Neutrinos and Flavor Symmetries

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Outline of the talk

Recent progress of Non-Abelian Discrete Symmetry for Lepton Flavors

1 Introduction
2 Flavor Symmetry: Neutrino Mixing and Masses
3 Flavor Symmetry: CP Violation
4 FLASY: Phenomenology
5 Summary and Prospect

1 Introduction

Experimental Data of
Neutrino Masses and Mixing Angles
give us big hints for

Flavor Symmetry

Before 2012 (no data for Θ_{13})

Neutrino Data suggested

Tri-bimaximal Mixing of Neutrinos

$$\sin^2\theta_{12} = 1/3,\ \sin^2\theta_{23} = 1/2,\ \sin^2\theta_{13} = 0,$$

$$U_{\text{tri-bimaximal}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

Harrison, Perkins, Scott (2002)

Conventional definition of Mixing Angles

$$U=\begin{bmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{bmatrix}\begin{bmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{-i\delta} & 0 & c_{13}\end{bmatrix}\begin{bmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$c_{ij}\equiv\cos\theta_{ij},\ s_{ij}\equiv\sin\theta_{ij}$$

Two Large Mixing Angles

$$\sin^2\theta_{12}\simeq\frac{1}{3}$$

$$\sin^2\theta_{23}\simeq\frac{1}{2}$$

Tri-bimaximal Mixing (TBM) is realized by

$$m_{TBM} = \frac{m_1+m_3}{2}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$$

in the diagonal basis of charged leptons.

Mixing angles are independent of neutrino masses.

Integer (inter-family related) matrix elements suggest Non-Abelian Discrete Flavor Symmetry.

Flavor (Family) Symmetry
FLASY

- **Abelian or Non-Abelian ?**

  Abelian : discriminate between families

  Non-Abelian : connect different families <u>2</u>, <u>3</u> …

- **Continuous or Discrete ?**

  Continuous : free rotation among families

  Discrete : definite meaning of families

Non-Abelian Discrete Symmetry is appropriate for lepton families in the standpoint of TBM !

Hint for the symmetry

$$m_{TBM} = \frac{m_1+m_3}{2}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

A_4 symmetric

Assign A_4 triplet $\underline{3}$ for $(\nu_e, \nu_\mu, \nu_\tau)_L$

E. Ma and G. RajaseKaran, PRD64(2001)113012

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{3} + \mathbf{3} + \mathbf{1} + \mathbf{1}' + \mathbf{1}''$$

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1} = a_1 b_1 + a_2 b_3 + a_3 b_2$$

The third matrix is A_4 symmetric !

The first and second matrices are Unit matrix and Democratic matrix, respectively, which could be derived from S_3 symmetry.

A lot of flavor models were proposed to realize the tri-bimaximal mixing.

The other mixing patterns were also discussed such as

Bi-maximal mixing

$$U_{\rm BM} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/2 & -1/2 & -1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix}$$

Barger, Pakvasa, Weiler, Whisnant, 1998

Golden-ratio

$$U_{\rm GR} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & -1/\sqrt{2} \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$\tan\theta_{12} = 1/\phi, \quad \phi = \frac{1+\sqrt{5}}{2}, \quad \theta_{12} = 31.7^\circ$$

Kajiyama, Raidal, Strumia, 2007
Everett, Stuart ,2009
Adulpravitchai, Blum, Rodejohann, 2009

In 2012

Reactor angle θ_{13} was measured by T2K, Daya Bay, MINOS, RENO, Double Chooz

$$\theta_{13} \simeq 9^\circ \simeq \theta_c/\sqrt{2}$$

Tri-bimaximal mixing was ruled out !

- Deviation from Tri-bimaximal mixing ?
- Different Anzatz ?
  Tri-maximal mixing, Tri-bimaximal Cabibbo ....

2 Flavor Symmetry

FLASY

Neutrino mixing and masses

Flavor Symmetry is still active for neutrino mixing with non-vanishing θ_{13}.

Many works !

Let us list Non-Abelian Discrete Group G

- Number of elements N_G is order of G, and finite
- Any two elements do not satisfy the commutativity

	N_G						
S_N	$N!$	:	S_3	S_4	……..		
A_N	$N!/2$	:	A_4	A_5	……..		
D_N	$2N$	:	D_4	D_5	D_6	D_{14}	………
Q_N	$2N$	:	Q_4	Q_6	……….		
T_N	$3N$	:	T_7	T_{13}	……….		
$\Sigma(2N^2)$		:	$\Sigma(18)$	……..			
$\Delta(3N^2)$		:	$\Delta(27)$	$\Delta(48)$	…….		
$\Sigma(3N^3)$		:	$\Sigma(81)$	……..			
$\Delta(6N^2)$		:	$\Delta(54)$	$\Delta(96)$	…….		

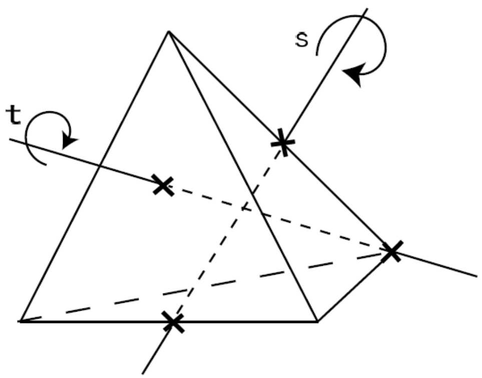
T' : Double covering group A_4

Ishimori, Kobayashi, Ohki, Shimizu, Okada, M.T
PTP supprem–ent, 183, 2010, arXiv1003.3552
Lect. Notes Physics (Springer) 858, 2012

A simple example: A_4 group

Even permutation group of four objects (1234)
12 elements (order 12) can be generated by S and T:
$S^2=T^3=(ST)^3=1$: S=(14)(23), T=(123)

C1: 1 h=1
C3: S, T^2ST, TST^2 h=2
C4: T, ST, TS, STS h=3
C4': T^2, ST^2, T^2S, ST^2S h=3



Symmetry of tetrahedron

Irreducible representations: 1, 1', 1", 3
$1^2+1^2+1^2+3^2=$ 12 elements

A_4 has subgroups:
three Z_2, four Z_3, one $Z_2 \times Z_2$ (klein four-group)

Z_2: {1,S}, {1,T^2ST}, {1,TST^2}
Z_3: {1,T,T^2}, {1,ST,T^2S}, {1,TS, ST^2}, {1,STS,ST^2S}
K_4: {1,S,T^2ST,TST^2}
Suppose A_4 is spontaneously broken to one of subgroups:
Neutrino sector preserves Z_2: {1,S}
Charged lepton sector preserves Z_3: {1,T,T^2}

$$S^T m_{LL}^{\nu} S = m_{LL}^{\nu}, \quad T^{\dagger} Y_e Y_e^{\dagger} T = Y_e Y_e^{\dagger}$$

$$\Downarrow$$

$$[S, m_{LL}^{\nu}] = 0, \quad [T, Y_e Y_e^{\dagger}] = 0$$

Mixing matrices diagonalise m_{LL}^{ν}, $Y_e Y_e^{\dagger}$ also diagonalize S and T, respectively !

For the triplet, the representations are given as

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}; \quad \omega = e^{2\pi i/3}$$

$$V_\nu^T S V_\nu = \mathrm{diag}(-1, 1, -1)$$

$$V_\nu = \begin{pmatrix} 2/\sqrt{6} & /1\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Independent of mass eigenvalues.

