Beam-beam simulations including Beamstrahlung in TLEP

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TLEP workshop at Fermilab, 25-26 July, 2013

Thanks to F. Zimmermann、K. Yokoya

Contents

- Beamstrahlung
- Weak-strong simulation to compare with Yokoya’s formula.
- Weak-strong simulation with self-consistent bunch length for TLEP.
- Strong-strong simulation for TLEP.

Beamstrahlung

- Synchrotron radiation during beam-beam interaction
- Number of photon

$$dn_\gamma = \frac{5\sqrt{3}\alpha\gamma}{6\rho} ds$$

- Photon energy

$$\langle u \rangle = \frac{8}{15\sqrt{3}} \hbar\omega_c = \frac{4}{5\sqrt{3}} \frac{\hbar c \gamma^3}{\rho} \qquad \omega_c = \frac{3c\gamma^3}{2\rho}$$

- Photon energy fluctuation

$$\langle u^2 \rangle = \frac{11}{27} (\hbar\omega_c)^2 = \frac{11}{12} \frac{\hbar^2 c^2 \gamma^6}{\rho^2}$$

- Energy loss and fluctuation of beam

$$\langle d\delta \rangle = dn_\gamma \langle u \rangle = \frac{2 r_e \gamma^3}{3\rho^2} ds \qquad \langle d\delta^2 \rangle = dn_\gamma \langle u^2 \rangle = \frac{55}{24\sqrt{3}} \frac{r_e \hbar}{mc} \frac{\gamma^5}{\rho^3} ds$$

- Beam motion

$$\frac{dp_{x,y}}{ds} = \frac{1}{\rho_{x,y}}$$

Simplified formula by K. Yokoya

- Gaussian flat beam, linear approximation

$$\frac{dp_x}{ds} = \frac{4n_e r_e}{\gamma} \frac{x}{\sigma_x(\sigma_x+\sigma_y)} = \frac{1}{\rho_x}$$

- Energy loss

$$\int \; d\delta_{BS} = \int_{-\infty}^{\infty} \frac{2r_e^3\gamma}{3}\left(\frac{4n_e(s)}{\sigma_x+\sigma_y}\right)^2 ds = 0.864 r_e^3 \gamma \left(\frac{N_e}{\sigma_z(\sigma_x+\sigma_y)}\right)^2 \sigma_z$$

- Number of photon

n_e: number of electron containing in dz

$$\int \; dn_\gamma = \int_{-\infty}^{\infty} \frac{5\sqrt{3}\alpha r_e}{6} \frac{4n_e(s)}{\sigma_x+\sigma_y} ds = 2.12 \frac{\alpha r_e N_e}{\sigma_x+\sigma_y}$$

$$\sqrt{\langle d\delta_{BS}\rangle^2} = d\delta_{BS}\sqrt{0.1639 + \frac{5.129}{n_\gamma}}$$

Beamstrahlung in LEP3 and TLEP (Yokoya's formula)

- LEP3

$$d\delta_{BS} = 0.864 r_e^3 \gamma \left(\frac{N_e}{\sigma_z(\sigma_x + \sigma_y)}\right)^2 \sigma_z = 3.9 \times 10^{-4} \qquad n_\gamma = 2.12 \frac{\alpha r_e n_e}{\sigma_x + \sigma_y} = 0.61$$

diffusion rate $\sqrt{\langle d\delta_{BS}\rangle^2} = d\delta_{BS}\sqrt{0.1639 + \frac{5.129}{n_\gamma}} = 1.1 \times 10^{-3}$

energy spread $\sigma_{\delta.BS} = \sqrt{\tau_\epsilon / T_0}\sqrt{\langle d\delta_{BS}\rangle^2} = 5.5 \times 10^{-3}$

- TLEP

$$d\delta_{BS} = 0.864 r_e^3 \gamma \left(\frac{N_e}{\sigma_z(\sigma_x + \sigma_y)}\right)^2 \sigma_z = 3.5 \times 10^{-4} \qquad n_\gamma = 2.12 \frac{\alpha r_e n_e}{\sigma_x + \sigma_y} = 0.5$$

diffusion rate $\sqrt{\langle d\delta_{BS}\rangle^2} = 1.1 \times 10^{-3}$ energy spread $\sigma_{\delta,BS} = 1.2 \times 10^{-2}$

Simulation of the beamstrahlung

- Slice beam along bunch length, $s=(z-z_i)/2$ for collision of particle (z) and i-th slice.
- Calculate beam-beam kick

$$\frac{dp_x}{ds} = \frac{4n_e r_e}{\gamma} F_x \qquad F_y + iF_x = \sqrt{\frac{\pi}{2(\sigma_x^2 - \sigma_y^2)}}\left[w\left(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right) - exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right) w\left(\frac{(\sigma_y/\sigma_x)x + i(\sigma_x/\sigma_y)y}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right)\right] \quad \text{or} \quad F_{x(y)} = -\frac{\partial\phi_{PIC}}{\partial x(y)}$$

- Number of photon emitted by interaction of a slice

$$dn_\gamma = \frac{5\sqrt{3}\alpha\gamma}{6\rho} ds \qquad \frac{1}{\rho_{x,y}} = \frac{dp_{x,y}}{ds} \qquad \frac{1}{\rho^2} = \frac{1}{\rho_x^2} + \frac{1}{\rho_y^2}$$

- Energy loss and diffusion (photon energy and fluctuation)

