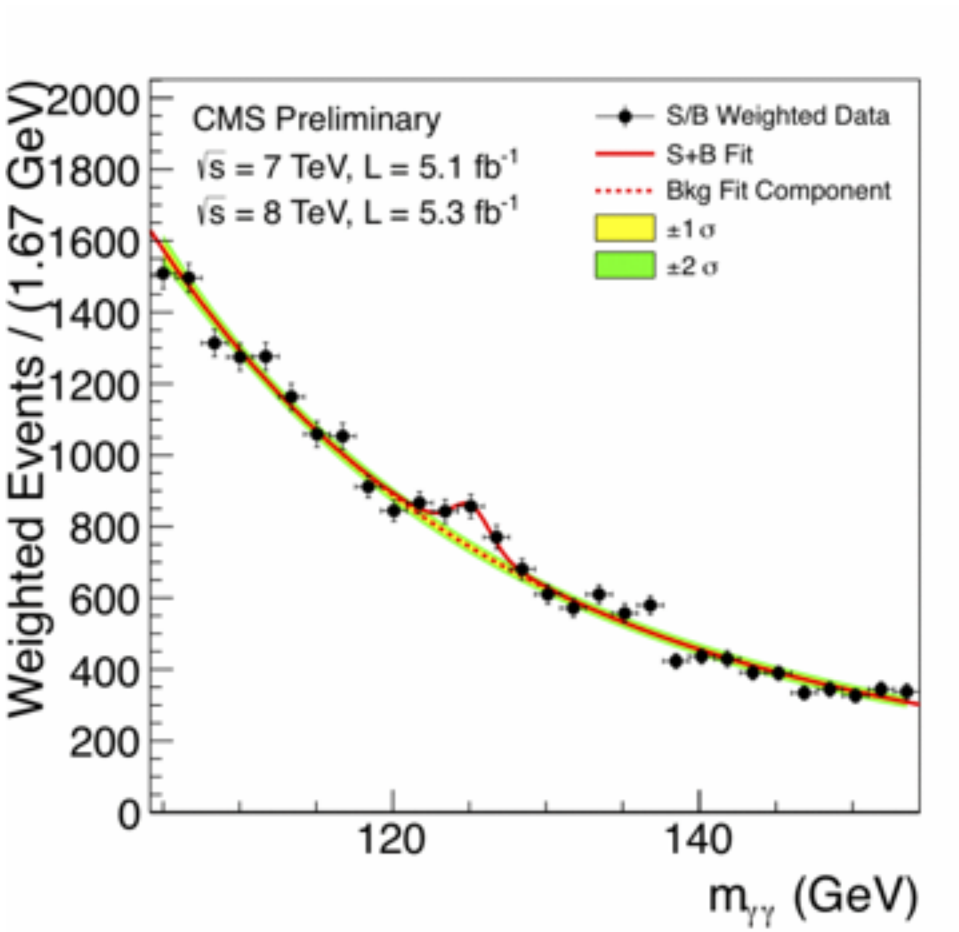
Naturalness and the Weak Gravity Conjecture

Clifford Cheung

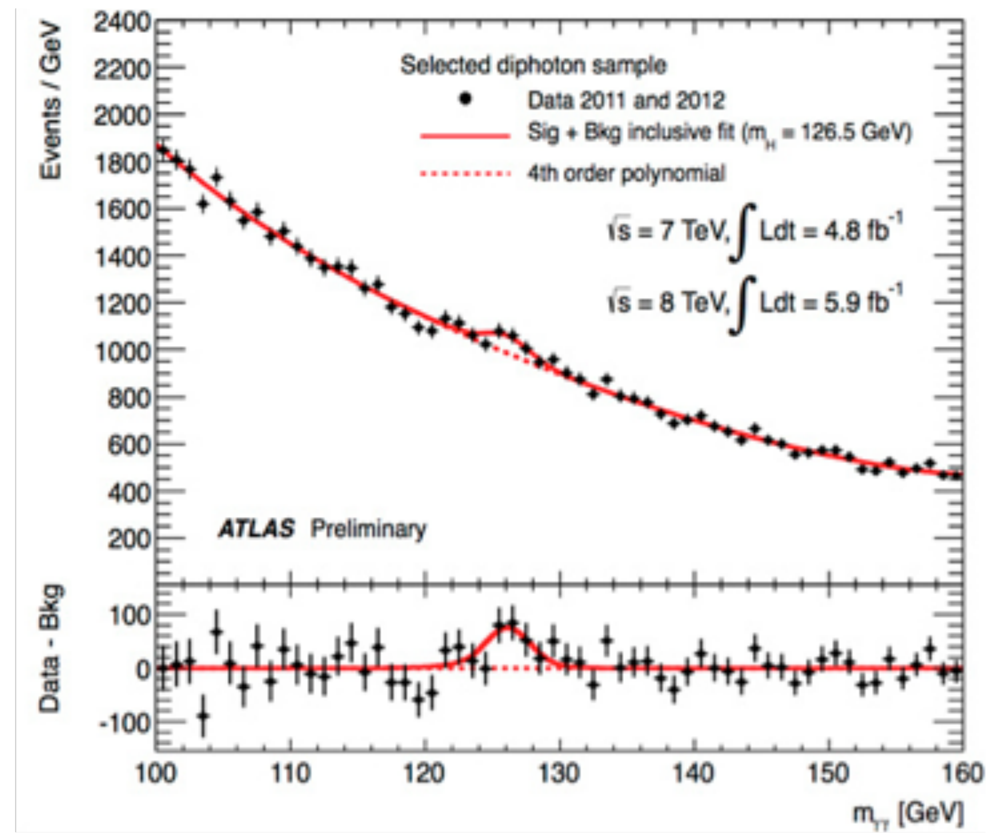
C. Cheung, G. N. Remmen (1402.2287, 1407.7865)

Nature exhibits a fundamental scalar…

CMS



ATLAS

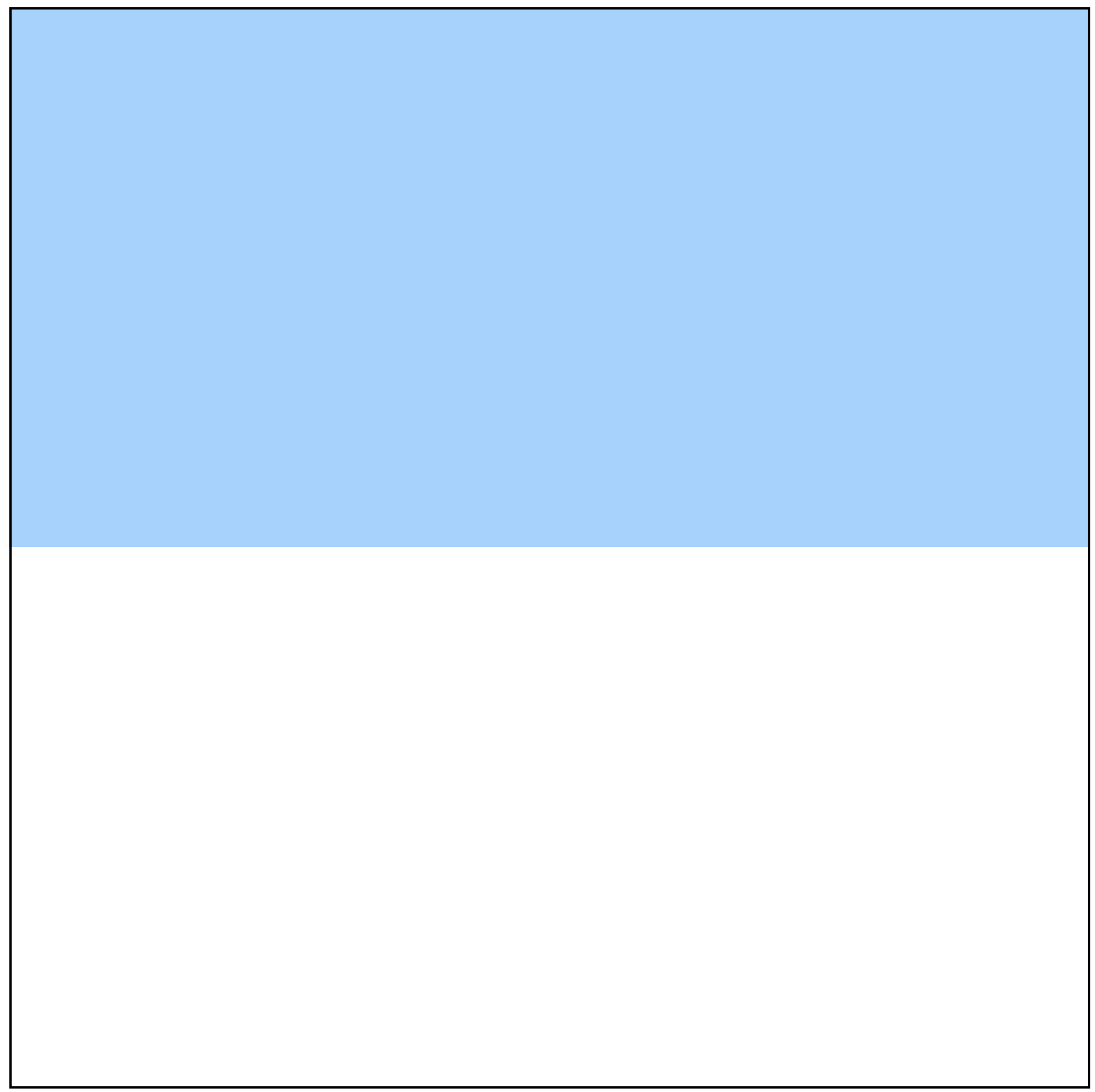


…but alas, no "naturalons," yet.

theory space

theory space

natural



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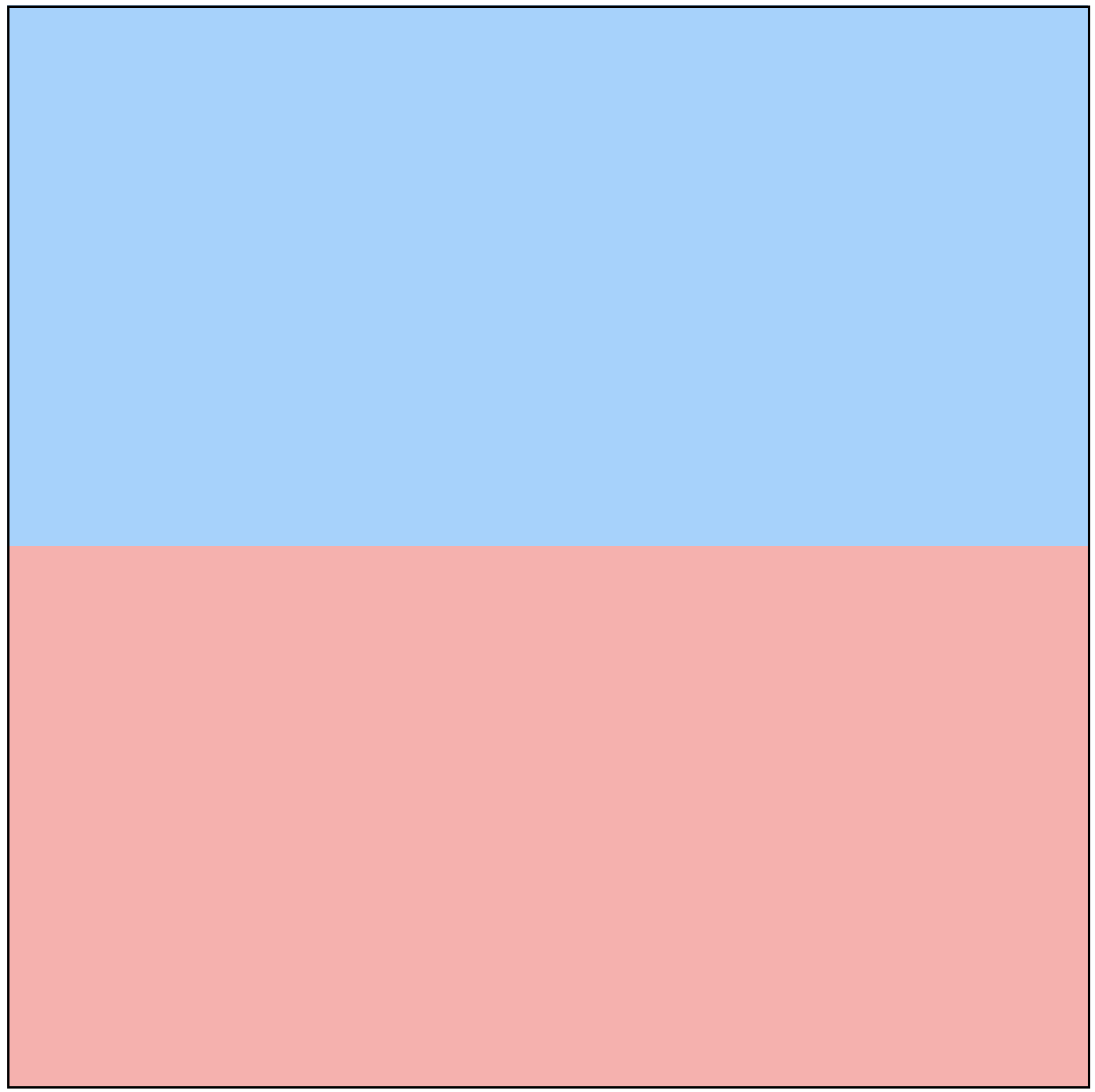
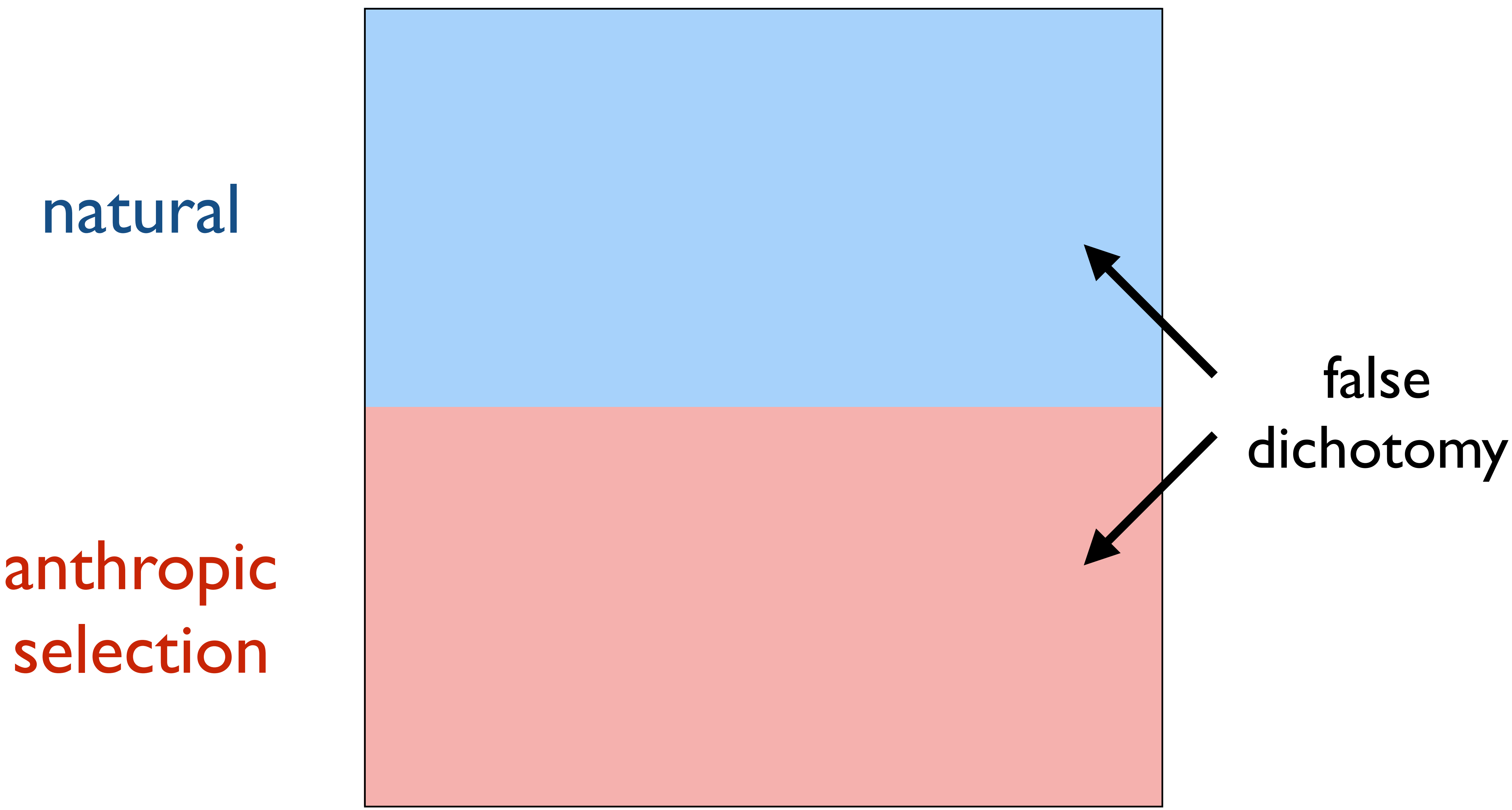


anthropic selection

theory space



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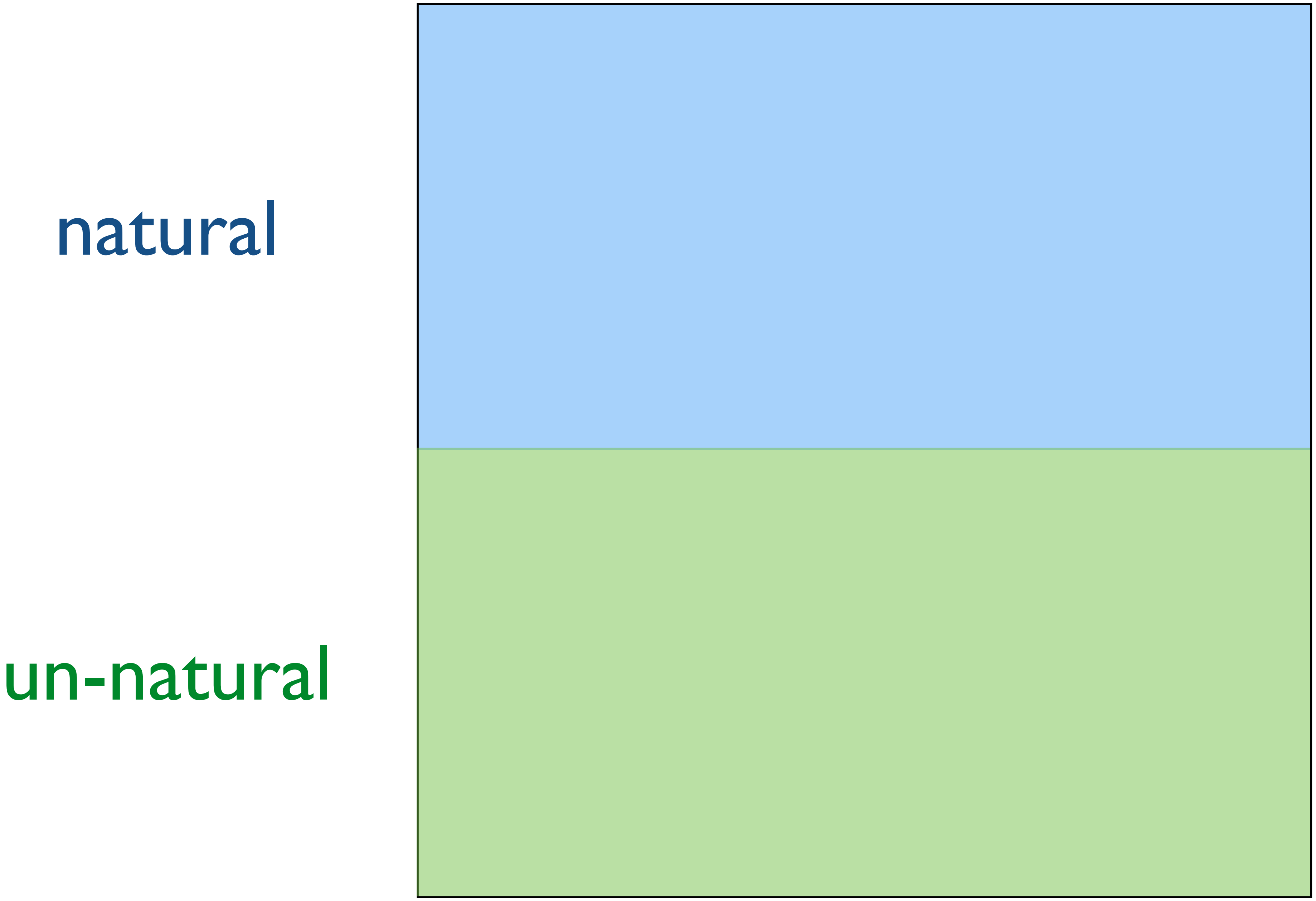

