Final Muon Cooling with Quadrupole Doublets to Feed a Potato Slicer

Terry Hart, John Acosta, Lucien Cremaldi, Sandra Oliveros, Don Summers
University of Mississippi-Oxford

David Neuffer, Fermilab

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6D Muon Ionization Cooling Channel Status

- **Two 6D channels cool by almost the 10^6 needed by a collider**
  More than enough longitudinally.
  But getting all the way to 0.025 mm-rad transversly is hard.
  3x more cooling plus emittance exchange is needed

	ϵ_x mm	ϵ_y mm	ϵ_z mm	ϵ_{6D} mm^3
Emittance after Phase Rotation	48.6	48.6	17.0	40,200
Helical (IPAC14 -TUPME016)	0.523	0.523	1.54	0.421
Rectlinear (IPAC14 -TUPME020)	0.28	0.28	1.57	0.123
Muon Collider (arXiv:0711.4275)	0.025	0.025	70	0.044

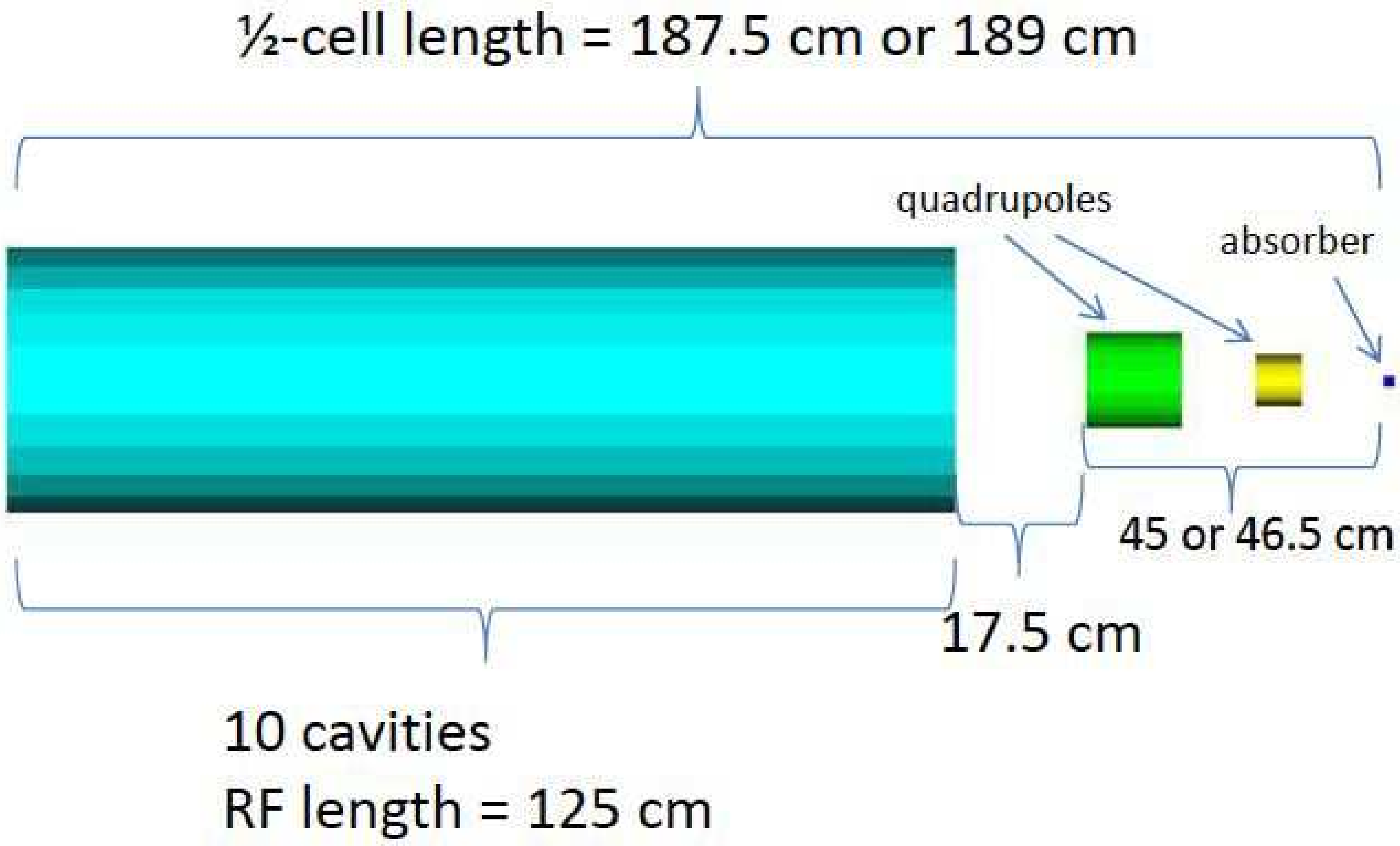
Final Muon Cooling in Three Steps

1. **Use quadrupole doublets for strong focusing. Fields below 14 Tesla.**

2. **Put dense low Z absorber in short low β^* regions.**

3. **Lower β^* as emittance falls, bigger $\beta_{\max}$ OK.**
   $\beta^* \beta_{\max} \propto$ **(focal length)^2**
   $\sigma_x = \sqrt{\epsilon_x^N \beta_x / (\beta\gamma)}$

Quadrupole Doublet Ionization Cooling Half Cell with RF



- **The RF is long to control the longitudinal emittance**

$$\epsilon_{L,N,eq} = \beta_L\, m_e c^2\, \gamma^2 (1-\beta^2/2)/[2 g_L \beta m_\mu c^2 [\ln[A/I]/\beta^2 - 1]]$$

$$\beta_L = \sqrt{\lambda_{\rm rf}\, \beta^3\, \gamma\, m_\mu c^2\, \alpha_p / (2\pi\, eV'\, \cos(\phi_s))} \quad A = 2 m_e c^2\, \gamma^2\, \beta^2$$

