PREDICTIONS OF LIGHT HADRONIC DECAYS OF HEAVY QUARKONIUM $^1D_2$ STATES IN NRQCD

Ying Fan
Peking University

In collaboration with Z.G.He, Y.Q.Ma & K.T.Chao
Phys.Rev.D80:014001,2009
Phys.Rev.Lett.101:112001,2008
Phys.Rev.D81:074032,2010

May 19th,2010 QWG7 at Fermilab

Outline

- Introduction and motivation
- Decay width determination in NRQCD
- Results and Discussions
- Non-$D\bar{D}$ annihilation decay of Ψ(3770)
- Summary

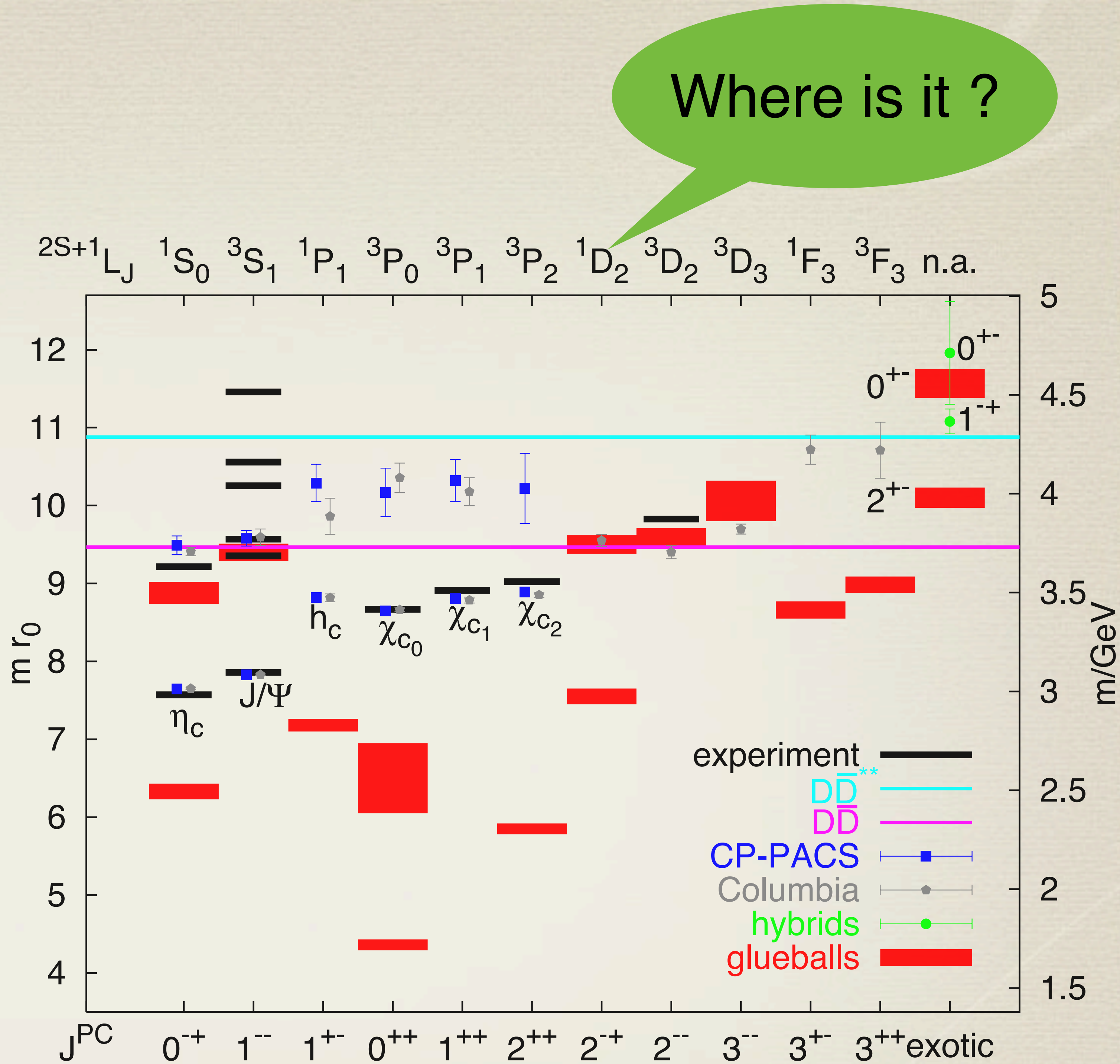
INTRODUCTION & MOTIVATION

☆ the only missing low-lying spin-singlet state in the charmonium family

☆ mass (between 3.80 and 3.84 GeV) below $D^*\bar{D}$ threshold (3.87GeV)

