Fermilab

LARP

# Numerical Simulations Of Beam Transverse Diffusion

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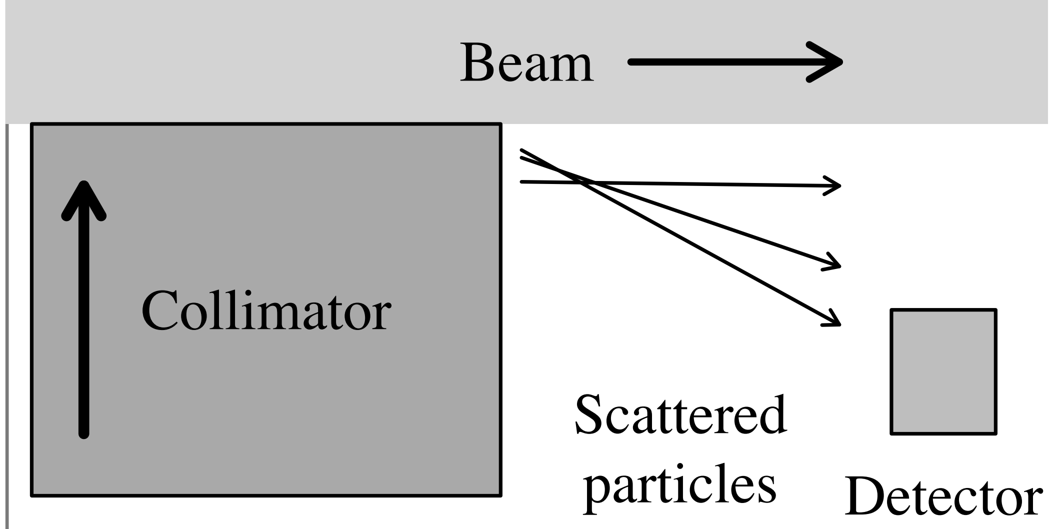
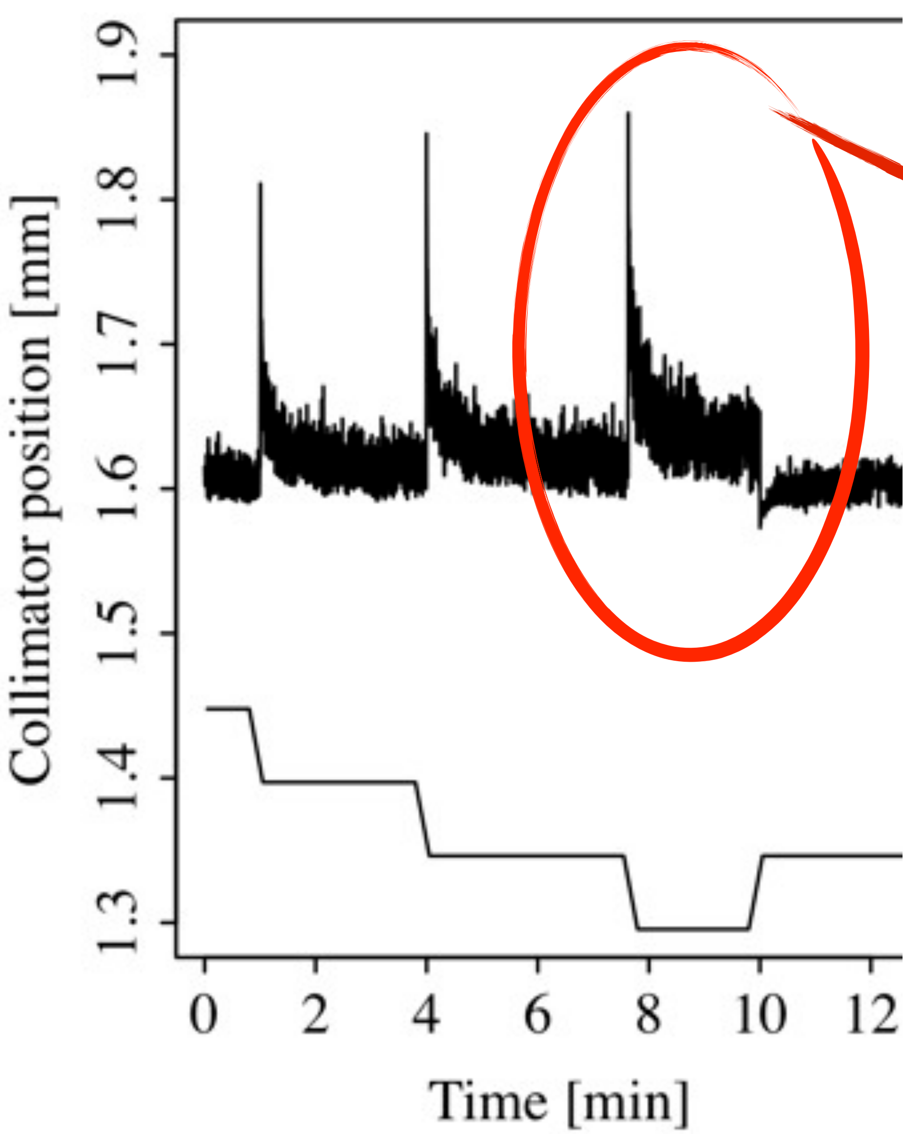
# Experimental Measurements

- Assuming that the diffusion equation regulates the time evolution of the particle amplitudes, it is possible to correlate the beam losses produced by a small collimator movement with the local diffusion coefficient.
- "Baby step" collimator measurements on the antiproton halo for different collimator positions, with and without BB, with and without Electron Lens
- analysis done by G. Stancari

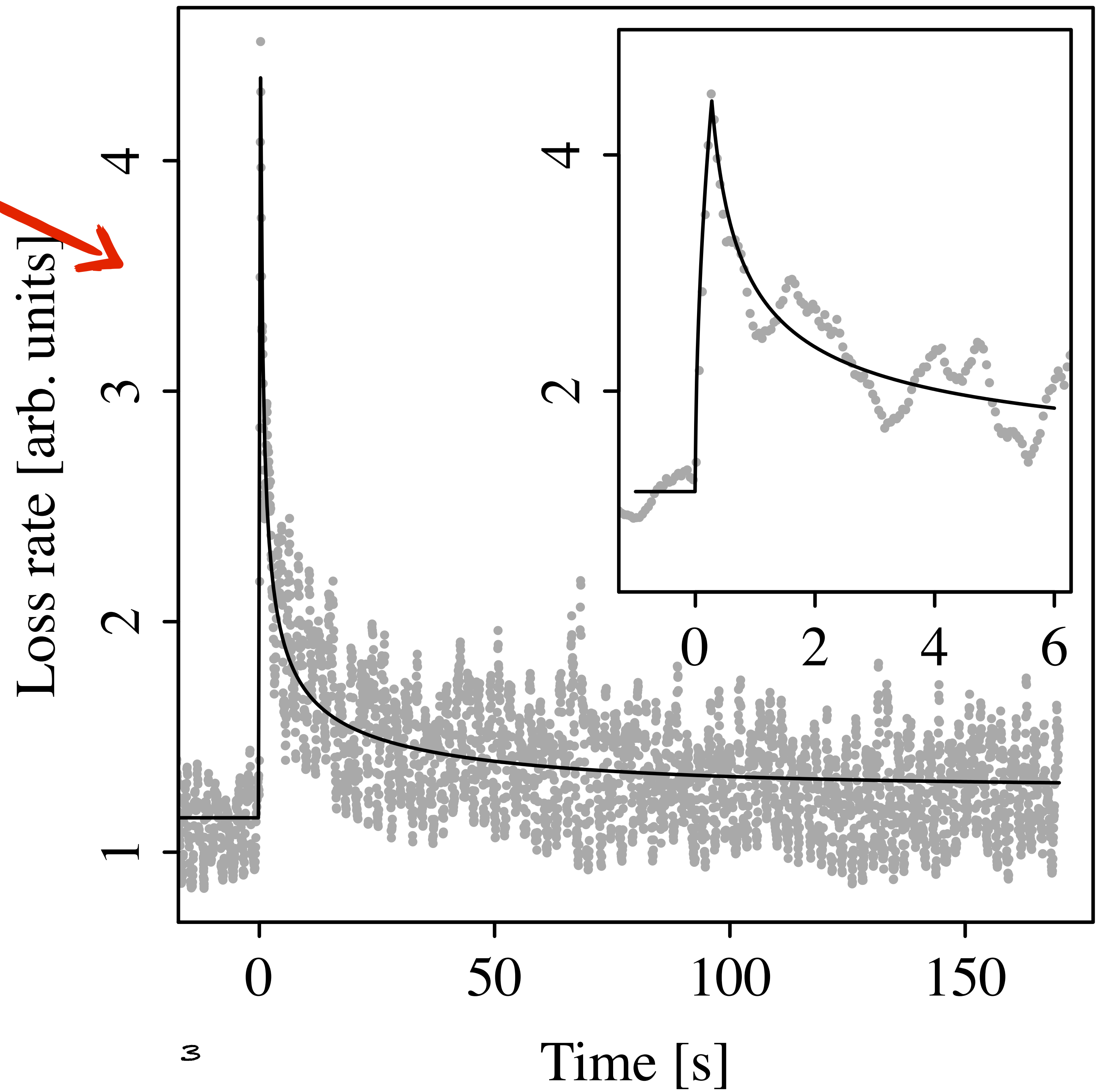



# Example

The diffusion coefficient has been calculated for different collimator positions.




courtesy of G. Stancari



# Experimental Measurements

- Assuming that the diffusion equation regulates the time evolution of the particle amplitudes, it is possible to correlate the beam losses produced by a small collimator movement with the local diffusion coefficient.
- "Baby step" collimator measurements on the antiproton halo for different collimator positions, with and without BB, with and without Electron Lens
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➡ my task: reproduce the diffusion coefficient with lifetrack

# Some Background: The Diffusion Model

density function

flux

$$\frac{\delta \rho}{\delta t} = -\nabla \phi$$

Continuity equation

$$\phi = -D \cdot \nabla \rho$$

diffusion coefficient

Flick's law*

*valid only for a brownian motion

$$\frac{\delta \rho}{\delta t} = \nabla (D \cdot \nabla \rho)$$

Diffusion equation

# Diffusion equation

$$\frac{\delta\rho}{\delta t} = \nabla(D \cdot \nabla\rho)$$

$$\boxed{\begin{aligned} \rho &= \rho(?) \\ D &= D(?) \end{aligned}}$$

# Diffusion equation

$$\frac{\delta\rho}{\delta t} = \nabla(D \cdot \nabla\rho)$$

$$\boxed{\begin{array}{c} \rho = \rho(?) \\ D = D(?) \end{array}}$$

## *Syphers*

1- only one dimension

2- assume D is scalar value

3- consider the density function in x, xp

4- transform in cylindrical coordinates

(only cylindrical symmetric distributions in x, xp)

5- Consider the invariant $W = [x^2 + xp^2]/\beta$

$$\frac{\delta\rho}{\delta t} = \frac{4D}{\beta}\frac{\delta}{\delta W}(W\frac{\delta\rho}{\delta W})$$

## *Seidel/Stancari*

1- only one dimension

2-Consider the invariant $J = x^2_{max}/(4\beta)$

3- consider the density function in J

$$\frac{\delta\rho}{\delta t} = \frac{\delta}{\delta J}(D(J)\frac{\delta\rho}{\delta J})$$

# Some (useful) Notation

| | coordinates | | invariant |
|---|---|---|---|
| physical | $x = n\sqrt{\epsilon\beta} \cdot \cos\phi$<br>$x' = n\sqrt{\epsilon\beta} \cdot (\cos\phi - \alpha\sin\phi)$ | | |
| Floquet | $\xi = x/\sqrt{\epsilon\beta}$<br>$\xi' = (\alpha x + \beta x')/\sqrt{\epsilon\beta}$ | $R = \sqrt{\xi^2 + \xi'^2}$ | $R^2$ |
| Sypher | $x = n\sqrt{\epsilon\beta} \cdot \cos\phi$<br>$xp = (\alpha x + \beta x')$ | $r = \sqrt{x^2 + xp^2} =$<br>$= R\sqrt{\epsilon\beta}$ | $W = r^2/\beta$<br>$= R^2\varepsilon$ |
| Seidel/ Stancari | $x = n\sqrt{\epsilon\beta} \cdot \cos\phi$ | | $J = x^2_{max}/(4\beta)$ |

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| Sypher | $x = n\sqrt{\epsilon\beta} \cdot \cos\phi$ <br> $xp = (\alpha x + \beta x')$ | $r = \sqrt{x^2 + xp^2} =$ <br> $= R\sqrt{\epsilon\beta}$ | $W = r^2/\beta$ <br> $= R^2\varepsilon$ |
| Seidel/ Stancari | $x = n\sqrt{\epsilon\beta} \cdot \cos\phi$ | | $J = x^2_{max}/(4\beta)$ |

equivalent

$$J = \frac{\varepsilon}{4} R^2 = \frac{1}{4} W$$

# Diffusion equation

$$\frac{\delta\rho}{\delta t} = \nabla(D \cdot \nabla\rho)$$

$$\boxed{\begin{array}{c}\rho = \rho(?)\\ D = D(?)\end{array}}$$

## *Syphers*

1- only one dimension

2- assume D is scalar value

3- consider the density function in x, xp

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## *Seidel/Stancari*

1- only one dimension

2-Consider the invariant $J = x_{max}^2/(4\beta)$

3- consider the density function in J

$$\frac{\delta\rho}{\delta t} = \frac{\delta}{\delta J}(D(J)\frac{\delta\rho}{\delta J})$$

$$J = \frac{\varepsilon}{4}R^2 = \frac{1}{4}W$$

$$D_J(J) = \frac{D_{sy}(J)J}{\beta}$$

for brownian motion,

$D_{sy}$ =const. -> $D_J$ / J const

*Seidel/Stancari*

$$J = \frac{\varepsilon_1}{4} R_1^2$$

$$\frac{\delta\rho}{\delta t} = \frac{\delta}{\delta J}\left(D(J)\frac{\delta\rho}{\delta J}\right)$$

$$\frac{\delta\rho}{\delta t} = \frac{\delta D}{\delta J}\cdot\frac{\delta\rho}{\delta J} + D\frac{\delta^2\rho}{\delta^2 J}$$

*Seidel/Stancari*

$$J = \frac{\varepsilon_1}{4} R_1^2$$

$$\frac{\delta\rho}{\delta t} = \frac{\delta}{\delta J}\left(D(J)\frac{\delta\rho}{\delta J}\right)$$

$$\frac{\delta\rho}{\delta t} = \cancel{\frac{\delta D}{\delta J}\frac{\delta\rho}{\delta J}} + D\frac{\delta^2\rho}{\delta^2 J}$$

considering delta-like initial particle distributions in the action space, we can assume the D coefficient for be constant over the considered J range

$$\frac{\delta\rho}{\delta t} = D\frac{\delta^2\rho}{\delta^2 J}$$

for this equation, according to Seidel, the diffusion coefficient is:

$$D = \frac{\langle \Delta J^2 \rangle}{2\Delta t}$$

change of $\rho(J)$ width in time

*Caveat! Devil's in details… (and footnotes)*

# Some (useful) Notation

strictly true for linear, uncoupled machines. The Tevatron is not!!

