

# *First Measurement of the Beam Normal Single Spin Asymmetry in $\Delta$ Resonance Production by Q-weak*

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for the Q-weak Collaboration

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in conjunction with the 48<sup>th</sup> Fermilab Users Meeting



# *Beam Normal Single Spin Asymmetry*

- Beam Normal Single Spin Asymmetries ( $B_n$ ) are generated when transversely polarized electrons scatter from unpolarized targets
- $B_n$  is parity conserving and is time-reversal invariant

$B_n$  is also known  
as transverse  
asymmetry

$$B_n = \frac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}}$$

↑ spin UP

↓ spin DOWN

# Beam Normal Single Spin Asymmetry

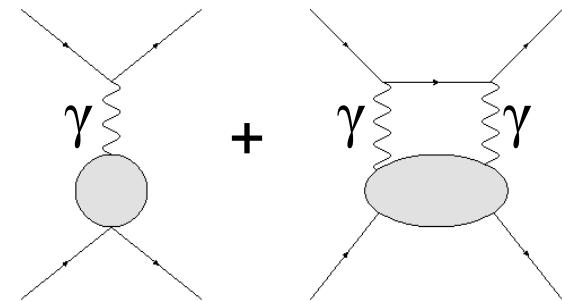
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↑ spin UP  
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$T_{1\gamma}$  – amplitude for 1-photon exchange  
 $T_{2\gamma}$  – amplitude for 2-photon exchange



$B_n$  arises from the interference of 2-photon exchange with 1-photon exchange in e-N scattering

- $B_n$  provides direct access to the imaginary part of the two-photon exchange amplitude

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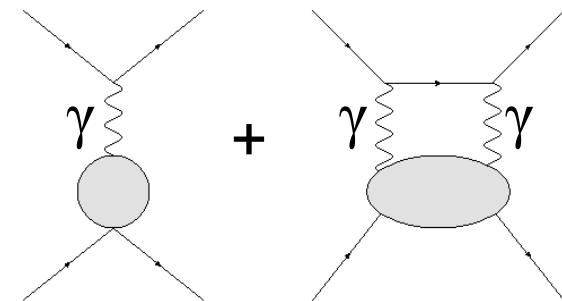
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Contains information about the Intermediate states of the nucleon

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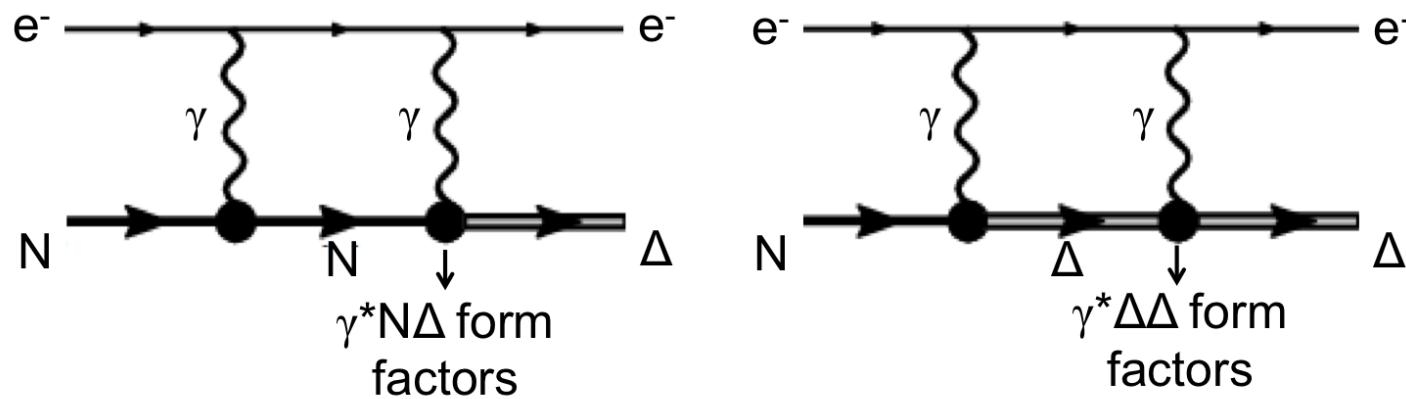


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# $B_n$ in the Production of the $\Delta$ Resonance

Measuring  $B_n$  in  $e^- + p \rightarrow e^- + \Delta$

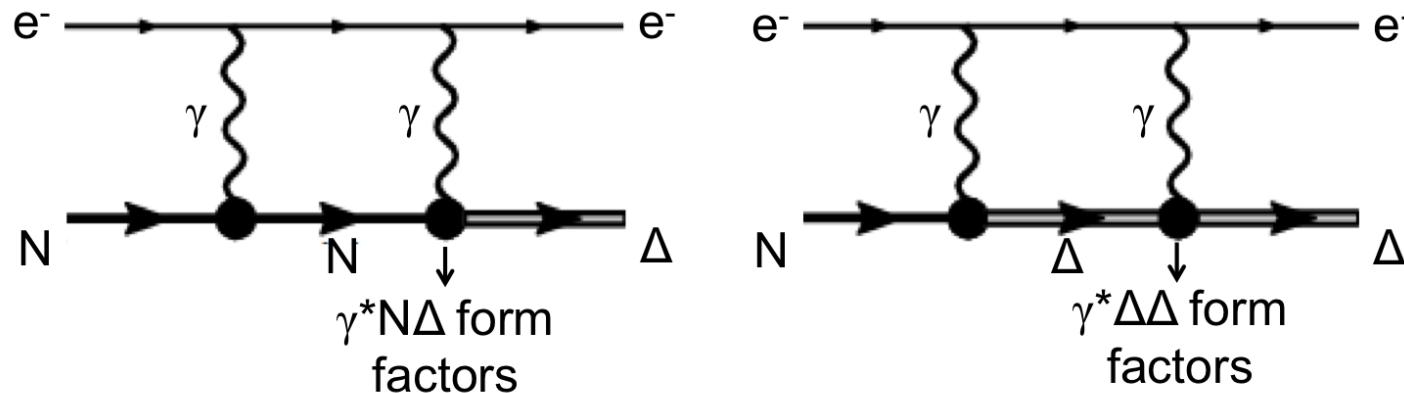


# $B_n$ in the Production of the $\Delta$ Resonance

Measuring  $B_n$  in  $e + p \rightarrow e + \Delta$

## Physics Motivation:

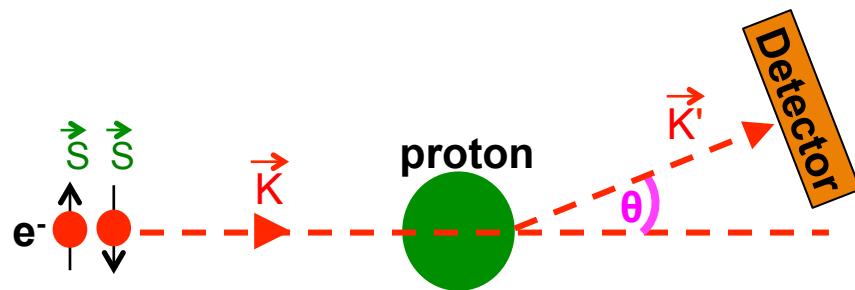
For p and  $\Delta$  intermediate hadrons, vertices are known  
- except for  $\gamma^* \Delta\Delta$  electromagnetic (EM) vertex



- Proton EM FF well known.  $N \rightarrow \Delta$  EM transition FF fairly well known
- Unique tool to study  $\gamma^* \Delta\Delta$  form factors
- Potential to constrain charge radius and magnetic moment of  $\Delta$ !

# Asymmetry Measurement

Can be measured with a transversely polarized beam to determine the magnitude

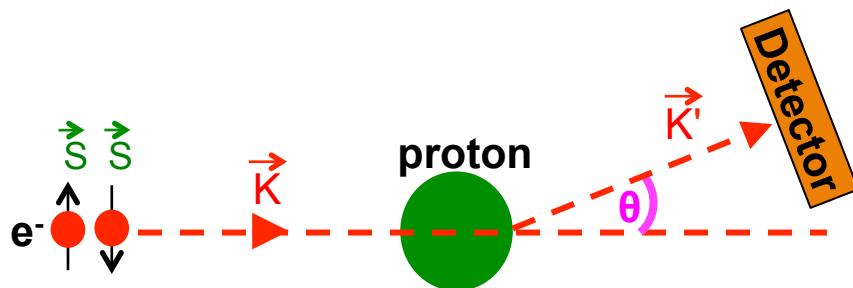


Measured asymmetry

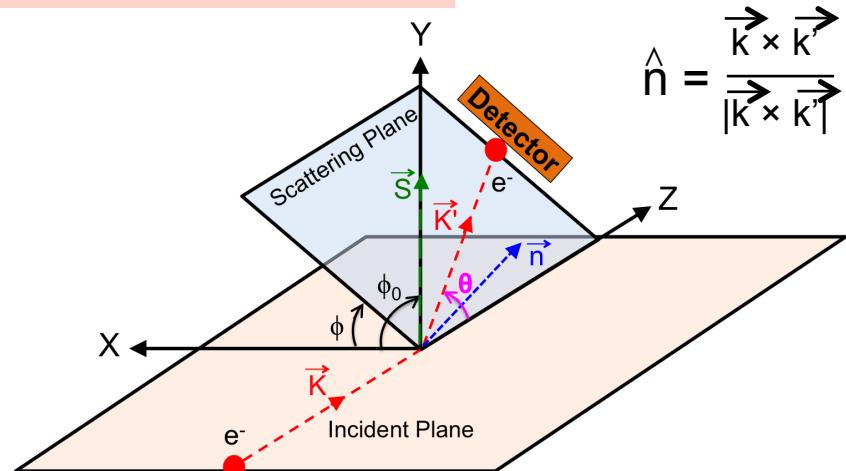
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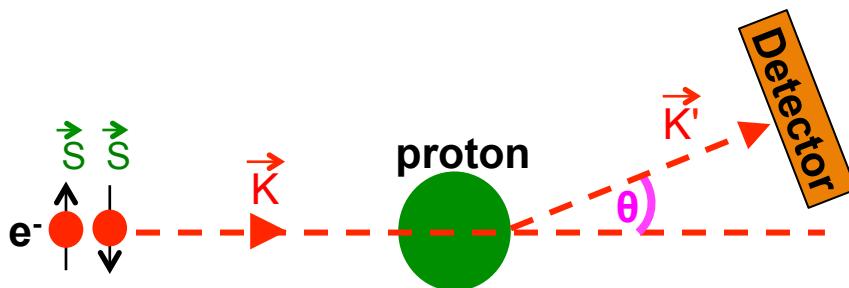
$$\epsilon_M(\phi) = \frac{N^{\uparrow} - N^{\downarrow}}{N^{\uparrow} + N^{\downarrow}} = -B_n \vec{S} \cdot \hat{n} = B_n S \sin(\phi - \phi_0) = B_n [P^V \cos(\phi) - P^H \sin(\phi)]$$

Measured asymmetry has a small azimuthal dependence

- Horizontal :  $P^H = S \cos(\phi_0)$
- Vertical :  $P^V = S \sin(\phi_0)$

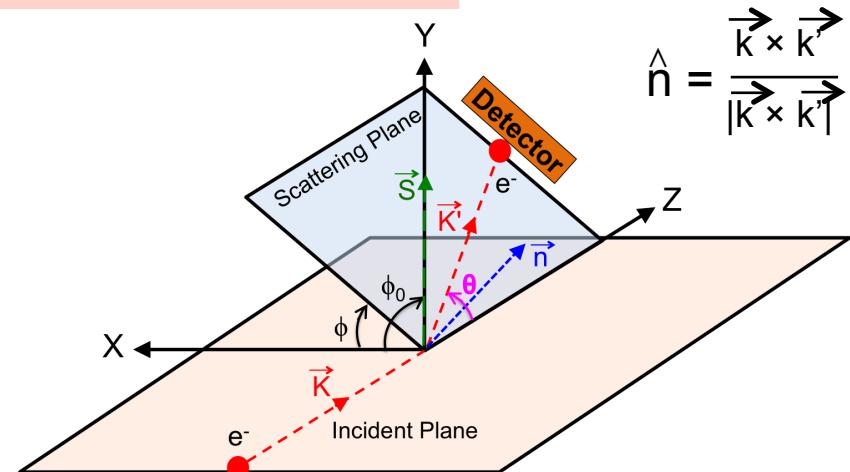
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Data taken on targets

- Hydrogen
- Aluminum
- Carbon

Transverse polarization:

- Horizontal
- Vertical

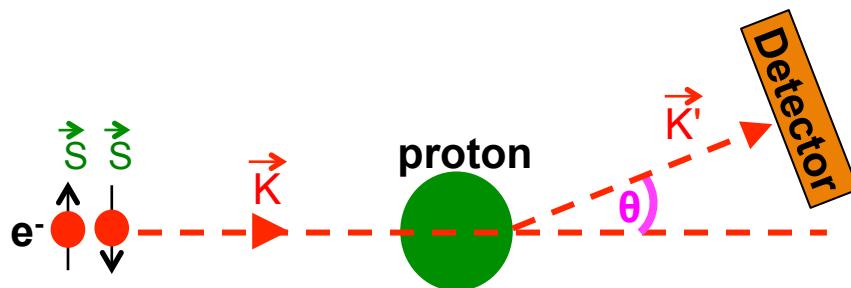
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Q-weak has data with different beam energy and physics process

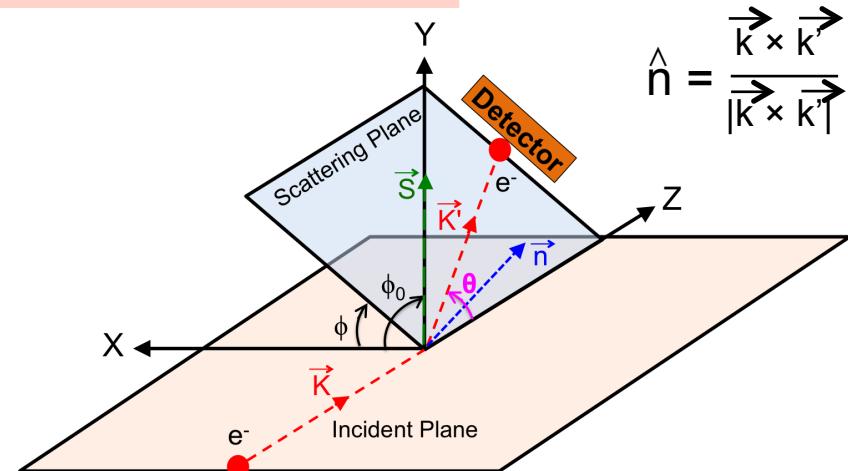
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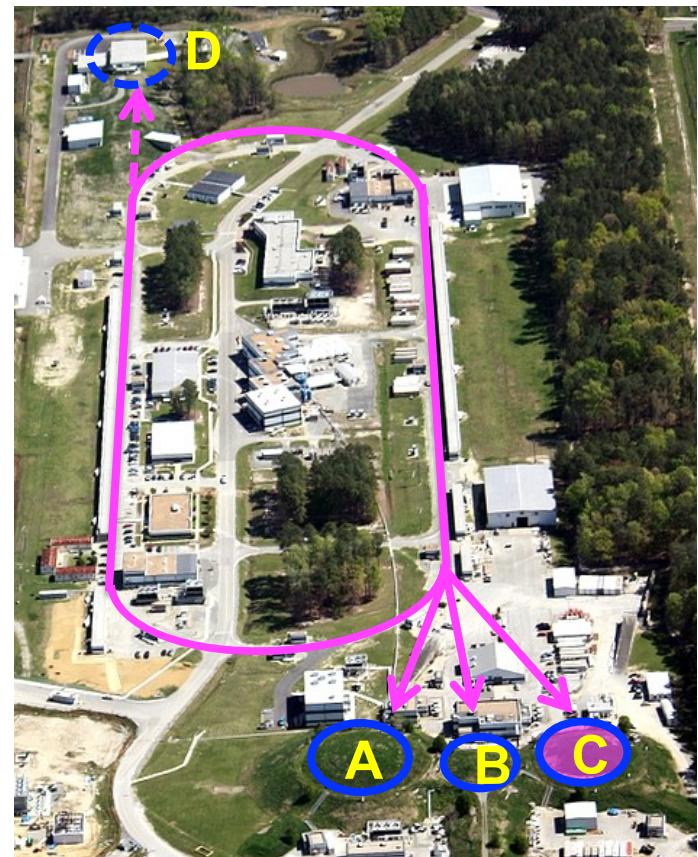
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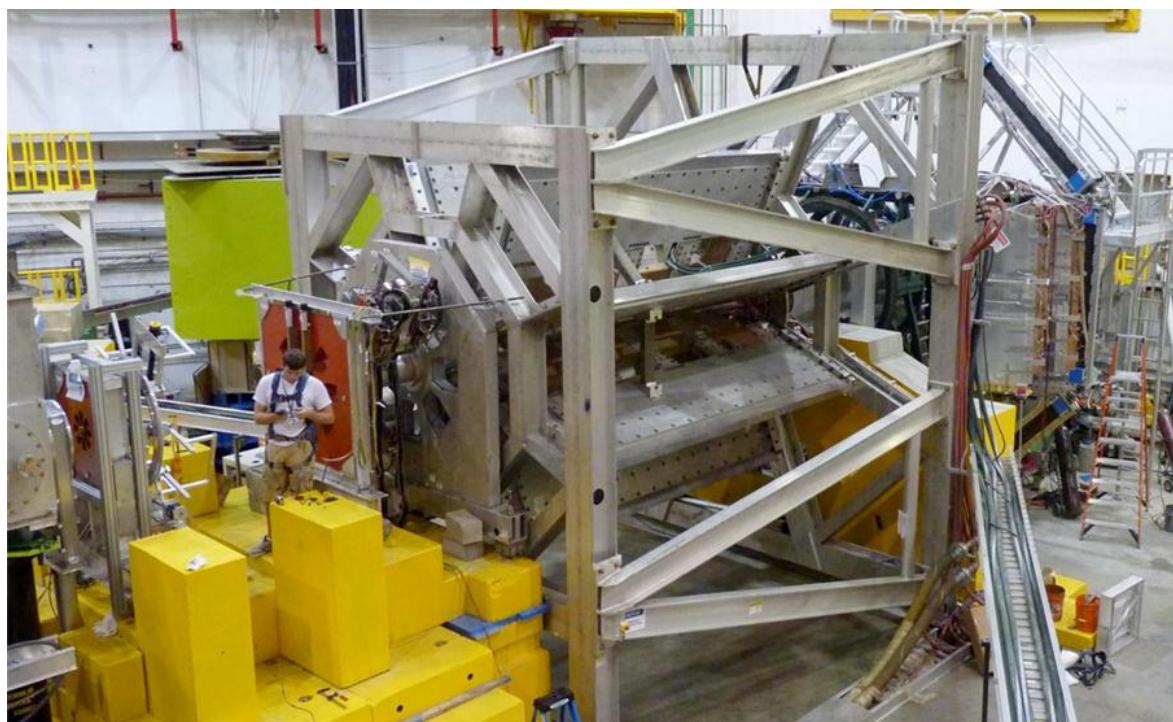
Q-weak has data with different beam energy and physics process

This talk: Inelastic e-p scattering with a  $\Delta(1232)$  final state at  $E = 1.16$  GeV

# *Jefferson Lab and Q-weak Setup*



Aerial view of Jefferson Lab



Q-weak setup inside Hall-C (during construction)

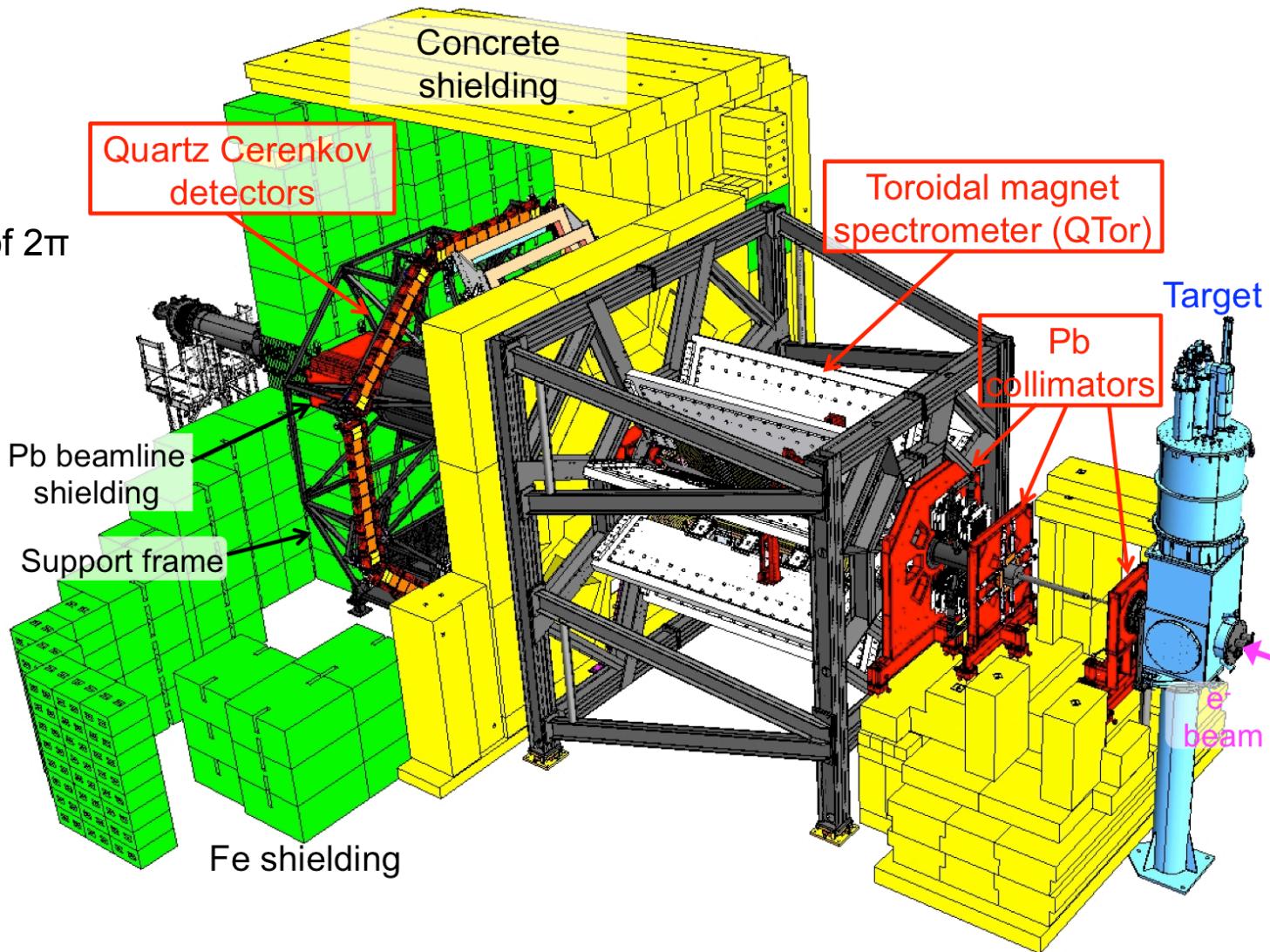
Q-weak experiment ran in Hall-C at the Thomas Jefferson National Laboratory in Newport News, Virginia from Jan 2010 – May 2012

Transverse measurements  
were taken from  
16 - 20 February, 2012

# *Q-weak Apparatus*

## Kinematics

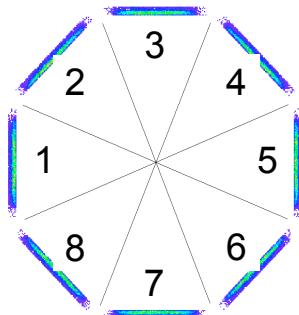
- $E_{beam} = 1.16 \text{ GeV}$
- $\langle\theta\rangle \sim 8.3^\circ$
- $Q^2 = 0.021 \text{ (GeV/c)}^2$
- $\phi$  coverage  $\sim 49\%$  of  $2\pi$
- Current =  $180 \mu\text{A}$
- Polarization = 88%



