

Neutrino Mass Generation

K.S. Babu

Oklahoma State University



Neutrino 2014

Boston, USA

June 2-7, 2014

Outline of the talk

This will be an attempt to paint with a broad brush a picture of theories and models of neutrino mass generation.

What fundamental physics have we learned?

What more can we learn?

Fermion Mass Spectrum

Fermion masses in units of m_t

$$m_t = 1.0$$

$$m_c = 3.6 \times 10^{-3}$$

$$m_u = 1.3 \times 10^{-5}$$

$$m_\tau = 1.0 \times 10^{-2}$$

$$m_\mu = 6.2 \times 10^{-4}$$

$$m_e = 3.0 \times 10^{-6}$$

$$m_b = 1.67 \times 10^{-2}$$

$$m_s = 3.1 \times 10^{-4}$$

$$m_d = 2.3 \times 10^{-5}$$

$$m_3 = 2.9 \times 10^{-13}$$

$$m_2 = 5.2 \times 10^{-14}$$

$$m_1 = < m_2 \quad \text{Normal hierarchy?}$$

$$V_q = \begin{pmatrix} 0.976 & 0.22 & 0.004 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{pmatrix}$$

$$U_\ell = \begin{pmatrix} 0.85 & -0.54 & 0.16 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & -0.59 & -0.70 \end{pmatrix}$$

$$\text{Im} \left(\frac{V_{ub} V_{cs}}{V_{us} V_{cb}} \right) = 0.34$$

$$\delta_{CP} = ?$$

Pressing Questions for Neutrinos

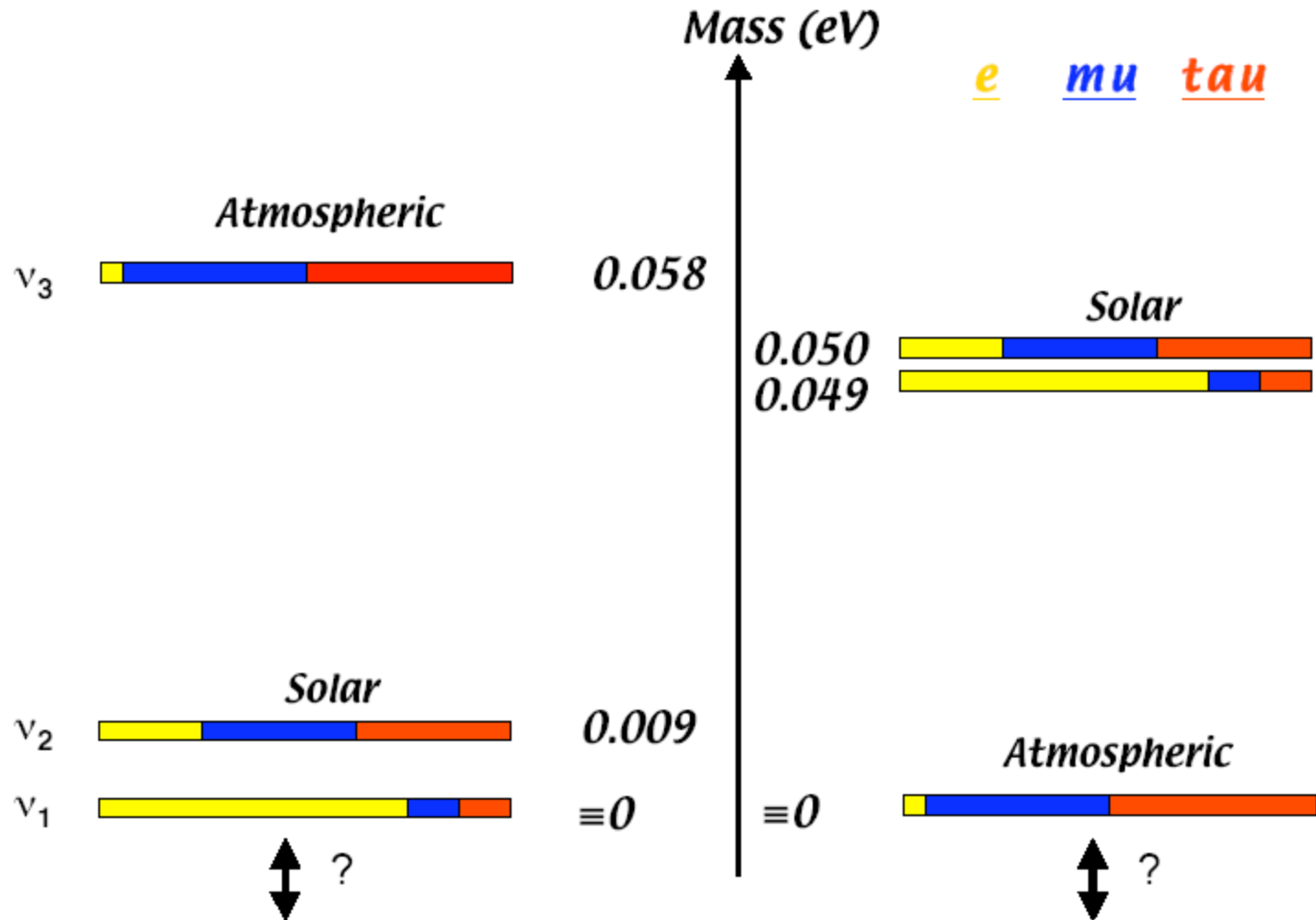
- Are neutrinos their own antiparticles?
- Is there CP violation in neutrino oscillations?
- Is the mass hierarchy normal or inverted?
- Are there light sterile neutrinos?
- What is the scale of neutrino mass generation?
- What explains the pattern of neutrino mixings?
- Can neutrinos be unified with quarks?
- Is neutrino CP violation related to baryon asymmetry?

A global fit to neutrino oscillation data

parameter	best fit	1σ range	2σ range	3σ range
Δm_{21}^2 [10^{-5}eV^2]	7.60	7.42–7.79	7.26–7.99	7.11–8.18
$ \Delta m_{31}^2 $ [10^{-3}eV^2] (NH)	2.48	2.41–2.53	2.35–2.59	2.30–2.65
$ \Delta m_{31}^2 $ [10^{-3}eV^2] (IH)	2.38	2.32–2.43	2.26–2.48	2.20–2.54
$\sin^2 \theta_{12}/10^{-1}$	3.23	3.07–3.39	2.92–3.57	2.78–3.75
$\sin^2 \theta_{23}/10^{-1}$ (NH)	5.67 (4.67) ^a	4.39–5.99	4.13–6.23	3.92 – 6.43
$\sin^2 \theta_{23}/10^{-1}$ (IH)	5.73	5.30–5.98	4.32–6.21	4.03–6.40
$\sin^2 \theta_{13}/10^{-2}$ (NH)	2.34	2.14–2.54	1.95–2.74	1.77–2.94
$\sin^2 \theta_{13}/10^{-2}$ (IH)	2.40	2.21–2.59	2.02–2.78	1.83–2.97
δ/π (NH)	1.34	0.96–1.98	0.0–2.0	0.0–2.0
δ/π (IH)	1.48	1.16–1.82	0.0–0.14 & 0.81–2.0	0.0–2.0

Forero, Tortola, Valle (2014)