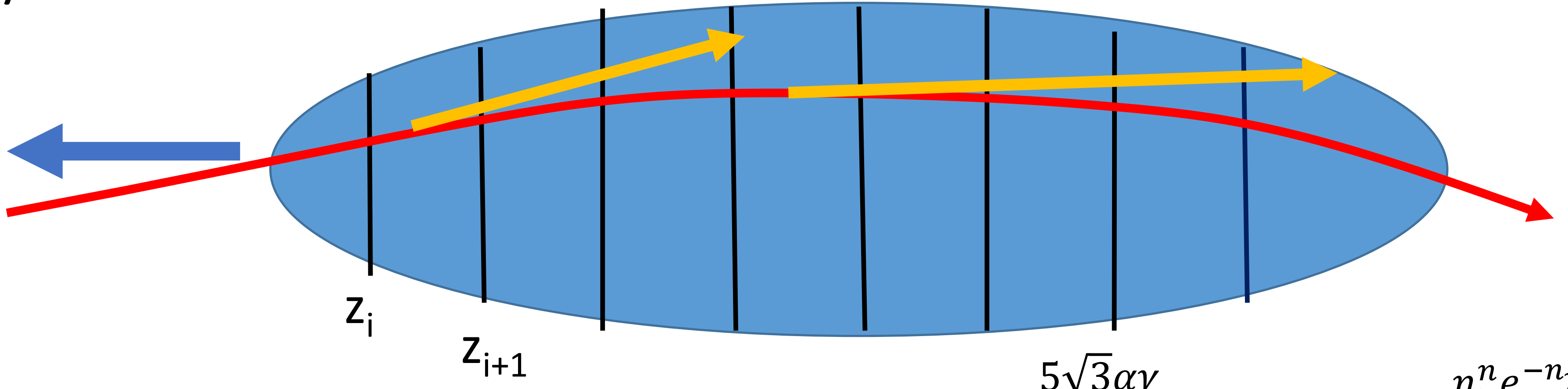
$$\langle u \rangle = \frac{8}{15\sqrt{3}}\hbar\omega_c = \frac{4}{5\sqrt{3}}\frac{\hbar c\gamma^3}{\rho} \qquad \langle u^2 \rangle = \frac{11}{27}(\hbar\omega_c)^2 = \frac{11}{12}\frac{\hbar^2 c^2 \gamma^6}{\rho^2} \qquad \langle u^2 \rangle - \langle u \rangle^2 = \frac{211}{300}\frac{\hbar^2 c^2\gamma^6}{\rho^2}$$

$$\langle d\delta \rangle = dn_\gamma \langle u \rangle = \frac{2r_e\gamma^3}{3\rho^2} ds \qquad \langle d\delta^2 \rangle = dn_\gamma \langle u^2 \rangle = \frac{55}{24\sqrt{3}}\frac{r_e\hbar}{mc}\frac{\gamma^5}{\rho^3} ds$$

- Poisson process for n_γ emission $\quad P(n) = \dfrac{n_\gamma^n e^{-n_\gamma}}{n!}$

Schematic view of the simulation

- Calculate trajectory interacting with colliding beam.
- Particles emit synchrotron radiation due to the momentum kick dp/ds.



$$ds = \frac{z_i - z_{i+1}}{2}$$

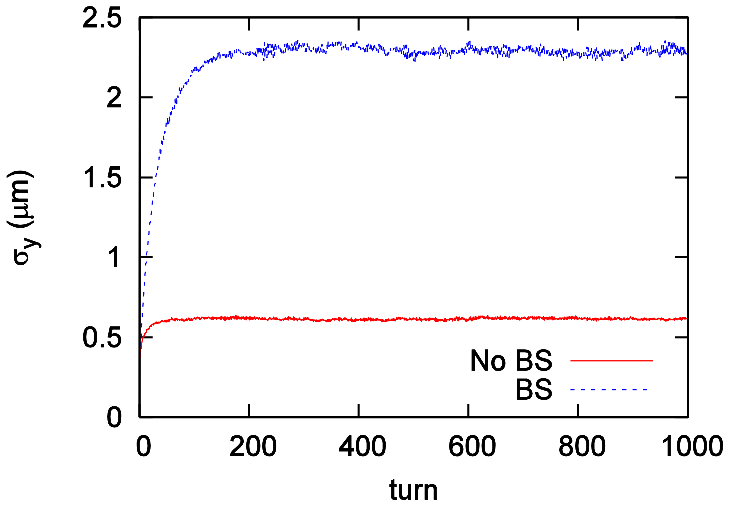
$$\frac{1}{\rho_{x,y}} = \frac{dp_{x,y}}{ds}$$

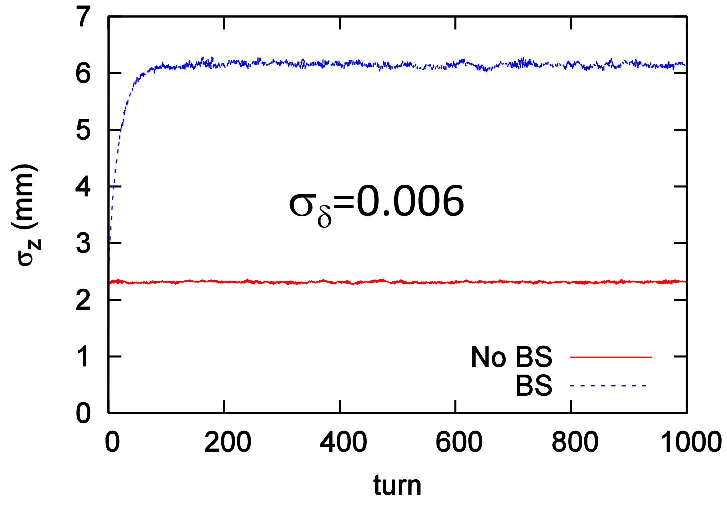
$$dn_\gamma = \frac{5\sqrt{3}\alpha\gamma}{6\rho} ds \qquad P(n) = \frac{n_\gamma^n e^{-n_\gamma}}{n!}$$