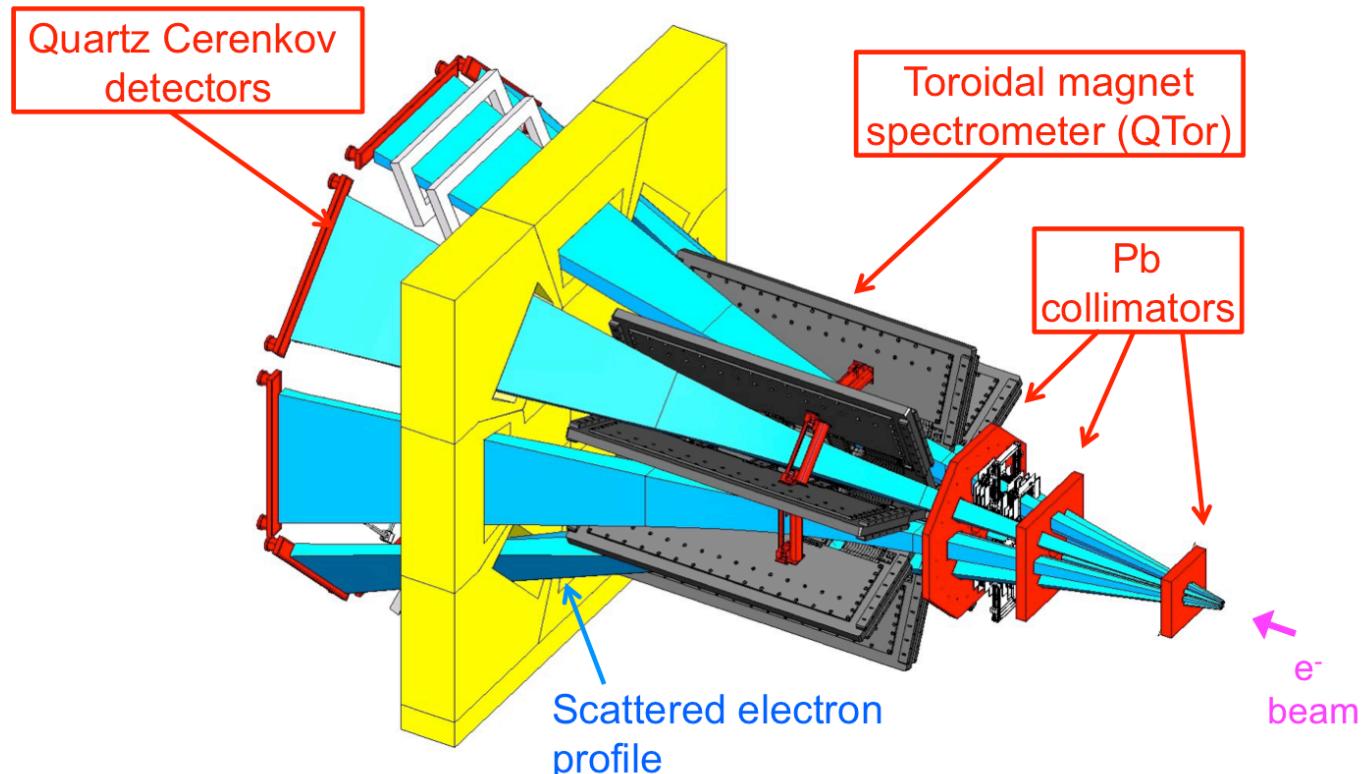
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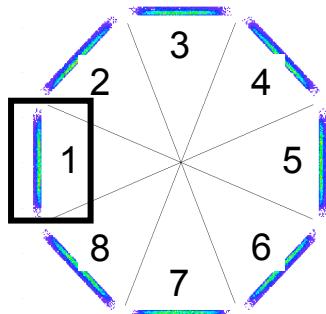
$$\phi \rightarrow (\text{octant } \# - 1) \times 45^\circ$$



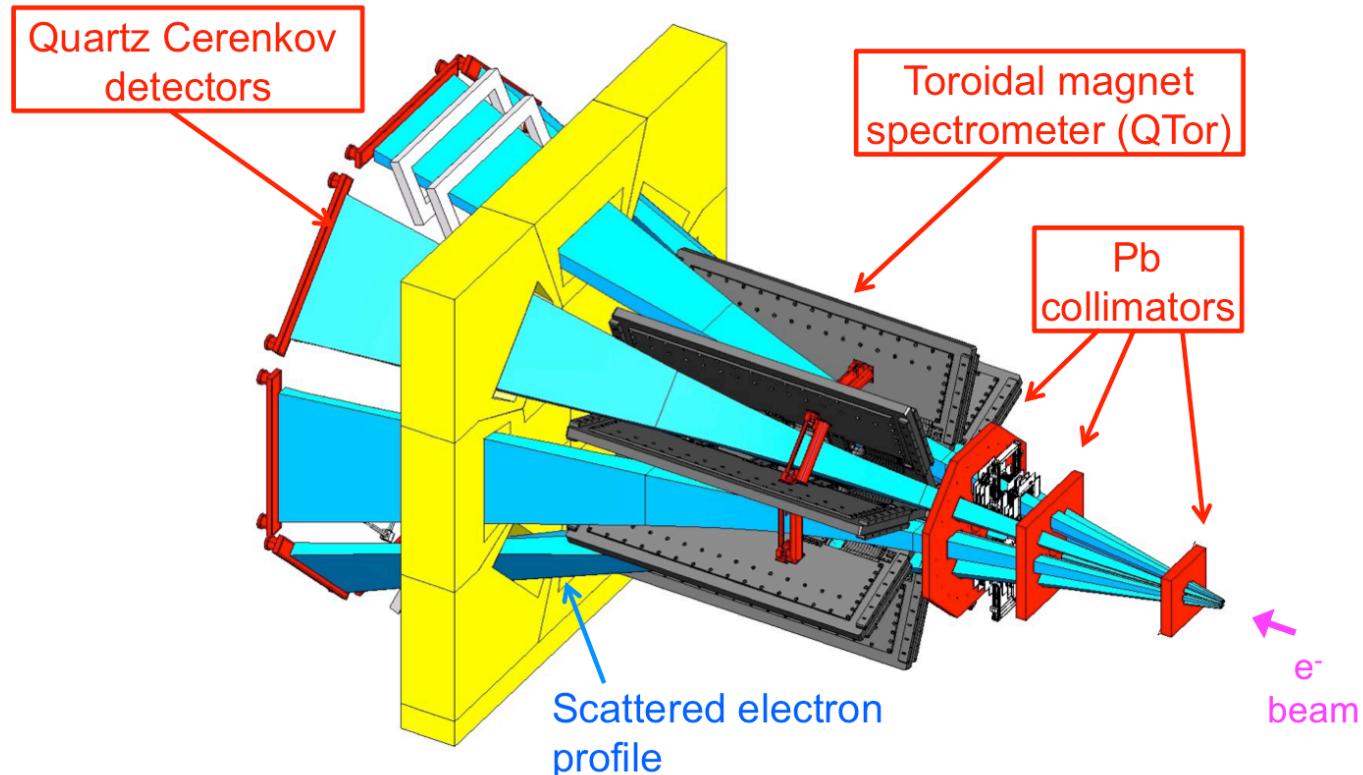
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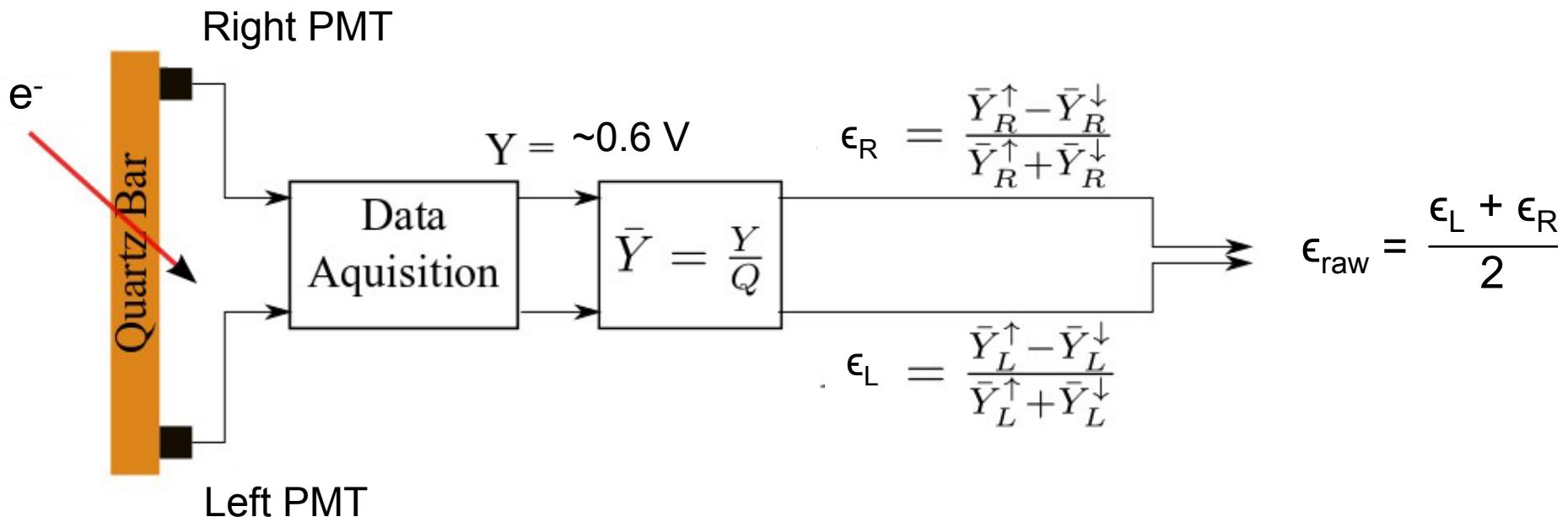
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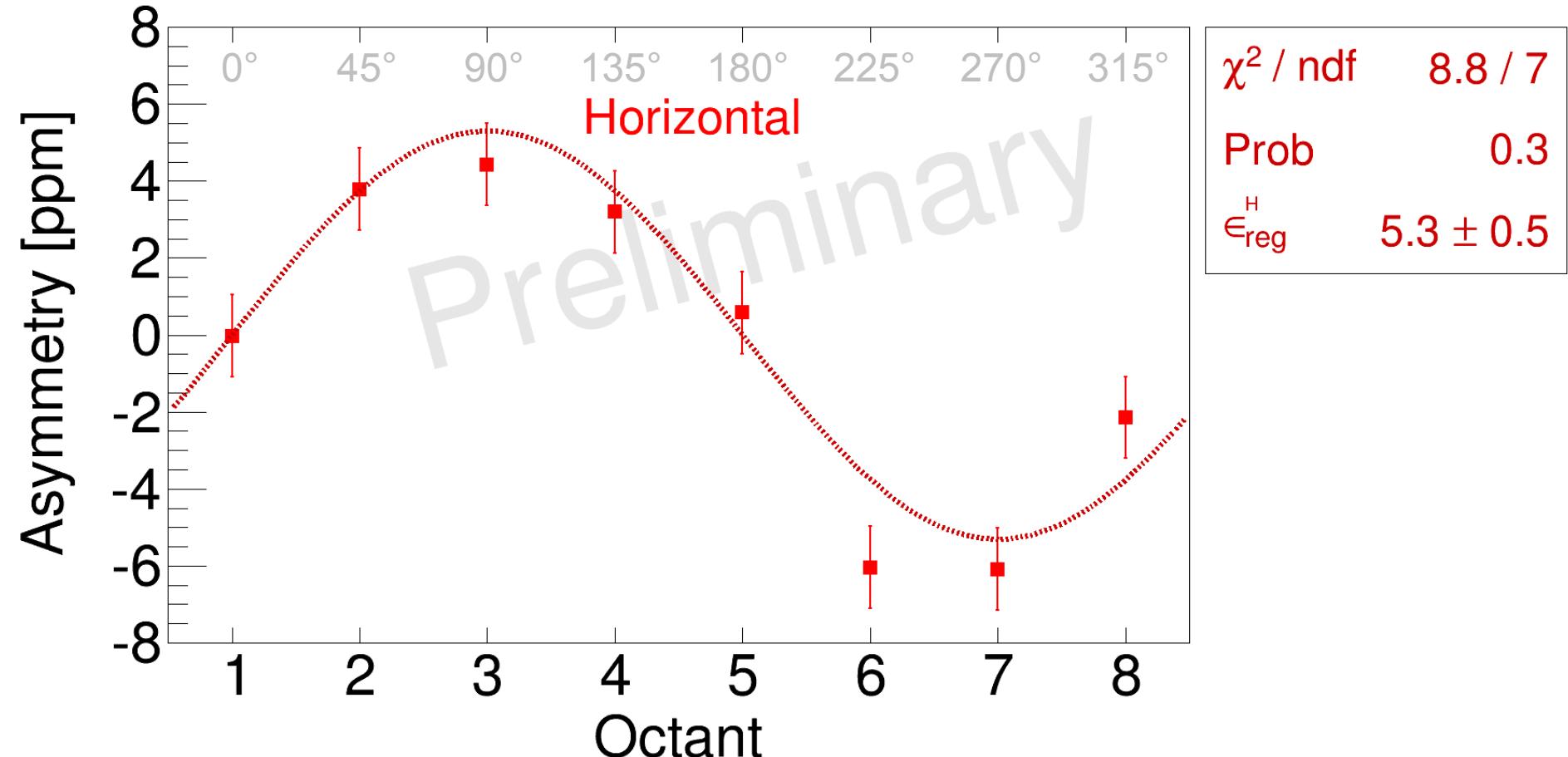
# Asymmetries from Cerenkov Detector Signals