☆ Odd parity forbids its decay to $D\bar{D}$

☆ a narrow resonance state



★ see CERN Yellow Report, CERN-2005-005 for this figure

Main Decay Modes

Decay Modes	Theoretical Evaluation (keV)	References
$\eta_{c2} \to \gamma\, h_c$	339-375	Barnes, Godfrey & Swanson (2005); Li &Chao (2009)
$\eta_{c2} \to \pi\,\pi\,\eta_c$	≈ 45	Eichten, Lane & Quigg (2002)
$\eta_{c2} \to$ light hadrons	110	Eichten, Lane & Quigg (2002)
	274-392	Fan, He, Ma & Chao (2009)

DECAY WIDTH DETERMINATION IN NRQCD

Decay width in NRQCD

$$\Gamma(H \to LH) = \sum_n \frac{2\,\mathrm{Im}\,\boxed{f_n(\mu_\Lambda)}}{m_Q^{d_n-4}} \boxed{\langle H | \boxed{O_n(\mu_\Lambda)} | H \rangle}$$

short-distance coefficients

long-distance matrix elements

$$|{}^1D_2\rangle = O(1)\boxed{|Q\bar{Q}({}^1D_2^{[1]})\rangle} + O(v)\boxed{|Q\bar{Q}({}^1P_1^{[8]})g\rangle} + O(v^2)\boxed{|Q\bar{Q}({}^1S_0^{[1,8]})gg\rangle} + \cdots$$

Color Singlet Model

Color-Octet Mechanism

$$\boxed{A(Q\bar{Q} \to Q\bar{Q})\Big|_{pertQCD}} = \sum_n \boxed{\frac{f_n(\mu_\Lambda)}{m_Q^{d_n-4}}} \boxed{\langle Q\bar{Q}|O_n(\mu_\Lambda)|Q\bar{Q}\rangle\Big|_{pertNRQCD}}$$

Optical theorem, covariant projection method, real corrections, virtual corrections

Dim. Reg.
D dimensions

Mathematica
FeynCalc, FeynArts

method of regions, expansion to NLO in α_s

Residual soft divergence $1/\varepsilon$ & Coulomb singularity $1/v$

Cancel
Exactly

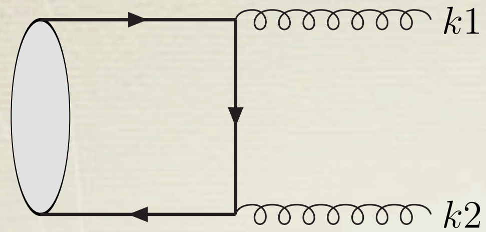
soft divergence $1/\varepsilon$ & Coulomb singularity $1/v$

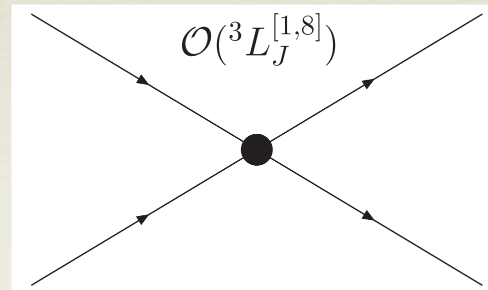
long-distance matrix elements by wave function & operator evolution equation

Decay width $= \sum$ $\boxed{\text{finite } \mathrm{Im} f_n(\mu_\Lambda)}$ $\times$ long-distance matrix elements

