EMC Effect in Electron and Neutrino Scattering

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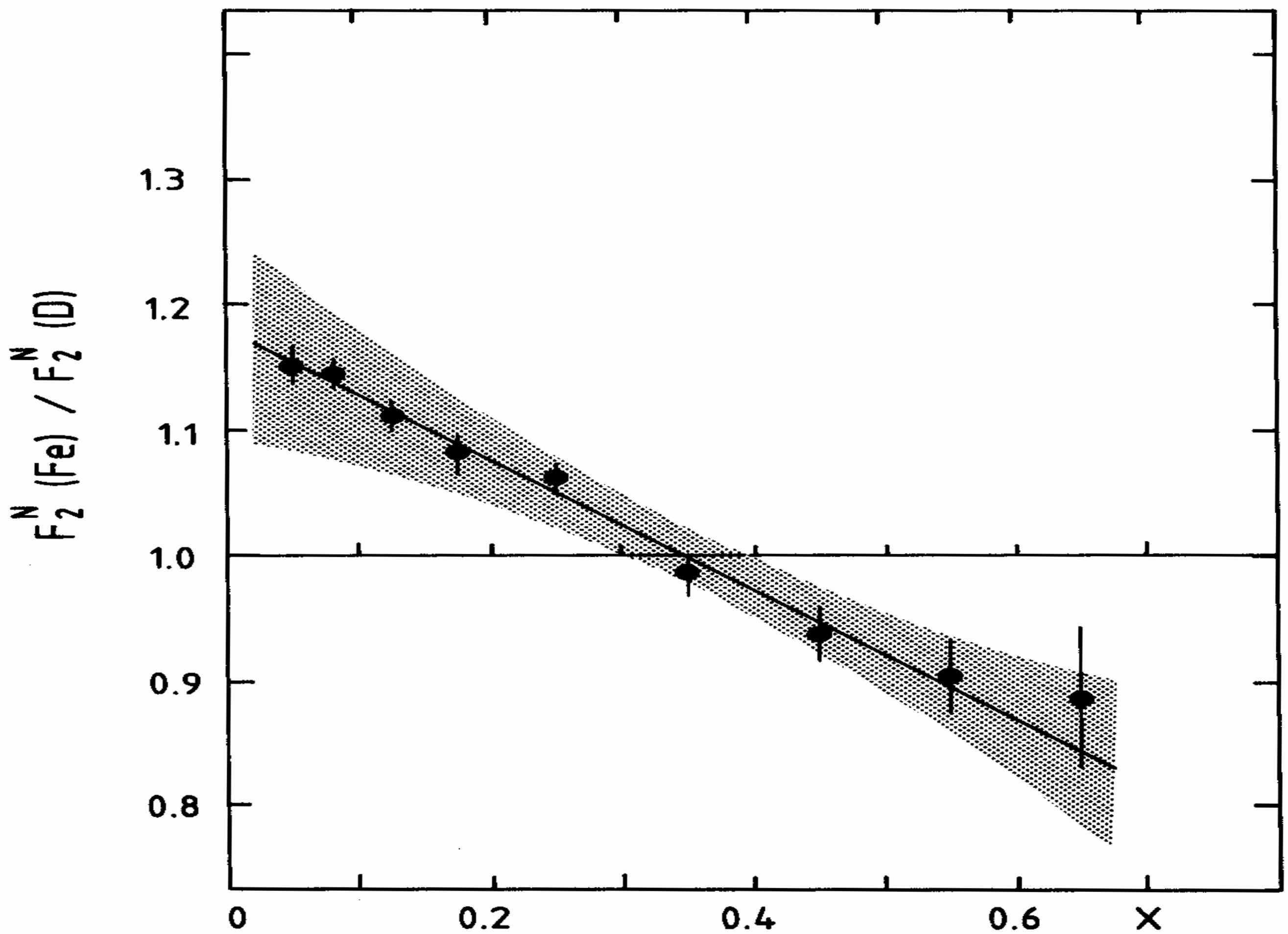
Theme

- Are nucleon properties modified by the nuclear medium?
  - Of fundamental importance.
  - Remains an open question.
- Areas where medium modifications seem important:
  - Quenching of g_A in-medium
  - Nuclear magnetic moments
  - Nuclear Form Factors (e.g.^{4}He)
- Most importantly nuclear structure functions, this is the EMC effect.

EMC Effect



- J. J. Aubert *et al.* [European Muon Collaboration], Phys. Lett. B **123**, 275 (1983).

Hadronic Tensor

- Bjorken limit and Callen-Gross relations (e.g. $F_2 = 2x\, F_1$)

  - For $J = \frac{1}{2}$ target

$$W_{\mu\nu} = \left(g_{\mu\nu}\frac{P\cdot q}{q^2} + \frac{P_\mu P_\nu}{P\cdot q}\right) F_2(x_A, Q^2) - i\,\frac{\varepsilon_{\mu\nu\lambda\sigma} q^\lambda P^\sigma}{\nu} F_3(x_A, Q^2)$$

  - For arbitrary J ($2\lfloor J \rfloor + 2$ structure functions)

$$W^H_{\mu\nu} = \left(g_{\mu\nu}\frac{P\cdot q}{q^2} + \frac{p_\mu p_\nu}{P\cdot q}\right) F_2^{JH}(x_A, Q^2) - i\,\frac{\varepsilon_{\mu\nu\lambda\sigma} q^\lambda P^\sigma}{\nu} F_3^{JH}(x_A, Q^2)$$

$$F_2^{JH} = F_2^{J\,-H}, \quad F_3^{JH} = F_3^{J\,-H}, \quad \left(q_s^{JH} = q_{-s}^{J\,-H}\right)$$

Parton Model Structure Functions

- Parton model expressions

$$F_1^{W^+\,JH} = \bar{u}^{JH} + d^{JH} + s^{JH} + \bar{c}^{JH}$$

$$F_3^{W^+\,JH} = -\bar{u}^{JH} + d^{JH} + s^{JH} - \bar{c}^{JH}$$

$$F_i^{W^+}(x) \equiv \frac{1}{2J+1} \sum_{H=-J}^{J} F_i^{W^+\,JH}(x).$$

- $\lfloor J \rfloor + 1$ quark distributions for nucleus spin J.
- Twist-2 in QCD

$$F_2^{W^+}(x) = C_q(x,\alpha_s) \otimes [\bar{u}(x) + d(x) + s(x) + \bar{c}(x)]$$

Calculation

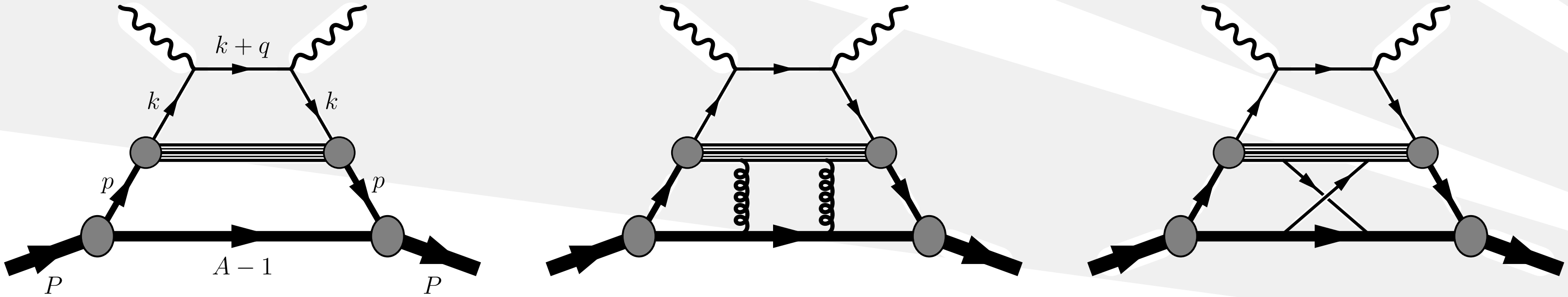
- Finite Nuclei quark distributions

$$q_A^{JH}(x_A) = \frac{P^+}{A}\int \frac{d\xi^-}{2\pi} e^{iP^+ x_A \xi^-/A} \langle A, P, H|\overline{\psi}(0)\,\gamma^+\,\psi(\xi^-)|A, P, H\rangle .$$