Not corrected for

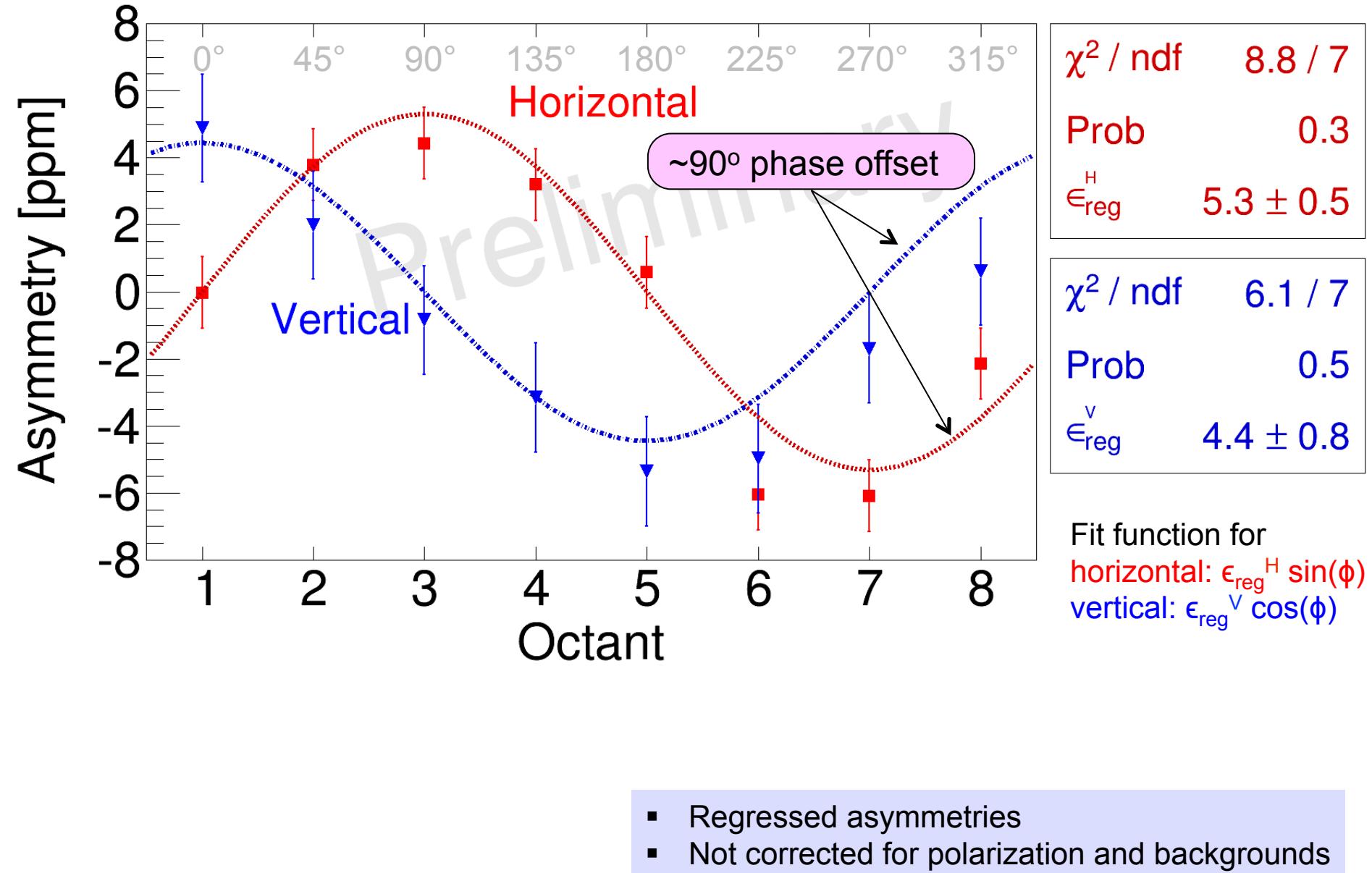
- False beam asymmetries
- Polarization
- Backgrounds

# Regressed Transverse Asymmetries



- Regressed asymmetries
- Not corrected for polarization and backgrounds

# Regressed Transverse Asymmetries

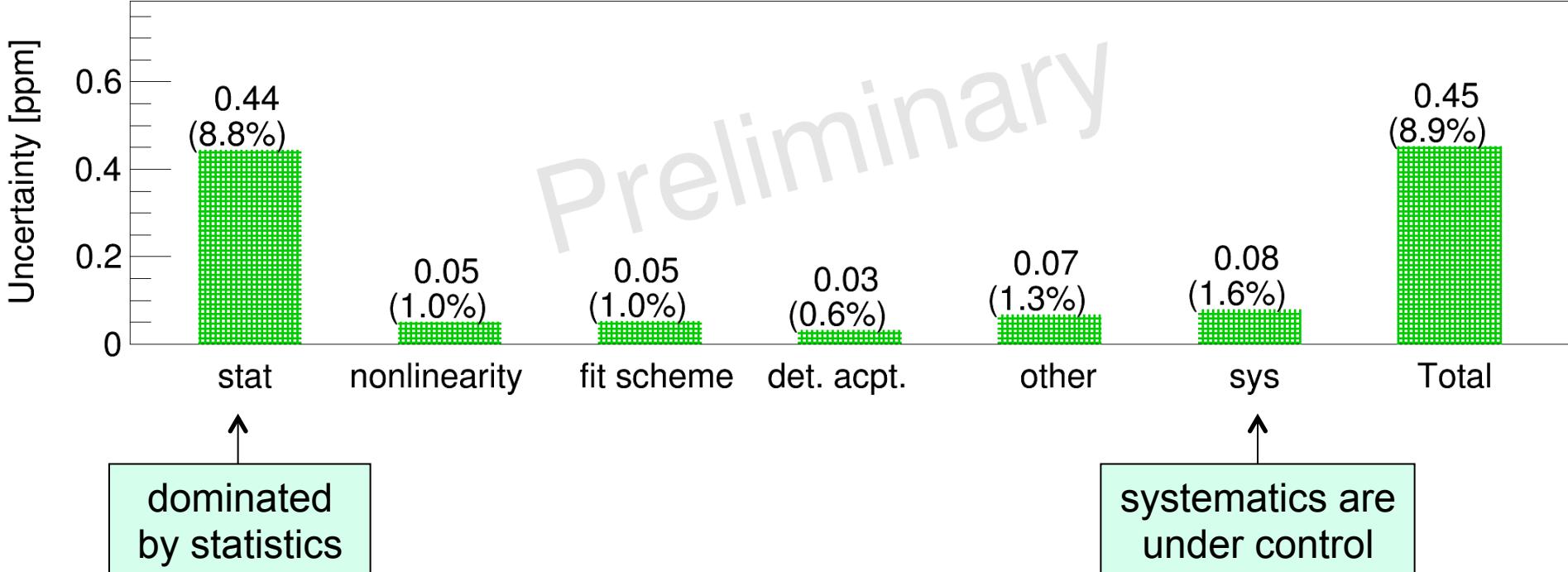


# Summary of Uncertainties on $\epsilon_{reg}$

Measured asymmetry

$$\epsilon_{reg} = 5.1 \pm 0.4 \text{ (stat)} \pm 0.1 \text{ (sys) ppm}$$

- Error weighted (H and V) regressed asym.
- Corrected for detector acceptance
- Not corrected for polarization and backgrounds



# *Extraction of Physics Asymmetry*

**Beam Normal Single  
Spin Asymmetry**

$$B_n = M_{\text{kin}} \left[ \frac{\frac{\epsilon_{\text{reg}}}{P} - B_{A1}f_{A1} - B_{BB}f_{BB} - B_{Q\text{Tor}}f_{Q\text{Tor}} - B_{el}f_{el}}{1 - f_{A1} - f_{BB} - f_{Q\text{Tor}} - f_{el}} \right]$$

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Extracting  $B_n$  from the experimental measured asymmetry by  
◆ removing false asymmetries

$$\text{◆ } \epsilon_{\text{reg}} = \epsilon_{\text{raw}} - \sum_i \frac{\partial \epsilon_{\text{raw}}}{\partial T_i} \Delta T_i, \text{ cor. for det. acpt.}$$

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Extracting  $B_n$  from the experimental measured asymmetry by

- ◆ removing false asymmetries
- ◆ correcting for the beam polarization

◆  $\epsilon_{\text{reg}} = \epsilon_{\text{raw}} - \sum_i \frac{\partial \epsilon_{\text{raw}}}{\partial T_i} \Delta T_i$ , cor. for det. acpt.

# *Extraction of Physics Asymmetry*

**Beam Normal Single Spin Asymmetry**

$$B_n = M_{\text{kin}} \left[ \frac{\frac{\epsilon_{\text{reg}}}{P} - [B_{\text{Al}}f_{\text{Al}} - B_{\text{BB}}f_{\text{BB}} - B_{\text{QTor}}f_{\text{QTor}} - B_{\text{el}}f_{\text{el}}]}{1 - [f_{\text{Al}} - f_{\text{BB}} - f_{\text{QTor}} - f_{\text{el}}]} \right]$$

Extracting  $B_n$  from the experimental measured asymmetry by

- ◆ removing false asymmetries
- ◆ correcting for the beam polarization
- ◆ removing background asymmetries

- ◆  $\epsilon_{\text{reg}} = \epsilon_{\text{raw}} - \sum_i \frac{\partial \epsilon_{\text{raw}}}{\partial T_i} \Delta T_i$ , cor. for det. acpt.
- ◆  $B_{\text{bi}}$  = Background asymmetries
- ◆  $f_{\text{bi}}$  = dilution factors
  - Al : aluminum window backgrounds
  - BB : scattering from the beamline
  - QTor: neutral particles in the magnet acpt.
  - el : elastic radiative tail

# *Extraction of Physics Asymmetry*

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$$B_n = \boxed{M_{\text{kin}}} \left[ \frac{\frac{\epsilon_{\text{reg}}}{P} - B_{\text{Al}}f_{\text{Al}} - B_{\text{BB}}f_{\text{BB}} - B_{\text{QTor}}f_{\text{QTor}} - B_{\text{el}}f_{\text{el}}}{1 - f_{\text{Al}} - f_{\text{BB}} - f_{\text{QTor}} - f_{\text{el}}} \right]$$

Extracting  $B_n$  from the experimental measured asymmetry by

- ◆ removing false asymmetries
- ◆ correcting for the beam polarization
- ◆ removing background asymmetries
- ◆ correcting for radiative tails and other kinematic correction

- ◆  $\epsilon_{\text{reg}} = \epsilon_{\text{raw}} - \sum_i \frac{\partial \epsilon_{\text{raw}}}{\partial T_i} \Delta T_i$ , cor. for det. acpt.
- ◆  $B_{\text{bi}}$  = Background asymmetries
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  - el : elastic radiative tail
- ◆  $M_{\text{kin}}$  = kinematic correction

# Summary of Uncertainties

Beam Normal Single  
Spin Asymmetry

$$B_n = M_{\text{kin}} \left[ \frac{\frac{\epsilon_{\text{reg}}}{P} - B_{\text{Al}} f_{\text{Al}} - B_{\text{BB}} f_{\text{BB}} - B_{\text{QTor}} f_{\text{QTor}} - B_{\text{el}} f_{\text{el}}}{1 - f_{\text{Al}} - f_{\text{BB}} - f_{\text{QTor}} - f_{\text{el}}} \right]$$

$$B_n = 43 \pm 16 \text{ ppm}$$

Preliminary

at kinematics

- $\langle E \rangle = 1.16 \text{ GeV}$
- $\langle W \rangle = 1.2 \text{ GeV}$
- $\langle \theta \rangle = 8.3^\circ$
- $\langle Q^2 \rangle = 0.021 \text{ GeV}^2$