Divergence Cancellation

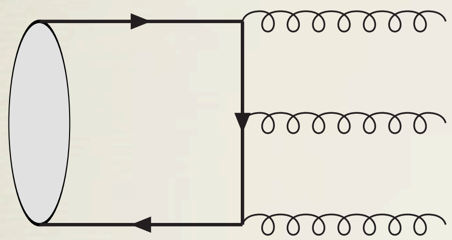
α_s^2

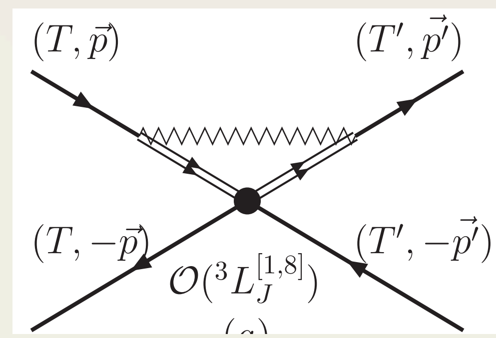




soft

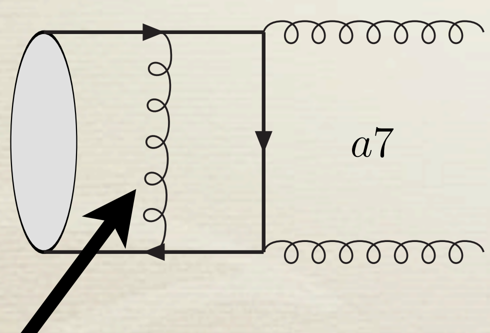
α_s^3

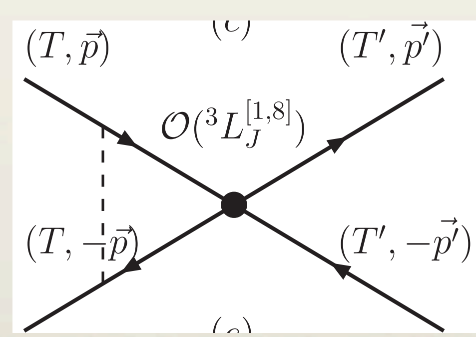




$$= \frac{\alpha_s}{3\pi m_Q^2}\left(\frac{1}{\varepsilon_{UV}} - \frac{1}{\varepsilon}\right)\vec{p}\cdot\vec{p}'$$

$$\left\langle O^R({}^1P_1^{[8]})\right\rangle_{NLO} = \cdots + \frac{4\alpha_s\left(\frac{\mu}{\mu_\Lambda}\right)^{2\varepsilon}}{3\pi m_Q^2}\left(-\frac{1}{\varepsilon} - \ln 4\pi + \gamma_E\right)C_F\left\langle O({}^1D_2^{[1]})\right\rangle_{LO}$$





$$= \frac{\alpha_s\pi}{4v}\left(1 - \frac{i}{\pi}\left(\frac{1}{\varepsilon} - \ln\left(\frac{m_Q^2v^2}{\pi\mu^2}\right) - \gamma_E\right)\right)$$

Coulomb

$$\left\langle O^R({}^1D_2^{[1]})\right\rangle_{NLO} = \left(1 + \frac{\alpha_s\pi}{2v}C_F\right)\left\langle O({}^1D_2^{[1]})\right\rangle_{LO}$$

Results and Discussions

- The long-distance matrix element of D-wave color-singlet operator is related with the second derivative of radial wave function at the origin:

$$\langle n^1D_2 | O(n^1D_2) | n^1D_2 \rangle = \frac{15 |R''_{nD}(0)|^2}{8\pi}$$

- P-wave and S-wave matrix elements are given by operator evolution equations

$$\langle 1^1D_2 | O^R(^1P_1^{[8]})(\mu_\Lambda) | 1^1D_2 \rangle = \frac{8C_F}{3m_Q^2 b_0} \ln \frac{\alpha_s(\mu_{\Lambda_0})}{\alpha_s(\mu_\Lambda)} \langle 1^1D_2 | O(^1D_2^{[1]}) | 1^1D_2 \rangle$$

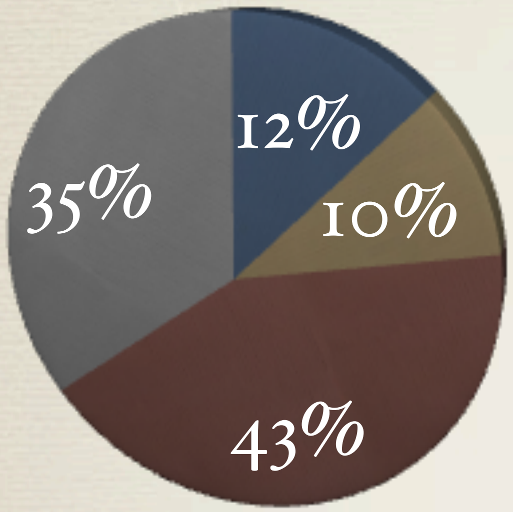
- m_c=1.5GeV, v^2=0.3, $\mu_{\Lambda 0}=m_c v$, $\mu_\Lambda=2m_c$, $\alpha_s(2m_c)$=0.249, N_f=3, Λ_{QCD}=390MeV

Subprocess	short-distance	long-distance(MeV)
$(^1S_0)_1 \to$ LH	$8.38\alpha_s^2 + 29.6\alpha_s^3$	0.0368
$(^1S_0)_8 \to$ LH	$2.62\alpha_s^2 + 11.4\alpha_s^3$	0.0920
$(^1P_1)_8 \to$ LH	$1.57\alpha_s^2 + 6.18\alpha_s^3$	0.680
$(^1D_2)_1 \to$ LH	$1.12\alpha_s^2 + 1.67\alpha_s^3$	0.786