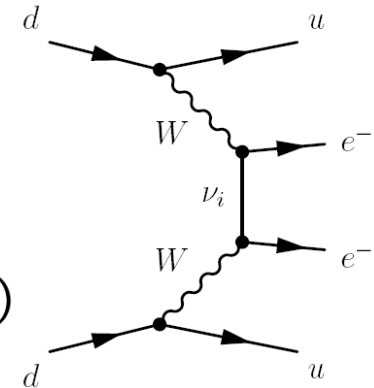
Neutrino Mass Ordering



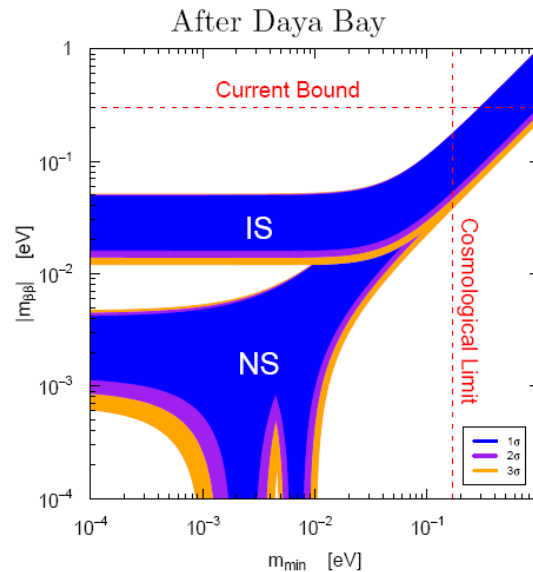
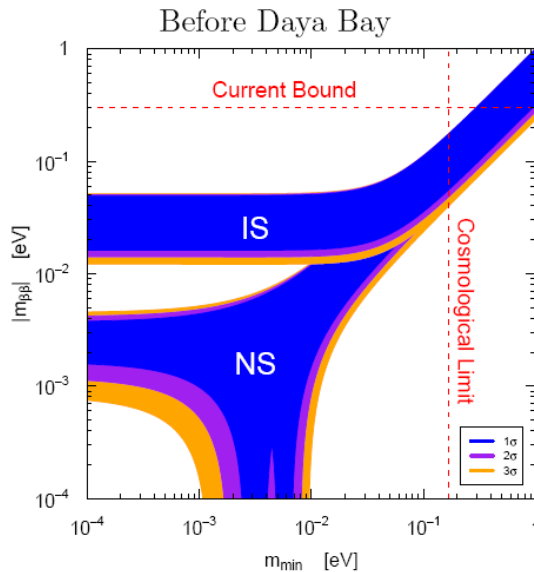
Neutrinoless Double Beta Decay

$$N(A, Z) \rightarrow N(A, Z + 2) + e^- + e^-$$

(Assumes neutrino mass is the only contribution)



$$|m_{\beta\beta}| = \left| \cos^2 \theta_{12} \cos^2 \theta_{13} m_1 + e^{2i\alpha_{12}} \sin^2 \theta_{12} \cos^2 \theta_{13} m_2 + e^{2i\alpha_{13}} \sin^2 \theta_{13} m_3 \right|$$



Bilenky, Giunti, 2012

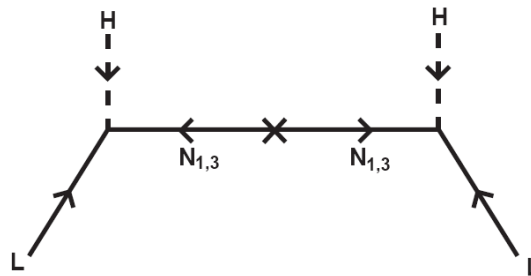
Origin of small Majorana neutrino mass

Seesaw mechanism

$$\mathcal{L}_{\text{eff}} = \frac{LLHH}{M} \Rightarrow m_\nu \sim \frac{v^2}{M}$$

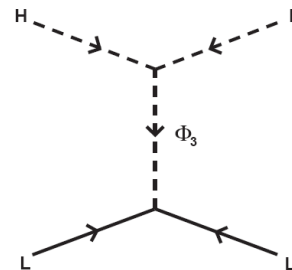
Neutrino oscillations probe $M \sim 10^{14}$ GeV

Type (I,III) seesaw



$$N_1 : (1, 1, 0), N_3 : (1, 3, 0)$$

Type II seesaw

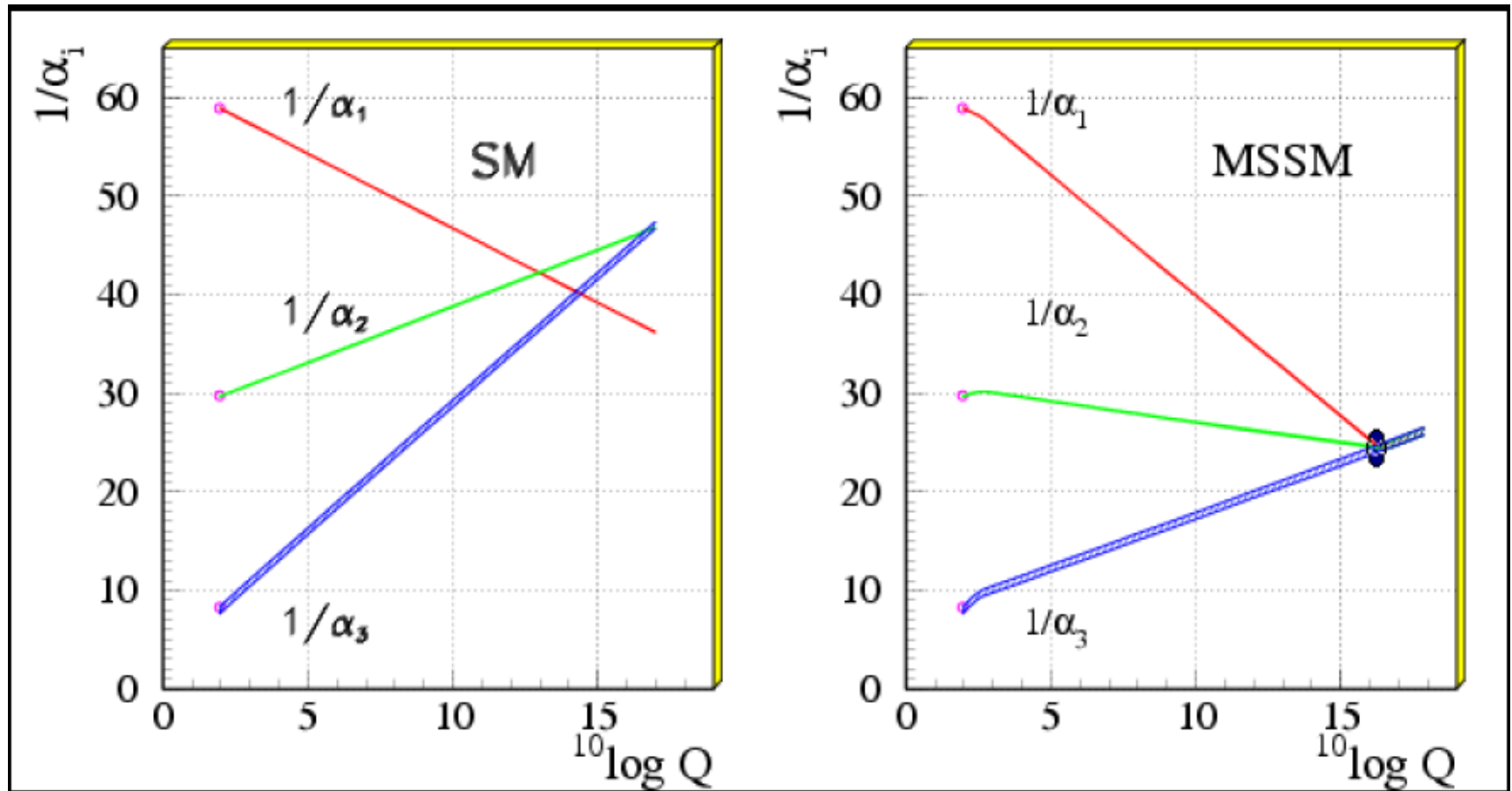


$$\Phi_3 : (1, 3, +1)$$

Neutrino Masses in Grand Unified Theories

- Electric charge quantization
 - ◇ $Q_p = -Q_e$ to better than 1 part in 10^{21}
- Miraculous cancellation of anomalies
- Quantum numbers of quarks and leptons
- **Existence of ν_R and thus neutrino mass via seesaw**
- Unification of gauge couplings with low energy SUSY
- $b - \tau$ unification
- Baryon asymmetry of the universe via leptogenesis

