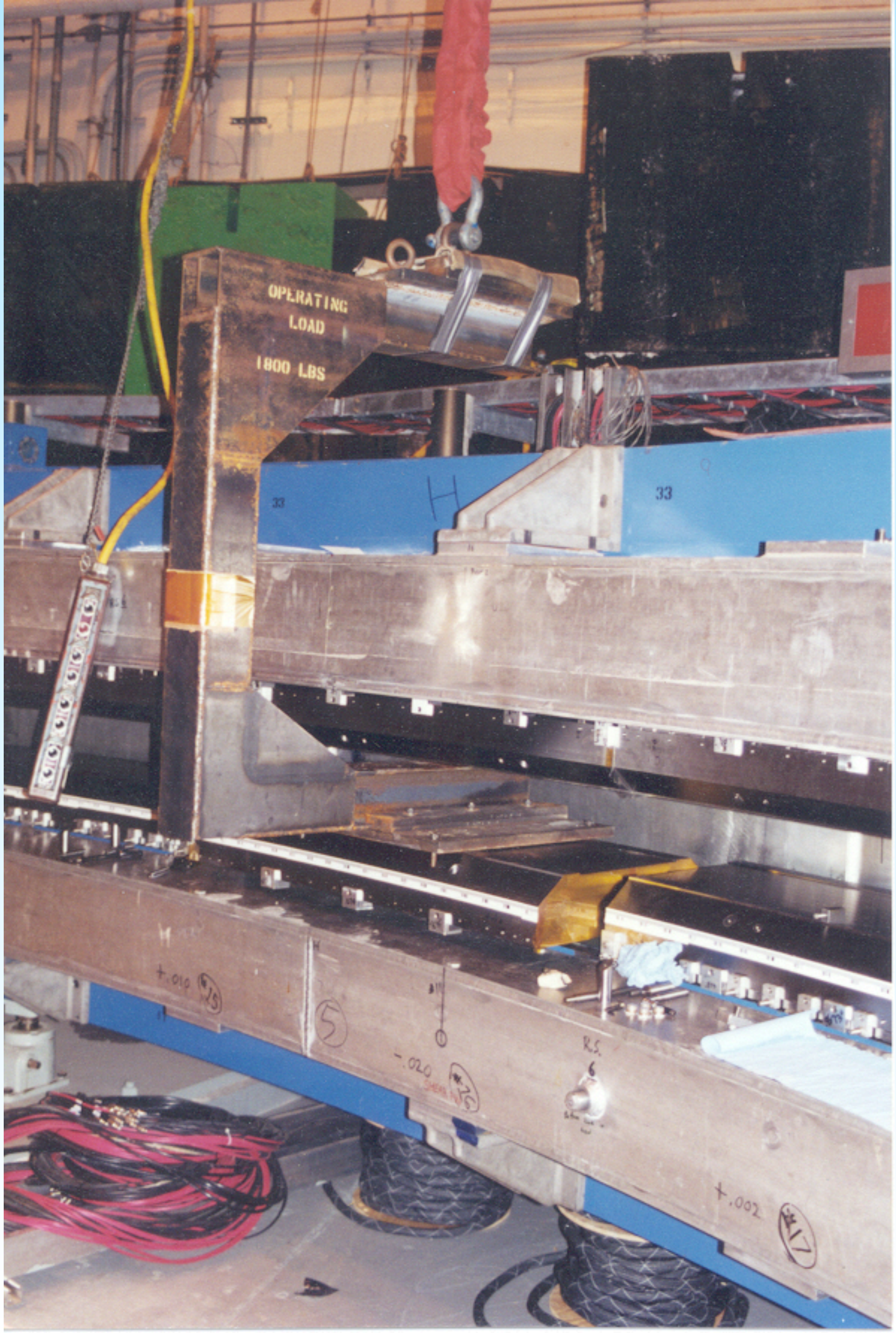
Winding a coil



Installing an inner coil into its cryostat

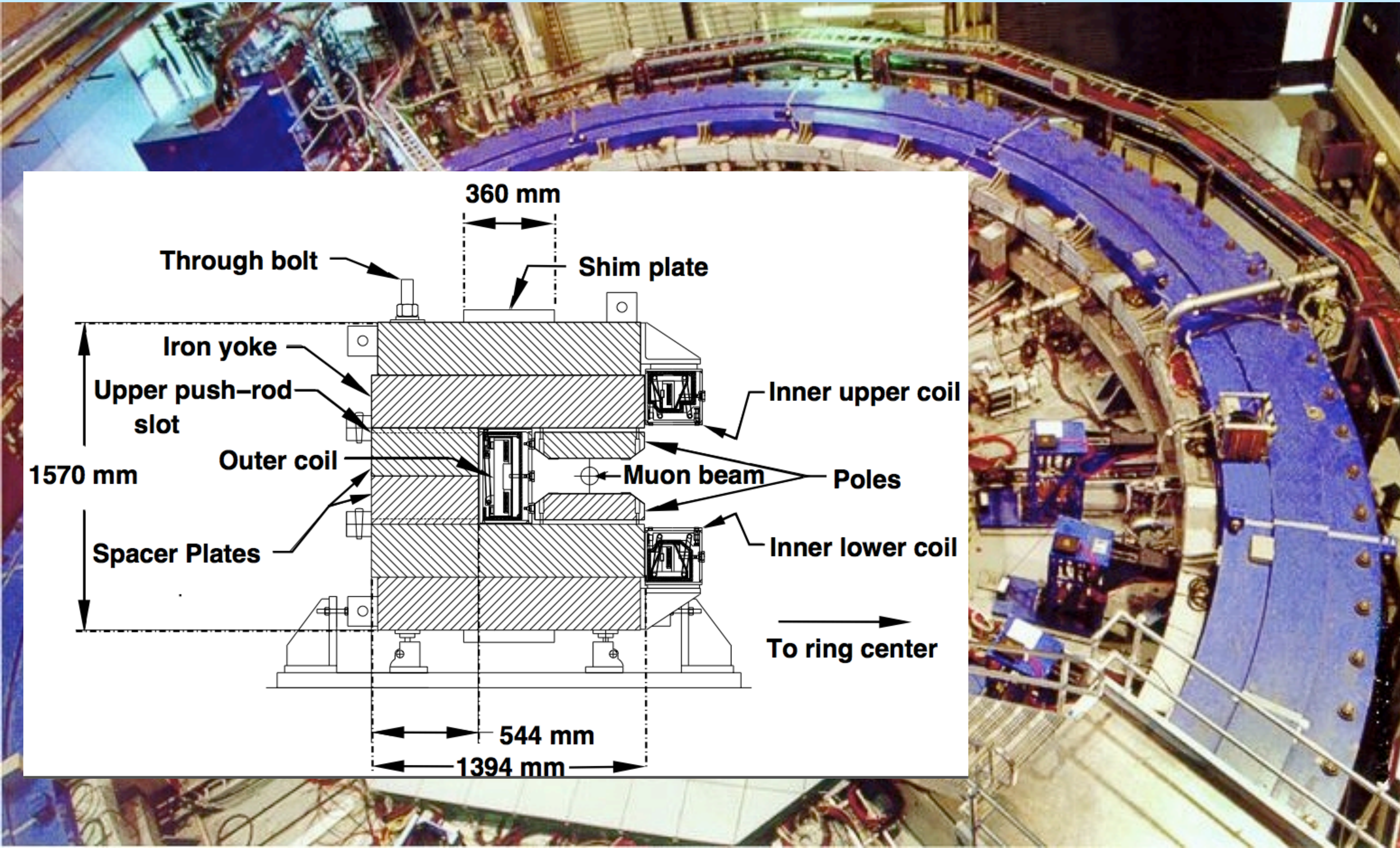


Installing a pole piece



The Magnet

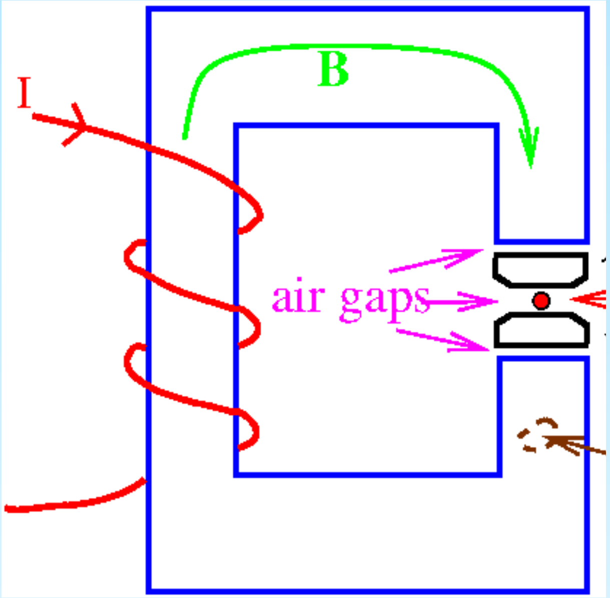
The Magnet



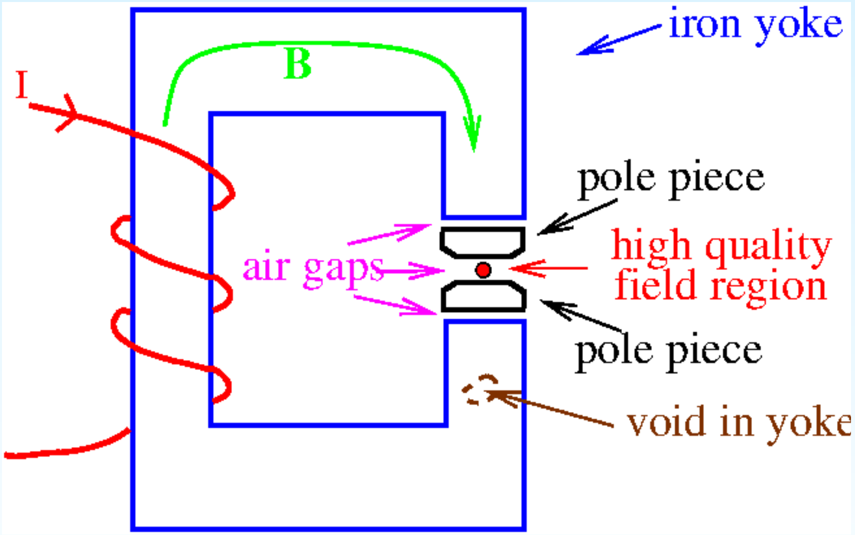
360 mm
Through bolt
Shim plate
Iron yoke
Upper push–rod slot
Inner upper coil
Outer coil
1570 mm
Muon beam
Poles
Spacer Plates
Inner lower coil
To ring center
544 mm
1394 mm