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# Some (useful) Notation

| | coordinates | | invariant |
|---|---|---|---|
| physical | $(x, x')$ | | |
| Floquet | $(\xi_1, \xi_1', \xi_2, \xi_2', \xi_3, \xi_3') = N \cdot (x, x', y, y', z, dpp)^T$ | $R_1 = \sqrt{\xi_1^2 + \xi_1'^2}$<br>$R_2 = \sqrt{\xi_2^2 + \xi_2'^2}$ | $R_1^2$<br>$R_2^2$ |
| Sypher | $(x_1, xp_1, x_2, xp_2, x_3, x_3) = N_2 \cdot (x, x', y, y', z, dpp)^T$ | $r_1 = \sqrt{x_1^2 + xp_1^2}$<br>$r_2 = \sqrt{x_2^2 + xp_2^2}$ | $W_1 = r_1^2/\beta_1$<br>$W_2 = r_2^2/\beta_2$ |
| Seidel/ Stancari | | | $J = x_{max}^2/(4\beta)$ |

... however for coupled linear machines it is possible to define three planes (eigenvectors of the one turn matrix) for whom the motion is uncoupled. Normalization is still possible.

# Some (useful) Notation

To compensate for amplitude beating introduced by strong non linearities (BB) we consider an average amplitude over 50K turns

... however for coupled ~~linear~~ machines it is possible to define three planes (eigenvectors of the one turn matrix) for whom the motion is uncoupled. Normalization is still possible.

| | coordinates | | |
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# Some (useful) Notation

simulations

planes 1,2: normalized, uncoupled

| | coordinates | | invariant |
|---|---|---|---|
| physical | $(x, x')$ | | |
| Floquet | $(\xi_1, \xi_1', \xi_2, \xi_2', \xi_3, \xi_3') = N \cdot (x, x', y, y', z, dpp)^T$ | $R_1 = \sqrt{\xi_1^2 + \xi_1'^2}$<br>$R_2 = \sqrt{\xi_2^2 + \xi_2'^2}$ | $R_1^2$<br>$R_2^2$ |
| Sypher | $(x_1, xp_1, x_2, xp_2, x_3, x_3) = N_2 \cdot (x, x', y, y', z, dpp)^T$<br>we are omniscient!!! | $r_1 = \sqrt{x_1^2 + xp_1^2}$<br>$r_2 = \sqrt{x_2^2 + xp_2^2}$ | $W_1 = r_1^2/\beta_1$<br>$W_2 = r_2^2/\beta_2$ |
| Seidel/ Stancari | $x$ | | $J = x_{max}^2/(4\beta_c)$ |

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| Seidel/ Stancari | $x$ | | $J = x_{max}^2/(4\beta_c)$ |

simulations

planes 1,2: normalized, uncoupled

experiment

we don't know much...

planes x,y: physical, coupled

# how do we compare the two worlds?

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simulations

planes 1,2: normalized, uncoupled

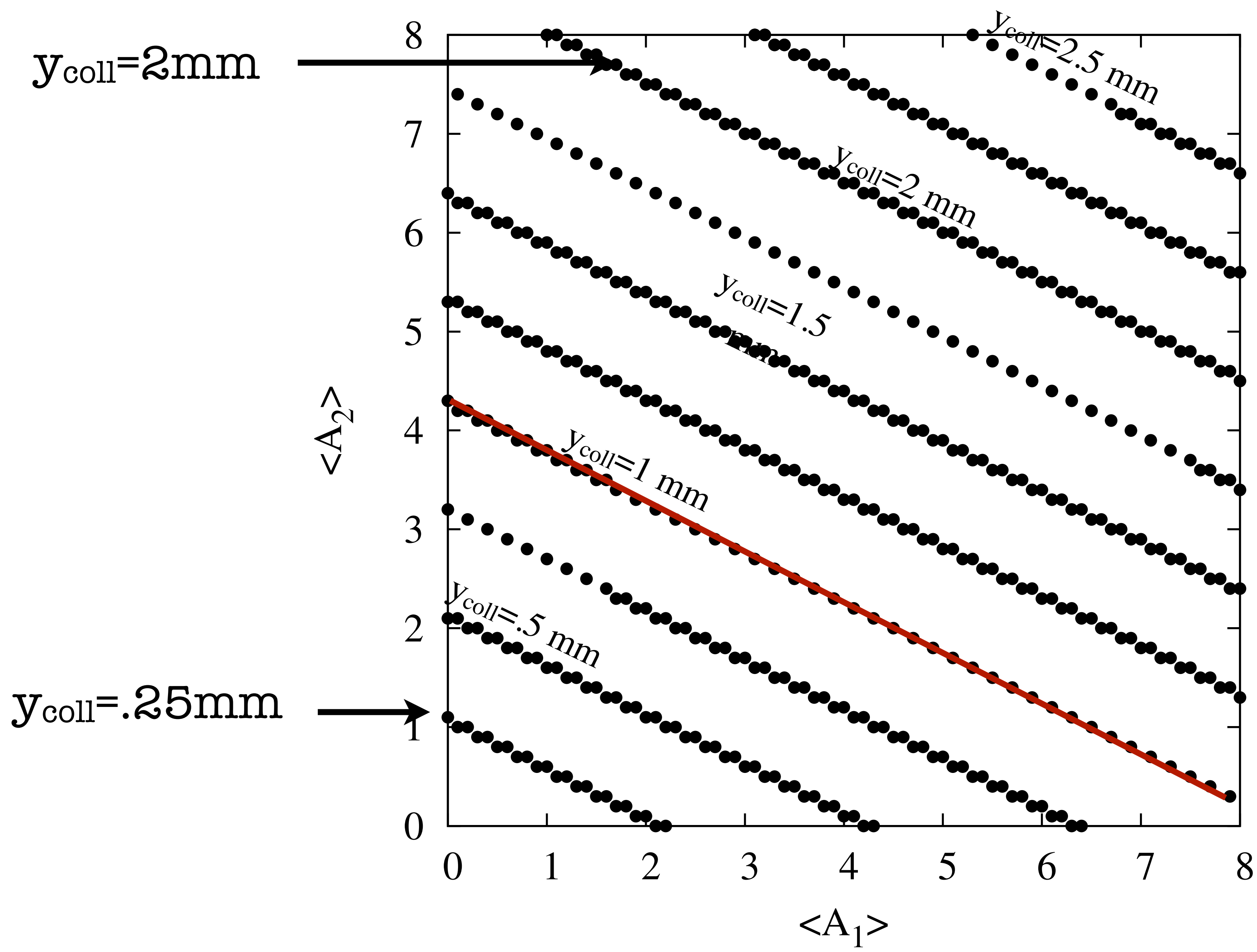
experiment

planes x,y: physical, coupled

# how do we compare the two worlds?

1. where is the collimator in the $<A_1>$, $<A_2>$ space?
2. how do we pass from the diffusion coefficient in the normalized direction to the diffusion coefficient in the vertical direction, for each point in the $<A_1>$, $<A_2>$ space?
3. how do we calculate the overall diffusion coefficient seen by the collimator?

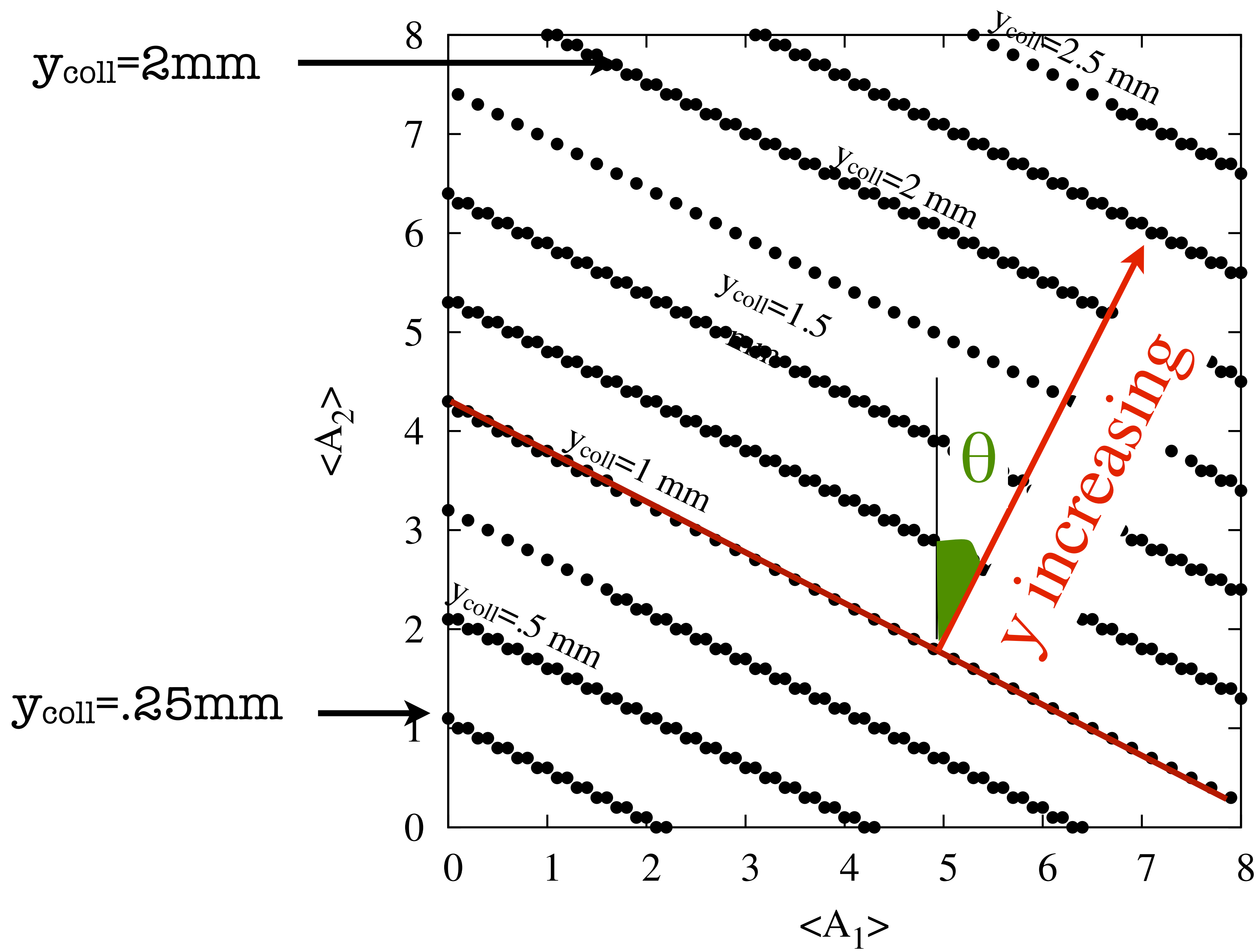
1. where is the collimator in the $\langle A_1 \rangle$, $\langle A_2 \rangle$ space?




no BB

the collimator edge is a skew line in the $\langle A_1 \rangle$, $\langle A_2 \rangle$ plane.

1. where is the collimator in the $\langle A_1 \rangle$, $\langle A_2 \rangle$ space?