Gauge coupling unification



From S. Raby, PDG Review

SO(10) Grand Unification

Unifies all members of a family into a single 16-plet

$u_r : \{-+++-\}$	$d_r : \{-++-+\}$	$u_r^c : \{+- -++\}$	$d_r^c : \{+- - --\}$
$u_b : \{+-+ +- \}$	$d_b : \{+-+ -+ \}$	$u_b^c : \{-+-++ \}$	$d_b^c : \{-+- -- \}$
$u_g : \{++- +- \}$	$d_g : \{++- -+ \}$	$u_g^c : \{- -++ \}$	$d_g^c : \{- -+ -- \}$
$\nu : \{--- +- \}$	$e : \{--- -+ \}$	$\nu^c : \{+++ ++ \}$	$e^c : \{+++ -- \}$

Predicts right-handed neutrino and thus neutrino masses

First 3 spins refer to color, last 2 are weak spins

$$Y = \frac{1}{3}\Sigma(C) - \frac{1}{2}\Sigma(W)$$

$$\text{Eg: } Y(e^c) = \frac{1}{3}(3) - \frac{1}{2}(-2) = 2$$

Such an elegant arrangement very strongly suggestive of GUTs

Minimal SO(10) Model

$$\mathcal{L}_{\text{Yukawa}} = Y_{10} \mathbf{16} \mathbf{16} \mathbf{10}_H + Y_{126} \mathbf{16} \mathbf{16} \overline{\mathbf{126}}_H$$

Two Yukawa matrices determine all fermion masses and mixings, including the neutrinos

$$M_u = \kappa_u Y_{10} + \kappa'_u Y_{126}$$

$$M_d = \kappa_d Y_{10} + \kappa'_d Y_{126}$$

$$M_\nu^D = \kappa_u Y_{10} - 3\kappa'_u Y_{126}$$

$$M_l = \kappa_d Y_{10} - 3\kappa'_d Y_{126}$$

$$M_{\nu R} = \langle \Delta_R \rangle Y_{126}$$

$$M_{\nu L} = \langle \Delta_L \rangle Y_{126}$$

Model has only 11 real parameters plus 7 phases

Babu, Mohapatra (1993)

Fukuyama, Okada (2002)

Bajc, Melfo, Senjanovic, Vissani (2004)

Fukuyama, Ilakovac, Kikuchi, Meljanac, Okada (2004)

Aulakh et al (2004)

Bertolini, Frigerio, Malinsky (2004)

Babu, Macesanu (2005)

Bertolini, Malinsky, Schwetz (2006)

Dutta, Mimura, Mohapatra (2007)

Bajc, Dorsner, Nemevsek (2009)

Specific Example: Type I Seesaw

Input at the GUT scale:

$$\begin{array}{lll} m_u = 0.0006745 & m_c = 0.3308 & m_t = 97.335 \\ m_d = 0.0009726 & m_s = 0.02167 & m_b = 1.1475 \\ m_e = 0.000344 & m_\mu = 0.0726 & m_\tau = 1.350 \text{ GeV} \\ s_{12} = 0.2248 & s_{23} = 0.03278 & s_{13} = 0.00216 \\ & \delta_{CKM} = 1.193 . \end{array}$$

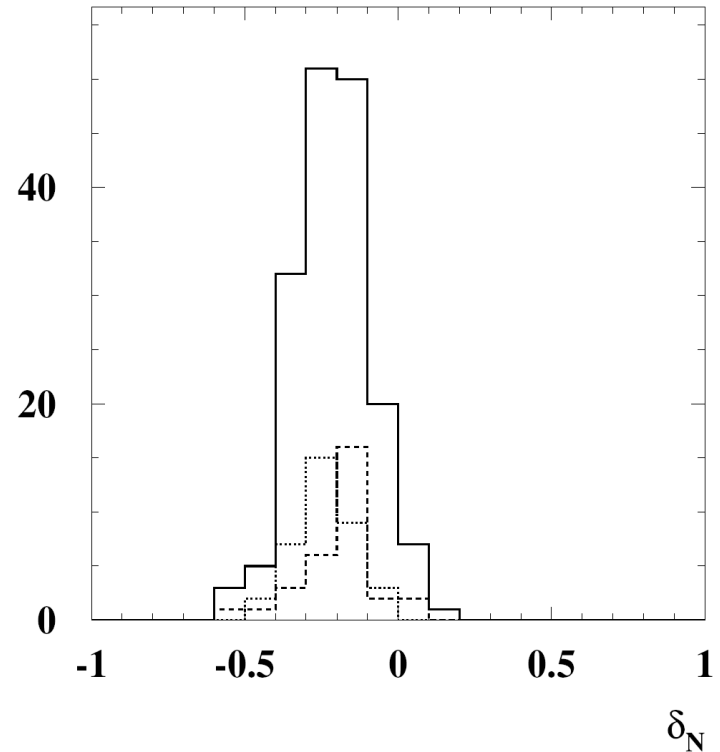
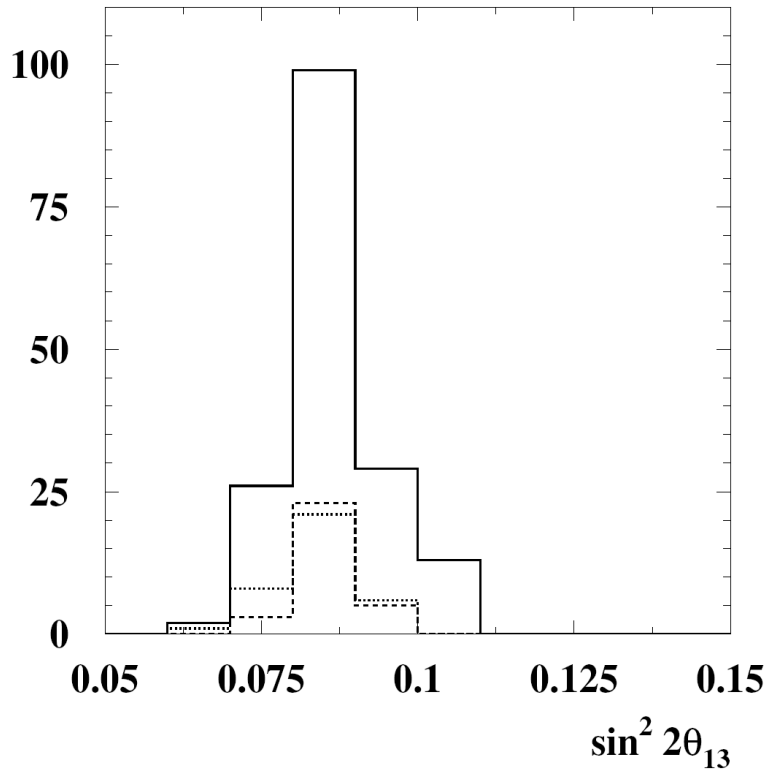
Output for neutrinos:

$$\sin^2 \theta_{12} \simeq 0.27, \quad \sin^2 2\theta_{23} \simeq 0.90, \quad \sin^2 2\theta_{13} \simeq 0.08$$

$$m_i = \{0.0021e^{0.11i}, 0.0098e^{-3.08i}, 0.048\} \text{ eV}$$

$$\Delta m_{23}^2 / \Delta m_{12}^2 \simeq 24$$

Theta(13) in Minimal SO(10)