Magnetic Circuits

$$\oint \vec{H}\cdot d\vec{\ell} = NI; \;\; \vec{B} = \mu\vec{H}$$



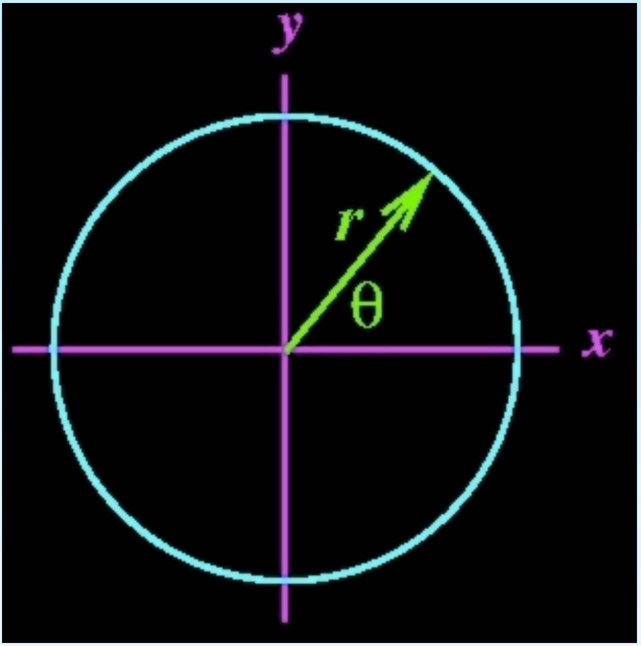
$$\Phi \underbrace{\oint \frac{d\ell}{\mu A}}_{\text{reluctance}} = \underbrace{NI}_{\text{Magnetomotive force}}$$



$\mu \approx 1$ **for air,** $\mu \rightarrow 10^4$ **for steel**

The Magnet

$$\langle B \rangle = \int M(r,\theta) B(r,\theta) r dr d\theta$$



A circular aperture minimizes the effect of higher multipoles on $\langle B \rangle$

$$B(r,\theta) = \sum_{n=0}^{\infty} r^n \left[c_n \cos n\theta + s_n \sin n\theta \right]$$

$$M(r,\theta) = \sum \left[\gamma_m(r) \cos m\theta + \sigma_m(r) \sin m\theta \right]$$

Fig. 4.8

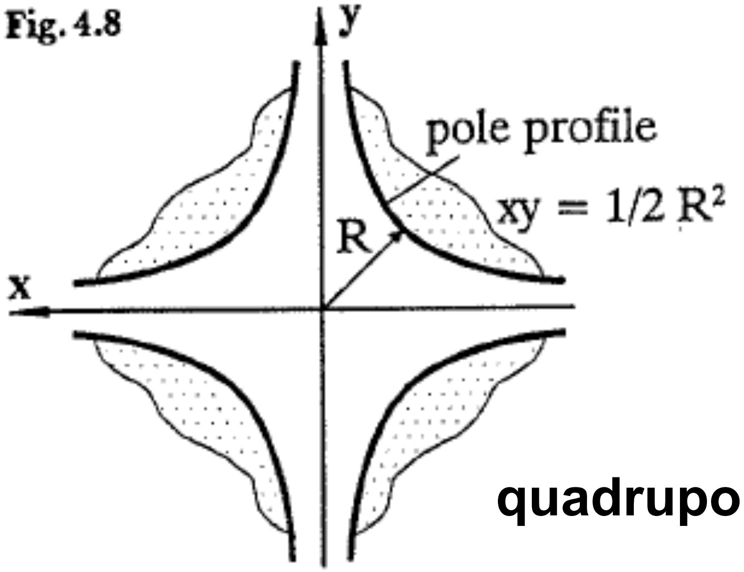
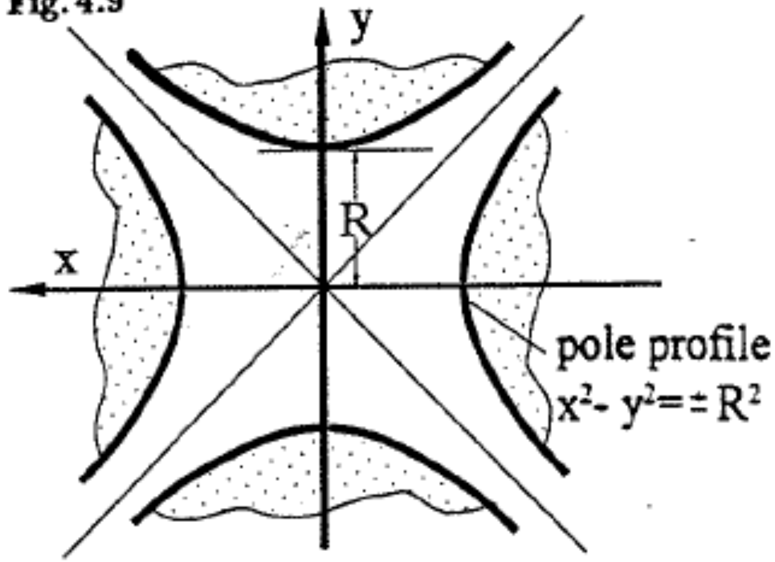


Fig. 4.9



quadrupole

normal

skew

Fig. 4.11

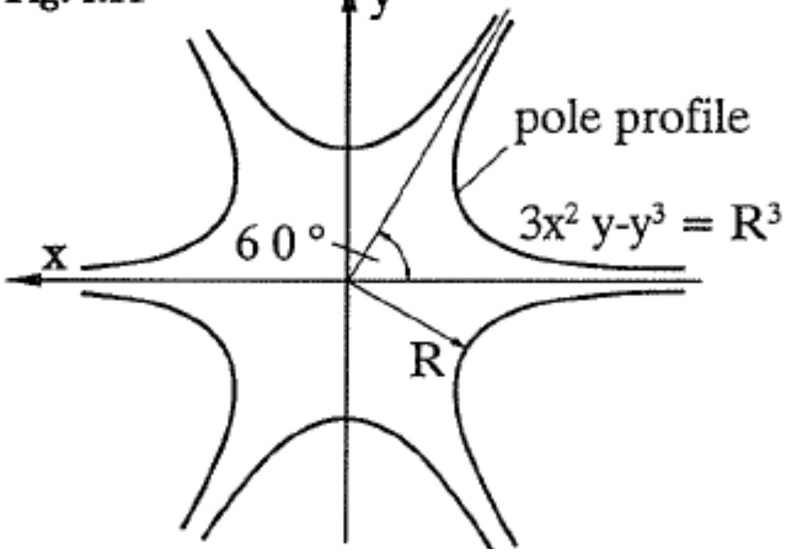
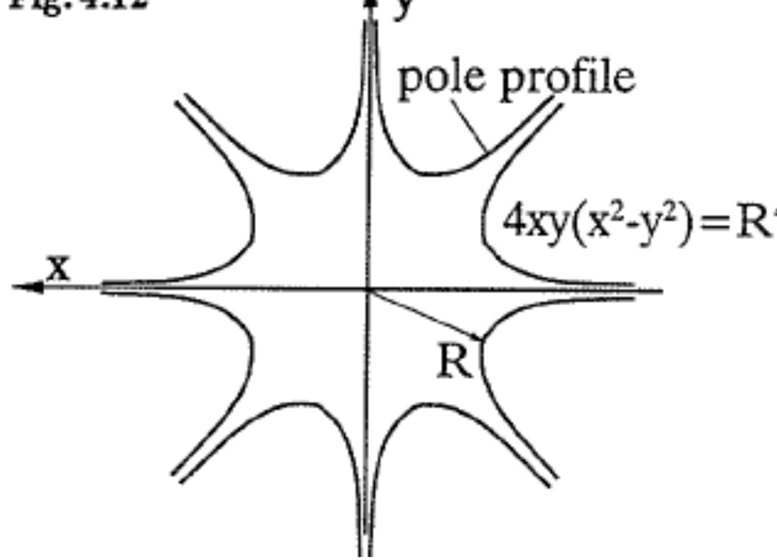