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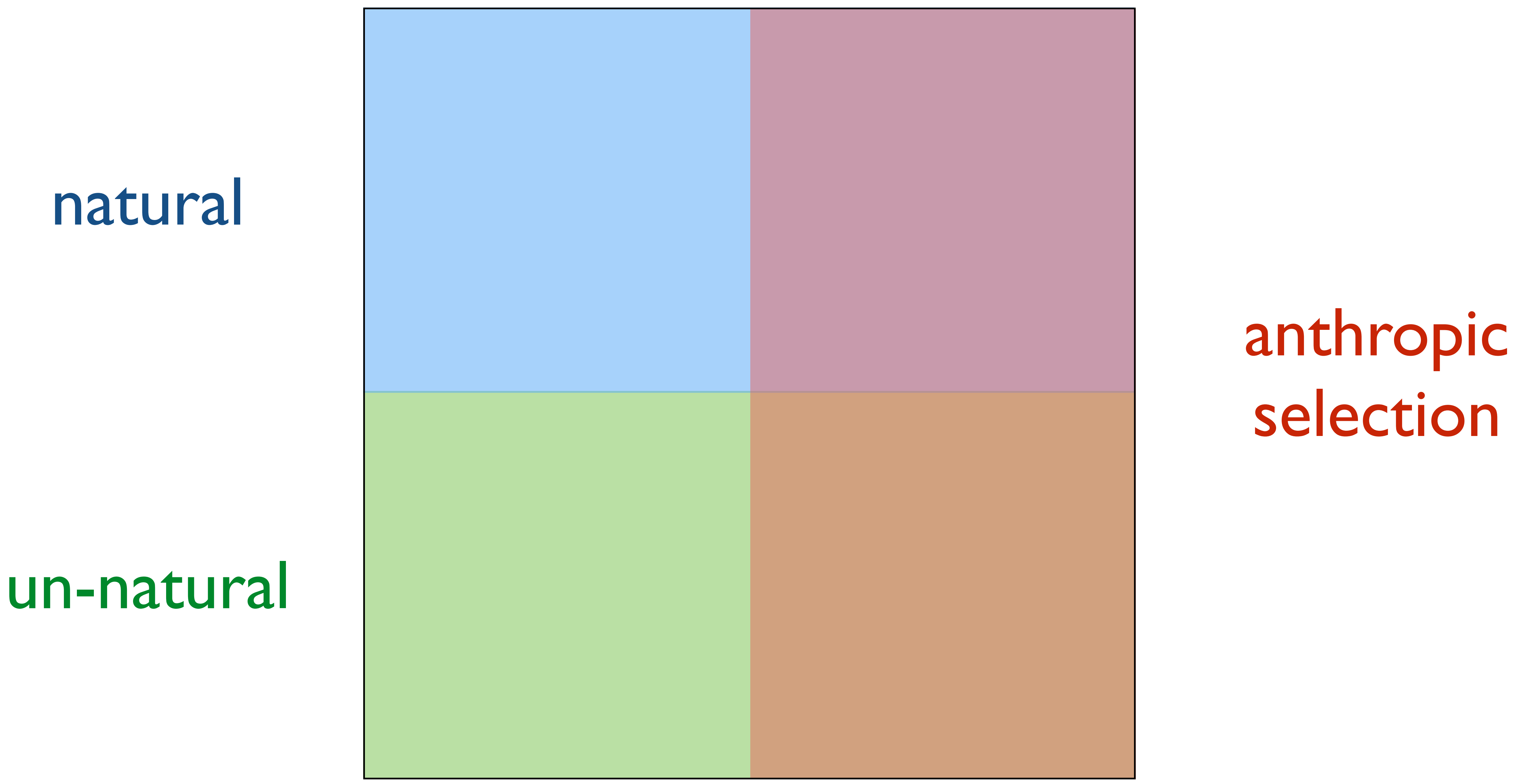

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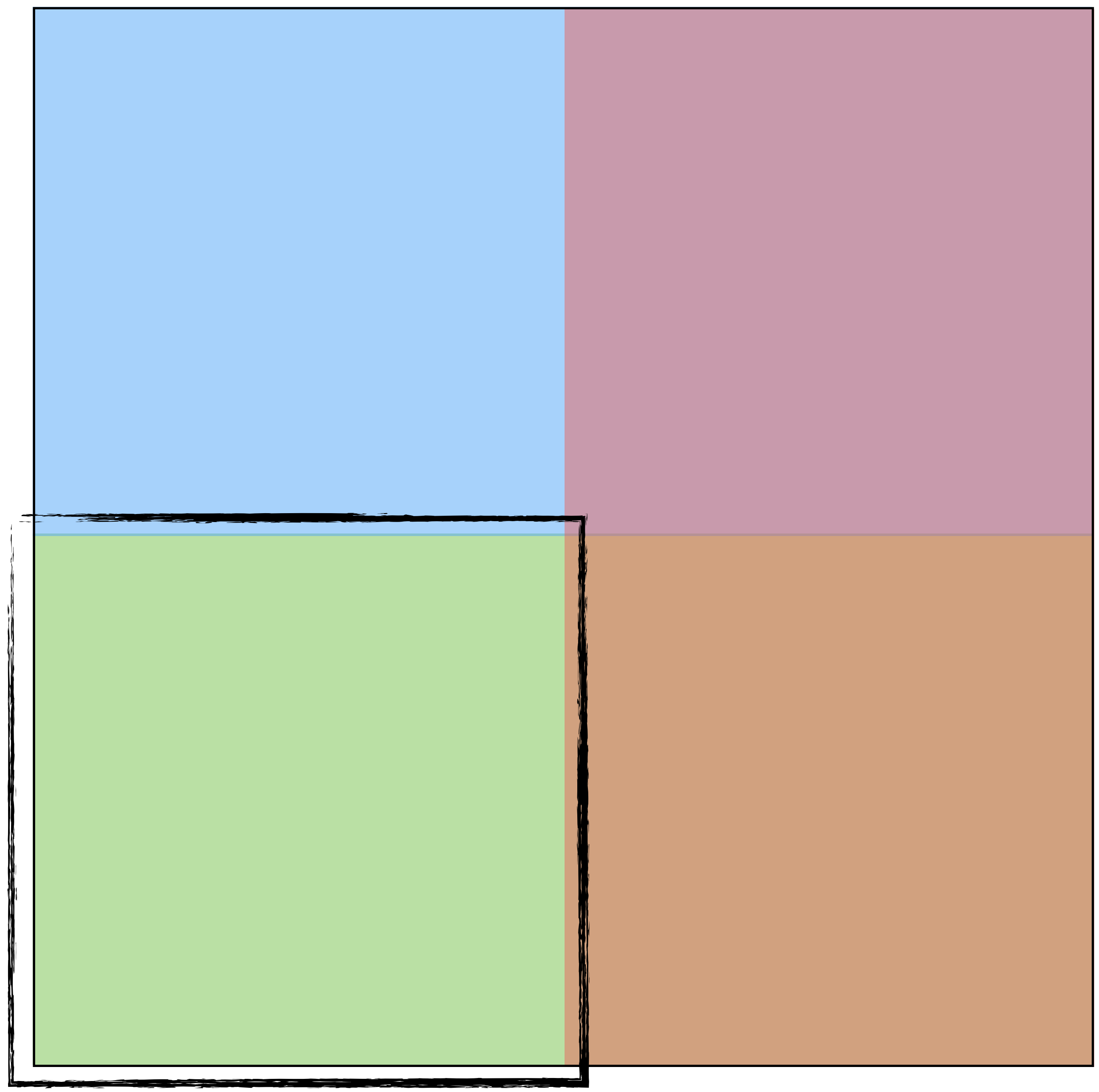

anthropic selection

un-natural

“The 3rd Way”

Imagine a "desert" above the weak scale.

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$$m_H \sim 10^{-16} m_{\rm Pl} \qquad\qquad V_0^{1/4} \sim 10^{-30} m_{\rm Pl}$$

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Possible lessons?

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Possible lessons?

- We suck (at computing symmetry unprotected dimensionful parameters).

- Don't modify gravity, understand tuning.

What is the invariant meaning of naturalness?

Consider an EFT for the hierarchy problem.

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_0^2\phi^2 \quad \longleftarrow \text{(light state)}\\ &+\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}M_0^2\chi^2 \quad \longleftarrow \text{(heavy state)}\\ &-\frac{1}{4}\lambda_0\phi^2\chi^2 + \ldots\end{aligned}$$

Claim: even pulling every dirty trick in the book, there's still an irreducible tuning.

Computing with a hard cutoff …

$$\delta m_0^2 \sim \frac{\lambda_0}{16\pi^2}(\Lambda^2 + M_0^2 \log M_0^2/\Lambda^2 + M_0^2 + \ldots)$$

… or dimensional regularization…

$$\delta m_0^2 \sim \frac{\lambda_0}{16\pi^2}\left(\frac{M_0^2}{\epsilon} + M_0^2 \log M_0^2/\mu^2 + M_0^2 + \ldots\right)$$

… the problem persists.

Regulator abra cadabra won't fix ubiquitous **tree-level** hierarchy problems.

ultraviolet symmetries $\longleftrightarrow$ dangerous symmetry breaking parameters

The issue is a generalization of the doublet-triplet splitting problem.

$$\delta m_0^2 \sim \lambda_0 \langle \chi \rangle^2 \longleftarrow \text{GUT scale, PQ scale}$$

Let's re-examine the terms.

$$\delta m_0^2 \sim \frac{\lambda_0}{16\pi^2}\left(\frac{M_0^2}{\epsilon} + \underbrace{M_0^2 \log M_0^2/\mu^2}_{\text{scheme independent}} + \underbrace{M_0^2}_{\text{scheme dependent}} + \ldots\right)$$

Finite and 1/ε contributions are unobservable and can be absorbed entirely into pole mass.

However, the running is physical.

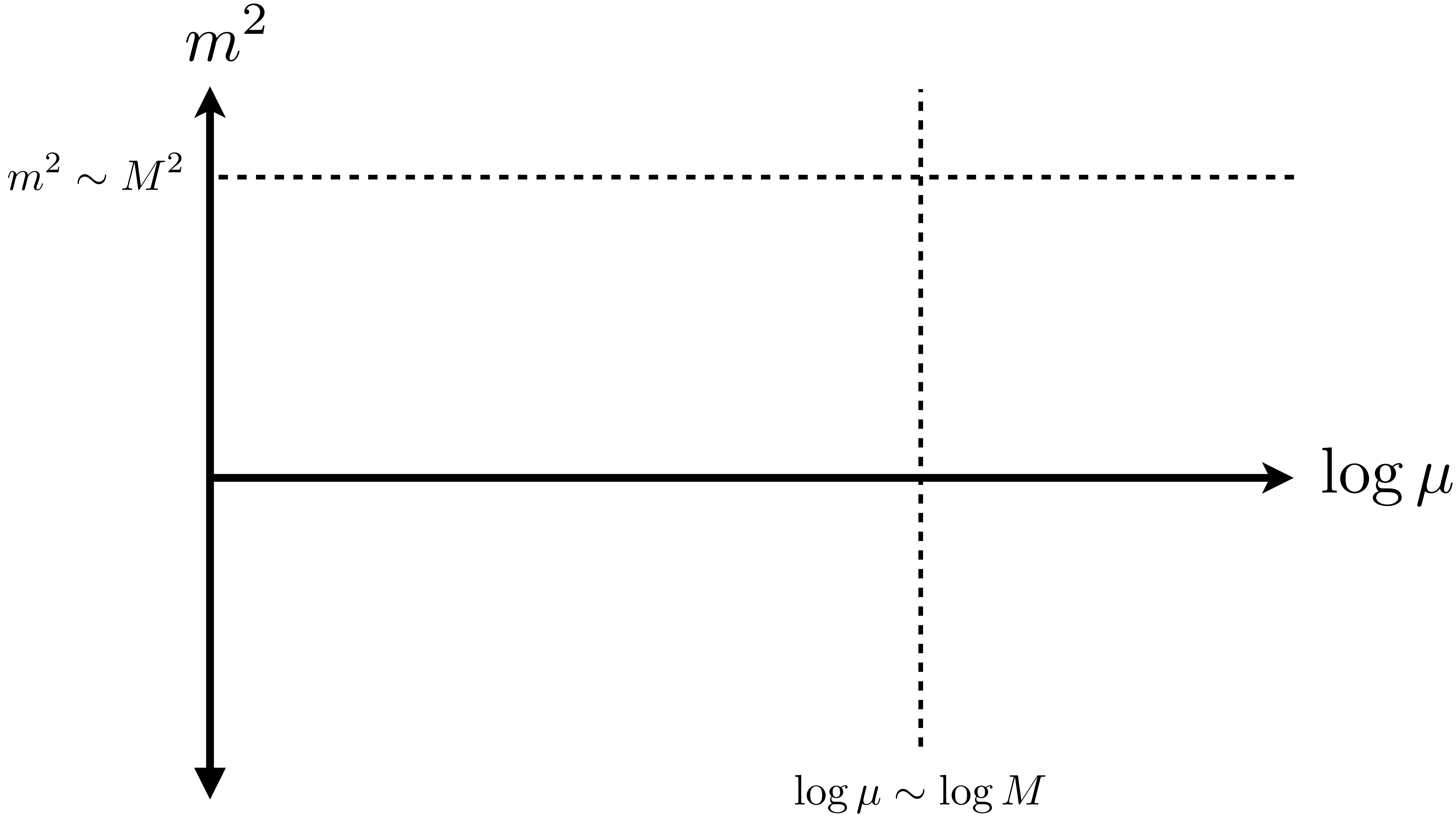
criteria of "running naturalness"

Theory is fine-tuned if the mass changes by orders of magnitude in an e-fold of running.

$$\frac{dm^2}{d\log\mu} = \frac{\lambda M^2}{16\pi^2}$$

This is a part of the hierarchy problem that won't ever go away.

Consider the running of the light mass.



