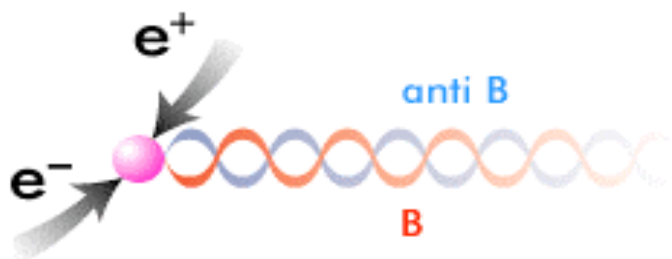
Lecture 2

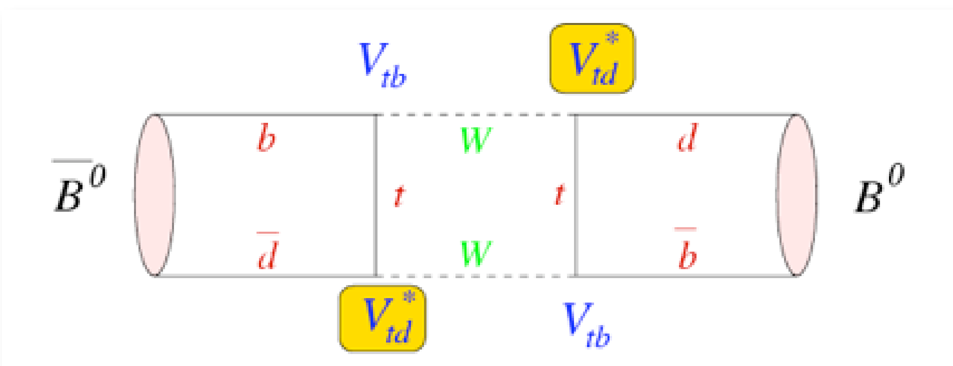
- $B\text{-}\bar{B}$ mixing amplitude
- Inclusive processes: OPE and applications ($B \to X_{c,u} l\nu$, $B \to X_s \gamma$)
- Exclusive processes: trees and penguins, CP violation, searches for New Physics

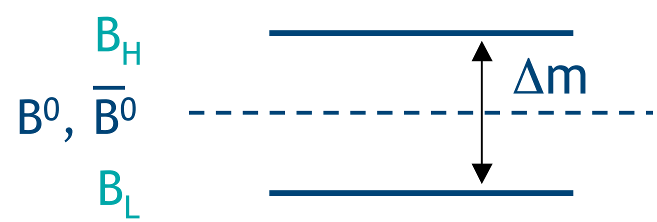
B-$\overline{\text{B}}$ Mixing



Oscillations of neutral mesons

- Neutral mesons can be transformed into their antiparticles by second-order weak processes
- Analogy with quantum-mechanical system of coupled pendulums: state B^0 at t=0 develops into a superposition of states B^0 and $\bar{B}^0$ with time-oscillating amplitudes





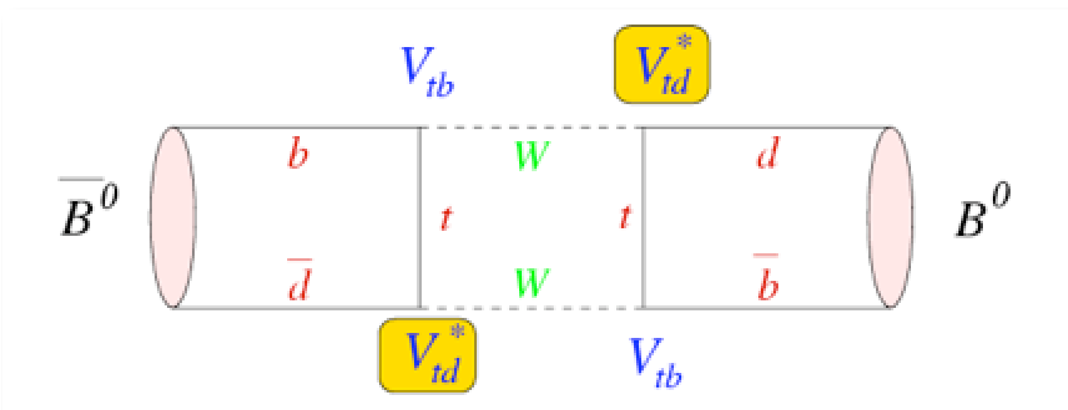
Quantum-mechanical treatment

- Time evolution of an initial (at t=0) $\bar{B}^0$ state:

$$\boxed{|\psi(t)\rangle \propto \cos\left(\frac{\Delta m}{2}\,t\right)|\bar{B}^0\rangle + i e^{2i\beta}\,\sin\left(\frac{\Delta m}{2}\,t\right)|B^0\rangle}$$

where:

$$e^{2i\beta}\,\frac{\Delta m}{2} = \frac{1}{2m_B}\,\langle B^0|\,\mathcal{H}_{\text{eff}}^{\Delta B=2}\,|\bar{B}^0\rangle$$



$\propto (V_{tb}V_{td}^*)^2 \propto e^{2i\beta}$

Calculation of the mass difference

- Master formula:

$$\boxed{\Delta m = \frac{G_F^2 M_W^2}{16\pi^2}\,|V_{tb}V_{td}^*|^2\, S_0\!\left(\frac{m_t^2}{M_W^2}\right)\eta_{\rm QCD}\,\frac{1}{m_B}\,\langle B^0|\,(\bar d b)_{V-A}(\bar d b)_{V-A}\,|\bar B^0\rangle}$$

$S_0(x_t) \approx 0.784\, x_t^{0.76}$

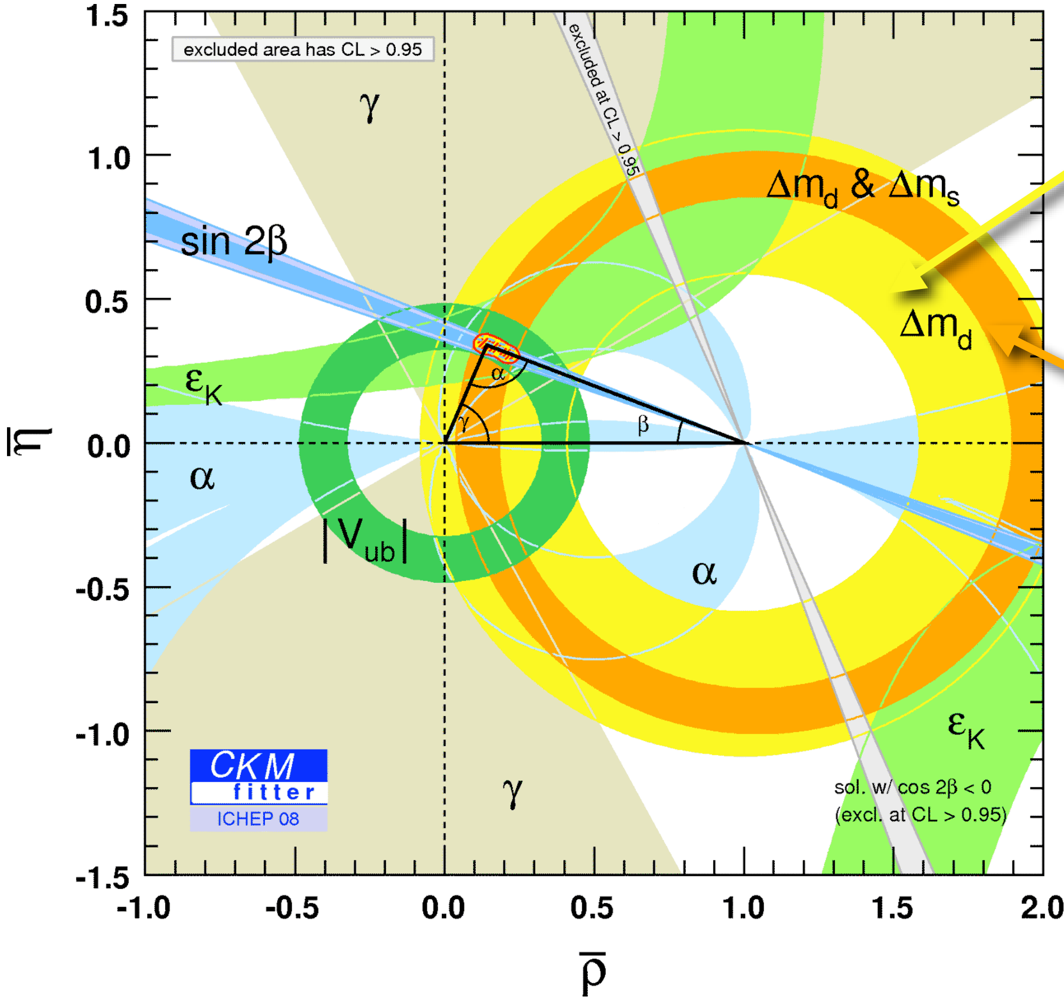
perturbative QCD correction

$\equiv \frac{8}{3} B_B f_B^2 m_B^2$

(from lattice QCD)

- Discovery of B-$\bar{\text{B}}$ mixing (ARGUS experiment, 1987) pointed to a very heavy top quark!