Fig. 4.12



sextupole

octupole

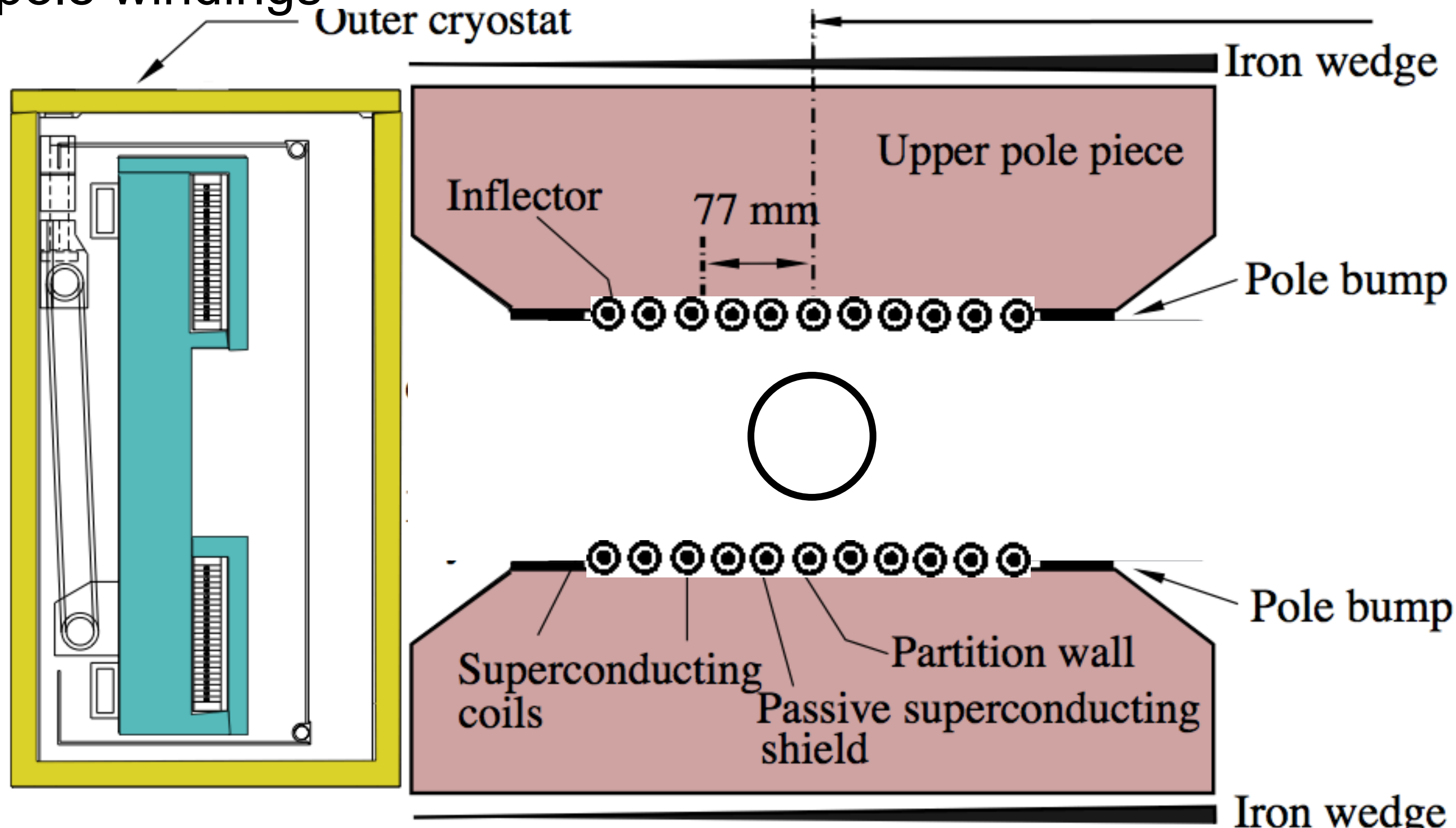
Magnet shimming tools (Gordon Danby's design):

wedges – radial motion dipole;

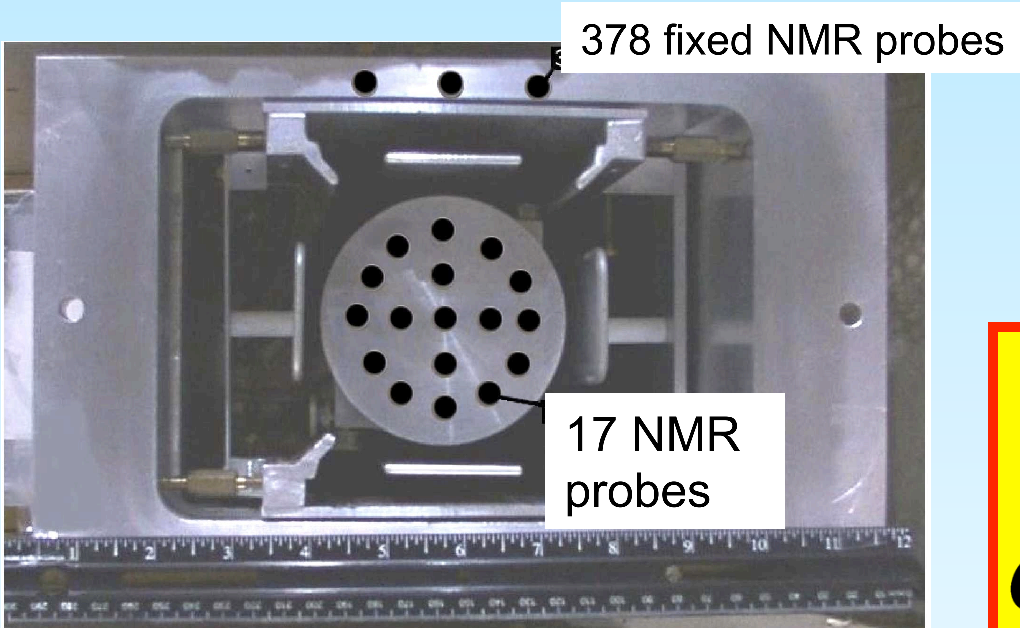
wedge angle – quadrupole

pole bumps – sextupole

pole windings



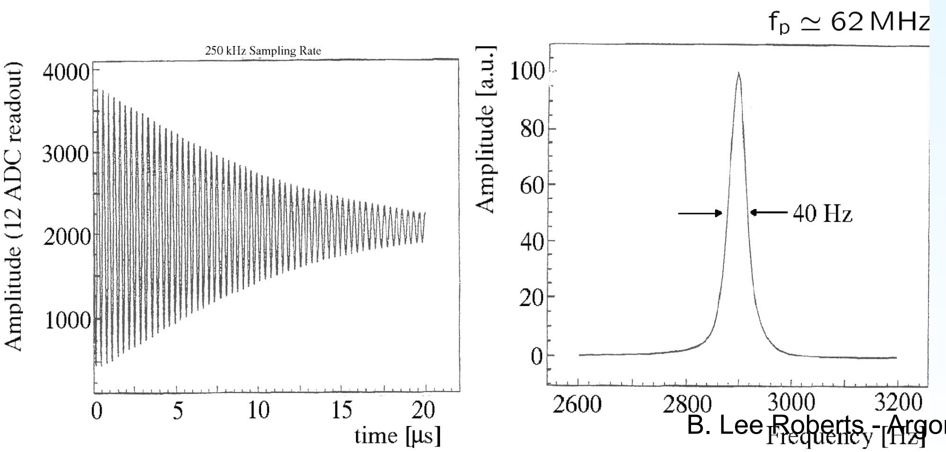
The magnetic field is measured and controlled using pulsed NMR and the free-induction decay.



Calibration to a spherical water sample that ties the field to the Larmor frequency of the free proton ω_p.

$$a_\mu = \frac{\frac{\omega_a}{\omega_p}}{\frac{\mu_\mu}{\mu_p} - \frac{\omega_a}{\omega_p}}$$

Free induction decay signals:



$\frac{\mu_\mu}{\mu_p}$ from $\mu^+ e^-$ atom

Absolute calibration: Spherical water sample

The Larmor frequency of a proton in a spherical water sample is related to that of the free proton through

$$f_{\mathsf{L}}(\mathsf{sph} - \mathsf{H_2O}, T) = [1 - \sigma(\mathsf{H_2O}, T)]\, f_{\mathsf{L}}(\mathsf{free})$$

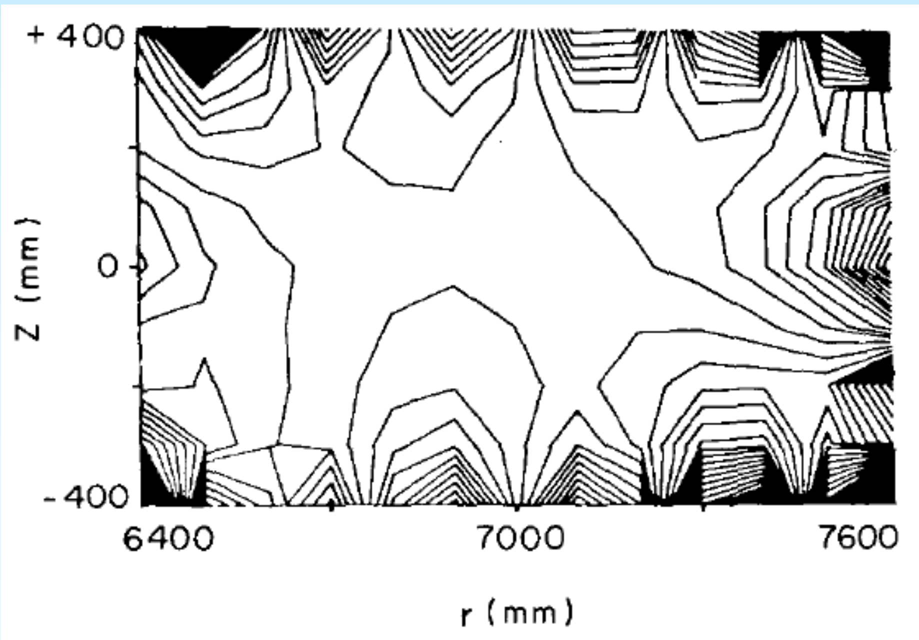
where $\sigma(H_2O, T) = 25.790(14) \times 10^{-6}$ is from the diamagnetic shielding of the proton in the water molecule, determined from

$$\sigma(\mathsf{H_2O}, 34.7^\circ C) = 1 - \frac{g_p(\mathsf{H_2O}, 34.7^\circ C)}{g_J(H)} \frac{g_J(H)}{g_p(H)} \frac{g_p(H)}{g_p(\mathsf{free})}$$