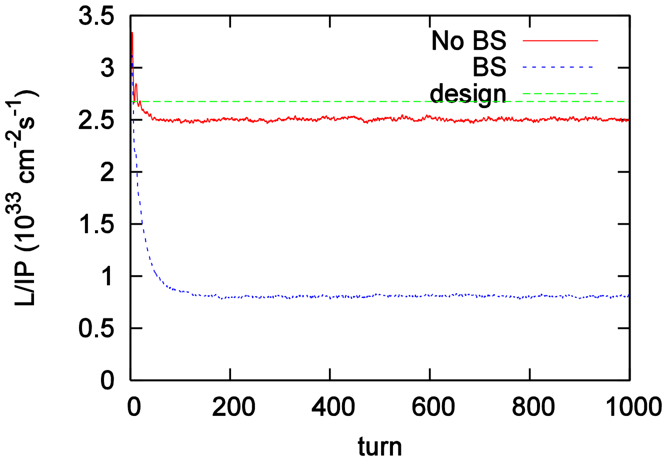
$$\langle d\delta \rangle = dn_\gamma \langle u \rangle = \frac{2 r_e \gamma^3}{3\rho^2} ds$$

$$\langle d\delta^2 \rangle = dn_\gamma \langle u^2 \rangle = \frac{55}{24\sqrt{3}} \frac{r_e \hbar}{mc} \frac{\gamma^5}{\rho^3} ds$$

Test using weak-strong model (LEP3)







$$\sigma_{\delta.BS} = \sqrt{\tau_\epsilon/T_0}\sqrt{\langle d\delta_{BS}\rangle^2} = 5.5 \times 10^{-3}$$

The equilibrium σ_δ agrees with Yokoya's formula.

Note σ_z is not self-consistent.