Determination of $|V_{td}|$

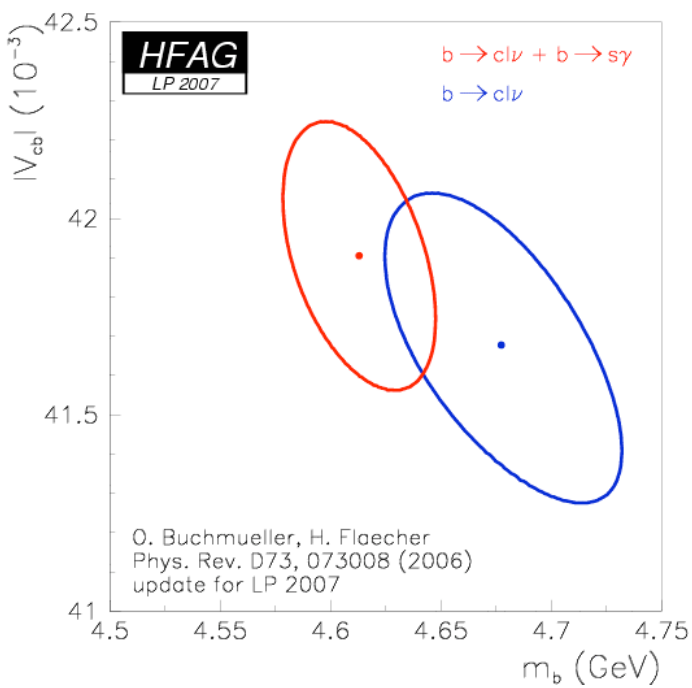


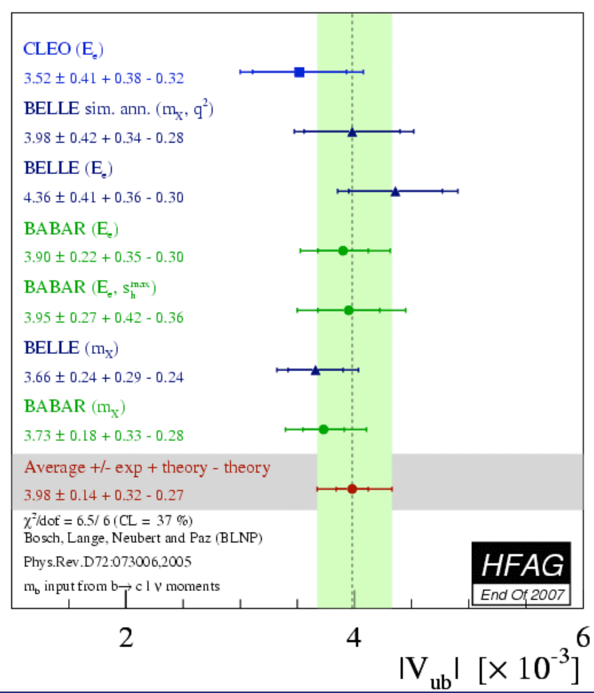
result derived from B_d mixing alone (large theoretical uncertainties)

result derived from ratio of B_d and B_s mixing frequencies (reduced theoretical uncertainties)

$$\frac{\Delta m(B_d)}{\Delta m(B_s)} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{B_{B_d} f_{B_d}^2 m_{B_d}}{B_{B_s} f_{B_s}^2 m_{B_s}}$$

Inclusive Decays of B Mesons





OPE for inclusive rates

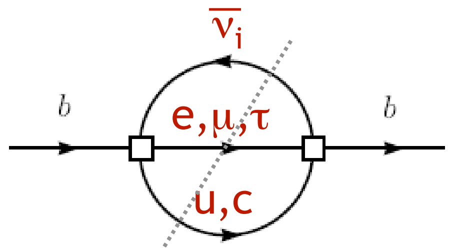
- Optical theorem: relation between total rate and discontinuity of forward matrix element

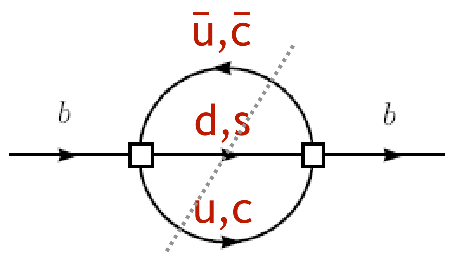
$$\Gamma(\bar{B} \to X) = \frac{1}{2m_B} \sum_X (2\pi)^4 \, \delta^4(p_B - p_X) \, |\langle X | H_{\rm eff} | \bar{B} \rangle|^2$$
$$= \frac{1}{2m_B} \, \text{Disc} \, \langle \bar{B} | \, i \int d^4x \, T\{ H_{\rm eff}(x) \, H_{\rm eff}(0) \} | \bar{B} \rangle$$

- Quark-hadron duality:
  - many accessible final states X
  - their sum can be approximated by a partonic analysis, c.f. calculation of $\sigma(e^+e^- \to \text{hadrons})$

OPE for inclusive rates

- Leading diagrams for some examples:

$\Gamma_{SL}(B) = \langle B|$  $|B\rangle$

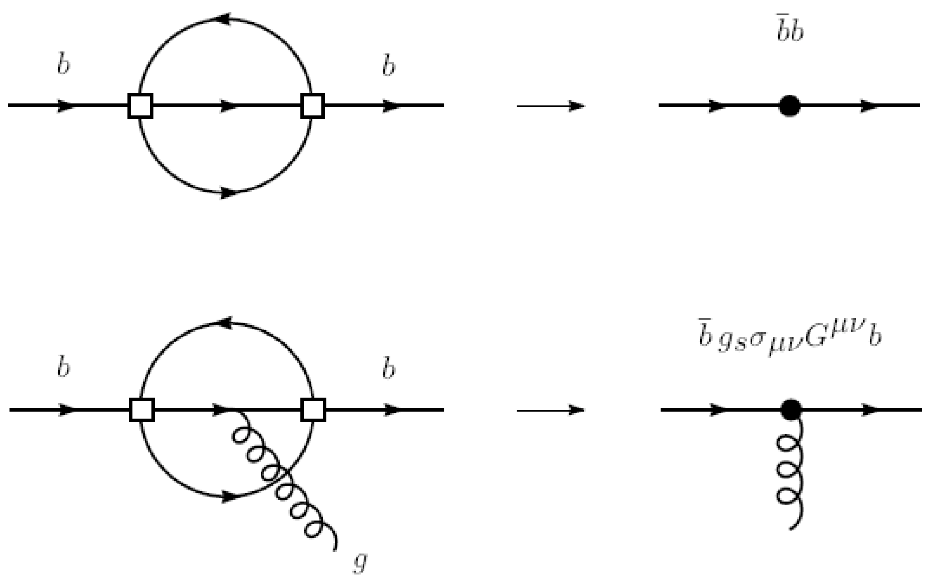
$\Gamma_{had}(B) = \langle B|$  $|B\rangle$

By selecting a subset of operators in the effective Hamiltonian one can study partially inclusive rates, not just the total rate!