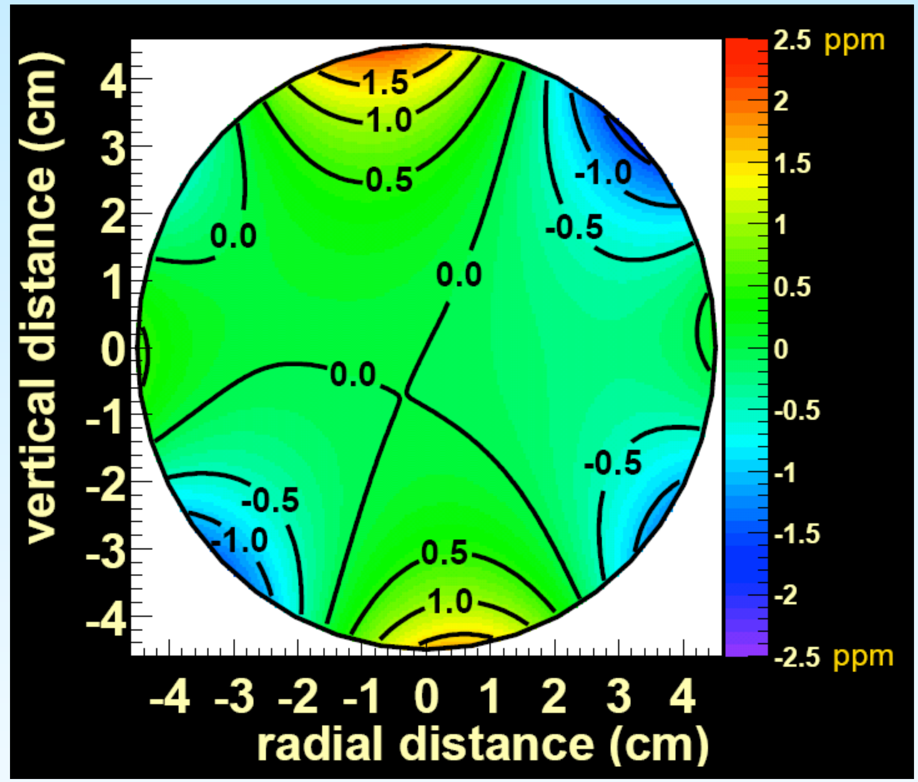
1. the ratio of the g-factors of the proton in a spherical water sample to that of the electron in the hydrogen ground state
2. the ratio of electron to proton g-factors in hydrogen
3. the bound-state correction relating the g-factor of the proton bound in hydrogen to the free proton

B field averaged over azimuth

CERN: 2 ppm contours

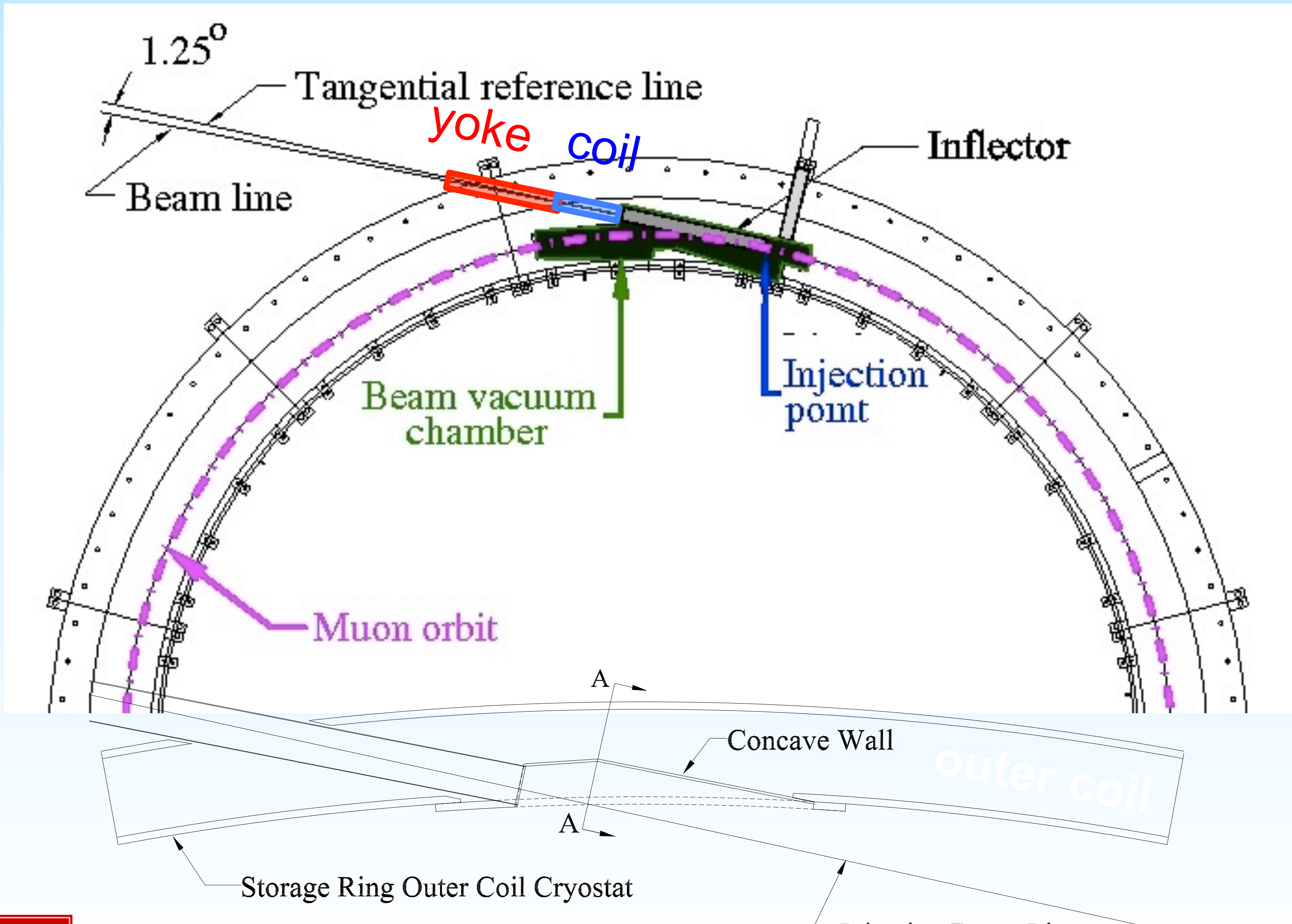


E821: 0.5 ppm contours

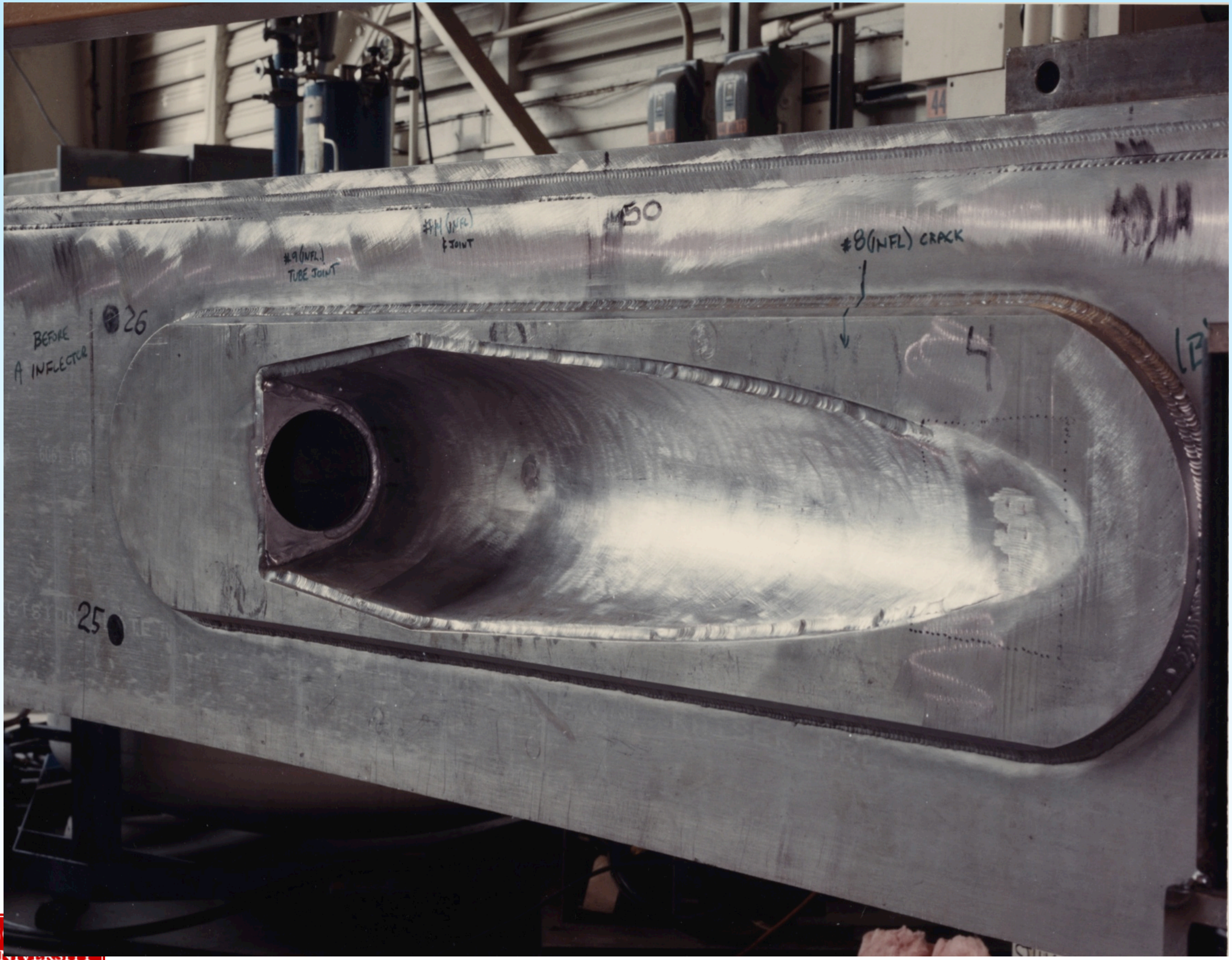


$$\langle B \rangle_{\mu-\text{dist}} = \int M(r,\theta) B(r,\theta) r dr d\theta$$