Parameter list for TLEP

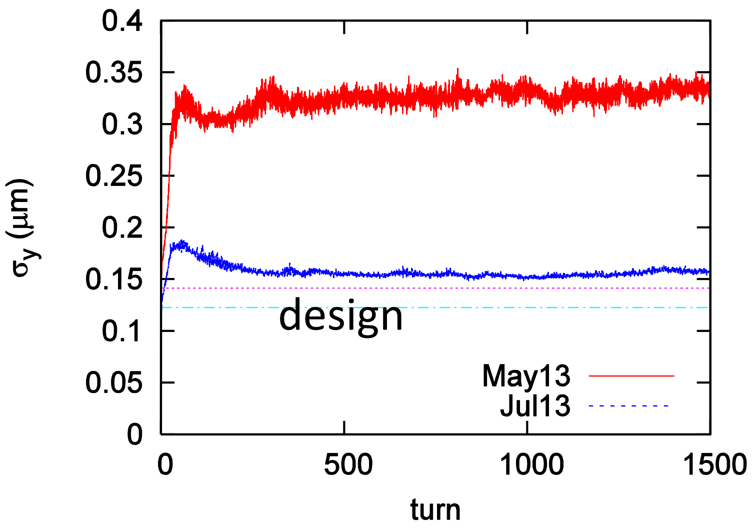
July 2013

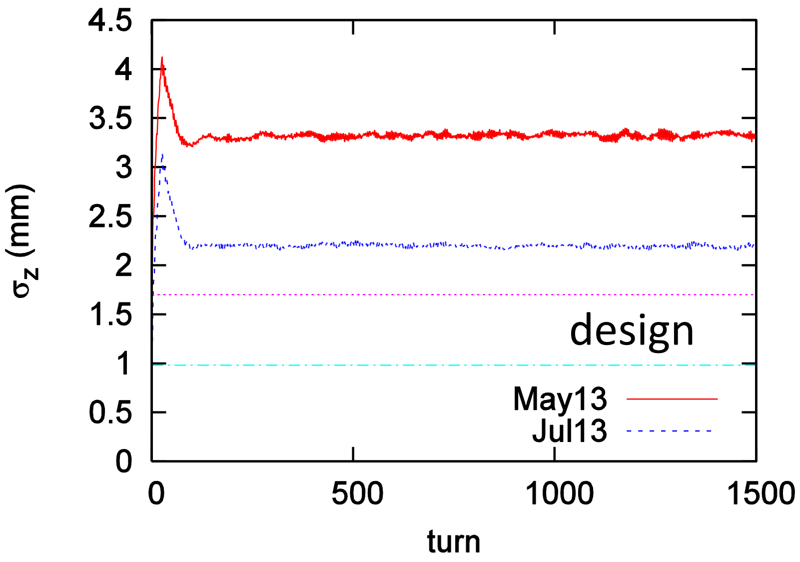
Courtesy of F. Zimmermann

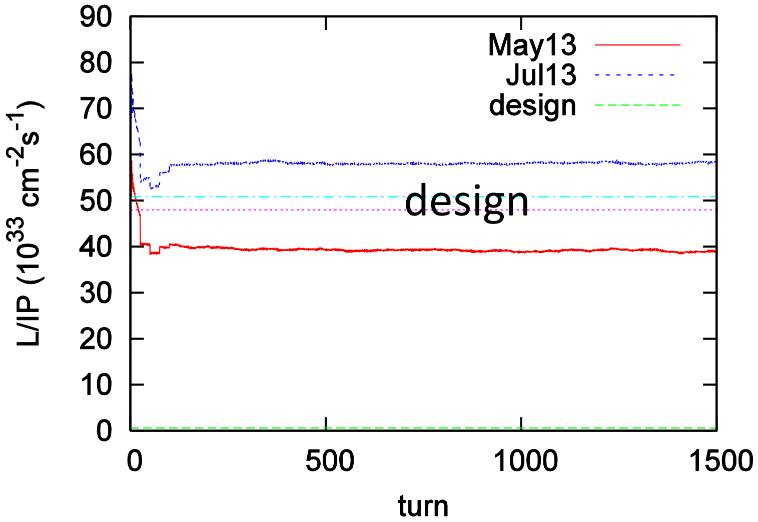
	TLEP Z	TLEP W	TLEP H	TLEP t
E_{beam} [GeV]	45	80	120	175
circumf. [km]	100	100	100	100
beam current [mA]	1440	154	29.8	6.7
#bunches/beam	7500	3200	167	160
#e−/bunch [10^{11}]	4.0	1.0	3.7	0.88
horiz. emit. [nm]	29.2	3.3	7.5	2.0
vert. emit. [nm]	0.06	0.017	0.015	0.002
mom. c. α_c [10^{-5}]	3.6	0.4	0.4	0.1
β^*_x [m]	0.5	0.2	0.5	1.0
β^*_y [mm]	1.0	1.0	1.0	1.0
σ^*_x [µm]	121	26	61	45
σ^*_y [µm]	0.25	0.13	0.12	0.045
δ^{SR}_{rms} [%]	0.05	0.09	0.14	0.20
$\sigma^{SR}_{z,rms}$ [mm]	1.16	0.91	0.98	0.68
δ^{tot}_{rms} [%]	0.13	0.20	0.30	0.23
$\sigma^{tot}_{z,rms}$ [mm]	2.93	1.98	2.11	0.77
V_{RF},tot [GV]	2	2	6	12
$t_{\|\|}$ (turns)	1319	242	72	23
ξ_x/IP	0.068	0.086	0.094	0.057
ξ_y/IP	0.068	0.086	0.094	0.057
ν_s	0.257	0.0633	0.09	0.0468
$\mathcal{L}$ /IP[10^{32}cm^{-2}s^{-1}]	5860	1640	508	132
number of IPs	4	4	4	4
beam lifetime [min] (rad. Bhabha)	99	38	24	21
beam lifetime [min] (beamstrahlung Telnov with h=2%)	>10^{25}	>10^{6}	38	14

Weak-strong with Self consistent bunch length (TLEP)

- σ_z is revised by the average of every 100 bb interactions.







The bunch length and energy spread agree with the formula given by F. Zimmermann.

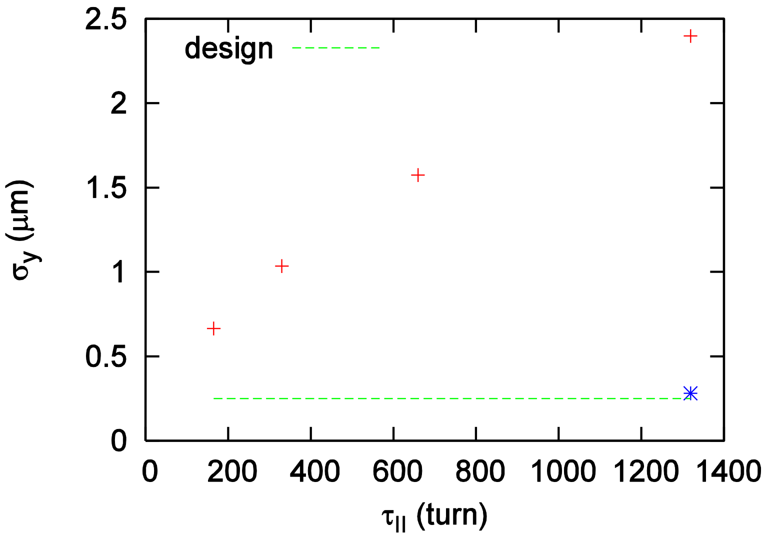
Parameters May13, which is old one, is not shown in detail here.

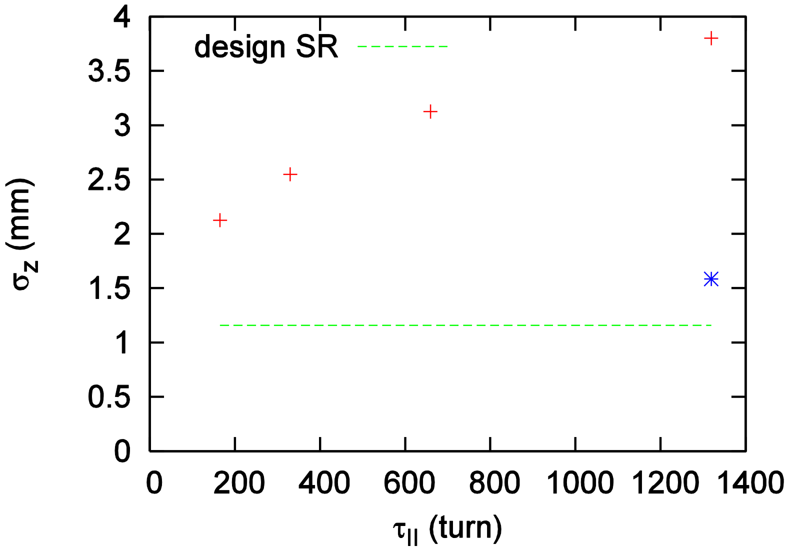
Luminosity and bunch length by weak-strong