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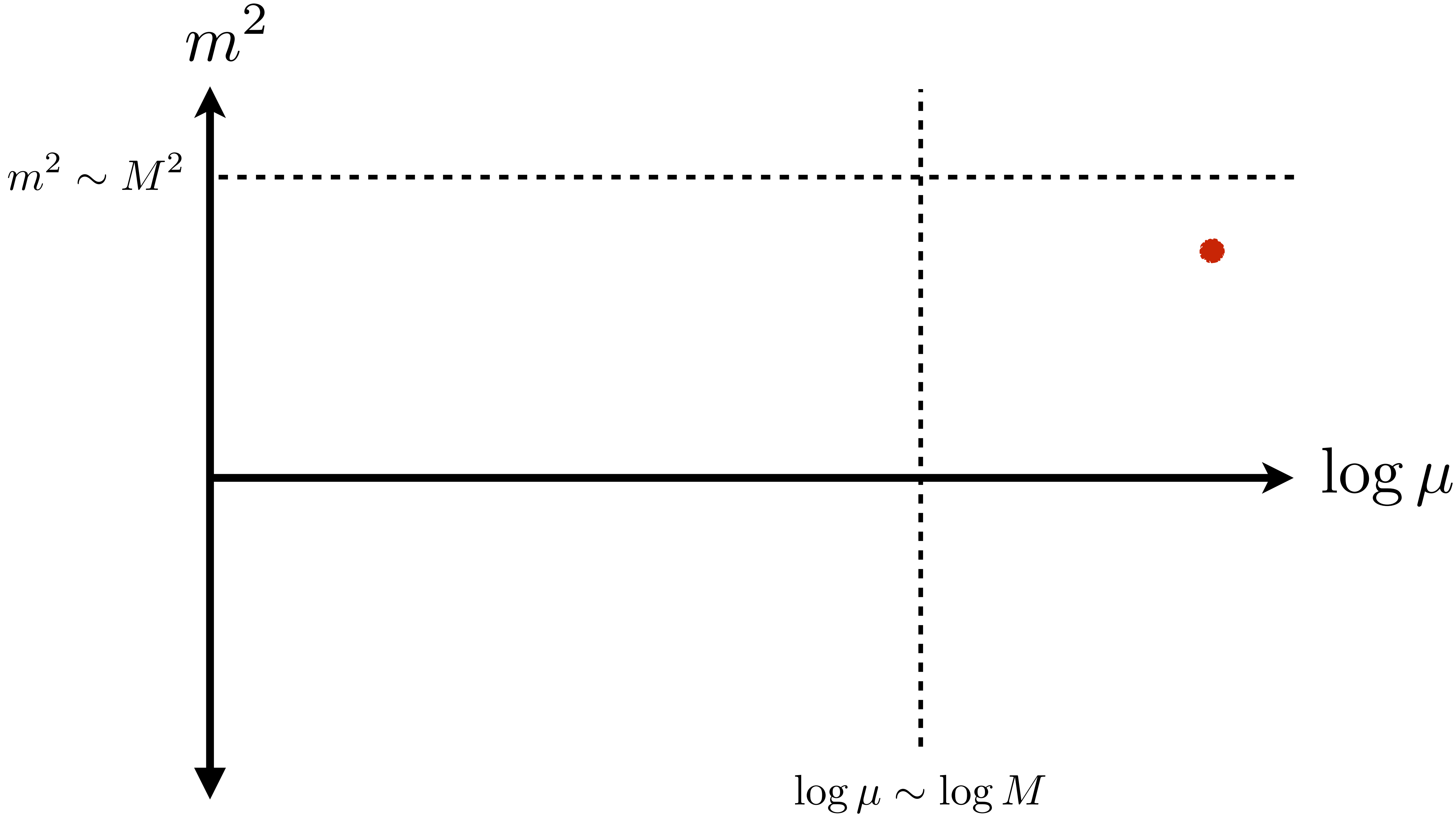
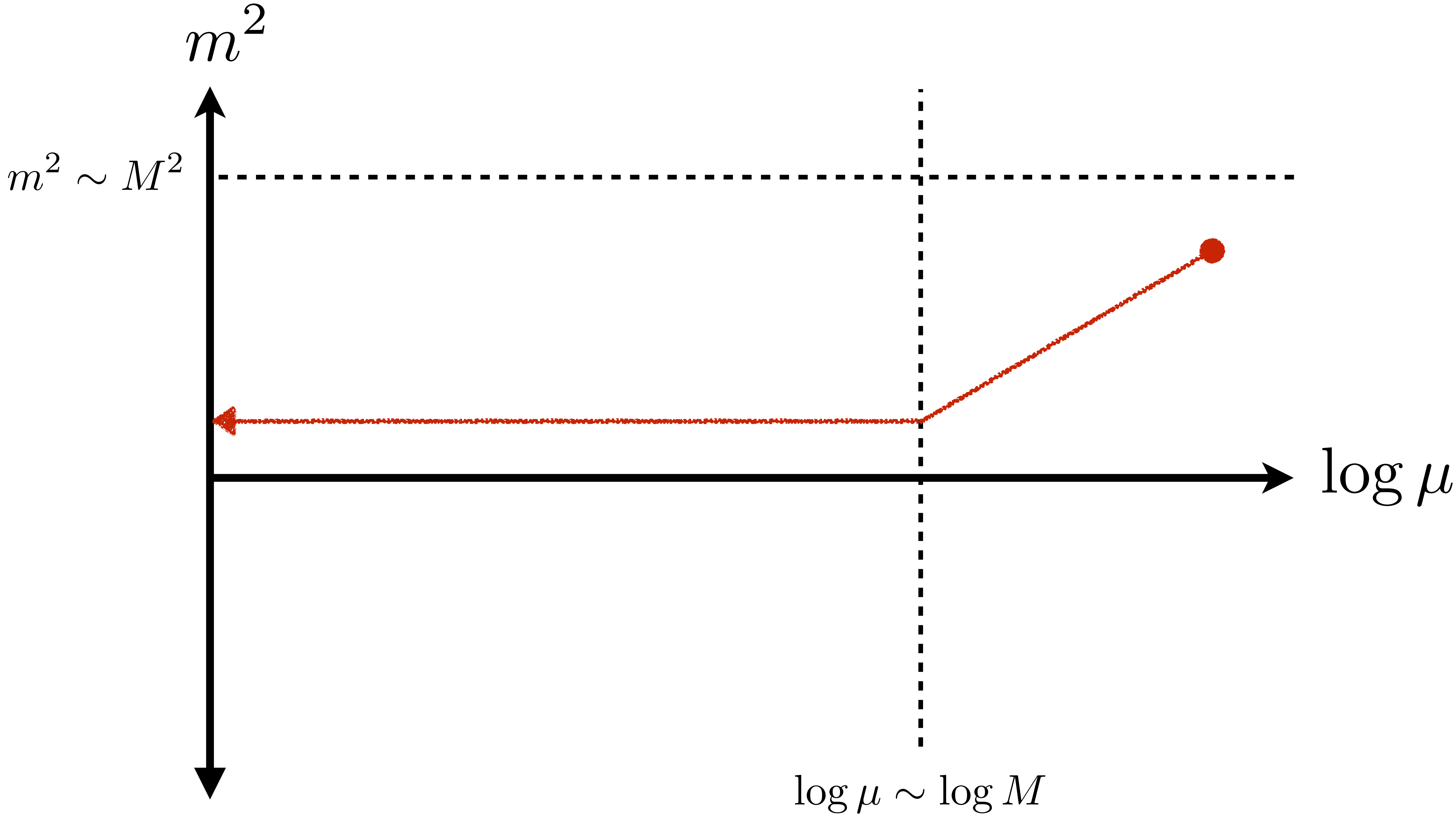
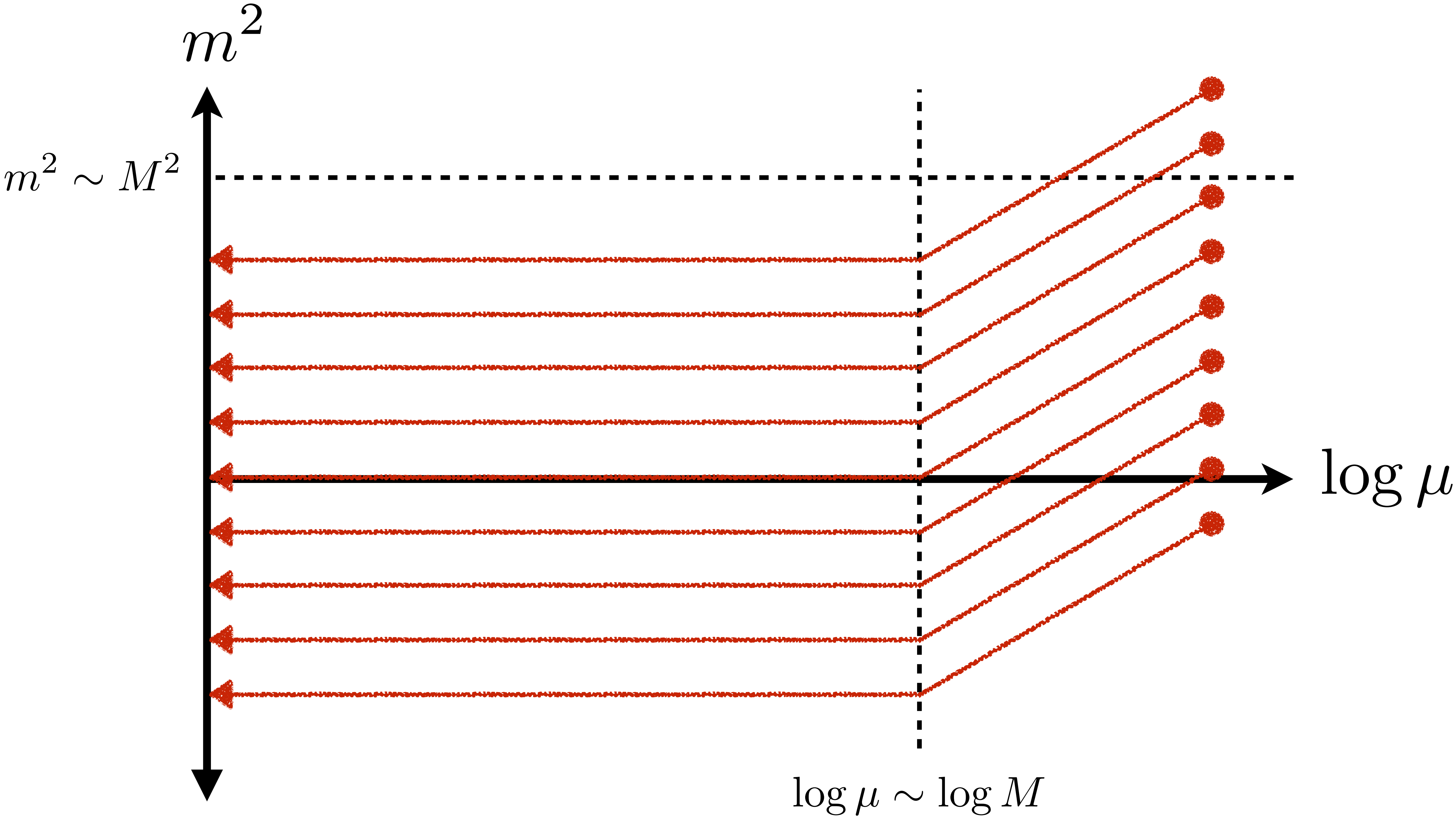


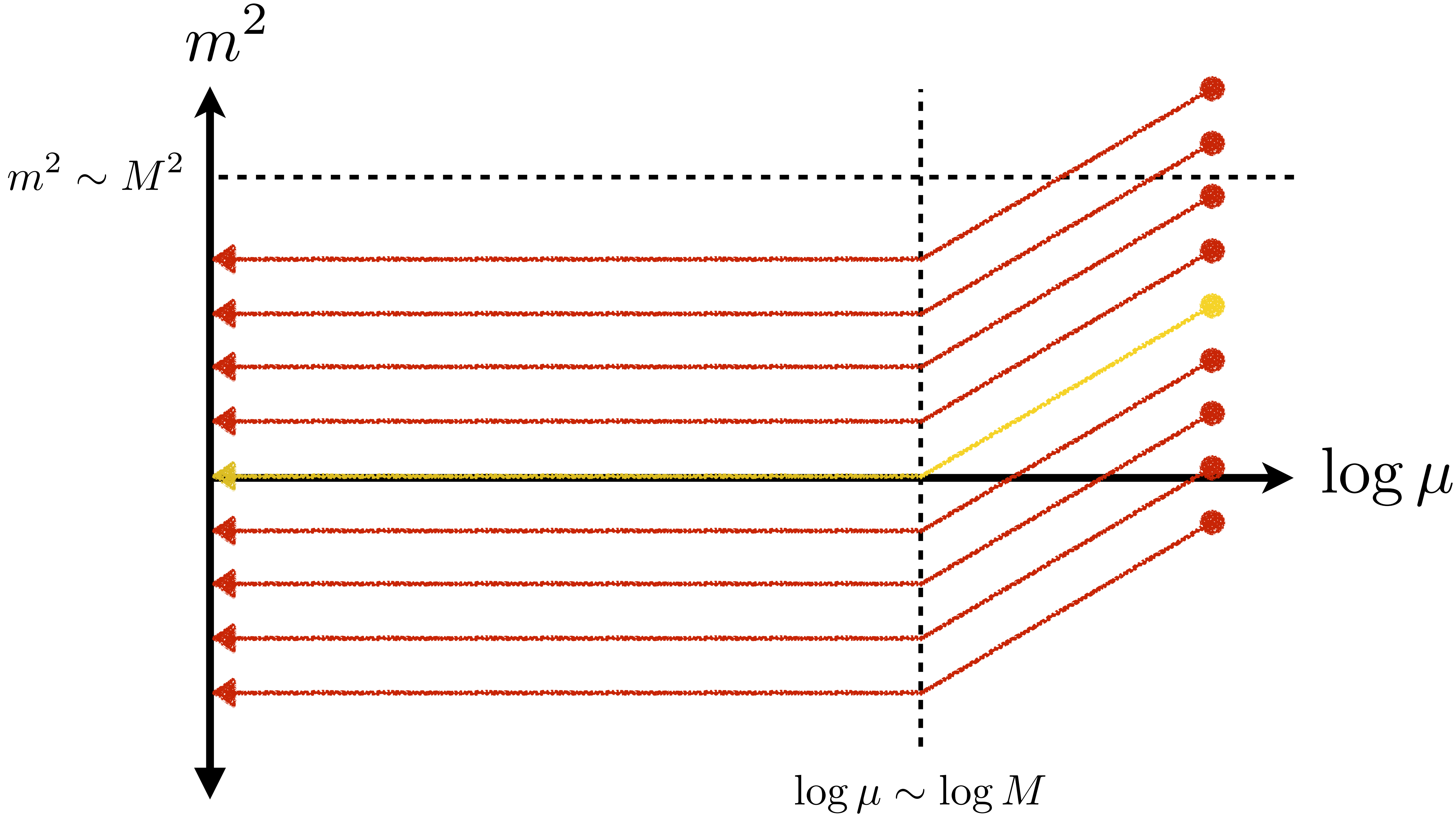
Fix a UV boundary condition.



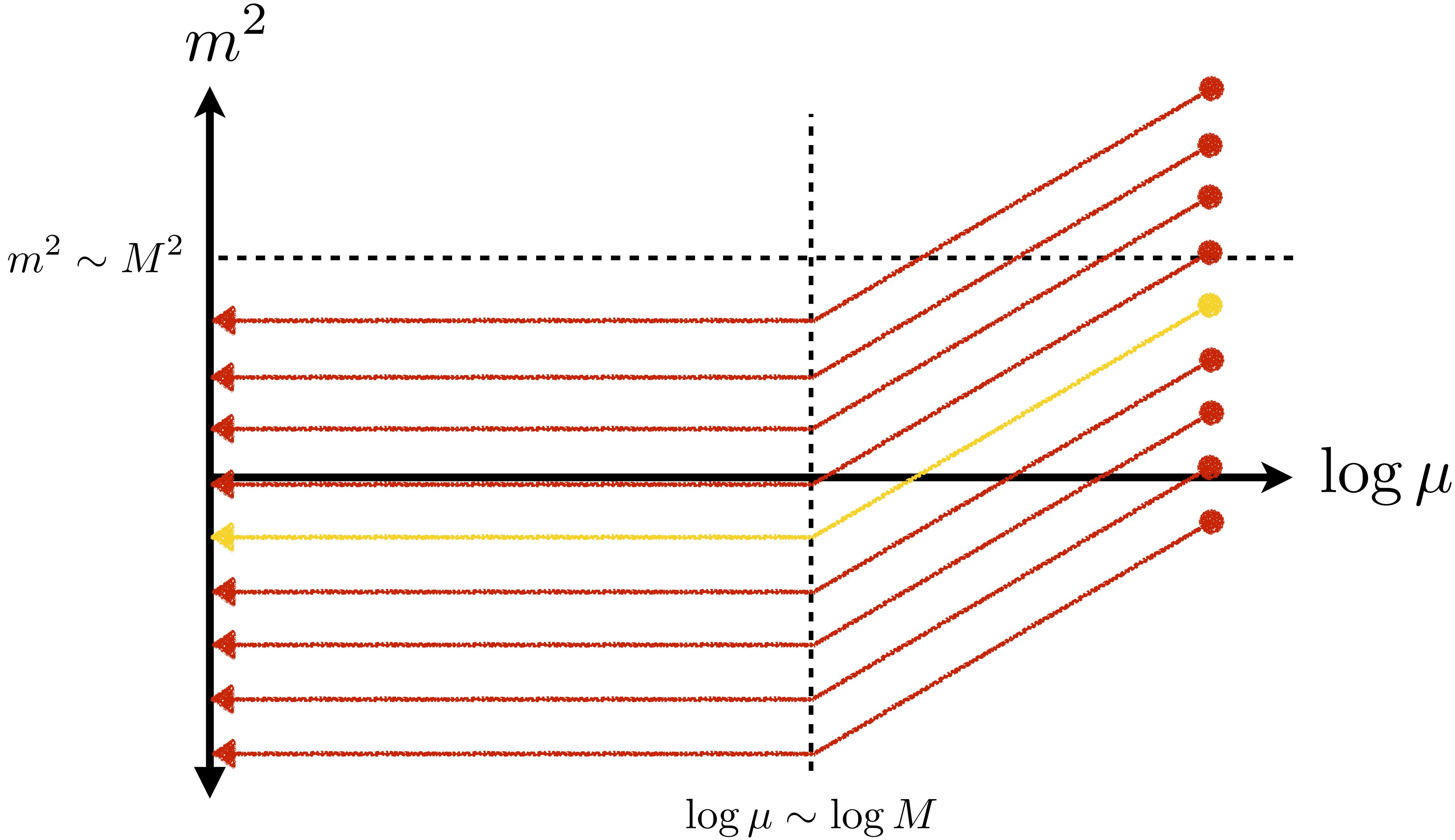
Run down to obtain pole mass.



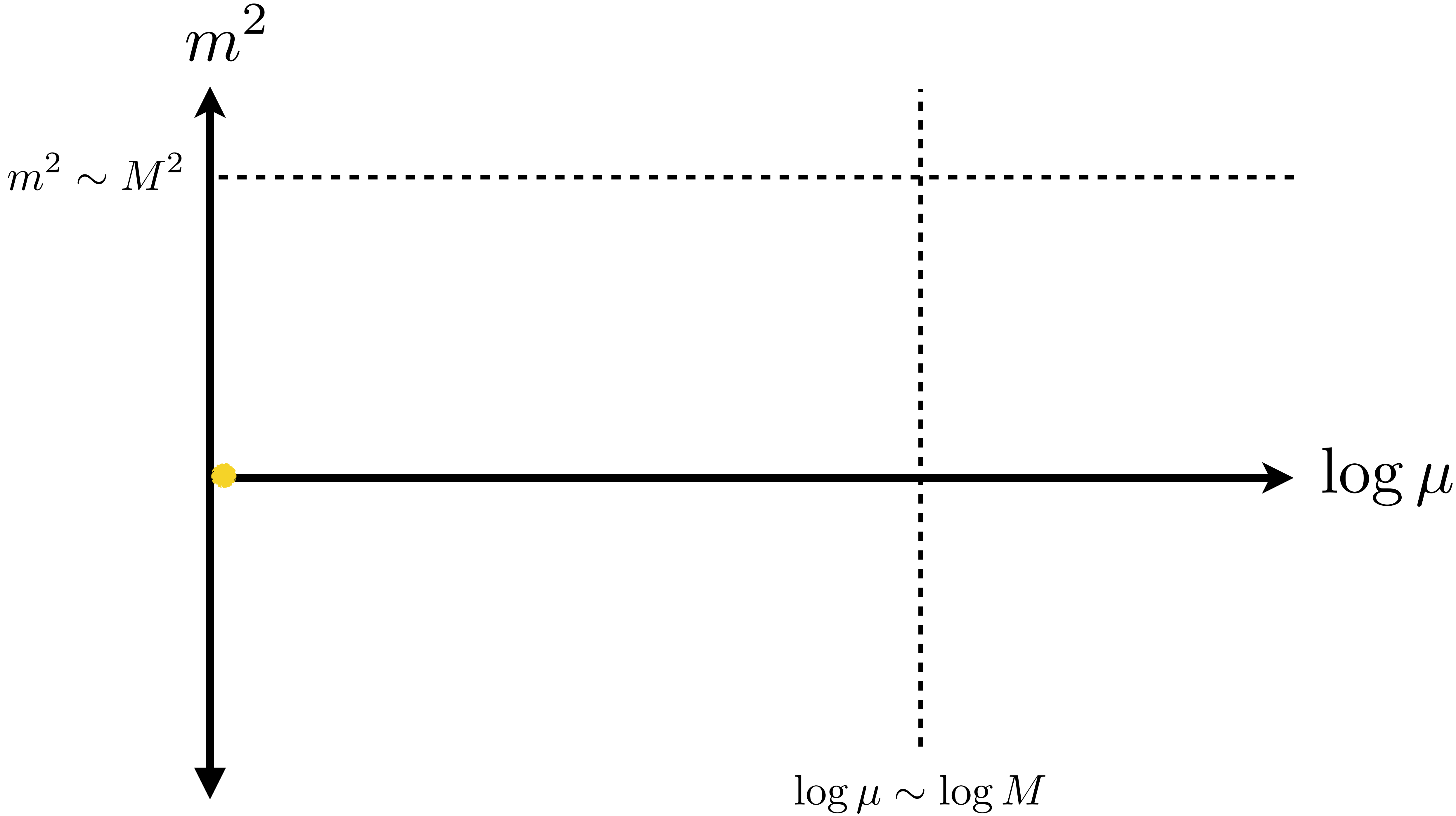
Now, throw darts in the UV.