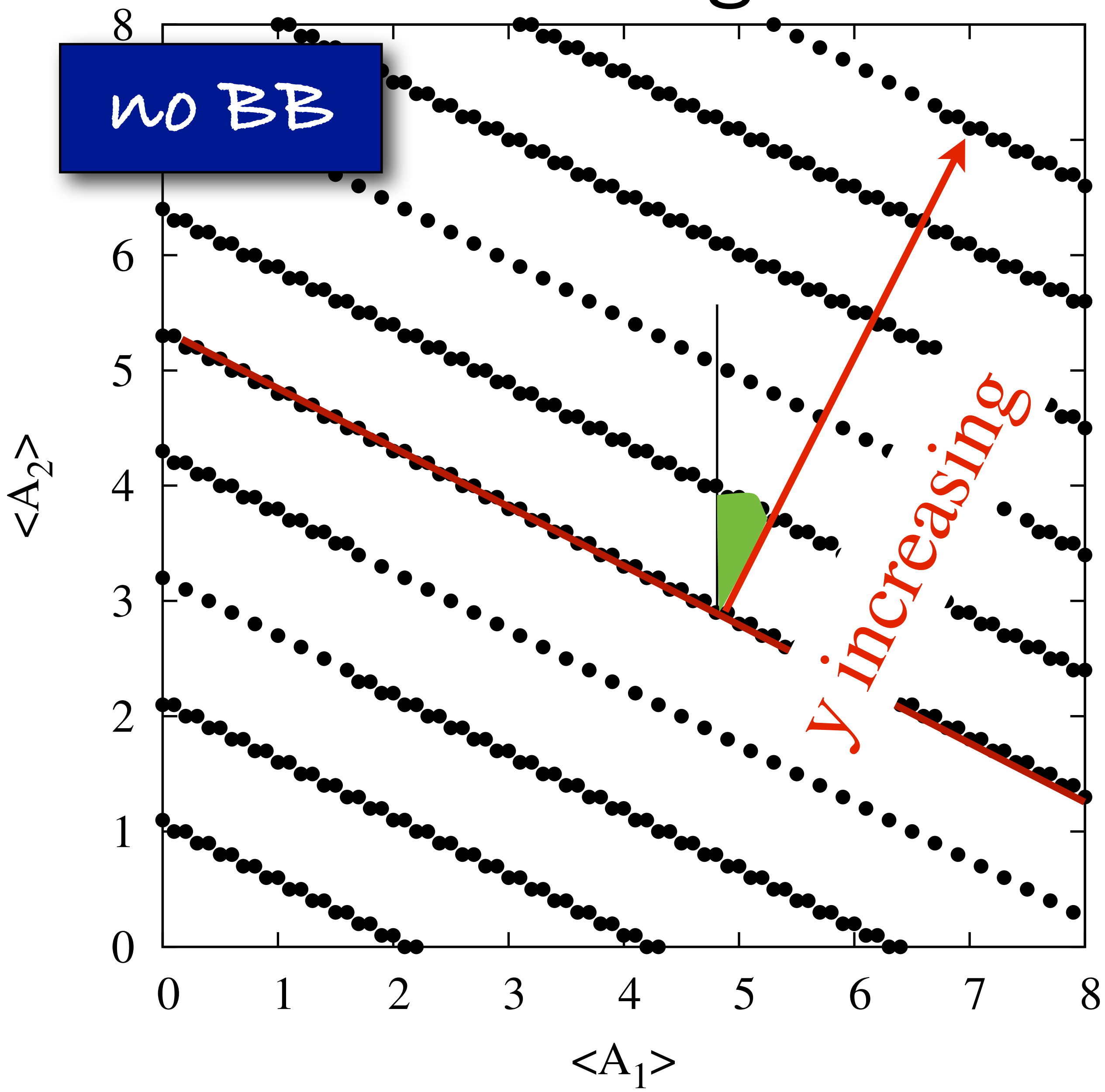

no BB

the collimator edge is a skew line in the $\langle A_1 \rangle$, $\langle A_2 \rangle$ plane.

2. how do we pass from the diffusion coefficient in the normalized direction to the diffusion coefficient in the vertical direction, for each point in the $\langle A_1 \rangle$, $\langle A_2 \rangle$ space?

$$\boxed{D_y(A_1, A_2) = D_1 \ \cos\theta + D_2 \ \sin\theta}$$

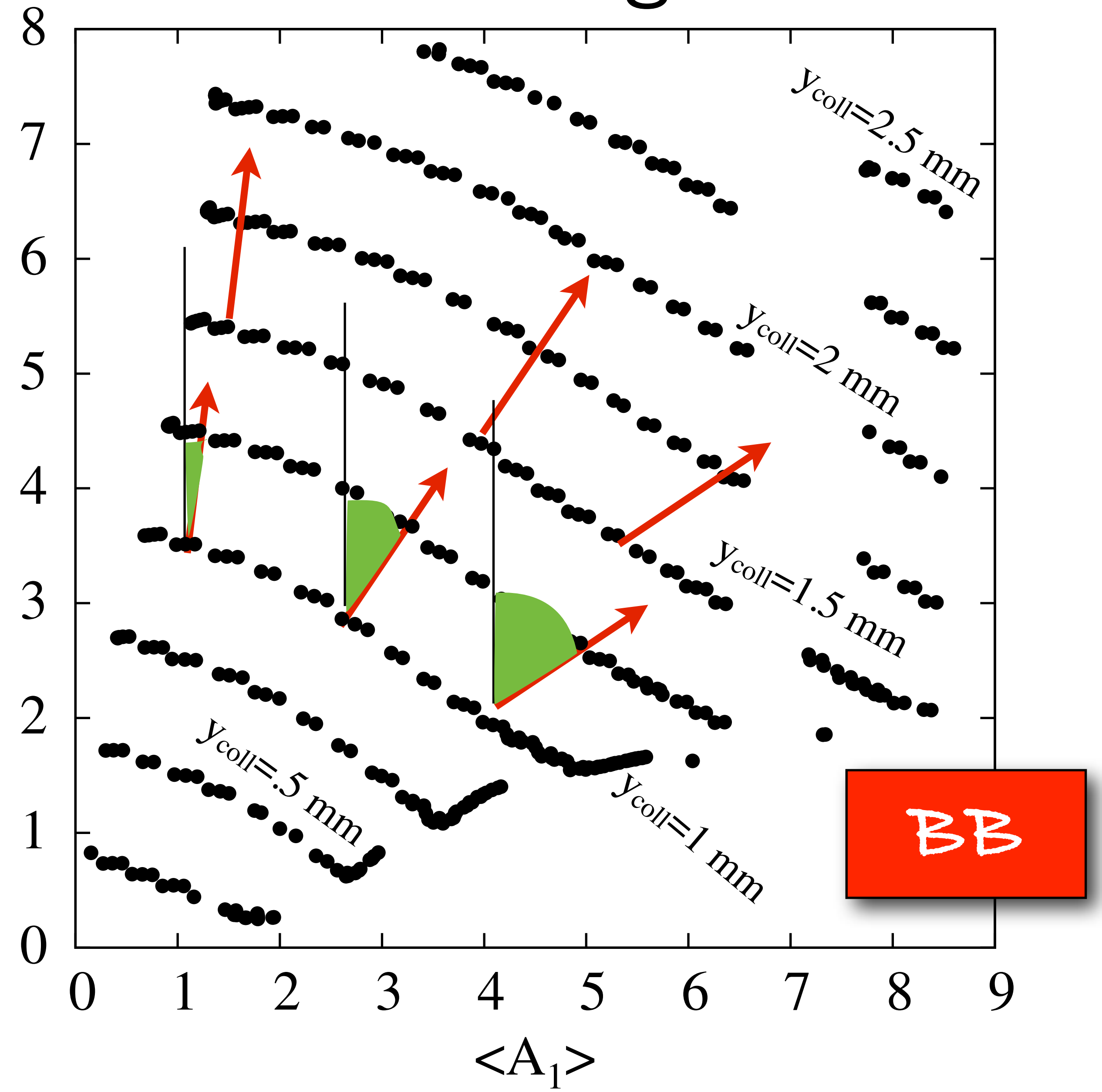
# collimator edge @ F48




for the linear case, the collimator edge is a skew line in the <$A_1$>, <$A_2$> plane.

## the angle θ is constant

# collimator edge @ F48




when BB is present, the collimator edge is not a linear function of <$A_1$>, <$A_2$>

## the angle θ changes along the collimator edge

3. how do we calculate the overall diffusion coefficient seen by the collimator?

in principle, we should calculate the diffusion coefficient for each point along the collimator line and integrate keeping in consideration the population of each point.

$$\boxed{D_{exp}(x_{coll}) = \int_{x_{coll}} D(A_1, A_2) \cdot \rho(A_1, A_2)\ dx_{coll}}$$

$$D_{exp}(x_{coll}) = \int_{x_{coll}} D(A_1, A_2) \cdot \rho(A_1, A_2)\ dx_{coll}$$

calculate the diffusion coefficient would mean:

1. get $D_1$,$D_2$: sample the whole space $A_1$, $A_2$ and calculate $D_1(A_1,A_2)$ and $D_2(A_1,A_2)$ for each point
2. calculate $\theta(A_1,A_2)$
3. assume some particle distribution
4. make the weighted average and calculate the overall D coefficient

for the linear case, the normal modes are uncoupled

↓

D1=D1(A1)
D2=D2(A2)

+

$y_{coll}$=f(A,A2) is simply a line

✓ possible

$$D_{exp}(x_{coll}) = \int_{x_{coll}} D(A_1, A_2) \cdot \rho(A_1, A_2)\ dx_{coll}$$

beam beam effect couples the normal modes!

calculate the diffusion coefficient would mean:

1. get $D_1$,$D_2$: sample the whole space $A_1$, $A_2$ and calculate $D_1(A_1,A_2)$ and $D_2(A_1,A_2)$ for each point
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3. assume some particle distribution
4. make the weighted average and calculate the overall D coefficient

$D1=D1(A1,A2)$
$D2=D2(A1,A2)$

$y_{coll}$ is a complicated function

$$D_{exp}(x_{coll}) = \int_{x_{coll}} D(A_1, A_2) \cdot \rho(A_1, A_2)\ dx_{coll}$$

beam beam effect couples the normal modes!

calculate the diffusion coefficient would mean:

1. get $D_1$,$D_2$: sample the ... $A_2$ and calculate $D_1(A_1$, ... for each point


2. calculate ...

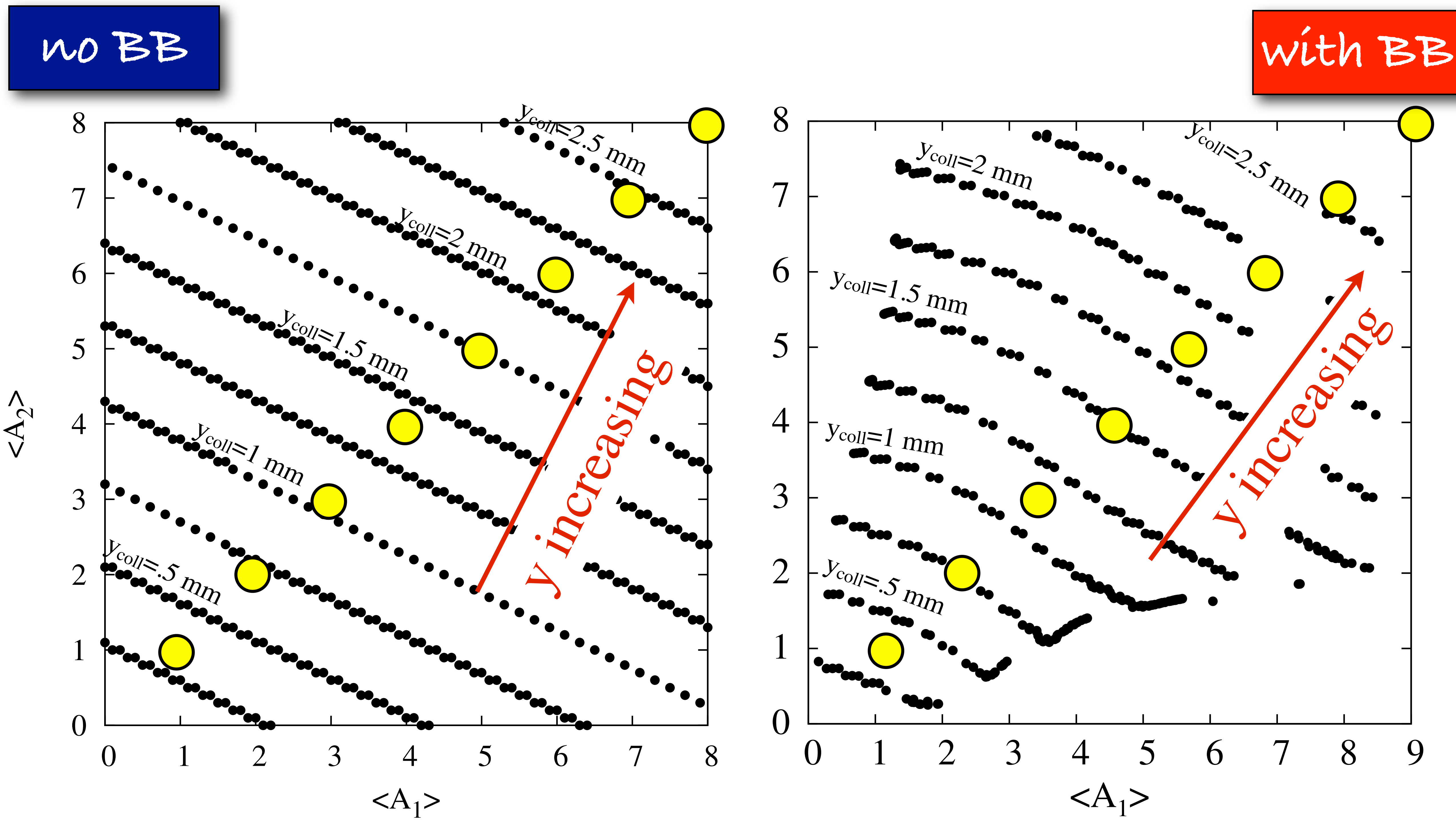
3. assume some particle distribution

4. make the weighted average and calculate the overall D coefficient

D1=D1(A1,A2)
D2=D2(A1,A2)

$y_{coll}$ is a complicated function

# Shortcut N.1



no BB
with BB
$y_{coll}$=2.5 mm
$y_{coll}$=2 mm
$y_{coll}$=1.5 mm
$y_{coll}$=1 mm
$y_{coll}$=.5 mm
y increasing
$\langle A_1 \rangle$
$\langle A_2 \rangle$