Cooling calculations follow D. Neuffer, arXiv:1312.1266

Dense low Z absorbers fit short low β^*_{xy} regions

- **Muon Equilibrium emittance at 200 MeV/c**

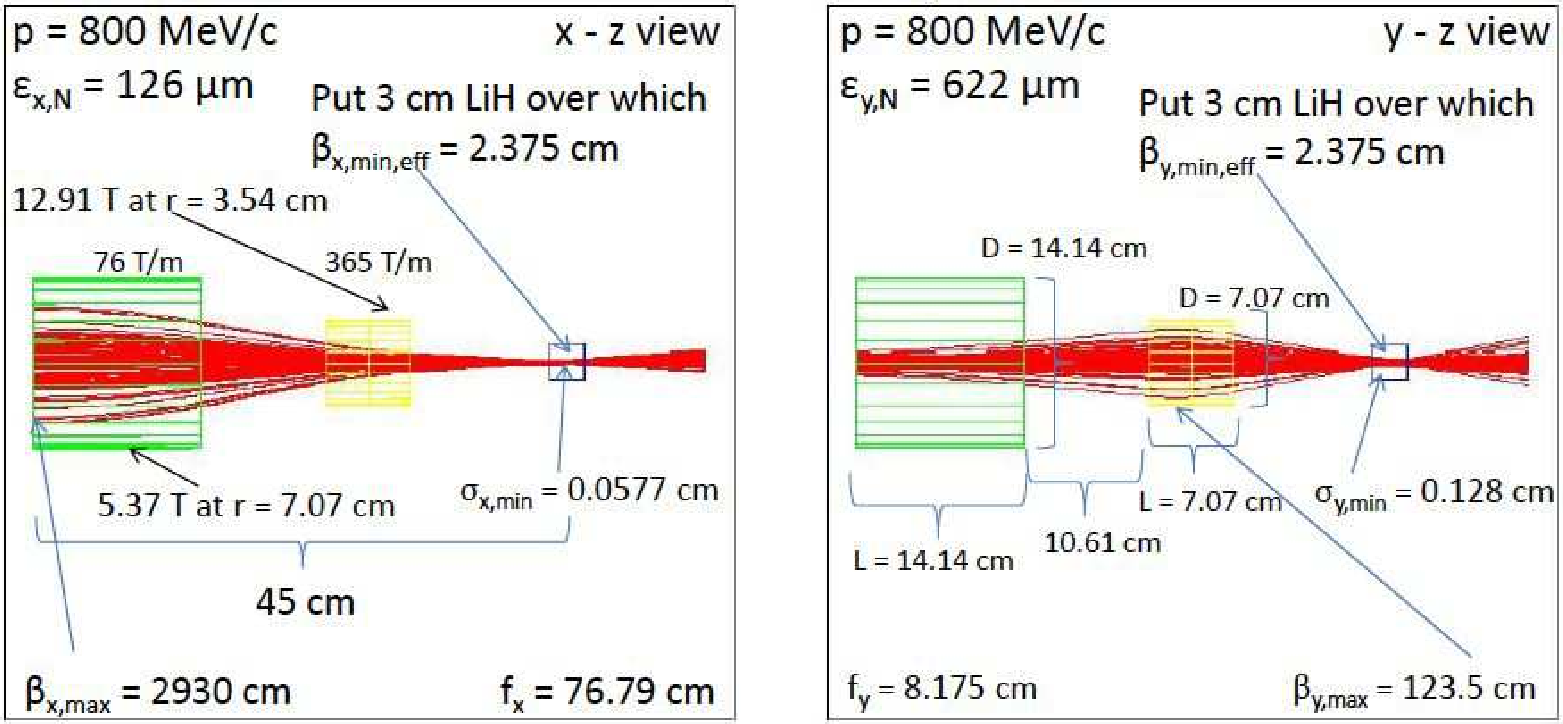
$$\epsilon_\perp = \beta^* E_s^2 / (2 g_x \beta m_\mu c^2 (dE/ds) L_R)$$

$\beta^* = 1$ cm, $E_s = 13.6$ MeV, $g_x = 1$, β = v/c = 0.88

$m_\mu c^2 = 105.7$ MeV, L_R = radiation length

Material	Density g/cm^3	L_R cm	dE/ds MeV/cm	$\epsilon_\perp$ (mm - rad) (equilibrium)
H_2 gas	0.000084	750,000	0.00037	0.036
Li H	0.82	97	1.73	0.059
Be	1.85	35.3	3.24	0.087
B_4C	2.52	19.9	4.57	0.109
Diamond	3.52	12.1	6.70	0.123
Be O	3.01	13.7	5.51	0.132

