Mixed QCD $\times$ EW corrections to Drell–Yan processes in the resonance region

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in collaboration with

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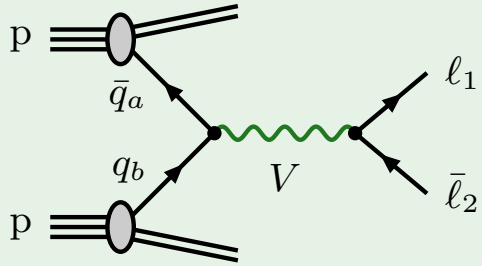
Outline

1. Motivation and introduction
2. Drell–Yan process @ NLO & the Pole Approximation
3. Pole expansion @ NNLO $\mathcal{O}(\alpha_s \alpha)$
4. Summary and outlook

$\mathrm{W}^{\pm}$ and Z production at the LHC

Large cross section & clean experimental signature



$$q\,\bar{q} \to \mathrm{Z}/\gamma^* \to \ell^-\,\ell^+$$
$$u\,\bar{d} \to \mathrm{W}^+ \to \nu_\ell\,\ell^+$$
$$d\,\bar{u} \to \mathrm{W}^- \to \ell^-\,\bar{\nu}_\ell$$

$\hookrightarrow$ One of the most precise probes to test the Standard Model (SM)

Important standard candles

- Detector calibration
- Luminosity monitor
- Constraining quark PDFs
- Z', W' searches
  (high $M_{\ell\ell}$, $M_{\mathrm{T},\nu\ell}$ tails)
- **Precision measurements**
  M_{W}, $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$

M_{W} measurement

- **Tevatron**: $M_{\mathrm{W}} = 80.387 \pm 0.016\ \mathrm{GeV}$
  (most precise measurement of M_{W} to date!)
- **LHC**: aimed precision of $\Delta M_{\mathrm{W}} \lesssim 10\ \mathrm{MeV}$
- Fits to kinematic distributions

Largest missing piece:
mixed QCD$\times$EW corrections
$\sim$ %-level effects expected!

$\mathcal{O}(\alpha_s\alpha)$ corrections needed!
(especially around the resonance)

Theoretical Status

QCD NNLO $\mathcal{O}(\alpha_s^2)$ (differential) [Melnikov, Petriello '06] [Catani, et al.'09]
resummation [Moch, Vermaseren, Vogt '05] [Laenen, Magnea '06]
PS matching [Frixione Webber] [Alioli, et al. '08] [Hamilton, Richardson, Tully '08]

EW NLO $\mathcal{O}(\alpha)$ W [Dittmaier, Krämer '02] [Baur, Wackeroth '04]
Z [Baur, et al. '02] [Dittmaier, Huber '10]

multi-photon radiation [Baur, Stelzer '00] [Placzek, Jadach '03] [Calame, et al. '04]

\+ much more...

Approaches to Combination

- Soft-gluon emission + final-state QED [Cao, Yuan '04]
- NLO EW + QCD parton-shower [Richardson, et al. '12]
- NLO (EW+QCD) + PS matching [Bernaciak, Wackeroth '12] [Barze, et al. '12, '13]

Steps towards NNLO QCD×EW $\mathcal{O}(\alpha_s\alpha)$ (far from complete)

- NLO EW $\mathcal{O}(\alpha)$ to V + jet (off-shell + decay) [Denner, Dittmaier, Kasprzik, Mück '09, '11]
- Decay widths [Czarnecki, Kühn '96](Z) [Kara '13](W)
- $Z\bar{f}f$ vertex [Kotikov, Kühn, Veretin '08]
- 2-loop on-shell V production [Bonciani '11]
- 2-loop virtual QCD×QED [Kilgore, Sturm '12]

1 Motivation and introduction

2 Drell–Yan process @ NLO & the Pole Approximation

3 Pole expansion @ NNLO $\mathcal{O}(\alpha_s \alpha)$

4 Summary and outlook

Pole expansion: Overview

[Stuart '91] [H.Veltman '94]
[Aeppli, v.Oldenborgh, Wyler '94]

Aim: Improve the theoretical prediction in resonance region
$\hookrightarrow$ Expansion about complex pole $\mu_V^2 = M_V^2 - \mathrm{i} M_V \Gamma_V$ $\rightsquigarrow$ leading: $(p_V^2 - \mu_V^2)^{-1}$

$$\mathcal{M} = \frac{W(p_V^2)}{p_V^2 - M_V^2 + \Sigma(p_V^2)} + N(p_V^2)$$

W: resonant part, N: non-resonant part, $\Sigma(p_V^2)$: self-energy of V

- $\left[p_V^2 - M_V^2 + \Sigma(p_V^2)\right]^{-1}$: Dyson-resummed propagator
  $\rightsquigarrow$ complex pole @ $p_V^2 = \mu_V^2$ **gauge invariant**
- Residue $W(\mu_V^2)/[1 + \Sigma'(\mu_V^2)]$ **gauge invariant**

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$$\mathcal{M} = \frac{W(p_V^2)}{p_V^2 - M_V^2 + \Sigma(p_V^2)} + N(p_V^2)$$

$$= \underbrace{\frac{W(\mu_V^2)}{p_V^2 - \mu_V^2} \frac{1}{1 + \Sigma'(\mu_V^2)}}_{\text{“factorizable” corrections}} + \underbrace{\left[\frac{W(p_V^2)}{p_V^2 - M_V^2 + \Sigma(p_V^2)} - \frac{W(\mu_V^2)}{p_V^2 - \mu_V^2} \frac{1}{1 + \Sigma'(\mu_V^2)} \right]}_{\text{“non-factorizable” corrections + non-res.}} + N(p_V^2)$$

- $\left[p_V^2 - M_V^2 + \Sigma(p_V^2)\right]^{-1}$: Dyson-resummed propagator
  $\rightsquigarrow$ complex pole @ $p_V^2 = \mu_V^2$ **gauge invariant**
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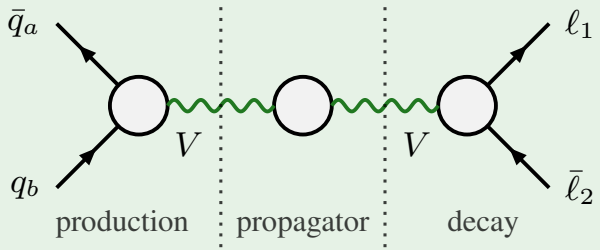
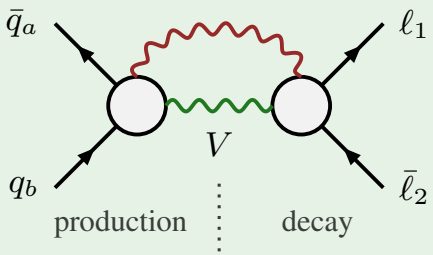
Aim: Improve the theoretical prediction in resonance region
$\hookrightarrow$ Expansion about complex pole $\mu_V^2 = M_V^2 - \mathrm{i} M_V \Gamma_V$ (gauge-invariant!)

$$\mathcal{M} = \underbrace{\frac{W(\mu_V^2)}{p_V^2 - \mu_V^2} \frac{1}{1+\Sigma'(\mu_V^2)}}_{\text{“factorizable” corrections}} + \underbrace{\left[\frac{W(p_V^2)}{p_V^2 - M_V^2 + \Sigma(p_V^2)} - \frac{W(\mu_V^2)}{p_V^2 - \mu_V^2} \frac{1}{1+\Sigma'(\mu_V^2)}\right]}_{\text{“non-factorizable” corrections + non-res.}} + N(p_V^2)$$



- on-shell production & decay
  $\hookrightarrow$ fact. ini $(2 \to 1)$
  $\hookrightarrow$ fact. fin $(1 \to 2)$
  $\hookrightarrow$ propagator $(1 \to 1)$
  (taken care by on-shell scheme)



- connect production & decay
  resonant contribution
  $\leftrightarrow$ only soft-photon exchange
  $\hookrightarrow$ non-fact. $(2 \to 2)$

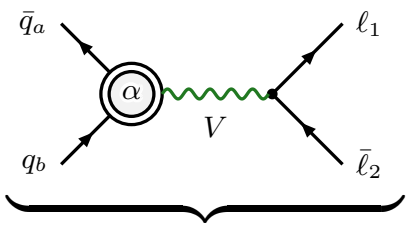
$\Rightarrow$ Simplifications compared to the full off-shell calculation $(2 \to 2)$ [Dittmaier, Krämer '01], etc.

Virtual factorizable corrections

Cut a V propagator $\longrightarrow$ two disconnected diagrams

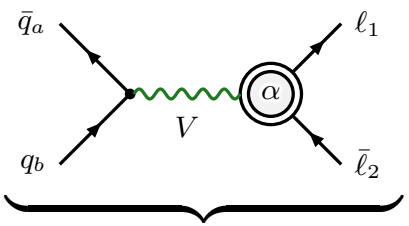
$$\mathcal{M}^{\bar{q}_a q_b \to \ell_1 \bar{\ell}_2}_{\mathrm{V_{ew}},\mathrm{fact}} = \sum_\lambda \frac{\mathcal{M}^{\bar{q}_a q_b \to V}_{\mathrm{V_{ew}}}(\lambda)\, \mathcal{M}^{V \to \ell_1 \bar{\ell}_2}_{\mathrm{B}}(\lambda) \;+\; \mathcal{M}^{\bar{q}_a q_b \to V}_{\mathrm{B}}(\lambda)\, \mathcal{M}^{V \to \ell_1 \bar{\ell}_2}_{\mathrm{V_{ew}}}(\lambda)}{p_V^2 - \mu_V^2}$$

$=$



$\rightsquigarrow$ fact. ini

$+$

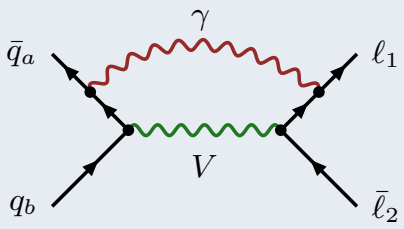


$\rightsquigarrow$ fact. fin

- explicit propagator $(p_V^2 - \mu_V^2)^{-1}$
- $\sum_\lambda$: spin correlation
- gauge invariance $\leftrightarrow$ on-shell projection $(p_V^2 = M_V^2)$

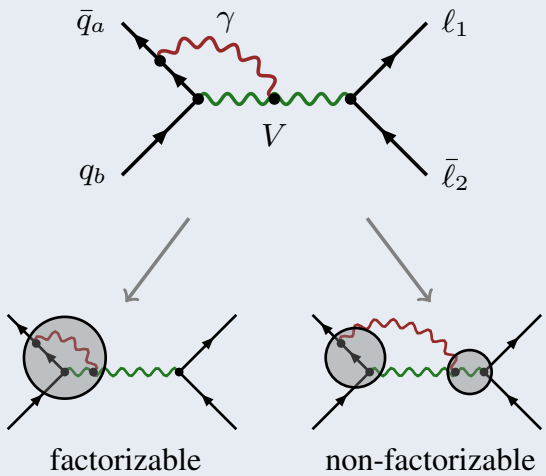
Virtual non-factorizable corrections

Manifestly non-factorizable



- no "explicit" propagator $(p_V^2 - \mu_V^2)^{-1}$

Not manifestly non-factorizable

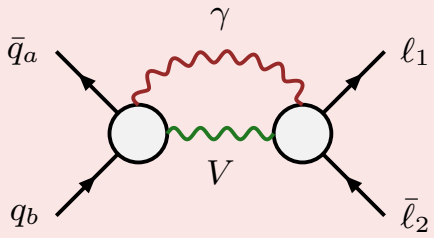


- explicit propagator $(p_V^2 - \mu_V^2)^{-1}$
- gauge-invariant definition: $\Big[\text{full} - \text{fact.}\Big]_{p_V^2 \to M_V^2}$
- origin:

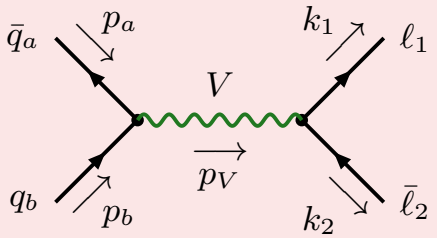
$$\left\{ \begin{array}{c} \text{infrared regularization } (1/\epsilon) \\ \text{vs.} \\ \text{on-shell limit } (p_V^2 \to M_V^2) \end{array} \right\} \text{ do not commute!}$$

Only soft region ($|q^\mu| \lesssim \Gamma_V$) leads to resonant contributions!