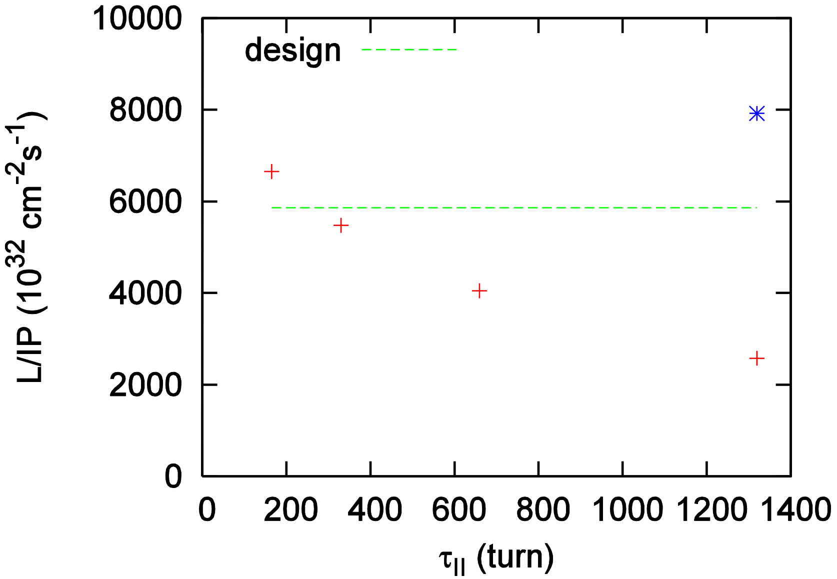
	TLEP-Z	TLEP-Z -II	TLEP-W	TLEP-H	TLEP-t
E (GeV)	45	45	80	120	175
#e−/bunch [10^{11}]	4.0	2.0	1.0	3.7	0.88
Nbunch	7500	15000	3200	167	160
Damping time (turn)	1318	1319	242	72	23
$\sigma x/\sigma y$ (μm)	121/0.25	61/0.125	26/0.13	61/0.12	45/0.045
σ_z/σ_δ(SR, mm/%)	1.16/0.05	0.73/0.05	0.91/0.09	0.98/0.14	0.68/0.20
L (design, $cm^{-2}s^{-1}$)/IP	58.6×10^{34}	58.6×10^{34}	16.4×10^{34}	5.08×10^{34}	1.32×10^{34}
Weak strong with BS	σ_z self-consistent				
σ_z/σ_δ(mm/%)	3.8/0.16	1.6/0.11	2.1/0.21	2.2/0.31	0.79/0.23
$\sigma x/\sigma y$ (μm)	99/2.4	99/0.25	20/0.18	48/0.16	36/0.044
L ($cm^{-2}s^{-1}$)/IP	25.7×10^{34}	$80.\times10^{34}$	18.4×10^{34}	5.8×10^{34}	1.56×10^{34}

Luminosity of TLEP-Z was lower than the design. TLEP-Z-II : $\varepsilon_x/4$, $\varepsilon_y/4$, $N_e/4$, $\alpha_p/2.5$, N_{bunch}x4 was investigated. Beamstrahlung is mild.

TLEP-Z (scan for the damping time)







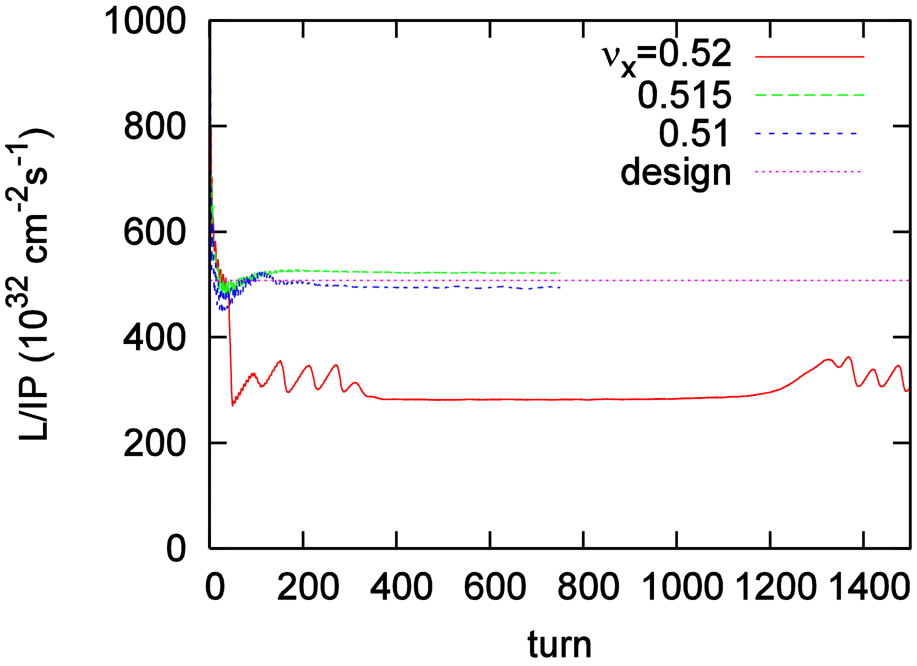
Luminosity of TLEP-Z was lower than the design. Scan parameters to find better performance.