- Using Convolution formalism

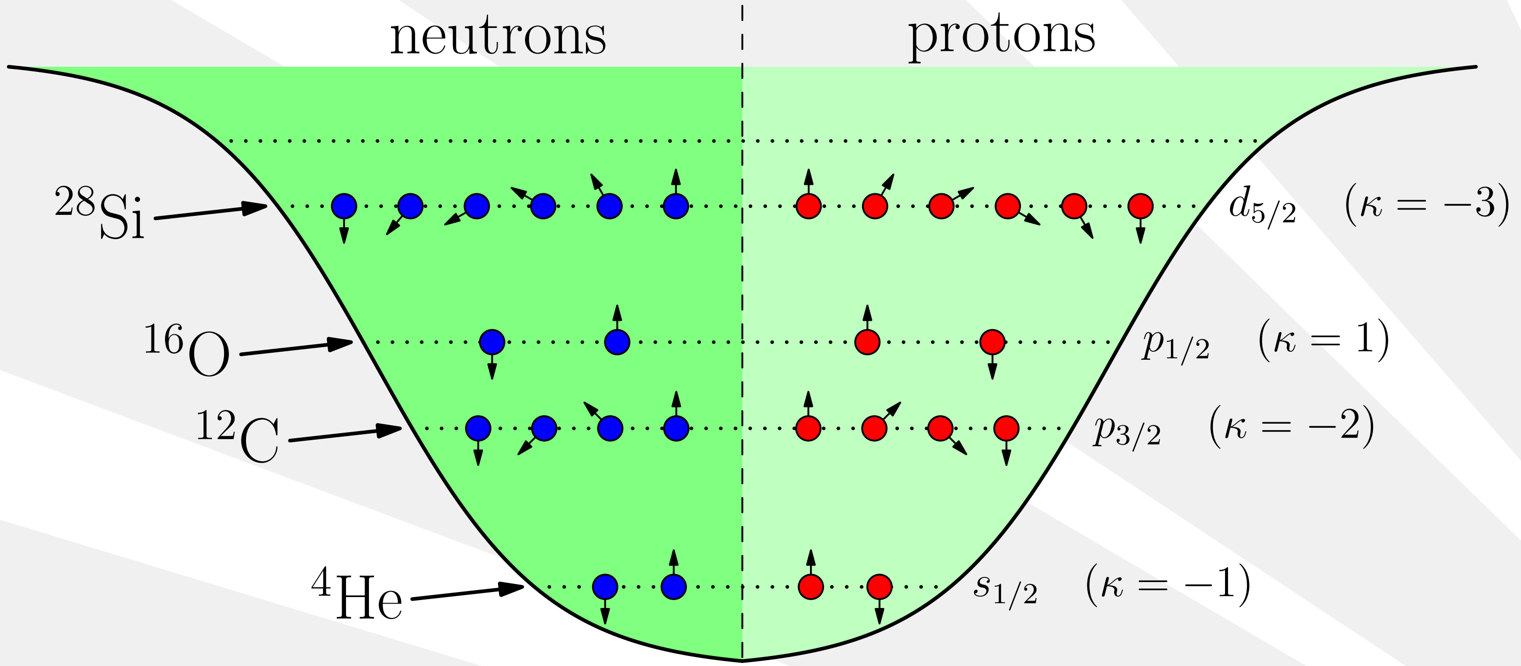
$$q_A^{JH}(x_A) = \sum_{\kappa,m}\int dy_A \int dx\ \delta(x_A - y_A\, x)\ f_{\kappa,m}^{(JH)}(y_A)\ q_\kappa(x)\,.$$

- Diagrammatically



(a) (b) (c)

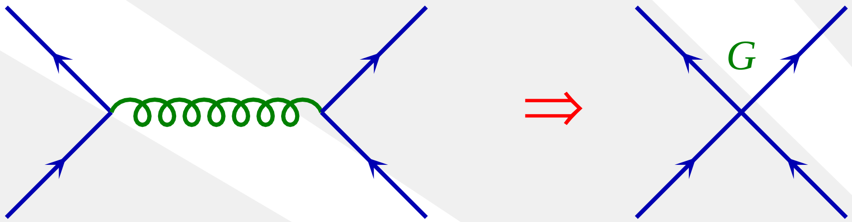
Shell Model



$$q_A^{JH}(x_A) = \sum_{\kappa,m} \int dy_A \int dx \; \delta(x_A - y_A\, x)\, f_{\kappa,m}^{(JH)}(y_A)\; q_\kappa(x)\,.$$

Nambu–Jona-Lasinio Model

- Low energy effective theory

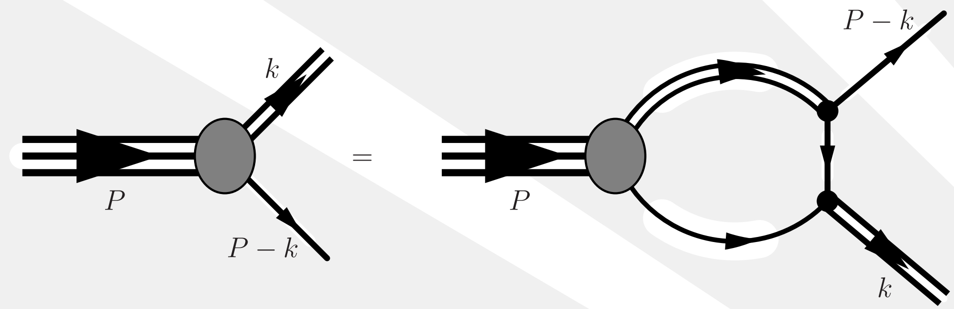


- Investigate the role of quark degrees of freedom.
- Lagrangian has same symmetries as QCD:
  - Importantly chiral symmetry and CSB,
    $\longrightarrow$ Dynamically generated quark masses,
    $\longrightarrow$ Non-zero chiral condensate.
- Lagrangian ($\Gamma =$ Dirac, colour, isospin matrices)

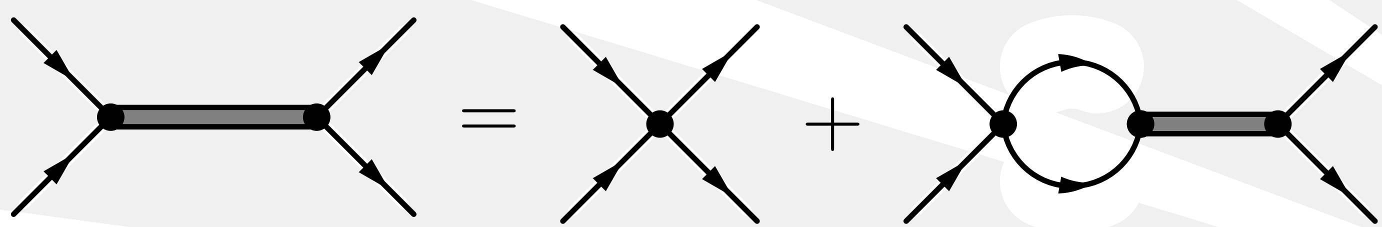
$$\mathcal{L}_{NJL} = \overline{\psi}\,(i\,\partial\!\!\!/ - m)\,\psi + G\left(\overline{\psi}\Gamma\psi\right)^2.$$