- Phase-space integrations provide "smearing" needed for applications of quark-hadron duality

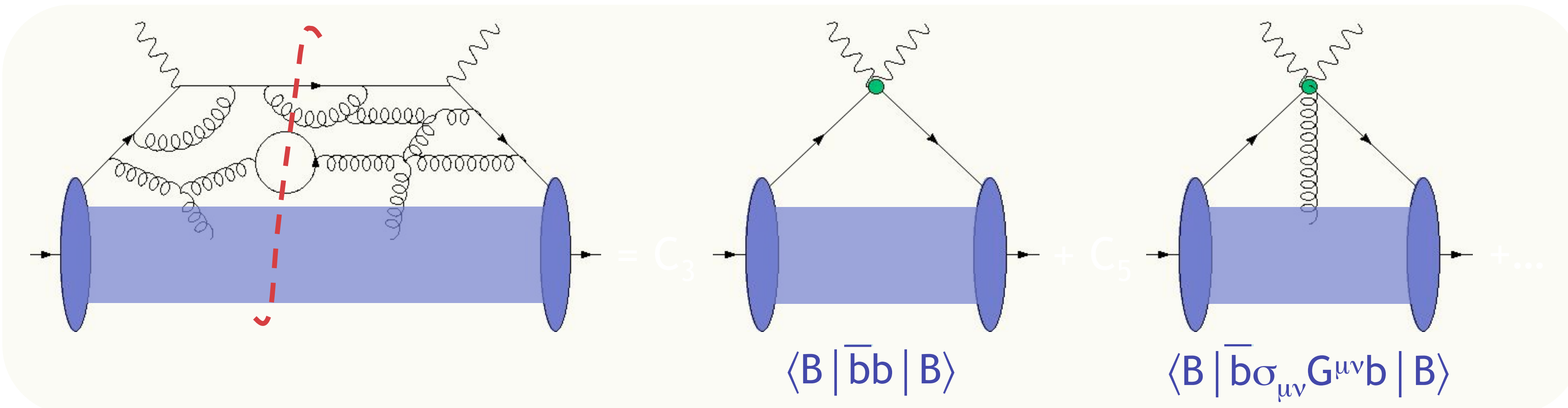
OPE for inclusive rates

- Integrals over the discontinuity can be deformed into the complex momentum plane, where internal lines are far off-shell, and the matrix elements can be expanded in local operators:



OPE for inclusive rates

- More realistic picture:



OPE for inclusive rates

- Heavy-quark expansion of these matrix elements introduces few hadronic parameters:

$$\frac{\langle \bar{B}|\,\bar{b}b\,|\bar{B}\rangle}{2m_B} = 1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^3}{m_b^3}\right)$$

$$\frac{\langle \bar{B}|\,\bar{b}g_s\sigma_{\mu\nu}G^{\mu\nu}b\,|\bar{B}\rangle}{2m_B} = 2\mu_G^2 + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^3}{m_b}\right)$$

from fits to data: $\mu_\pi^2 \approx 0.40\ \text{GeV}^2$, $\mu_G^2 \approx 0.35\ \text{GeV}^2$

- Radiative corrections from hard gluons ($\mu \sim m_b$) can be calculated and included in Wilson coefficients

Semileptonic decays: $|V_{cb}|$, m_b etc.

- Master formula:

$$\Gamma(\bar{B} \to X_c\, l\, \bar{\nu}_l) = \frac{G_F^2 m_b^5}{192\pi^3} \boxed{|V_{cb}|^2} \left\{ \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2}\right) \left[f\left(\frac{m_c}{m_b}\right) + \frac{\alpha_s(m_b)}{\pi}\, g\left(\frac{m_c}{m_b}\right) + \dots \right] - \frac{2\mu_G^2}{m_b^2} \left(1 - \frac{m_c^2}{m_b^2}\right)^4 + \dots \right\}$$

- Similar formulae exist for moments of the lepton energy and hadronic invariant mass spectra

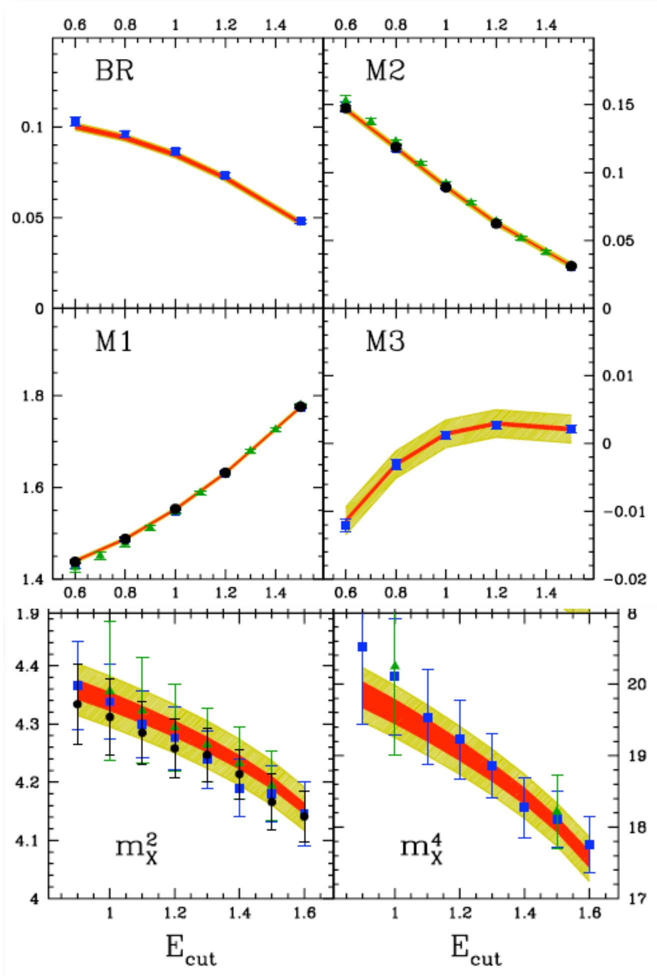
$\Rightarrow$ extract $|V_{cb}|$, m_b, m_c, μ_π^2, μ_G^2 from a global fit

Semileptonic decays: $|V_{cb}|$, m_b etc.

- State of the art:
  - leading term at $O(\alpha_s^2)$
  - $1/m_b^2$ terms (almost) at $O(\alpha_s)$
  - $1/m_b^3$ terms at tree level

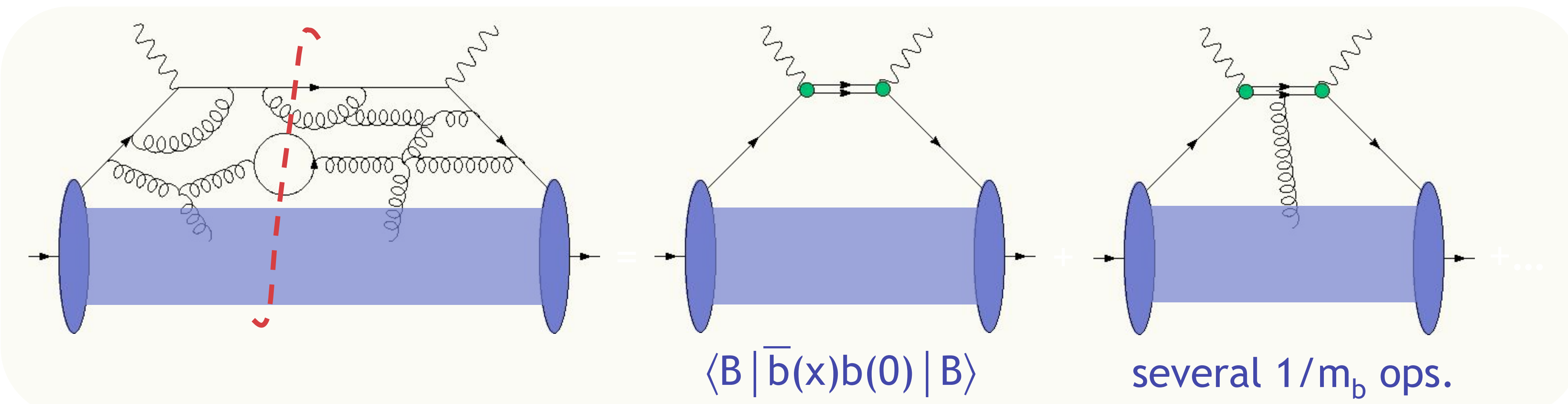
- Results:
  - $|V_{cb}|=(41.7\pm0.4\pm0.6)\cdot10^{-3}$ (HFAG '07)
  - $m_b^{kin}=(4.677\pm0.053)$ GeV

known to better than 1.5%



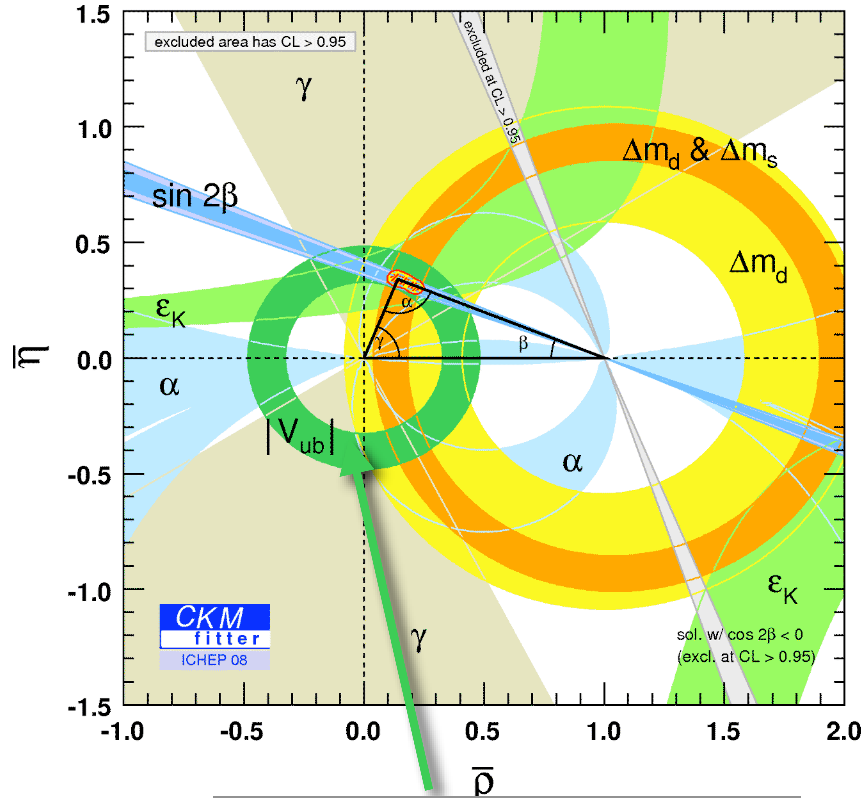
Semileptonic decays: $|V_{ub}|$

- In principle similar analysis (replace $m_c \to 0$), but theoretical complications due to phase-space restrictions from experimental cuts (suppression of charm background)
- Light-cone expansion:



Semileptonic decays: $|V_{ub}|$

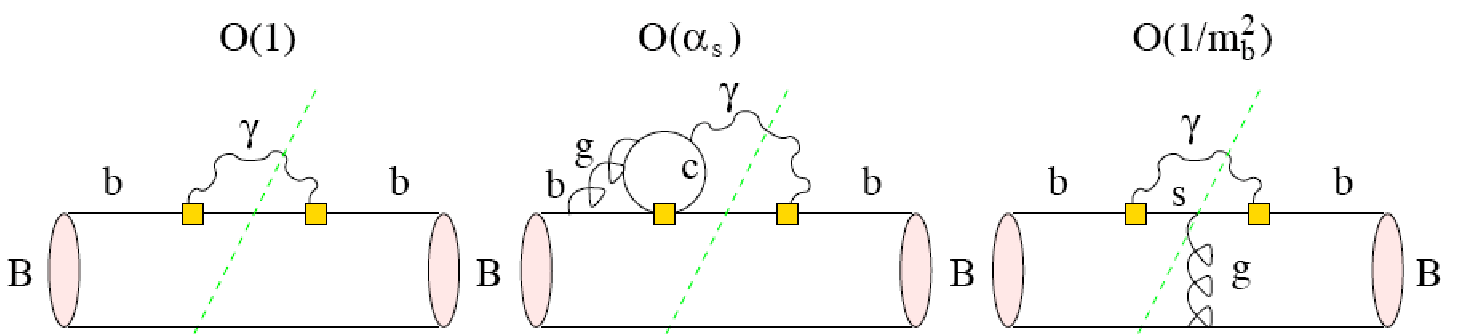
BLNP theory	$\lvert V_{ub}\rvert$ $[10^{-3}]$ (m_b=4.71±0.06 GeV)
CLEO, E_e>2.1 GeV	3.52±0.41±0.35
Belle, E_e>1.9 GeV	4.36±0.41±0.33
BaBar, E_e>2.0 GeV	3.90±0.22±0.33
BaBar, E_e>2.0 GeV, s_h^{max}<3.5 GeV^2	3.95±0.27±0.39
Belle, M_X<1.7 GeV	3.66±0.24±0.27
Belle, M_X>1.7 GeV, q^2>8 GeV^2	3.98±0.42±0.31
BaBar, M_X>1.55 GeV	3.73±0.18±0.31
Average (HFAG '07)	3.98±0.14±0.30



result derived from inclusive and exclusive semileptonic B decays

Inclusive radiative decay $B \rightarrow X_s \gamma$

- Prototype FCNC process, very sensitive to physics beyond SM
- Important constraint on model building
- Leading graphs in the OPE:



  - dominated by electromagnetic dipole operator $Q_{7\gamma}$

Inclusive radiative decay $B \to X_s \gamma$

- Decay rate:

$$\Gamma(\bar{B} \to X_s \gamma) = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{tb} V_{ts}^*|^2 \left\{ |C_{7\gamma}(m_b)|^2 + \mathcal{O}\left(\alpha_s, \frac{\Lambda_{\rm QCD}^2}{m_b^2} \right) \right\}$$

$$= (3.15 \pm 0.23) \cdot 10^{-4}$$

  - good agreement with experimental results $(3.55 \pm 0.25) \cdot 10^{-4}$ (HFAG '07) and $(3.31 \pm 0.19 \pm 0.37) \cdot 10^{-4}$ (Belle '08)

- Good agreement between theory and data implies strict bounds on parameters of New Physics models!

Inclusive radiative decay B→X$_s$γ

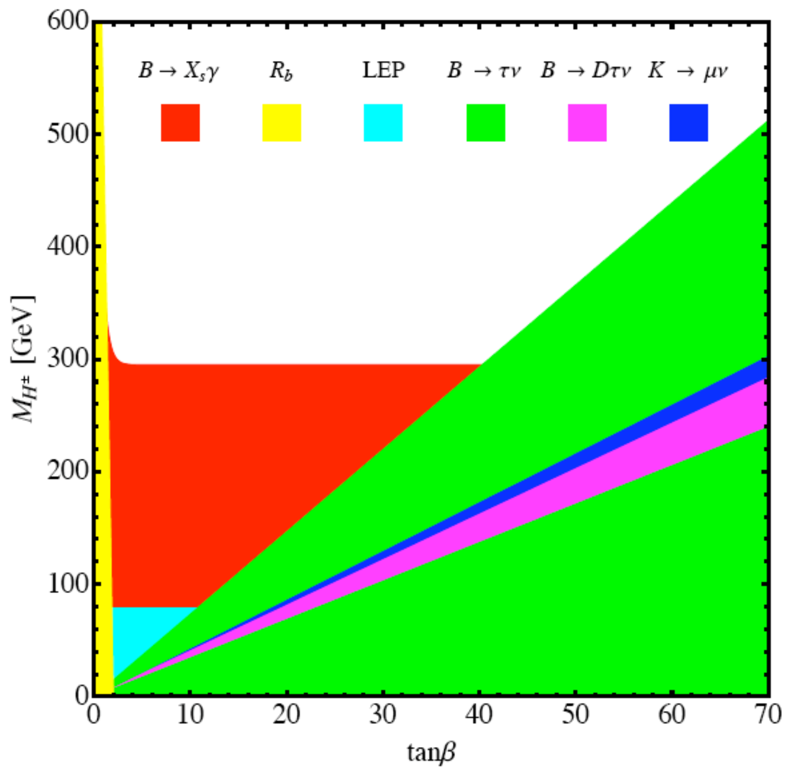
- Constraints on BSM models: (U. Haisch, May 2008)

Model	Accuracy	Effect	Bound
THDM type II	NLO [39, 40]	$\Uparrow$	$M_H^\pm > 295\,\text{GeV}$ (95% CL) [16]
MFV MSSM	NLO [40–45]	$\Updownarrow$	—
MFV SUSY GUTs	NLO [46]	$\Downarrow$	—
LR	NLO [40]	$\Updownarrow$	—
general MSSM	LO [47]	$\Updownarrow$	$\lvert(\delta_{23}^d)_{LL}\rvert \lesssim 4\times10^{-1}$, $\lvert(\delta_{23}^d)_{RR}\rvert \lesssim 8\times10^{-1}$, $\lvert(\delta_{23}^d)_{LR}\rvert \lesssim 6\times10^{-2}$, $\lvert(\delta_{23}^d)_{RL}\rvert \lesssim 2\times10^{-2}$ [47]
UED5	LO [49]	$\Downarrow$	$1/R > 600\,\text{GeV}$ (95% CL) [50]
UED6	LO [51]	$\Downarrow$	$1/R > 650\,\text{GeV}$ (95% CL) [51]
RS	LO [52]	$\Uparrow$	$M_{\text{KK}} \gtrsim 2.4\,\text{TeV}$
LH	LO [53]	$\uparrow$	—
LHT	LO [54]	$\updownarrow$	—