Stage 1: Quadrupole Doublet Cooling with $\beta^*_{xy} = 2$ cm

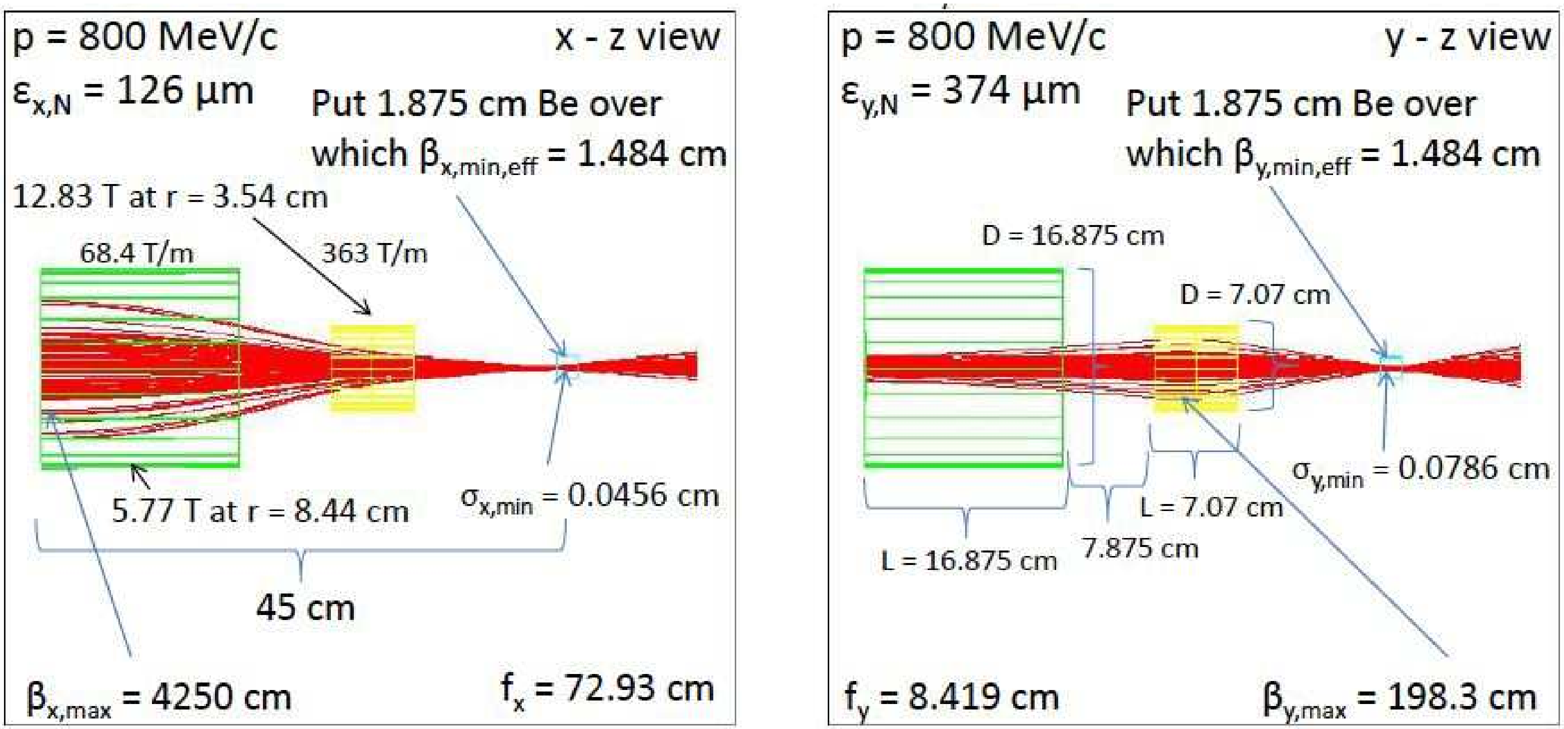


- $\beta^* \beta_{\rm max} \propto$ (focal length)^2 $\quad \sigma_x = \sqrt{\epsilon_x^N \beta_x / (\beta\,\gamma)}$
- Input a flat muon beam

  Option 1: Slice and recombine to get a flat beam

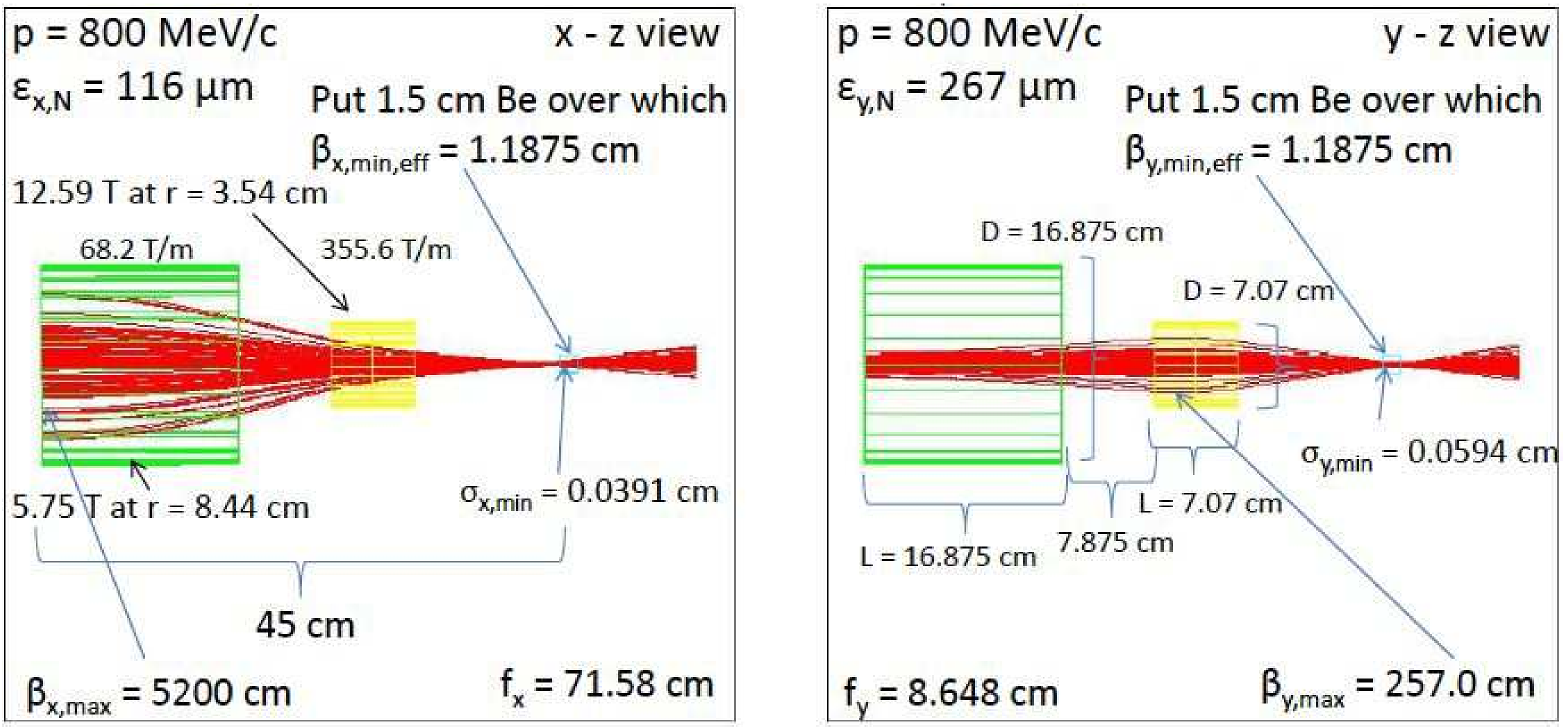
  Option 2: Round to flat skew quadrupole triplet works, if solenoid/absorber can spin beam, arXiv:1504.03972
- Quadrupole Transmission = $\pm 3.2\sigma$. $\pm 4\sigma$ would be better.
- Quadrupole Separation = 10.61 cm. More would be better.