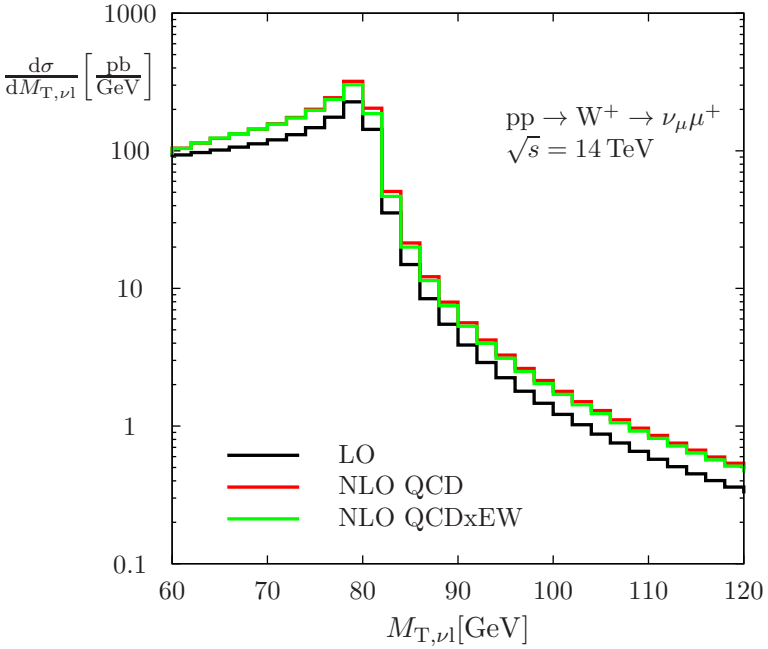
$\Rightarrow$



$\sim \delta^{\bar{q}_a q_b \to \ell_1 \bar{\ell}_2}_{\mathrm{V_{ew}, nf}}$

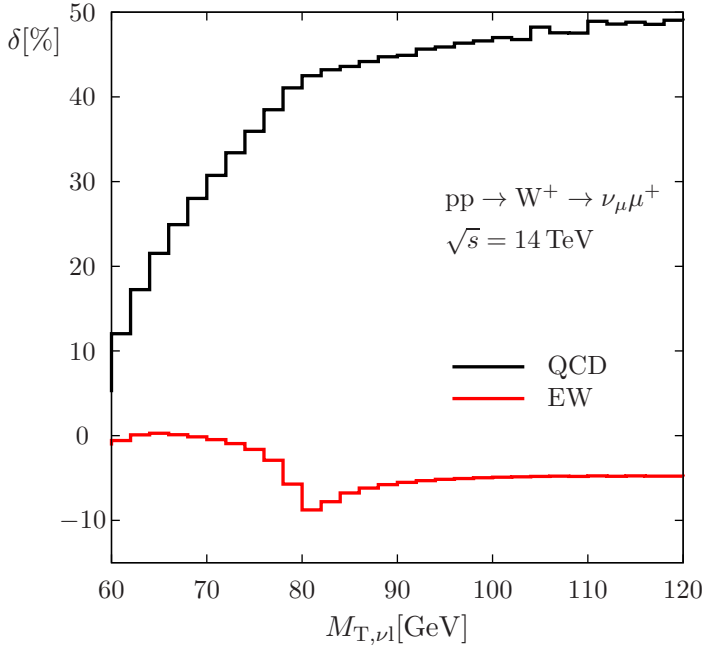


W^+: $M_{\mathrm{T},\ell\nu}$ distribution



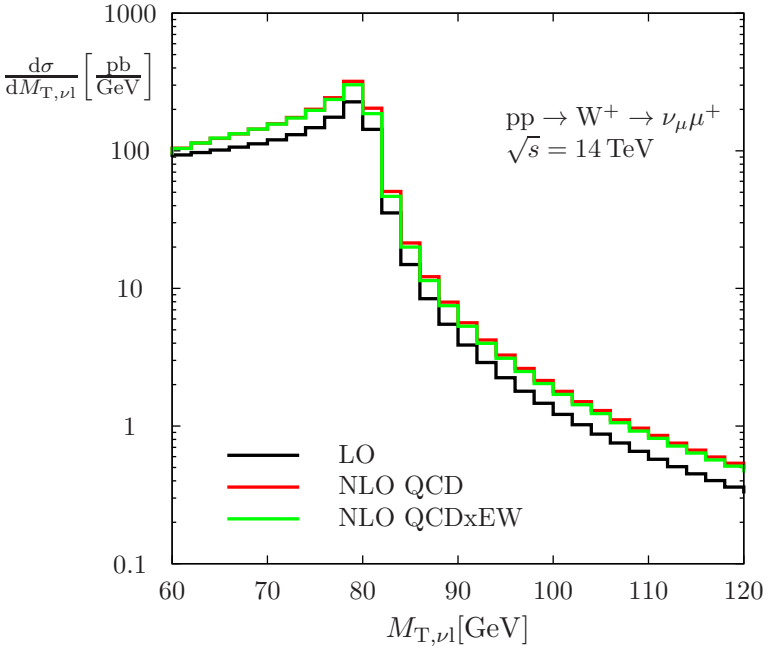
$$M_{\mathrm{T},\ell\nu} = \sqrt{2(E_{\mathrm{T},\ell}\not{E}_{\mathrm{T}} - \mathbf{p}_{\mathrm{T},\ell}\cdot\not{\mathbf{p}}_{\mathrm{T}})}$$

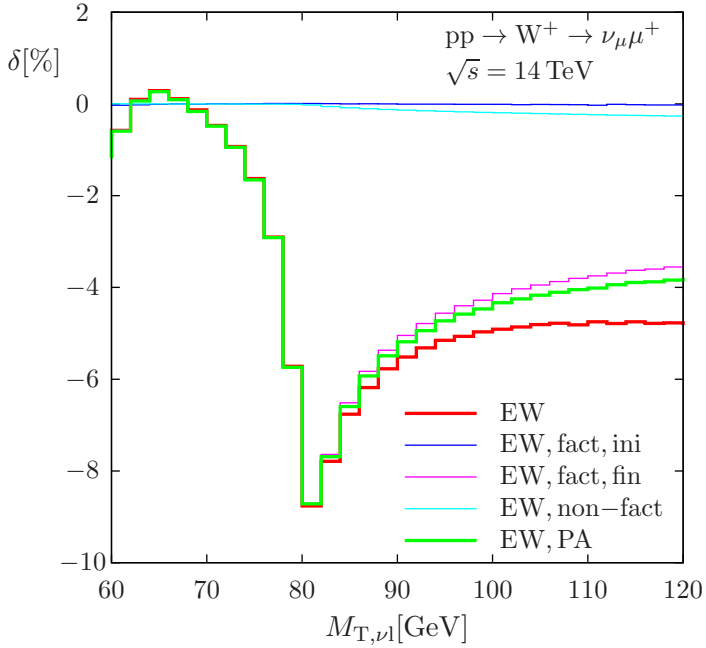
- most important distribution for the determination of M_{W}



- **EW**: significant shape distortion

W^+: $M_{T,\ell\nu}$ distribution



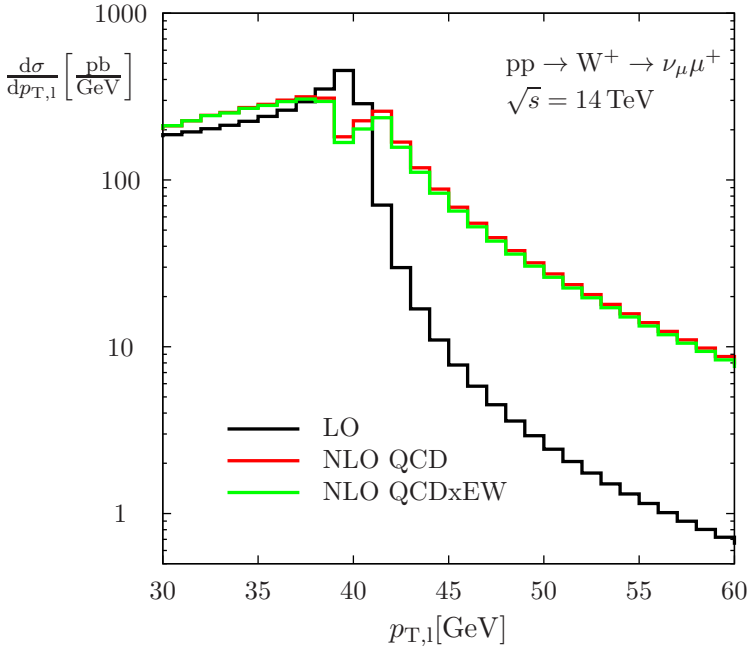


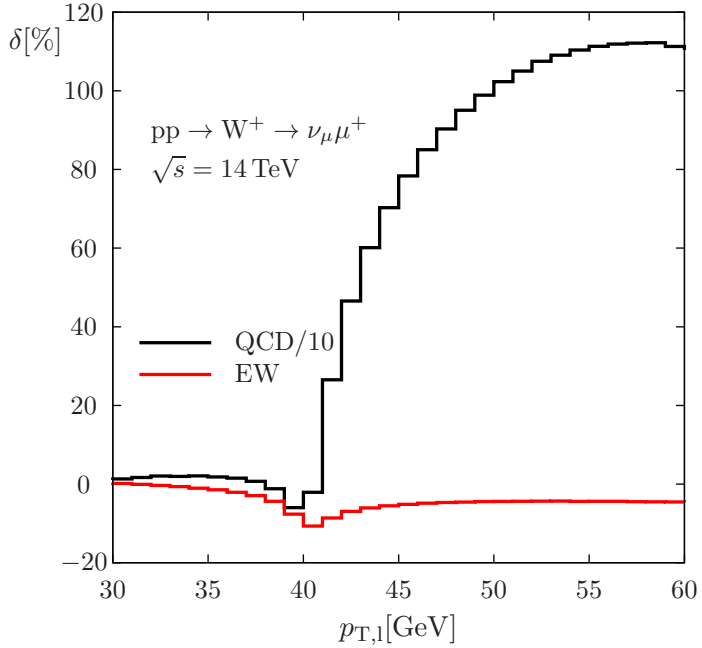
$$M_{T,\ell\nu} = \sqrt{2(E_{T,\ell}\not{E}_T - \mathbf{p}_{T,\ell}\cdot\not{\mathbf{p}}_T)}$$

- most important distribution for the determination of M_W

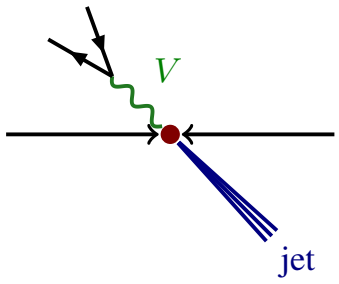
- good agreement between **full** & **PA**
- **fact. ini** & **non-fact.** small and flat
- corrections mainly from **fact. fin**

W^+: $p_{T,\ell}$ distribution



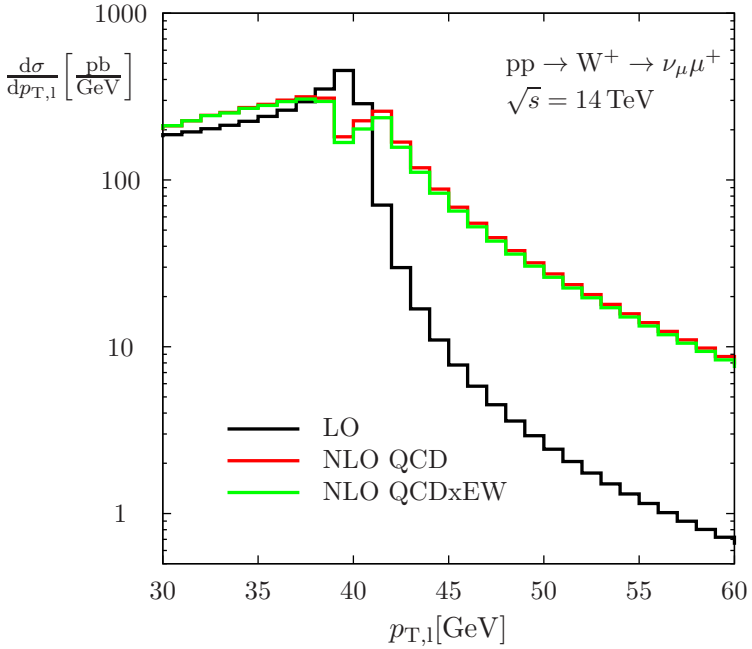


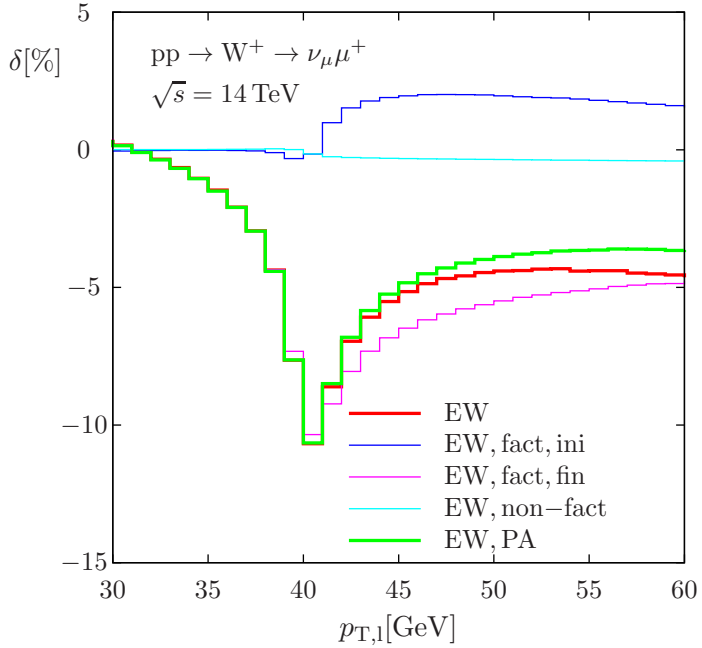
- also important for M_W measurement
- sensitive to initial-state radiation
- jet veto