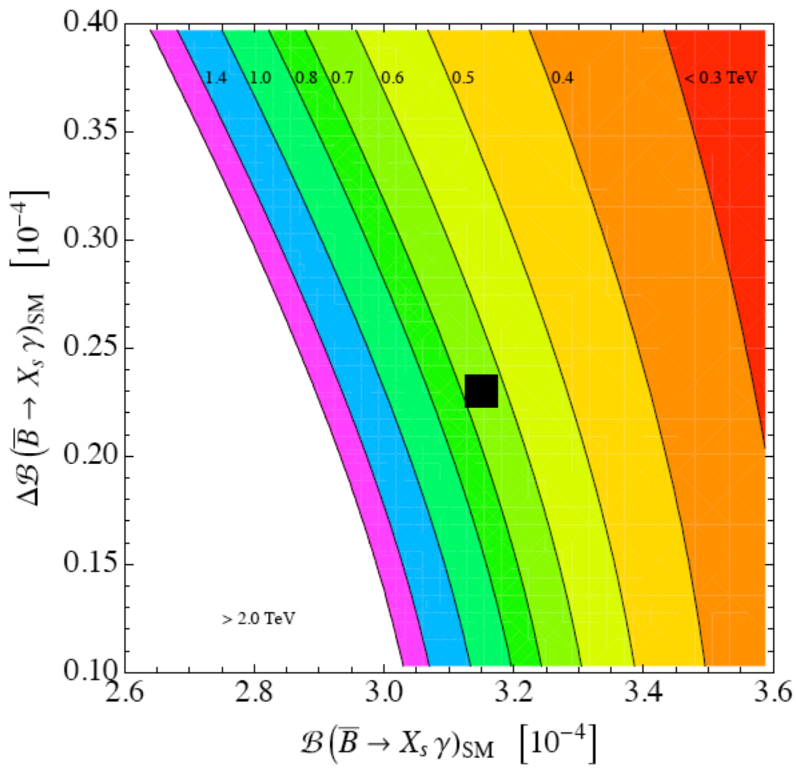
Table I Theoretical accuracy, effect on $\mathcal{B}(\bar{B}\to X_s\gamma)$ relative to the SM prediction, and if applicable, constraint on the parameter space following from $\bar{B}\to X_s\gamma$ in popular BSM scenarios. Arrows pointing upward (downward) indicate that the BSM effects interfere constructively (destructively) with the SM $b\to s\gamma$ amplitude. Single (double) arrows specify whether the maximal possible shift is smaller (larger) than the theoretical uncertainty of the SM expectation. See text for details.

Inclusive radiative decay $B \to X_s\gamma$

- Constraints on BSM models: (U. Haisch, May 2008)



type-II two-Higgs-doublet model



universal extra dimension model

Exclusive Decays and New Physics

Rare two-body decays

- Weak effective Hamiltonian also describes exclusive processes
- Many of them are rare: either proceed via $b \to u$ transition or via penguin loops
- Typical branching ratios ~ few 10^{-6}, sensitive probes of physics beyond SM
- Examples (~100 modes explored):
  - $B \to \pi\pi$, $B \to \pi\rho$, $B \to \pi K$, $B \to \phi K$, $B \to \eta' K$, etc.

Rare two-body decays

- Structure of H_{eff} (many operators with different CKM factors) implies that decay amplitudes are, in general, superpositions of several partial amplitudes A_i carrying different phases
  $\rightarrow$ possibility to observe CP violation!
- Weak (CP-odd) phases:

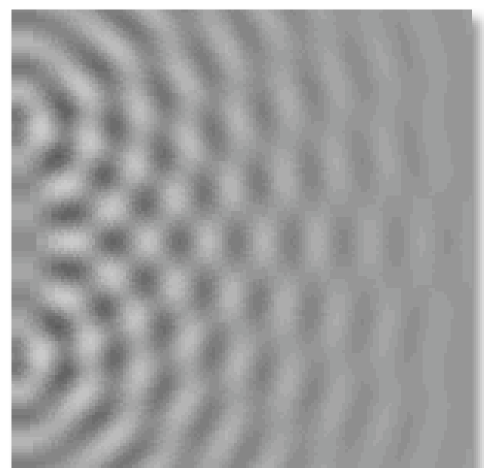
$$\varphi_i \xrightarrow{\text{CP}} -\varphi_i$$

complex parameters in Lagrangian (δ_{CKM}, θ_{QCD} ?)

- Strong (CP-even) phases:

$$\delta_i \xrightarrow{\text{CP}} \delta_i$$

due to final-state rescattering (poles in propagators)



Rare two-body decays

- Decay amplitudes:

$$A(\bar{B} \to \bar{f}) = \sum_i A_i \, e^{i\theta_i} \, e^{i\delta_i}$$

$$\bar{A}(B \to f) = \sum_i A_i \, e^{-i\theta_i} \, e^{i\delta_i}$$

- Squared amplitudes:

$$|A(\bar{B} \to \bar{f})|^2 = \sum_i A_i^2 + \sum_{i \neq j} 2 A_i A_j \cos[(\theta_i - \theta_j) + (\delta_i - \delta_j)]$$

$$|\bar{A}(B \to f)|^2 = \sum_i A_i^2 + \sum_{i \neq j} 2 A_i A_j \cos[-(\theta_i - \theta_j) + (\delta_i - \delta_j)]$$

Rare two-body decays

- Direct CP asymmetry:

$$A_{\mathrm{CP}}(f) = \frac{\Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f)}{\Gamma(\bar{B} \to \bar{f}) + \Gamma(B \to f)}$$
$$= -\frac{\sum_{i \neq j} 2A_i A_j \sin(\theta_i - \theta_j) \sin(\delta_i - \delta_j)}{\sum_i A_i^2 + \sum_{i \neq j} 2A_i A_j \cos(\theta_i - \theta_j) \cos(\delta_i - \delta_j)}$$

- Requires at least two partial amplitudes that differ in both their weak and strong phases

SM analysis of rare decays

- Consider b→s(d)+X decays for concreteness:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=u,c} V_{qb} V_{qs}^* \left(C_1\, Q_1^{(q)} + C_2\, Q_2^{(q)} \right) - V_{tb} V_{ts}^* \sum_{i=3,\dots,10,7\gamma,8g} C_i\, Q_i \right]$$

W-boson exchange

penguin and box graphs

- CKM factors (with standard phase conventions):

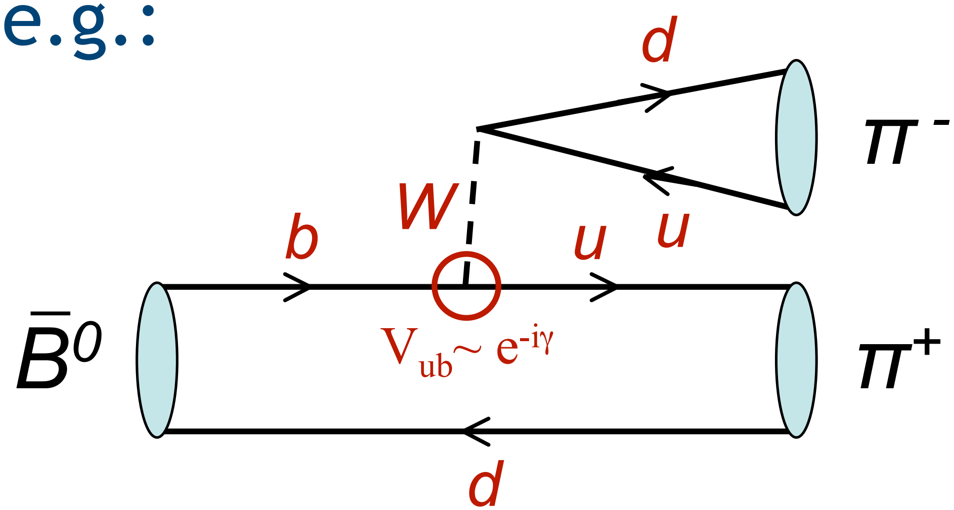
$$V_{cb}V_{cs}^* = \text{real} \qquad\qquad V_{ub}V_{us}^* \propto e^{-i\gamma}$$

$$-V_{tb}V_{ts}^* = V_{cb}V_{cs}^* + V_{ub}V_{us}^* \quad \text{(unitarity)}$$

⇒ only two independent weak phases $\theta=\{0,\gamma\}$ exist in SM!

SM analysis of rare decays

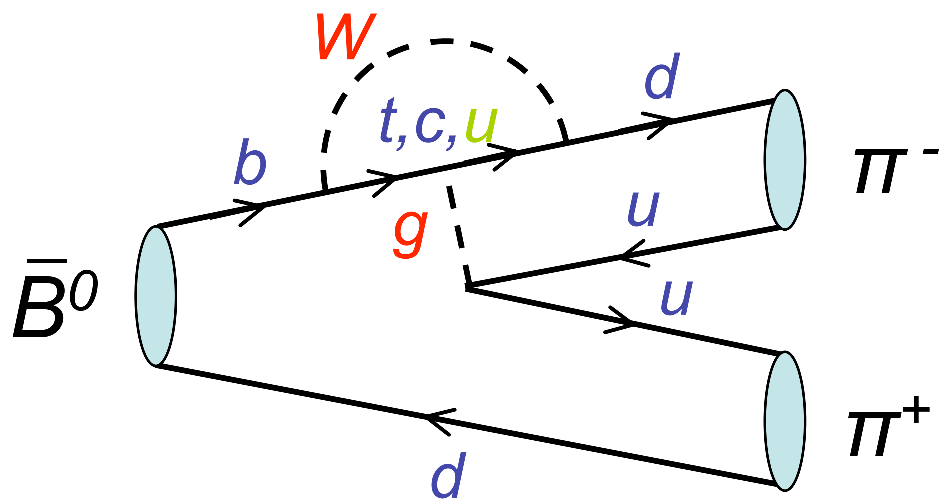
- For rare (i.e., charmless) hadronic B decays, terms $\sim e^{-i\gamma}$ are dominated by tree-level W-boson exchange, e.g.:



- All such terms are called the “tree amplitude T” (also includes small terms from penguin graphs!)