Freedom of the rotation between 1st and 3rd column.

$$V_\nu = \begin{pmatrix} 2c/\sqrt{6} & 1/\sqrt{3} & 2s/\sqrt{6} \\ -c/\sqrt{6}+s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}-c/\sqrt{2} \\ -c/\sqrt{6}+s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}+c/\sqrt{2} \end{pmatrix}$$

$$c=\cos\theta,\ \ s=\sin\theta$$

Tri-maximal mixing : TM2

θ is fixed by the experimental data.

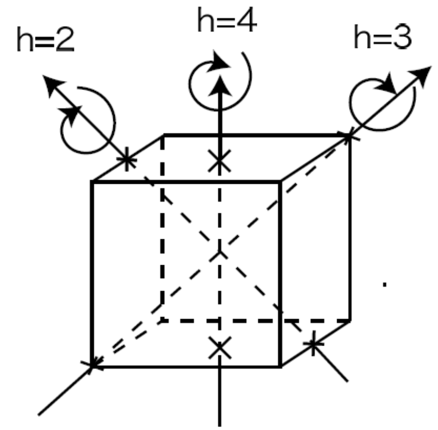
S_4 group:

All permutations among four objects, 4 ! =24 elements

24 elements can be generated by S,T and U:

$S^2=T^3=U^2=1, \quad ST^3=(SU)^2=(TU)^2=(STU)^4=1$

Irreducible representations: 1, 1', 2, 3, 3'



Symmetry of a cube

$$U = \mp \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}; \quad \omega = e^{2\pi i/3}$$

S_4 has subgroups:

nine Z_2, four Z_3, three Z_4, four $Z_2 \times Z_2$ (K_4)

Suppose S_4 is spontaneously broken to one of subgroups:

Neutrino sector preserves S and U (K_4)

Charged lepton sector preserves T (Z_3)

$$S^T m^\nu_{LL} S = m^\nu_{LL}, \quad U^T m^\nu_{LL} U = m^\nu_{LL}, \quad T^\dagger Y_e Y_e^\dagger T = Y_e Y_e^\dagger$$

$$\Downarrow$$

$$[S, m^\nu_{LL}] = 0, \quad [U, m^\nu_{LL}] = 0, \quad [T, Y_e Y_e^\dagger] = 0$$

$$V_\nu = \begin{pmatrix} 2/\sqrt{6} & /1\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Tri-bimaximal mixing

C.S.Lam, PRD98(2008)
arXiv:0809.1185

So called "Direct approach of Flavor Symmetry"

Suppose S_4 is spontaneously broken to another subgroups:

Neutrino sector preserves SU (Z_2)

Charged lepton sector preserves T (Z_3)

$$(SU)^T m_{LL}^{\nu} SU = m_{LL}^{\nu}, \quad T^\dagger Y_e Y_e^\dagger T = Y_e Y_e^\dagger$$

$$\Downarrow$$

$$[SU, m_{LL}^{\nu}] = 0, \quad [T, Y_e Y_e^\dagger] = 0$$

Tri-maximal mixing TM1

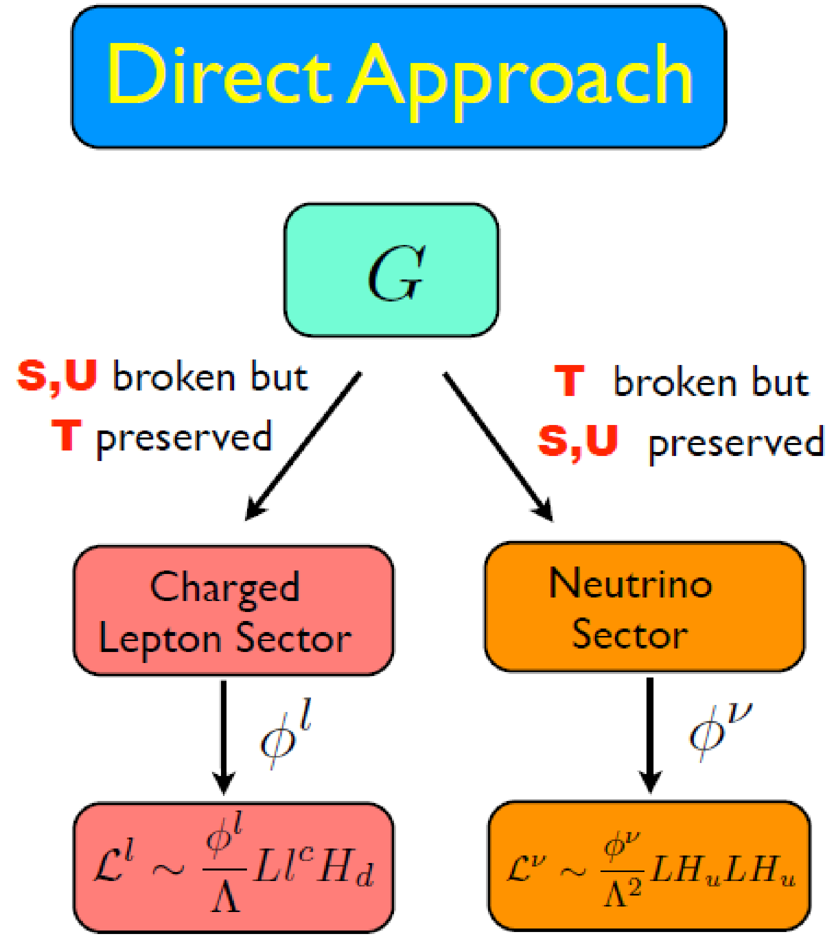
$$V_\nu = \begin{pmatrix} 2/\sqrt{6} & c/\sqrt{3} & s/\sqrt{3} \\ -1/\sqrt{6} & c/\sqrt{3} - s/\sqrt{2} & -s/\sqrt{3} - c/\sqrt{2} \\ -1/\sqrt{6} & c/\sqrt{3} + s/\sqrt{2} & -s/\sqrt{3} + c/\sqrt{2} \end{pmatrix}$$

$c = \cos\theta, \ s = \sin\theta$

Different predictions in mixing angles between TM1 and TM2.

Direct approach of FLASY

Different subgroups of G are preserved in the neutrino sector and charged lepton sector, respectively.



S.F.King

arXiv: 1402.4271 King, Merle, Morisi, Simizu, M.T

Indirect Approach

Flavor symmetry G is completely broken in T,S,U by flavon (SU_2 singlet scalors) VEV's.
Flavor symmetry controls couplings among leptons and flavons with special vacuum alignments.