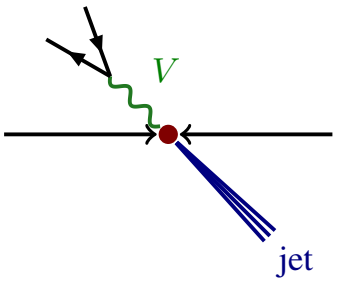
- **QCD**: huge corrections above threshold $\leftrightarrow$ recoil of the jet
- **EW**: also shape distortion

W^+: $p_{T,\ell}$ distribution





- also important for M_W measurement
- sensitive to initial-state radiation
- jet veto



- good agreement between **full** & **PA**
- **non-fact.** again flat and small
- **fact. ini** contributes significantly
  $\leftrightarrow$ sensitivity to initial-state radiation
- again largest contribution from **fact. fin**

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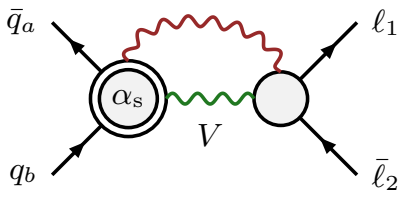
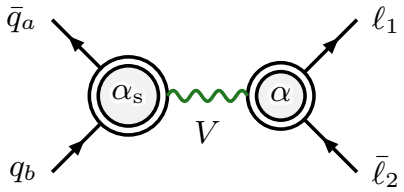
Contributions



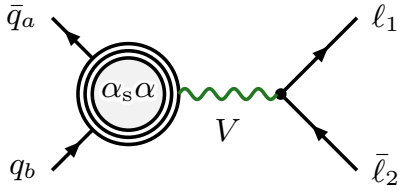
Non-factorizable (nf) corrections*

- **discussed in the following**



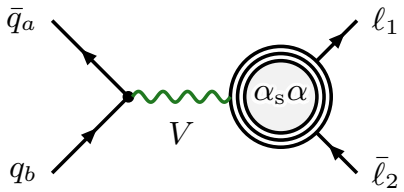
Factorizable initial–final corrections*

- large corrections
  & shape distortion expected
- work in progress, preliminary results



Factorizable initial–initial corrections*

- no significant shape distortion expected
  c.f. M_{T} distribution for fact. ini corrections $\mathcal{O}(\alpha)$
- no $\mathcal{O}(\alpha_s\alpha)$ PDFs



Factorizable final–final corrections*

- only a constant factor: $\mathcal{O}(\alpha_s\alpha)$ counterterm
  $\hookrightarrow$ no impact on shape

*only virtual contributions indicated $\rightsquigarrow$ also real-, double-real emission, interferences,...

Non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections

$$\hat{\sigma}_{\text{nf}}^{\text{QCD}\otimes\text{EW}} = \iint\limits_{3+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\mathrm{V_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\mathrm{C_s}\otimes\mathrm{R_{ew}}}$$

$$+ \int_3 \mathrm{d}\sigma_{\text{nf}}^{\mathrm{R_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma_{\text{nf}}^{\mathrm{V_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma_{\text{nf}}^{\mathrm{C_s}\otimes\mathrm{V_{ew}}}$$

NLO QCD

$$\hat{\sigma}^{\text{QCD}} = \int_3 \mathrm{d}\sigma^{\mathrm{R_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}$$

NLO EW: non-factorizable corrections

$$\hat{\sigma}_{\text{nf}}^{\text{EW}} = \iint\limits_{2+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\mathrm{R_{ew}}} + \int_2 \mathrm{d}\sigma_{\text{nf}}^{\mathrm{V_{ew}}}$$

Non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections

$$\hat{\sigma}_{\text{nf}}^{\text{QCD}\otimes\text{EW}} = \iint\limits_{3+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\mathrm{V_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\mathrm{C_s}\otimes\mathrm{R_{ew}}}$$

$$+ \int_3 \mathrm{d}\sigma_{\text{nf}}^{\mathrm{R_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma_{\text{nf}}^{\mathrm{V_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma_{\text{nf}}^{\mathrm{C_s}\otimes\mathrm{V_{ew}}}$$

NLO QCD

$$\hat{\sigma}^{\text{QCD}} = \int_3 \mathrm{d}\sigma^{\mathrm{R_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}$$

NLO EW: non-factorizable corrections

$$\hat{\sigma}_{\text{nf}}^{\text{EW}} = \iint\limits_{2+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\mathrm{R_{ew}}} + \int_2 \mathrm{d}\sigma_{\text{nf}}^{\mathrm{V_{ew}}} = \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\text{B}}\, \delta_{\mathrm{R_{ew}},\text{nf}}^{2\to 2+\gamma} + \int_2 \mathrm{d}\sigma^{\text{B}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\text{nf}}^{2\to 2}\right\}$$

based on eikonal currents modified by off-shell effects

Non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections

$$\hat{\sigma}_{\mathsf{nf}}^{\mathsf{QCD}\otimes\mathsf{EW}} = \iint\limits_{3+\gamma} \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{V_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{C_s}\otimes\mathrm{R_{ew}}}$$

$$+ \int_3 \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{R_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{V_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{C_s}\otimes\mathrm{V_{ew}}}$$

NLO QCD

$$\hat{\sigma}^{\mathsf{QCD}} = \int_3 \mathrm{d}\sigma^{\mathrm{R_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}},$$

$$\mathrm{d}\sigma_a^{\mathrm{C_s}} = \frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\frac{1}{\epsilon}\left(\frac{4\pi\mu^2}{\mu_F^2}\right)^{\epsilon}\sum_b \int_0^1 \mathrm{d}z\ \mathrm{d}\sigma_b^{\mathsf{B}}\ P^{ab}(z)$$

$\mathrm{d}\sigma^{\mathrm{C_s}}$: collinear subtraction term $\rightsquigarrow$ NLO PDFs

NLO EW: non-factorizable corrections

$$\hat{\sigma}_{\mathsf{nf}}^{\mathsf{EW}} = \iint\limits_{2+\gamma} \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{R_{ew}}} + \int_2 \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{V_{ew}}} = \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathsf{B}}\ \delta_{\mathrm{R_{ew}},\mathsf{nf}}^{2\to 2+\gamma} + \int_2 \mathrm{d}\sigma^{\mathsf{B}}\ 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathsf{nf}}^{2\to 2}\right\}$$

Non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections

$$\hat{\sigma}_{\mathsf{nf}}^{\mathsf{QCD}\otimes\mathsf{EW}} = \iint\limits_{3+\gamma} \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{V_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\, \delta_{\mathrm{R_{ew}},\mathsf{nf}}^{2\to 2+\gamma}$$

$$+ \int_3 \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{R_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{V_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathsf{nf}}^{2\to 2}\right\}$$

NLO QCD

$$\hat{\sigma}^{\mathsf{QCD}} = \int_3 \mathrm{d}\sigma^{\mathrm{R_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}},$$

$$\mathrm{d}\sigma_a^{\mathrm{C_s}} = \frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\frac{1}{\epsilon}\left(\frac{4\pi\mu^2}{\mu_F^2}\right)^{\epsilon}\sum_b \int_0^1 \mathrm{d}z\, \mathrm{d}\sigma_b^{\mathsf{B}}\, P^{ab}(z)$$

$\mathrm{d}\sigma^{\mathrm{C_s}}$: collinear subtraction term $\rightsquigarrow$ NLO PDFs

NLO EW: non-factorizable corrections

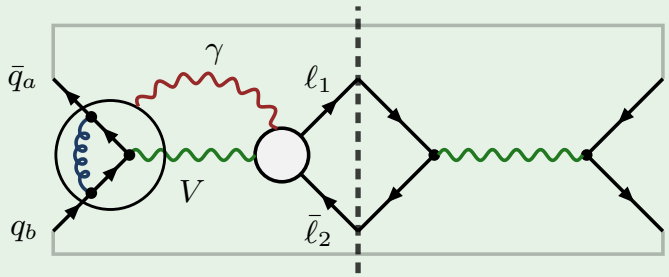
$$\hat{\sigma}_{\mathsf{nf}}^{\mathsf{EW}} = \iint\limits_{2+\gamma} \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{R_{ew}}} + \int_2 \mathrm{d}\sigma_{\mathsf{nf}}^{\mathrm{V_{ew}}} = \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathsf{B}}\, \delta_{\mathrm{R_{ew}},\mathsf{nf}}^{2\to 2+\gamma} + \int_2 \mathrm{d}\sigma^{\mathsf{B}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\mathsf{nf}}^{2\to 2}\right\}$$

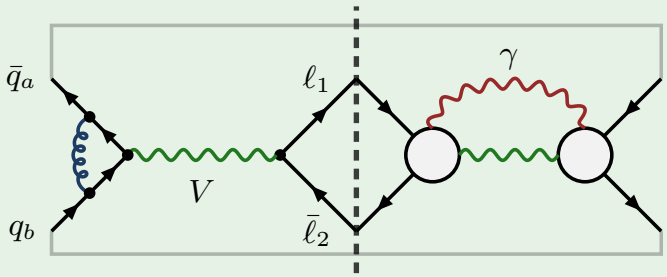
Non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections

$$\hat{\sigma}_{\text{nf}}^{\text{QCD}\otimes\text{EW}} = \iint\limits_{3+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\mathrm{V_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\, \delta_{\mathrm{R_{ew}},\text{nf}}^{2\to 2+\gamma}$$

$$+ \int_3 \mathrm{d}\sigma_{\text{nf}}^{\mathrm{R_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma_{\text{nf}}^{\mathrm{V_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\text{nf}}^{2\to 2}\right\}$$

(Virtual QCD)×(Virtual EW)





- $\propto \delta_{\mathrm{V_{ew}},\text{nf}}^{2\to 2}$ due to non-trivial cancellations
- c.f. $\mathcal{O}(\alpha)$ corrections $\propto \delta_{\mathrm{V_{ew}},\text{nf}}^{2\to 2}$

2-loop diagrams calculated by expanding in the loop-momentum (γ):
$|q^\mu| \sim \Gamma_V \sim (p_V^2 - \mu_V^2)/M_V \to 0$
different methods applied $\to$ results agree

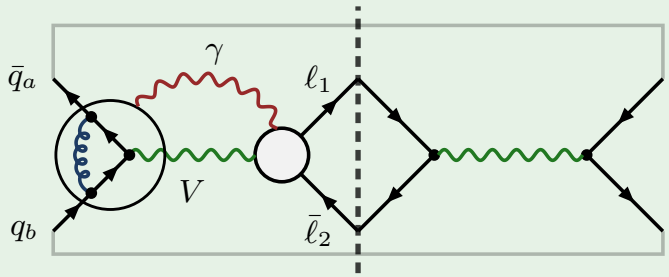
- Mellin–Barnes representation
- effective-field-theory inspired approach [Beneke, et al. '03,'04]
- gauge-invariance argument by generalization of [Yennie, Frautschi, Suura '61]