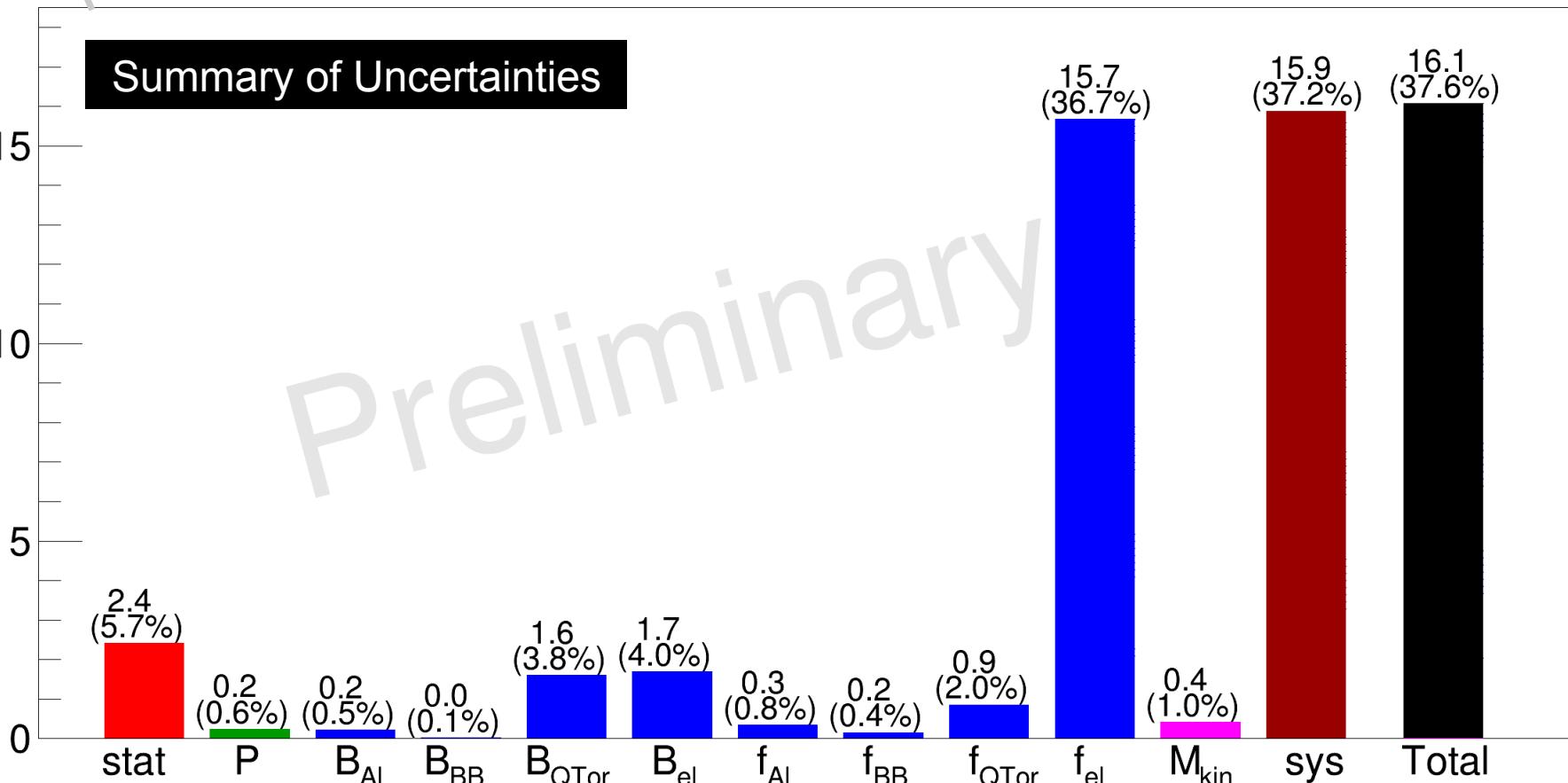
~ 38% measurement of beam normal single asymmetry in  $\Delta$  resonance production

# Summary of Uncertainties

Beam Normal Single  
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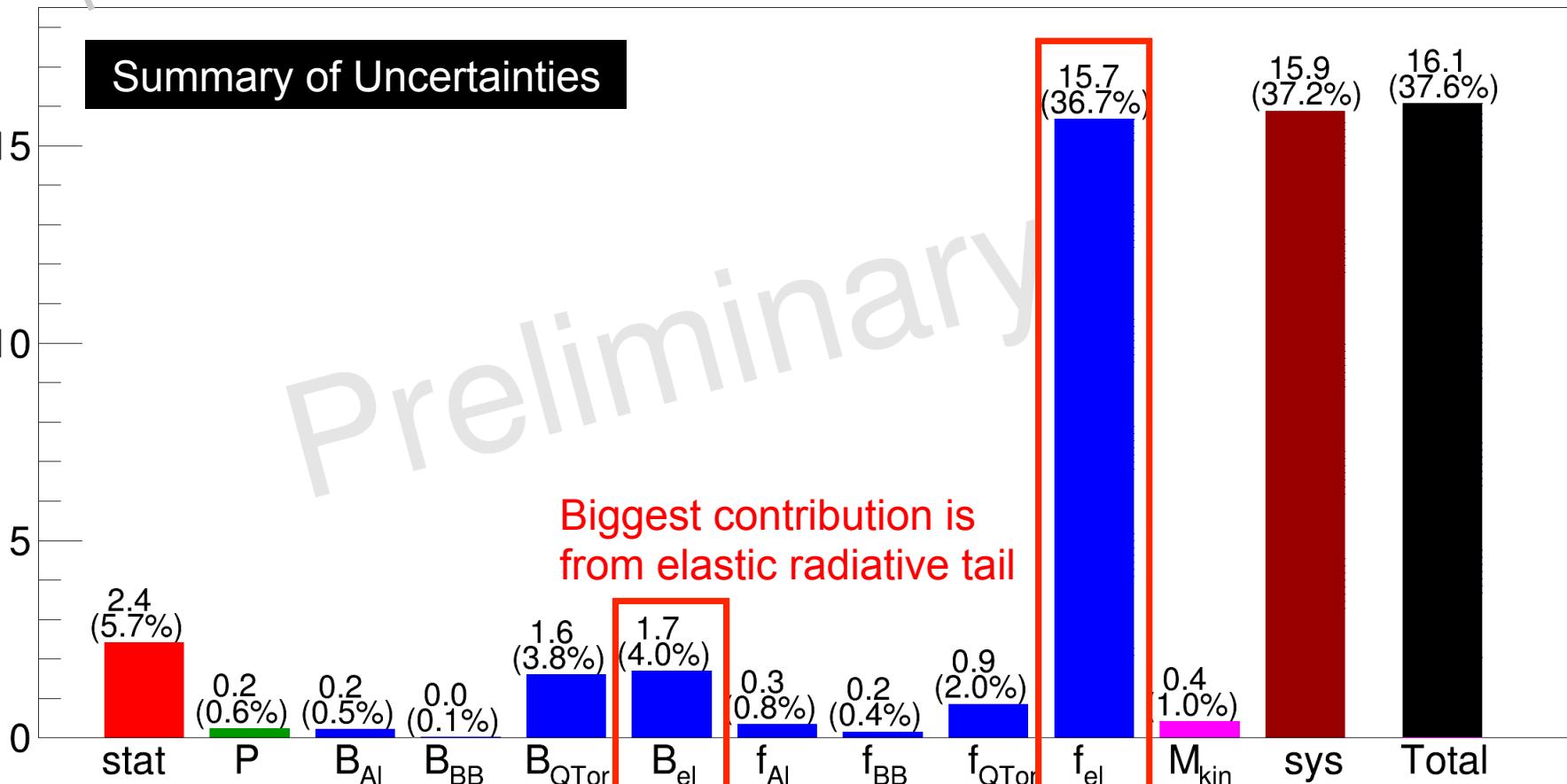


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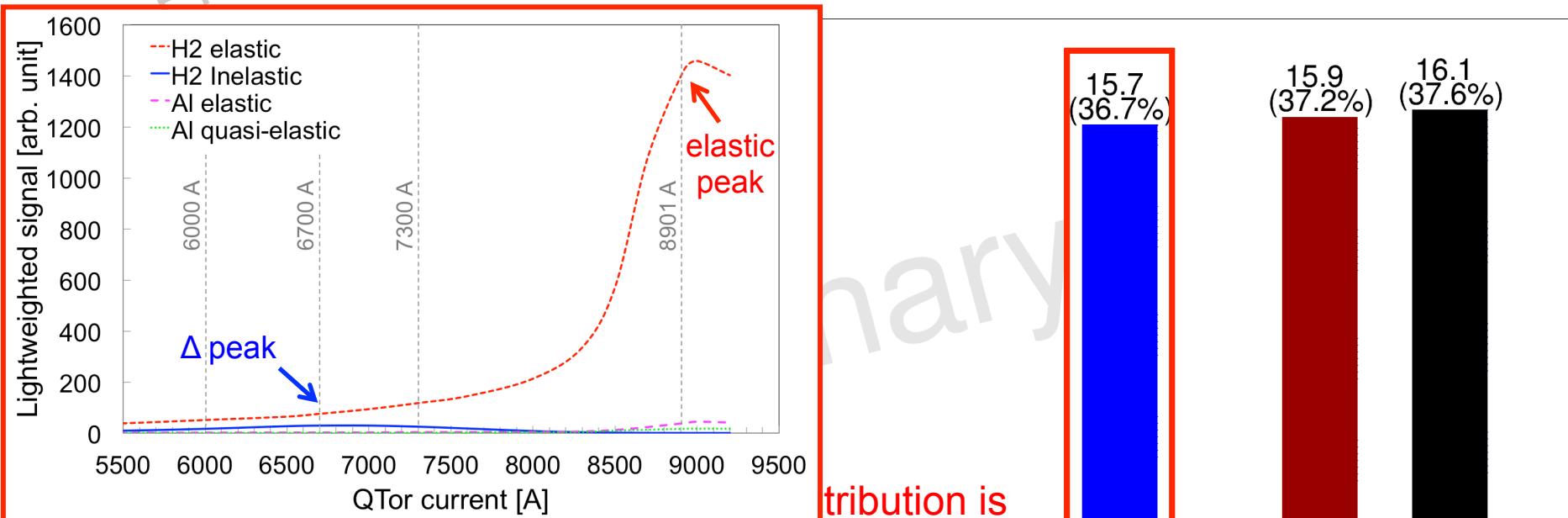


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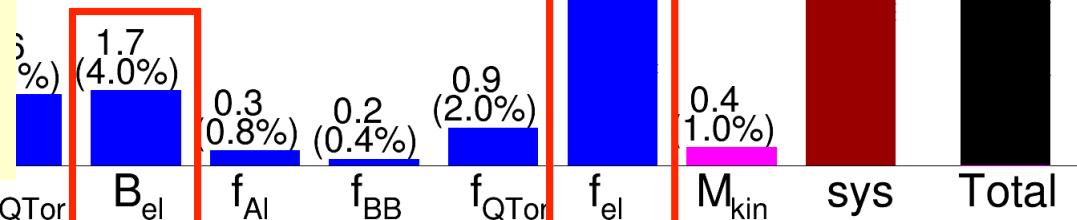
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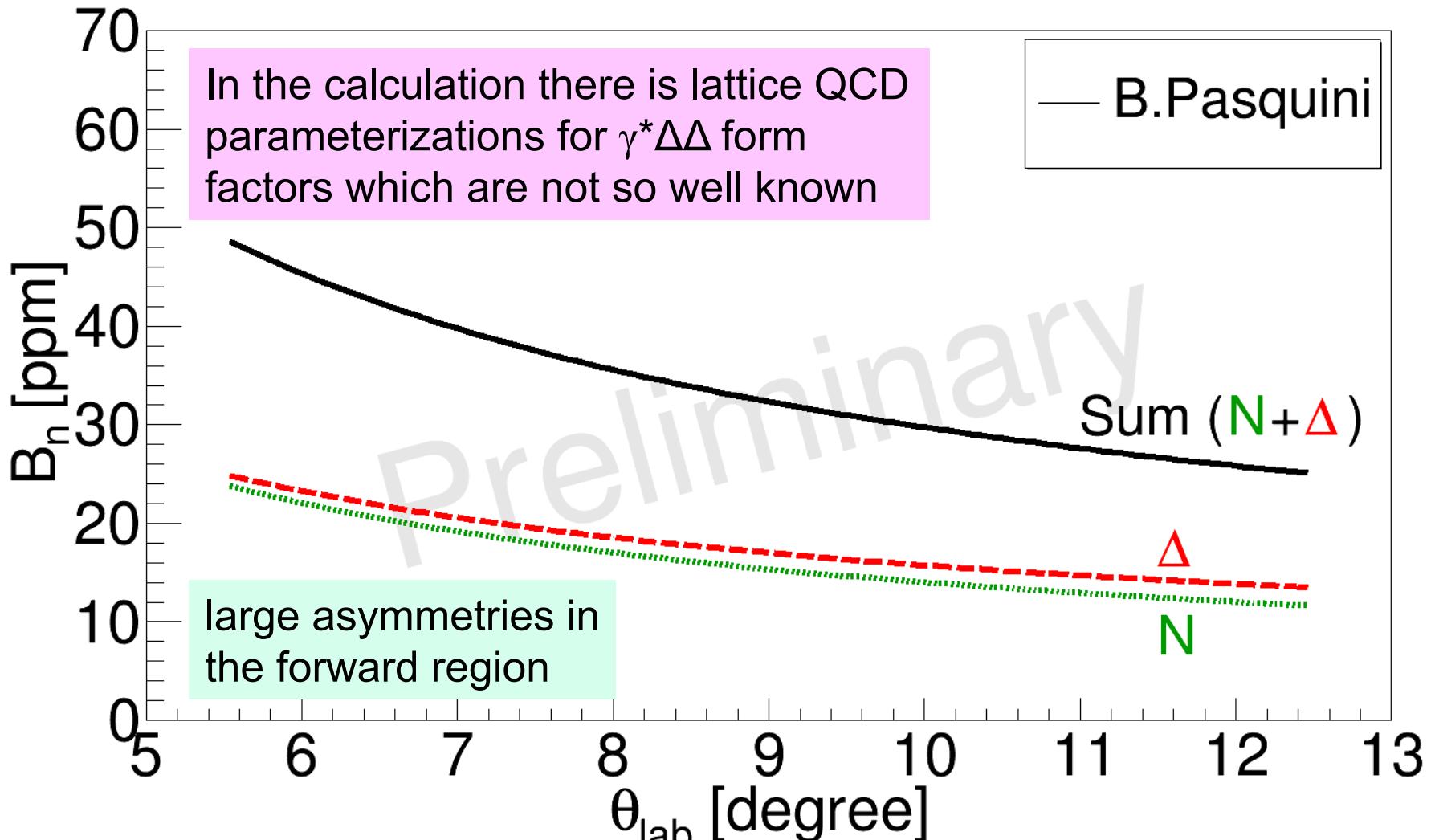
tribution is

from elastic radiative tail

This prelim. analysis takes 10% additional uncertainty due to discrepancy between data and simulation (incomplete)