# Simulation Inputs

- lifetrac code
- standard optics, chromaticity on, collisions OFF
- random noise matrix, independent on particle amplitude
- simulations with and without Beam Beam, with and without electron lens
- electron lens: typical TEL2 parameters
- 1K particle with narrow distribution in the $A_1$,$A_2$ space (about 0.02 sigma).
- Center of the distribution between 1 and 8 sigma
- steps of 250K turns, about 25 steps (about 2.5 min)

from few slides before...

considering delta-like initial particle distributions in the action space, we can assume the D coefficient for be constant over the considered J range

for each skew plane we can write the diffusion equation and, according to Seidel, the diffusion coefficient is:

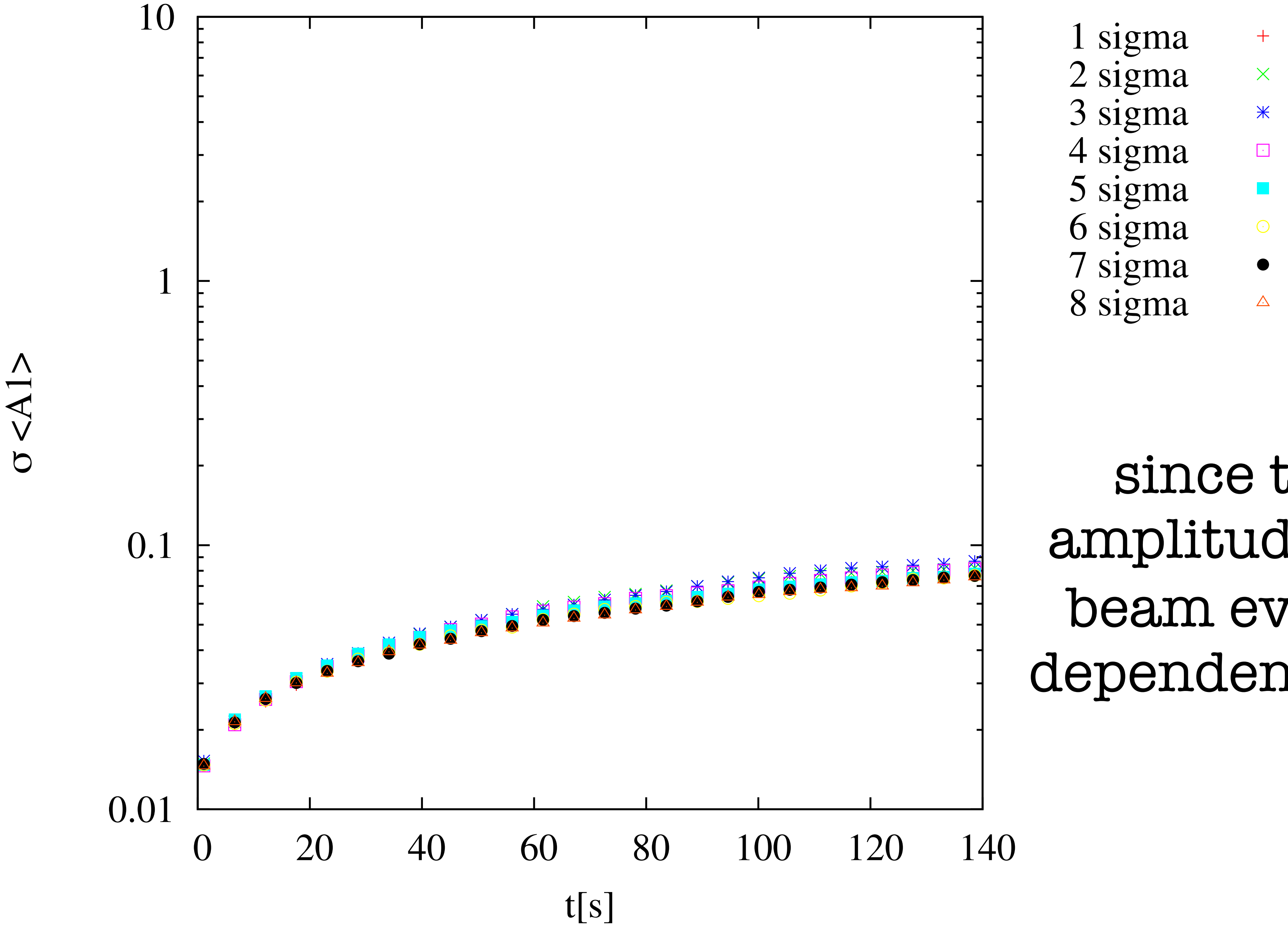
$$D_{1,2} = \frac{\langle \Delta J_{1,2}^2 \rangle}{2\Delta t}$$

change of $\rho(J)$ width in time

$$J_{1,2} = \frac{\varepsilon_{1,2}}{4} R^2_{1,2}$$

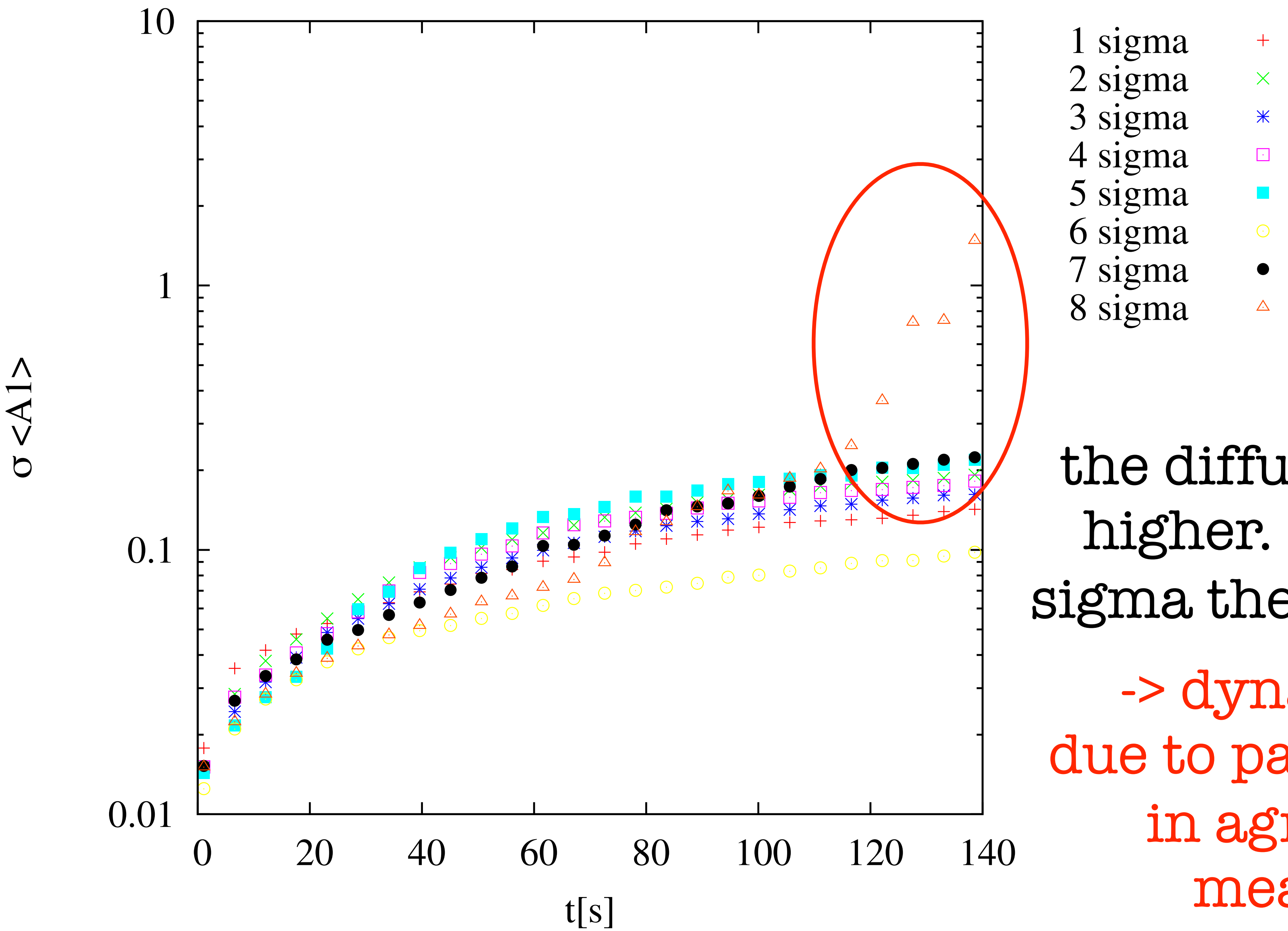
# Amplitude Evolution

random noise




since the noise is not amplitude dependent, the beam evolution does not dependent on amplitude as well

# Amplitude Evolution



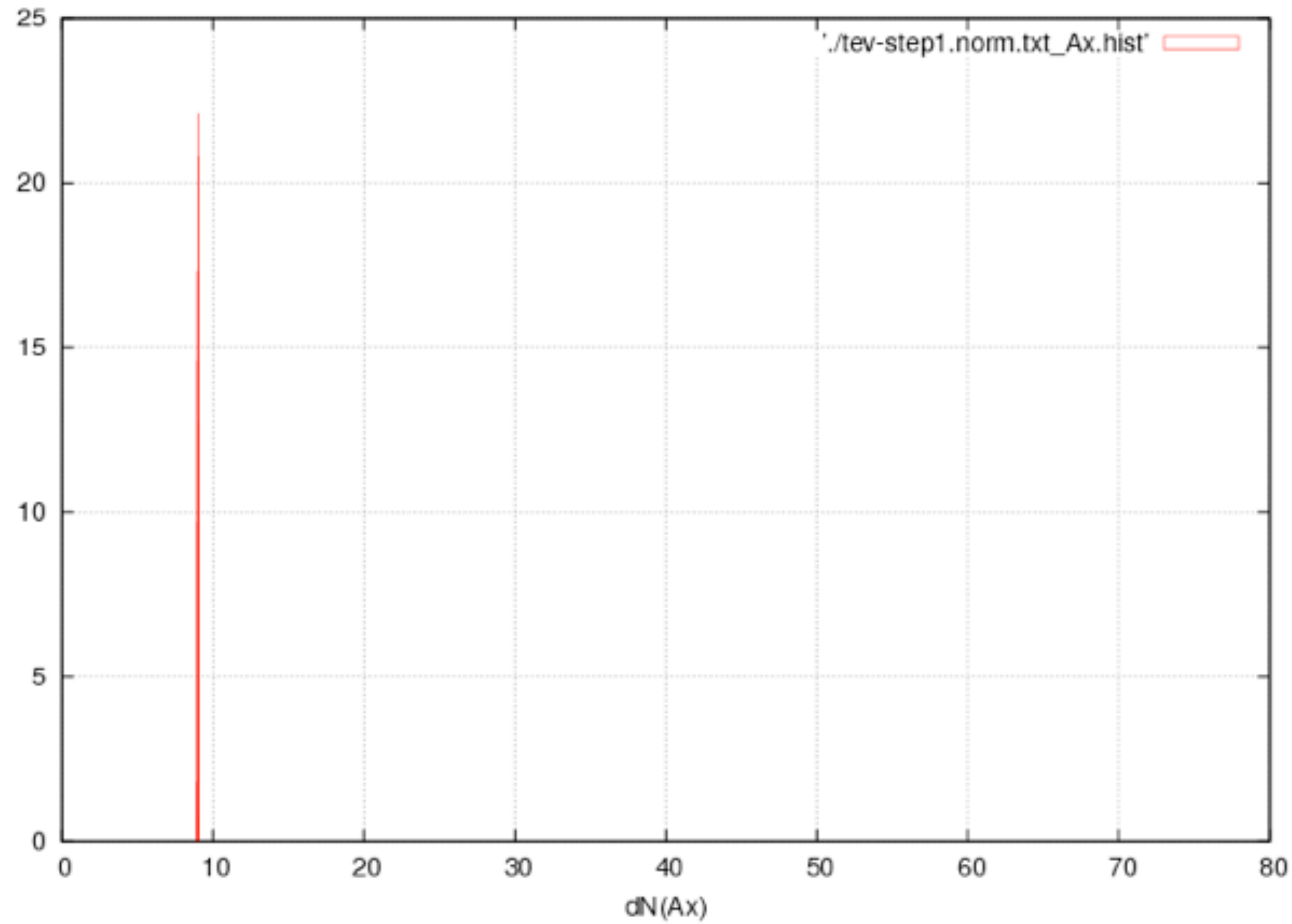

random noise

BB effect

the diffusion is generally higher. Starting from 8 sigma the beam "explodes"