Nucleon in the NJL model

- Nucleon approximated as quark-diquark bound state.
- Use relativistic Faddeev approach:



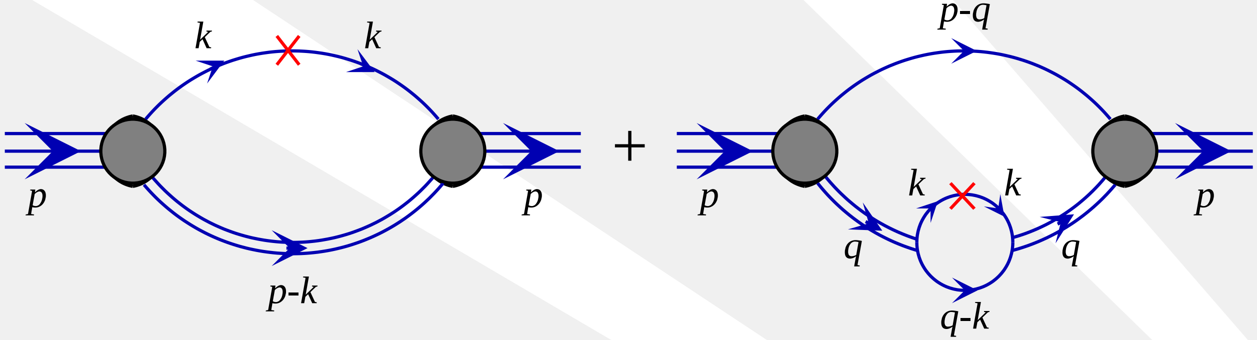
- Diquark - bound state of two quarks:
- Solve Bethe-Salpeter equation for diquark.



- We include scalar and axial-vector diquarks.

Nucleon quark distributions

- Associated with a Feynman diagram calculation.



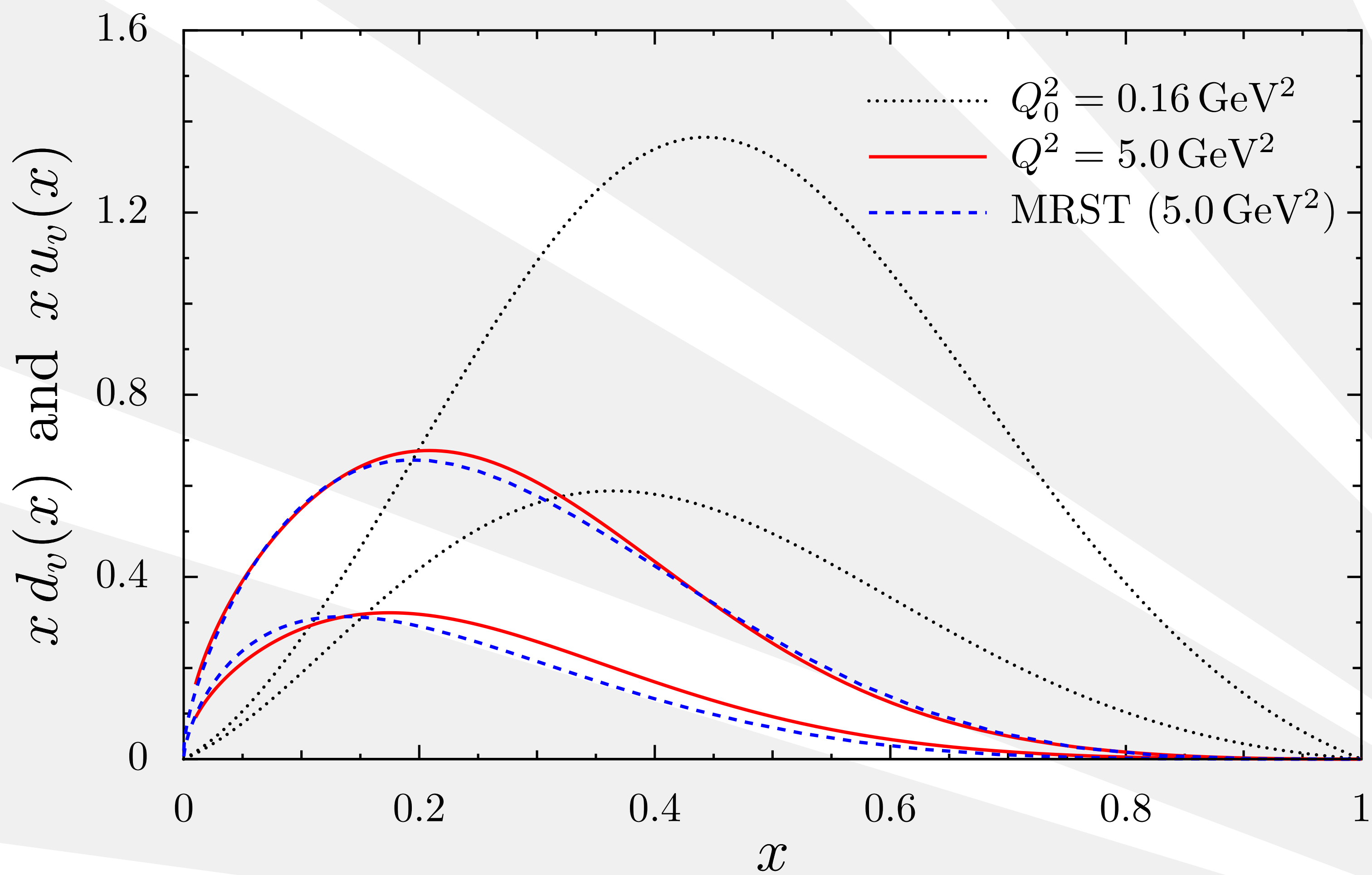
- $q(x) \rightarrow \mathsf{X} = \gamma^+ \, \delta(x - \frac{k^+}{p^+})$

- Covariant and gives correct support.

- Formalism satisfies baryon and momentum sum rules.