Stage 2: Quadrupole Doublet Cooling with $\beta^*_{xy} = 1.25$ cm



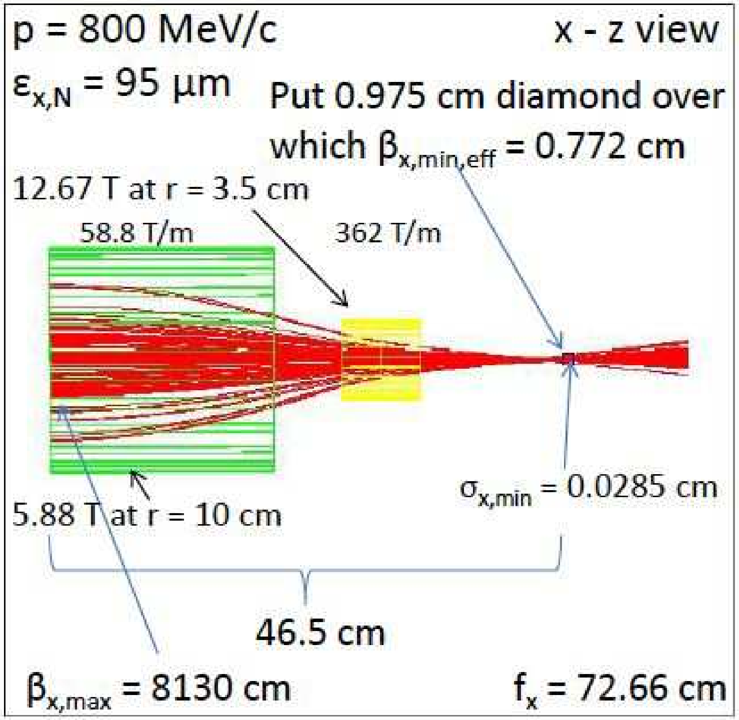
- **Quadrupole Transmission = $\pm 3.2\sigma$. Beryllium absorber.**

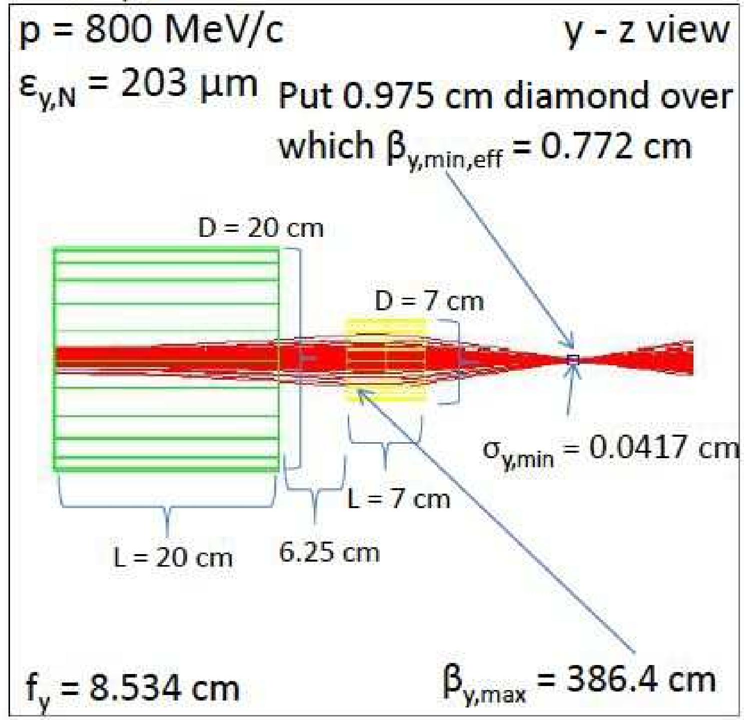
Stage 3: Quadrupole Doublet Cooling with $\beta^*_{xy} = 1$ cm