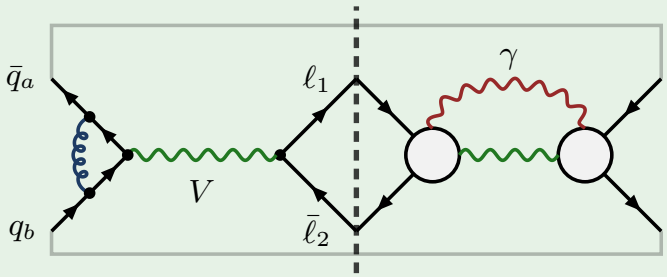
Non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections

$$\hat{\sigma}_{\text{nf}}^{\text{QCD}\otimes\text{EW}} = \iint\limits_{3+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\mathrm{V_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\, \delta_{\mathrm{R_{ew}},\text{nf}}^{2\to 2+\gamma}$$

$$+ \int_3 \mathrm{d}\sigma_{\text{nf}}^{\mathrm{R_s}\otimes\mathrm{V_{ew}}} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\text{nf}}^{2\to 2}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\text{nf}}^{2\to 2}\right\}$$

(Virtual QCD)×(Virtual EW)





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- c.f. $\mathcal{O}(\alpha)$ corrections $\propto \delta_{\mathrm{V_{ew}},\text{nf}}^{2\to 2}$

$$\mathrm{d}\sigma_{\text{nf}}^{\mathrm{V_s}\otimes\mathrm{V_{ew}}} = 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\text{nf}}^{2\to 2}\right\} \mathrm{d}\sigma^{\mathrm{V_s}}$$

Non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections

$$\hat{\sigma}_{\mathrm{nf}}^{\mathrm{QCD}\otimes\mathrm{EW}} = \iint\limits_{3+\gamma} \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{V_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\, \delta_{\mathrm{R_{ew}},\mathrm{nf}}^{2\to 2+\gamma}$$

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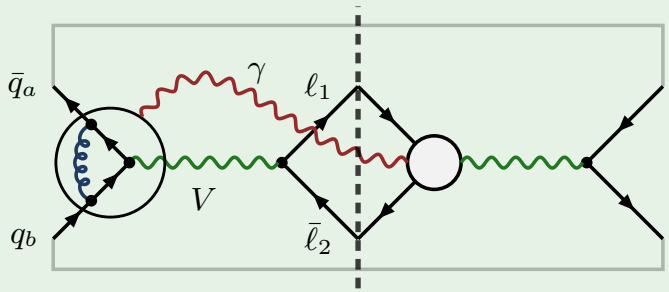
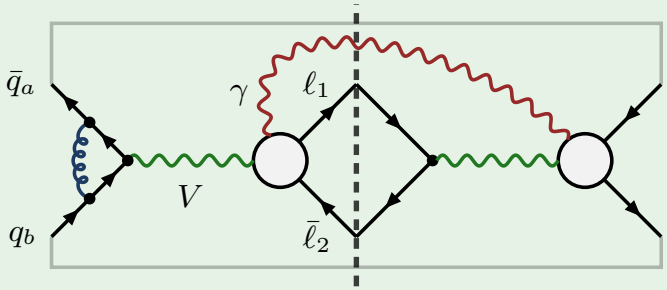
(Virtual QCD)×(Real EW)





- same modified eikonal currents as in $\mathcal{O}(\alpha)$ corrections
  (only depends on momenta & charges of external legs)

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$$\hat{\sigma}_{\text{nf}}^{\text{QCD}\otimes\text{EW}} = \iint\limits_{3+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{V_s}}\, \delta_{\mathrm{R_{ew}},\text{nf}}^{2\to2+\gamma} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\, \delta_{\mathrm{R_{ew}},\text{nf}}^{2\to2+\gamma}$$

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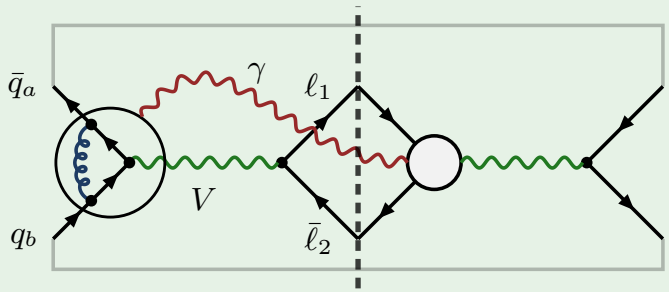
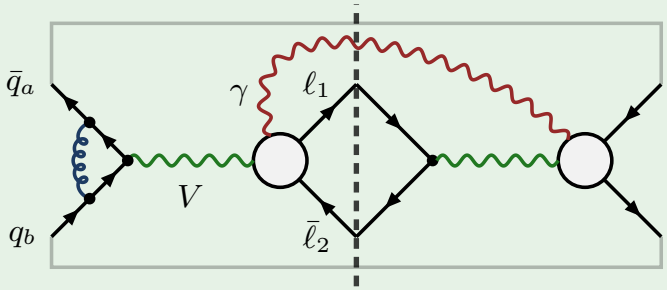
(Virtual QCD)×(Real EW)





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$$\mathrm{d}\sigma_{\text{nf}}^{\mathrm{V_s}\otimes\mathrm{R_{ew}}} = 2\,\mathrm{Re}\left\{\delta_{\mathrm{R_{ew}},\text{nf}}^{2\to2+\gamma}\right\}\mathrm{d}\sigma^{\mathrm{V_s}}$$

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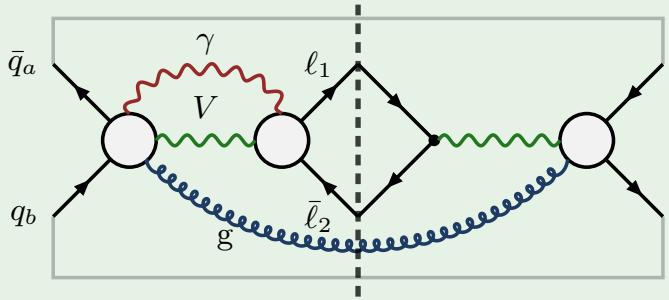
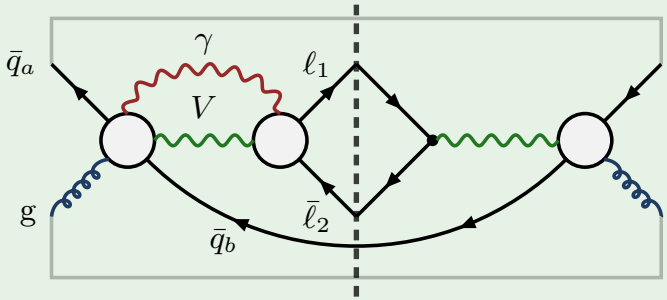
(Real QCD)×(Virtual EW)





- additional kinematic dependence from QCD emission
- new feature in qg channels: soft photon exchange between two final-state legs!
  $\hookrightarrow$ enhancements?

Non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections

$$\hat{\sigma}_{\text{nf}}^{\text{QCD}\otimes\text{EW}} = \iint\limits_{3+\gamma} \mathrm{d}\sigma_{\text{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{V_s}}\, \delta_{\mathrm{R_{ew}},\text{nf}}^{2\to2+\gamma} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\, \delta_{\mathrm{R_{ew}},\text{nf}}^{2\to2+\gamma}$$

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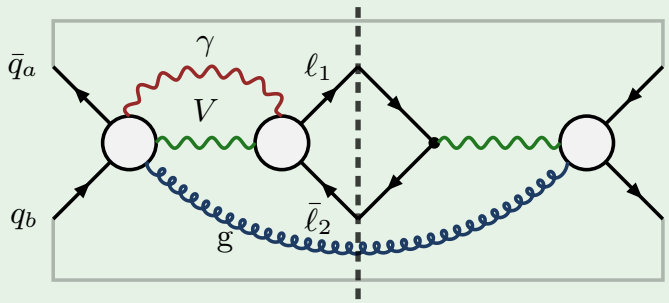

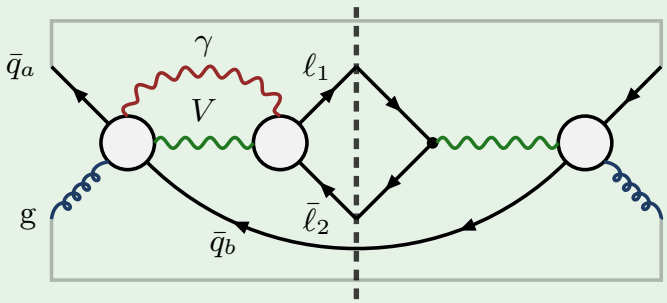

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$$\mathrm{d}\sigma_{\text{nf}}^{\mathrm{R_s}\otimes\mathrm{V_{ew}}} = 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\text{nf}}^{2\to3}\right\} \mathrm{d}\sigma^{\mathrm{R_s}}$$

Non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections

$$\hat{\sigma}_{\mathrm{nf}}^{\mathrm{QCD}\otimes\mathrm{EW}} = \iint\limits_{3+\gamma} \mathrm{d}\sigma_{\mathrm{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{V_s}}\, \delta_{\mathrm{R_{ew}},\mathrm{nf}}^{2\to2+\gamma} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\, \delta_{\mathrm{R_{ew}},\mathrm{nf}}^{2\to2+\gamma}$$

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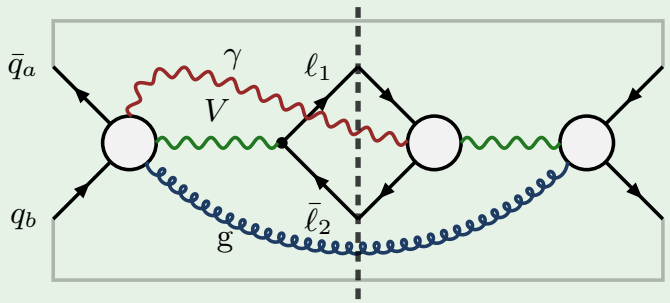
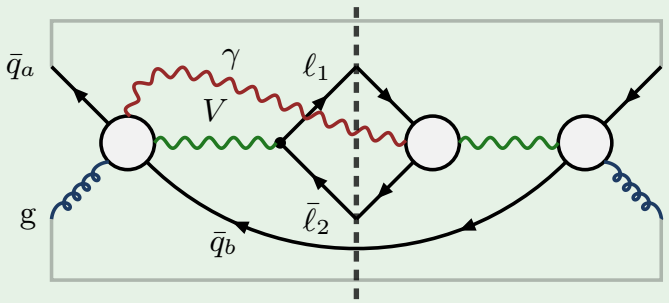
(Real QCD)×(Real EW)





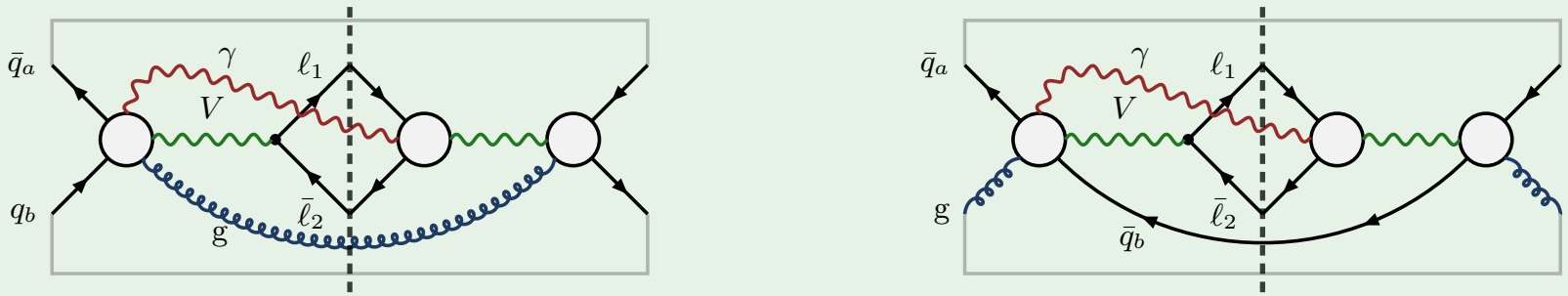
- new kinematic dependence on gluon emission (tree level)