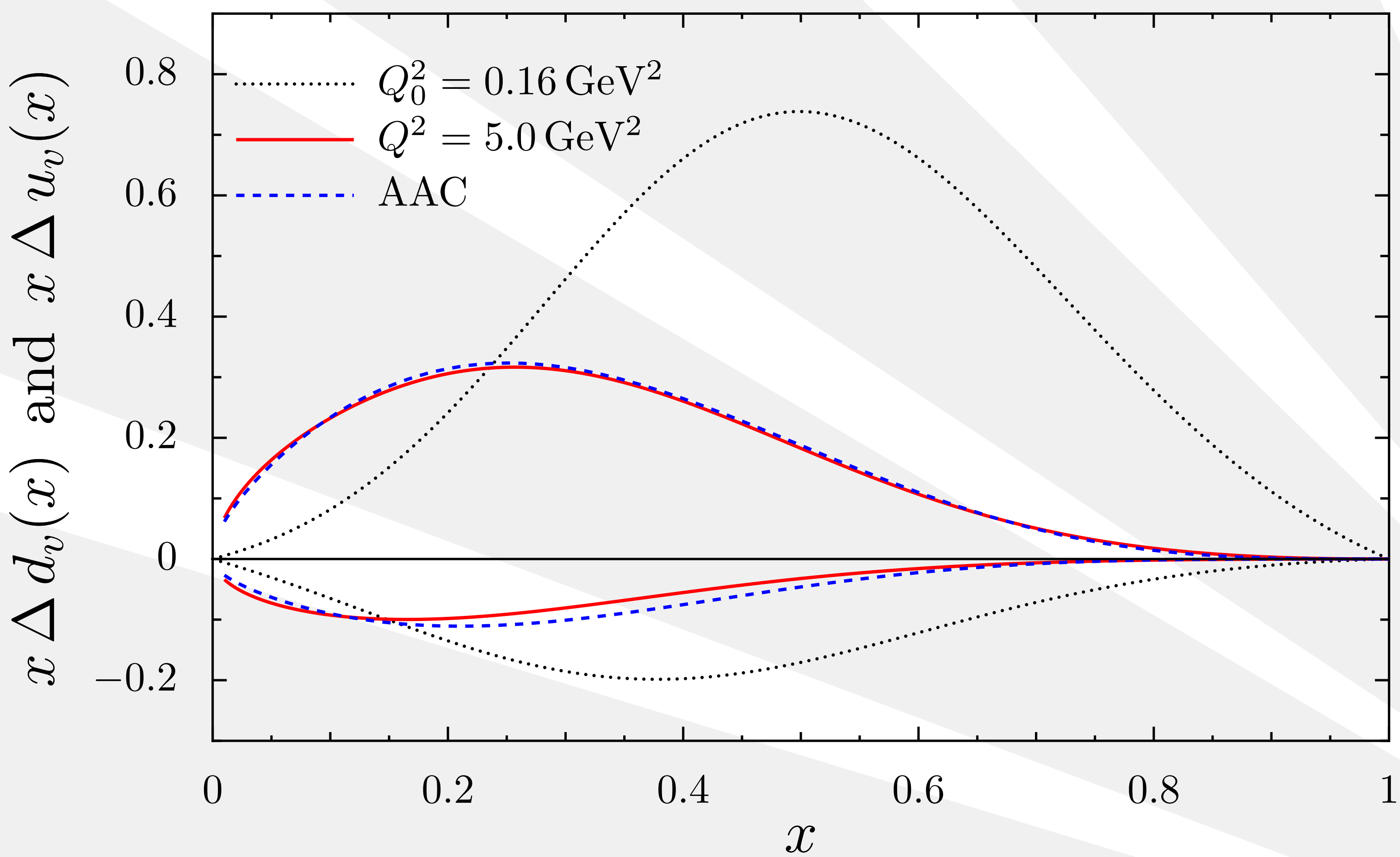
$$\int_0^1 dx\, q(x) = N_q, \qquad \int_0^1 dx\, x\, [u(x) + d(x)] = 1.$$

$u_v(x)$ and $d_v(x)$ distributions



- MRST, Phys. Lett. B **531**, 216 (2002).

$\Delta u_v(x)$ and $\Delta d_v(x)$ distributions



- M. Hirai, S. Kumano and N. Saito, Phys. Rev. D **69**, 054021 (2004).

NJL Model at Finite Density

- Re-calculate diagrams

$$\mathcal{L} = \overline{\psi}\,(i\,\partial\!\!\!/ - M^* - V\!\!\!/\,)\,\psi + \mathcal{L}'_I$$

- Equivalent to:
  - ❖ Scalar field: via effective masses
  - ❖ Vector field: via scale transformation

- Nuclear Matter $(\varepsilon_F = E_F + 3V_0)$

$$q_A\,(x_A) = \frac{\varepsilon_F}{E_F}\, q_{A0}\left(\frac{\varepsilon_F}{E_F} x_A - \frac{V_0}{E_F}\right)$$

- Finite Nuclei $(\hat{M}_{N\kappa} = \overline{M}_N - 3V_\kappa)$

$$q_{A,\kappa}(x_A) = \frac{\overline{M}_N}{\hat{M}_N}\, q_{A0,\kappa}\left(\frac{\overline{M}_{N\kappa}}{\hat{M}_{N\kappa}} x_A - \frac{V_\kappa}{\hat{M}_{N\kappa}}\right)$$

Nucleon distribution functions

- Definition

$$f_{\kappa m}(y_A) = \frac{\sqrt{2}\,\overline{M}_N}{A} \int \frac{d^3p}{(2\pi)^3} \delta\left(p^3 + \varepsilon_\kappa - \overline{M}_N\, y_A\right) \overline{\Psi}_{\kappa m}(\vec{p})\ \gamma^+\ \Psi_{\kappa\, m}(\vec{p})\,,$$

- Central Potential Dirac eigenfunctions

$$\Psi_{\kappa m}(\vec{p}) = (-i)^\ell \begin{bmatrix} F_\kappa(p)\,\Omega_{\kappa m}(\theta,\phi) \\ -G_\kappa(p)\,\Omega_{-\kappa m}(\theta,\phi) \end{bmatrix},$$

  - Dirac Equation

$$\left[-i\,\vec{\alpha}\cdot\vec{\nabla} + \beta\,[M(r) - V_s(r)] + V_v(r)\right] \psi_\kappa(r) = \varepsilon_\kappa\,\psi_\kappa(r)$$