- **Quadrupole Transmission = $\pm 3.0\sigma$. Beryllium absorber.**

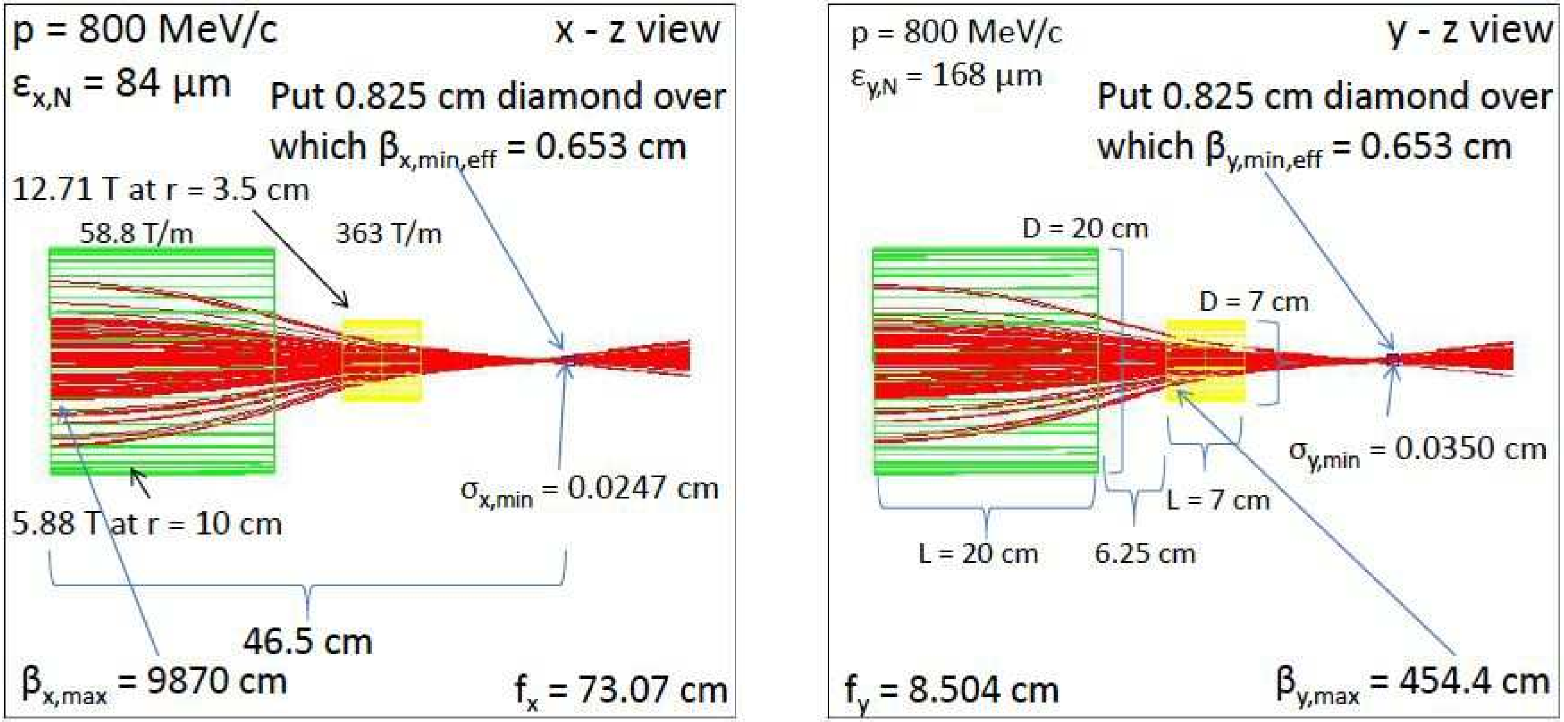
Stage 4: Quadrupole Doublet Cooling with $\beta^*_{xy} = 0.65$ cm





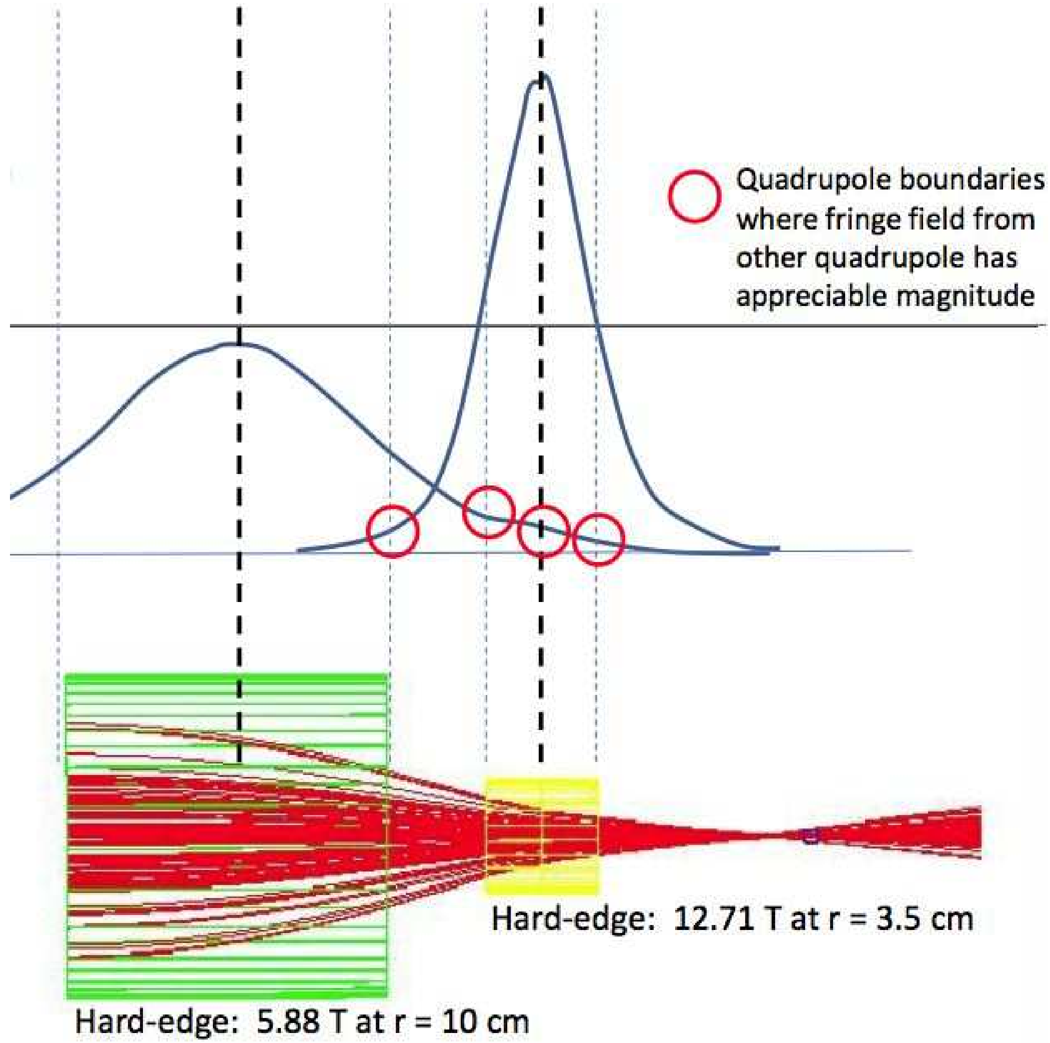
- **Quadrupole Transmission = $\pm 3.1\sigma$. Diamond absorber.**