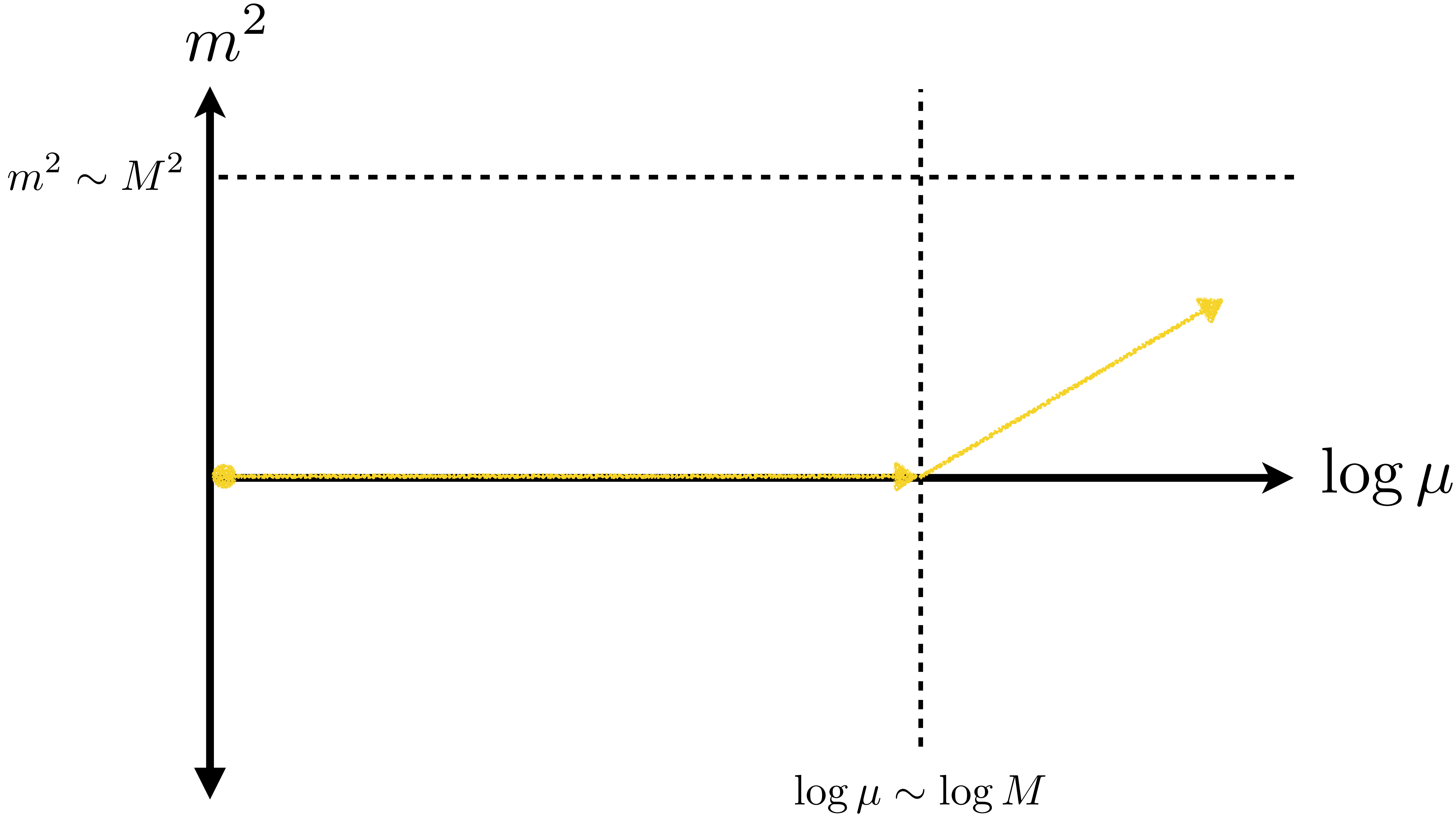
A light pole mass is exceedingly rare.



Moreover, it is unstable to variations of M.



What if pole mass is the boundary condition?



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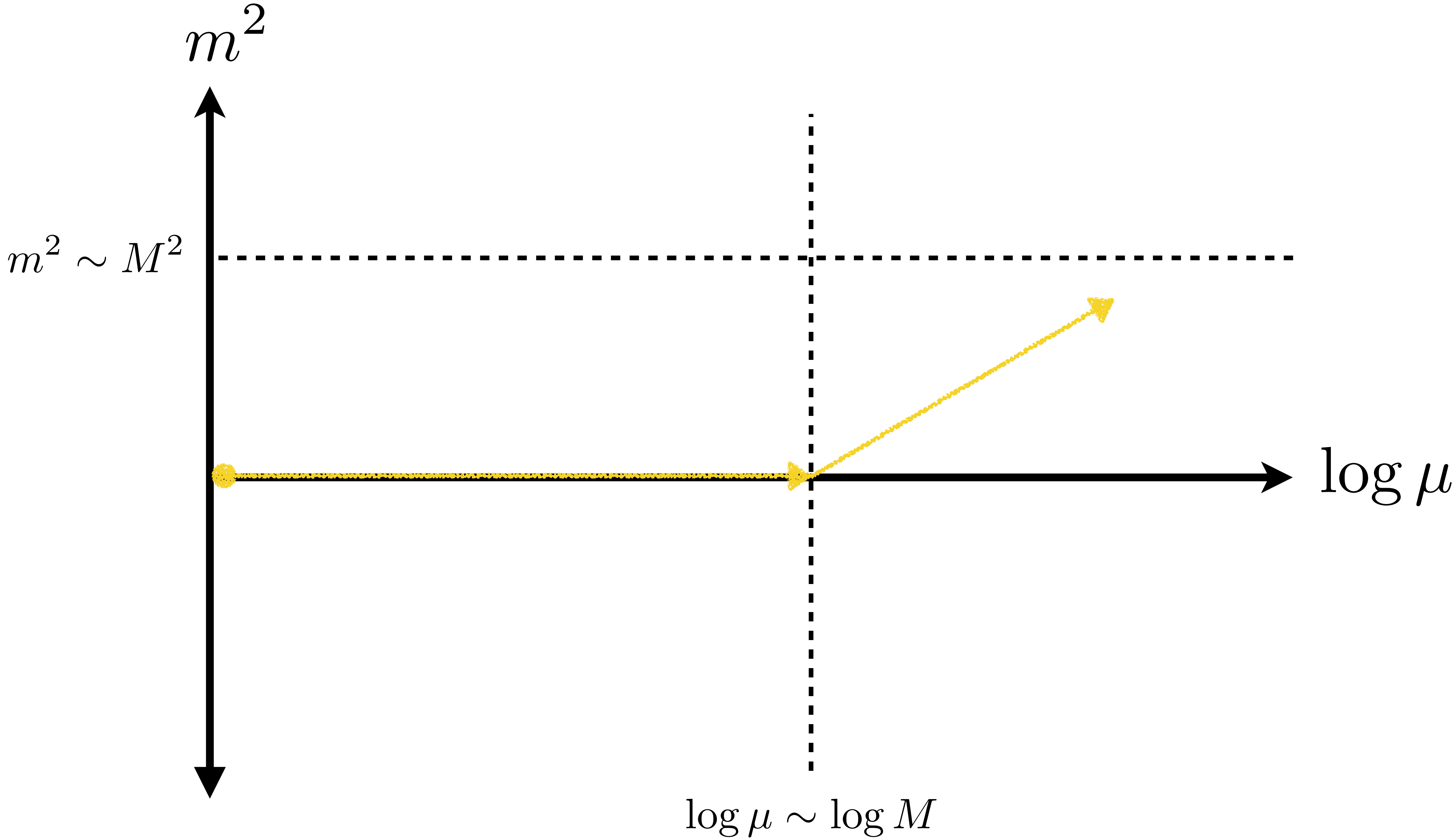


Then varying M doesn't do much.

Fine-tuning comes from throwing darts evenly in the UV. Possibilities for "3rd Way":

- Maybe the more "fundamental" boundary condition is **on-shell**, not **UV**. Indeed, S-matrix is only observable in quantum gravity.

- Maybe the darts aren't thrown evenly. There are constraints on theories mandated by consistency.

weak gravity conjecture

weak gravity conjecture (WGC)

(Arkani-Hamed, Motl, Nicolis, Vafa)

A long-range U(1) coupled consistently to gravity *requires* a state with

$$q > m/m_{\mathrm{Pl}}$$

which is a non-perturbative, highly non-trivial criterion for healthy theories.

"Gravity is the weakest force."

evidence #1

The WGC is satisfied by a litany of healthy field and string theories.

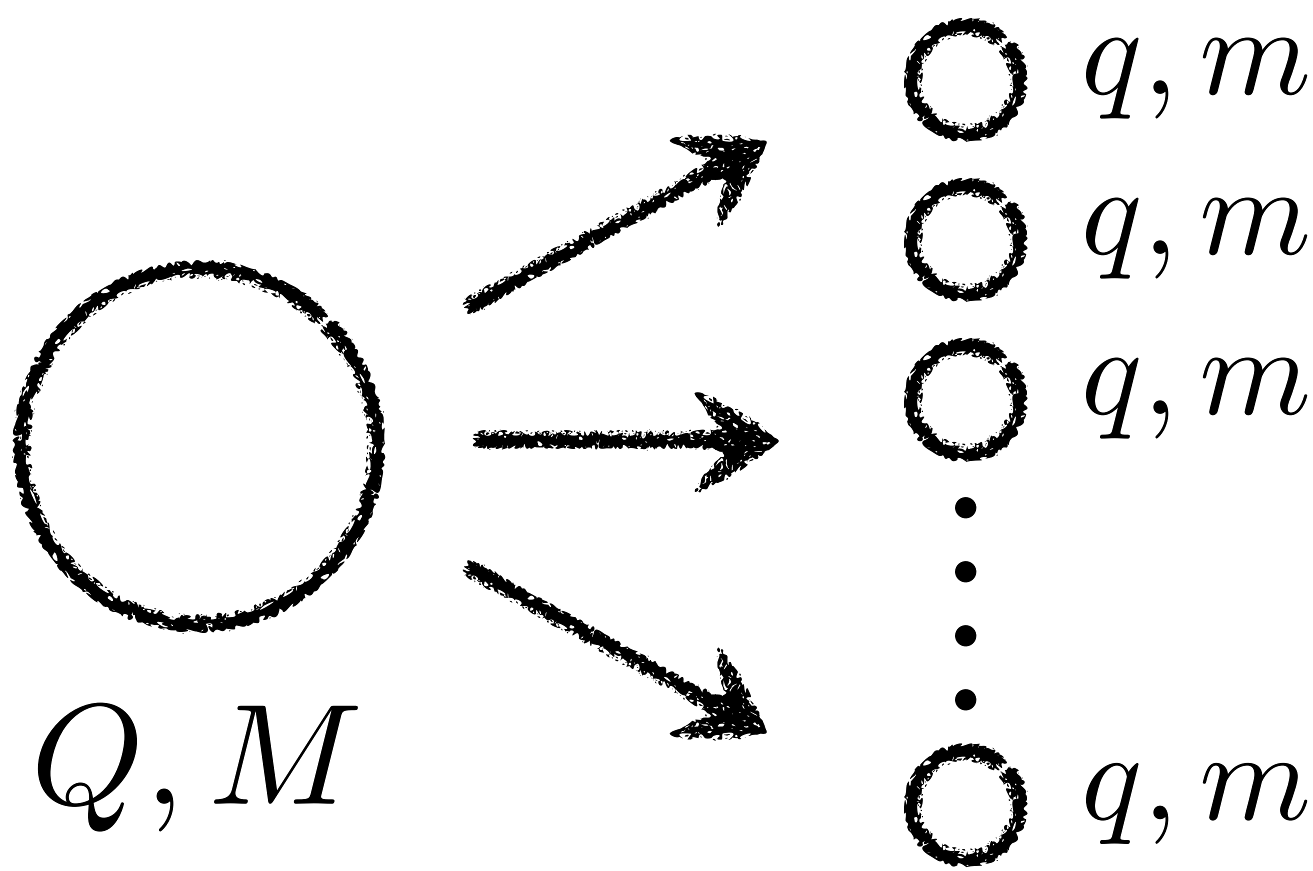
For example, for SU(2) → U(1) gauge theory,

$$g > m_W / m_{\mathrm{Pl}} \xrightarrow{(m_W = gv)} m_{\mathrm{Pl}} > v$$

and similarly for the monopoles.

evidence #2

The authors of the WGC justified it with a Gedanken experiment with black holes:



number of particles in final state $= \quad Q/q$ conservation of charge

total rest mass in final state $= \quad mQ/q < M$ conservation of energy

For an extremal black hole, $Q = M/m_{\mathrm{Pl}}$, so

$$q > m/m_{\mathrm{Pl}}$$

When the WGC criterion fails, extremal black holes are exactly stable.

In such a theory there will be a huge number of stable black hole remnants.

This yields serious pathologies:

- thermodynamic catastrophes
- tension with holography

But we have ignored a crucial effect, which is that charges and masses are renormalized!

$$q(\mu) > m(\mu)/m_{\rm Pl}$$

renormalized
quantities

We should evaluate quantities at pole mass.

Note: WGC can bound a radiatively *unstable* quantity (mass) by a *stable* one (charge).

scalar QED

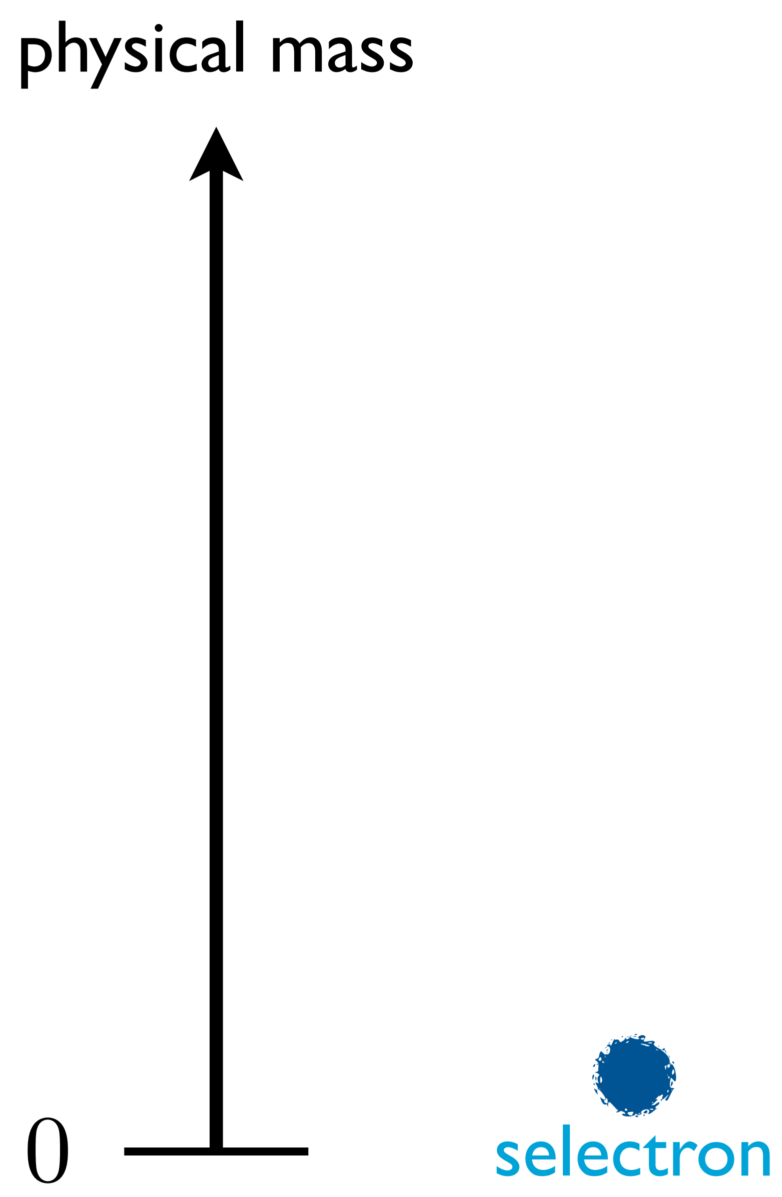
Take the very simplest case of a U(1) charged particle with a hierarchy problem:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + |D_\mu\phi|^2 - m^2|\phi|^2 - \frac{\lambda}{4}|\phi|^4$$

where the "selectron" has charge q:

$$D_\mu = \partial_\mu + iqA_\mu$$

There is fundamental tension between naturalness and the WGC.