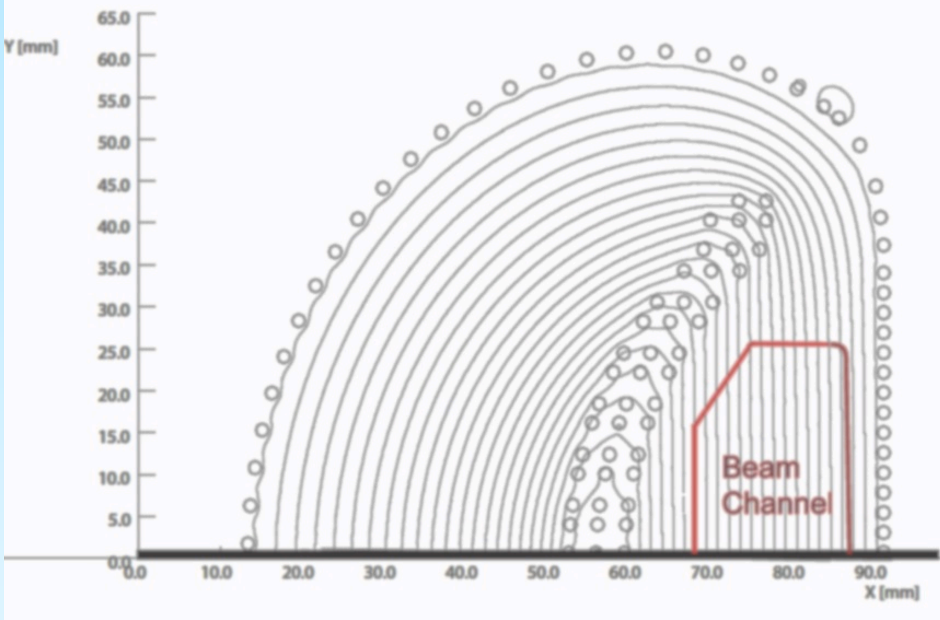
The need for an inflector magnet

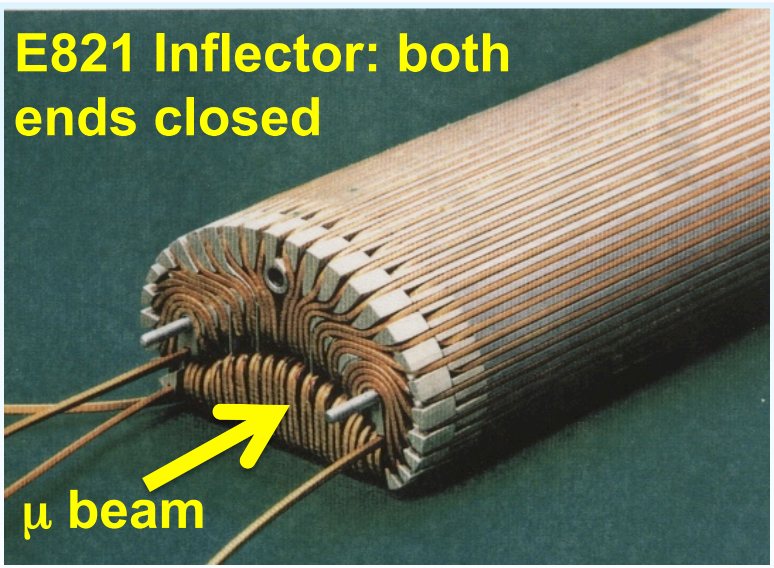


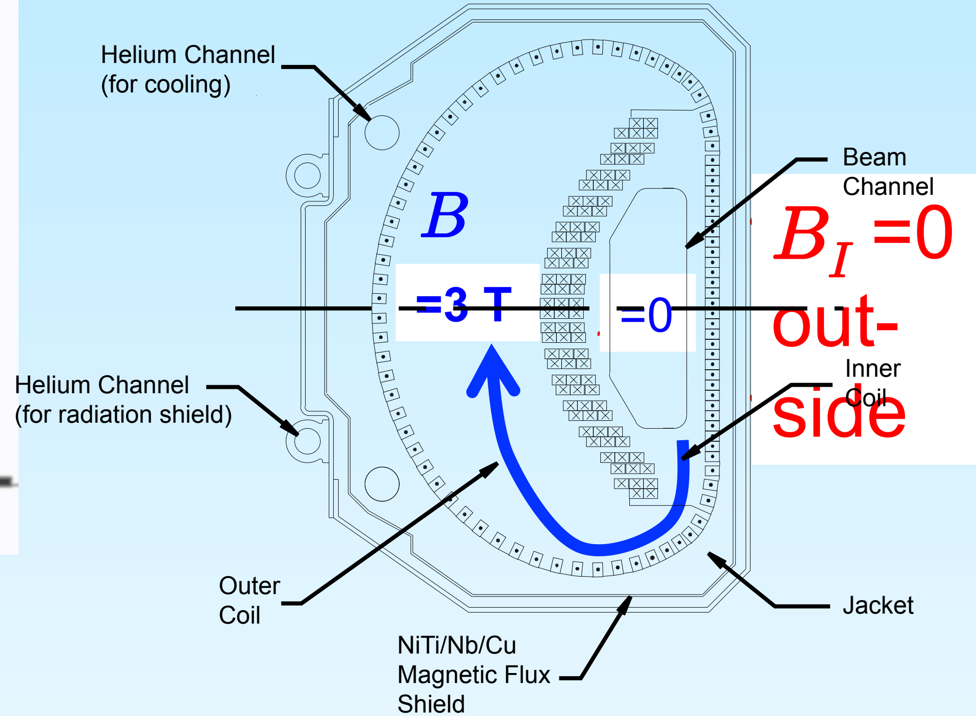
The beam penetration through the outer coil