Blue: TLEP-Z-II , $\varepsilon_x/4$, $\varepsilon_y/4$, $N_e/4$, $\alpha_p/2.5$, N_{bunch}x4

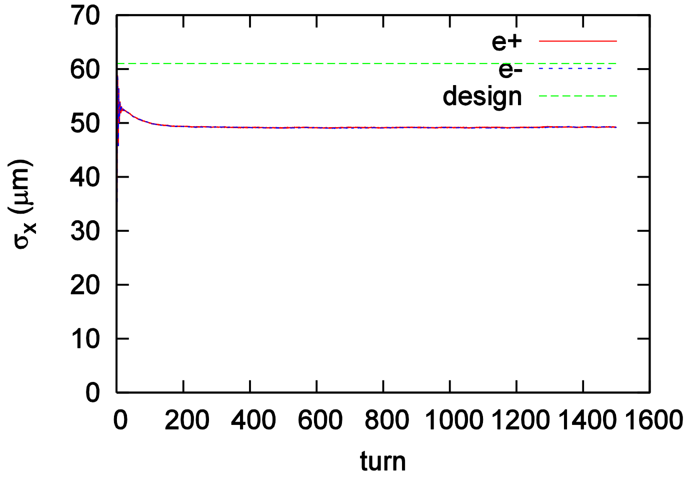
Strong-strong simulation including beamstrahlung

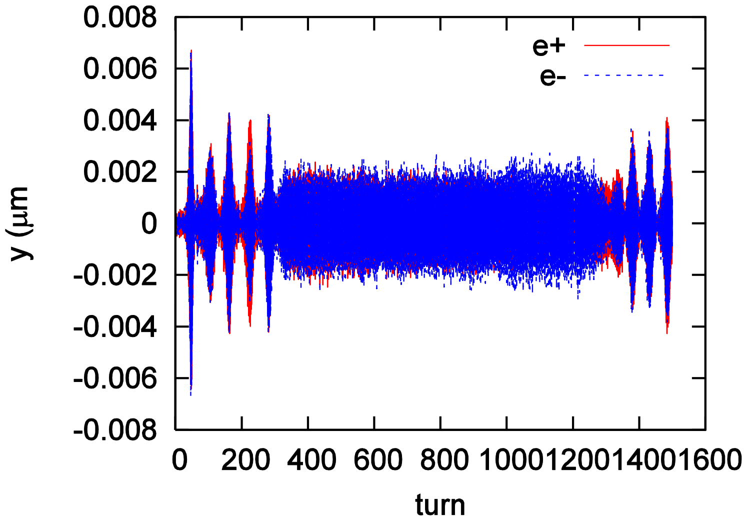
- 1/ρ is calculated by the potential in PIC.

$$\frac{1}{\rho_{x,y}} = \frac{dp_{x,y}}{ds} = -\frac{\partial\varphi}{\partial x}$$



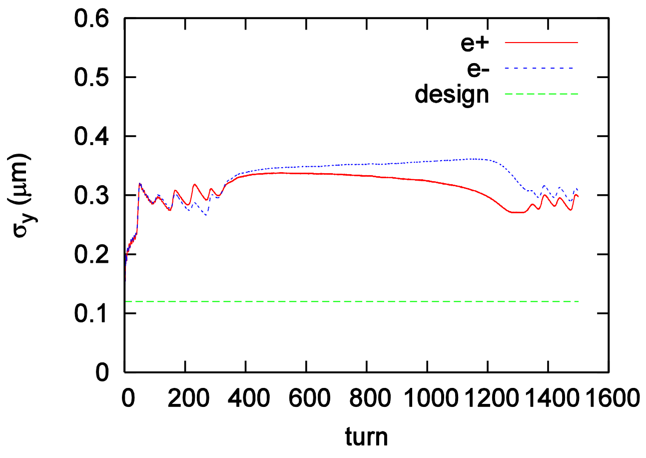
Instability was seen for (νx, νy)/IP=(0.52,0.58)