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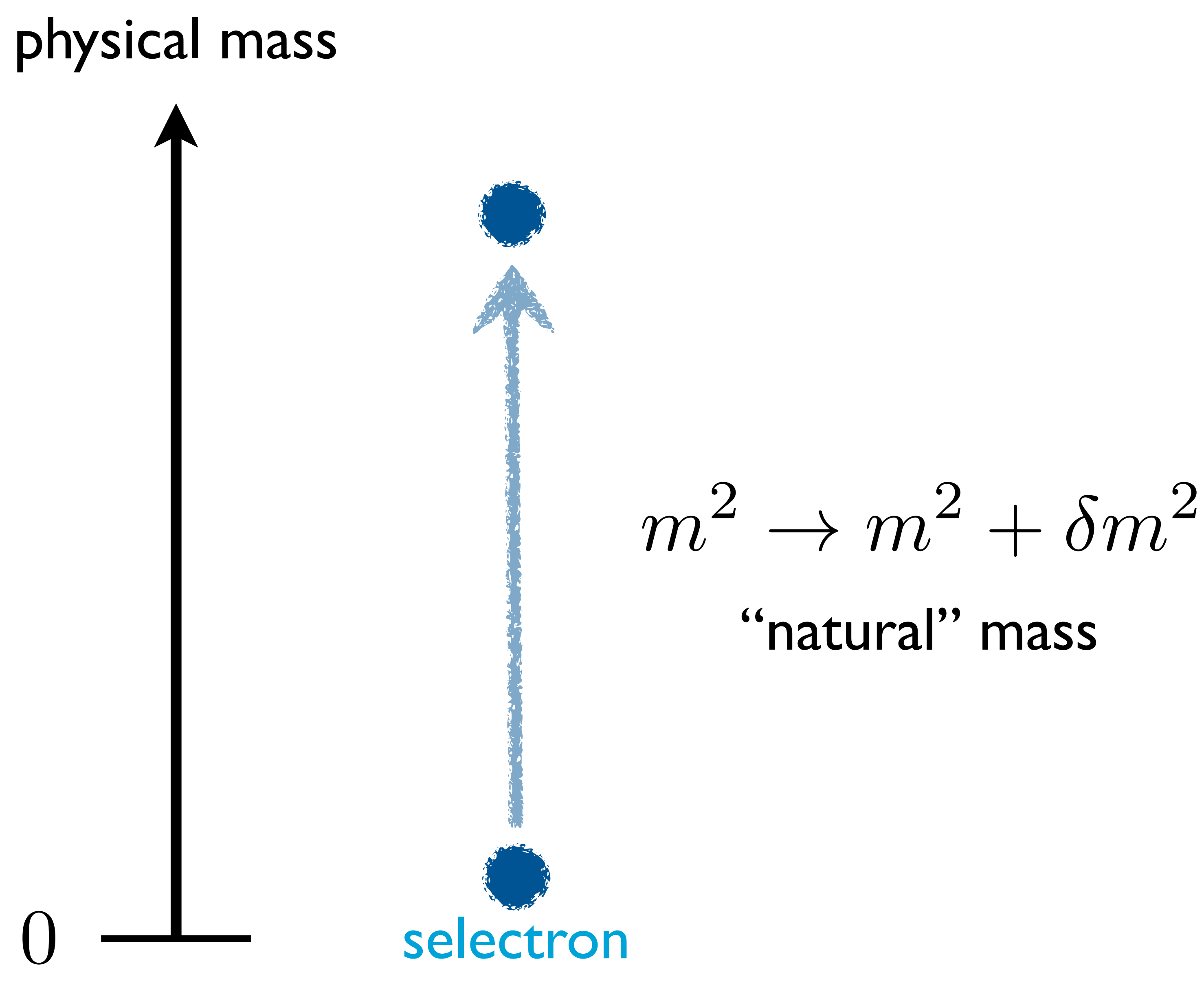

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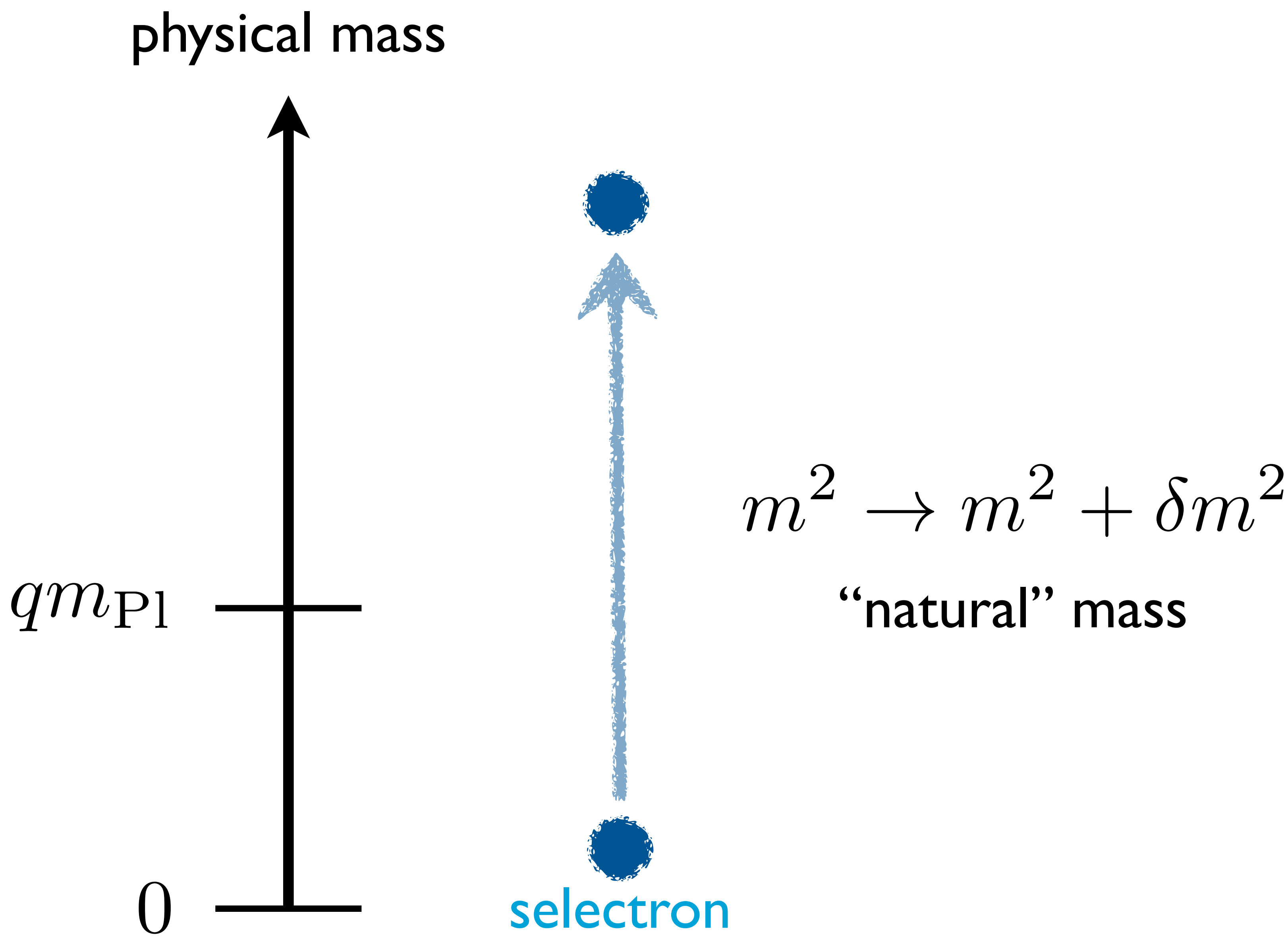

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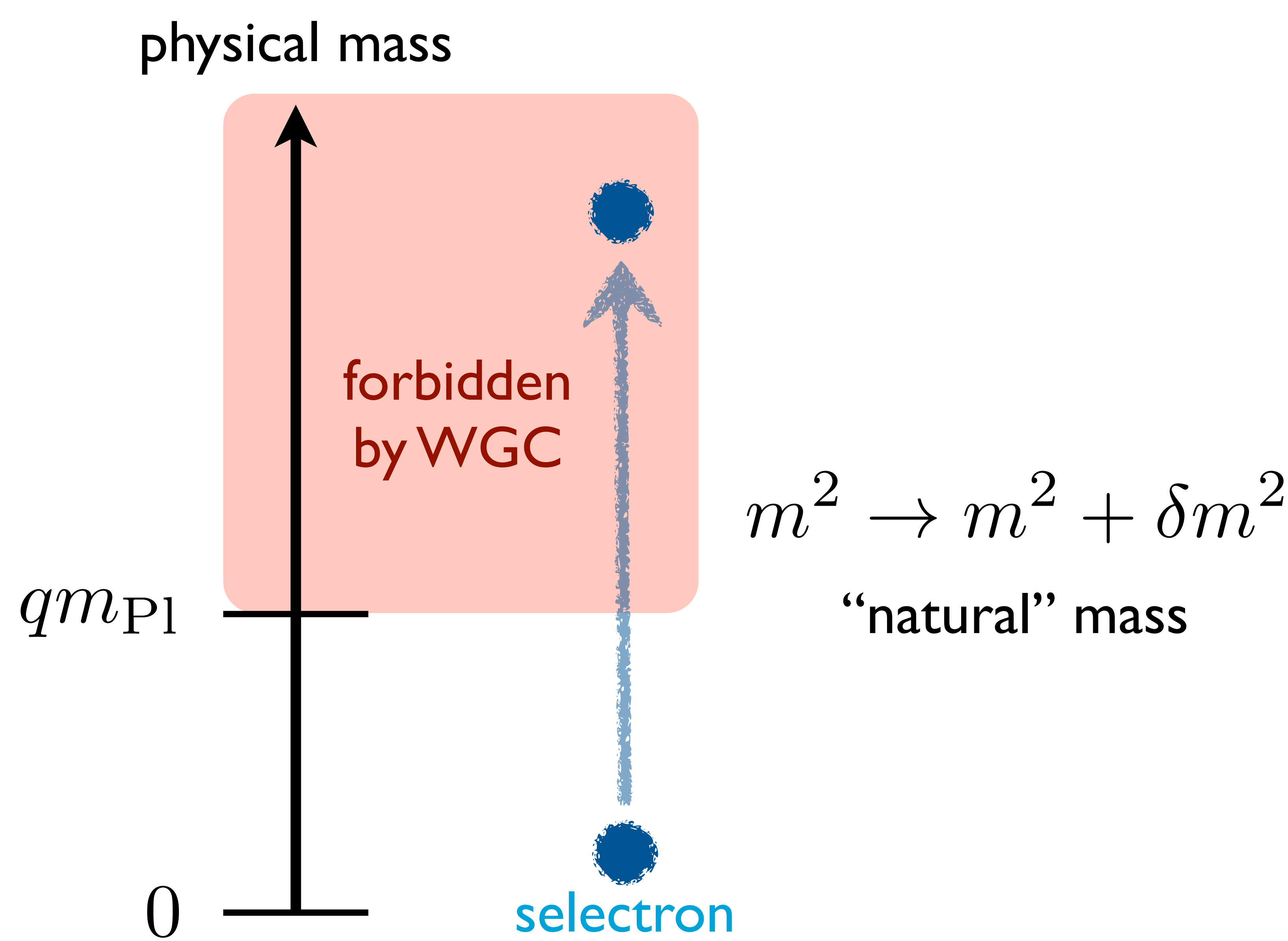

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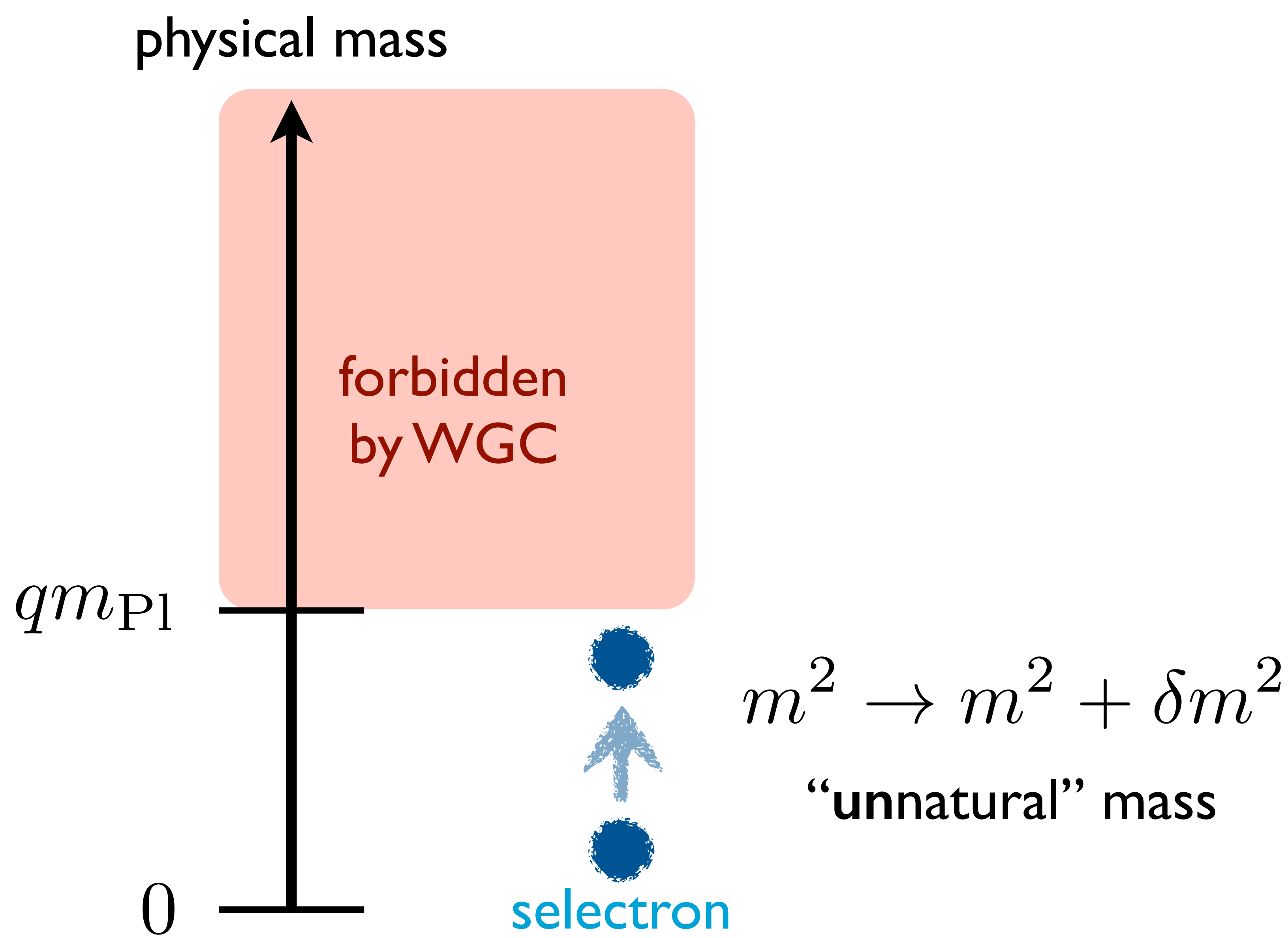

Let's quantify the tension.

$$m^2 \to m^2 + \delta m^2$$

$$\delta m^2 = \frac{\Lambda^2}{16\pi^2}(aq^2 + b\lambda)$$

incalculable coefficients

Naturalness principle: absent symmetries, the physical mass squared is $\sim \delta m^2$, so a, b are $\mathcal{O}(1)$ coefficients.

Setting the physical mass equal to its natural value yields a charge to mass ratio

$$z = q m_{\rm Pl}/m$$

$$= \frac{4\pi m_{\rm Pl}}{\Lambda} \frac{1}{\sqrt{a + b\lambda/q^2}}$$

Since the charged scalar is the only state in the spectrum, the WGC implies

$$z > 1$$

So, the loop cutoff is bounded from above.

$$\Lambda < \frac{4\pi m_{\mathrm{Pl}}}{\sqrt{a}} \qquad q^2 \gg \lambda$$

$$\Lambda < 4\pi m_{\mathrm{Pl}}\sqrt{\frac{q^2}{b\lambda}} \qquad q^2 \ll \lambda$$

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reasonable: cutoff below Planck

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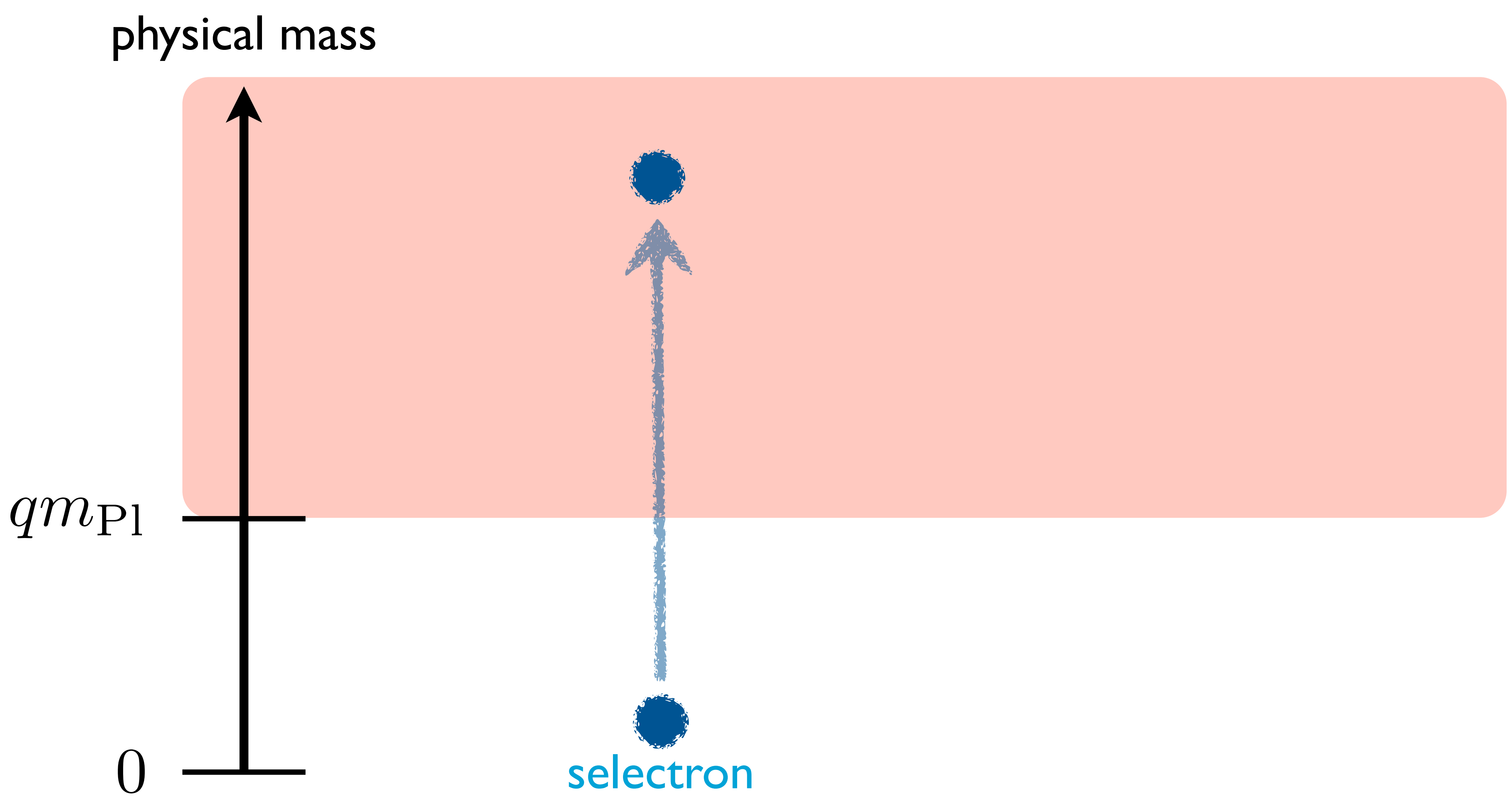
parametrically low cutoff!
(conjectured in original paper)

Naturalness and WGC *can* be reconciled if we revisit and *modify* our premises.