A_4 model

	Leptons	flavons
A_4 triplets	(L_e, L_μ, L_τ)	$\phi_\nu(\phi_{\nu 1}, \phi_{\nu 2}, \phi_{\nu 3})$
A_4 singlets	$e_R : \mathbf{1}$ $\mu_R : \mathbf{1}''$ $\tau_R : \mathbf{1}'$	$\xi : \mathbf{1}$

$\mathbf{3}_L \times \mathbf{3}_L \times \mathbf{3}_{flavon} \rightarrow \mathbf{1}$, $\mathbf{3}_L \times \mathbf{3}_L \times \mathbf{1}_{flavon} \rightarrow \mathbf{1}$

G. Altarelli, F. Feruglio, Nucl.Phys. B720 (2005) 64

$$m_{\nu LL} \sim y_1 \begin{pmatrix} 2\langle\phi_{\nu1}\rangle & -\langle\phi_{\nu3}\rangle & -\langle\phi_{\nu2}\rangle \\ -\langle\phi_{\nu3}\rangle & 2\langle\phi_{\nu2}\rangle & -\langle\phi_{\nu1}\rangle \\ -\langle\phi_{\nu2}\rangle & -\langle\phi_{\nu1}\rangle & 2\langle\phi_{\nu3}\rangle \end{pmatrix} + y_2\langle\xi\rangle \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

assume Vacuum Alingnment $\langle\phi_{\nu1}\rangle = \langle\phi_{\nu2}\rangle = \langle\phi_{\nu3}\rangle$

$$m_{\nu LL} = 3a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - a \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Tri-bimaximal mixing

$$m_\nu = 3a+b,\ b,\ 3a-b \ \Rightarrow m_{\nu_1} - m_{\nu_3} = 2m_{\nu_2}$$

Neutrino mass sum rule

A_4 Model to realize large θ_{13}

Modify G. Altarelli, F. Feruglio, Nucl.Phys. B720 (2005) 64

	(l_e, l_μ, l_τ)	e^c	μ^c	τ^c	$h_{u,d}$	ϕ_l	ϕ_ν	ξ	ξ'
$SU(2)$	2	1	1	1	2	1	1	1	1
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}'$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω

Y. Simizu, M. Tanimoto, A. Watanabe, PTP 126, 81(2011)

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1} \quad = a_1 * b_1 + a_2 * b_3 + a_3 * b_2$$
$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1}' \quad = a_1 * b_2 + a_2 * b_1 + a_3 * b_3$$
$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1}'' \quad = a_1 * b_3 + a_2 * b_2 + a_3 * b_1$$

ξ $\qquad\qquad$ ξ'

$$\mathbf{1} \times \mathbf{1} \Rightarrow \mathbf{1} \quad , \qquad \mathbf{1}'' \times \mathbf{1}' \Rightarrow \mathbf{1}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Additional Matrix

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + d\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

$$a = \frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{\Lambda}, \qquad b = -\frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{3\Lambda}, \qquad c = \frac{y^\nu_{\xi}\alpha_\xi v_u^2}{\Lambda}, \qquad d = \frac{y^\nu_{\xi'}\alpha_{\xi'} v_u^2}{\Lambda}$$

$$a = -3b$$

Both normal and inverted mass hierarchies are possible.

$$M_\nu = V_{\text{tri-bi}}\begin{pmatrix} a+c-\frac{d}{2} & 0 & \frac{\sqrt{3}}{2}d \\ 0 & a+3b+c+d & 0 \\ \frac{\sqrt{3}}{2}d & 0 & a-c+\frac{d}{2}\end{pmatrix} V^T_{\text{tri-bi}}$$

Tri-maximal mixing: TM2

3 Flavor Symmetry : CP Violation

Possibility of predicting CP phase δ in FLASY

Geometrical CP violation

A hint : under μ - τ symmetry $|U_{\mu i}| = |U_{\tau i}| \ i = 1, 2, 3$

$$\cos\theta_{23} = \sin\theta_{23} = \frac{1}{\sqrt{2}}$$

$$\sin\theta_{13}\cos\delta = 0$$

$\delta = \pm\frac{\pi}{2}$ is predicted since we know $\theta_{13} \neq 0$

Ferreira, Grimus, Lavoura, Ludl, JHEP2012,arXiv: 1206.7072

☆CP is conserved in HE theory before FLASY is broken.
☆CP is a dicrete symmetry.

Branco, Felipe, Joaquim, Rev. Mod. Physics 84(2012), arXiv: 1111.5332
Mohapatra, Nishi, PRD86, arXiv: 1208.2875
Holthhausen, Lindner, Schmidt, JHEP1304(2012), arXiv:1211.6953
Feruglio, Hagedorn, Ziegler, JHEP 1307, arXiv:1211.5560,
Eur.Phys.J.C74(2014), arXiv 1303.7178
E. Ma, PLB 723(2013), arXiv:1304.1603
Ding, King, Luhn, Stuart, JHEP1305, arXiv:1303.6180
Ding, King, Stuart, JHEP1312, arXiv:1307.4212,
Ding, King, 1403.5846
Meroni, Petcov, Spinrath, PRD86, 1205.5241
Girardi, Meroni, Petcov, Spinrath, JHEP1042(2014), arXiv:1312.1966
Li, Ding, Nucl. Phys. B881(2014), arXiv:1312.4401
Ding, Zhou, arXiv:1312.522

Generalized CP Symmetry

$$\varphi(x) \xrightarrow{\text{CP}} X_{\mathbf{r}}\varphi^*(x'), \quad x' = (t, -\mathbf{x})$$

X_r must be consistent with FLASY.

$$\varphi(x) \xrightarrow{\text{g}} \rho_{\mathbf{r}}(g)\varphi\ (x), \quad g \in G_f$$

$\rho_{\mathbf{r}}(g)$ **:Representaion matrix for g in the irreducible representation r.**

$$\begin{tikzcd}
\varphi(x) \arrow[r, "CP"] \arrow[d, "G_f"'] & X_{\mathbf{r}}\varphi^*(x') \arrow[d, "G_f"] \\
\rho_{\mathbf{r}}(g')\varphi(x) = X_{\mathbf{r}}\rho_{\mathbf{r}}^*(g)X_{\mathbf{r}}^{-1}\varphi(x) & X_{\mathbf{r}}\rho_{\mathbf{r}}^*(g)\varphi^*(x') \arrow[l, "CP^{-1}"]
\end{tikzcd}$$

Holthhausen, Lindner, Schmidt,
JHEP1304(2012), arXiv:1211.6953

Consistency condition

$$\boxed{X_{\mathbf{r}}\ \rho_{\mathbf{r}}^*(g)\ X_{\mathbf{r}}^{-1} = \rho_{\mathbf{r}}(g'), \quad g, g' \in G_f}$$

Suppose a symmetry including FLASY and CP symmetry:

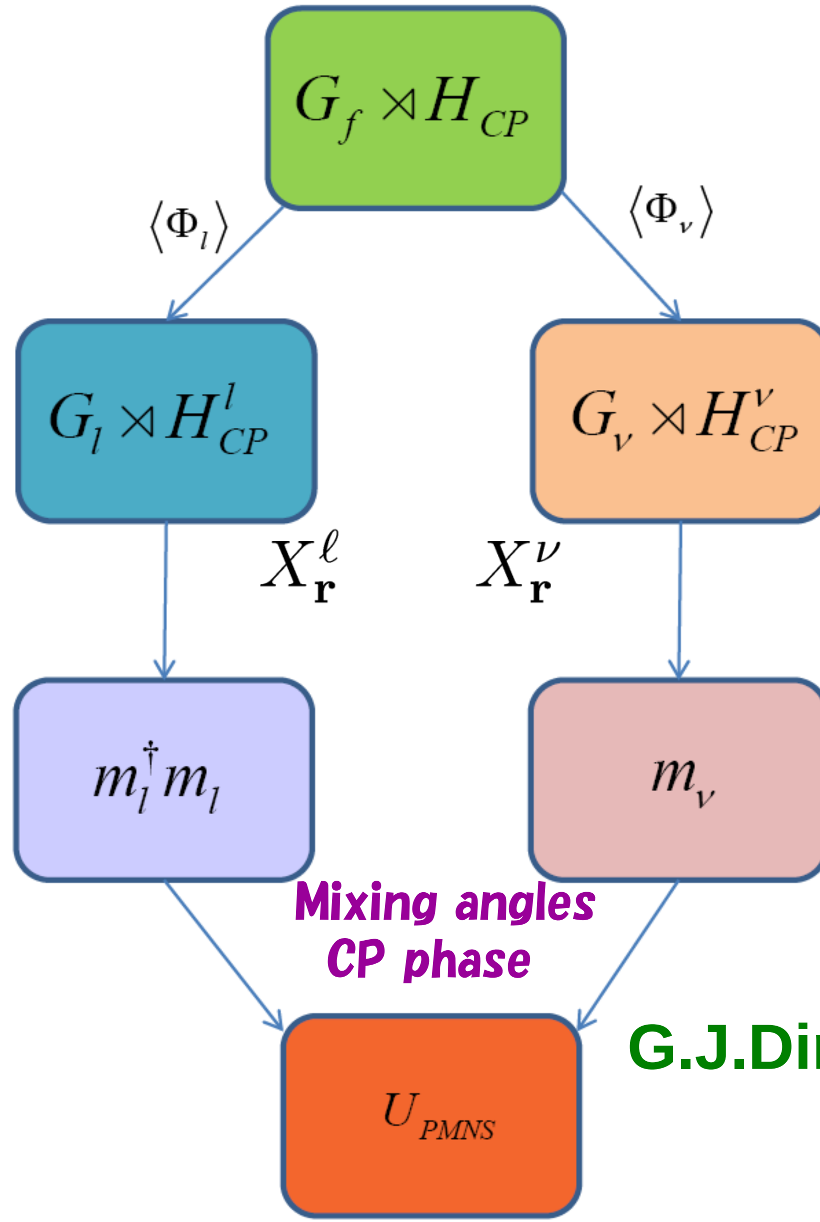
$$G_{CP} = G_f \rtimes H_{CP}$$

is broken to the subgroups in neutrino sector and charged lepton sector.

CP symmetry gives

$$X_{\mathbf{r}}^{\nu T} m_{\nu LL} X_{\mathbf{r}}^{\nu} = m_{\nu LL}^{*}$$

$$X_{\mathbf{r}}^{\ell \dagger} (m_{\ell}^{\dagger} m_{\ell}) X_{\mathbf{r}}^{\ell} = (m_{\ell}^{\dagger} m_{\ell})^{*}$$



G.J.Ding

An example of S_4 model

Ding, King, Luhn, Stuart, JHEP1305, arXiv:1303.6180

$G_\nu=\{1,S\}$ and $X_3^{\nu}=\{U, SU\}$, $X_3^{l}=\{1\}$

satisfy the consistency condition

$$X_{\mathbf{r}}\ \rho^*_{\mathbf{r}}(g)\ X^{-1}_{\mathbf{r}} = \rho_{\mathbf{r}}(g'), \quad g, g' \in G_f$$

$$m_{\nu LL} = \alpha \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \beta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + \epsilon \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

respects $G_\nu=\{1,S\}$

CP symmetry $X^{\nu T}_{\mathbf{r}} m_{\nu LL} X^{\nu}_{\mathbf{r}} = m^*_{\nu LL}$

$\Downarrow$

α, β, γ are real, ε is imaginary.

$$V_\nu = \begin{pmatrix} 2c/\sqrt{6} & 1/\sqrt{3} & 2s/\sqrt{6} \\ -c/\sqrt{6}+is/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}-ic/\sqrt{2} \\ -c/\sqrt{6}+is/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}+ic/\sqrt{2} \end{pmatrix}$$

$c = \cos\theta, \ \ s = \sin\theta$

$$\sin^2\theta_{13} = \tfrac{2}{3}\sin^2\theta, \ \ \sin^2\theta_{12} = \tfrac{1}{2+\cos 2\theta}, \ \ \sin^2\theta_{23} = \tfrac{1}{2}$$

$$|\sin\delta_{CP}| = 1, \quad \sin\alpha_{21} = \sin\alpha_{31} = 0$$

$$\delta_{CP} = \pm\pi/2$$

The predicton of CP phase depends on
the respected Generators of FLASY and CP symmetry.
Typically, it is simple value, 0, π, ±π/2 .

T' , Δ(6N²)

4 FLASY: phenomenology

Mixing sum rules

TM2
A_4, S_4

$$V_\nu = \begin{pmatrix} 2c/\sqrt{6} & 1/\sqrt{3} & 2s/\sqrt{6} \\ -c/\sqrt{6}+s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}-c/\sqrt{2} \\ -c/\sqrt{6}+s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}+c/\sqrt{2} \end{pmatrix}$$

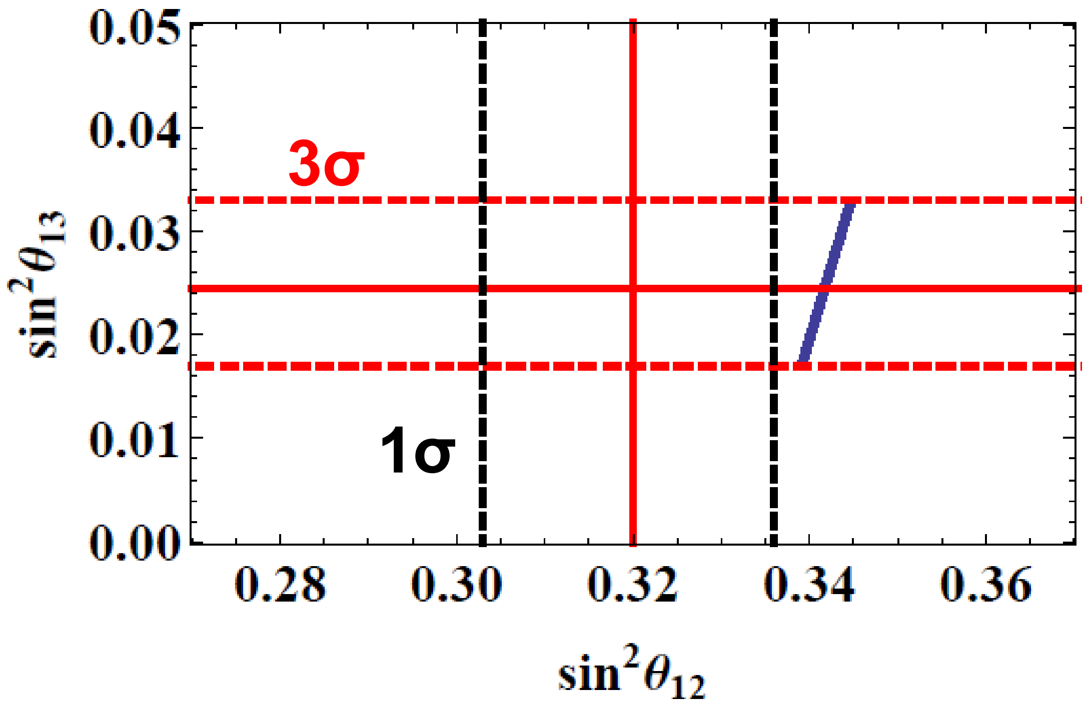
TM1
S_4

$$V_\nu = \begin{pmatrix} 2/\sqrt{6} & c/\sqrt{3} & s/\sqrt{3} \\ -1/\sqrt{6} & c/\sqrt{3}-s/\sqrt{2} & -s/\sqrt{3}-c/\sqrt{2} \\ -1/\sqrt{6} & c/\sqrt{3}+s/\sqrt{2} & -s/\sqrt{3}+c/\sqrt{2} \end{pmatrix}$$