Stage 5: Quadrupole Doublet Cooling with $\beta^*_{xy} = 0.55$ cm



- **Quadrupole Transmission $= \pm 3.0\sigma$. Diamond absorber.**

Fringe Fields

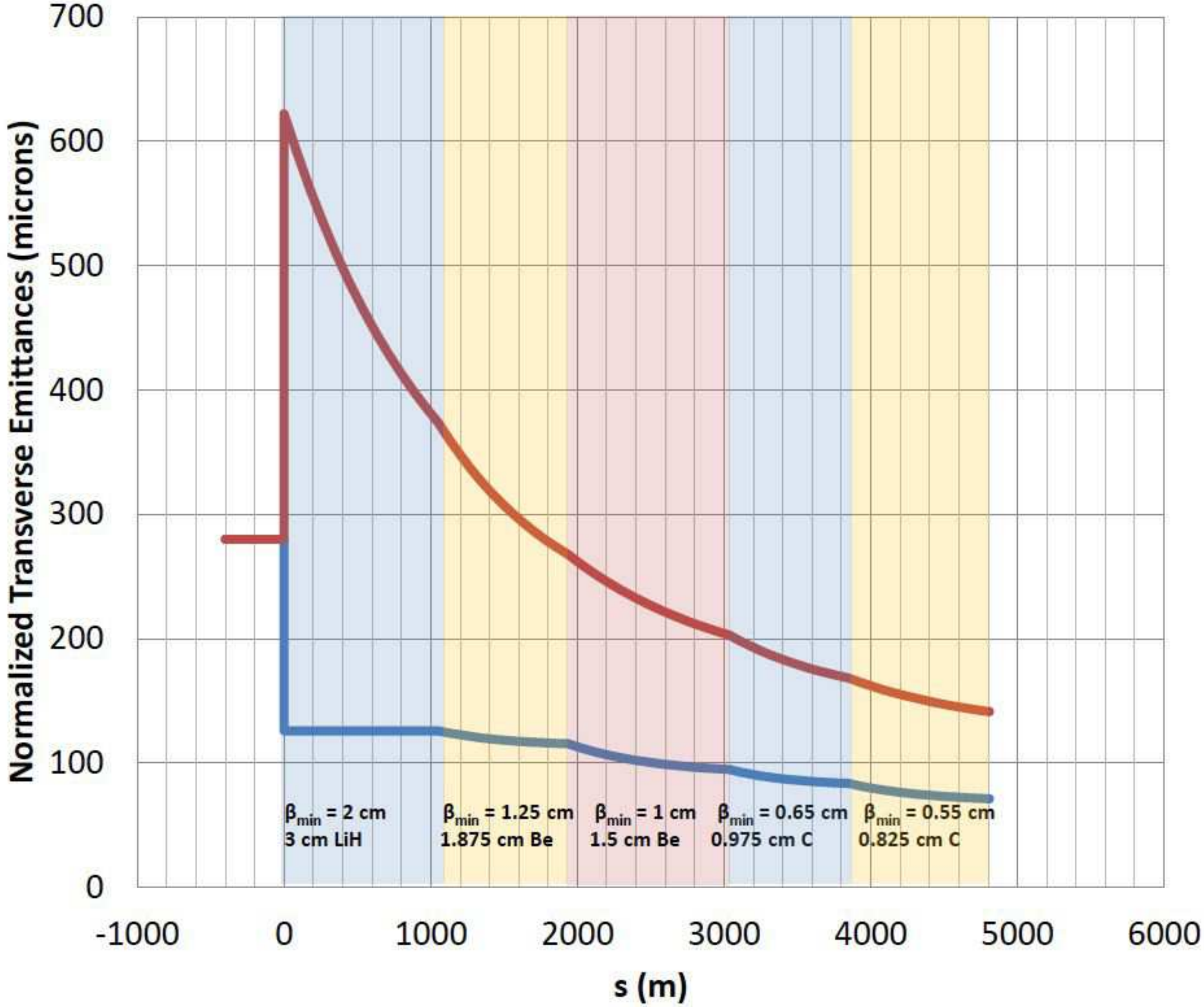


- **Worst case: 6% amplitude in neighboring quad center. Johnstone, Berz, Errede, Makino, NIM A519 (2004) 282**

Quadrupole Doublet Ionization Cooling Parameters

Stage	1	2	3	4	5
No. of Cells	281	235	294	215	254
Cell Length (m)	3.75	3.75	3.75	3.78	3.78
Stage Length (m)	1052	882	1102	812	960
Doublet Length (cm)	45	45	45	46.5	46.5
RF Phase ϕ_0	4.72^0	5.63^0	4.50^0	6.16^0	5.21^0
Wedge Angle	68.0^0	45.8^0	37.3^0	24.7^0	21.0^0
β^*_{xy} (cm)	2.0	1.25	1.0	0.65	0.55
$\beta_x^{\max}$ (cm)	2930	4250	5200	8130	9870
Absorber	LiH	Be	Be	C	C
Absorber Length (cm)	3.0	1.875	1.5	0.975	0.825
Quad Doublet Bore σ	± 3.2	± 3.2	± 3.0	± 3.1	± 3.0
Quad Doublet Gap (cm)	10.61	7.875	7.875	6.25	6.25
Final ϵ_x (mm-rad)	0.126	0.116	0.095	0.084	0.071
Final ϵ_y (mm-rad)	0.374	0.268	0.203	0.168	0.141
Final ϵ_z (mm-rad)	2.058	2.273	2.347	2.400	2.418