There is an obvious strategy.

Add new degrees of freedom below cutoff.



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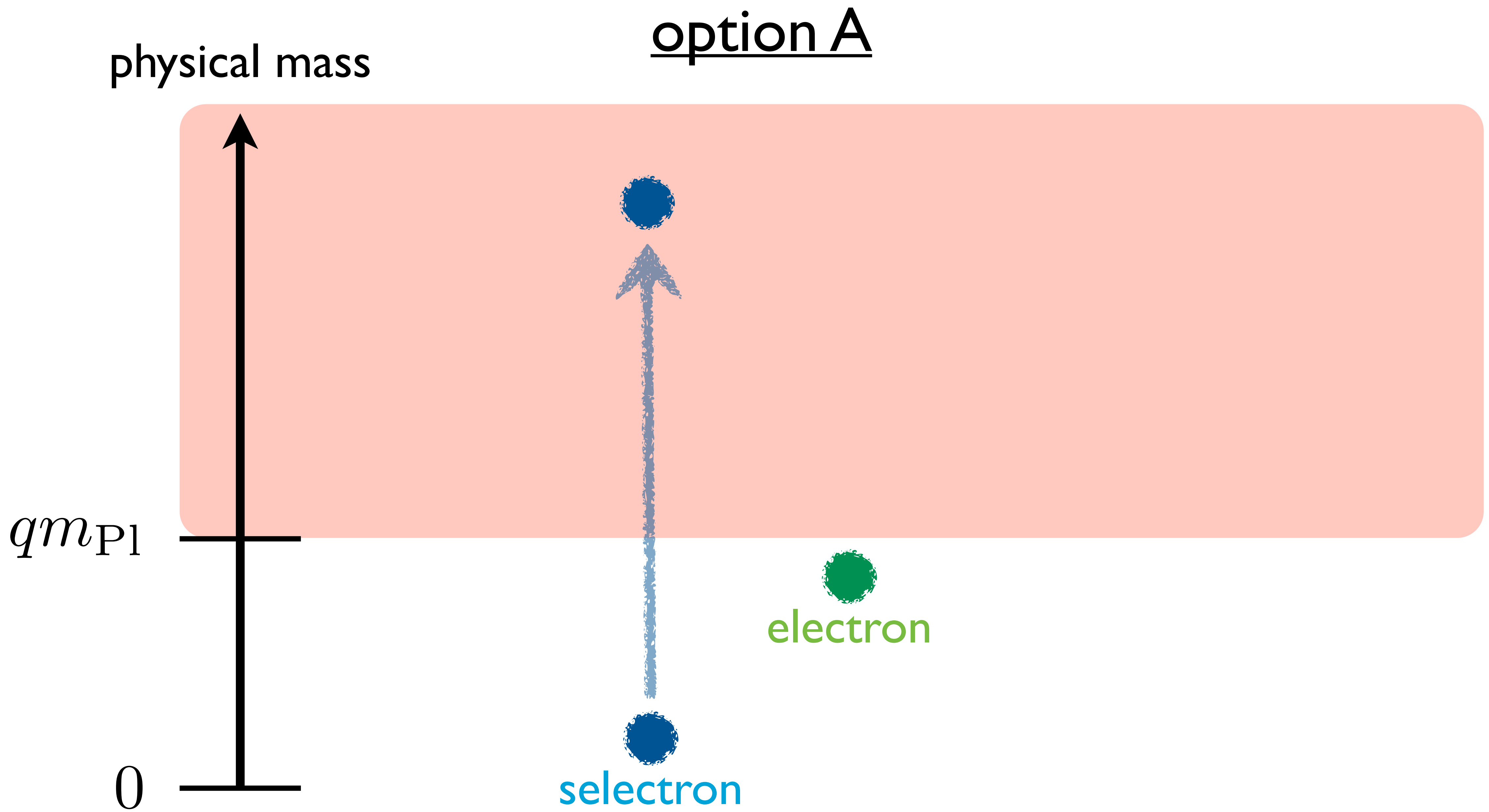

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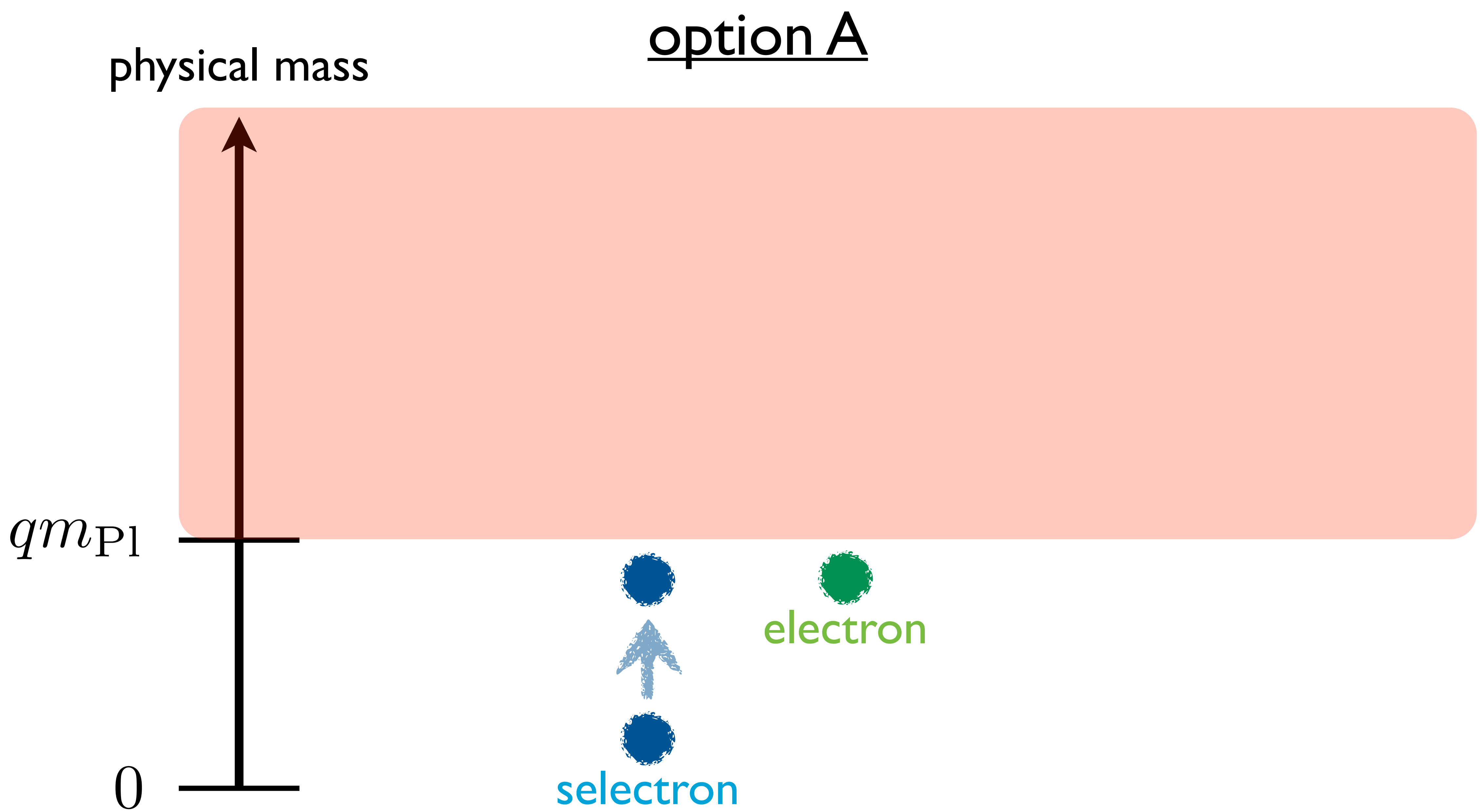

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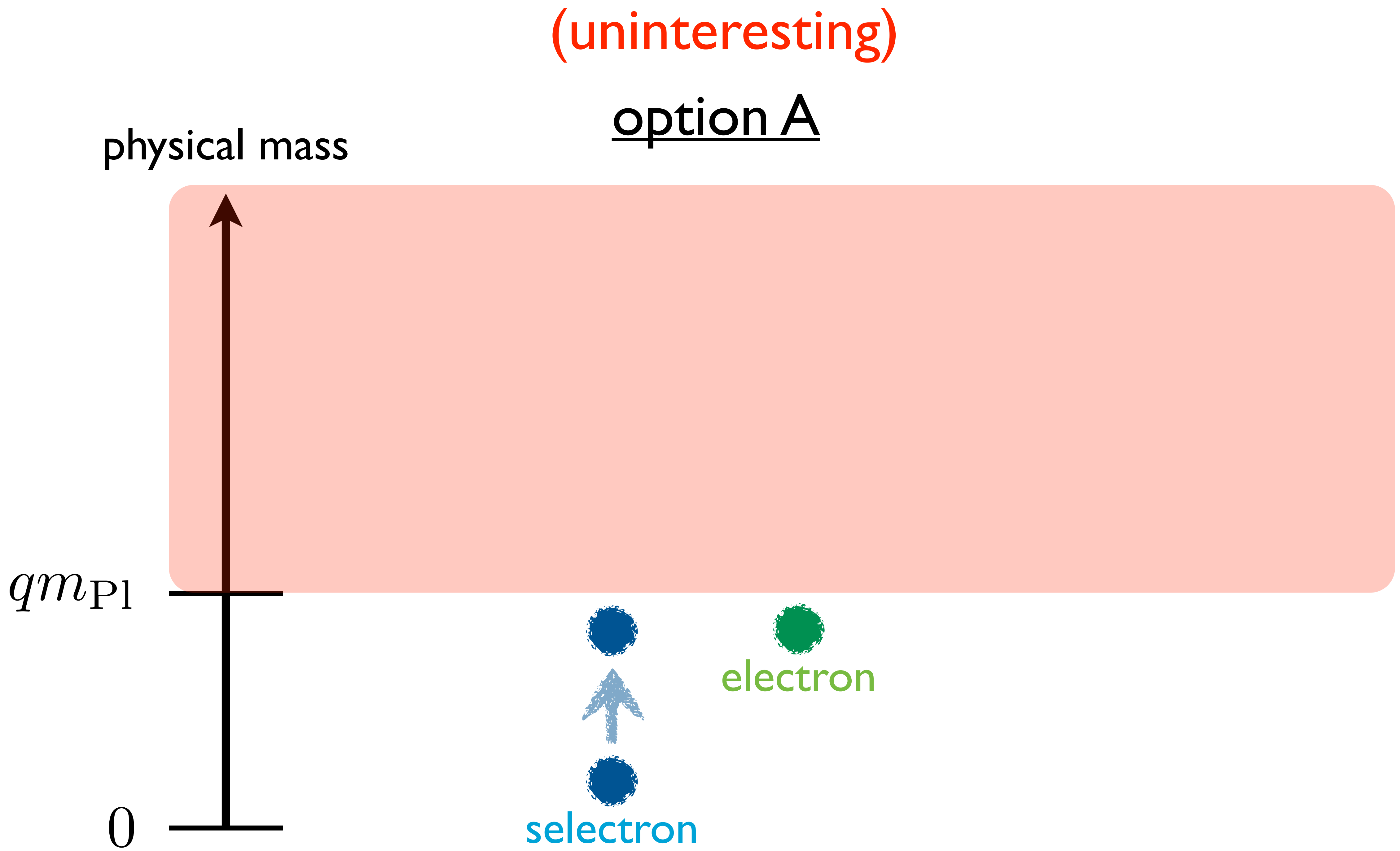

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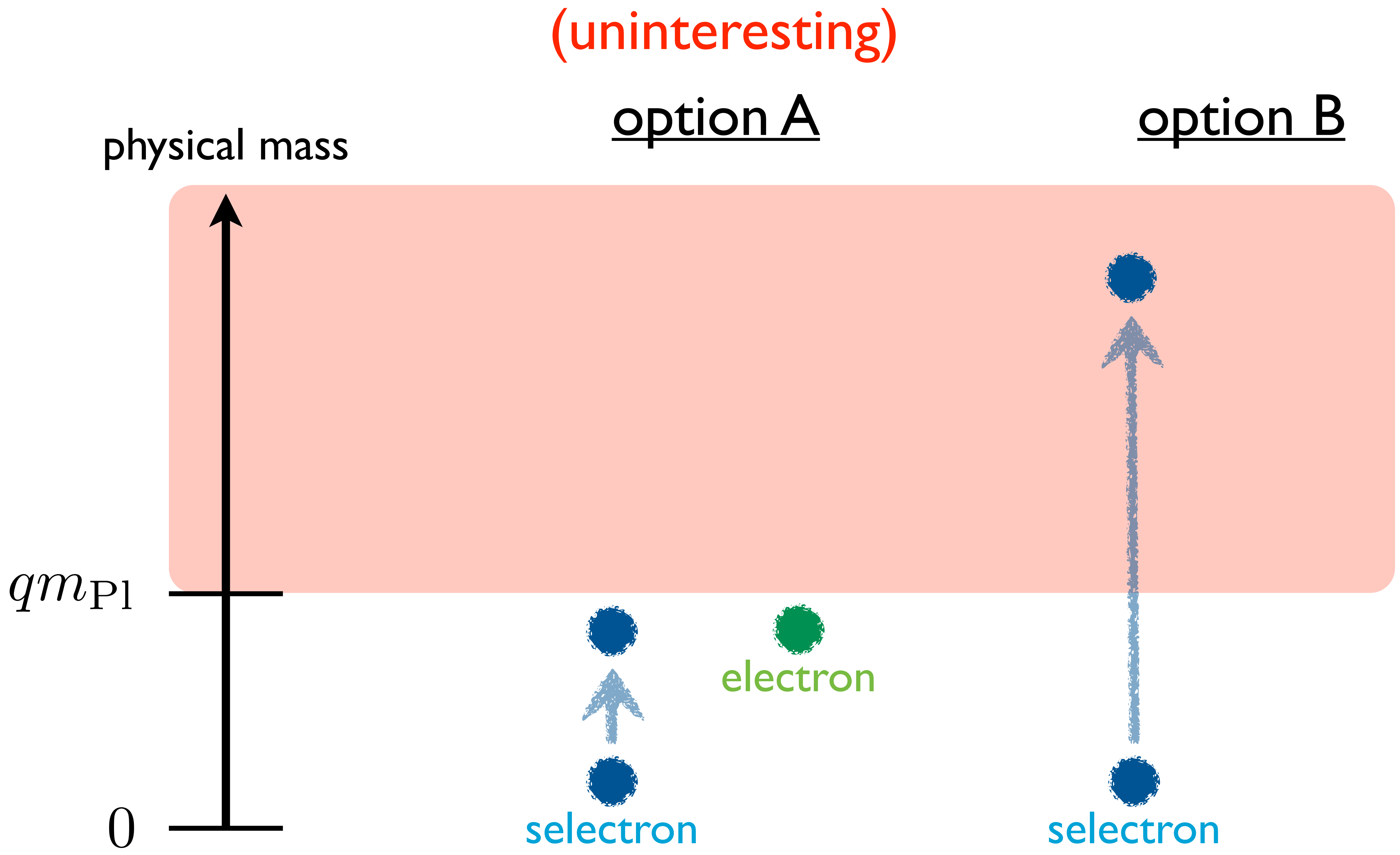

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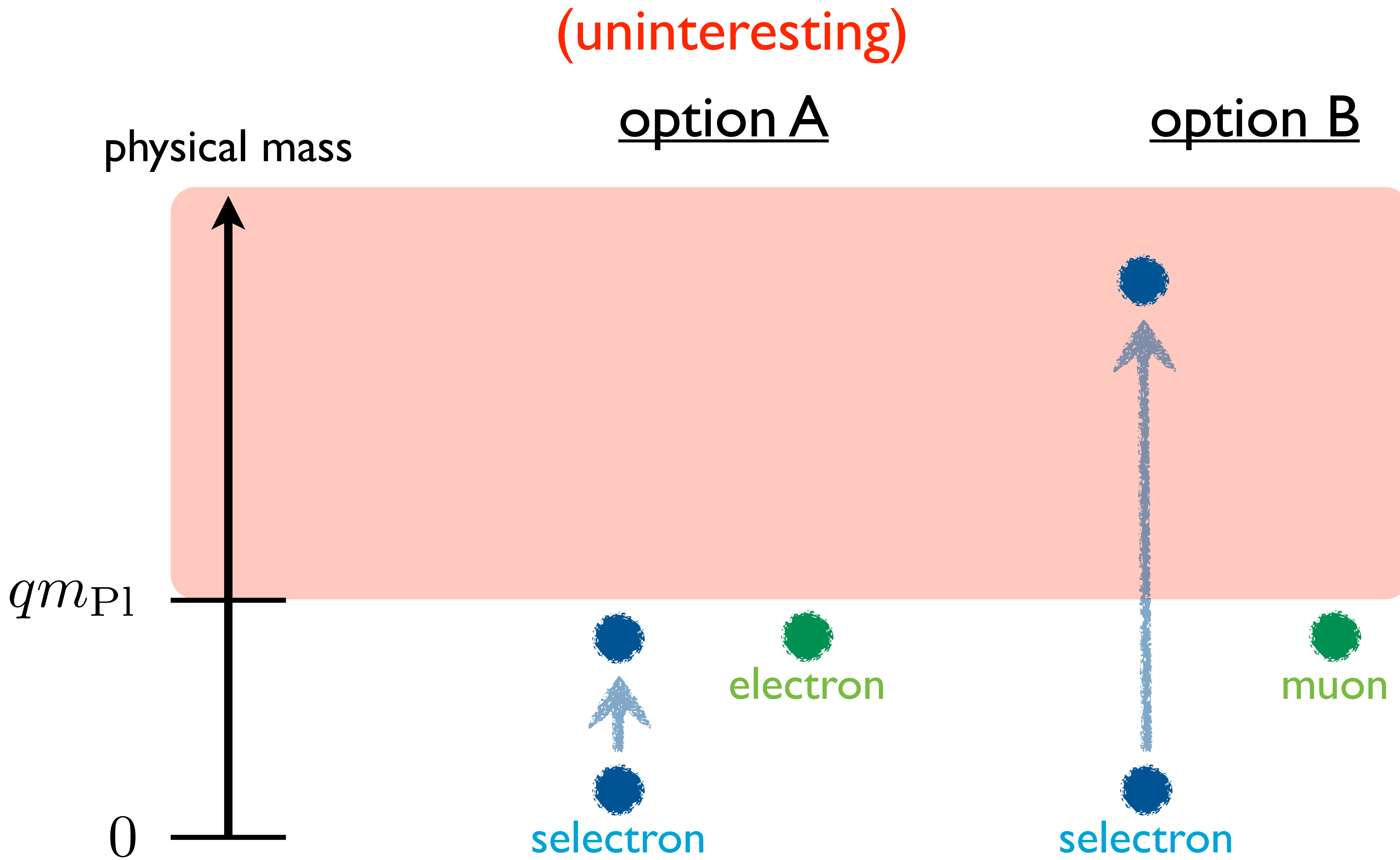