$\sin^2 2\theta_{13}$ and CP violating phase δ_N

K.S. Babu and C. Macesanu (2005)

$$\sin^2 2\theta_{13} = 0.089 \pm 0.010 \pm 0.005 \quad \text{Daya Bay (2012)}$$

Large Neutrino Mixing From Lopsided Matrices

Quark and Lepton Mass hierarchy:

$$m_d : m_s : m_b \sim m_e : m_\mu : m_\tau \sim \epsilon_1 : \epsilon_2 : \epsilon_3$$

$$m_u : m_c : m_t \sim \epsilon_1^2 : \epsilon_2^2 : \epsilon_3^2$$

This motivates:

$$\begin{aligned} U &= H^T U_0 H \\ D &= D_0 H \\ L &= H^T L_0 \\ N &= N_0 \end{aligned}$$

$$H = \text{Diag}(\epsilon_1, \epsilon_2, \epsilon_3) \quad \epsilon_1 \ll \epsilon_2 \ll \epsilon_3$$

10_i of $SU(5)$ carry flavor charge, $\bar{5}_i$ do not.

Leads to large left-handed charged lepton mixing and large right-handed down quark mixing.

K.S. Babu, S. Barr, 1995

Albright, Babu and Barr, 1998

Sato and Yanagida, 1998

Irges, Lavignac, Ramond, 1998

Altarelli, Feruglio, 1998

Flavor $U(1)$ charges:

$$10_i : (3, 2, 0) \quad \bar{5}_i : (p, p, p) \quad 1_i : (q, q, q)$$

$$U_{ij} = \begin{pmatrix} \epsilon^6 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} H_u, \quad D_{ij} = \begin{pmatrix} \epsilon^3 & \epsilon^3 & \epsilon^3 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ 1 & 1 & 1 \end{pmatrix} \epsilon^p H_d,$$
$$L_{ij} = \begin{pmatrix} \epsilon^3 & \epsilon^2 & 1 \\ \epsilon^3 & \epsilon^2 & 1 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \epsilon^p H_d, \quad \nu_{ij}^D = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \epsilon^{p+q} H_u$$

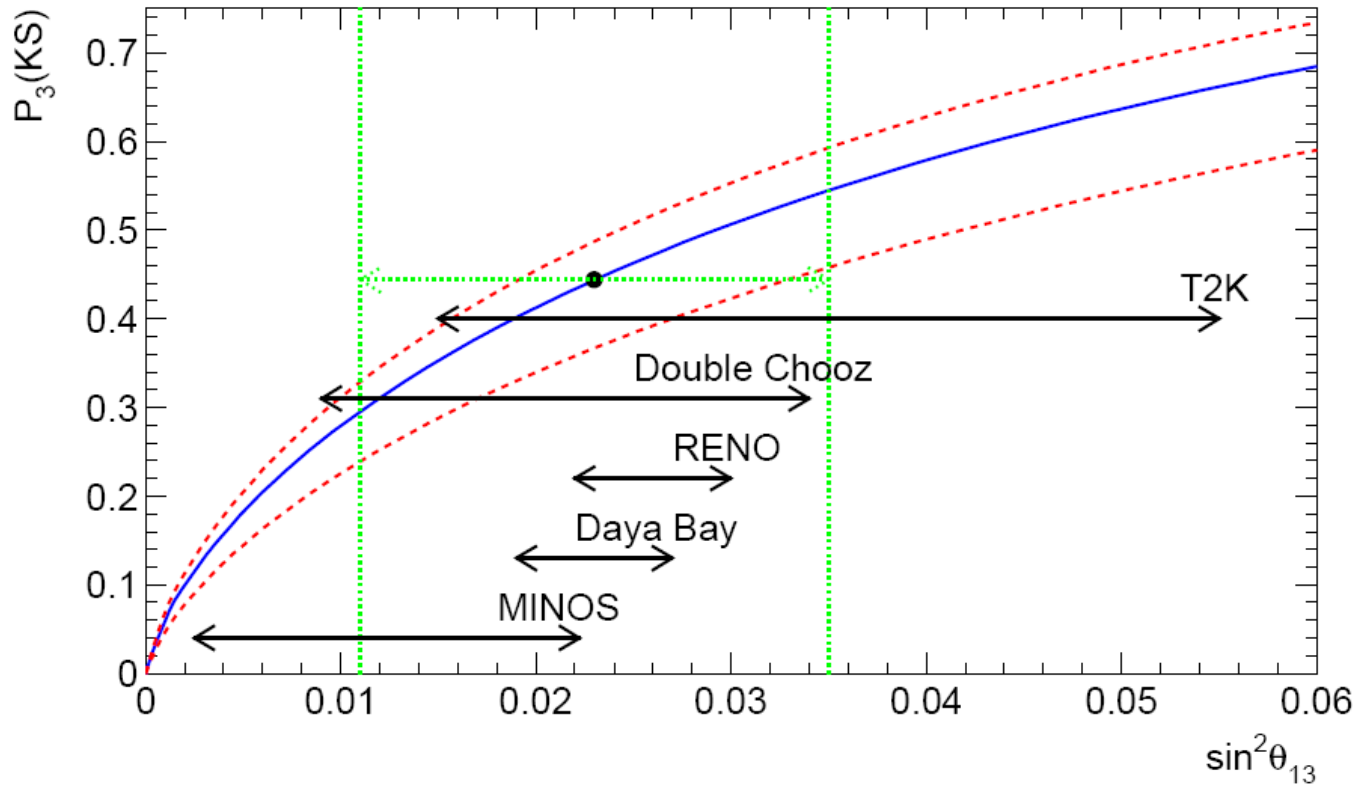
$$(M_\nu)_{ij} \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \epsilon \sim 0.2$$

All features of fermion masses and CKM mixing reproduced

No particular hierarchy in neutrino masses

θ_{13} predicted to be large

Neutrino Mass Anarchy



De Gouvea, Murayama (2012)

Lepton mixing matrix described by random draw of numbers in a unitary matrix

Radiative mass generation

If the mass of a fermion is zero due to a symmetry or the particle content in a renormalizable model, then either its mass remains zero, or it acquires a mass that is finite.

First example due to 't Hooft in 1971 in the classic paper showing renormalizability of spontaneously broken gauge theories.

A toy model where the electron mass was zero at tree-level due to a symmetry, but was induced and finite proportional to the muon mass as a one-loop radiative correction.

No counter-term for m_e is permitted in the model, so the induced m_e is finite and thus “calculable”.

Application of this idea to neutrino mass generation has many interesting and testable features.