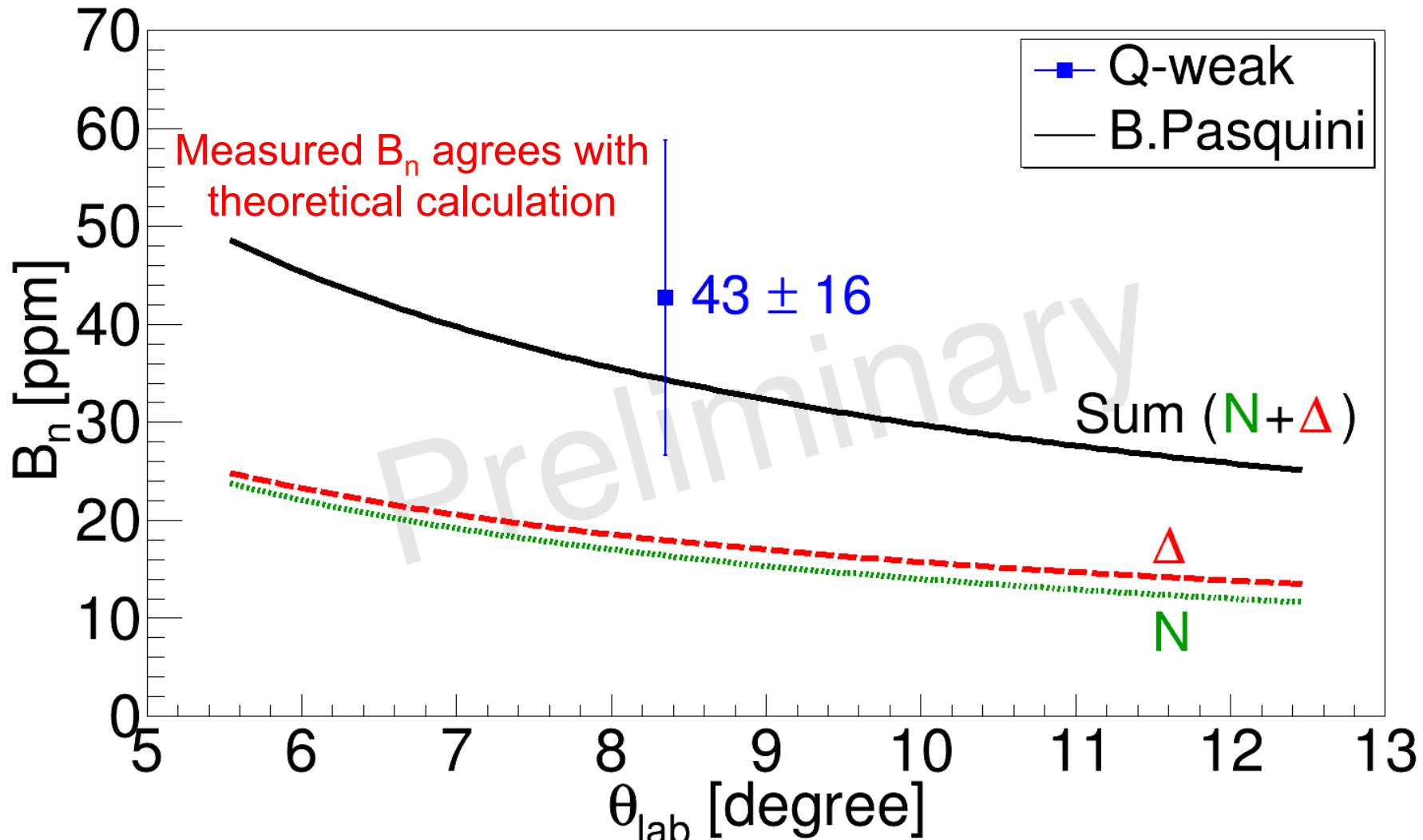


# Comparison of $B_n$ to Theory Calculation



- sensitive to  $\gamma^* \Delta\Delta$  form factors

# Comparison of $B_n$ to Theory Calculation



- sensitive to  $\gamma^* \Delta\Delta$  form factors
- Q-weak transverse dataset along with world data has potential to constrain models and study charge radius and magnetic moment of  $\Delta$

# Summary

Q-weak has measured  $B_n$  in the N-to- $\Delta$  transition on  $H_2$

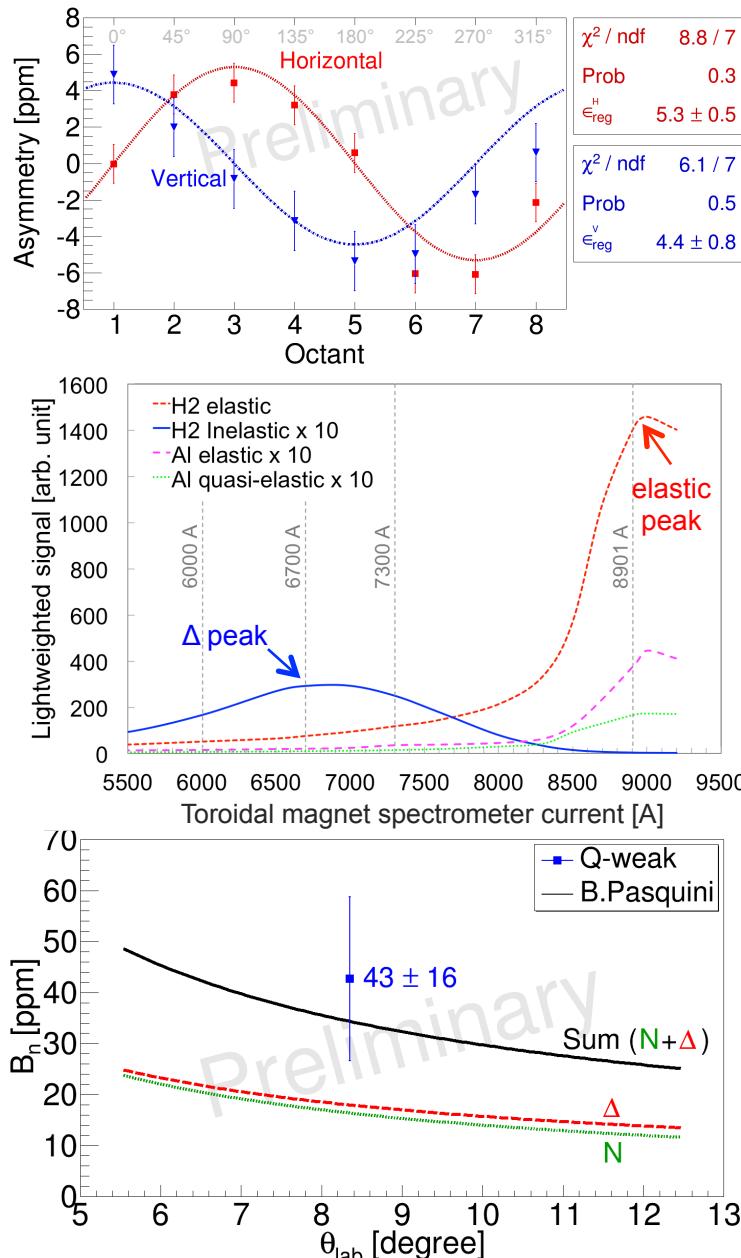
$$43 \pm 16 \text{ ppm}$$

$$\langle E \rangle = 1.16 \text{ GeV}, \langle W \rangle = 1.2 \text{ GeV}, \langle \theta \rangle = 8.3^\circ, \langle Q^2 \rangle = 0.021 \text{ GeV}^2$$

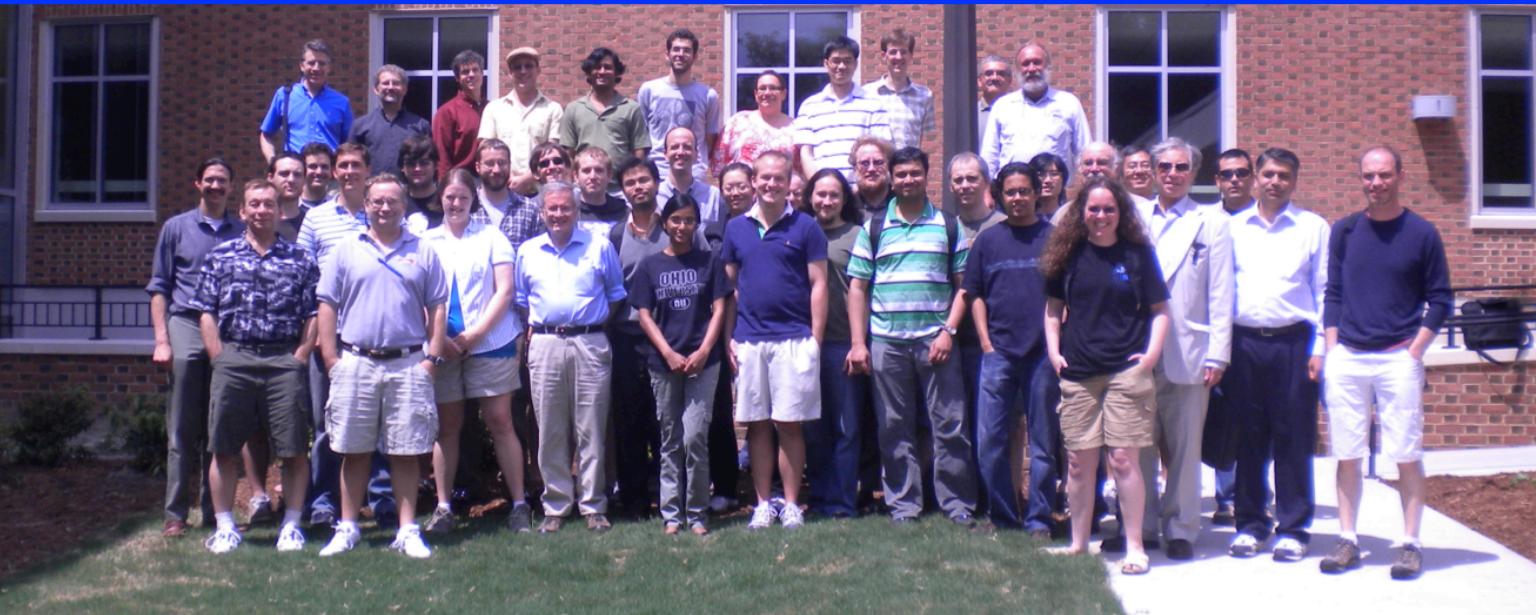
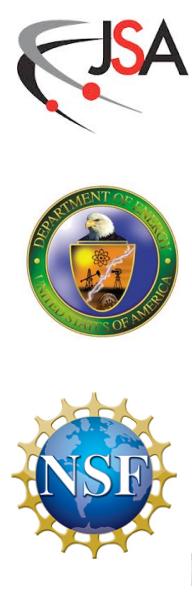
- preliminary result shows agreement with a theoretical calculation
- physics implications of the model needs investigation
- sensitive to  $\gamma^* \Delta\Delta$  form factors
- working towards the improvement in systematic uncertainty

Data for  $B_n$  at low  $Q^2$  in elastic and inelastic scattering with a  $\Delta(1232)$  final state from several targets and energy are available.