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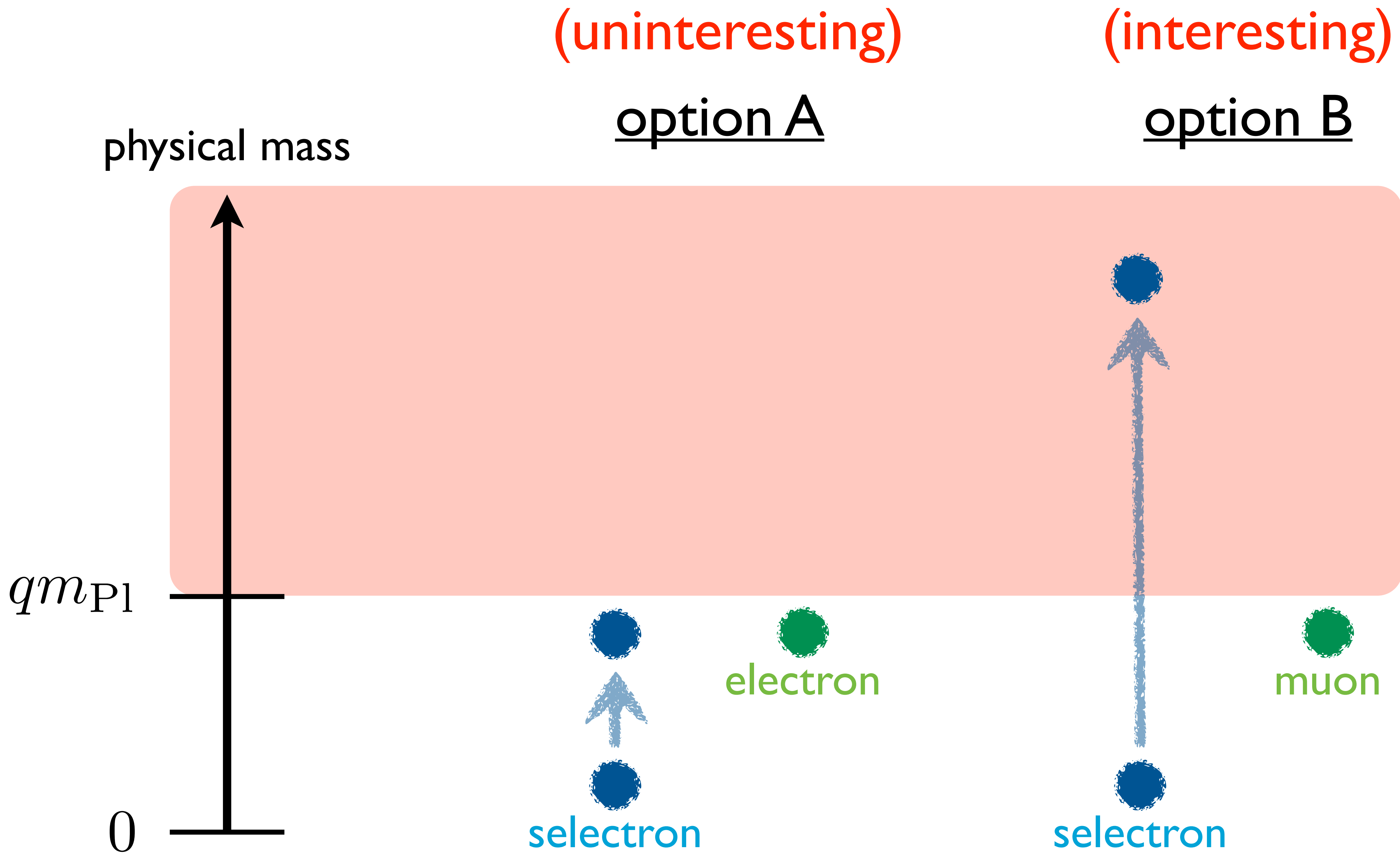

(technically, another option : Higgs phase)

$$\delta m^2 < 0$$

The WGC is ambiguous in the Higgs phase because $[q, m] \neq 0$. Whose mass, charge?

More importantly, black holes do not have Higgsed U(1) hair. No justification for WGC!

Some lessons from scalar QED:

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A new experimental test of naturalness:

False

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A new experimental test of naturalness:

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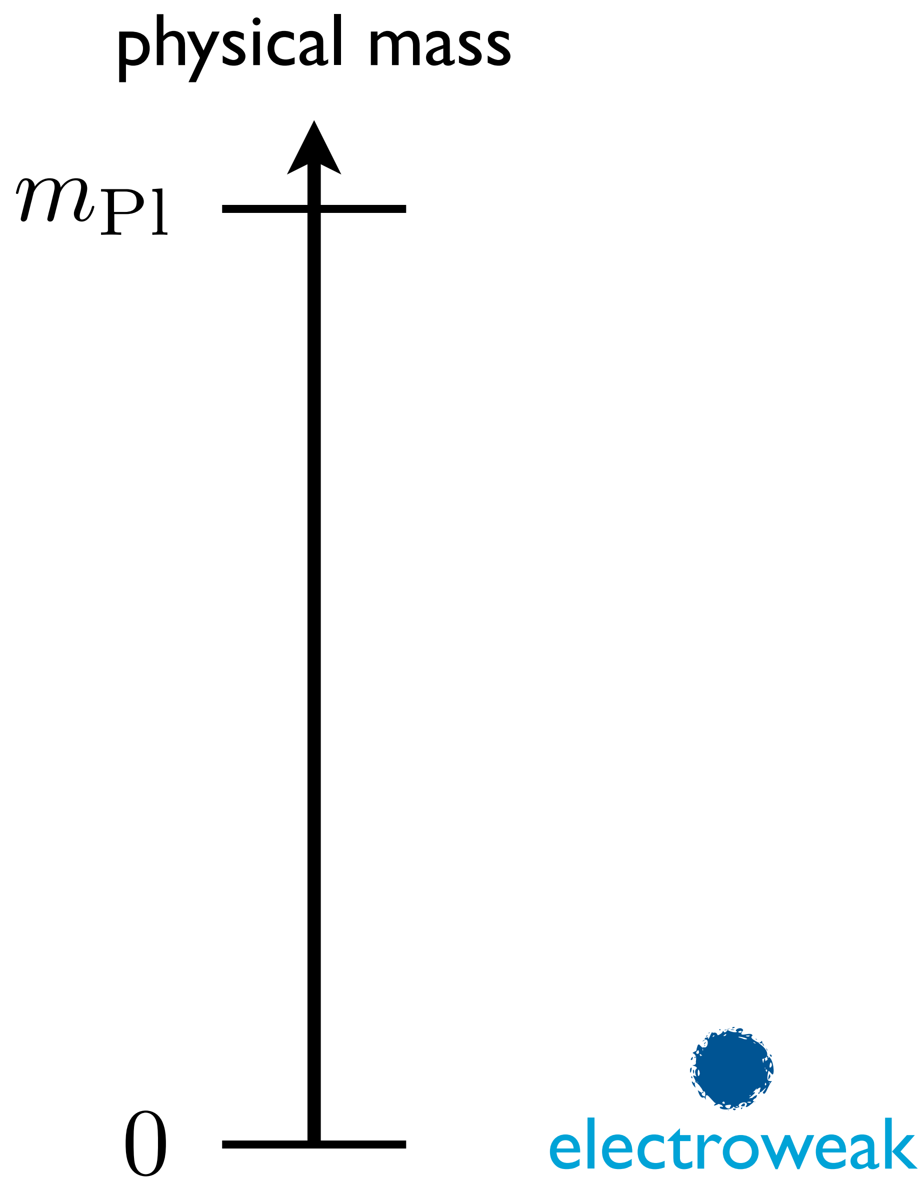
failure of naturalness

Let's apply this to the hierarchy problem.



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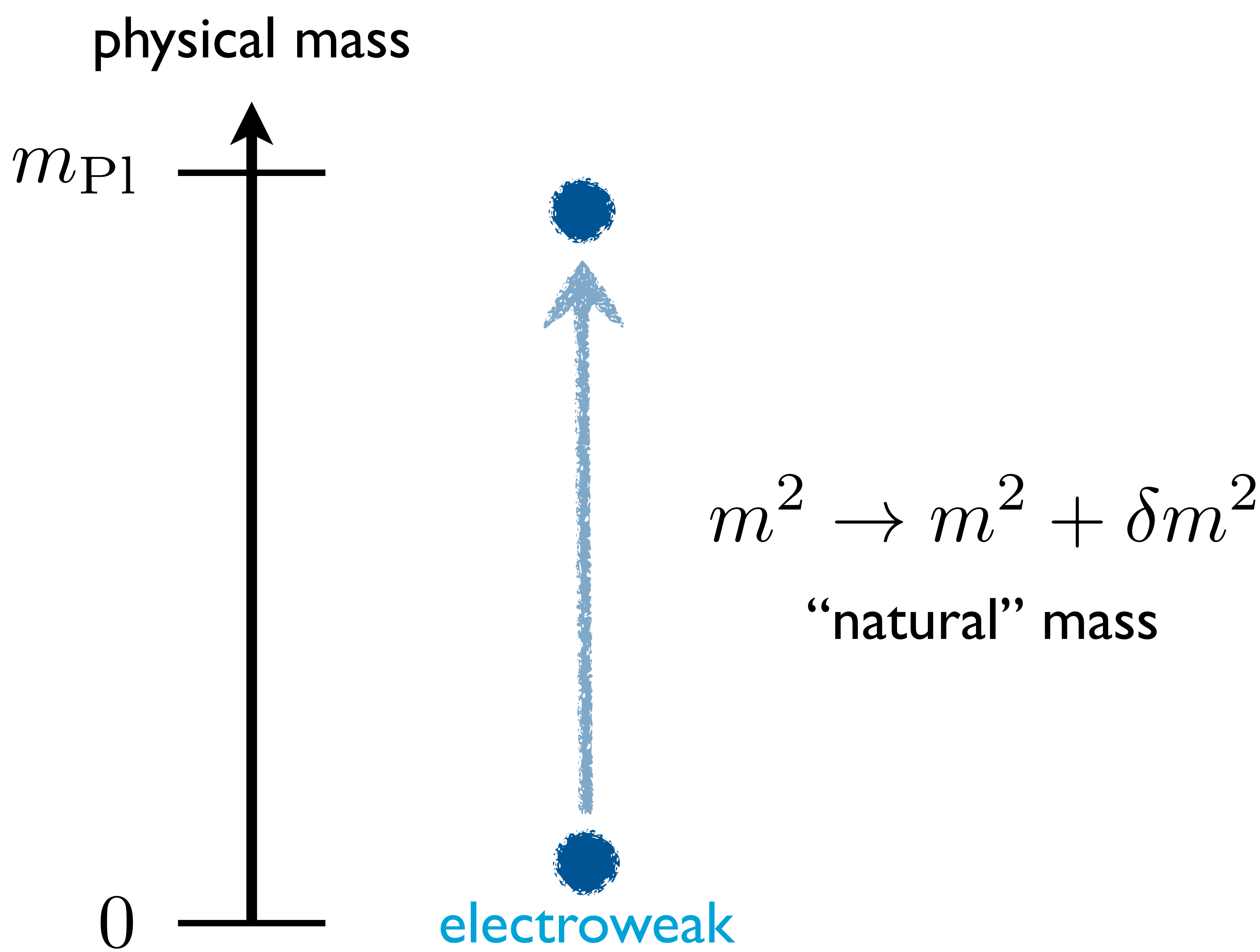

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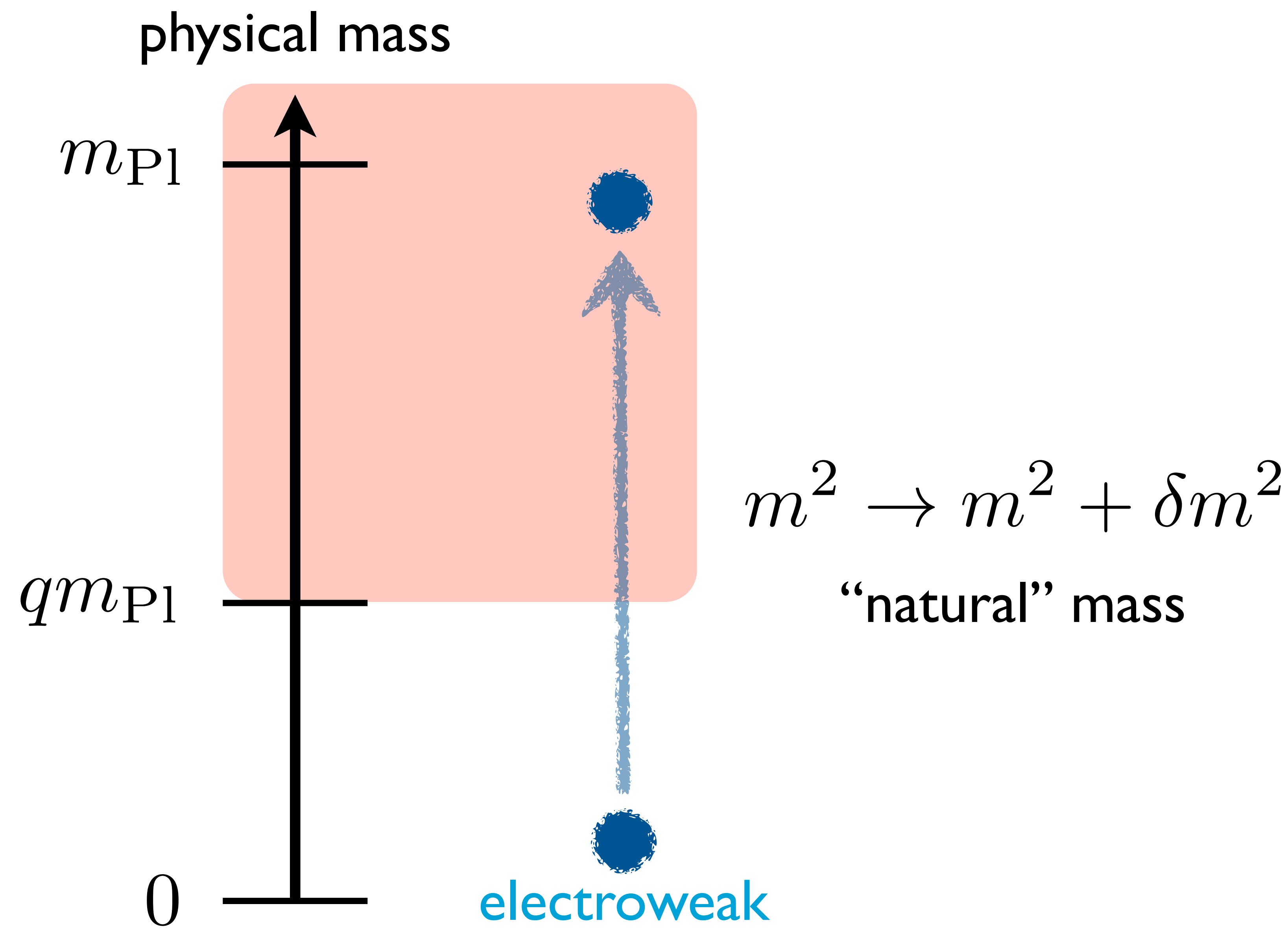