Zee Model of Neutrino Mass

A. Zee (1980)

Simplest example of using $d = 7$ operator

Neutrino masses induced at one-loop

Loop and chiral suppression $\Rightarrow$ scale can be low

No right-handed neutrinos introduced, so the seesaw operator $\mathcal{O}_1 = L^i L^j H^k H^l \epsilon_{ik} \epsilon_{jl}$ absent at tree-level

Effective operator $\mathcal{O}_2 = L^i L^j L^k e^c H^l \epsilon_{ij} \epsilon_{kl}$ induces neutrino mass

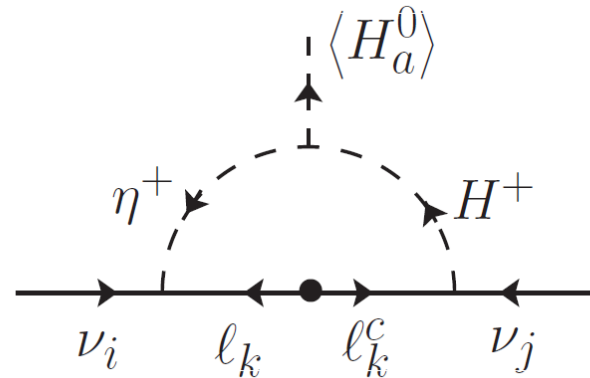
Introduces a second Higgs doublet and a charged singlet scalar η^+

Zee Model of Neutrino Mass (cont.)

$$H_a(1, 2, -\frac{1}{2}), \eta^+(1, 1, 1)$$

$$\mathcal{L}_{\text{Yuk}} = f_{ij} L_i L_j \eta^+ + Y'_{ij} L_i e_j^c H_2 + h.c.$$

$$V = \mu H_1 H_2 \eta^+ + h.c. + \dots$$



$$f_{ij} = -f_{ji}$$

$$M_\nu = \kappa \left(\hat{f} M_\ell^{\text{diag}} \hat{Y}^T + \hat{Y} M_\ell^{\text{diag}} \hat{f}^T \right)$$

$$\kappa = \frac{\sin 2\gamma}{16\pi^2} \log \left(\frac{M_1^2}{M_2^2} \right)$$

γ : $\eta^+ - H^+$ mixing angle, $M_{1,2}$: charged Higgs masses

In the Zee model, both Higgs doublets couple to leptons $\Rightarrow$ Flavor changing neutral currents at tree-level mediated by Higgs bosons

$$M_\nu = \kappa \left(\hat{f} M_\ell^{\text{diag}} \hat{Y}^T + \hat{Y} M_\ell^{\text{diag}} \hat{f}^T \right)$$

$\hat{Y}$ is arbitrary, so quantitative predictions difficult

Wolfenstein suggested a discrete Z_2 symmetry that allows only one Higgs doublet to couple to leptons

FCNC avoided, $\hat{Y} = \frac{M_\ell^{\text{diag}}}{v}$ **L. Wolfenstein (1980)**

Zee-Wolfenstein model very predictive for neutrinos

Smirnov, Tanimoto (1997)

Jarlskog, Matsuda, Skaldhauge, Tanimoto (1999)

Frampton, Glashow (1999)

Zee-Wolfenstein model:

$$M_\nu = \frac{\kappa}{v} \left[\hat{f} (M_\ell^{\text{diag}})^2 + (M_\ell^{\text{diag}})^2 \hat{f}^T \right]$$

$$M_\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}$$

3 real parameters explain all neutrino oscillation data

Compatible with **bimaximal** neutrino mixing

KamLand and solar neutrino data excluded this possibility

Koide (2001)
X.G. He (2004)

New Predictive Realization

- Complete absence of FCNC too restrictive –
- General Zee model too arbitrary –
- Choose an intermediate scenario – **Babu, Julio (2013)**

Impose a Z_4 symmetry that is family-dependent

$$L_i : (-i, i, i); \quad e_i^c : (-i, -i, -i);$$

$$H_1 : +1; \quad H_2 : -1; \quad \eta^+ : -1$$

$$Q_i : (-i, -i, -i), \quad u_i^c : (i, i, i), \quad \text{and} \quad d_i^c : (i, i, i)$$

- Neutrino mass hierarchy is inverted

- $\delta_{CP} = 0$ or π

Predictions:

- Majorana phases 0 or π

- $|U_{\tau 1}| = |U_{\tau 2}|$

- $m_3 = \frac{1}{2} \frac{\Delta m_{\text{solar}}^2}{|\Delta m_{\text{atm}}^2|^{1/2}} \simeq 7.5 \times 10^{-4} \text{ eV}$

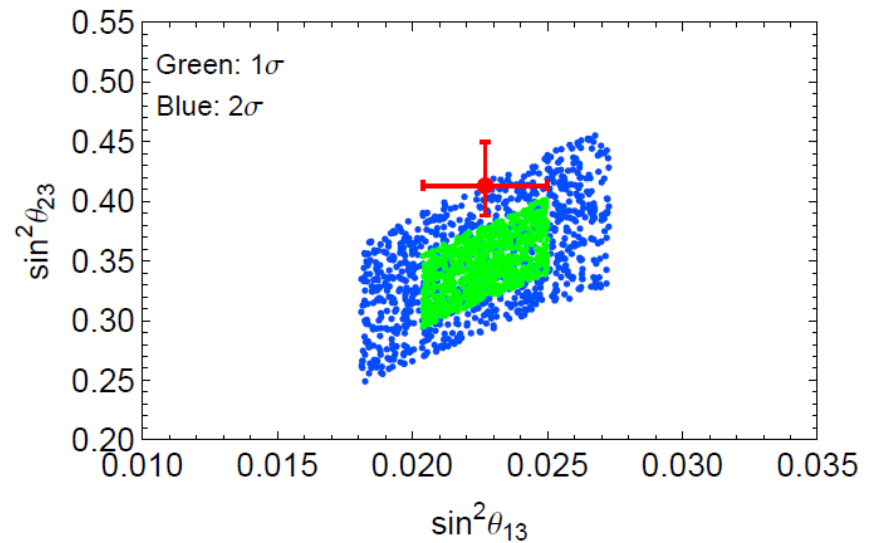
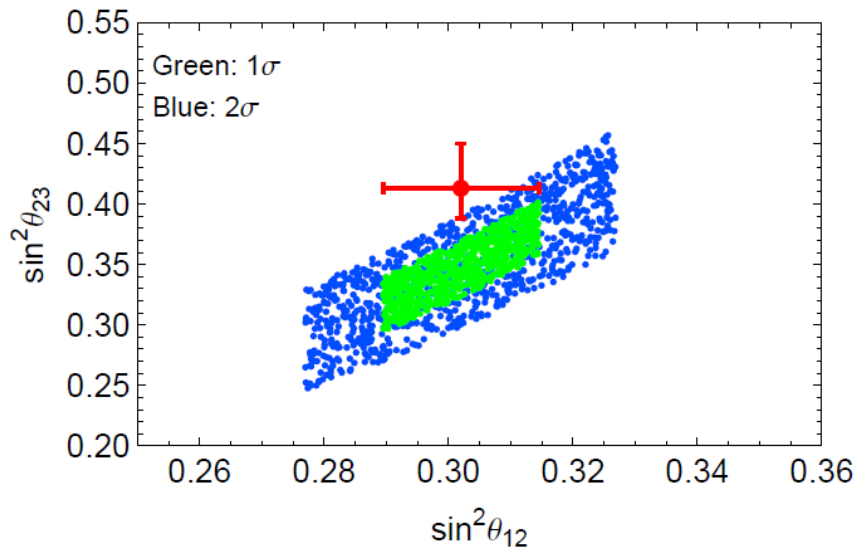
$$U_{\text{PMNS}} = \begin{pmatrix} \frac{1}{\sqrt{2}}(C_\chi C_\psi + S_\psi) & \frac{1}{\sqrt{2}}(C_\chi C_\psi - S_\psi) & -S_\chi C_\psi \\ \frac{1}{\sqrt{2}}(C_\chi S_\psi - C_\psi) & \frac{1}{\sqrt{2}}(C_\chi S_\psi + C_\psi) & -S_\chi S_\psi \\ \frac{S_\chi}{\sqrt{2}} & \frac{S_\chi}{\sqrt{2}} & C_\chi \end{pmatrix}$$

$|U_{\tau 1}| = |U_{\tau 2}|$ relation:

$$s_{13} = t_{23} \frac{1 - t_{12}}{1 + t_{12}}, \quad \text{or} \quad s_{13} = -t_{23} \frac{1 + t_{12}}{1 - t_{12}} \quad (\delta_{CP} = \pi) ;$$

$$s_{13} = t_{23} \frac{1 + t_{12}}{1 - t_{12}}, \quad \text{or} \quad s_{13} = -t_{23} \frac{1 - t_{12}}{1 + t_{12}} \quad (\delta_{CP} = 0)$$