Nucleon distributions: Results

- Spin-independent nucleon distribution

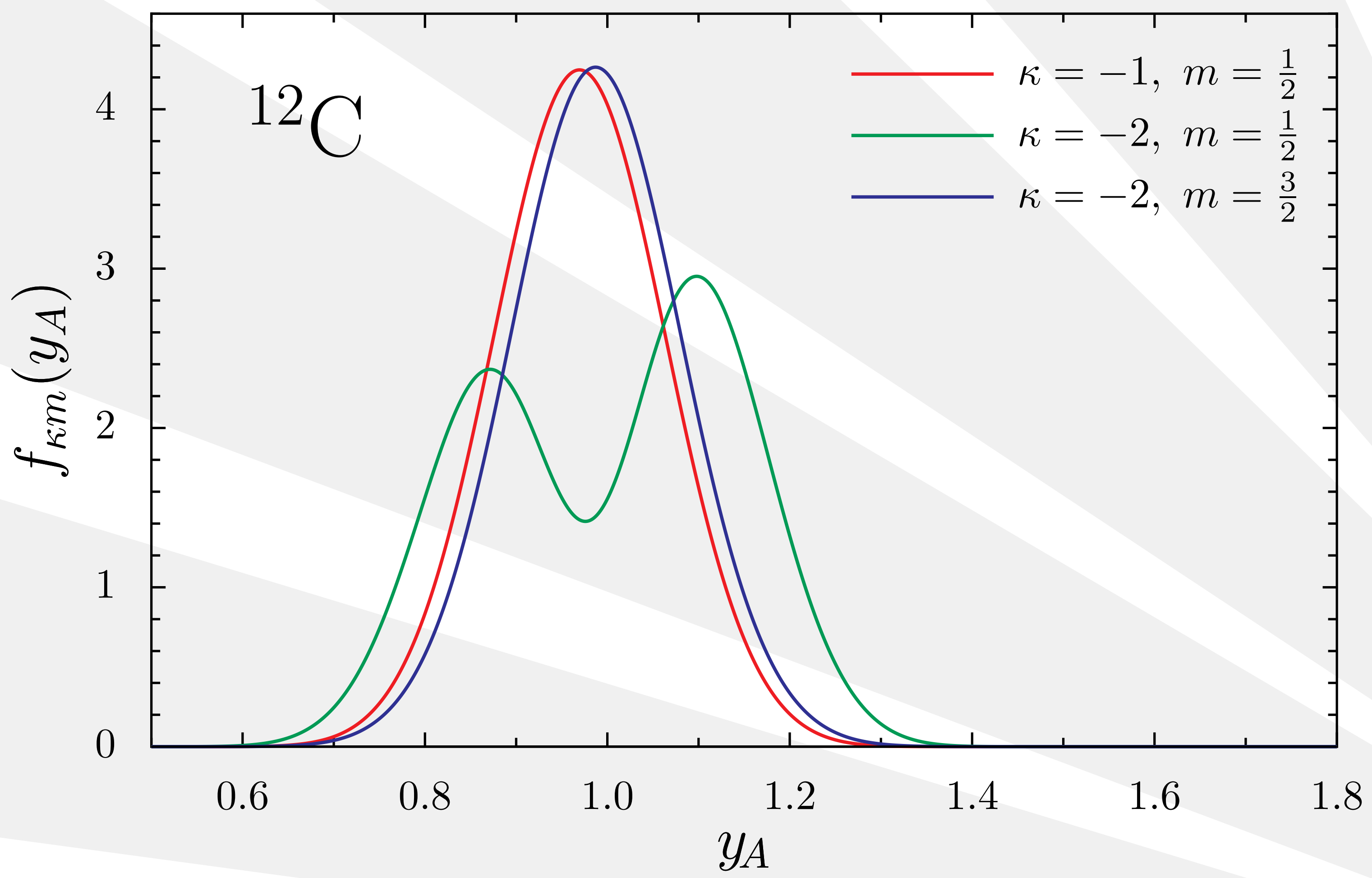
$$f_{\kappa,m}(y_A)=\sum_{k=0,2,\dots,2j}(-1)^{j-m}\sqrt{2k+1}\begin{pmatrix} j & j & k \\ m & -m & 0 \end{pmatrix}$$
$$(-1)^{j+\frac{1}{2}}(2j+1)(2\ell+1)\sqrt{2k+1}\begin{pmatrix} \ell & k & \ell \\ 0 & 0 & 0 \end{pmatrix}\begin{Bmatrix} \ell & k & \ell \\ j & s & j \end{Bmatrix}$$
$$\frac{\overline{M}_N}{16\pi^3}\int_\Lambda^\infty dp\, p\left[F_\kappa(p)^2+G_\kappa(p)^2+\frac{2}{p}\left(\varepsilon_k-\overline{M}_N\, y_A\right)F_\kappa(p)G_\kappa(p)\right] P_k\left(\frac{\overline{M}_N\, y_A-\varepsilon_\lambda}{p}\right)$$

  - ❖ $\Lambda = |\overline{M}_N\, y_A - \varepsilon_\kappa|$

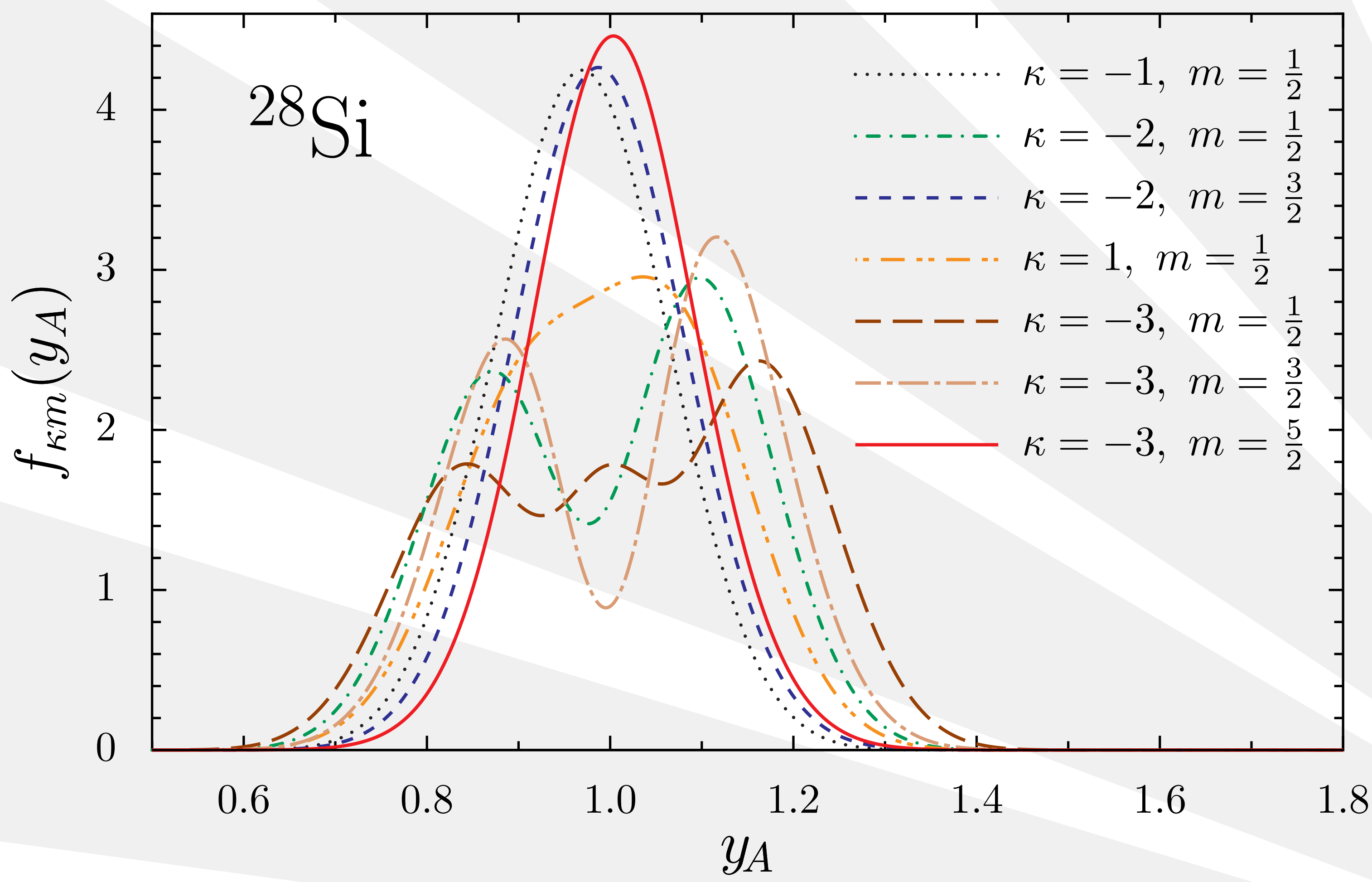
- Infinite nuclear matter

$$f(y_A)=\frac{3}{4}\left(\frac{\varepsilon_F}{p_F}\right)^3\left[\left(\frac{p_F}{\varepsilon_F}\right)^2-(1-y_A)^2\right].$$