$$c = \cos\theta, \;\; s = \sin\theta$$

TM2

$$V_\nu = \begin{pmatrix} 2c/\sqrt{6} & 1/\sqrt{3} & 2s/\sqrt{6} \\ -c/\sqrt{6}+s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}-c/\sqrt{2} \\ -c/\sqrt{6}+s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}+c/\sqrt{2} \end{pmatrix}$$



$$\sin^2\theta_{13} = \tfrac{2}{3}\sin^2\theta$$

$$\sin^2\theta_{12} = \frac{1}{2+\cos^2\theta}$$

$$\boxed{\sin^2\theta_{12} > \tfrac{1}{3}}$$

Y.Shimizu, M.T, arXiv 1405.1521

TM1

$$V_\nu = \begin{pmatrix} 2/\sqrt{6} & c/\sqrt{3} & se^{-i\sigma}/\sqrt{3} \\ -1/\sqrt{6} & c/\sqrt{3} - se^{i\sigma}/\sqrt{2} & -se^{-i\sigma}/\sqrt{3} - c/\sqrt{2} \\ -1/\sqrt{6} & c/\sqrt{3} + se^{i\sigma}/\sqrt{2} & -se^{-i\sigma}/\sqrt{3} + c/\sqrt{2} \end{pmatrix}$$

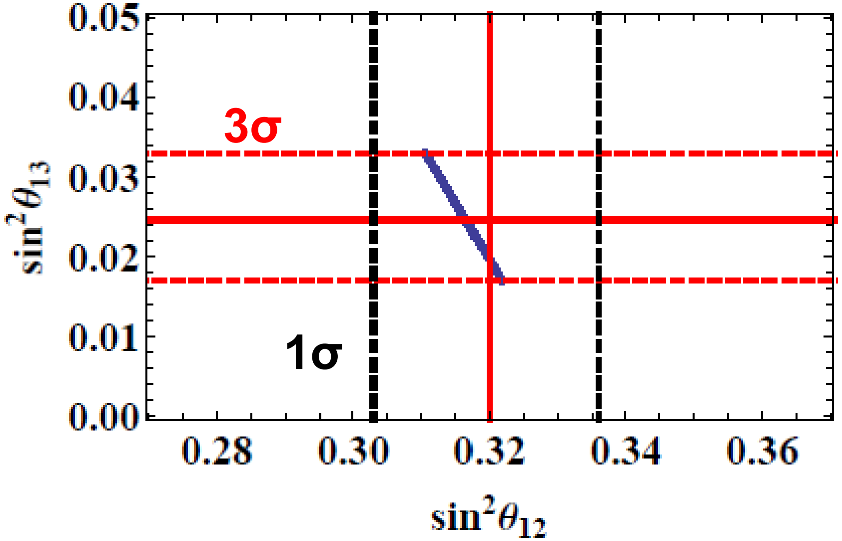
$$\sin^2\theta_{13} = \tfrac{1}{3}\sin^2\theta, \quad \sin^2\theta_{12} = 1 - \tfrac{1}{2+\cos^2\theta}$$

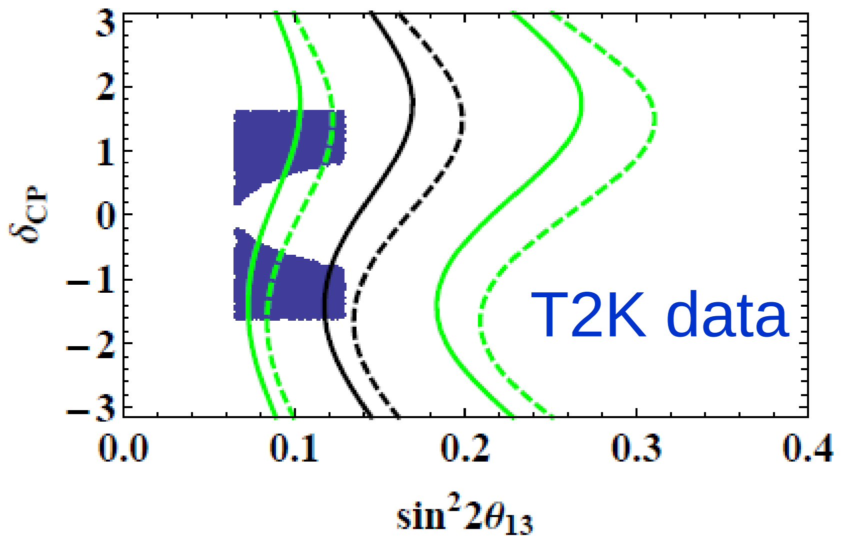
$$\sin^2\theta_{23} = \tfrac{1}{2}\left(1 - \tfrac{\sqrt{6}\sin 2\theta\cos\sigma}{2+\cos^2\theta}\right)$$

Rodejohann, Zhang,
PRD86(2012) arXiv:093008

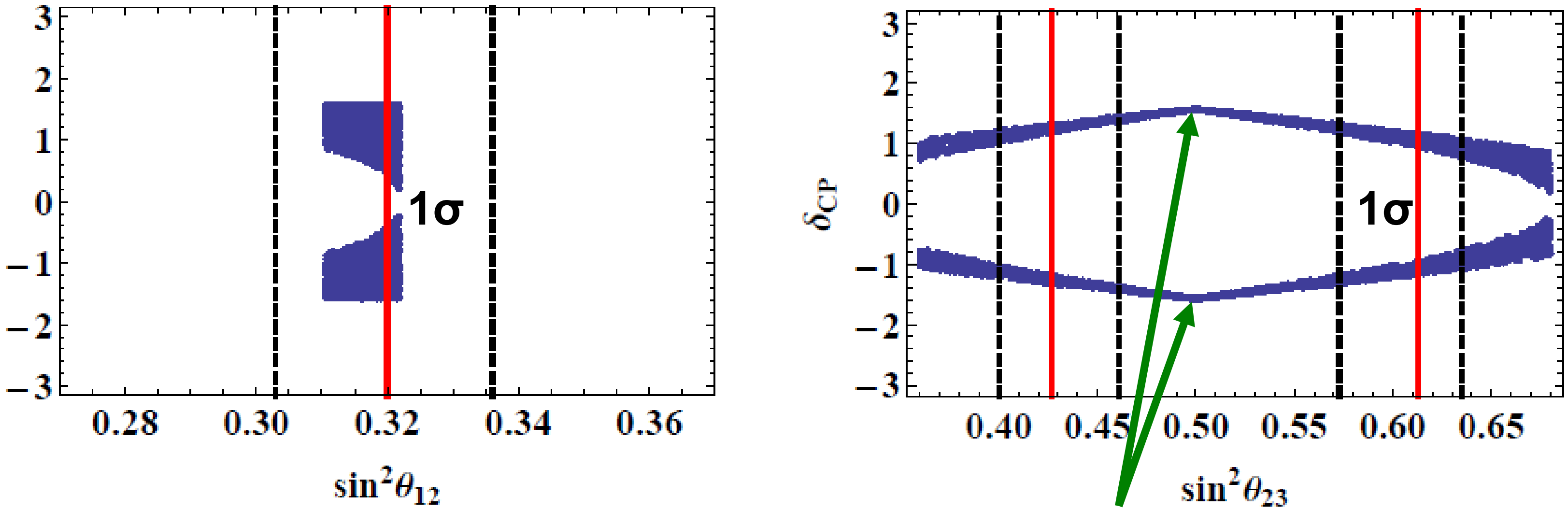
$$\boxed{\sin^2\theta_{12} < \tfrac{1}{3}}$$