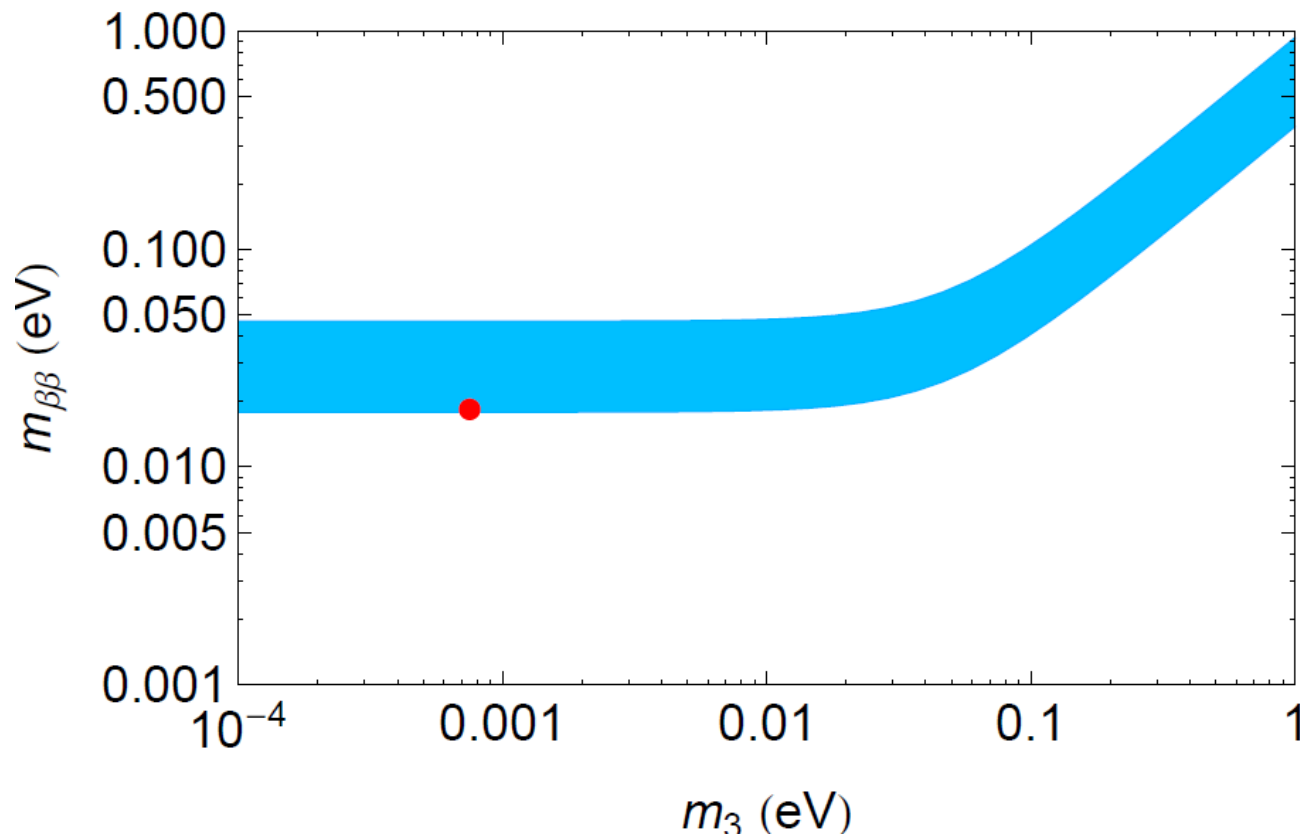
$\delta_{CP} = \pi$ required



Gonzalez-Garcia et al (2012)
Fogli et al (2012)

Effective mass in neutrinoless double beta decay:

$$m_{\beta\beta} \equiv \left| \sum_{i=1-3} U_{ei}^2 m_i \right| = (17.6 - 18.5) \text{ meV}$$

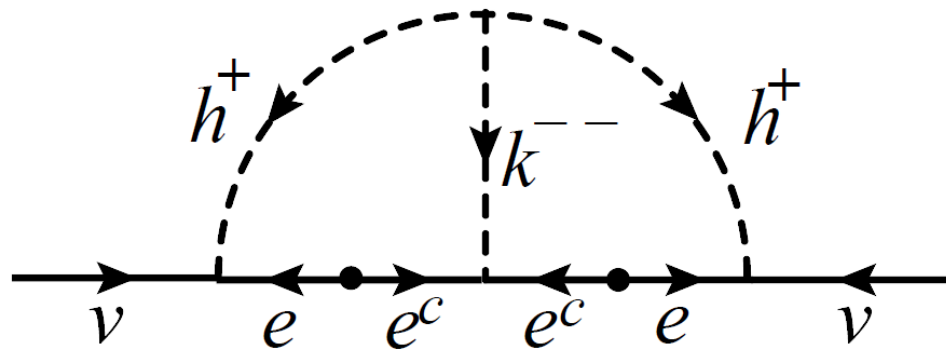


Two-loop neutrino mass generation

$$\mathcal{L} = f_{ij} L_i^a L_j^b h^+ \epsilon_{ab} + g_{ij} e_i^c e_j^c k^{--} + \mu h^+ h^+ k^{--} + \text{h.c.}$$



$$\mathcal{O}_9 = L^i L^j L^k e^c L^l e^c \epsilon_{ij} \epsilon_{kl}$$



A. Zee, (1985)
Babu (1988)

Consistent with all neutrino oscillation data

Predicts doubly charged Higgs boson with TeV mass

One neutrino is nearly massless

CP violation in neutrino oscillation is expected

Two-loop neutrino mass model (cont.)

Fits to neutrino masses and mixing angles, consistent with perturbativity and boundedness of potential as well as FCNC limits sets constraints on h^+ and k^{++} masses:

$$\text{NH :} \quad 306 \text{ GeV} < m_{k^{++}} < 177 \text{ TeV}; \quad 779 \text{ GeV} < m_{h^+} < 63 \text{ TeV}$$

$$\text{IH :} \quad 997 \text{ GeV} < m_{k^{++}} < 25 \text{ TeV}; \quad 2.3 \text{ TeV} < m_{h^+} < 7.9 \text{ TeV}$$

Since couplings are essentially fixed from neutrino masses, charged lepton flavor violation can be predicted

$\mu \rightarrow e\gamma$ and $\tau \rightarrow 3\mu$ limits used as input

Lower limits on branching ratios for $\mu \rightarrow 3e$, $\mu - e$ conversion in nuclei, as well as $\mu \rightarrow e\gamma$ and $\tau \rightarrow 3\mu$ follow

K.S. Babu, J. Julio (2013)