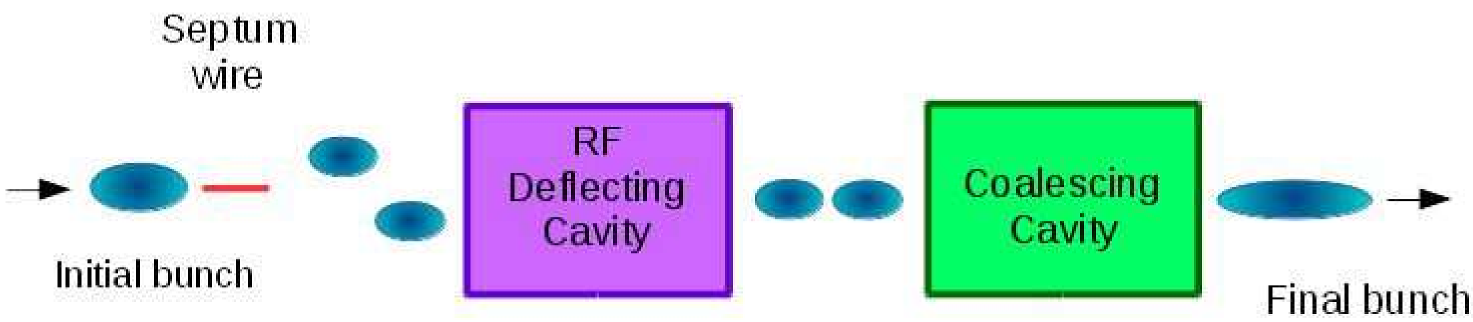
Flat Beam Quadrupole Doublet Ionization Cooling Summary



- $\epsilon_{xyz} =$ **(0.0714, 0.141, 2.418) mm.** $\epsilon_{6D} = 0.024 < 0.044\,\text{mm}^3$
  6D $\mu^+\mu^-$ collider cooling in hand. Transmission needs work.
  38% muon survival. More would be better.

Transverse to Longitudinal Emittance Exchange Introduction

- ϵ_{xyz}(0.0714, 0.141, 2.418) $\rightarrow$ (0.025, 0.025, 70) mm-rad



- Slice the bunch into 17 parts with septa (2 parts in cartoon). Like Fermilab Tevatron fixed target extraction.
- Form a 3.7 m long bunch train with RF deflector cavities. Like CERN tests for CLIC e^+e^-.
- Snap bunch coalesce muons into 1 bunch with RF in a ring. Like the Tevatron collider.

Septa slice wide beams with little loss

- Don Edwards and Mike Syphers, page 126
  "An Introduction to the Physics of High Energy Accelerators"
  0.1mm thick electrostatic septa, magnetic septa, Lambertson...
  Just like Tevatron extraction for fixed target

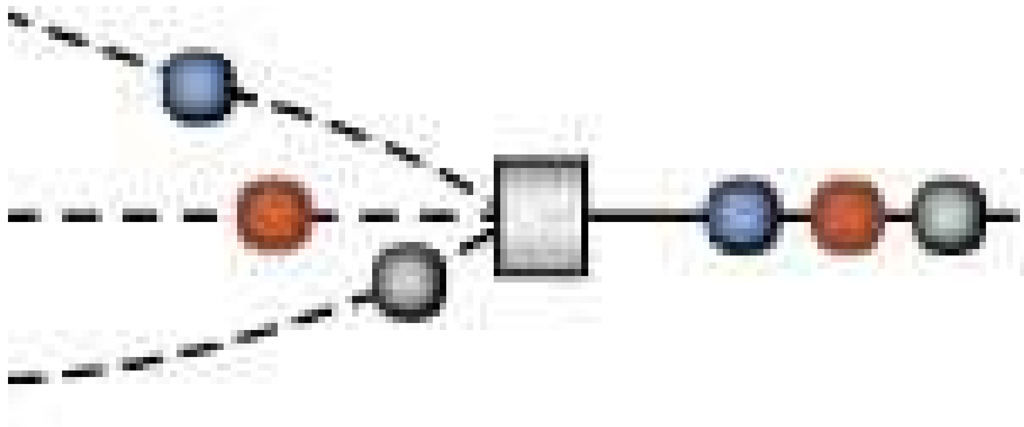
- For a 300 MeV/c beam.
  Use 2 vertical and 14 horizontal septa to slice into 17 parts.

$$x_{\rm max} = \sqrt{\frac{\epsilon_{N,x}\,\beta_x}{\beta\,\gamma}} = \sqrt{\frac{(0.100\,{\rm mm})(8,000\,{\rm mm})}{2.84}} = 17\,{\rm mm} \tag{1}$$

$$\text{Loss} = \frac{4\sqrt{2}\;w}{x_{\rm max}} = \frac{4\sqrt{2}\times 0.1\,{\rm mm}}{17\,{\rm mm}} = 0.03 \tag{2}$$

- The loss due to slicing is low.