- looking for model predictions !
- Q-weak transverse dataset along with world data has potential to constrain models and study charge radius and magnetic moment of  $\Delta$



# *Q-weak Collaboration*



D.S. Armstrong, A. Asaturyan, T. Averett, J. Balewski, J. Beaufait, R.S. Beminiwattha, J. Benesch, F. Benmokhtar, J. Birchall, R.D. Carlini<sup>1</sup>, J.C. Cornejo, S. Covrig, M.M. Dalton, C.A. Davis, W. Deconinck, J. Diefenbach, K. Dow, J.F. Dowd, J.A. Dunne, D. Dutta, W.S. Duvall, M. Elaasar, W.R. Falk, J.M. Finn<sup>1</sup>, T. Forest, D. Gaskell, M.T.W. Gericke, J. Grames, V.M. Gray, K. Grimm, F. Guo, J.R. Hoskins, K. Johnston, D. Jones, M. Jones, R. Jones, M. Kargantoulakis, P.M. King, E. Korkmaz, S. Kowalski<sup>1</sup>, J. Leacock, J. Leckey, A.R. Lee, J.H. Lee, L. Lee, S. MacEwan, D. Mack, J.A. Magee, R. Mahurin, J. Mammei, J. Martin, M.J. McHugh, J. Mei, R. Michaels, A. Micherdzinska, K.E. Myers, A. Mkrtchyan, H. Mkrtchyan, A. Narayan, L.Z. Ndukum, V. Nelyubin, Nuruzzaman, W.T.H van Oers, A.K. Opper, S.A. Page<sup>1</sup>, J. Pan, K. Paschke, S.K. Phillips, M.L. Pitt, M. Poelker, J.F. Rajotte, W.D. Ramsay, J. Roche, B. Sawatzky, T. Seva, M.H. Shabestari, R. Silwal, N. Simicevic, G.R. Smith<sup>2</sup>, P. Solvignon, D.T. Spayde, A. Subedi, R. Subedi, R. Suleiman, V. Tadevosyan, W.A. Tobias, V. Tvaskis, B. Waidyawansa, P. Wang, S.P. Wells, S.A. Wood, S. Yang, R.D. Young, S. Zhamkochyan

<sup>1</sup>Spokespersons    <sup>2</sup>Project Manager    Grad Students

# Backup Slides

# Transverse Dataset

Data on 3 types of targets

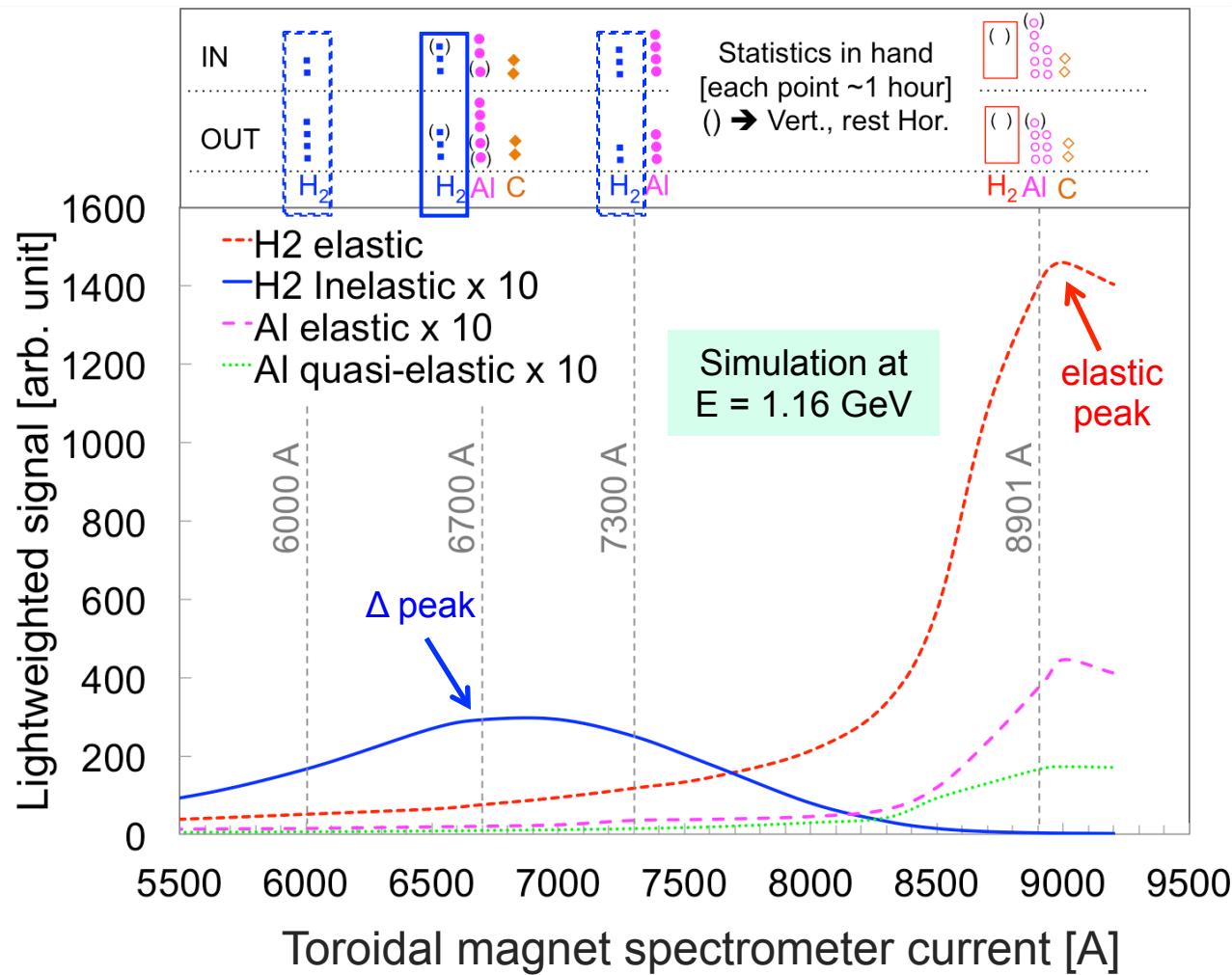
- Hydrogen
- Aluminum
- Carbon

Transverse polarization:

- Horizontal
- Vertical

This talk: Inelastic e-p scattering with a  $\Delta(1232)$  final state at  $E = 1.16$  GeV

Data on both side of the inelastic peak were taken to study the elastic dilution



Other datasets:

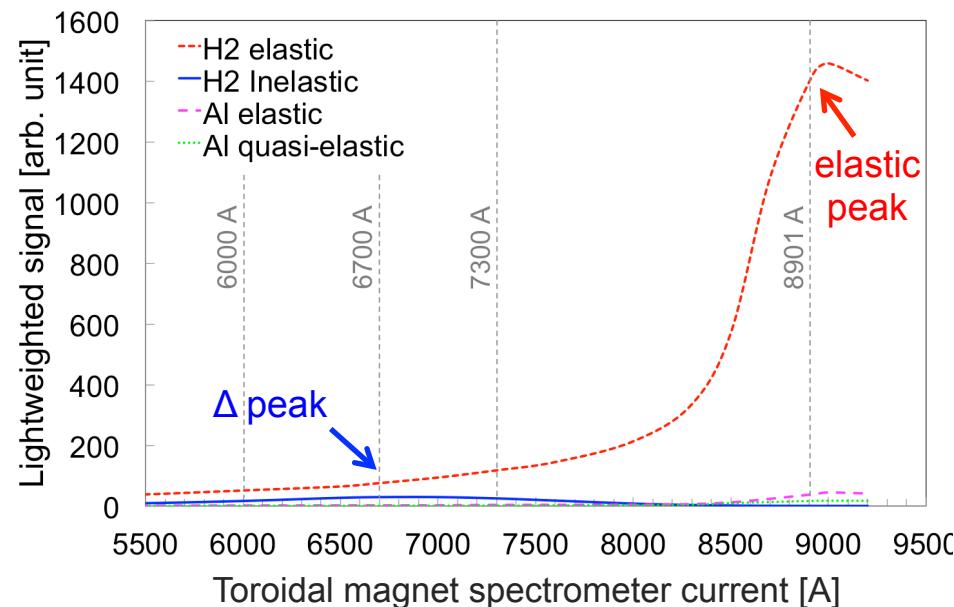
- in the  $N \rightarrow \Delta$  region (Al, C)
- in the  $N \rightarrow \Delta$  region ( $H_2$ , 0.877 GeV)
- elastic scattering ( $H_2$ , Al, C)
- elastic Møller scattering ( $H_2$ )
- in the DIS region (3.3 GeV)
- in pion photoproduction

# Elastic Radiative Tail

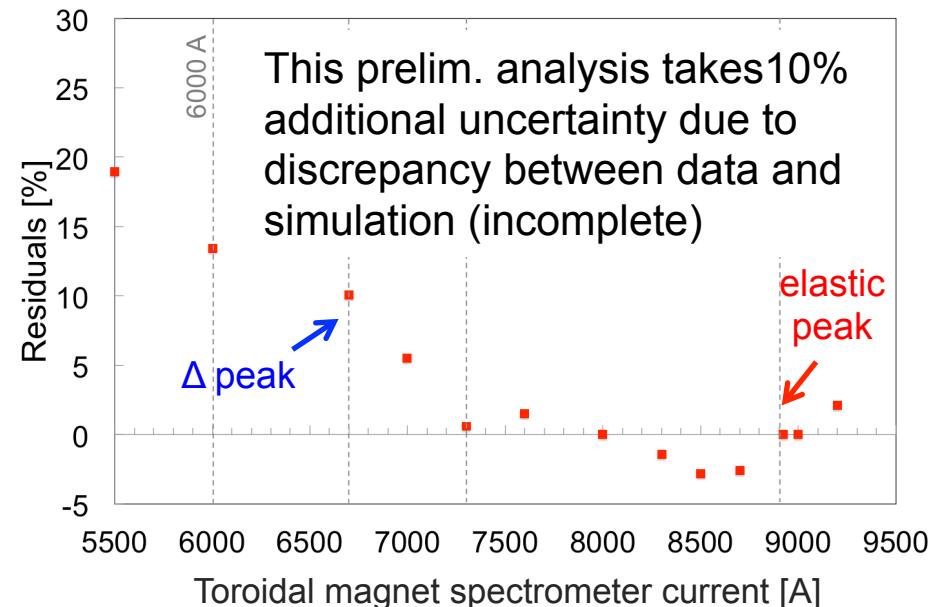
**Beam Normal Single Spin Asymmetry**

$$B_n = M_{\text{kin}} \left[ \frac{\frac{\epsilon_{\text{reg}}}{P} - B_{\text{Al}} f_{\text{Al}} - B_{\text{BB}} f_{\text{BB}} - B_{\text{QTor}} f_{\text{QTor}} - [B_{\text{el}} f_{\text{el}}]}{1 - f_{\text{Al}} - f_{\text{BB}} - f_{\text{QTor}} - [f_{\text{el}}]} \right]$$

## Background Corrections



## Elastic radiative tail



$$\text{Residual} = \frac{(\text{data} - \text{sim.})}{\text{data}}$$

$$B_{\text{el}} = -5.1 \pm 0.5 \text{ ppm}$$

$$f_{\text{el}} = 0.70 \pm 0.07$$

# $\Delta$ Elastic Form Factors

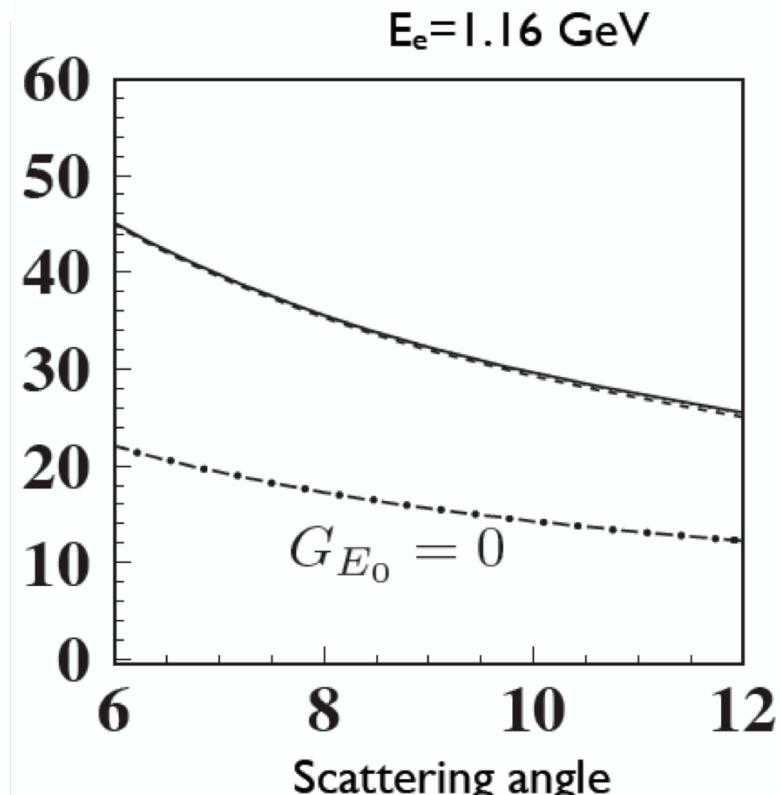
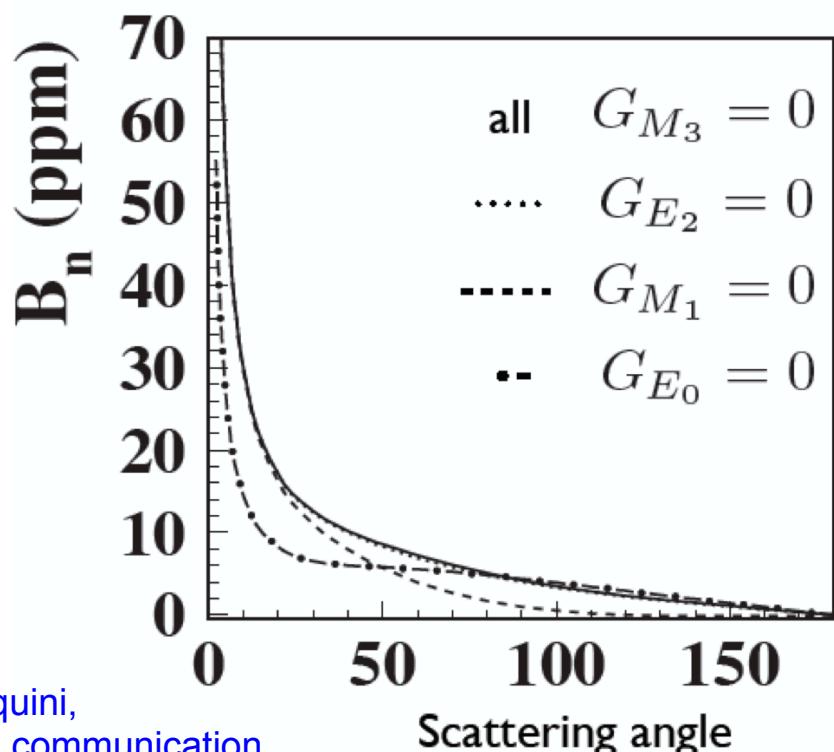
There are 4 elastic form factors (spin 3/2)