TM1





Y.Shimizu, M.T, arXiv 1405.1521



Geometrical CP prediction

$$G_{CP} = G_f \rtimes H_{CP}$$

Many models based on the A_4, S_4, A_5, T', $\Delta(96)$ …

Mass sum rules

Barry, Rodejohann, NPB842(2011) arXiv:1007.5217

Different types of neutrino mass spectra correspond to the neutrino mass generation mechanism.

$$\chi \tilde{m}_2 + \xi \tilde{m}_3 = \tilde{m}_1$$ (χ=2, ξ=1) (χ=-1, ξ=1)

$$\frac{\chi}{\tilde{m}_2} + \frac{\xi}{\tilde{m}_3} = \frac{1}{\tilde{m}_1}$$ **M_R structre in See-saw**

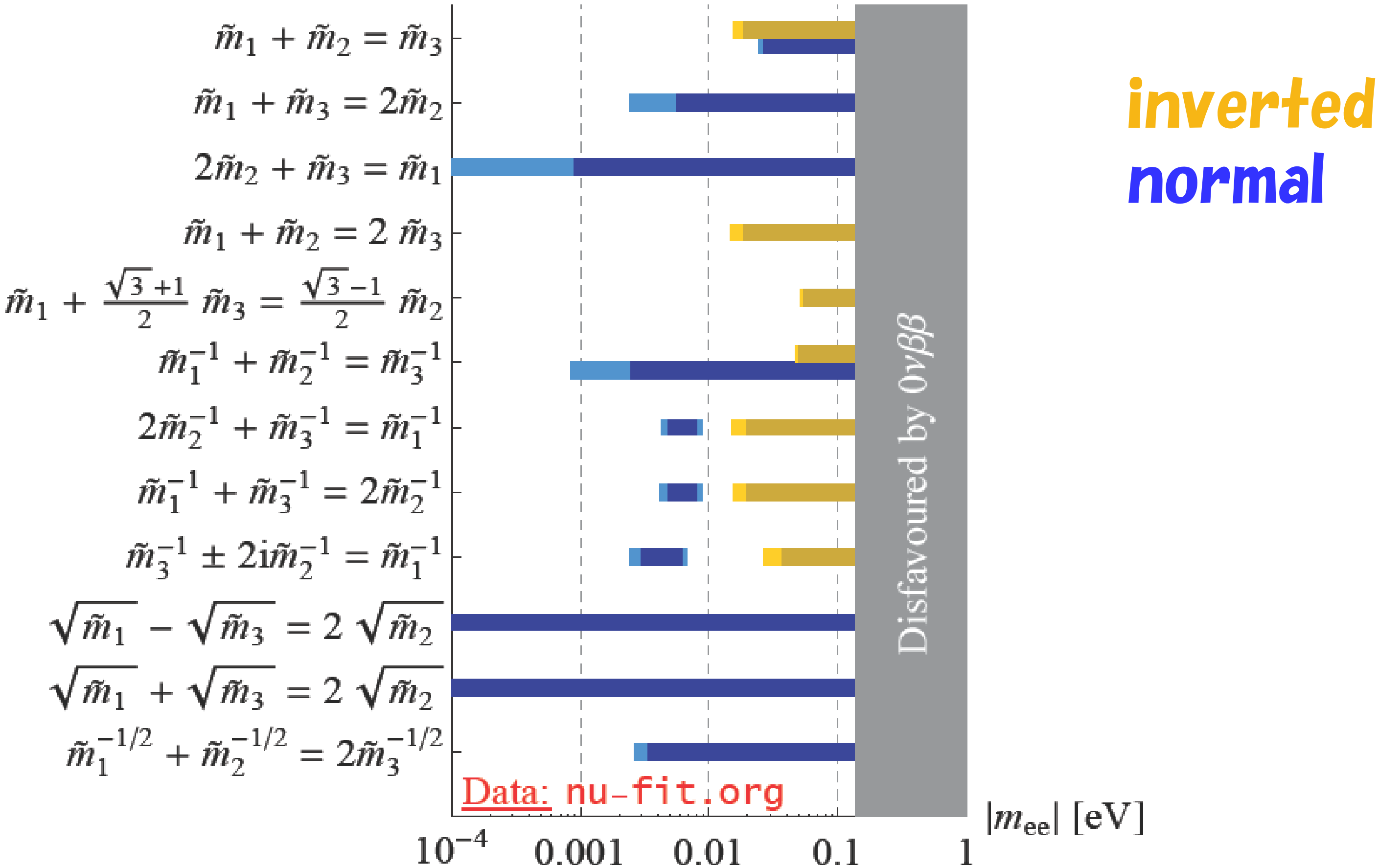
$$\chi\sqrt{\tilde{m}_2} + \xi\sqrt{\tilde{m}_3} = \sqrt{\tilde{m}_1}$$ **M_D structre in See-saw**

$$\frac{\chi}{\sqrt{\tilde{m}_2}} + \frac{\xi}{\sqrt{\tilde{m}_3}} = \frac{1}{\sqrt{\tilde{m}_1}}$$ **M_R in inverse See-saw**

χ and ξ are model specific complex parameters

King, Merle, Stuart, JHEP 2013, arXiv:1307.2901
King, Merle, Morisi, Simizu, M.T, arXiv: 1402.4271

Restrictions by mass sum rules on $|m_{ee}|$



King, Merle, Stuart, JHEP 2013, arXiv:1307.2901

5 Summary and Prospect

- FLASY predicts non-zero θ_{13}.
- FLASY with generalized CP symmetry predicts CP violating phase.
- FLASY predicts the mass sum rules.
- Predictions will be testable by the precise data of neutrino mixing angles, CP violating phase and m_{ee} .

- *Can one predict the CKM mixing of the qaurk sector ?*

$\Delta(6N^2)$: $\theta_c=\pi/N$

N = 14 is required !

Ishimori, King, 1403.4395

- *Challenging prediction of both θ_{13} and θ_c ?*

A_4, T', S_4 × SU_5^{GUT}

A_4, S_4 × SO_{10}^{GUT}

Meroni,Petcov,Spinrath2012
Antusch, Gross, Maurer, Sluka2013
King, Luhn, 2009, 2011
Hagedorn, King, Luhn,2010, 2012
Ishimori, Simizu, M.T, 2009, 2010
Bazzocchi, Frigerio, Morisi,2008 …..

SU(12) unification

$$SU(12) \Rightarrow SU(5) \times U(1)^7$$

Albright, Feger, Kephart, PRD86(2012), arXiv:1204.5471

- *Origin of Non-Abelian Discrete group:*

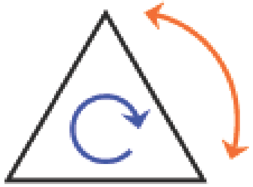
Stringy ?   Extra Dimension ?   SU(3)   ?

Extra Slides

Actually, TBM suggested S_3 or A_4 symmetry for the lepton flavor structure.