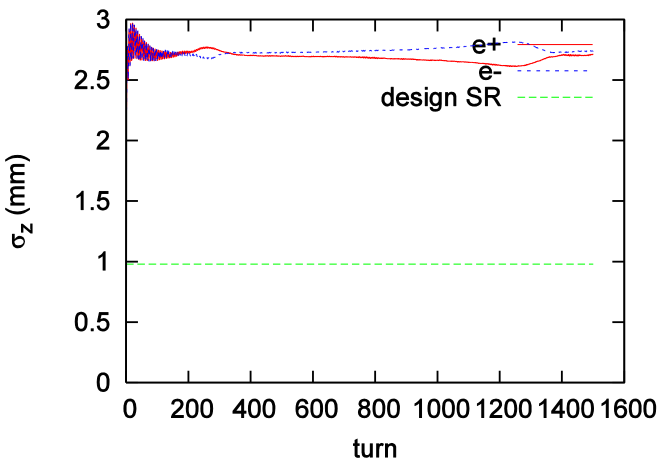




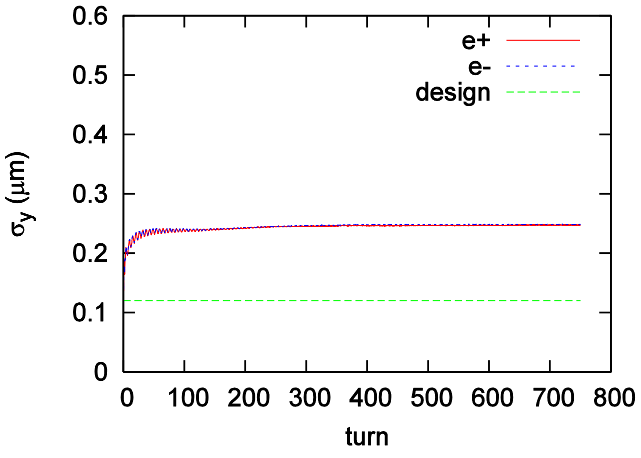
Beam size evolution for unstable and stable cases

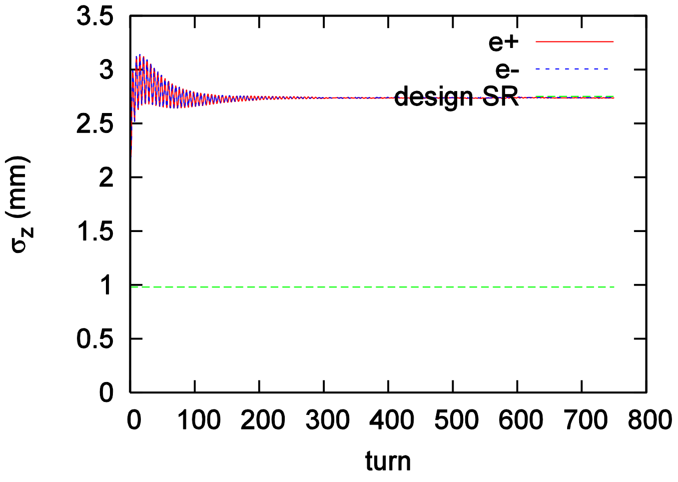
ν_x=0.52, unstable



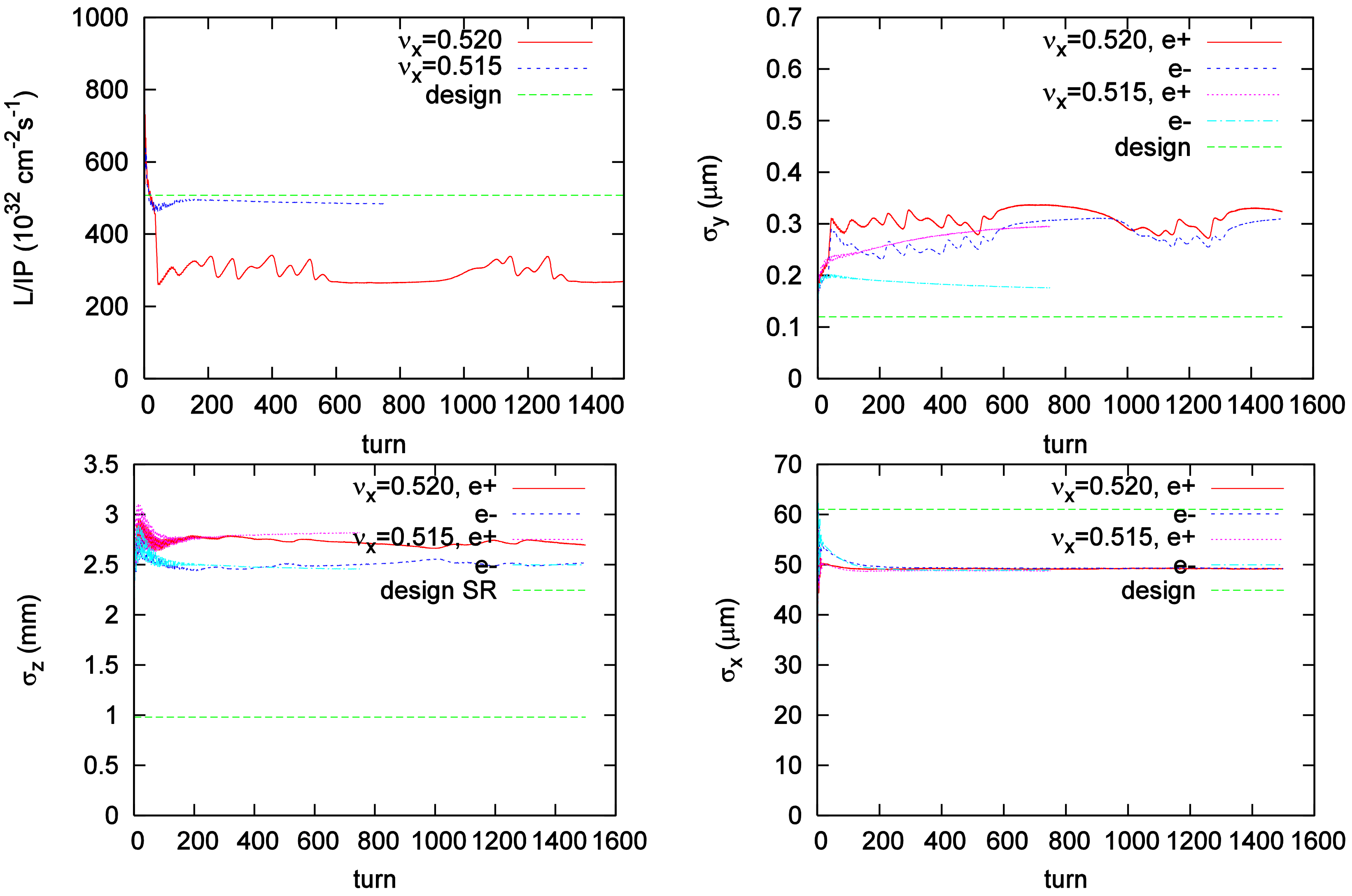


ν_x=0.515, stable





Small asymmetry (10% less e+ beam) of beam currents



In stable case ν_x=0.515, equilibrium of σ_y and σ_z is found.