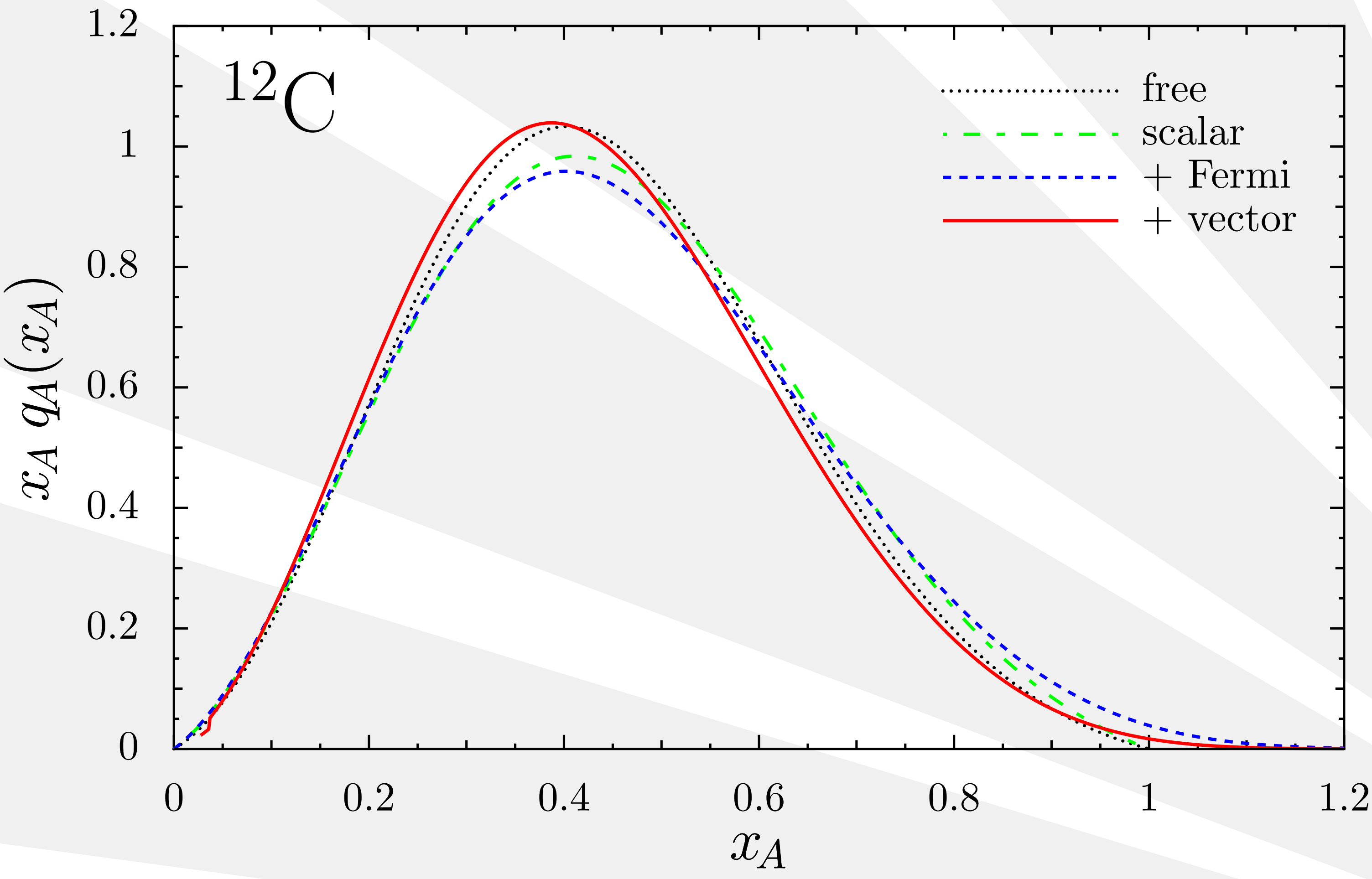
Nucleon distributions: ^{12}C



Nucleon distributions: ^{28}Si



Quark distribution in ^{12}C



EMC effect

- F_2 EMC ratio

$$R_{2A} = \frac{F_{2A}}{F_{2A}^{\text{naive}}} = \frac{F_{2A}}{Z\,F_{2p} + (A - Z)\,F_{2n}}$$

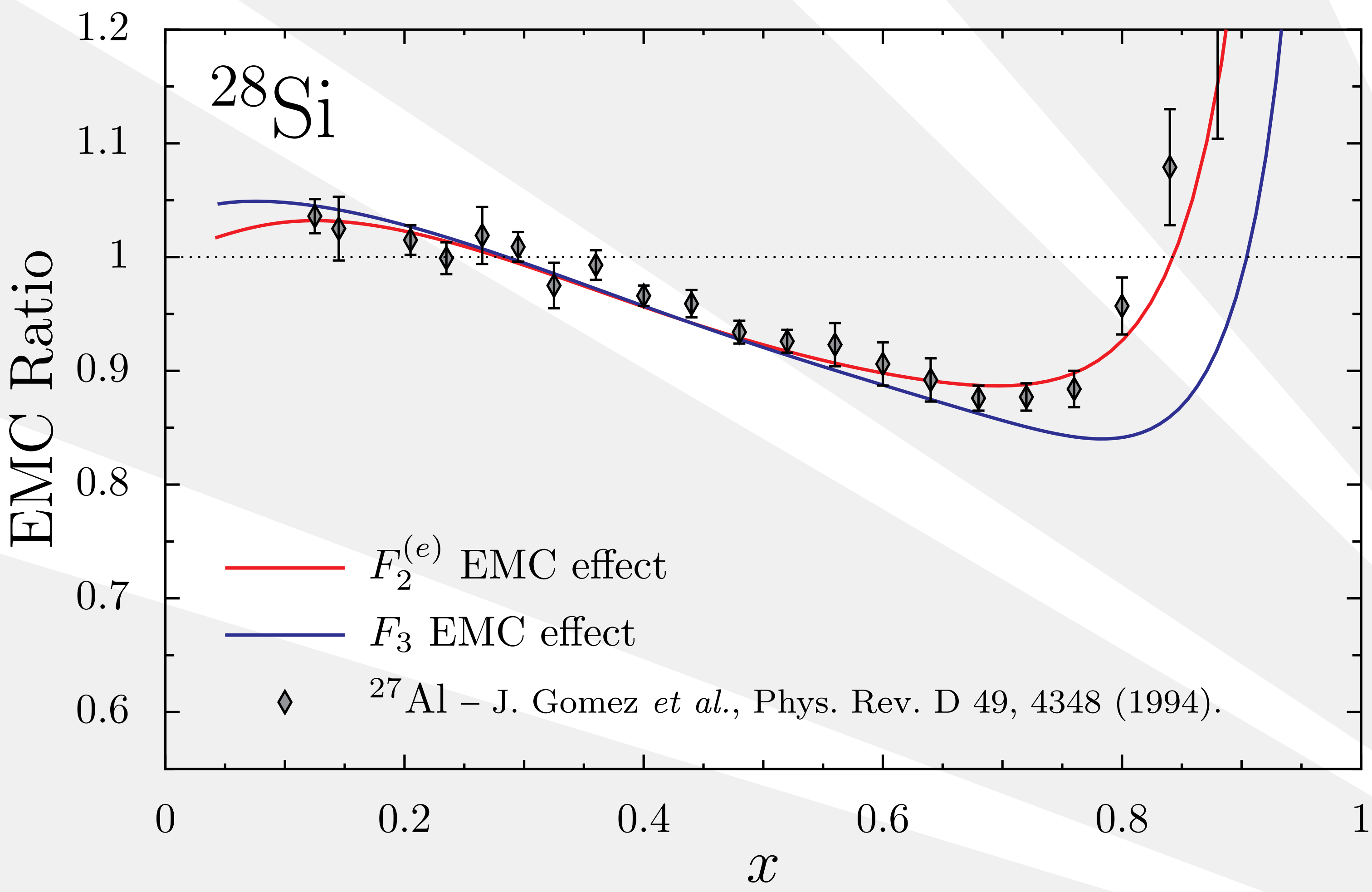
- Ratios equal 1 in non-relativistic and no-medium modification limit.