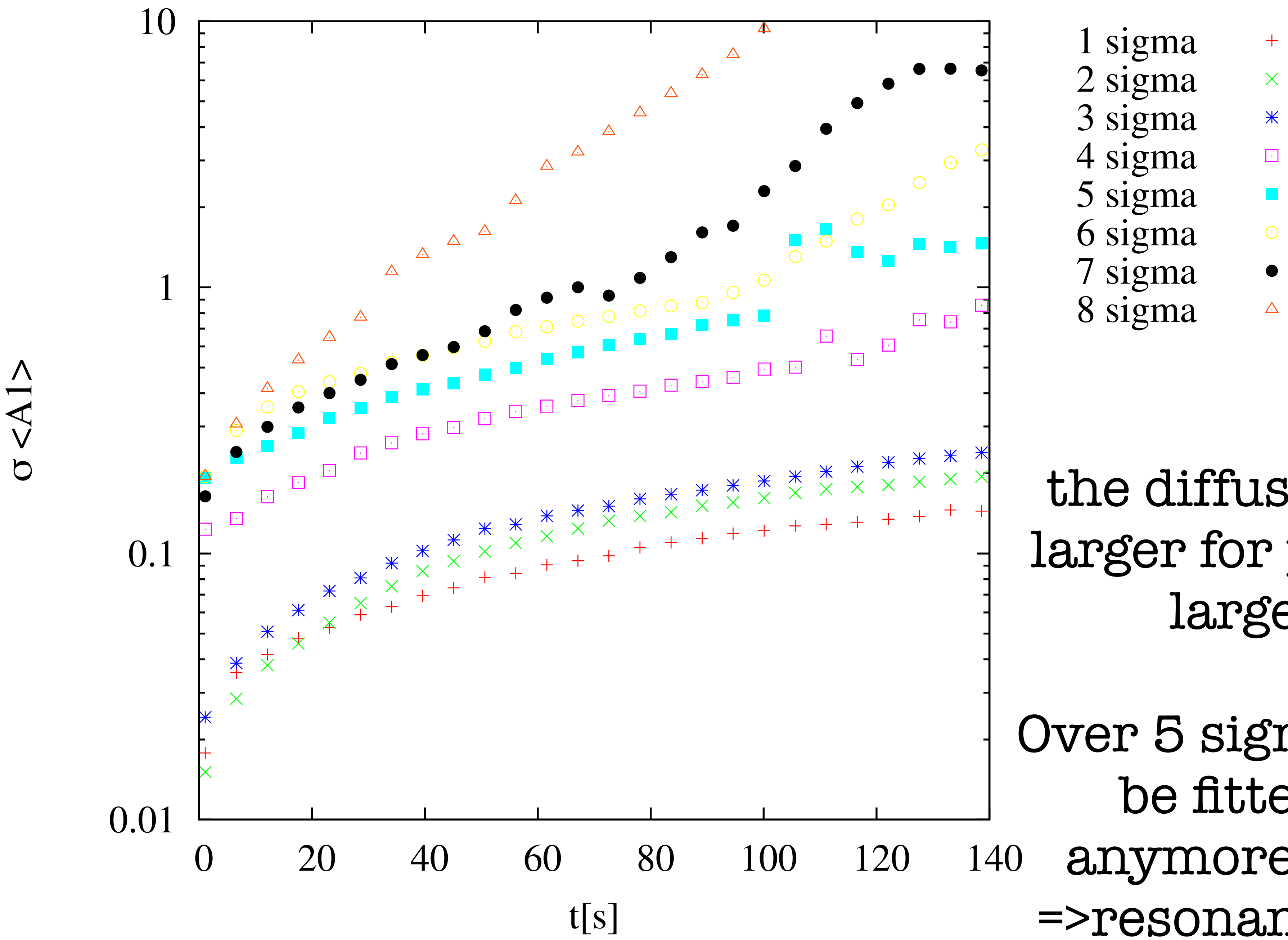
-> dynamic aperture due to parassitic crossing in agreement with measurements

# The Dynamic Aperture Limit




https://indico.fnal.gov/getFile.py/access?contribId=27&sessionId=12&resId=0&materialId=0&confId=5072

# Amplitude Evolution




random noise

BB effect

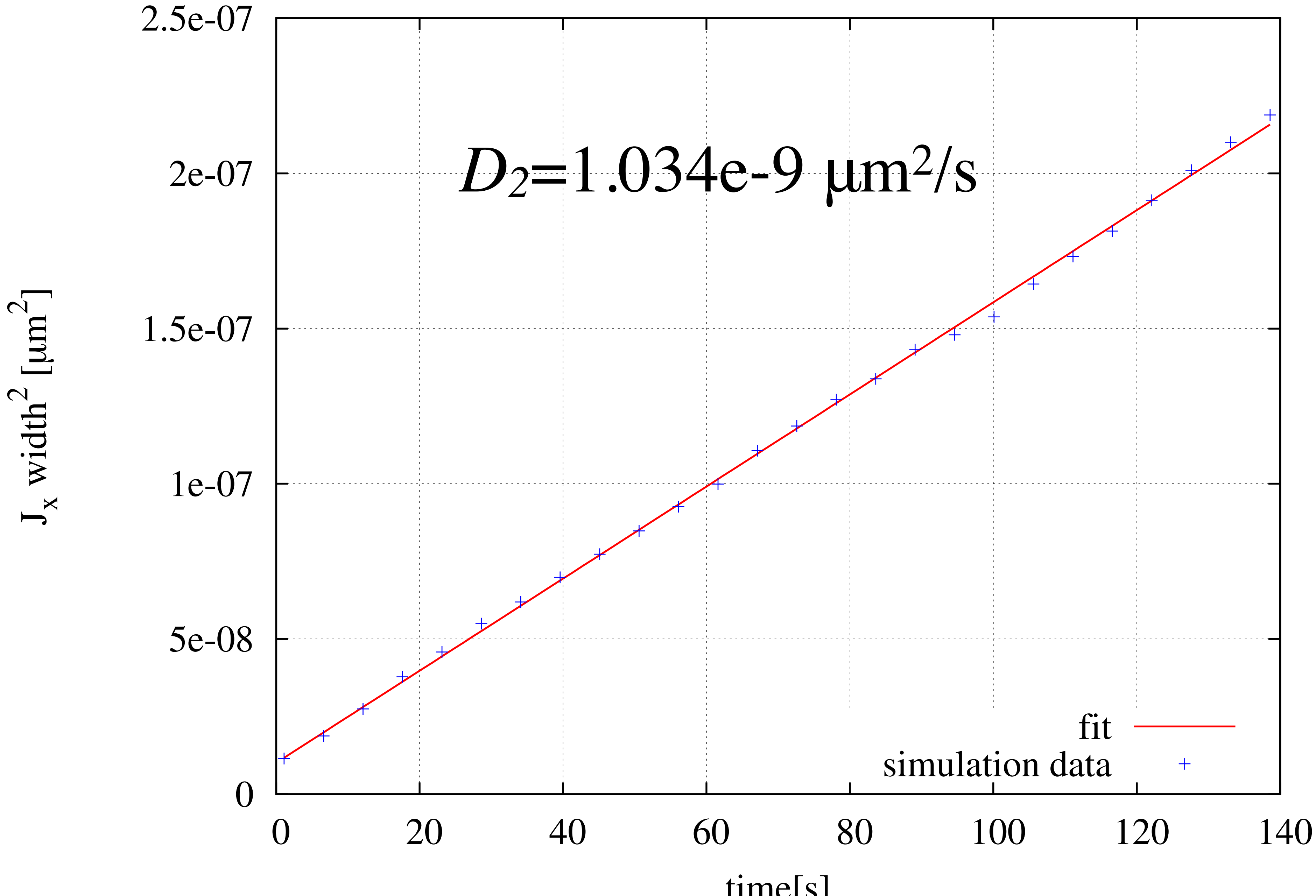
electron lens

the diffusion is considerably larger for particle amplitudes large than 3sigma.

Over 5 sigma the curve cannot be fitted to a quadratic anymore (diffusive model) =>resonance driven diffusion random noise

# Noise_Matr: NOISE

| | | | | | |
|---|---|---|---|---|---|
| 8.2E-15 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6.6E-18 | 0 | 0 | 0 | 0 |
| 0 | 0 | 4.1E-15 | 0 | 0 | 0 |
| 0 | 0 | 0 | 3.3E-18 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 4.5E-17 |

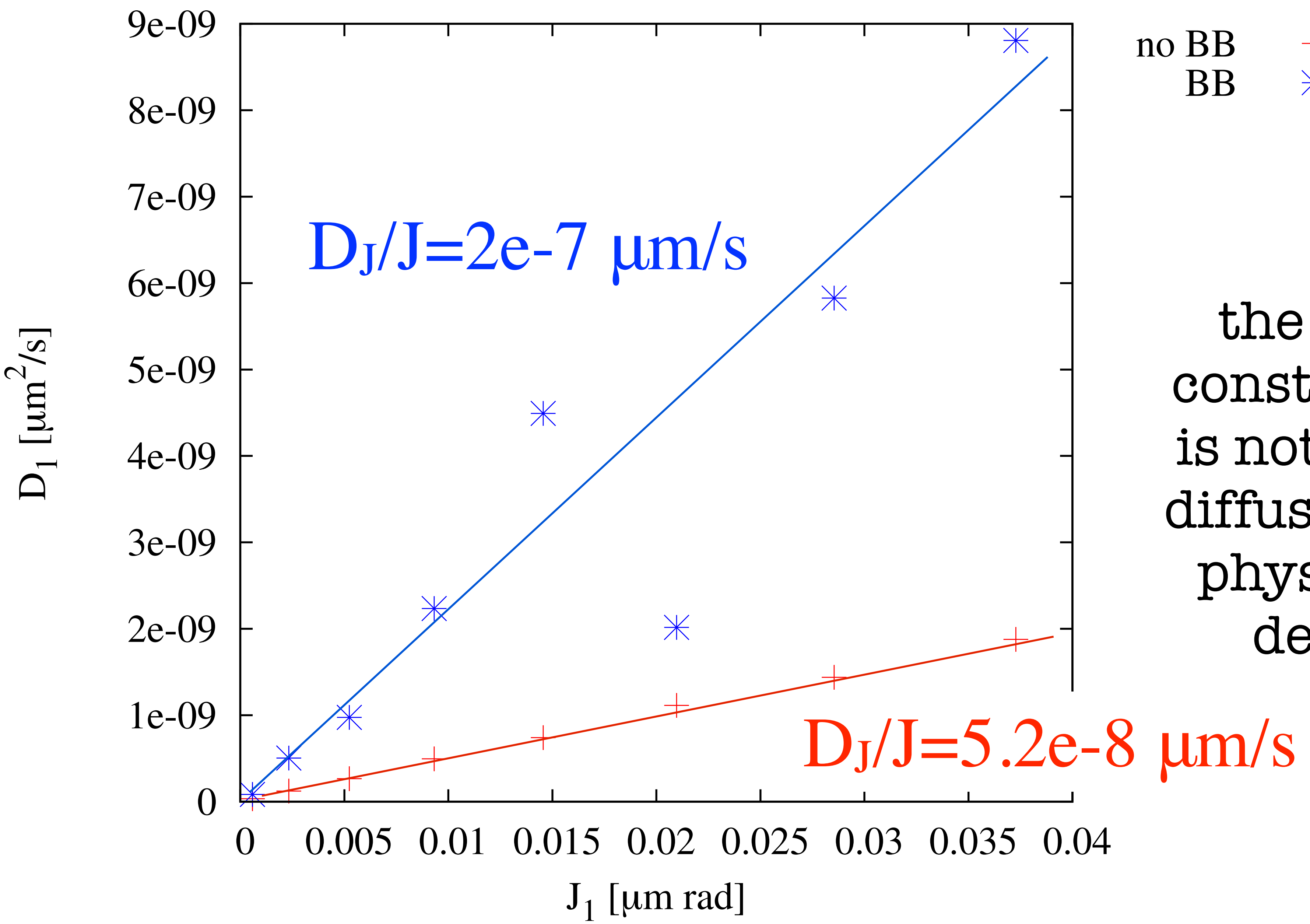



t [s]

$$D_{1,2} = \frac{\langle \Delta J_{1,2}^2 \rangle}{2\Delta t}$$

$A_1 = 5$ sigma

# D Vs J:

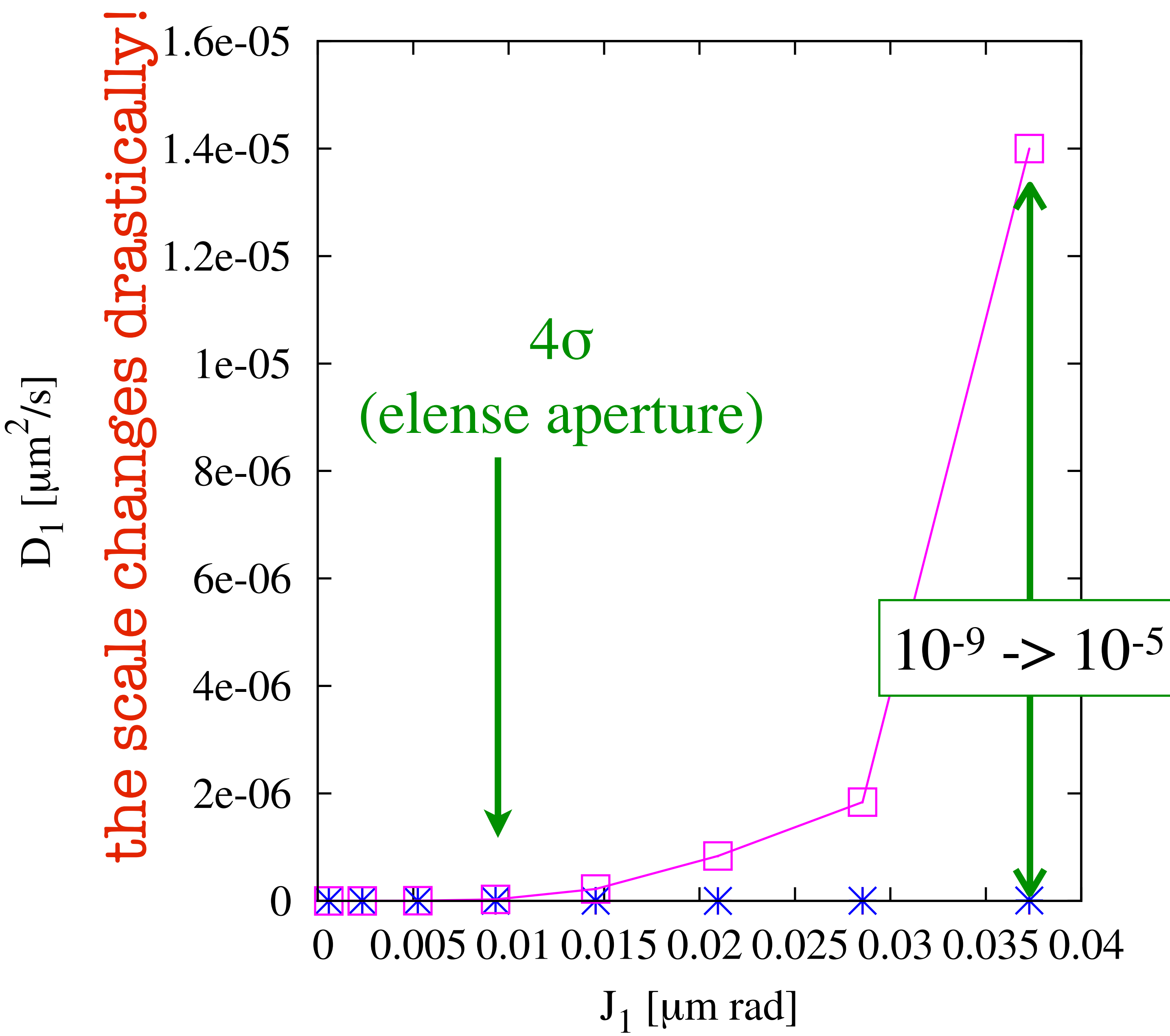



random noise

BB effect

the ratio $D_J/J$ is fairly constant when the elense is not there, meaning the diffusion coefficient in the physical space does not depend strongly on amplitudes.