Non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections

$$\hat{\sigma}_{\text{nf}}^{\text{QCD}\otimes\text{EW}} = \iint\limits_{3+\gamma} \mathrm{d}\sigma^{\mathrm{R_s}}\, \delta^{2\to3+\gamma}_{\mathrm{R_{ew}},\text{nf}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{V_s}}\, \delta^{2\to2+\gamma}_{\mathrm{R_{ew}},\text{nf}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\, \delta^{2\to2+\gamma}_{\mathrm{R_{ew}},\text{nf}}$$

$$+ \int_3 \mathrm{d}\sigma^{\mathrm{R_s}}\, 2\,\mathrm{Re}\left\{\delta^{2\to3}_{\mathrm{V_{ew}},\text{nf}}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}}\, 2\,\mathrm{Re}\left\{\delta^{2\to2}_{\mathrm{V_{ew}},\text{nf}}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}\, 2\,\mathrm{Re}\left\{\delta^{2\to2}_{\mathrm{V_{ew}},\text{nf}}\right\}$$

(Real QCD)×(Real EW)



- new kinematic dependence on gluon emission (tree level)

$$\mathrm{d}\sigma_{\text{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} = 2\,\mathrm{Re}\left\{\delta^{2\to3+\gamma}_{\mathrm{R_{ew}},\text{nf}}\right\}\mathrm{d}\sigma^{\mathrm{R_s}}$$

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$$+ \int_3 \mathrm{d}\sigma^{\mathrm{R_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\text{nf}}^{2\to 3}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\text{nf}}^{2\to 2}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}\, 2\,\mathrm{Re}\left\{\delta_{\mathrm{V_{ew}},\text{nf}}^{2\to 2}\right\}$$

Infrared singularities—QCD corrections: dipole subtraction formalism

$$\hat{\sigma}^{\text{QCD}} = \int_3 \mathrm{d}\sigma^{\mathrm{R_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}$$

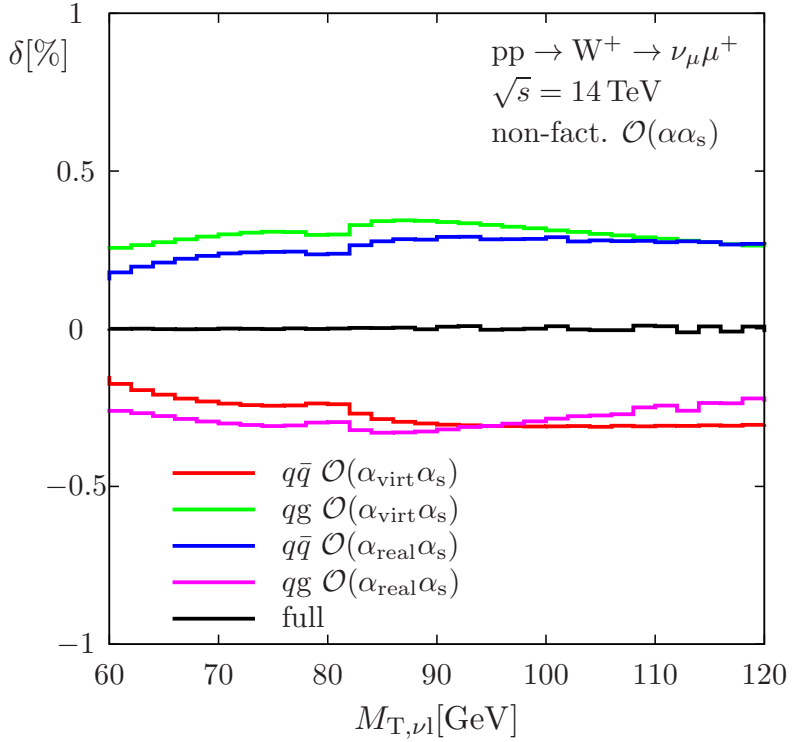
$$= \int_3 \left[\left(\mathrm{d}\sigma^{\mathrm{R_s}}\right)_{\epsilon=0} - \left(\mathrm{d}\sigma^{\mathrm{A_s}}\right)_{\epsilon=0}\right] + \int_2 \left[\mathrm{d}\sigma^{\mathrm{V_s}} + \mathrm{d}\sigma^{\mathrm{C_s}} + \int_1 \mathrm{d}\sigma^{\mathrm{A_s}}\right]_{\epsilon=0}$$

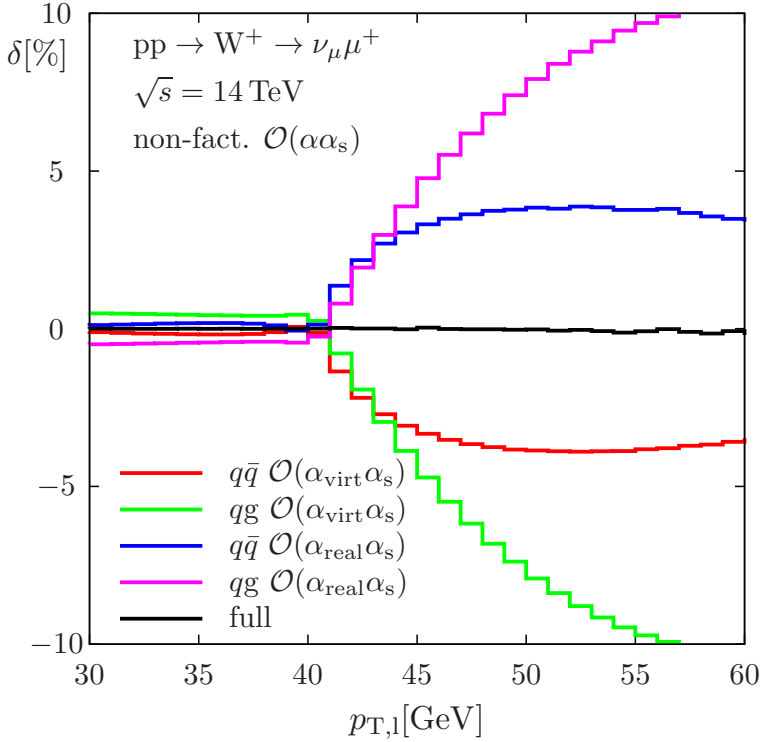
Infrared singularities—EW corrections: phase-space slicing method $\Delta E \ll \Gamma_V$

$$\int_\gamma \mathrm{d}\Phi_\gamma\, \mathrm{d}\sigma^{\text{QCD}}\, \delta_{\mathrm{R_{ew}},\text{nf}}^{\gamma} = \int\limits_{E_\gamma<\Delta E} \mathrm{d}\Phi_\gamma\, \mathrm{d}\sigma^{\text{QCD}}\, \delta_{\mathrm{R_{ew}},\text{nf}}^{\gamma} + \int\limits_{E_\gamma>\Delta E} \mathrm{d}\Phi_\gamma\, \mathrm{d}\sigma^{\text{QCD}}\, \delta_{\mathrm{R_{ew}},\text{nf}}^{\gamma}$$

$$= \underbrace{\int\limits_{E_\gamma<\Delta E} \mathrm{d}\Phi_\gamma\, \delta_{\text{eik}}^{\gamma}}_{=\,\delta_{\text{soft}}(\Delta E)}\, \mathrm{d}\sigma^{\text{QCD}} + \int\limits_{E_\gamma>\Delta E} \mathrm{d}\Phi_\gamma\, \mathrm{d}\sigma^{\text{QCD}}\, \delta_{\mathrm{R_{ew}},\text{nf}}^{\gamma}$$

W^+ distributions





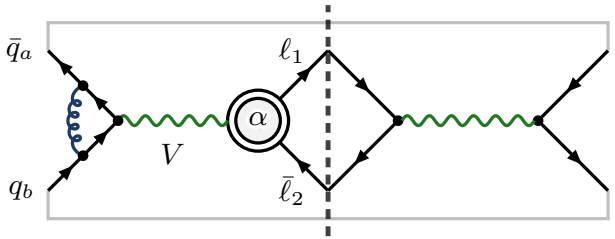
- $\mathcal{O}(\alpha_{\text{real}}\alpha_s)$: with cut $E_\gamma > \Delta E$, $\mathcal{O}(\alpha_{\text{virt}}\alpha_s)$: including $\delta_{\text{soft}}(\Delta E)$
- almost perfect cancellation between different contributions
- tiny & flat corrections!

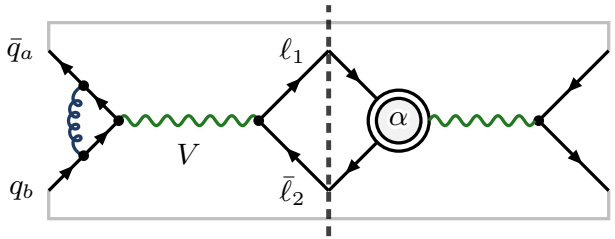
$\Rightarrow$ dominant contributions at $\mathcal{O}(\alpha_s\alpha)$ from the factorizable corrections!

Initial–final factorizable corrections (preliminary results)

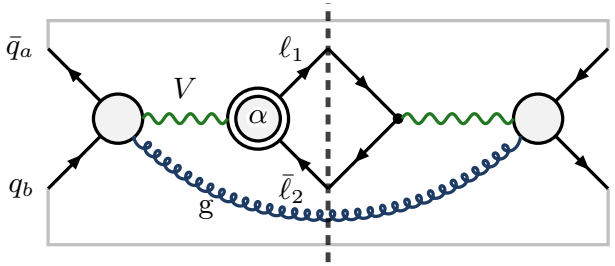
Contributions to the initial–final factorizable $\mathcal{O}(\alpha_s \alpha)$ corrections

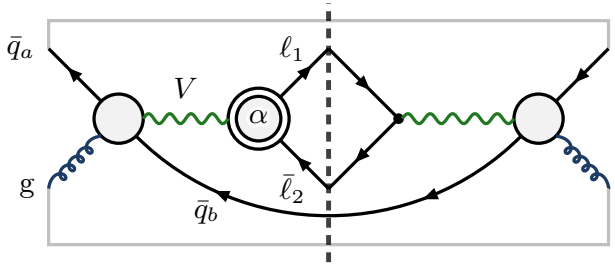
(Virtual QCD)
× (Virtual EW)





(Real QCD)
× (Virtual EW)





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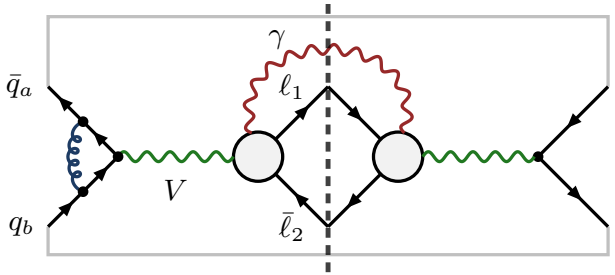