- Isoscalar EMC ratios

$$R_{2A}(x) \simeq \frac{u_A + \bar{u}_A + d_A + \bar{d}_A}{u + \bar{u} + d + \bar{d}}$$

$$R_{3A}(x) \simeq \frac{u_A - \bar{u}_A + d_A - \bar{d}_A}{u - \bar{u} + d - \bar{d}}$$

EMC ratios ^{28}Si



EMC effect for ν ($\bar{\nu}$)

- Definition of F_2 EMC ratio

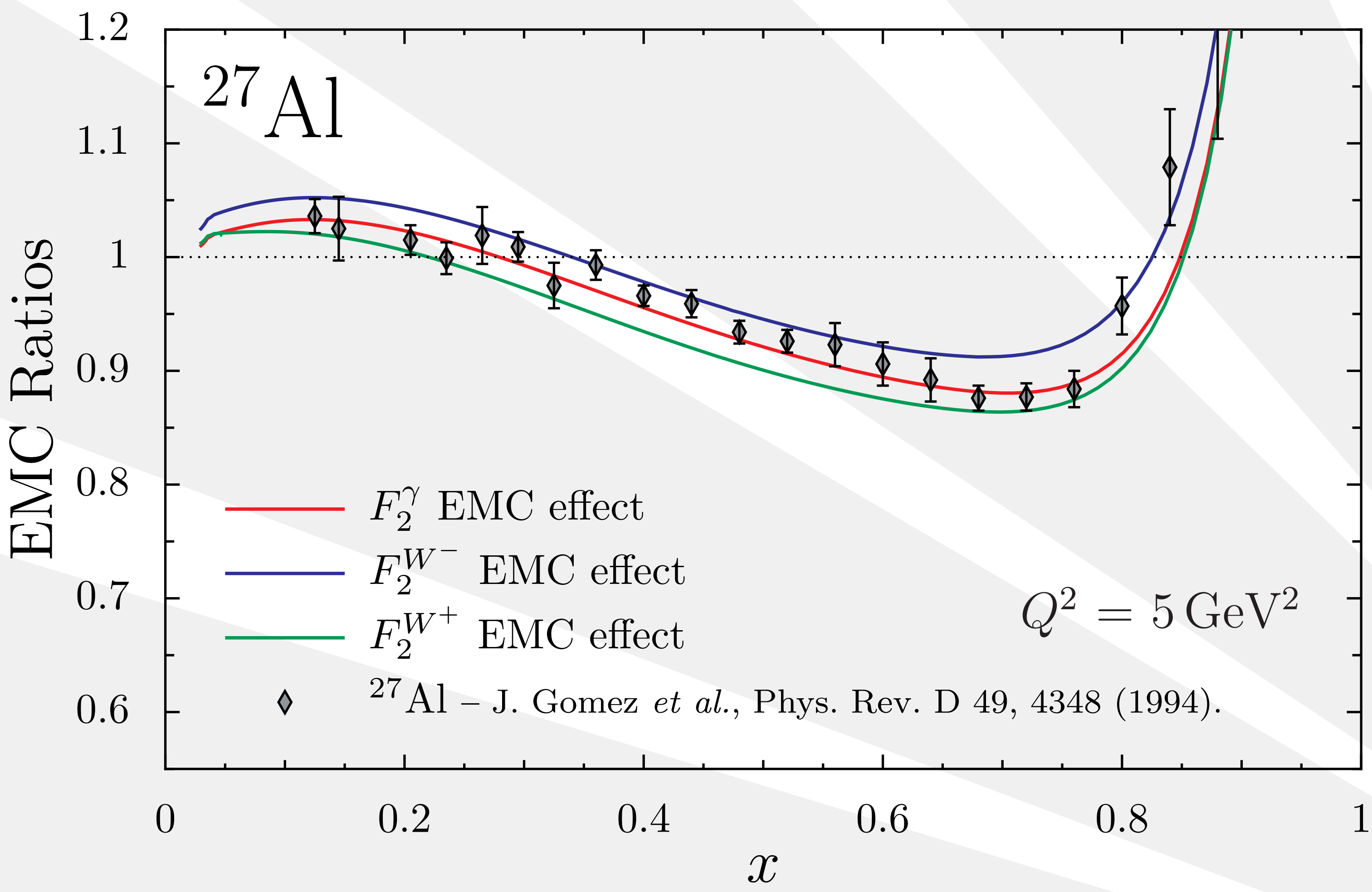
$$R^i_{2A} = \frac{F^i_{2A}}{F^{i,\,\text{naive}}_{2A}} = \frac{F^i_{2A}}{Z\,F^i_{2p} + (A - Z)\,F^i_{2n}}$$

- Neutrino EMC ratios

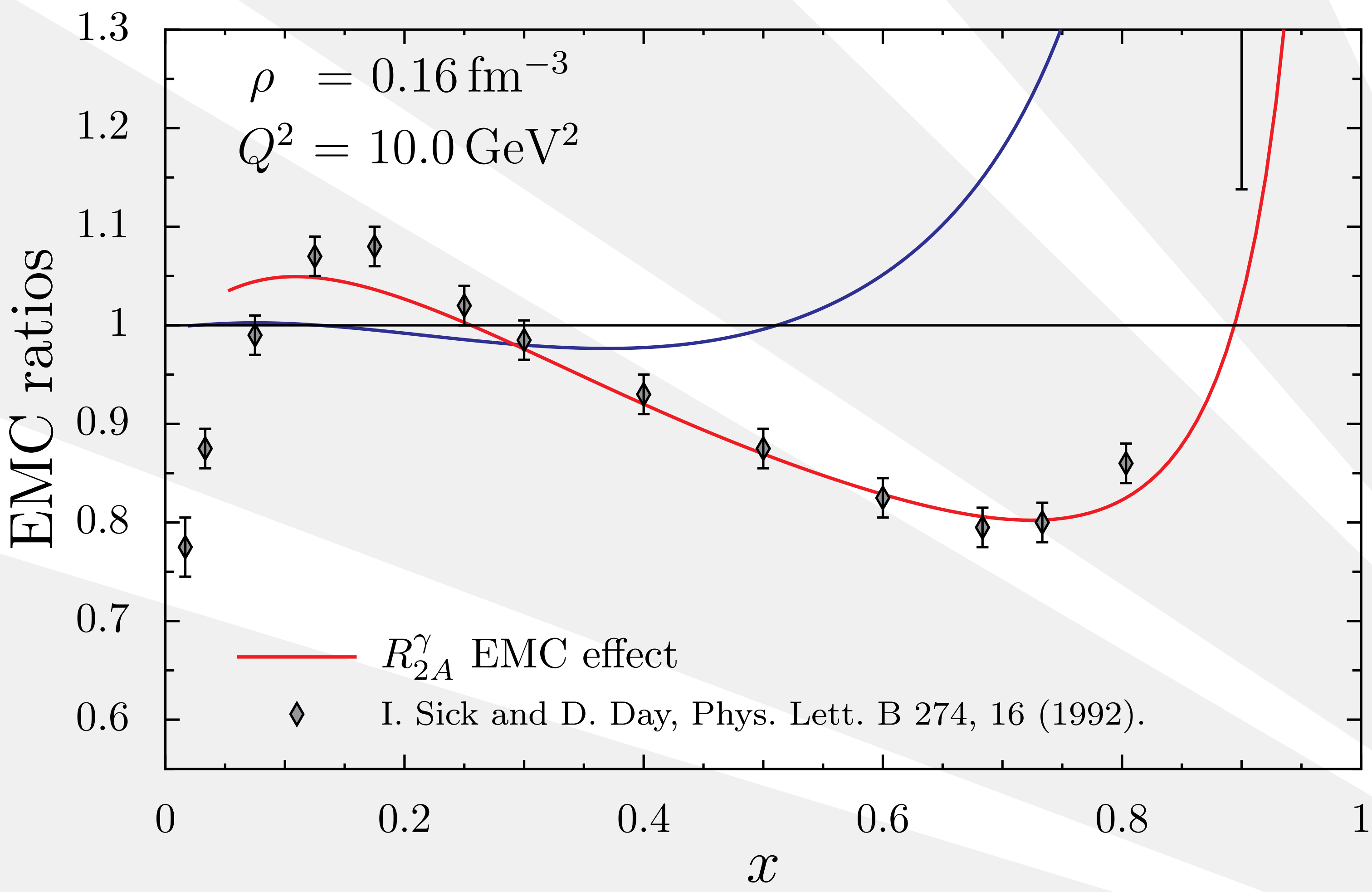
$$R^{W-}_{2A}(x) \simeq \frac{u_A + \bar{d}_A}{Z(u_p + \bar{d}_p) + N(u_n + \bar{d}_n)}$$