# D Vs J:




random noise

BB effect

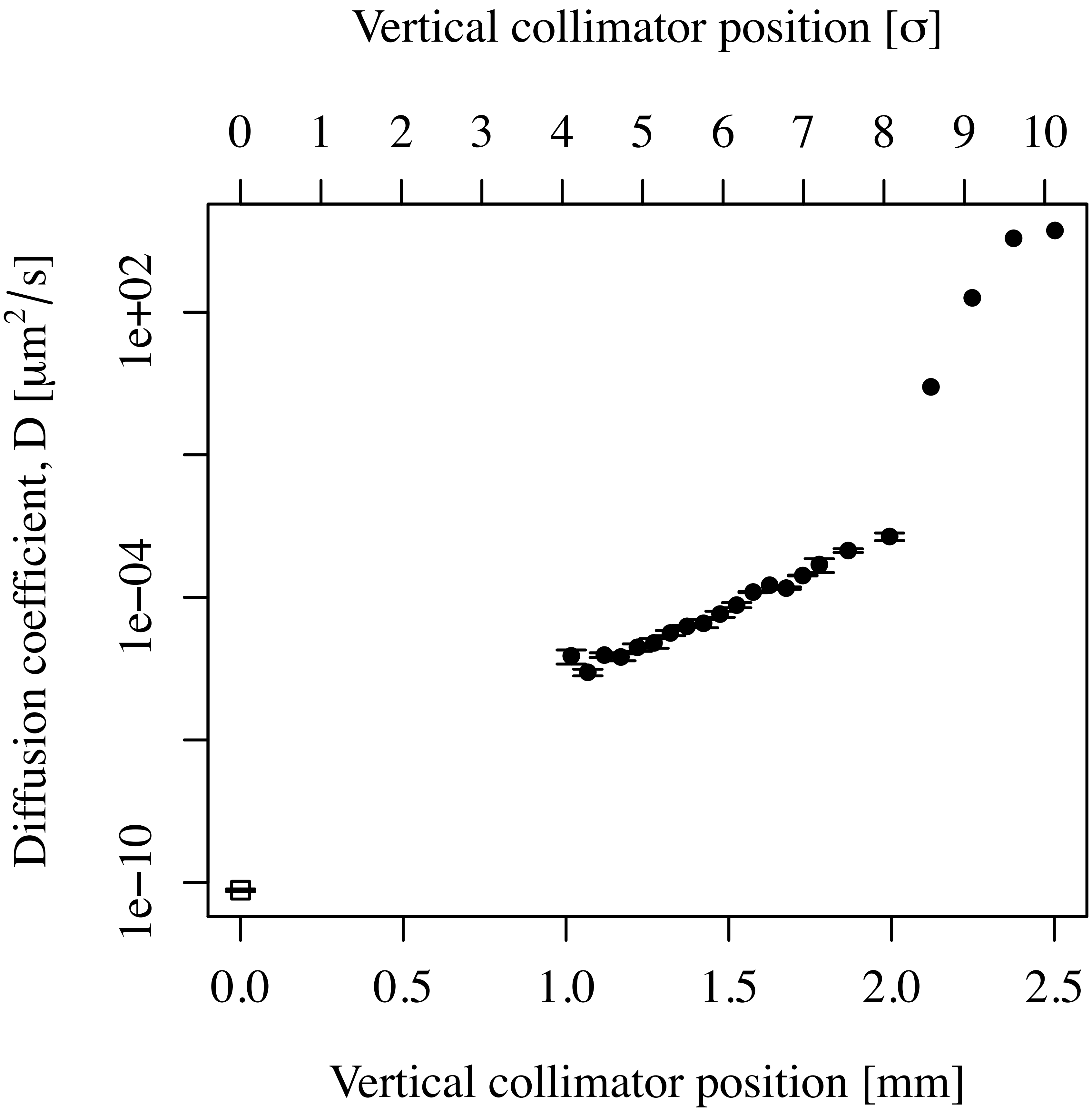
electron lens

the ratio $D_J/J$ is not constant anymore. Strong dependency on J.

- Up to 4 sigma the beam behaviour is unchanged

- Over 4 sigma the diffusion coefficient increases of up a factor 10K

# experimental conditions: BB but not elense




courtesy of G. Stancari

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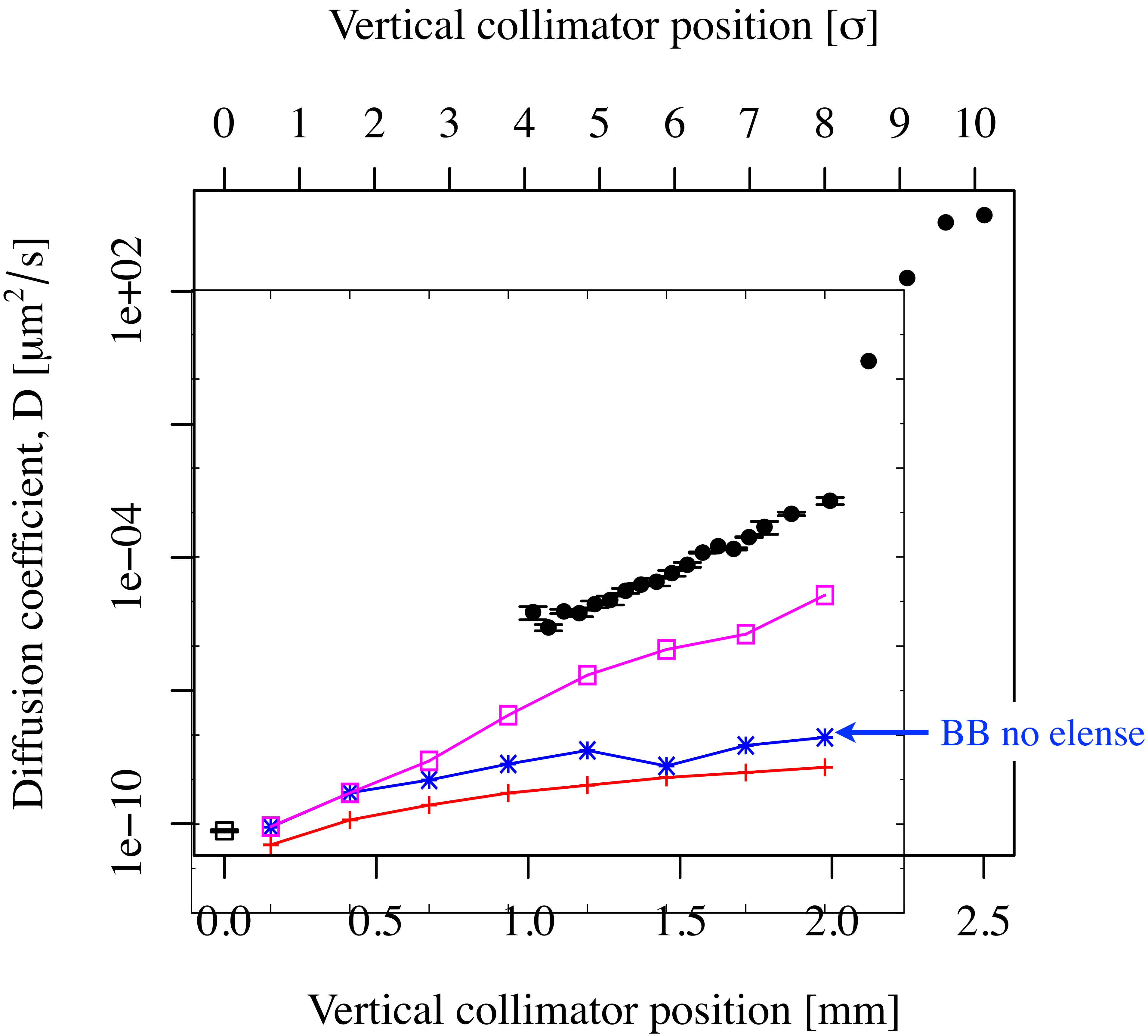


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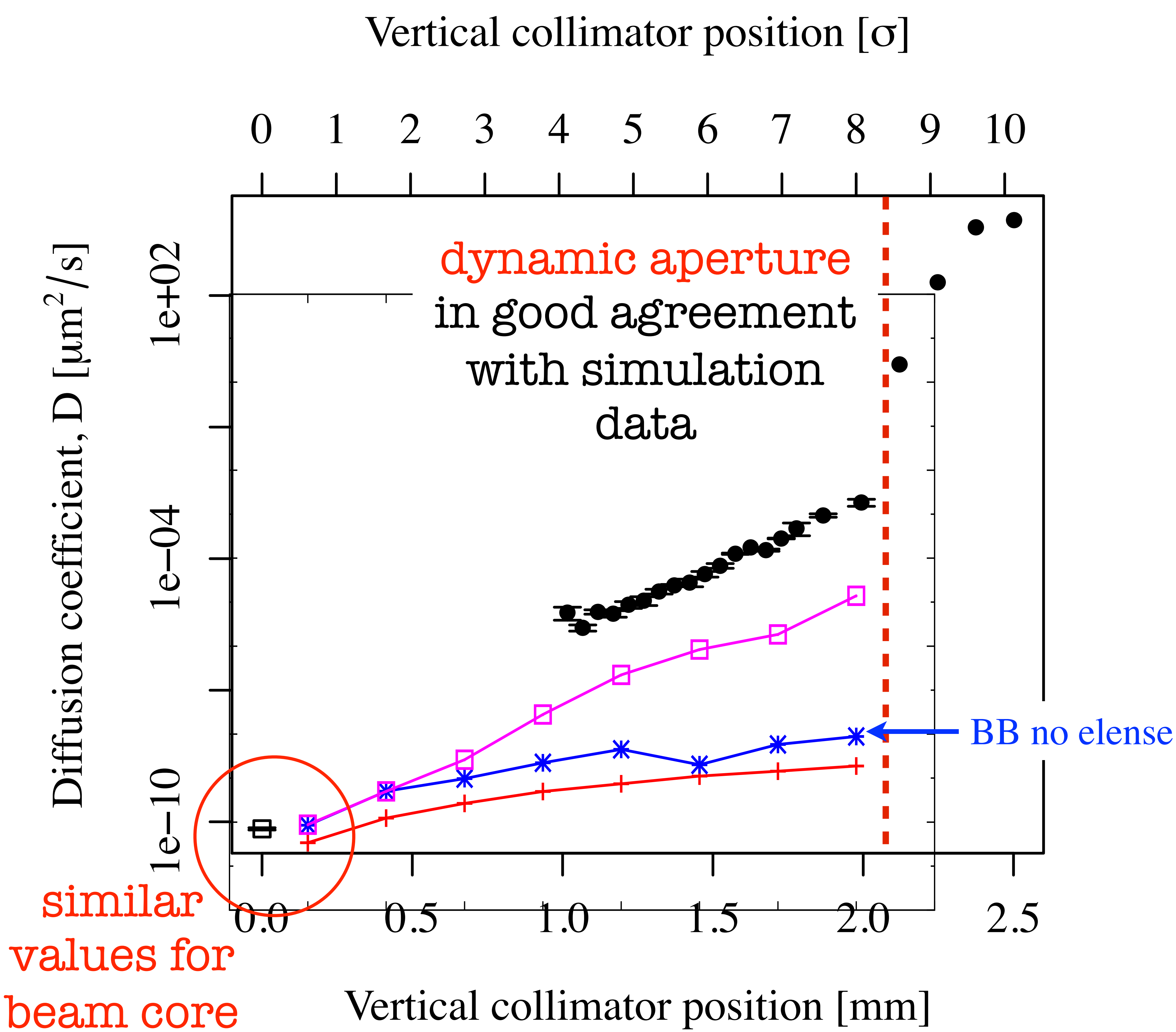
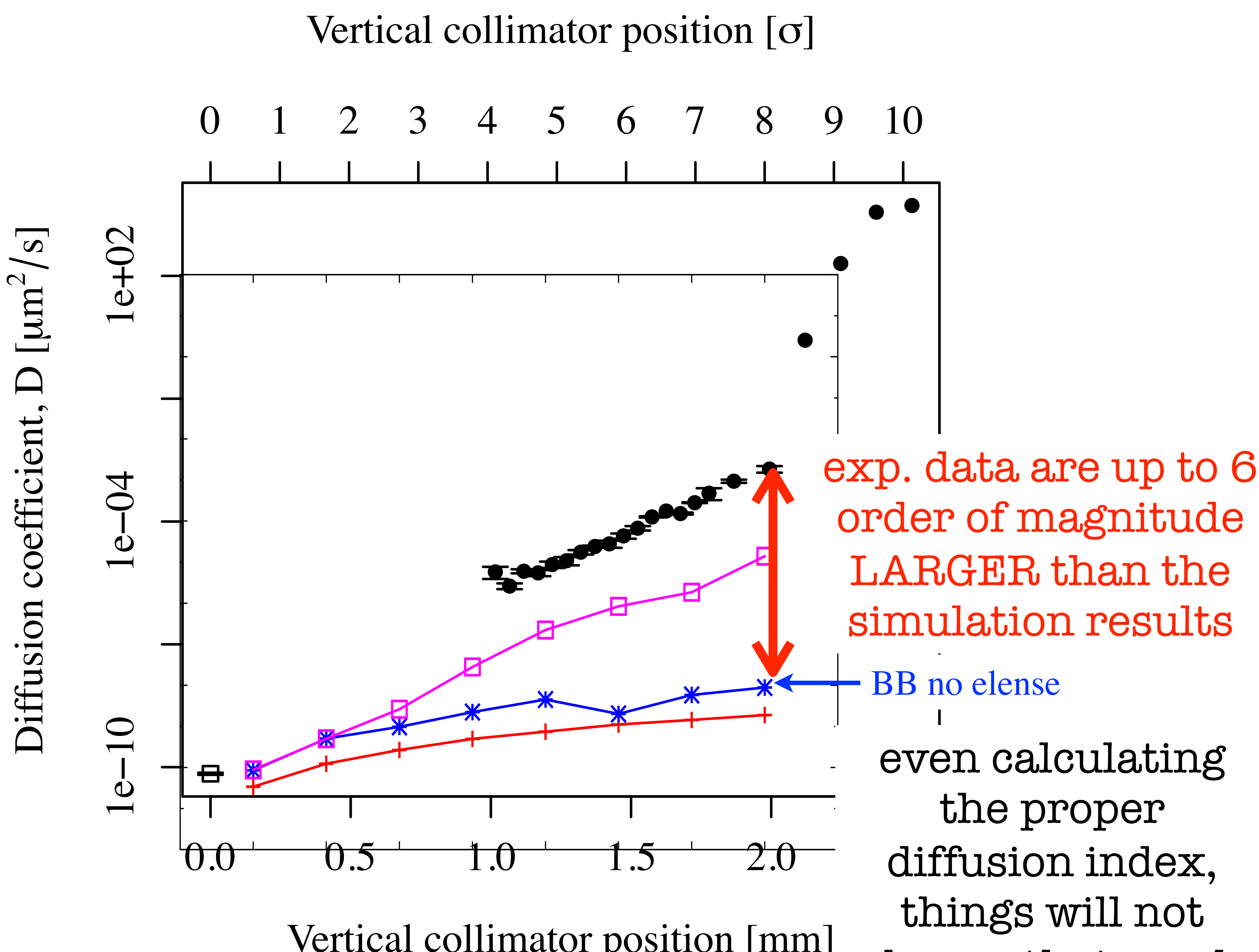