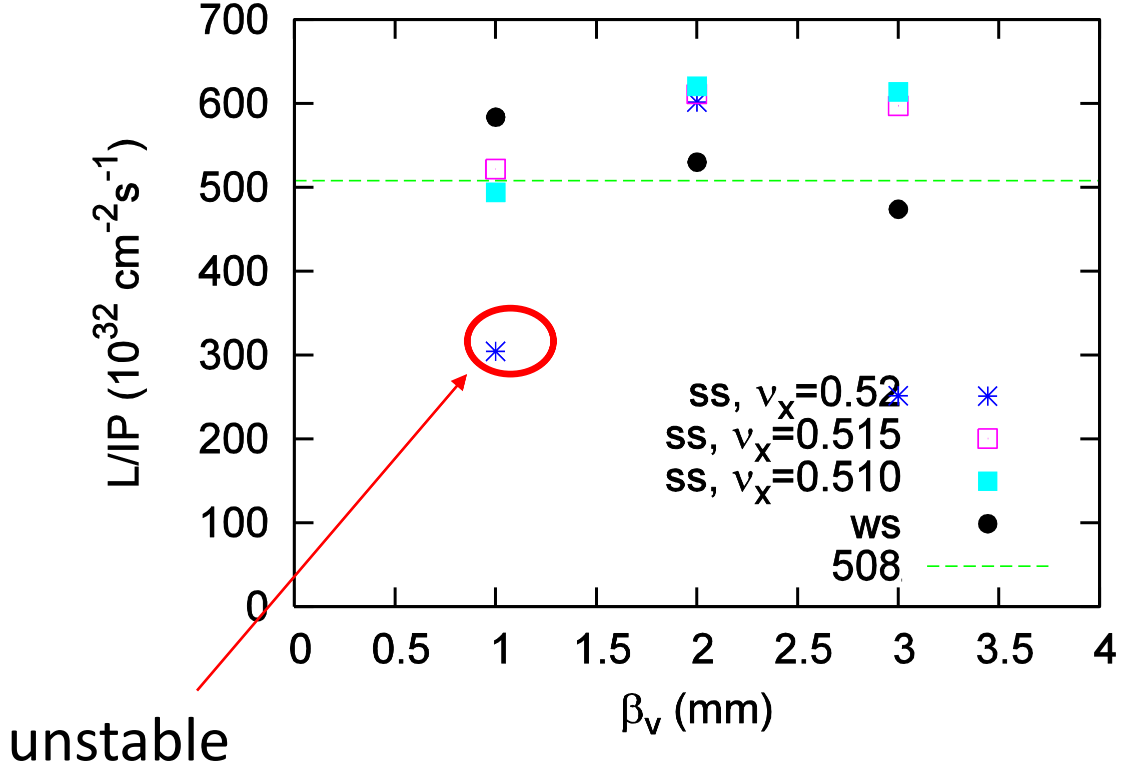
σ_z=2.5 and 2.7mm.

σ_y=0.18 and 0.3 μm.

Luminosity and bunch length

	TLEP-Z-II	TLEP-W	TLEP-H	TLEP-t
E (GeV)	45	80	120	175
#e−/bunch [10^{11}]	2.0	1.0	3.7	0.88
σx/σy (μm)	61/0.125	26/0.13	61/0.12	45/0.045
σ_z/σ_δ(SR, mm/%)	0.73/0.05	0.91/0.09	0.98/0.14	0.68/0.20
L (design, $cm^{-2}s^{-1}$)/IP	58.6×10^{34}	16.4×10^{34}	5.08×10^{34}	1.32×10^{34}
Weak strong with BS	σ_z self-consistent			
σ_z/σ_δ(mm/%)	1.6/0.11	2.1/0.21	2.2/0.31	0.79/0.23
σx/σy (μm)	99/0.25	20/0.18	48/0.16	36/0.044
L ($cm^{-2}s^{-1}$)/IP	$80. \times 10^{34}$	18.4×10^{34}	5.8×10^{34}	1.56×10^{34}
			Strong-strong	
σ_z/σ_δ(mm/%)			2.7/0.38	
σx/σy (μm)			50/0.25	
L ($cm^{-2}s^{-1}$)/IP			5.2×10^{34}	
(νx, νy)/IP			0.515,0.58	

Scan for β_y



Not sensitive for β_y.

Summary

- Beam-beam simulation including beamstrahlung was performed.
- The Weak-strong simulation agreed with analytic formula (20% longer bunch length and energy spread.
- In the Strong-strong simulation, bunch lengthening due to beamstrahung is enhanced. The horizontal beam size is squeezed by dynamic beta in this operating point.
- Unstable at ν_x, ν_y =0.52,0.58. Choice of the operating point.
- The simulated luminosity was satisfied to the TLEP design.
- Even asymmetry of two beams, reliable equilibrium was found.