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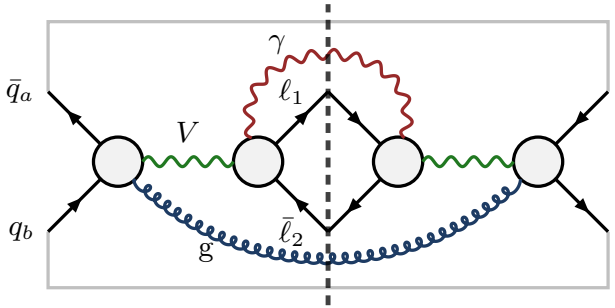
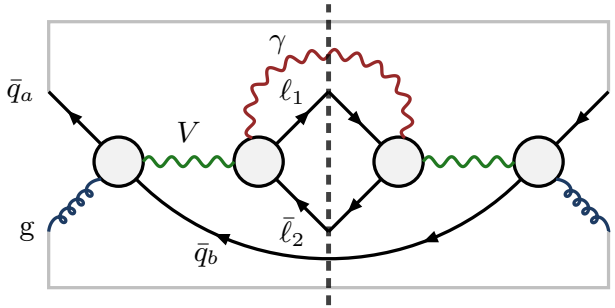
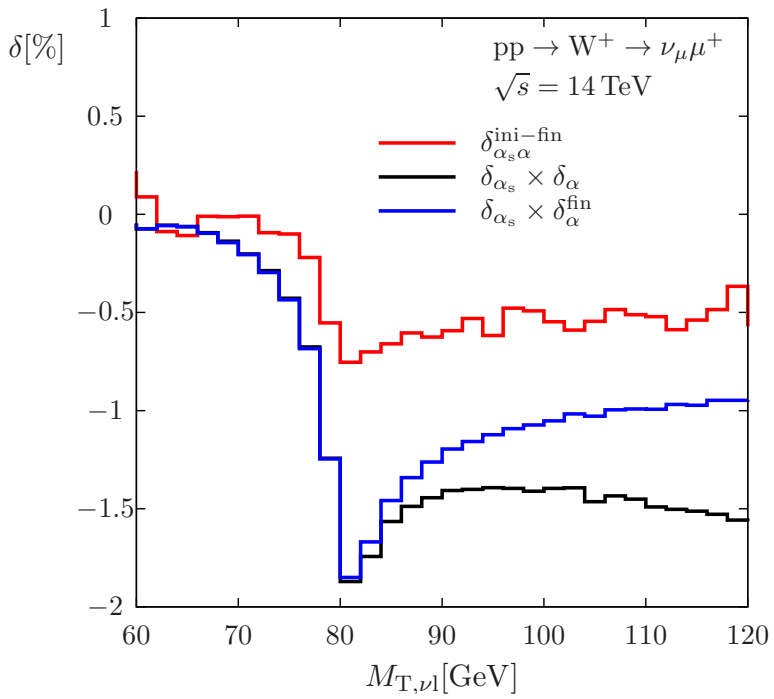
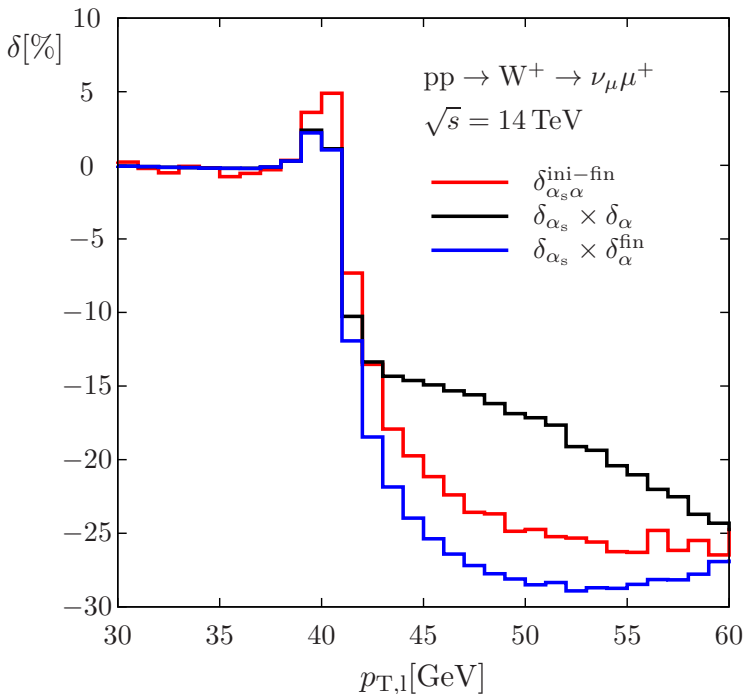




Initial–final factorizable corrections (preliminary results)

W^+ production with dressed leptons (with γ recombination)





- no naive factorization: $\delta^{\text{ini-fin}}_{\alpha_s\alpha} \neq \delta_{\alpha_s} \times \delta^{\text{fin}}_{\alpha}$

todo: generalize to non-collinear safe observables $\rightsquigarrow$ bare leptons

todo: estimate shifts in M_{W}, M_{Z}

1 Motivation and introduction

2 Drell–Yan process @ NLO & the Pole Approximation

3 Pole expansion @ NNLO $\mathcal{O}(\alpha_s \alpha)$

4 Summary and outlook

Summary and outlook

Largest theoretical unknown in Drell–Yan processes: $\mathcal{O}(\alpha_s \alpha)$
important in distributions around resonance (M_W measurement) $\rightsquigarrow$ Pole expansion

Pole approximation @ $\mathcal{O}(\alpha)$

PA reproduces full result near resonance

fact. ini:	small and flat in M_T distributions, larger for p_T
fact. fin:	dominant contribution
non-fact.:	small and flat

Pole approximation @ $\mathcal{O}(\alpha_s \alpha)$

- establish concept of PA at this order
- calculation of non-factorizable corrections $\rightarrow$ negligible
  $\hookrightarrow$ factorizable corrections are dominant
- largest contribution expected from
  (QCD initial state) $\times$ (EW final state) factorizable corrections
  $\hookrightarrow$ work in progress, preliminary results

Summary and outlook

Largest theoretical unknown in Drell–Yan processes: $\mathcal{O}(\alpha_s\alpha)$
important in distributions around resonance (M_W measurement) $\rightsquigarrow$ Pole expansion

Pole approximation @ $\mathcal{O}(\alpha)$

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- fact. fin: dominant contribution
- non-fact.: small and flat

Pole approximation @ $\mathcal{O}(\alpha_s\alpha)$

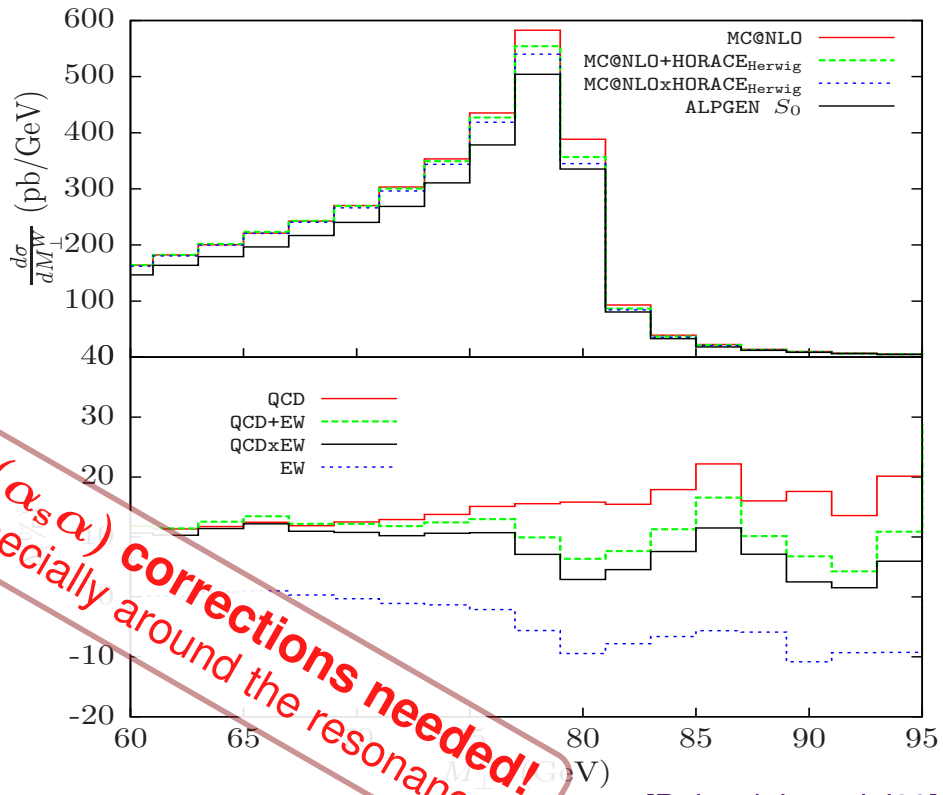
- establish concept of PA at this order
- calculation of non-factorizable corrections $\rightarrow$ negligible
  $\hookrightarrow$ factorizable corrections are dominant
- largest contribution expected from
  (QCD initial state) $\times$ (EW final state) factorizable corrections
  $\hookrightarrow$ work in progress, preliminary results

Thank you

Backup Slides

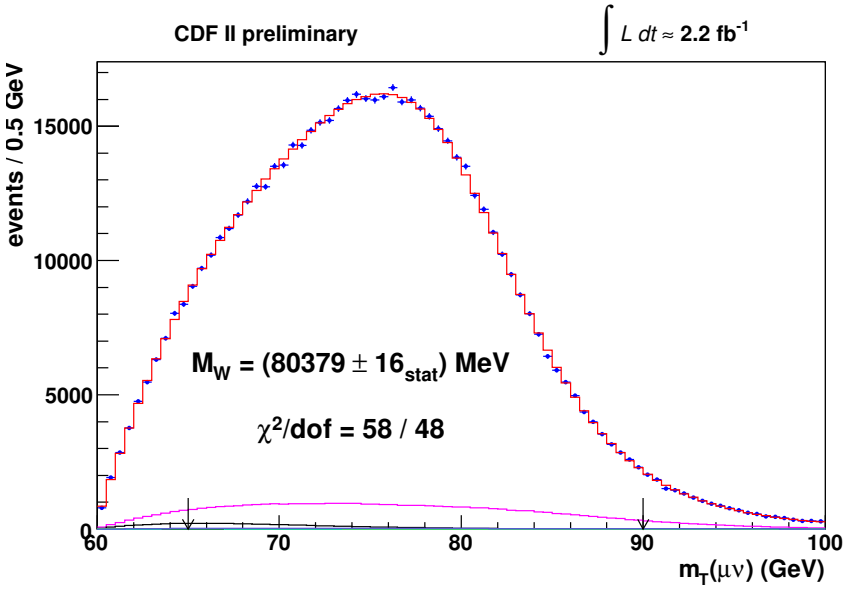
Motivation: M_{W} measurement

- **Tevatron**: $M_{\mathrm{W}} = 80.387 \pm 0.016\ \mathrm{GeV}$
  (most precise measurement of M_{W} to date!)
- **LHC**: aimed precision of
  $\Delta M_{\mathrm{W}} \lesssim 10\ \mathrm{MeV}$
- Fits to kinematic distributions



$\mathcal{O}(\alpha_s\alpha)$ corrections needed!
(especially around the resonance)

[Balossini, et al. '09]



Largest missing piece:
mixed QCD$\times$EW corrections
$\hookrightarrow$ compare two extremes:

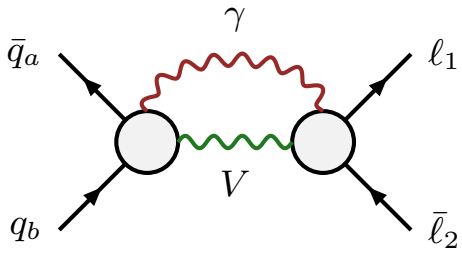
- - - - $(1 + \delta_{\mathrm{QCD}} + \delta_{\mathrm{EW}})$
——— $(1 + \delta_{\mathrm{QCD}}) \times (1 + \delta_{\mathrm{EW}})$

Difference $\sim$ %-level!

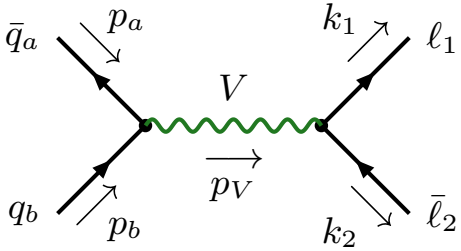
Virtual non-factorizable corrections

Only soft region ($|q^\mu| \lesssim \Gamma_V$) leads to resonant contributions!

- neglect q^μ (γ loop momentum) everywhere except in divergent propagators
- only scalar integrals
- corrections factorize off from the lower-order diagram



$\sim \delta^{\bar{q}_a q_b \to \ell_1 \bar{\ell}_2}_{\mathrm{V_{ew}},\mathrm{nf}}$



$\bar{d}u \to \mathrm{W}^+ \to \nu_\ell \ell^+$:

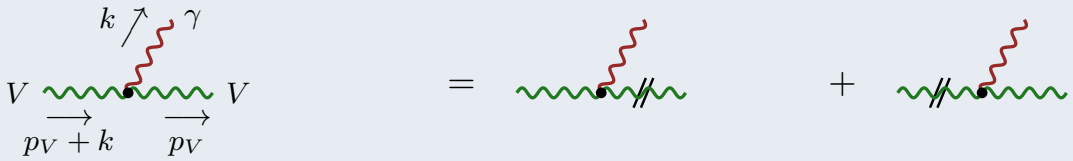
$$\delta^{\bar{d}u \to \nu_\ell \ell^+}_{\mathrm{V_{ew}},\mathrm{nf}} = -\frac{\alpha}{2\pi}\left\{-2 + Q_d\,\mathrm{Li}_2\left(1+\frac{M_\mathrm{W}^2}{\hat{t}_\mathrm{res}}\right) - Q_u\,\mathrm{Li}_2\left(1+\frac{M_\mathrm{W}^2}{\hat{u}_\mathrm{res}}\right)\right.$$
$$+\left[2\ln\left(\frac{\mu_\mathrm{W}^2-\hat{s}}{M_\mathrm{W}^2}\right) - \frac{c_\epsilon}{\epsilon} - \ln\left(\frac{\mu^2}{M_\mathrm{W}^2}\right)\right]$$
$$\left.\times\left[1 + Q_d\ln\left(-\frac{M_\mathrm{W}^2}{\hat{t}_\mathrm{res}}\right) - Q_u\ln\left(-\frac{M_\mathrm{W}^2}{\hat{u}_\mathrm{res}}\right)\right]\right\}$$