Interleave 3.7 m long bunch train with 10 RF deflector cavities

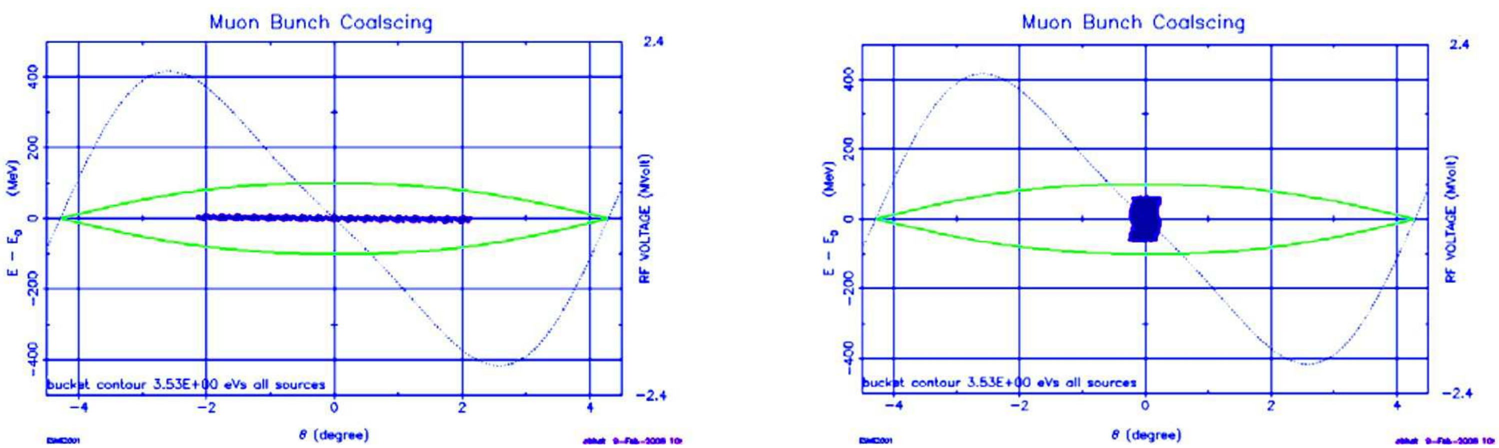


Number of Trains Interleaving	Number of RF Cavities	RF MHZ	RF Wave-length	Output Spacing in Wavelengths	Output Bunch Spacing
17 $\rightarrow$ 6	6	731	410 mm	9/4	923 mm
6 $\rightarrow$ 2	3	487	616 mm	3/4	462 mm
2 $\rightarrow$ 1	1	650	462 mm	1/2	231 mm

- Kick 4x greater than 300 MeV/c rms beam divergence.
  ϵ = 0.025 mm-rad, $\beta_\perp$ = 8000 mm
  4 $\sqrt{\epsilon/(\gamma\beta\beta_\perp)}$ = 4.2 mrad, which matches CLIC tests
  The $\pm 4\sigma$ beam diameter is $8\sqrt{\epsilon\beta_\perp/(\gamma\beta)}$ = 67 mm

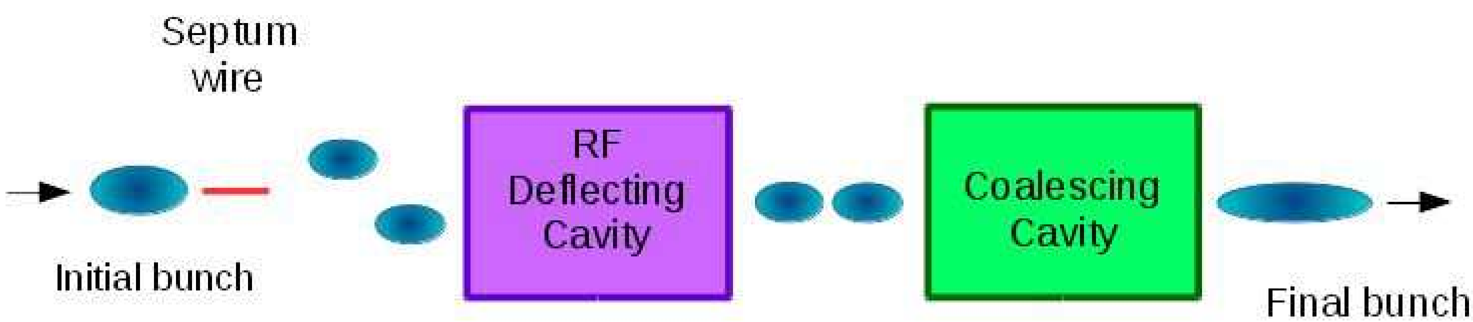
Snap bunch coalesce 17 bunches into one longer bunch

- **Snap bunch coalescence was used at the Tevatron collider.**
- **Inject a 3.7 m long train with 17 bunches into a 21 GeV ring**
  Long wavelength RF gives each bunch a different energy
  Bunches merge during 1/4 synchrotron period
  Capture with short λ RF. Muon decay loss is 13% in 55 μs.
  Chandra Bhat: Packing fraction can approach 87%
  2.418 mm x 17 / 0.87 = 47 mm. Better than 70 mm needed, but one expects some dilution.
- **ESME runs: PAC07-THPMN095, "Muon Bunch Coalescing" by Johnson, Ankenbrandt, Bhat, Popovic, Bogacz, Derbenev**
  http://accelconf.web.cern.ch/AccelConf/p07/PAPERS/THPMN095.PDF



Quadrupole Doublet Cooling & Emittance Exchange Summary

- **Final cooling: Dense low Z absorbers fit short low β^*_{xy} regions**
  6 and 13 Tesla quadrupole doublets provide low β^*_{xy} regions
  Flat muon beam fits the quadrupole doublets and achieves:
  $\epsilon_{xyz} = (0.0714, 0.141, 2.418)$ **mm.** $\epsilon_{6D} = 0.024 < 0.044\,\mathrm{mm}^3$
  6D $\mu^+\mu^-$ collider cooling in hand. Transmission is only 3.1σ.
  38% muon survival. Large p compaction rings may help.



- **Slice the bunch into 17 parts with septa (2 parts in cartoon).**
- **Form a 3.7 m long bunch train with 10 RF deflector cavities.**
- **Snap bunch coalesce muons into 1 bunch with RF in a ring.**
- ϵ_{xyz}**(0.0714, 0.141, 2.418) $\rightarrow$ (0.025, 0.025, 70) mm-rad**