$$G_{E0}(Q^2), G_{M1}(Q^2), G_{E2}(Q^2), G_{M3}(Q^2)$$

J. Segovia et. al. arXiv:1308.5225 (2013)

$Q^2 = 0$  define dimensionless multipole moments

- $G_{E0}(0) = e_\Delta$  charge
- $G_{M1}(0) \propto \mu_\Delta$  magnetic moment
- $G_{E2}(0) \propto D_\Delta$  dipole moment
- $G_{M3}(0) \propto O_\Delta$  octopole moment



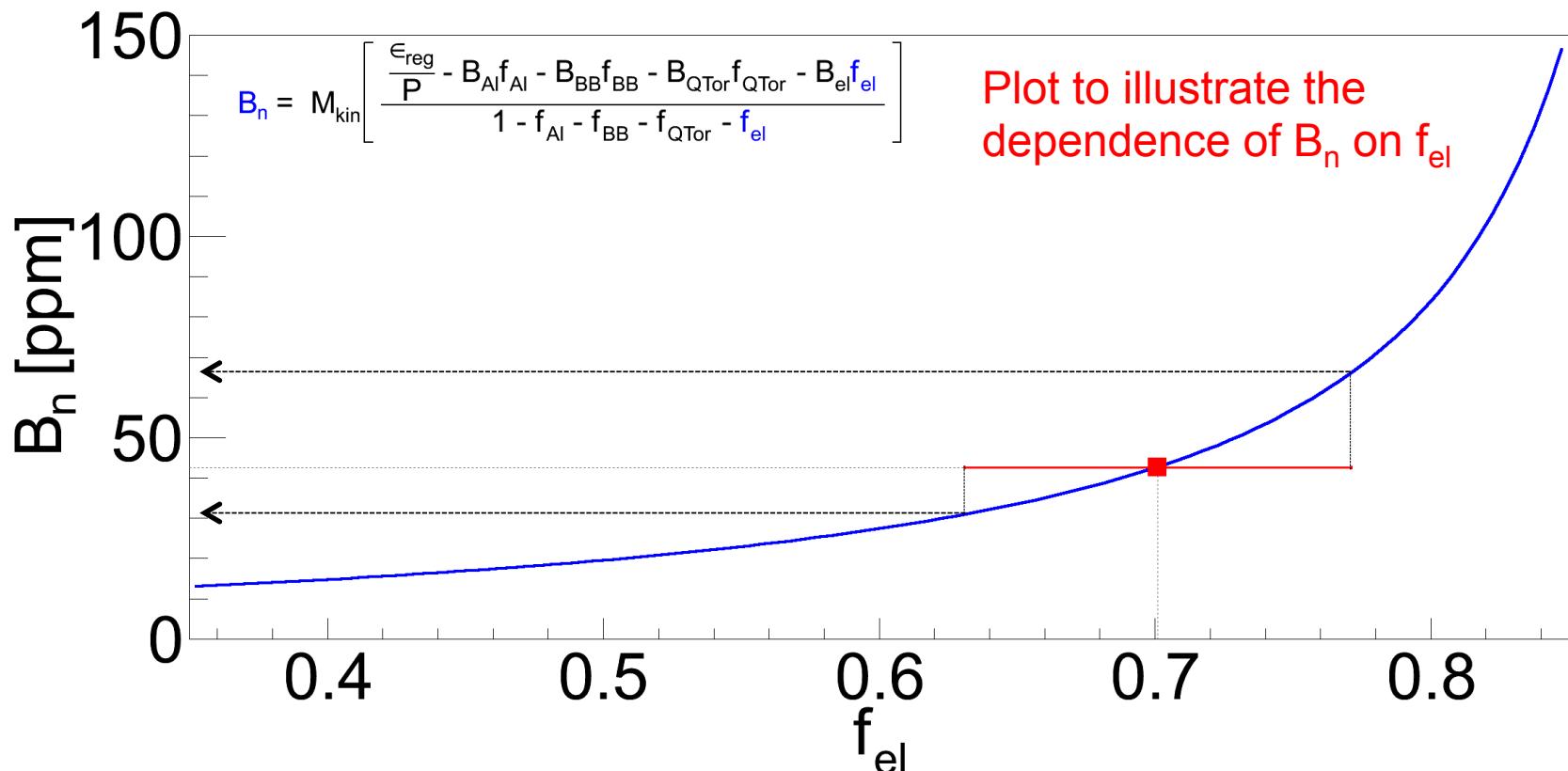
B Pasquini,  
private communication

# Asymmetry is Diluted by Elastic Radiative Tail

Simplifying\* the extraction equation  
(\* for illustrative purposes)

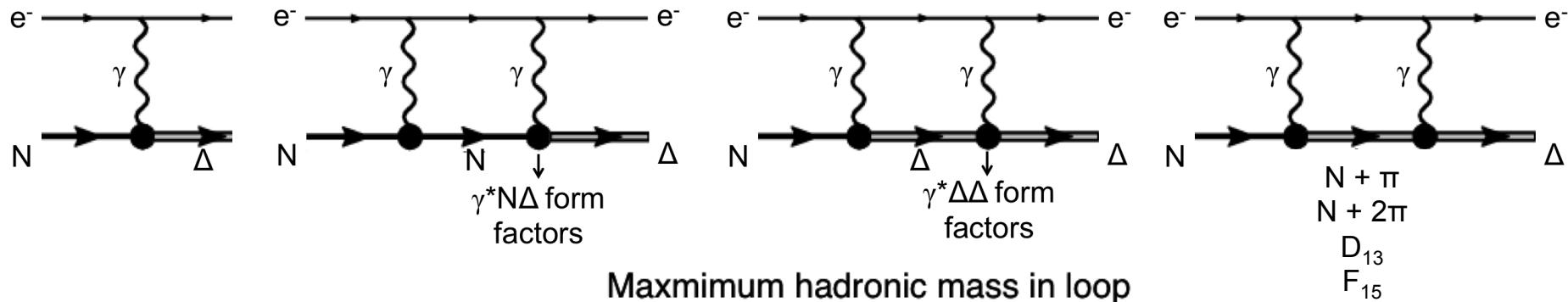
$$\frac{(1 - f_{\text{total}})}{M_{\text{kin}}} B_n = \frac{\epsilon_{\text{reg}}}{P} - B_{\text{el}} f_{\text{el}} - \text{others}$$

$$0.214 \times 43 \text{ ppm} = 9.2 \text{ ppm} \approx (5.1/0.875) - 0.70 \times (-5.1) - 0.3 = 5.8 - (-3.6) - 0.3 \text{ ppm}$$

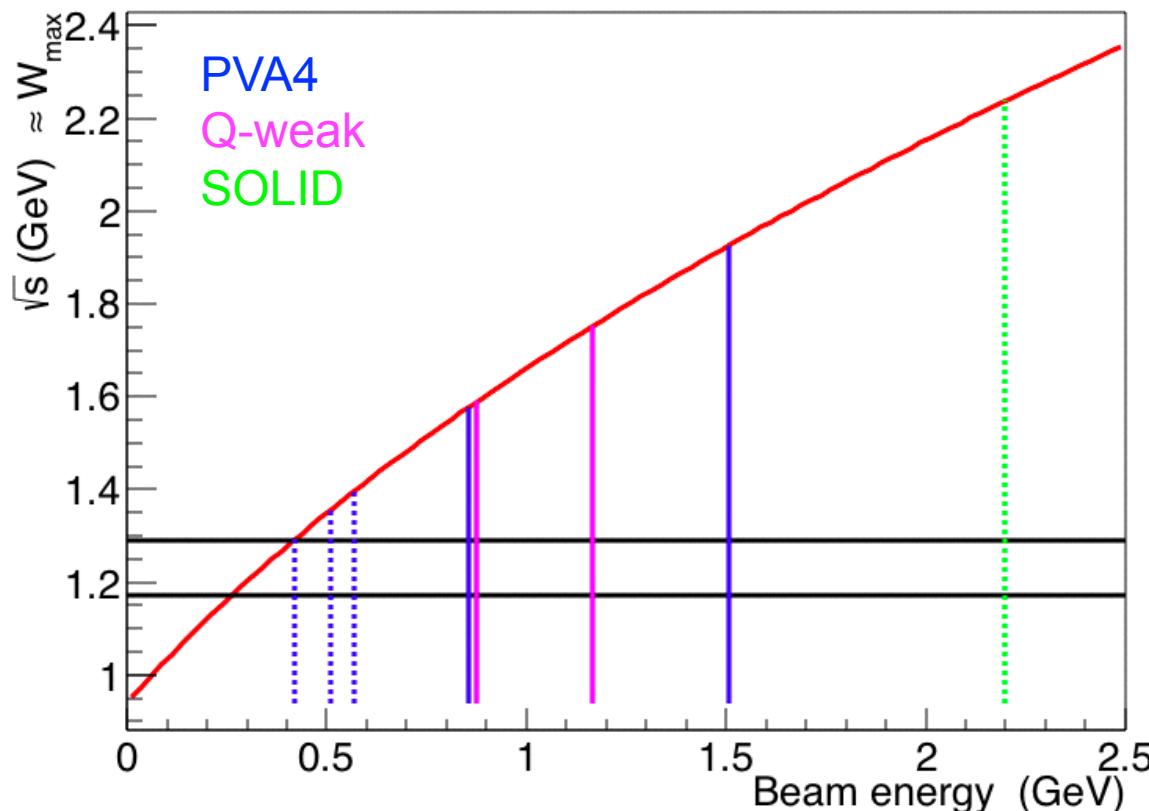


- The extraction of  $B_n$  depends strongly on the elastic dilution
- Careful study is ongoing to reduce uncertainty in  $f_{\text{el}}$

# Summary of Measurements



Higher beam energies allow more intermediate states



# Why $B_n=0$ at $\theta=0$ ?

$$B_n = \frac{\sigma \uparrow - \sigma \downarrow}{\sigma \uparrow + \sigma \downarrow} = \frac{2T_{1y} \times \text{Im } T_{2y}}{|T_{1y}|}$$

$$\varepsilon^{-1} = 1 + 2 \left[ 1 + \frac{v^2}{Q^2} \right] \tan^2 \frac{\theta_e}{2}$$

$$\begin{aligned} \sigma \uparrow - \sigma \downarrow &\propto \frac{2m_e}{Q} (1-\varepsilon) \underbrace{\left[ \frac{\varepsilon_L}{\varepsilon} \right]^{1/2}}_{\frac{Q}{v}} = \frac{2m_e}{Q} (1-\varepsilon) \frac{Q}{v} = \frac{2m_e}{v} (1-\varepsilon) \\ &= \frac{4m_e \varepsilon}{v} \left[ 1 + \frac{v^2}{Q^2} \right] \tan^2 \frac{\theta_e}{2} \end{aligned}$$

$$(1-\varepsilon) = \varepsilon(1/\varepsilon - 1)$$

$$= 2\varepsilon \left[ 1 + \frac{v^2}{Q^2} \right] \tan^2 \frac{\theta_e}{2}$$

$$v = K \bullet P$$

K is the average incoming four-momenta of the electron  
 P is the average outgoing four-momenta of the proton

$$v_{el} = 0.013$$

$$v_{in} = 0.348$$

$$v_{min} = 300 \text{ MeV} + K.E.$$

Private communication with  
 Carl Carlson