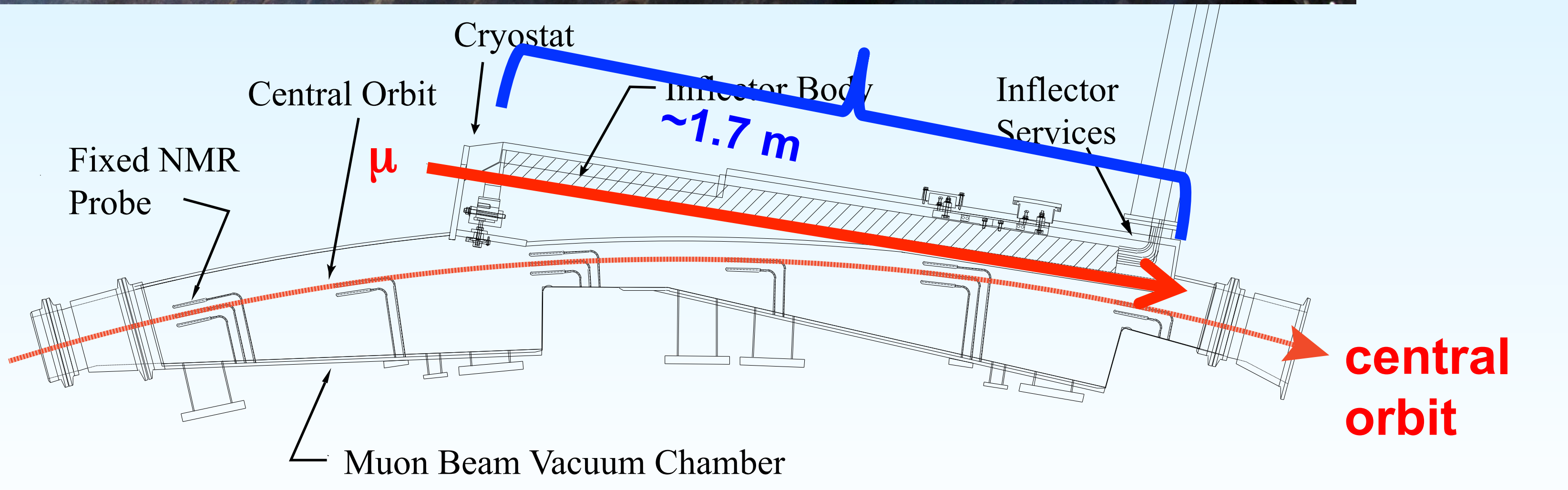
Inflector Design



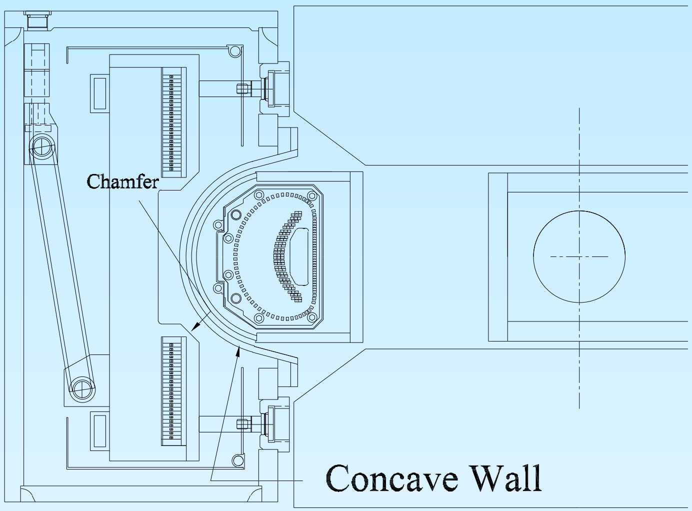


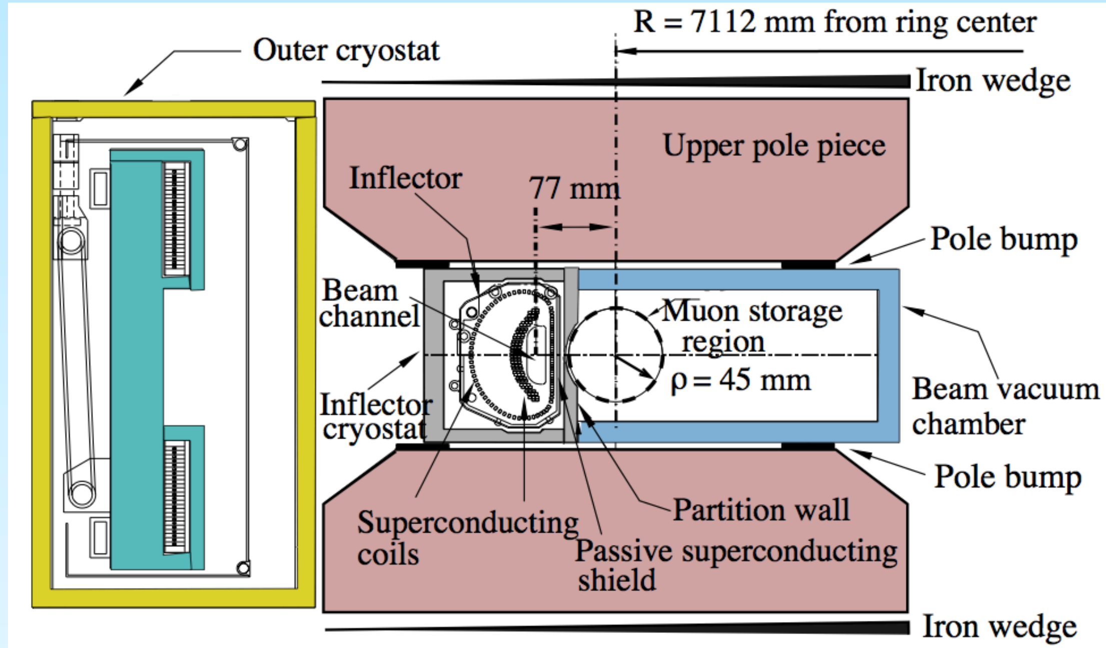


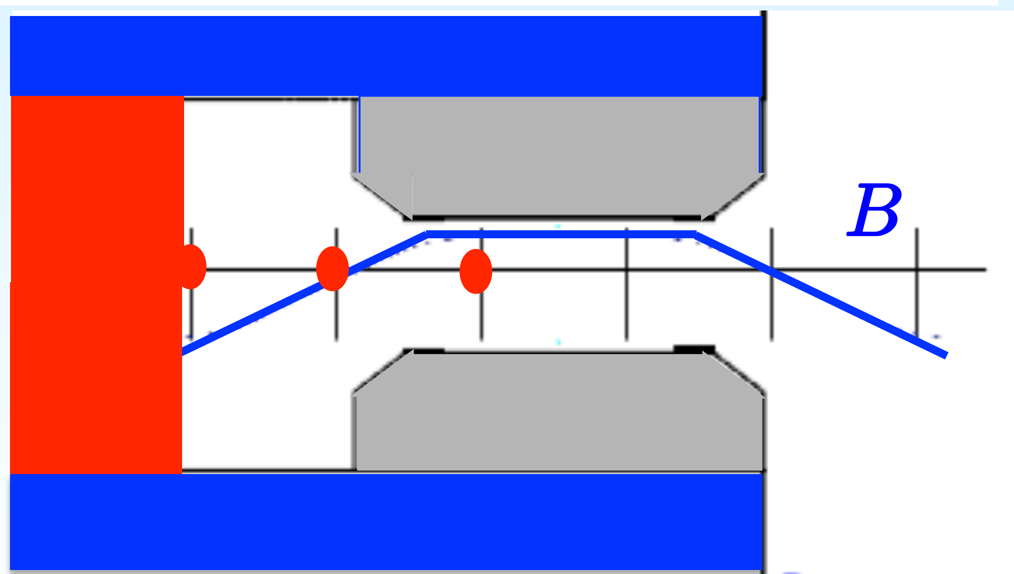
Inflector Geometry