$$R^{W+}_{2A}(x) \simeq \frac{\bar{u}_A + d_A}{Z(\bar{u}_p + d_p) + N(\bar{u}_n + d_n)}$$

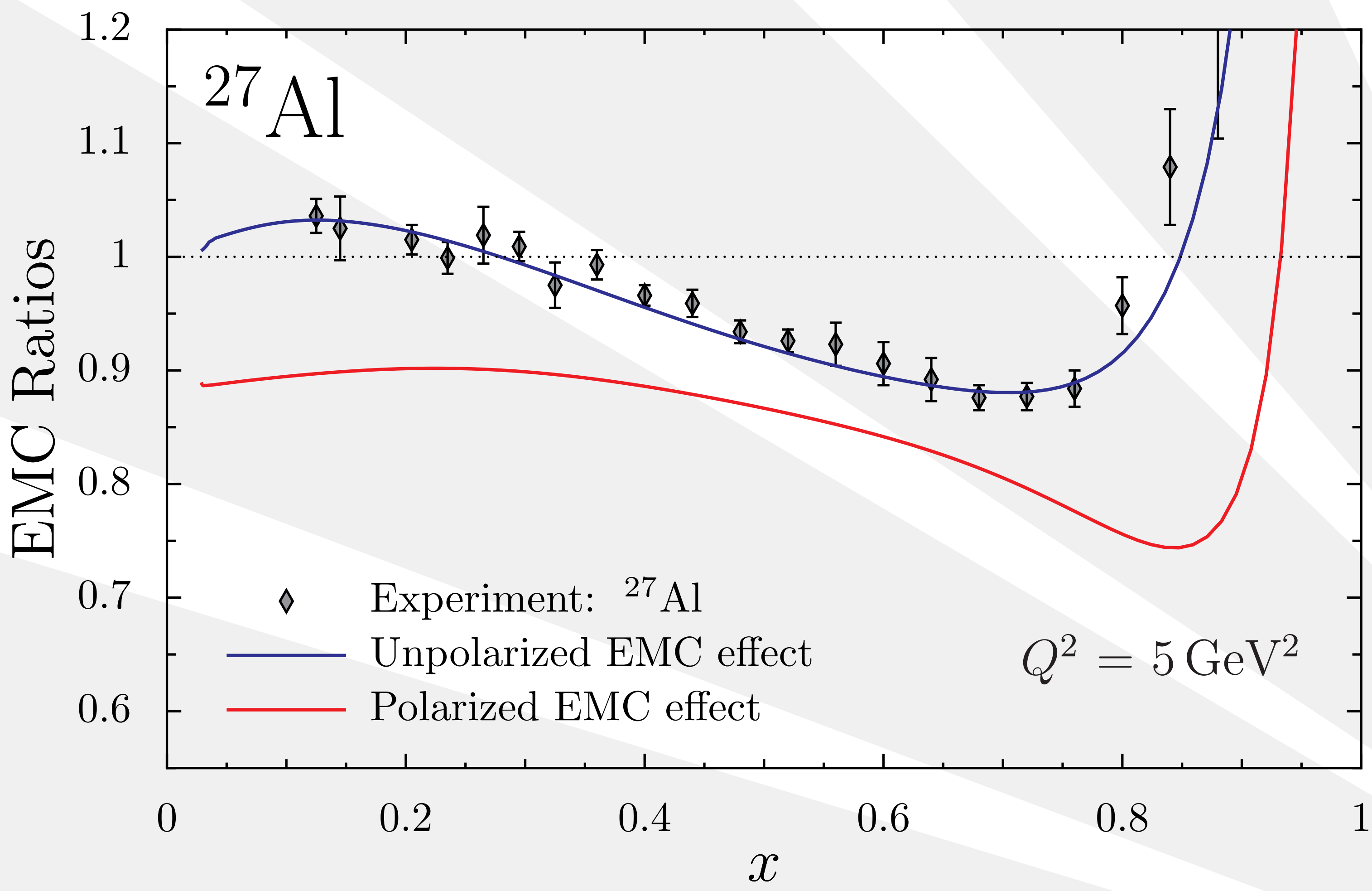
EMC ratios ^{27}Al



Is there medium modification



Polarized EMC ratio ^{27}Al



Conclusions

- Effective chiral quark theories can be used to incorporate quarks into many-body physics.
- Calculated nuclear quark distributions where the quarks bind to mean scalar and vector fields.
- Reproduced EMC effect.
- Essential to have medium-modified quark distributions.
- Hopefully neutrino DIS can help answer the question; are nucleon properties modified by the nuclear medium?

Regularization

- Proper-time regularization

$$\frac{1}{X^n} = \frac{1}{(n-1)!}\int_0^\infty d\tau\, \tau^{n-1}\, e^{-\tau X}$$

$$\longrightarrow \quad \frac{1}{(n-1)!}\int_{1/(\Lambda_{UV})^2}^{1/(\Lambda_{IR})^2} d\tau\, \tau^{n-1}\, e^{-\tau X}.$$

- Λ_{IR} eliminates unphysical thresholds for the nucleon to decay into quarks: $\rightarrow$ simulates confinement.
  - G. Hellstern, R. Alkofer and H. Reinhardt, Nucl. Phys. A **625**, 697 (1997).
- Needed for: nuclear matter saturation, Δ baryon.
  - W. Bentz, A.W. Thomas, Nucl. Phys. A **696**, 138 (2001)

Model Parameters

- Free Parameters: Λ_{IR}, Λ_{UV}, M_0, G_π, G_s and G_a.
- Constraints:
  - $f_\pi = 93$ MeV, $m_\pi = 140$ MeV and $M_N = 940$ MeV
  - $(\rho, E_B/A) = (0.16\,\text{fm}^{-3}, -15.7\text{ MeV})$
  - $\int_0^1 dx\ (\Delta u_v(x) - \Delta d_v(x)) = g_A = 1.267$
- We obtain:
  - $\Lambda_{IR} = 240$ MeV, $\Lambda_{UV} = 644$ MeV, $M_0 = 400$ MeV
  - $G_\pi = 19\text{ GeV}^{-2}$, $G_s = 7.5\text{ GeV}^{-2}$, $G_a = 2.8\text{ GeV}^{-2}$
  - $M_s = 690$ MeV, $M_a = 990$ MeV,