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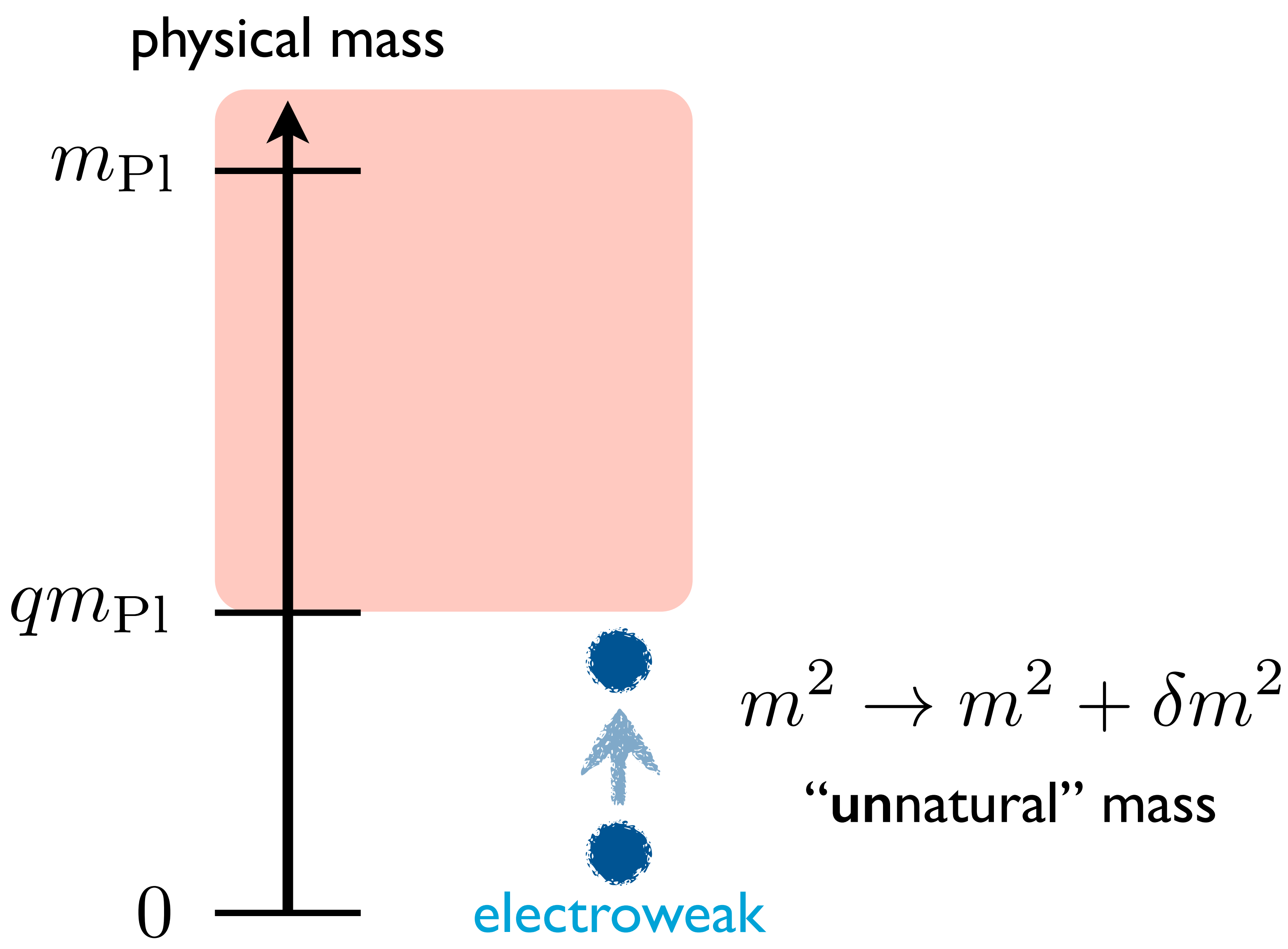

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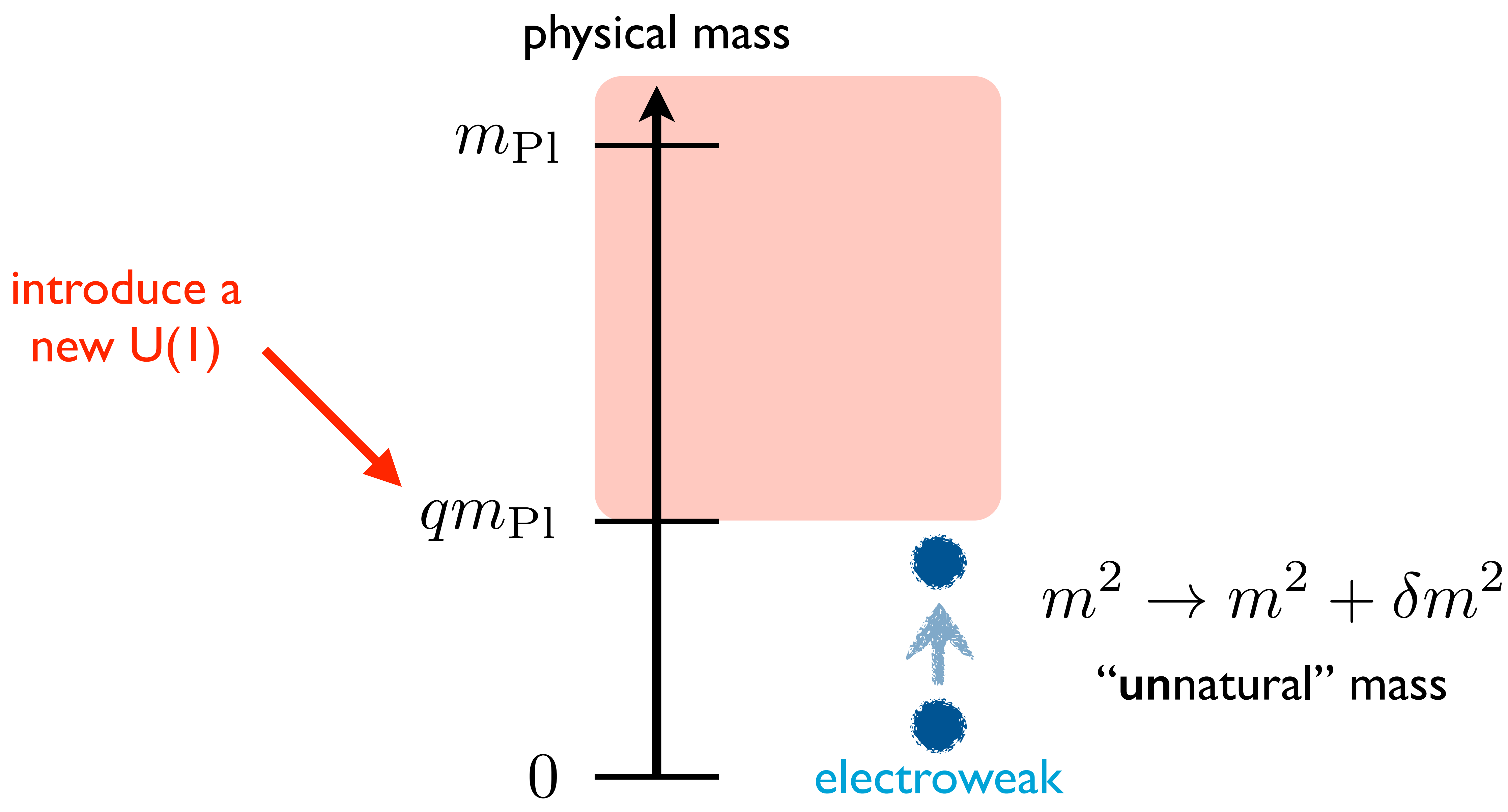

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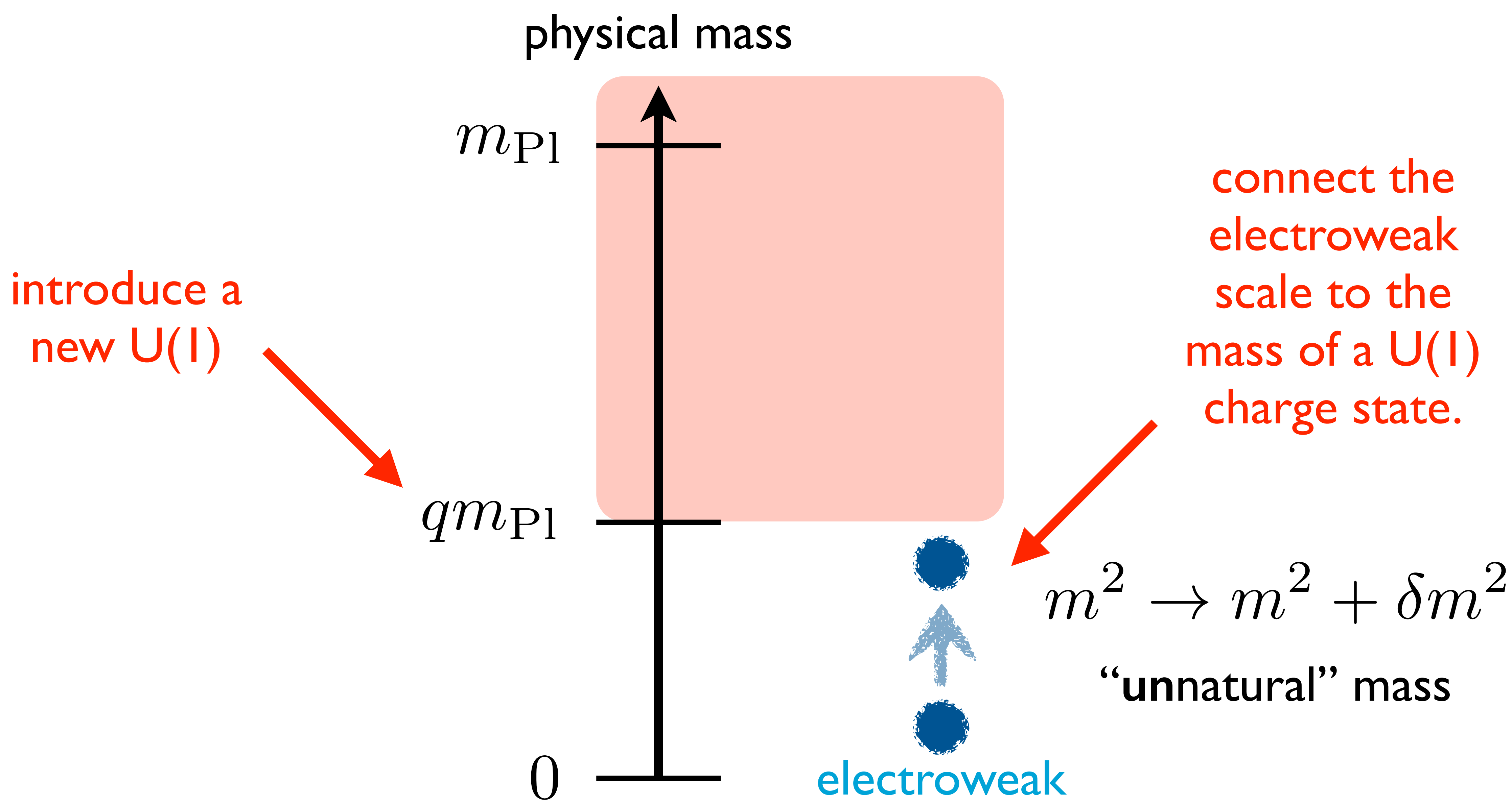
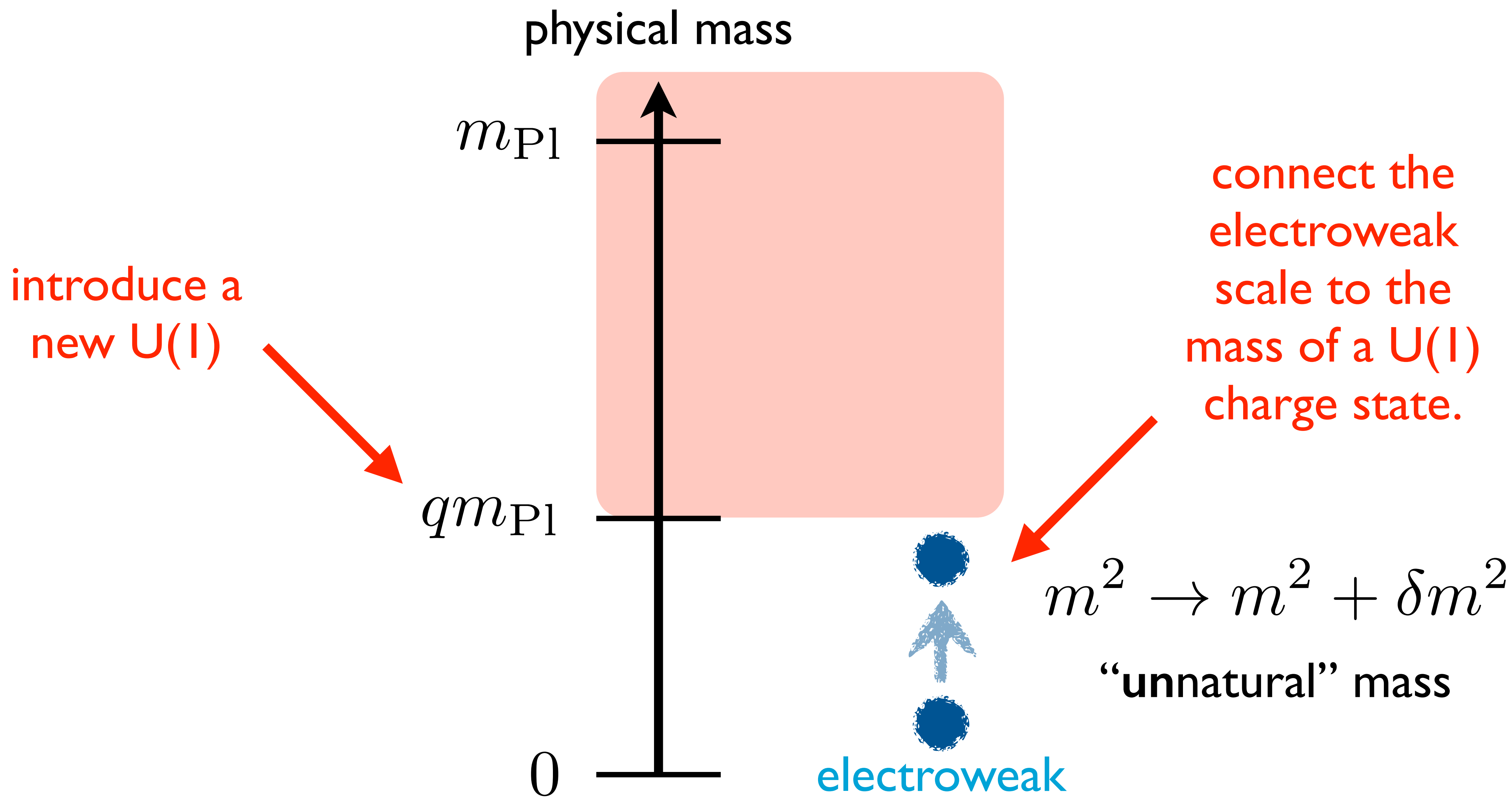


The electroweak scale is unnatural, but only because a natural value is forbidden!



a simple model

Weakly gauge $U(1)_{\mathrm{B-L}}$ with Dirac neutrinos.

$$-\mathcal{L} = m_\nu \bar{\nu}_L \nu_R + \mathrm{h.c.} \qquad m_\nu \sim y_\nu v$$

Assuming that $m_\nu \sim 0.1\ \mathrm{eV}$, we fix

$$q \sim 10^{-29} \quad (\sim m_\nu / m_{\mathrm{Pl}})$$

so that the WGC is marginally satisfied.

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(technically natural) $$q \sim 10^{-29} \quad (\sim m_\nu / m_{\mathrm{Pl}})$$

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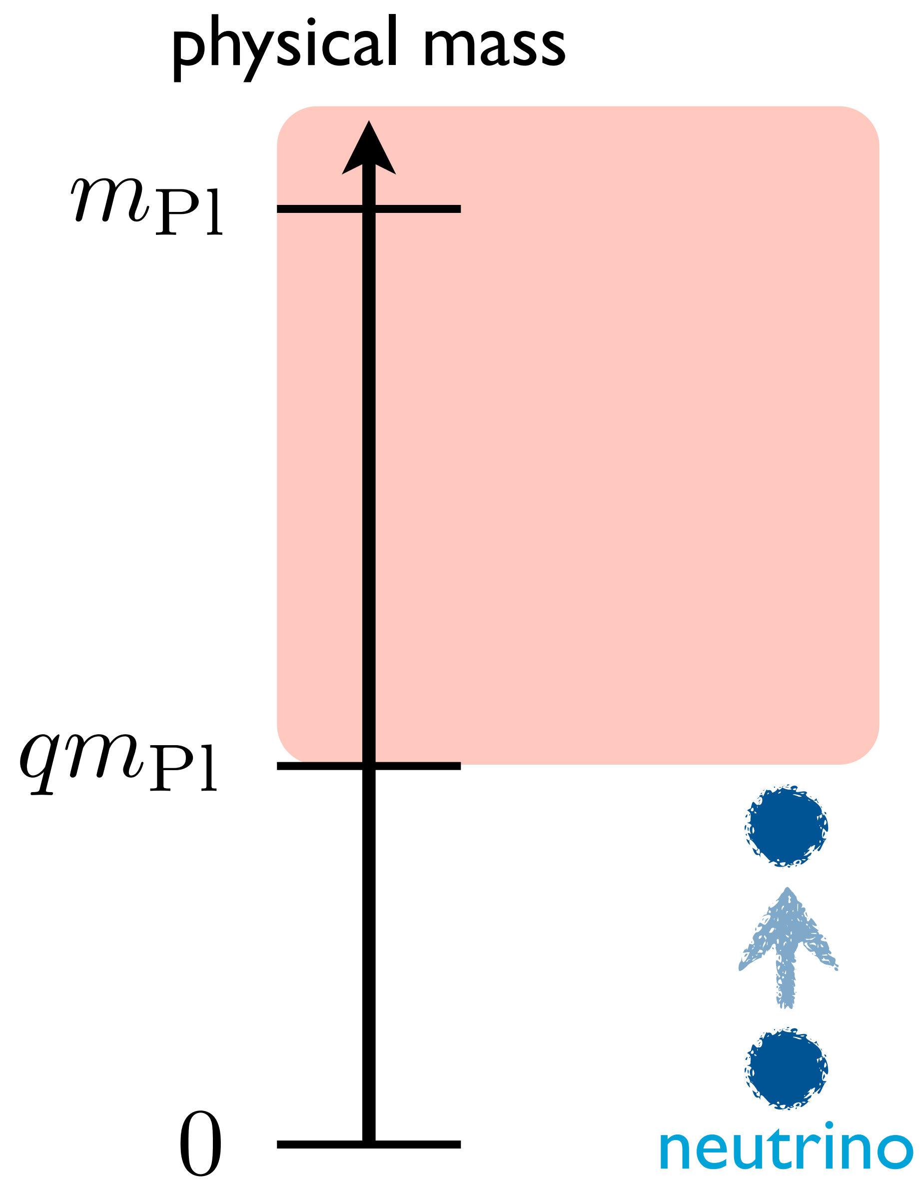
Fixing couplings, were the electroweak scale larger, then the WGC condition would fail.



The model is a proof of concept but it has has a prediction: a massless gauge boson.

There are very stringent limits of fifth forces and violation of equivalence principle:

$$q \lesssim 10^{-24}$$ (torsion balance)

An incredibly small charge!

But, naturalness & small charge → low cutoff.

$$q \lesssim 10^{-24} \quad \longrightarrow \quad \Lambda \lesssim q m_{\mathrm{Pl}} \lesssim \mathrm{keV}$$

Such an extremely low cutoff is not there!

Hence, literally any fifth force observation will exclude the low cutoff conjecture and invalidate the argument from naturalness.

conclusions

- “Running Naturalness” is an irreducible notion of fine-tuning.

- Still, fine-tuning presumes evenly thrown darts in the UV as boundary cond.

- Perhaps “on-shell” is somehow more fundamental than “UV” (cf. S-matrix).

- Perhaps there is UV selection. WGC forbids certain natural theories.

- Milli-charges offer an experimental test.

thanks!