Radial beam motion

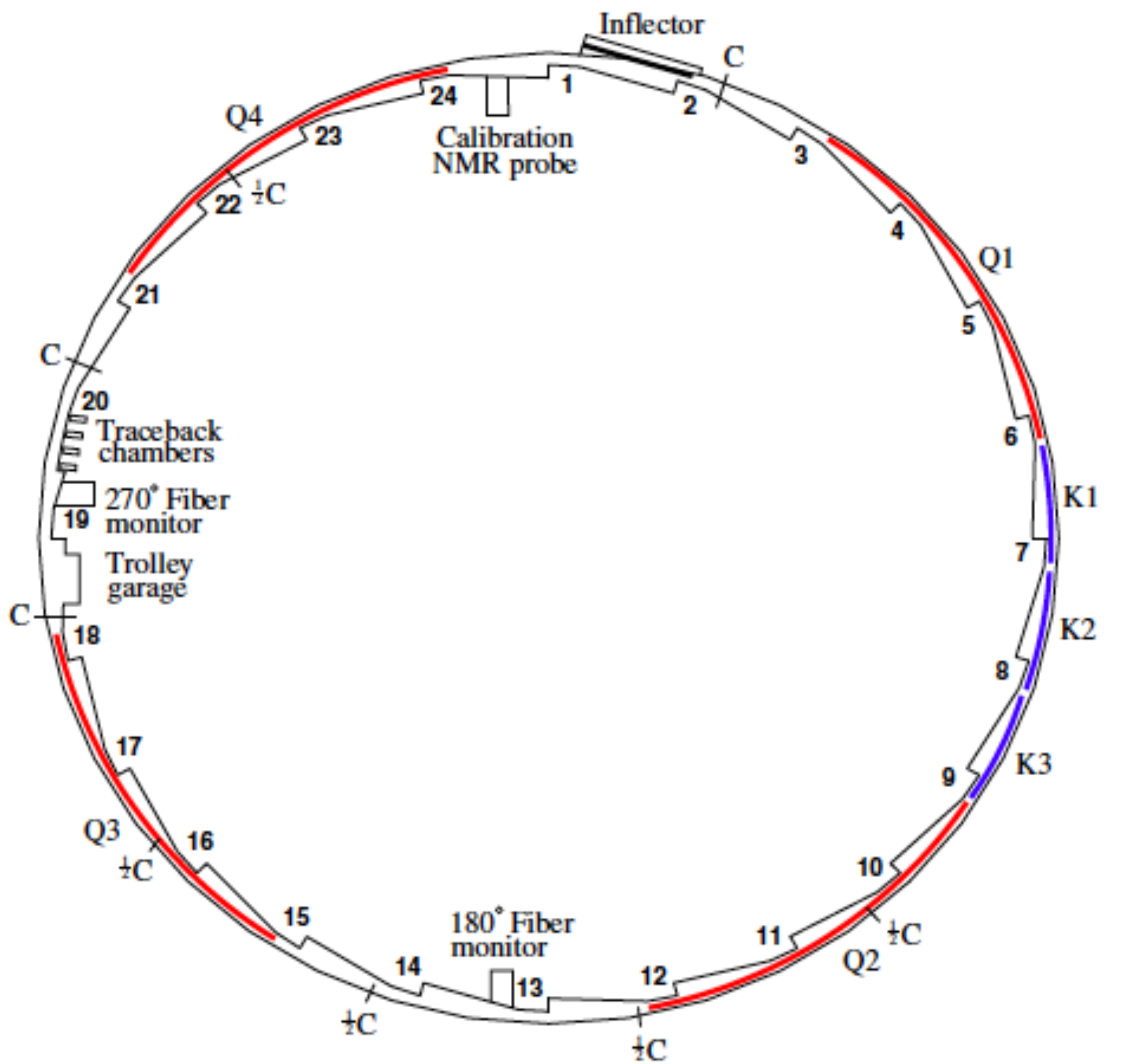




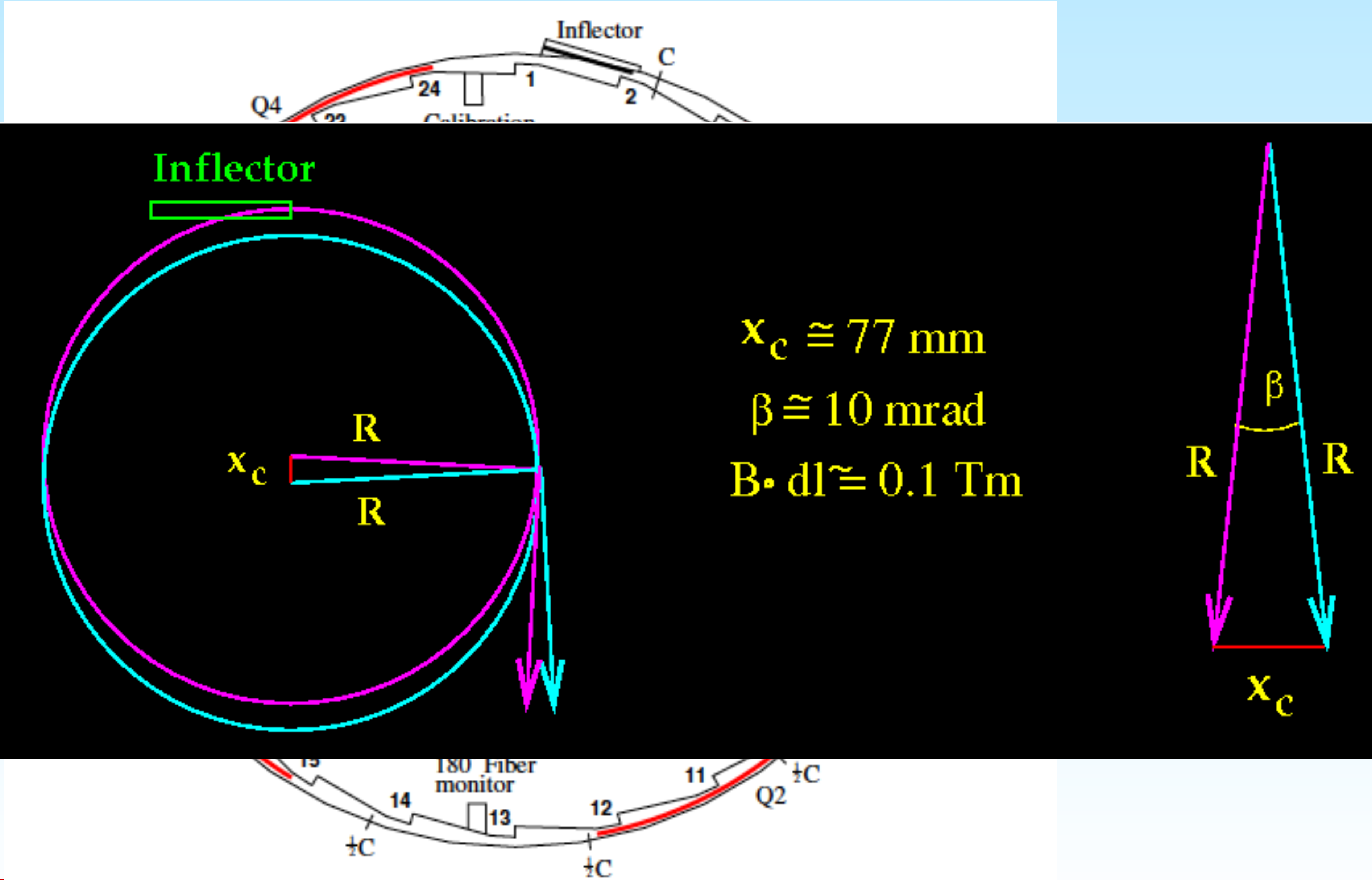


- Beam exits yoke
- Beam through outer coil
- Beam through Inflector
- Beam kicked onto orbit