[Dittmaier, Krämer '02]

Real corrections in the PA

Decompose into initial-state and final-state radiation

$$\frac{1}{(p_V+k)^2-\mu_V^2}\cdot\frac{1}{p_V^2-\mu_V^2} = \frac{1}{2p_V\cdot k}\left[\frac{1}{p_V^2-\mu_V^2} - \frac{1}{(p_V+k)^2-\mu_V^2}\right]$$



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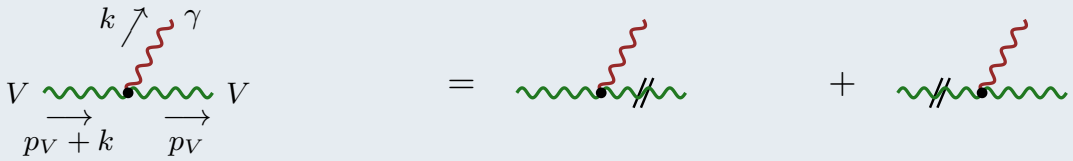

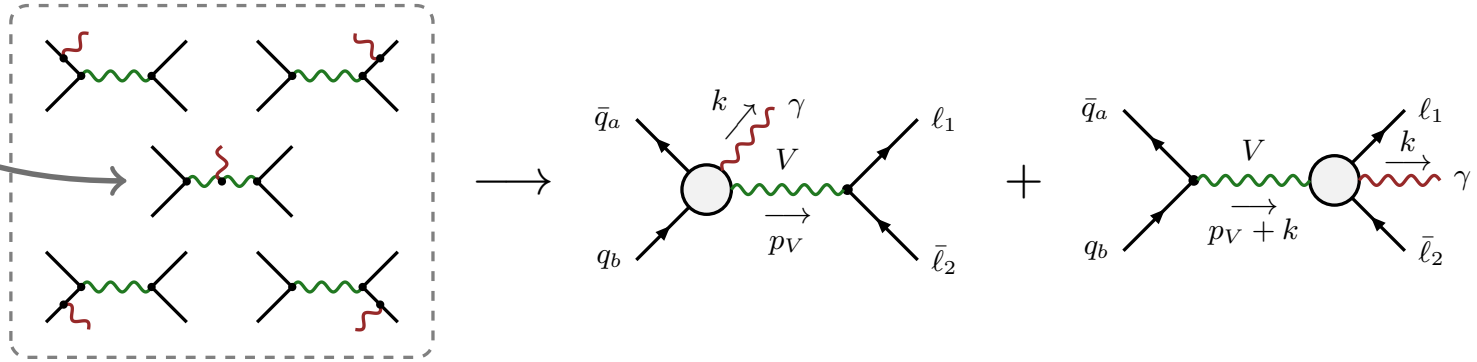

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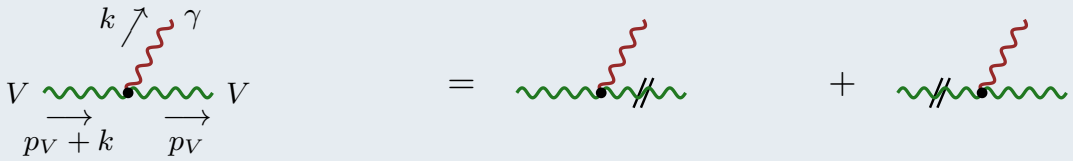
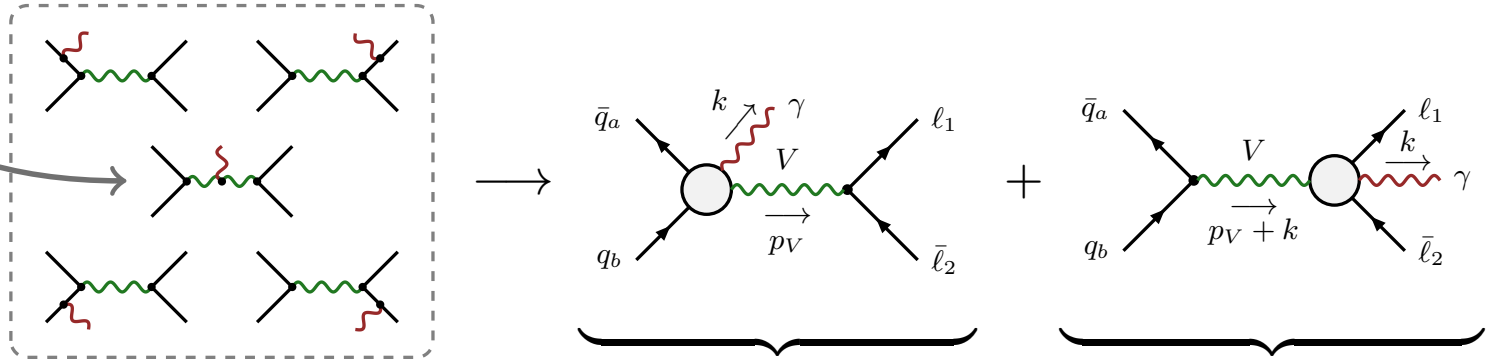
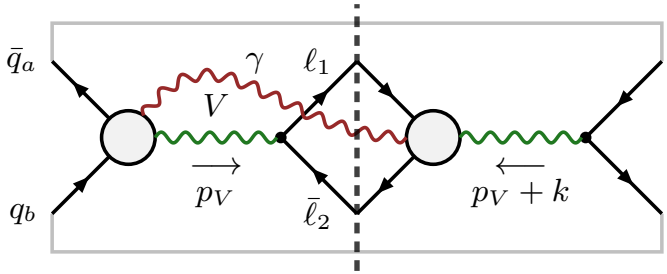




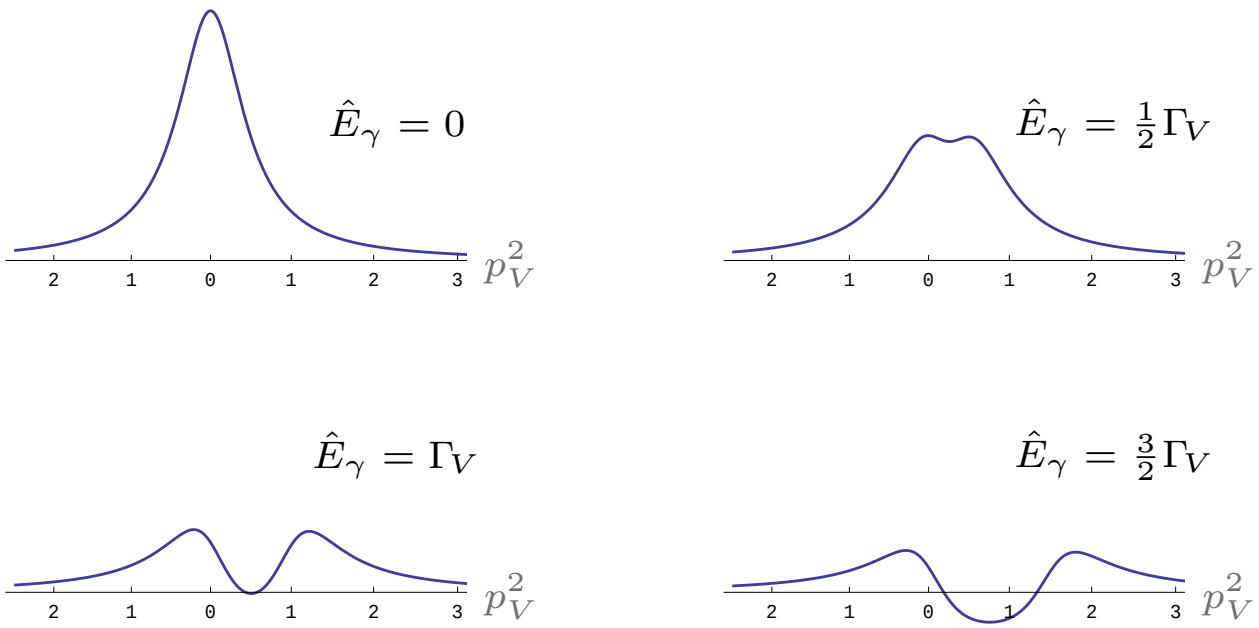
$|\ldots|^2 \rightsquigarrow$ fact. ini. $\qquad$ $|\ldots|^2 \rightsquigarrow$ fact. fin.

Real non-factorizable corrections

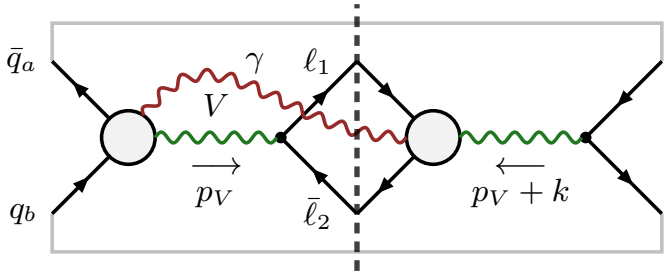


- only soft photons! $(E_\gamma \sim \Gamma_V \ll M_V)$

$$2\,\mathrm{Re}\left\{\frac{1}{p_V^2-\mu_V^2}\left(\frac{1}{(p_V+k)^2-\mu_V^2}\right)^*\right\}$$



Real non-factorizable corrections



- only soft photons! $(E_\gamma \sim \Gamma_V \ll M_V)$

modified eikonal currents $\rightsquigarrow$ factorizes off from diagram without γ emission

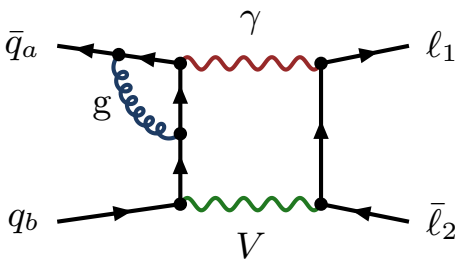
$$\left|\mathcal{M}_{\mathrm{Rew}}^{\bar{q}_a q_b \to \ell_1 \bar{\ell}_2 \gamma}\right|^2 \Big|_{\text{non-fact}} = \delta_{\mathrm{Rew},\text{nf}}^{\bar{q}_a q_b \to \ell_1 \bar{\ell}_2 \gamma} \left|\mathcal{M}_{\text{B}}^{\bar{q}_a q_b \to \ell_1 \bar{\ell}_2}\right|^2$$

$$\delta_{\mathrm{Rew},\text{nf}}^{\bar{q}_a q_b \to \ell_1 \bar{\ell}_2 \gamma} = -e^2 \; 2\,\mathrm{Re}\left\{ (J_{\text{prod}}^\mu)^* \, J_{\text{dec},\mu} \right\}$$

$$J_{\text{prod}}^\mu = -Q_a \frac{p_a^\mu}{k \cdot p_a} + Q_b \frac{p_b^\mu}{k \cdot p_b} + (Q_a - Q_b) \frac{(p_a + p_b)^\mu}{k \cdot (p_a + p_b)}$$

$$J_{\text{dec}}^\mu = \left[-Q_1 \frac{k_1^\mu}{k \cdot k_1} + Q_2 \frac{k_2^\mu}{k \cdot k_2} + (Q_1 - Q_2) \frac{(k_1 + k_2)^\mu}{k \cdot (k_1 + k_2)} \right] \frac{p_V^2 - \mu_V^2}{(p_V + k)^2 - \mu_V^2}$$

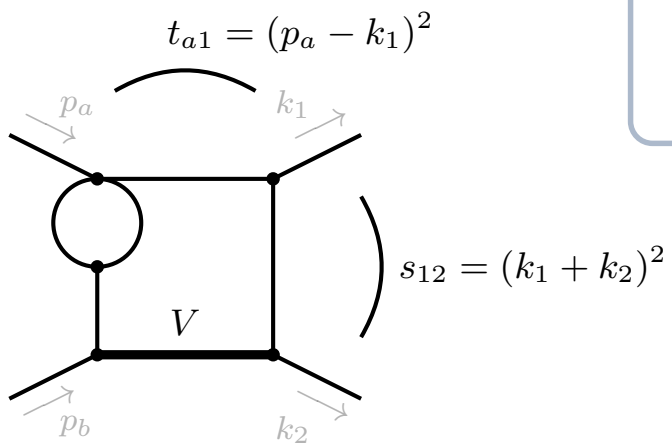
Example 2-Loop Diagram



$$\sim -\frac{C_F \alpha_s}{2\pi} \frac{Q_q Q_l \alpha}{2\pi} \mathcal{M}^0 \, (1-\epsilon)\,(-\hat{t})\,(\mu_V^2 - \hat{s}^2)\; I(\hat{s},\hat{t})$$