D. Schmitz, T.Schwetz, H. Zhang (2014)

J. Herrero-Garcia, M. Nebot, N. Rius, A.

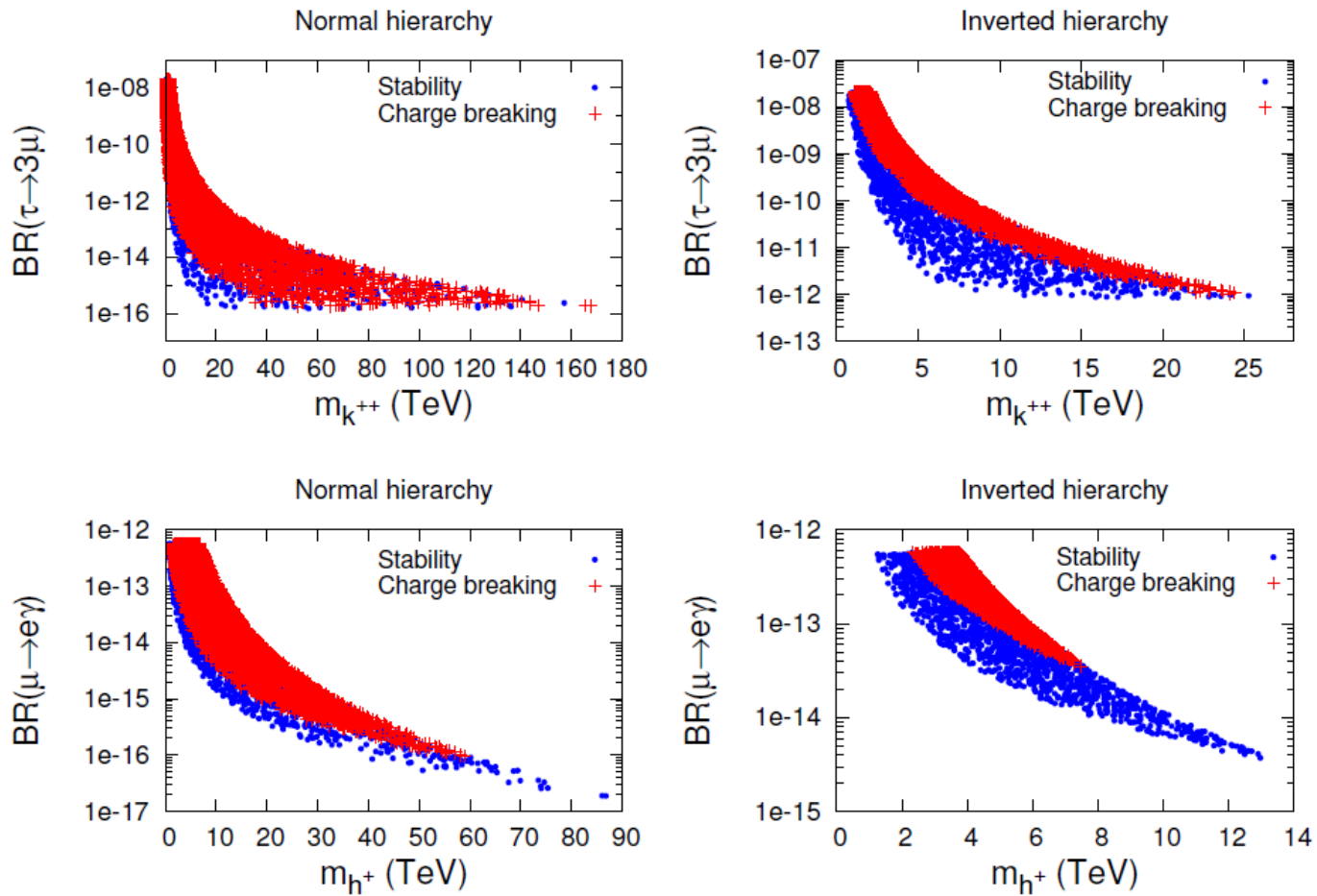
Santamaria (2014)

K.S. Babu, C. Macesanu (2005)

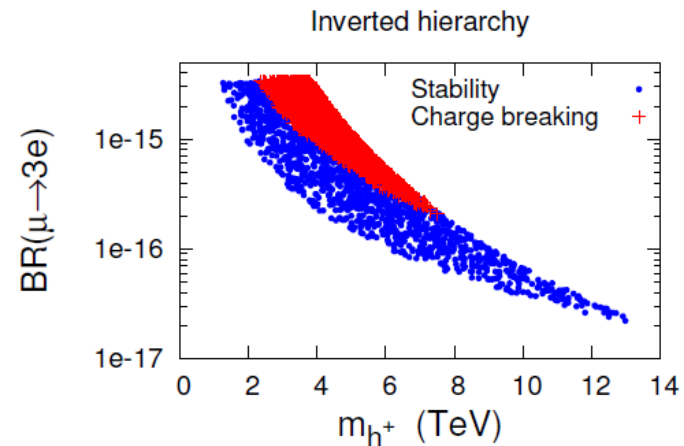
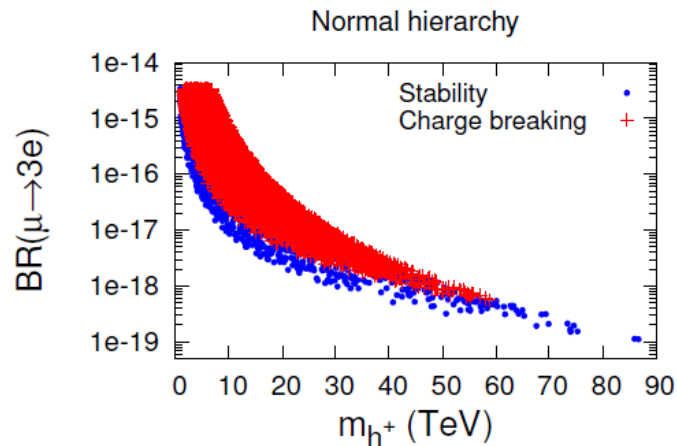
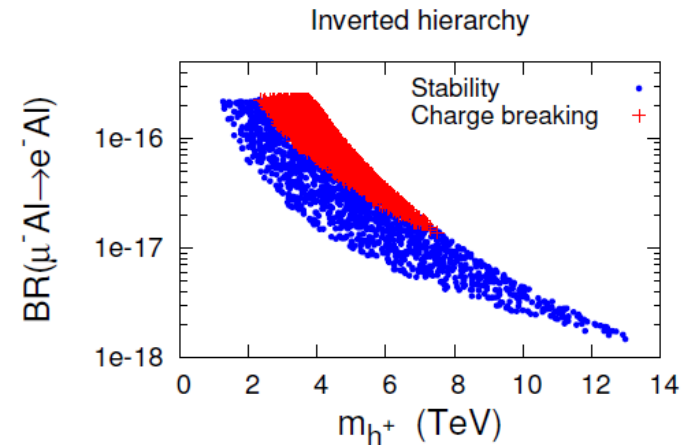
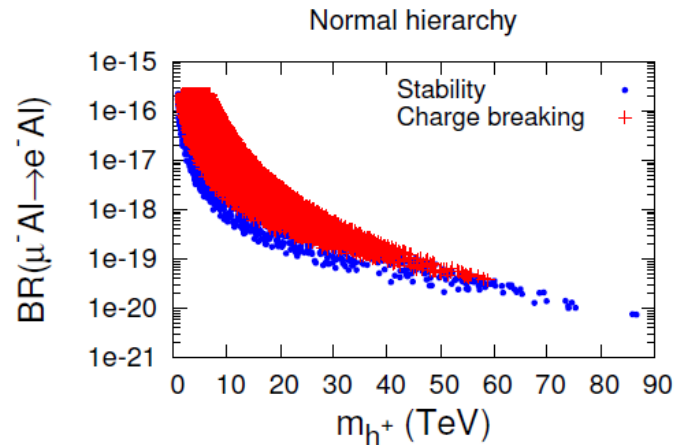
D. Sierra, M. Hirsch (2006)

M. Nebot, J. Oliver, D. Paolo, A. Santamaria (2008)

LFV in Radiative Neutrino Mass Model



LFV in Radiative Neutrino Mass Model (cont.)