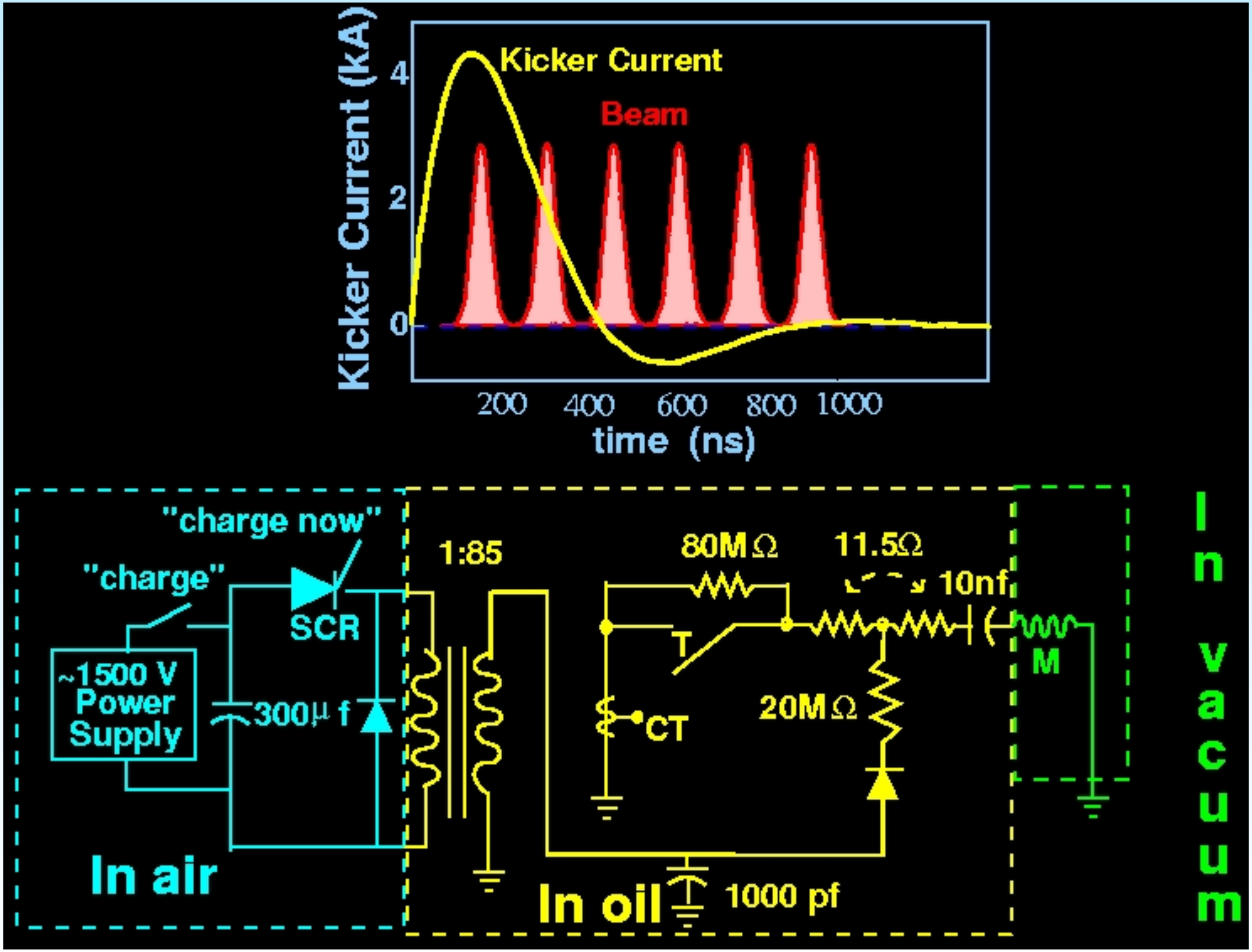
The muon kicker



The muon kicker

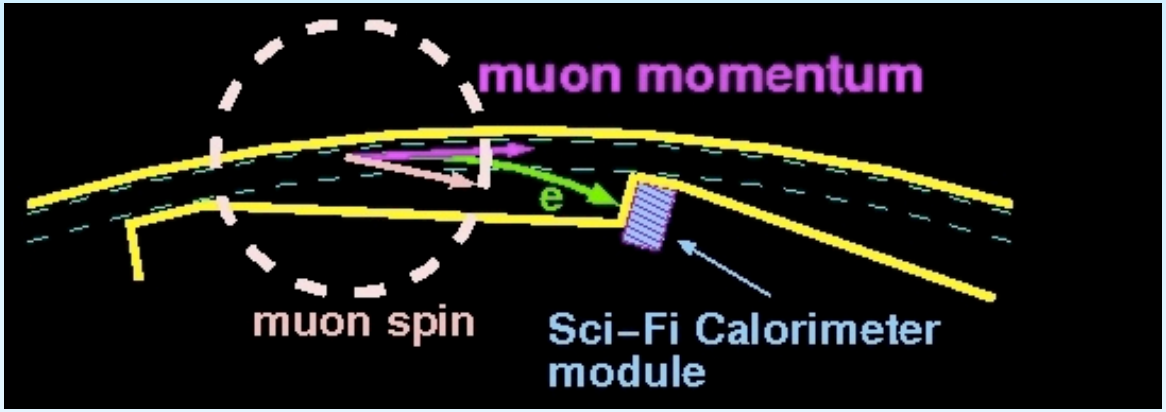


The muon kicker circuit

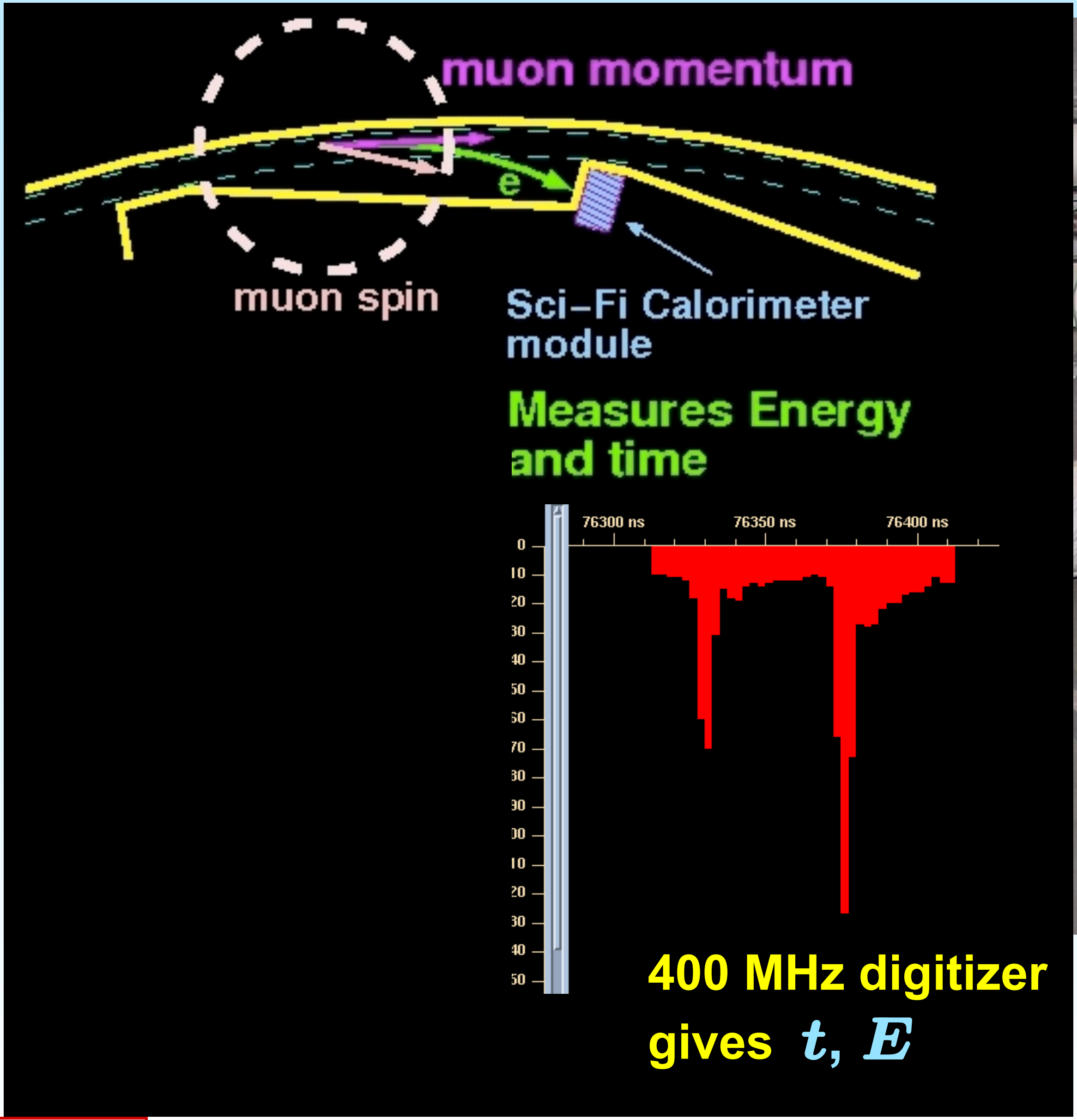


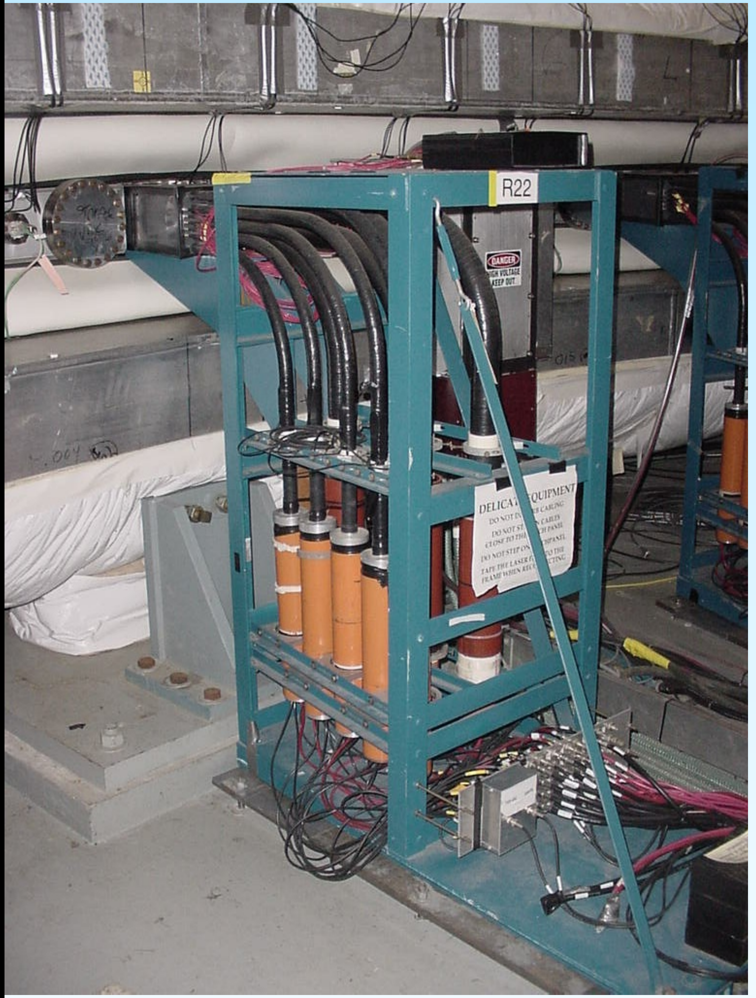
The kicker pulse forming network

- Scalloped vacuum chamber minimizes pre-showering, so the correct energy is measured – improves A, and thus NA^2

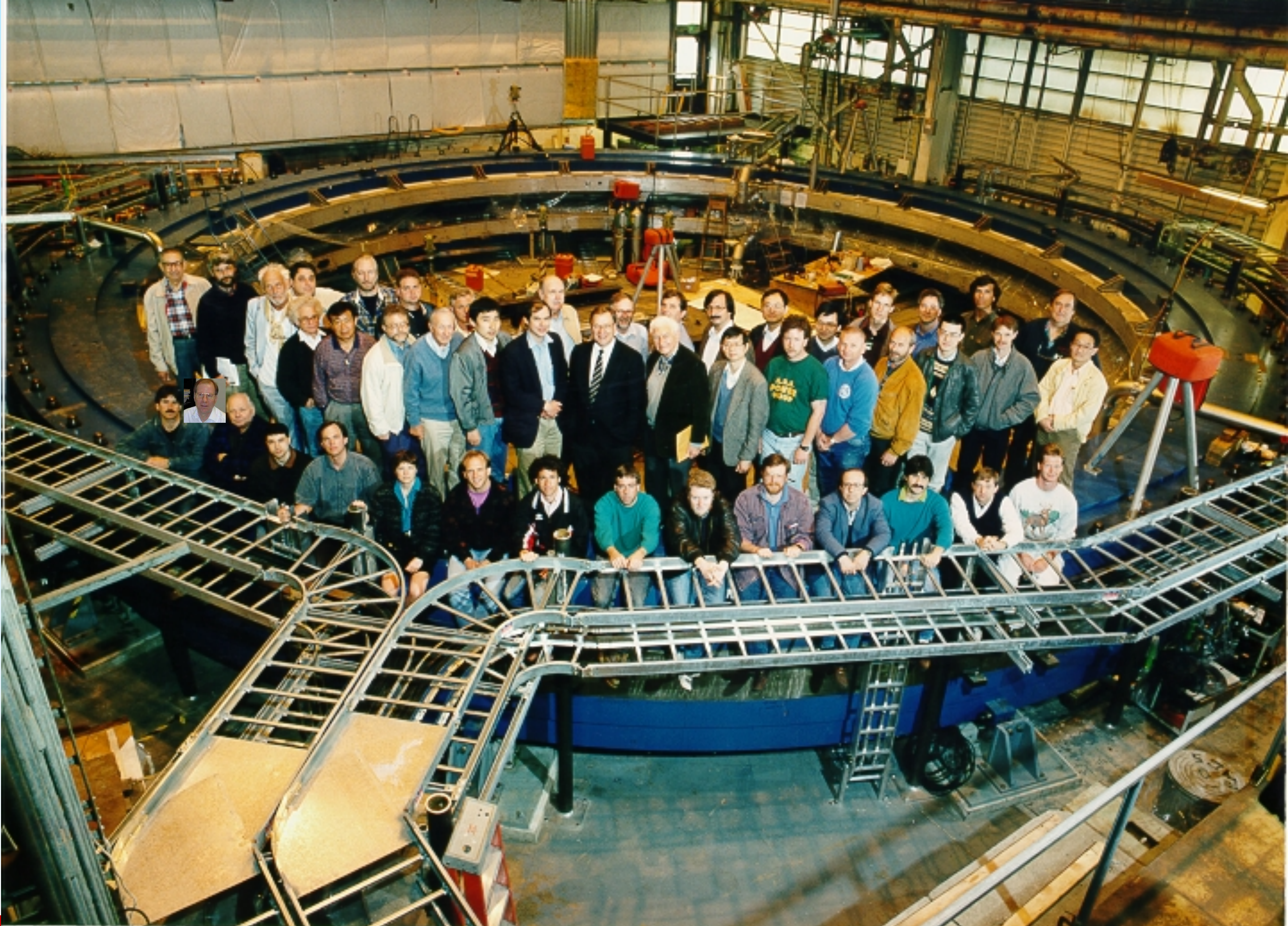


To measure ω_a, we used Pb-scintillating fiber calorimeters.





Count number of e^- with $E_e \geq 1.8$ GeV



To be continued …