courtesy of G. Stancari

# experimental conditions: BB but not elense




even calculating the proper diffusion index, things will not change that much

courtesy of G. Stancari

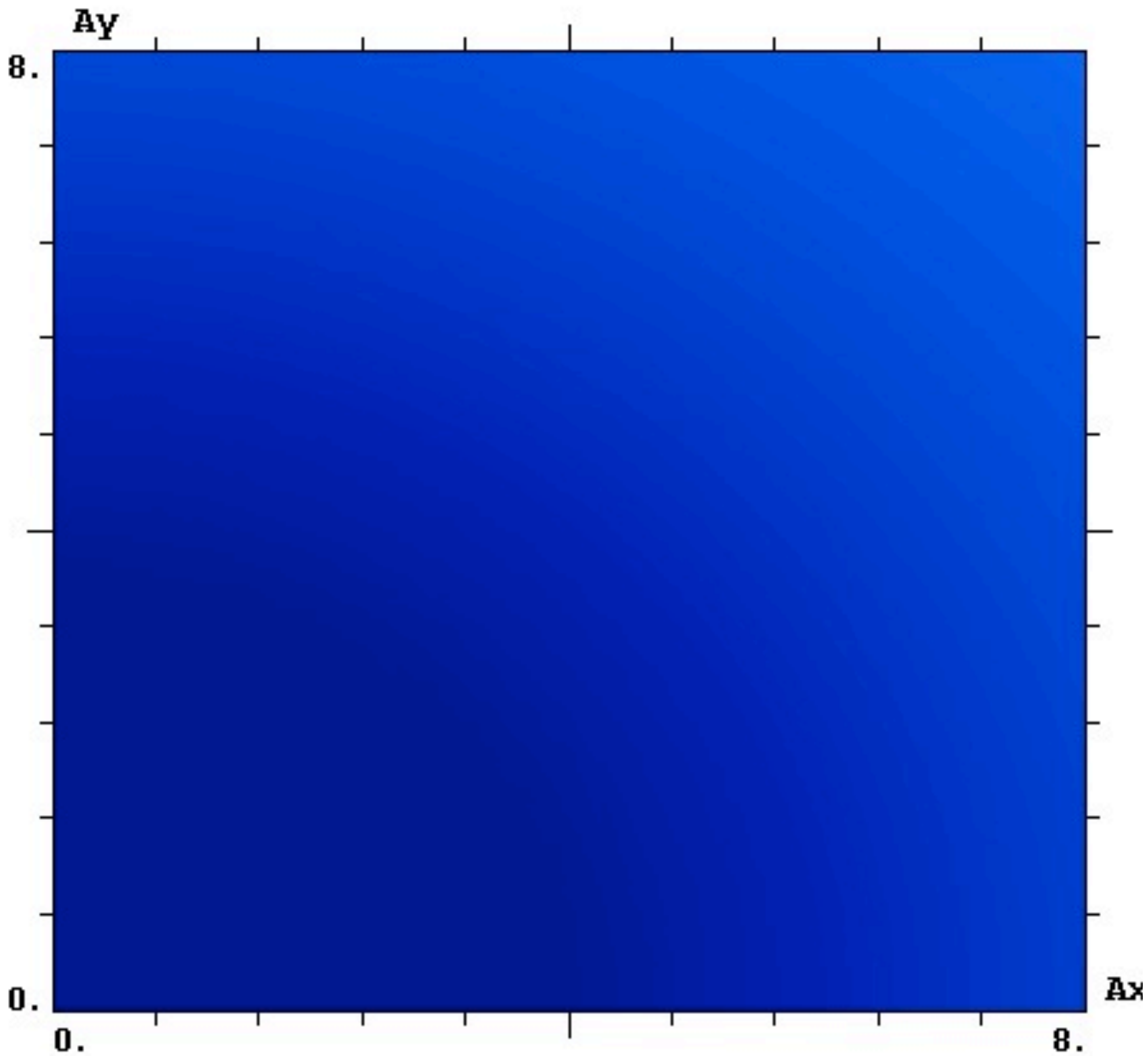
# Shortcut N.2: Fma Plots

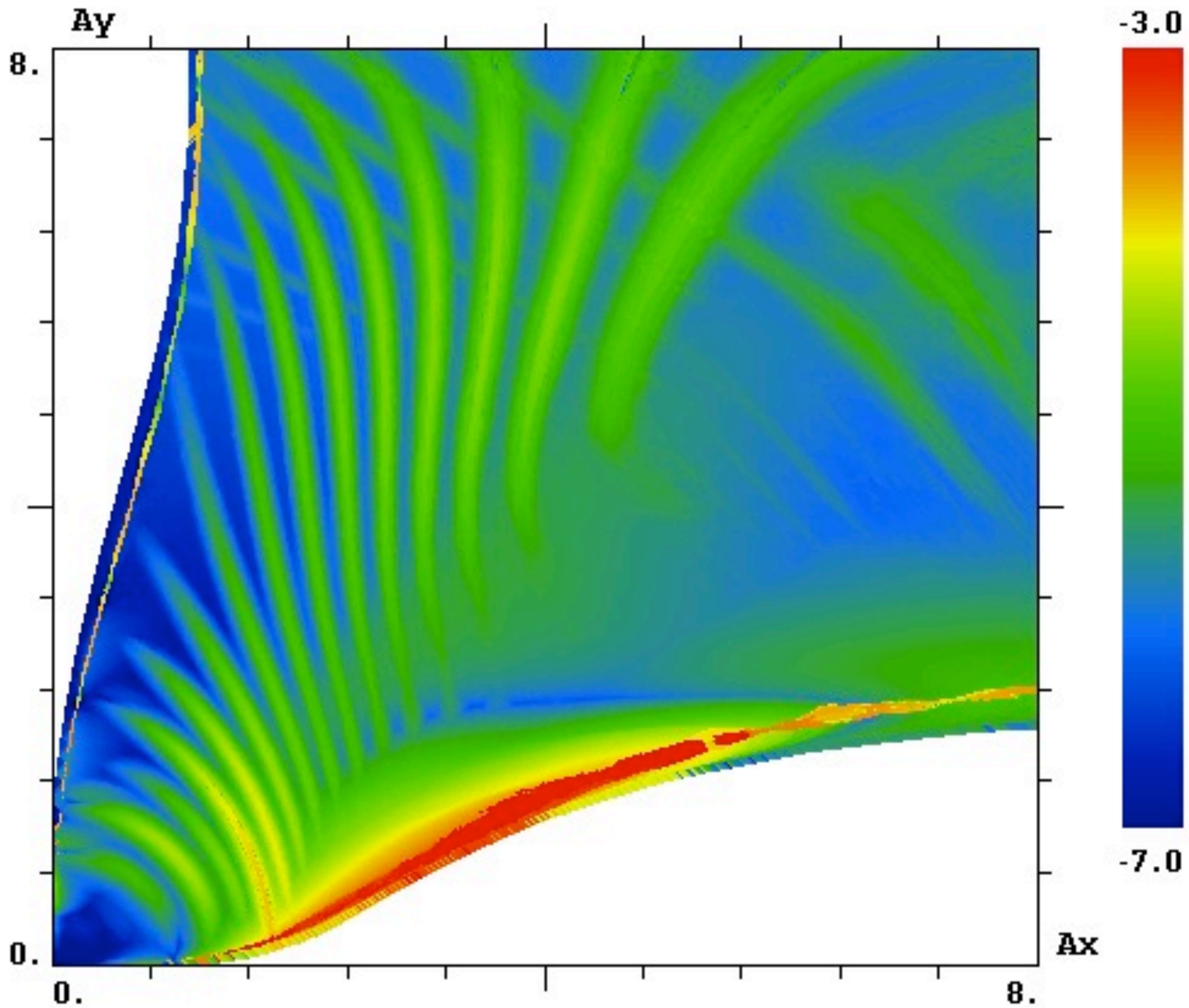
- FMA (frequency map analysis) measures the main betatron tune jitter in logarithmic scale (here presented in the average amplitude space)
- it shows the machine resonances
- the diffusion index i (tune jitter) is a qualitative measure of $D_J/J$- unfortunately we do not know exact formula to relate the two...
- it allows us to have a qualitative, but global picture of the change in diffusion