SM analysis of rare decays

- Terms ~1 are dominated by penguin graphs, e.g.:



- All terms ~1 are called the "penguin amplitude P" (*note*: part of top-quark loop and all of up-quark loop is included in T!)

SM analysis of rare decays

- This implies:

$$A(\bar{B} \to \bar{f}) = T\, e^{-i\gamma}\, e^{i\delta_T} + P\, e^{i\delta_P}$$

$$\bar{A}(B \to f) = T\, e^{i\gamma}\, e^{i\delta_T} + P\, e^{i\delta_P}$$

- And:

$$\boxed{A_{\mathrm{CP}}(f) = \frac{2TP \sin\gamma \sin(\delta_T - \delta_P)}{T^2 + P^2 + 2TP \cos\gamma \cos(\delta_T - \delta_P)}}$$

where $\gamma \approx 70^o$ is large

- If P~T (true in many cases), then A_{CP} can be large provided that $(\delta_T-\delta_P)$~O(1) is sizeable

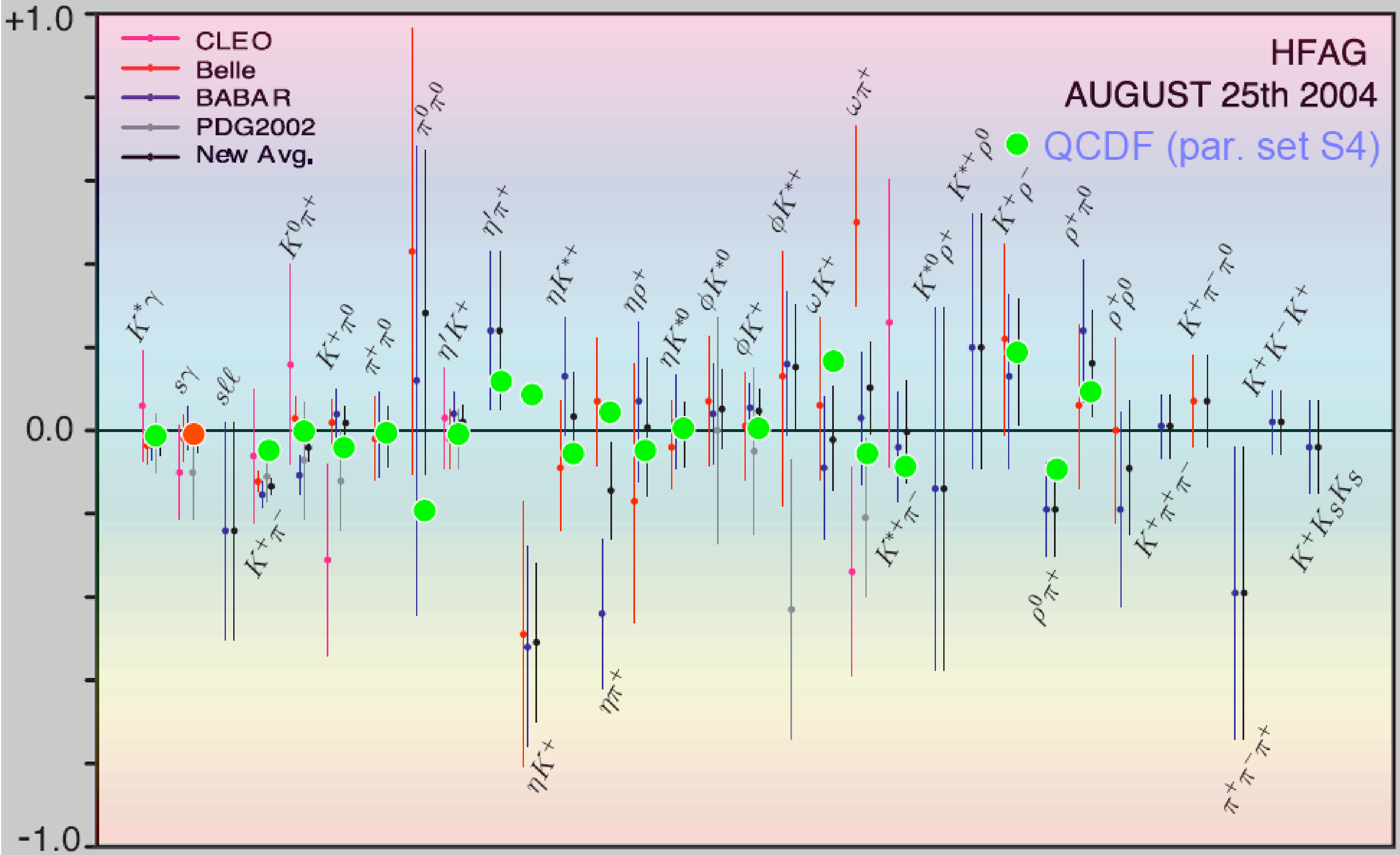
SM analysis of rare decays

- Theoretical prediction (QCD factorization, SCET):

$$(\delta_T - \delta_P) = O[\alpha_s(m_b),\ \Lambda_{QCD}/m_b]$$

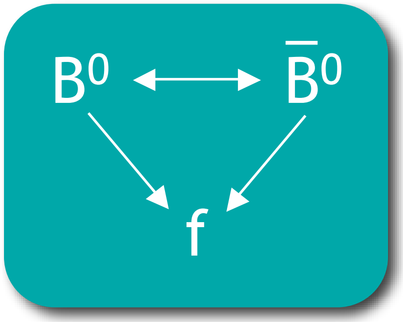
  - parametrically suppressed in heavy-quark limit
- Expect $A_{CP}(f) \lesssim 10\%$ for most decay modes, which is confirmed by experimental data

CP Asymmetry in Charmless B Decays



CP in Interference of Mixing and Decay

- Study interference of mixing and decay in neutral B decays into CP eigenstates
- Time-dependent CP asymmetry provides direct access to angles of the unitarity triangle!



- To see how this works, use previous result for time evolution of initial $\bar{B}^0$ state to get:

$$A_{\bar{B}^0\to f}(t) \propto A\cos\frac{\Delta m t}{2} + i\bar{A}\,e^{2i\beta}\sin\frac{\Delta m t}{2}$$

direct decay

indirect decay via mixing

CP in Interference of Mixing and Decay

- Time dependence of decay rate:

$$\Gamma_{\bar{B}^0\to f}(t) \propto |A|^2 \cos^2\frac{\Delta m t}{2} + |\bar{A}|^2 \sin^2\frac{\Delta m t}{2} - \mathrm{Im}(A^* \bar{A}\, e^{2i\beta}) \sin\Delta m t$$

$$\propto 1 + \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} \cos\Delta m t - \frac{2\mathrm{Im}(A^*\bar{A}\, e^{2i\beta})}{|A|^2 + |\bar{A}|^2} \sin\Delta m t$$

- Rate for CP-conjugate process $B^0 \to f$ given by same expression with $A \leftrightarrow \bar{A}$ and $\beta \to -\beta$

CP in Interference of Mixing and Decay

- Time-dependent CP asymmetry:

$$A_{\mathrm{CP}}(t) \equiv \frac{\Gamma_{\bar{B}^0\to f}(t) - \Gamma_{B^0\to f}(t)}{\Gamma_{\bar{B}^0\to f}(t) + \Gamma_{B^0\to f}(t)} = \mathcal{C}\cos(\Delta m\, t) - \mathcal{S}\sin(\Delta m\, t)$$

$$\boxed{\frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2}} \qquad \boxed{\frac{2\mathrm{Im}(A^*\bar{A}\,e^{2i\beta})}{|A|^2 + |\bar{A}|^2}}$$

(direct CP asymmetry)

- Special case: decay amplitude dominated by a single partial amplitude with weak phase φ_A