Irreducible representations

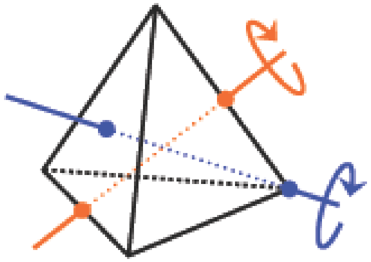
S_3 : permutation of 3 objects (3 generations) **2, 1, 1'**



A_4 : even permutation of 4 objects

$\begin{pmatrix} \text{3-dim representation x 1} \\ \text{1-dim representation x 3} \end{pmatrix}$

suitable for 3 generations



3, 1, 1', 1"

Comment:

S_4 is a group in the series of

$$\Delta(6N^2) = (Z_n \times Z'_N) \rtimes S_3$$

$$\Delta(6) = S_3, \quad \Delta(24) \sim S_4$$

$$\Delta(54), \Delta(96), \ldots$$

$\Delta(96), \cdots$ **have desired $Z_2 \times Z_2$ (K_4) subgroup to predicts non-vanishing Θ_{13} keeping preserved S, U, T,**

but for N=22 to be consistent with experimental data.

$$\sin\theta_{13} = \sqrt{\frac{2}{3}} \sin\frac{\pi}{N}$$

King, Neder, Stuart, arXiv: PLB(2013) 1305.3200

See-saw realization of neutrino masses in the indirect approach

$$m_{\nu LL} = -m_D M_R^{-1} m_D^T$$

Suppose: Two triplet flavons $\phi_{\rm atm}, \phi_{\rm sol}$ couple to ν_{R3}, ν_{R2}, respectively.

$$\mathcal{L}_Y = (\langle\phi_{\rm atm}\rangle \cdot L)\nu_{R3} + (\langle\phi_{\rm sol}\rangle \cdot L)\nu_{R2}$$

$$M_R = \begin{pmatrix} M_A & 0 \\ 0 & M_B \end{pmatrix},$$ the third one is supposed to be decoupled.

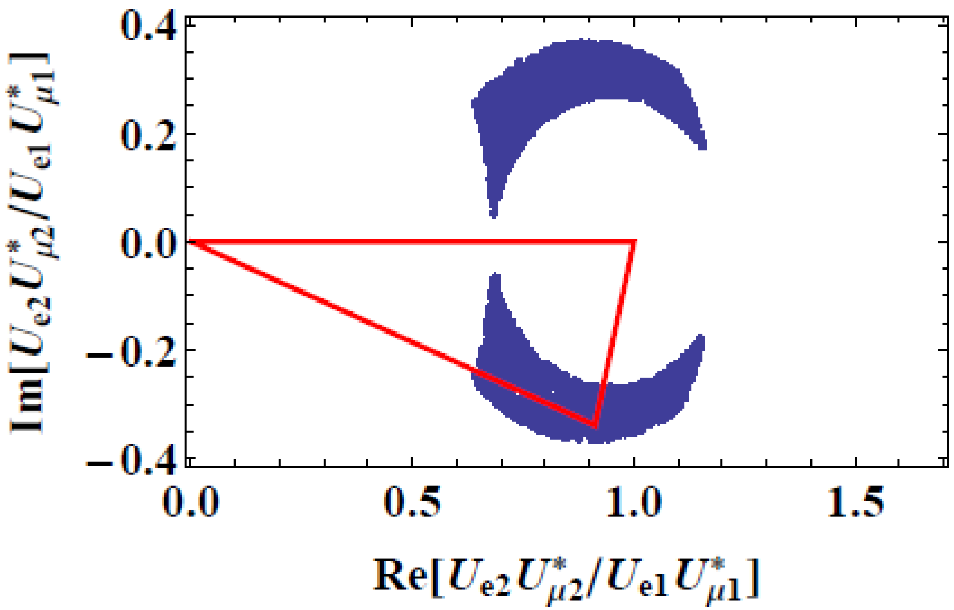
$$\boxed{m_{\nu LL} \sim \langle\phi_{\rm atm}\rangle\langle\phi_{\rm atm}\rangle^T / M_A + \langle\phi_{\rm sol}\rangle\langle\phi_{\rm sol}\rangle^T / M_B}$$

A simple example: $\boxed{\langle\phi_{\rm atm}\rangle^T = (0,1,1), \quad \langle\phi_{\rm sol}\rangle^T = (1,4,2)}$

King arXiv:1305.4846

Specific alignments of flavons determine the flavor structure of neutrino mass matrix !

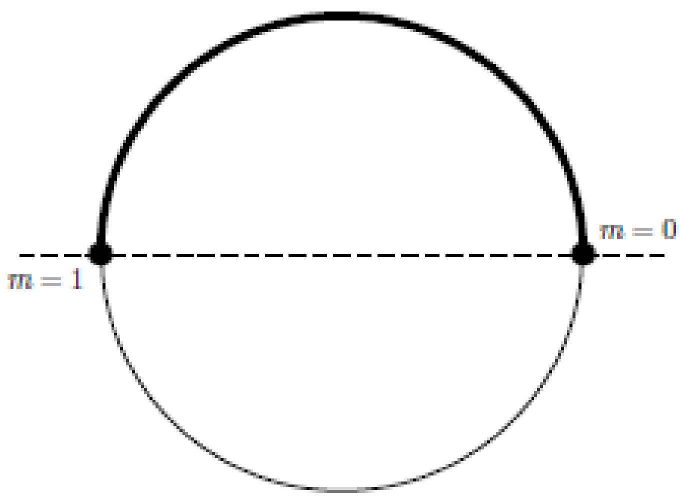
Lepton Unitarity Triangle



Stringy Origin of FLASY was proposed.

D_4 flavor symmetry is Typical symmetry of Stringy origin.

Stringy origin of non-Abelian discrete flavor symmetries;
T. Kobayashi, H. Nilles, F. Ploger, S.Raby, M. Ratz,
Nucl.Phys.B768(2007) 135, hep-ph/0611020