Zeros in fermion mass matrices

CKM mixing angles may be related to quark mass ratios

A two family example:

$$M_u = \begin{pmatrix} 0 & A_u \\ A_u^* & B_u \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & A_d \\ A_d^* & B_d \end{pmatrix}.$$

$$\Rightarrow |\sin \theta_C| \simeq \left| \sqrt{\frac{m_d}{m_s}} - e^{i\psi} \sqrt{\frac{m_u}{m_c}} \right|$$

Weinberg (1977)
Wilczek, Zee (1977)
Fritzsch (1977)

Here ψ is a free phase, but still θ_C is constrained

Two symmetries needed to enforce this structure:

- (i) Parity symmetry for hermiticity
- (ii) Family $U(1)$ symmetry for the zero in (1,1) entries

Texture zeros for neutrinos

$$A_1 : \begin{pmatrix} 0 & 0 & X \\ 0 & X & X \\ X & X & X \end{pmatrix}$$

$$A_2 : \begin{pmatrix} 0 & X & 0 \\ X & X & X \\ 0 & X & X \end{pmatrix}$$

$$B_1 : \begin{pmatrix} X & X & 0 \\ X & 0 & X \\ 0 & X & X \end{pmatrix}$$

$$B_2 : \begin{pmatrix} X & 0 & X \\ 0 & X & X \\ X & X & 0 \end{pmatrix}$$

$$B_3 : \begin{pmatrix} X & 0 & X \\ 0 & 0 & X \\ X & X & X \end{pmatrix}$$

$$B_4 : \begin{pmatrix} X & X & 0 \\ X & X & X \\ 0 & X & 0 \end{pmatrix}$$

$$C : \begin{pmatrix} X & X & X \\ X & 0 & X \\ X & X & 0 \end{pmatrix}$$

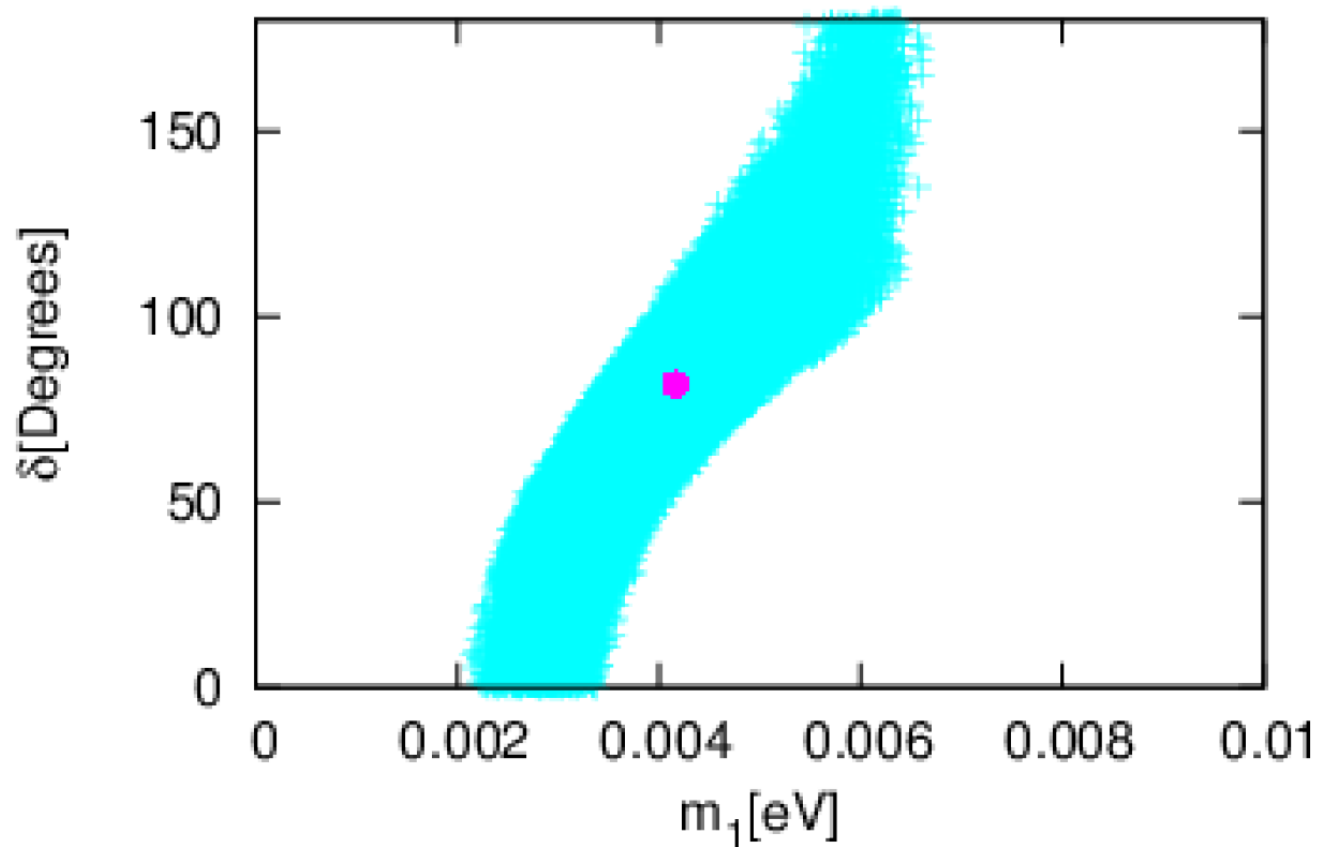
Frampton, Glashow, Marfatia (2002)

Xing (2002)

Merle, Rodejohann (2006)

Goswami et. al (2006)

Predictions for Model A1



K. Babu, Z. Devi, S. Goswami (2014)
J. Liao, D. Marfatia, K. Whisnant (2014)

Summary and Conclusions

- Neutrino experiments are probes of very high scale physics
- A class of GUTs naturally predict large neutrino mixings, including large θ_{13}
- New particles at TeV expected in many scenarios of radiative neutrino mass generation
- Lepton flavor violation provides complementary information on neutrino mass generation
- Zeros in neutrino mass matrix quite consistent and leads typically to large θ_{13}
- Neutrino CP violation, $m_{\beta\beta}^{0\nu}$, and mass hierarchy measurements will greatly enhance our fundamental understanding of Nature

Backup Slides

Specific Example for Quark & Lepton masses

Fit

Input at GUT scale

$$\tan \beta = 55$$

$$m_u = 0.85 \text{ MeV}$$

$$m_d = 1.08 \text{ MeV}$$

$$m_c = 222.3 \text{ MeV}$$

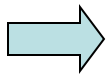
$$m_s = 34.3 \text{ MeV}$$

$$m_t = 85.5 \text{ GeV}$$

$$m_b = 1.549 \text{ GeV}$$

$$\delta_{CKM} = 1.508$$

$$V_{us} = 0.22 \quad V_{ub} = 0.0027 \quad V_{cb} = 0.036$$



Output: Type II Seesaw

$$\sin^2 2\theta_{\odot} = 0.635$$

$$\sin^2 2\theta_{e3} = 0.08$$

$$\sin^2 2\theta_{atm} = 0.892$$

$$\frac{\Delta m_{atm}^2}{\Delta m_{\odot}^2} = 15.2$$

Tri-bimaximal Neutrino Mixing

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot P$$

Harrison, Perkins, Scott (2002)

Neutrino mixing angles are geometrical!

Exact symmetry excluded by Daya Bay and RENO data at 5σ

A variety of models based on A_4 and other symmetries

Ma, Rajasekaran (2001)

Xing (2002)

Babu, Ma, Valle (2003)

Altarelli, Feruglio (2005)

He, Keum, Volkas (2006)

Mohapatra, Nasri, Yu (2006)

King, Malinsky (2007)

Verzielas, King, Ross (2007)

Chen, Mahanthappa (2007)

Honda, Tanimoto (2008)

Everett, Stuart (2009)

Grimus, Lavoura, Ludl (2009)

There is generically a vacuum alignment problem:

A_4 needs two triplet Higgs: $\langle \chi \rangle = (1, 1, 1)$ $\langle \phi \rangle = (0, 1, 0)$