$\Rightarrow$ $\mathcal{C} = 0$ and $\mathcal{S} = \sin[2(\beta-\varphi_A)]$

CP in Interference of Mixing and Decay

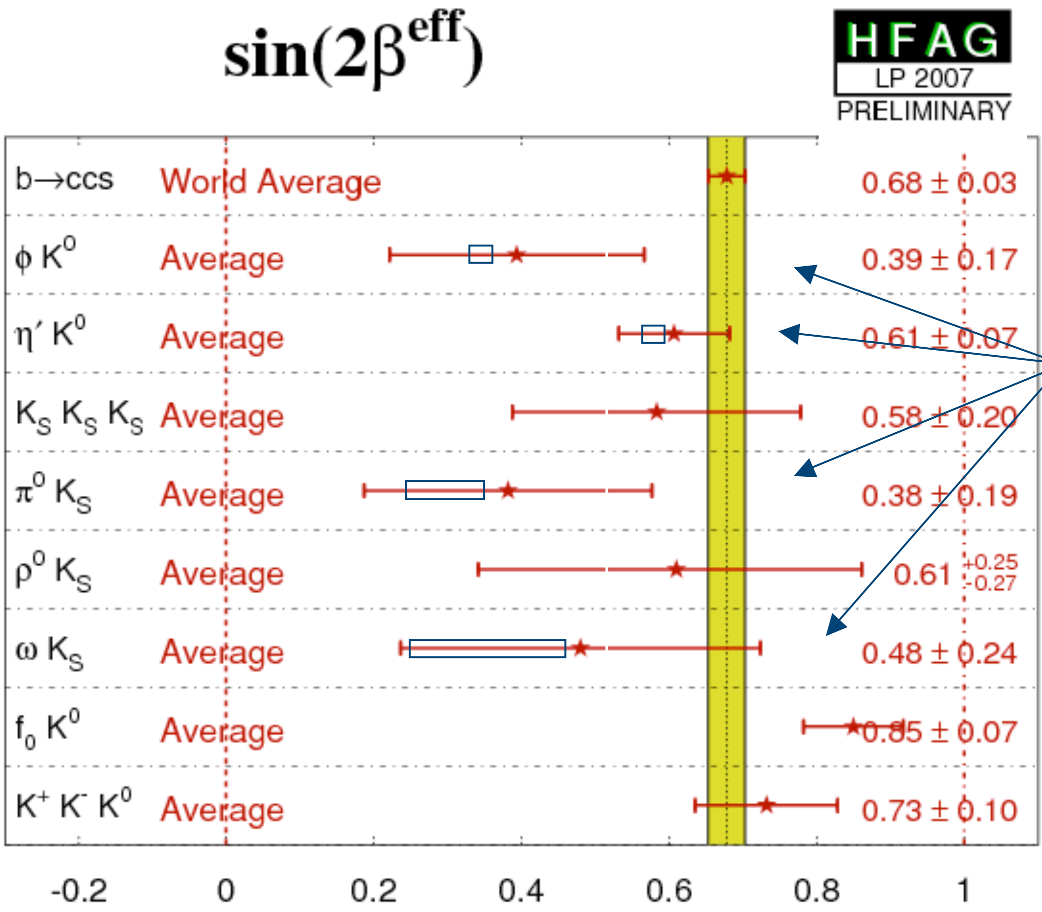
- Allows determination of a weak phase (almost) free of hadronic uncertainties!
- 2 possibilities in SM:

$\boxed{\varphi_A = 0 \quad \Rightarrow \quad S = \sin(2\beta)}$ (e.g. $B \to J/\psi K_S, \phi K_S$)

$\boxed{\varphi_A = -\gamma \quad \Rightarrow \quad S = \sin[2(\beta + \gamma)] = -\sin(2\alpha)}$ (e.g. $B \to \pi\pi, \rho\rho$)

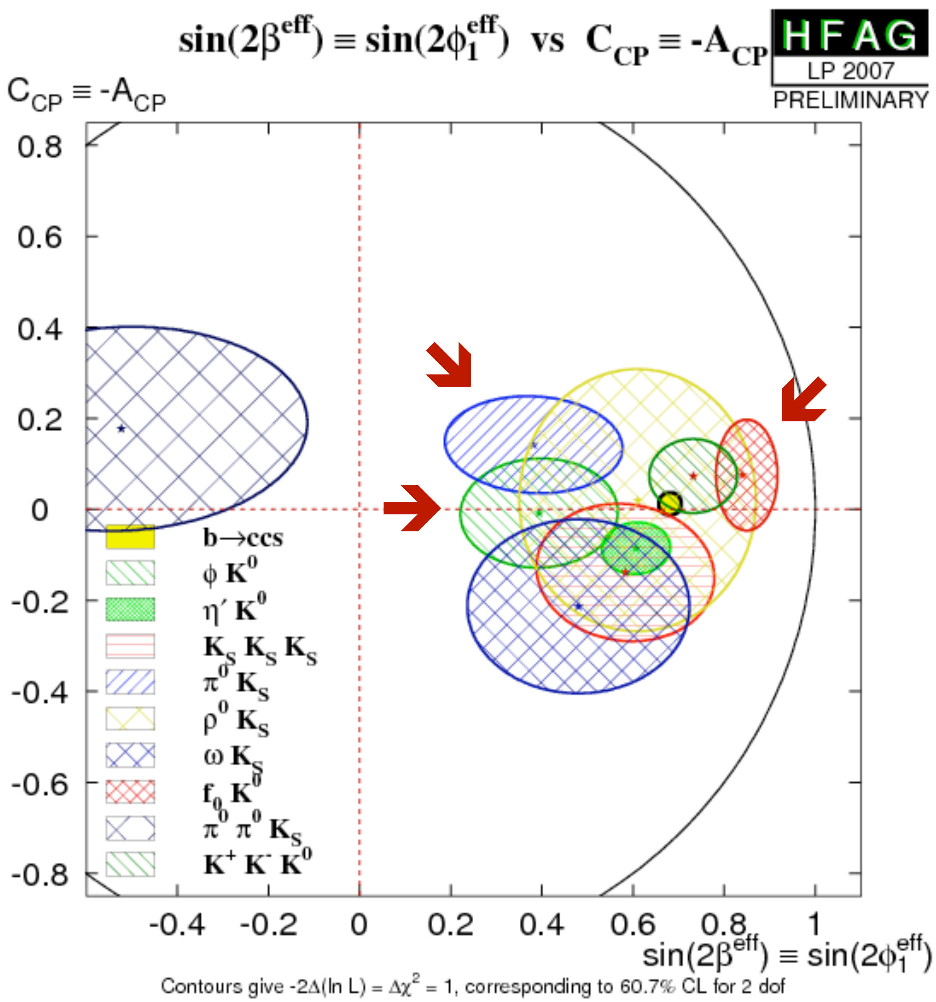
- Comparing $\sin 2\beta$ values extracted from tree-dominated vs. penguin-dominated modes is a sensitive probe for New Physics

$(\sin 2\beta)_{\text{tree}}$ vs. $(\sin 2\beta)_{\text{penguin}}$



theor. corrections increase deviations

$(\sin 2\beta)_{\text{tree}}$ vs. $(\sin 2\beta)_{\text{penguin}}$



- Present accuracy:

  $\sigma(\sin 2\beta_{\phi K s}) = 0.17$

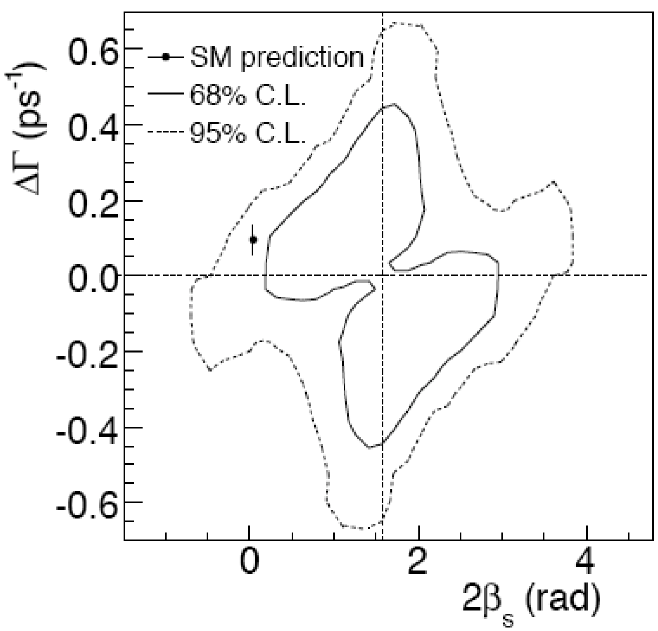
- LHC*b* capability with 10 fb^{-1}:

  $\sigma(\sin 2\beta_{\phi K s}) = 0.10$

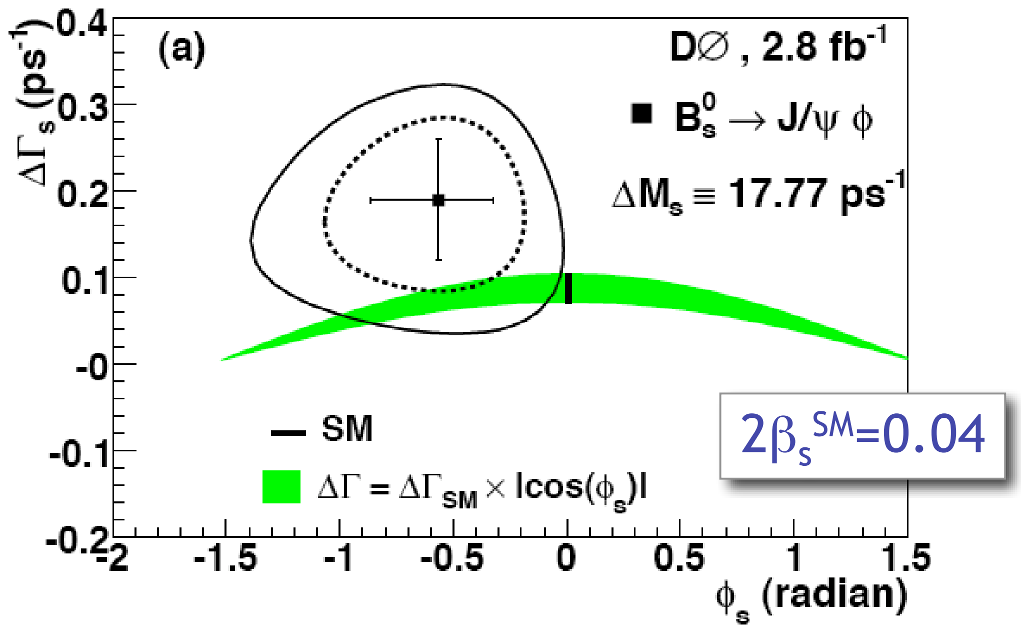
- Significant improvement requires a Super-B factory

New physics in B_s mixing ?

- Analogous study of mixing-induced CP violation in $B_s \to J/\psi\ \phi$ decay (Tevatron, LHC*b*) tests SM prediction that $\phi_s = 2\beta_s = 0.04$



CDF (Dec. 2007)



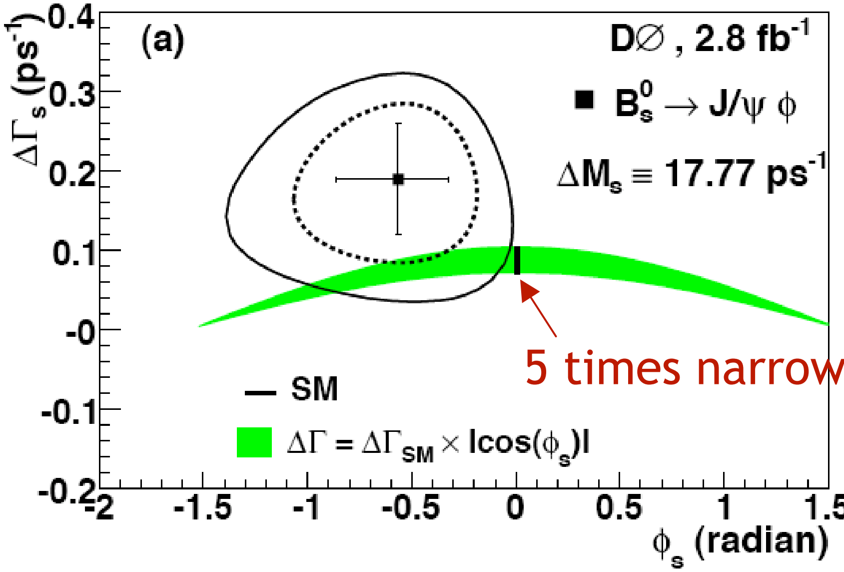
D0 (Feb. 2008)

New physics in B_s mixing ?

- Capabilities of LHC*b*:

Luminosity	$0.5\ fb^{-1}$ (~2009)	$2\ fb^{-1}$ (~2010)	$10\ fb^{-1}$ (~2013)
$\sigma(2\beta_s)$	0.046	0.021	0.009

- Precision:

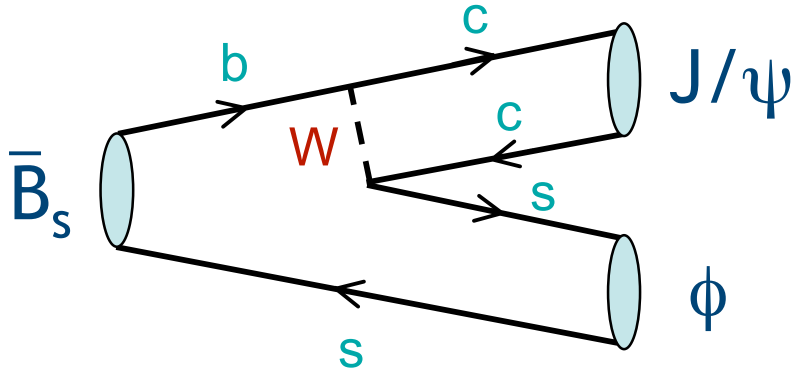


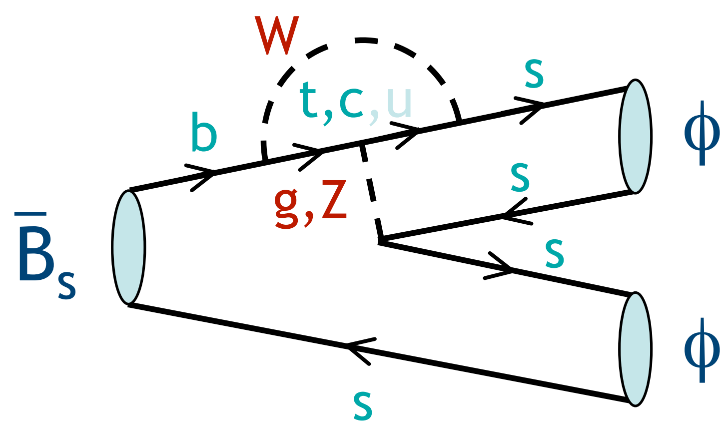
5 times narrower than this bar!

$2\beta_s^{SM}=0.04$

$(\sin 2\beta_s)_{tree}$ vs. $(\sin 2\beta_s)_{penguin}$

- LHC*b* can also do “tree vs. loop” test with B_s decays
- Compare $\sin 2\beta_s$ values extracted from $B_s \to J/\psi\ \phi$ vs. $B_s \to \phi\phi$



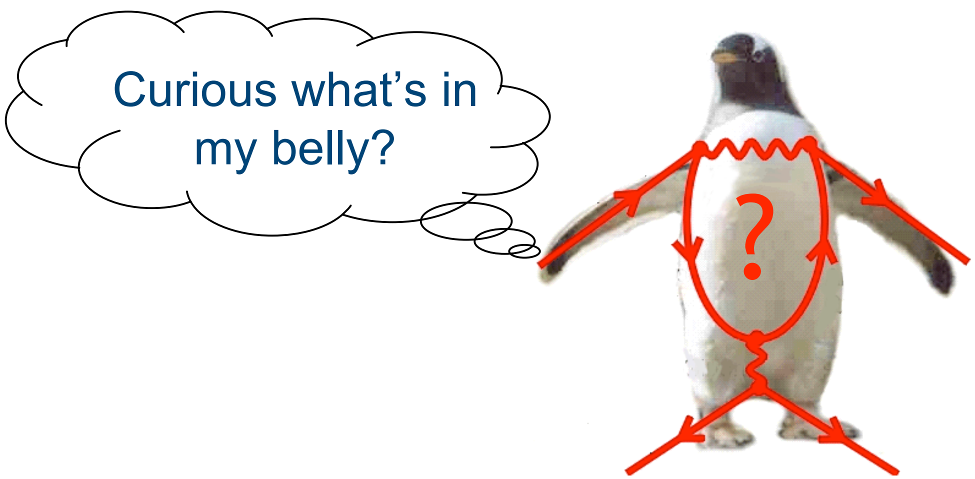


Luminosity	2 fb^{-1} (~2010)	10 fb^{-1} (~2013)
$\sigma(2\beta_s^{\phi\phi})$	0.11	0.04

$2\beta_s^{\phi\phi,SM}=0$

If any of these effects are real …

- Hints at O(1) new physics effects in mixing amplitudes and rare decay amplitude
- Requires large, O(1) new CP-violating phases



Check at ATLAS/CMS!

Summary

- Flavor physics addresses some key questions of modern particle physics
- Large potential for testing SM and searching for New Physics (complementary to searches at high-energy frontier)
- Interesting hints exist for potential new Physics effects in FCNC processes in the B_d and B_s systems
- Near future will tell whether any of these are real

Thank you!