storage mode

storage mode

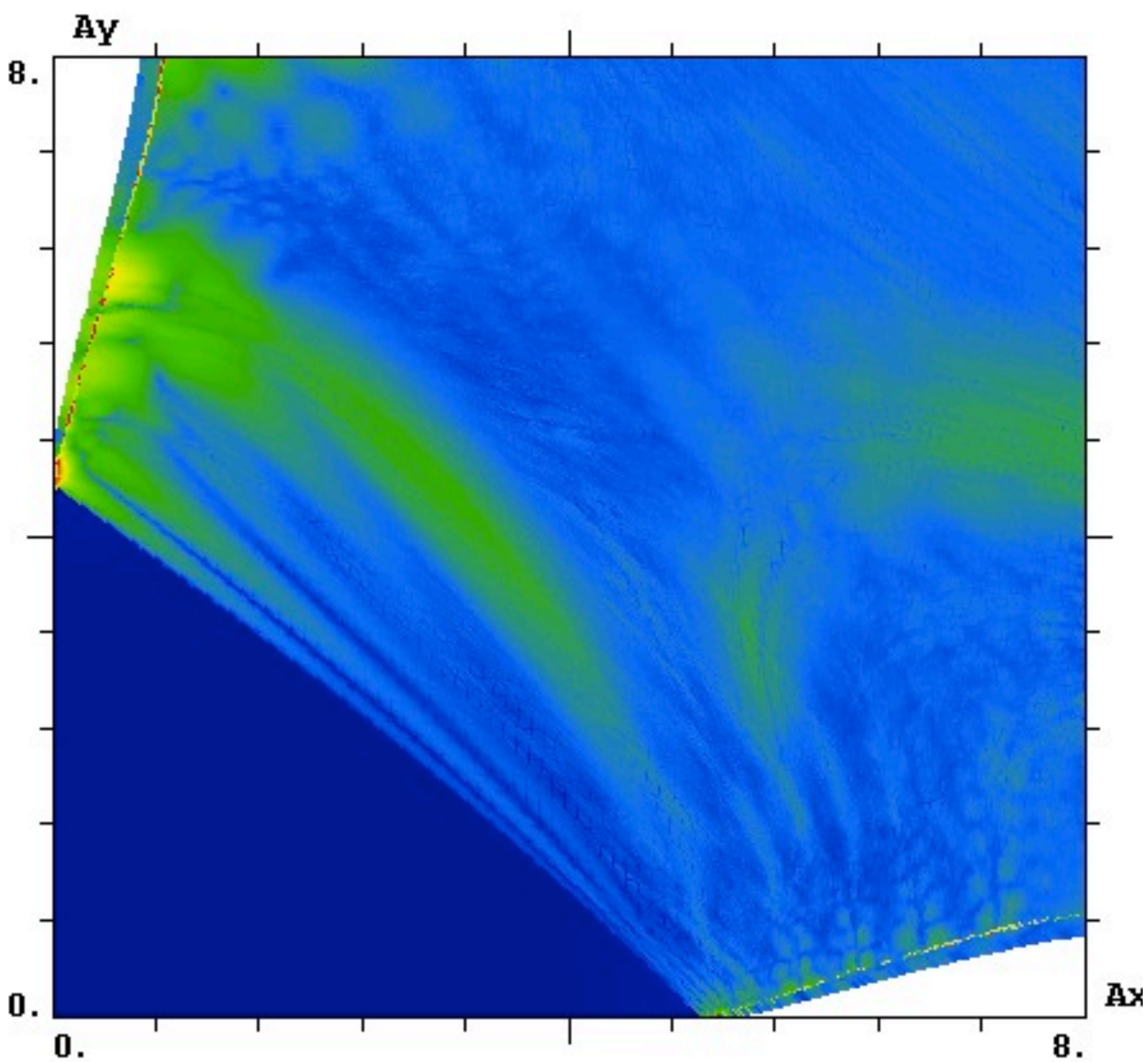
BB effect



storage mode

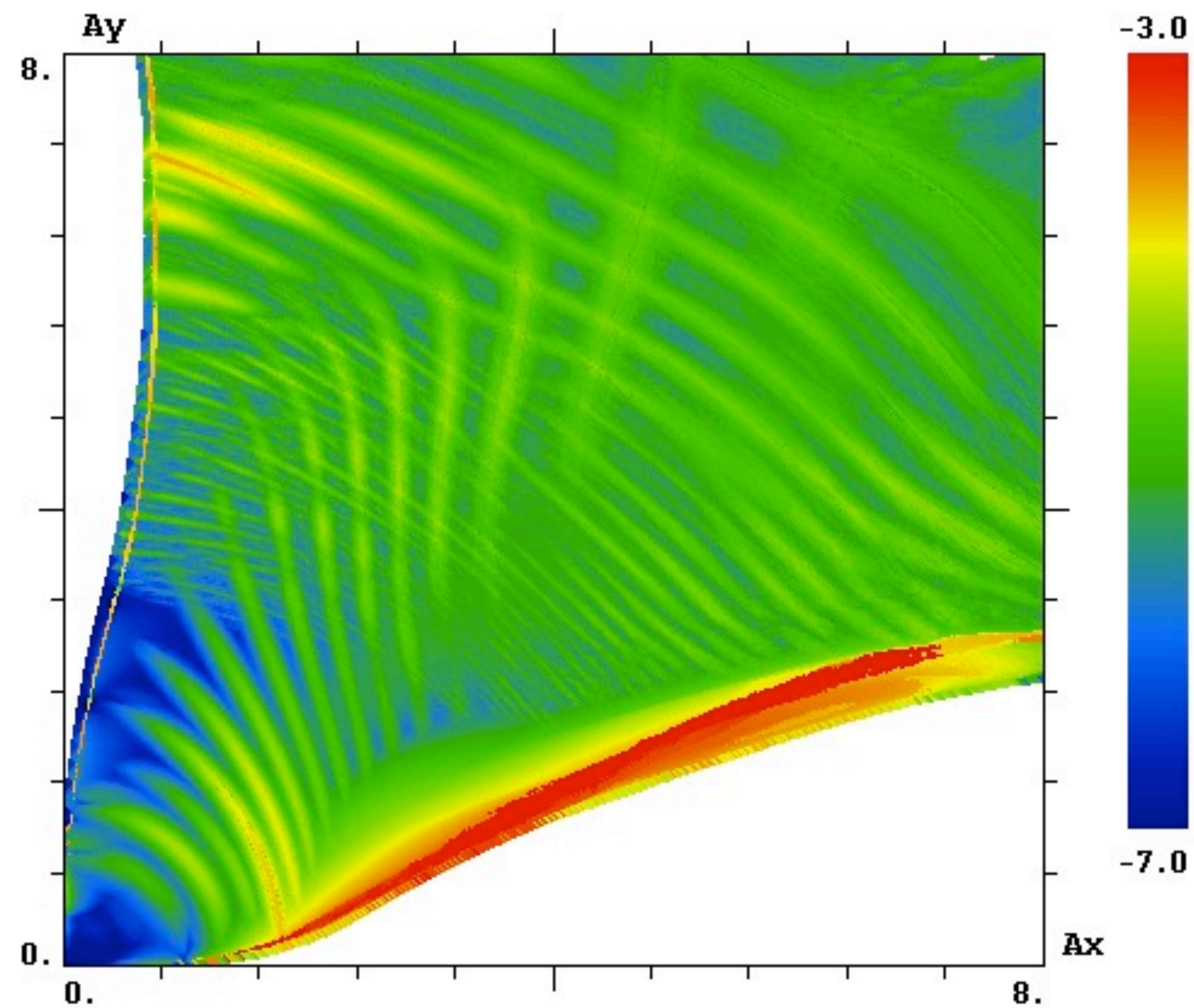
electron lens




storage mode

BB effect

electron lens



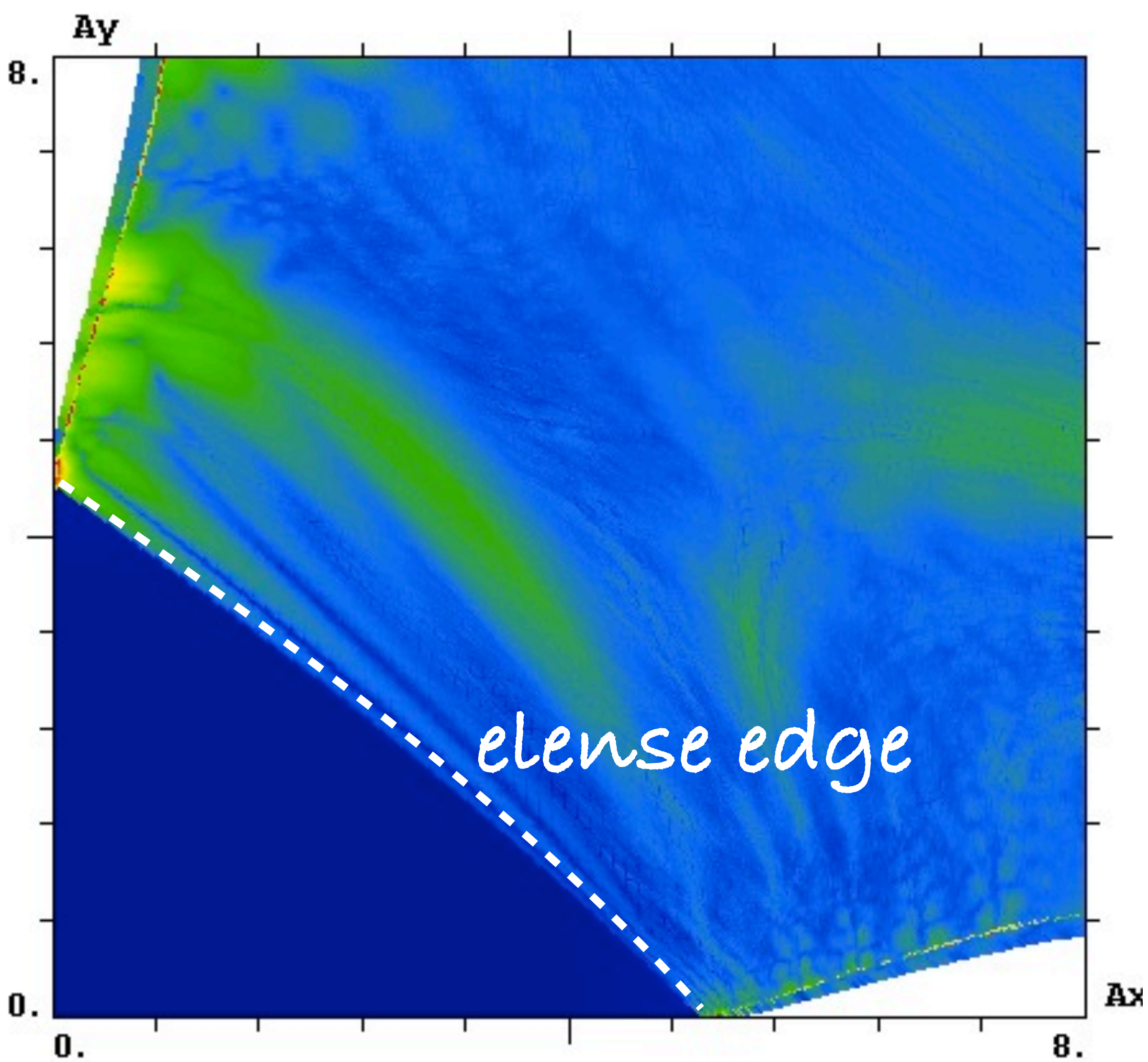
storage mode

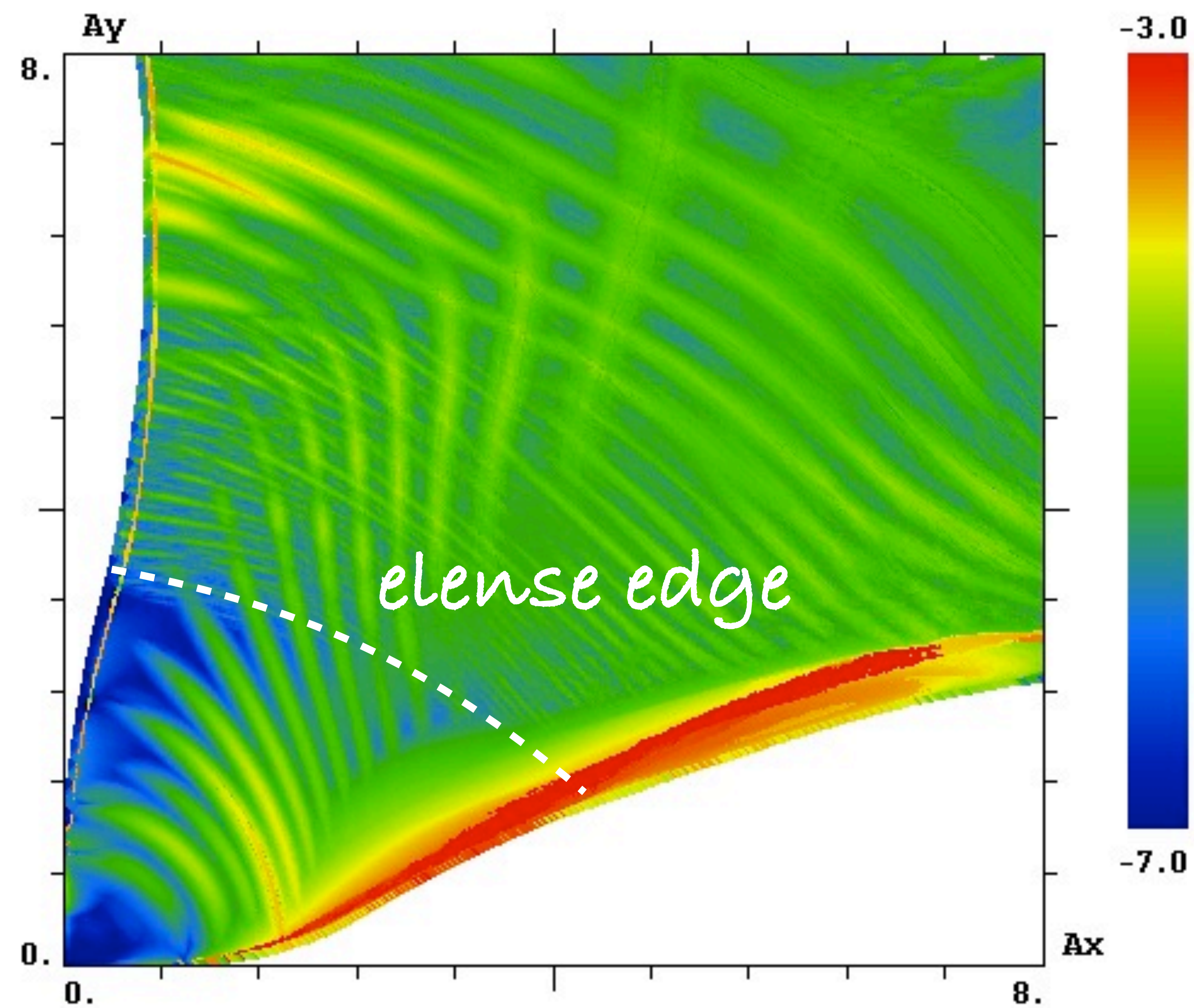
electron lens

storage mode

BB effect

electron lens




core is not touched

core is not touched

# A Different Approach

- Simulate the experiment directly!
- Does not provide insights

# Conclusions

- Diffusion coefficients have been calculated in the normalized planed for a sampling of the $<A_1>$,$<A_2>$ space
- As expected, in case of random noise, $D_J/J$ is constant, about 5e-8 μm/s
- With the BB case, a factor 2 to 10 in diffusion increase is estimated.
- Even with BB included, the diffusion coefficient n simulations is still many orders of magnitudes lower than the experimental one.
- Including a basic description of the electron lens, particles with amplitude larger than the inner electron lens radius diffuse with a much faster rate (10K x factor). The core is untouched.
- fma plots show a global picture of the diffusion index in the amplitude space, showing clearly the influence of elense.
- Different approaches are proposed.