Strings on orbifolds

S^1/Z^2 orbifold

$$D_4 = S_2 \ltimes (Z_2 \times Z_2)$$

Stringy origin of non-Abelian discrete flavor symmetries

T. Kobayashi, H. Niles, F. PloegerS, S. Raby, M. Ratz, hep-ph/0611020

D_4, $\Delta(54)$

Non-Abelian Discrete Flavor Symmetries from Magnetized/Intersecting Brane Models

H. Abe, K-S. Choi, T. Kobayashi, H. Ohki, 0904.2631

D_4, $\Delta(27)$, $\Delta(54)$

Non-Abelian Discrete Flavor Symmetry from T^2/Z_N Orbifolds

A.Adulpravitchai, A. Blum, M. Lindner, 0906.0468

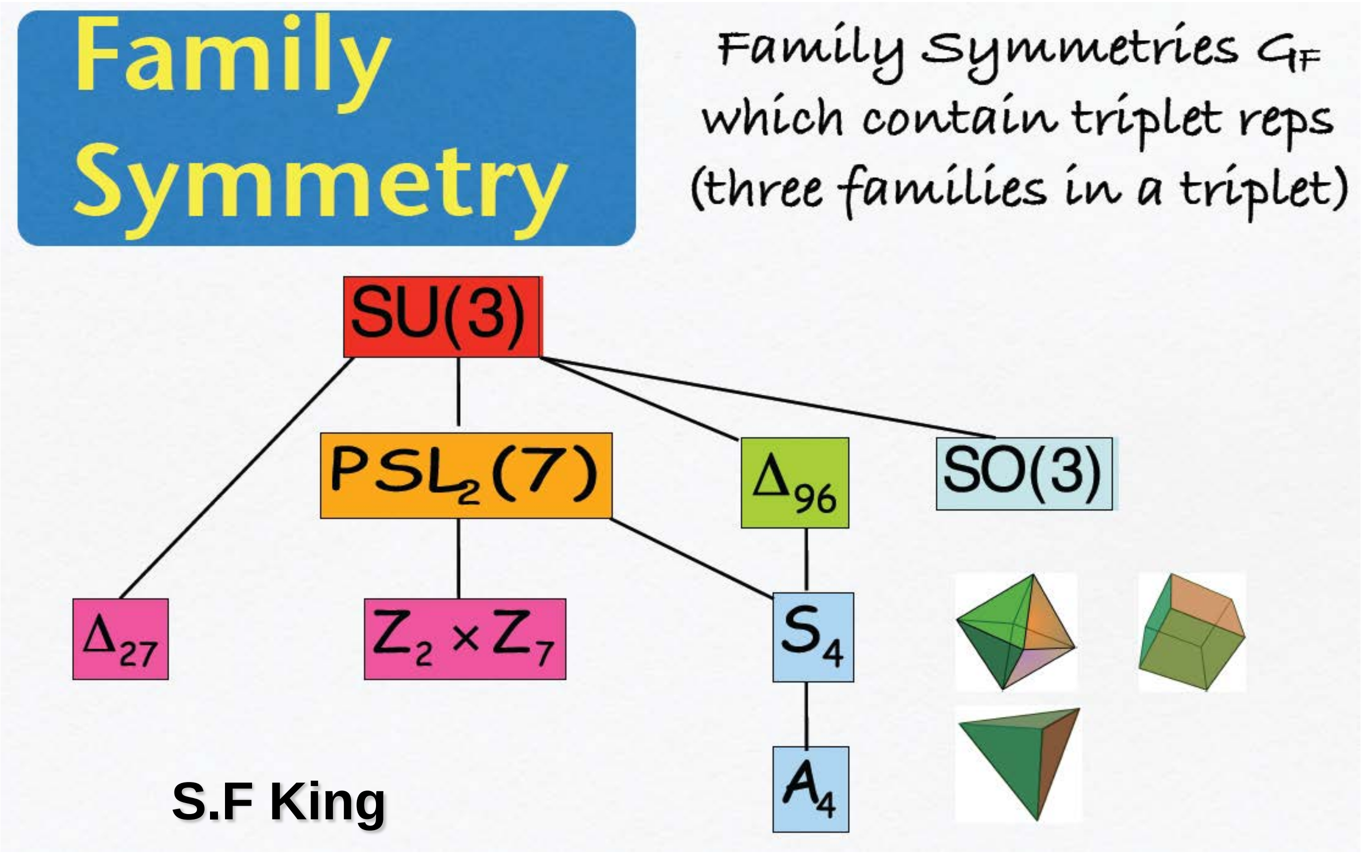
A_4, S_4, D_3, D_4, D_6

Non-Abelian Discrete Flavor Symmetries of 10D SYM theory with Magnetized extra dimensions

H. Abe, T. Kobayashi, H. Ohki, K.Sumita, Y. Tatsuta 1404.0137

S_3, $\Delta(27)$, $\Delta(54)$

Subgroups of SU(3)



C.Luhn, JHEP 1103, 108(2011) arXiv:1101.2417
A.Merle, R. Zwicky, JHEP, 1202 128(2012) 1110.4891

Domain wall solution : *"Flavon inflation"*

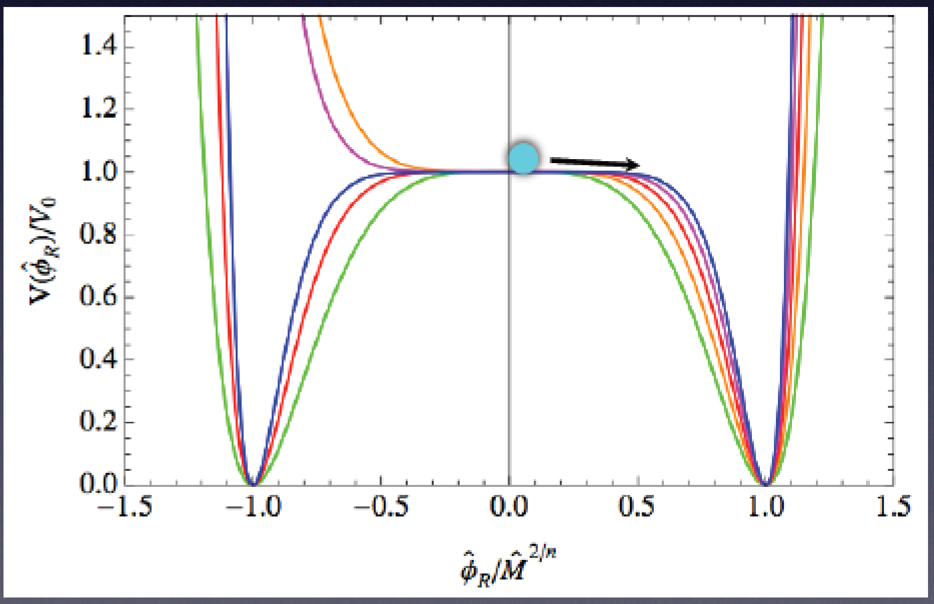
The flavons themselves can also act as inflaton fields in so-called 'new inflation':

S. A., S.F. King, M. Malinsky, L. Velasco Sevilla, I. Zavala (arXiv:0805.0325)

Antusch $$W_{\rm fl} = S_i \left([\Theta_i]^n - \mu_i^2 \right) + \ldots$$

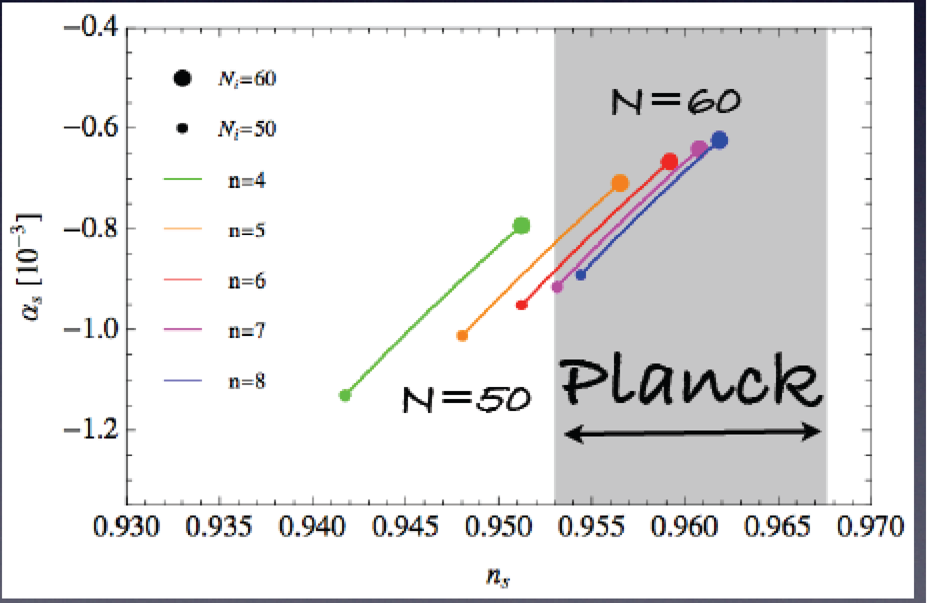
Flavon = Inflaton field

Inflaton potential for different n: Plateau for small Φ



Shown in the plot: Potential for n=4,5,6,7,8

S. A., F. Cefala (arXiv:1306.6825)



Very predicted in SUGRA with Heisenberg symmetry!