- Mellin–Barnes representation
- method of regions
- generalization of [Yennie, Frautschi, Suura '61]

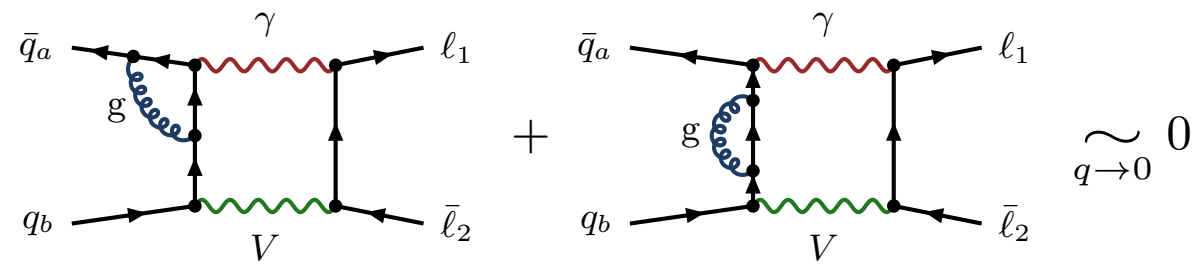
$I(\hat{s},\hat{t}) =$

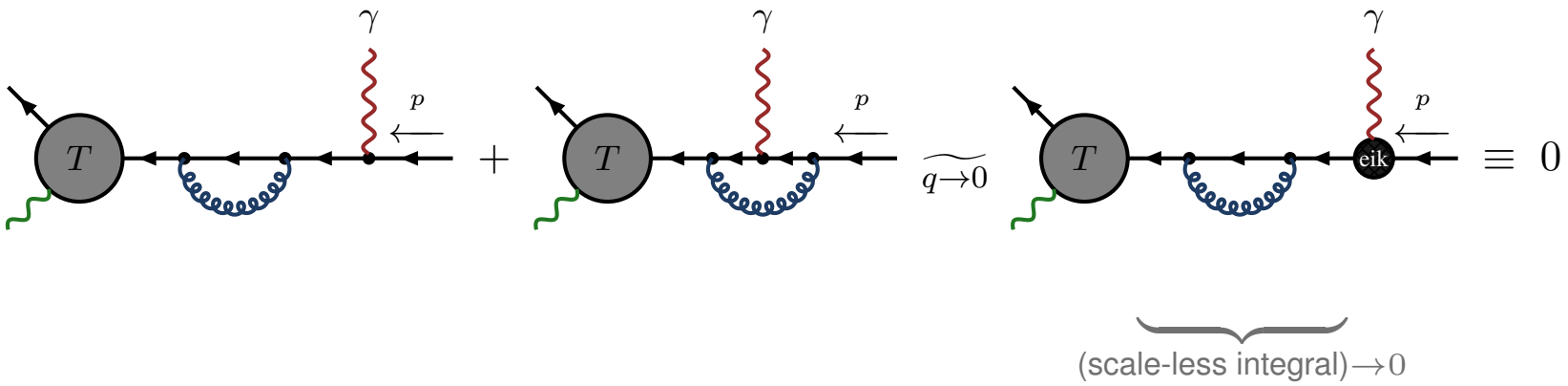


$$= \frac{(4\pi)^{2\epsilon}\Gamma^2(1+\epsilon)}{(\mu_V^2-\hat{s})(-\hat{t})} \left(\frac{\mu_V^2-\hat{s}}{M_V^2}\right)^{-3\epsilon} \left(\frac{-\hat{t}}{\mu^2}\right)^{-2\epsilon} \left\{ \frac{1}{2\epsilon^3} + \frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left[2 + \frac{5\pi^2}{12} + \mathrm{Li}_2\left(1+\frac{\hat{t}}{M_V^2}\right)\right] \right.$$
$$+ 2\,\mathrm{Li}_3\left(\frac{-\hat{t}}{M_V^2}\right) + \mathrm{Li}_3\left(1+\frac{\hat{t}}{M_V^2}\right) - 6\zeta(3) - 2\ln\left(\frac{-\hat{t}}{M_V^2}\right)\left[\frac{\pi^2}{6} - \mathrm{Li}_2\left(1+\frac{\hat{t}}{M_V^2}\right)\right]$$
$$\left. + \ln^2\left(\frac{-\hat{t}}{M_V^2}\right)\ln\left(1+\frac{\hat{t}}{M_V^2}\right) + \frac{5\pi^2}{6} + 2\,\mathrm{Li}_2\left(1+\frac{\hat{t}}{M_V^2}\right) + 4 + \mathcal{O}(\epsilon) + \mathcal{O}\left(\hat{s}-\mu_V^2\right) \right\}$$

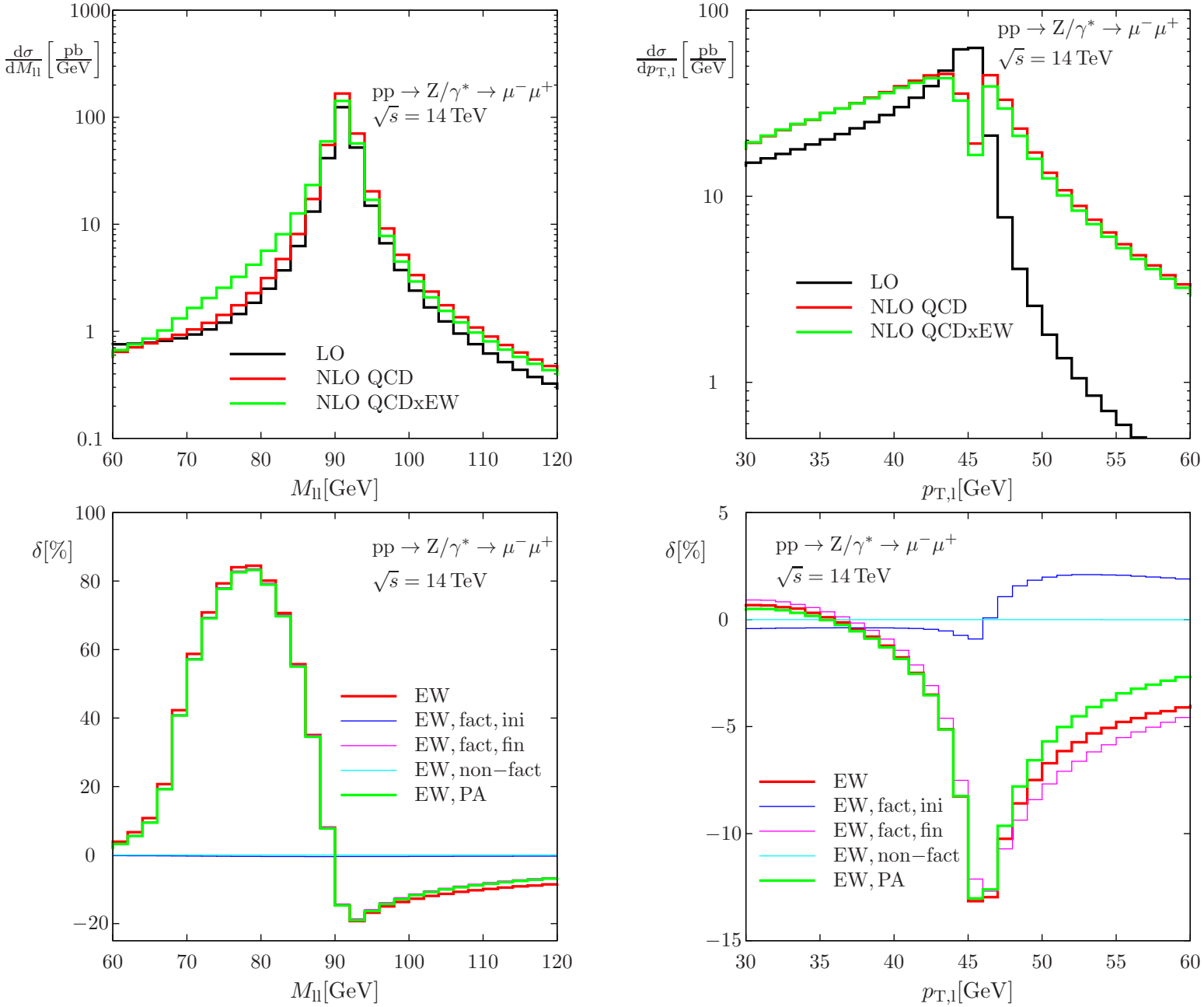
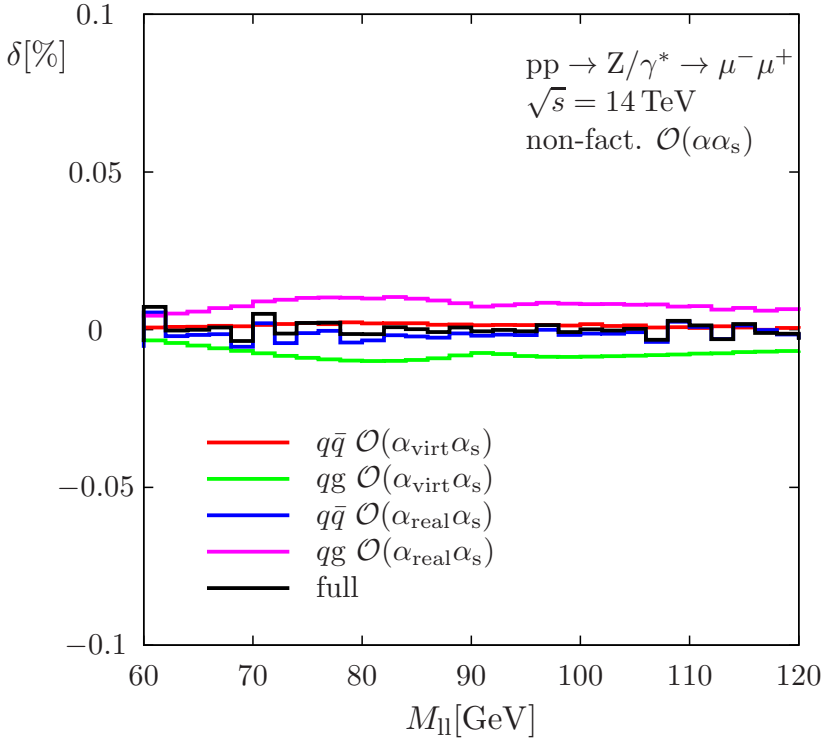
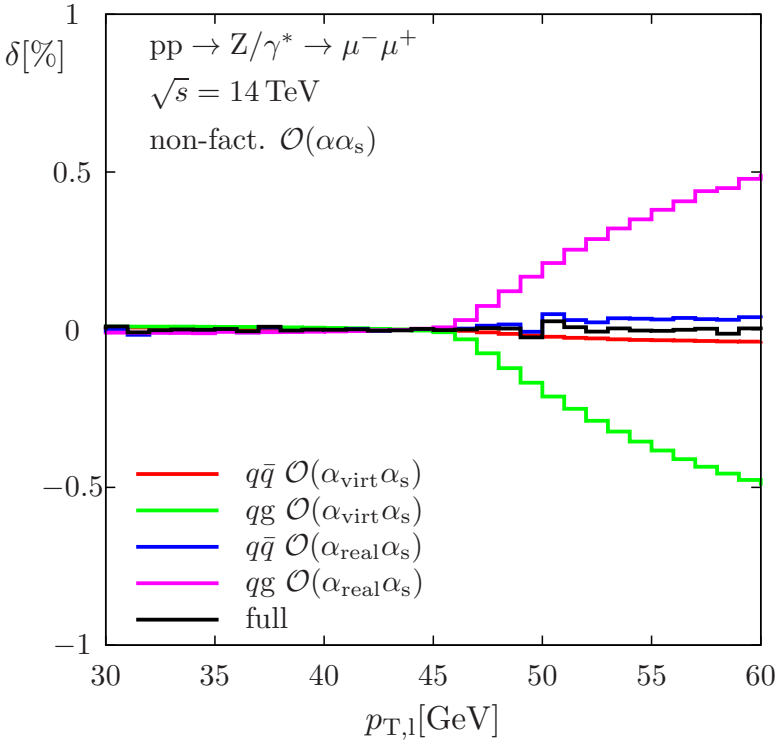
YFS argument

[Yennie, Frautschi, Suura '61] (photon momentum q)





Z distributions @ NLO



Z distributions (non-factorizable) @ NNLO