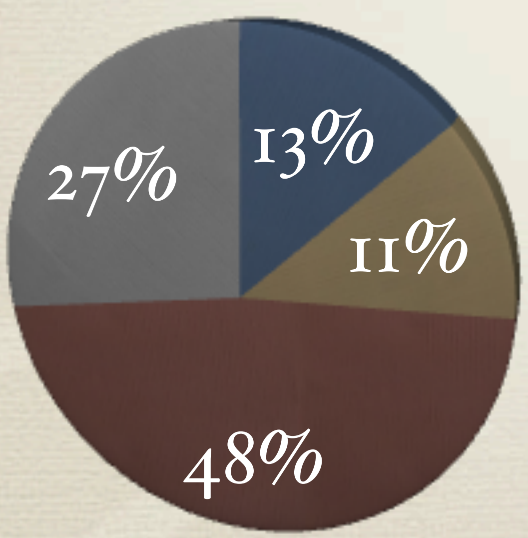
Results and Discussions

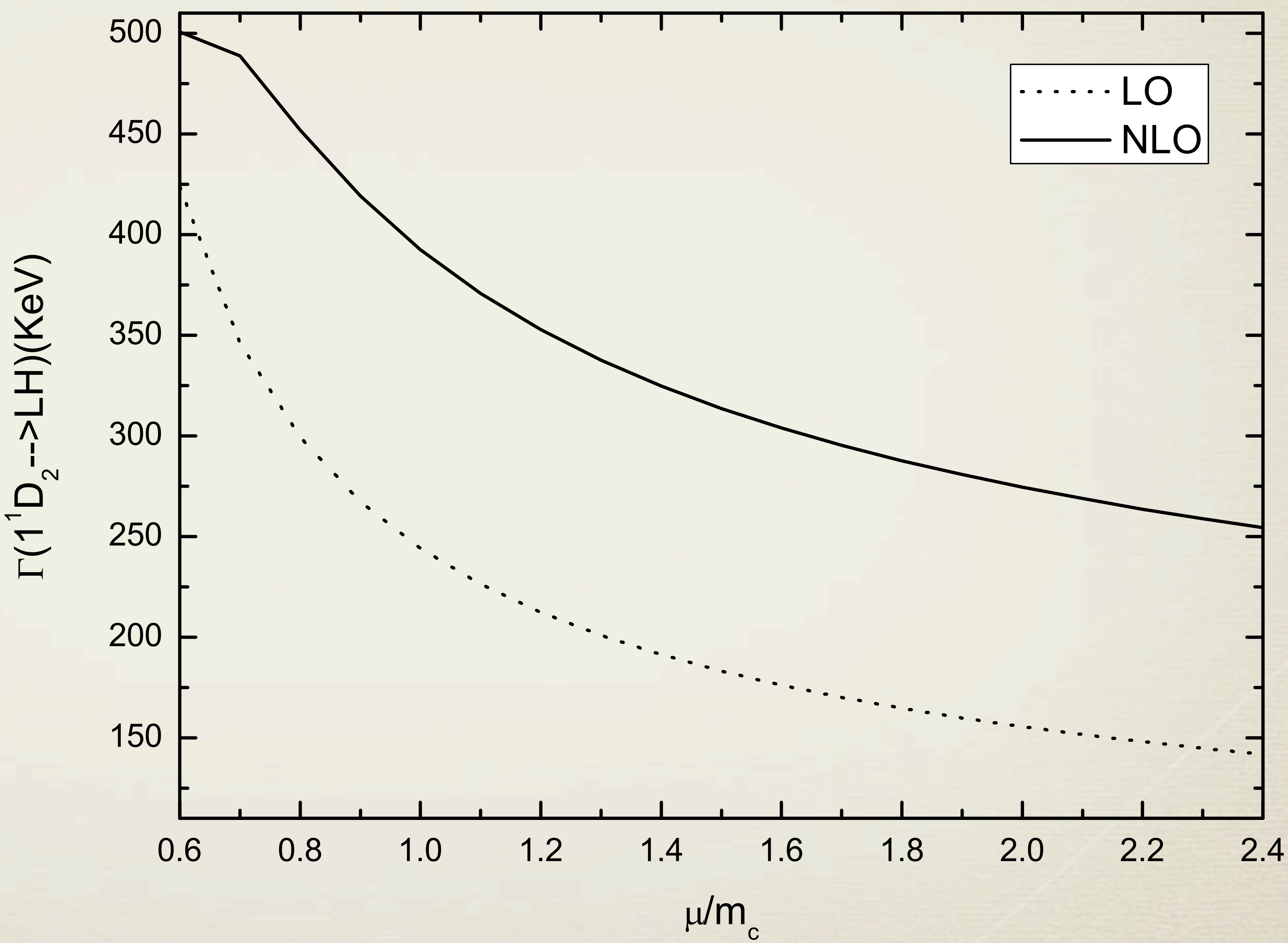
α_s^2 155 keV

S-singlet S-octet P-octet D-singlet



α_s^3 274 keV

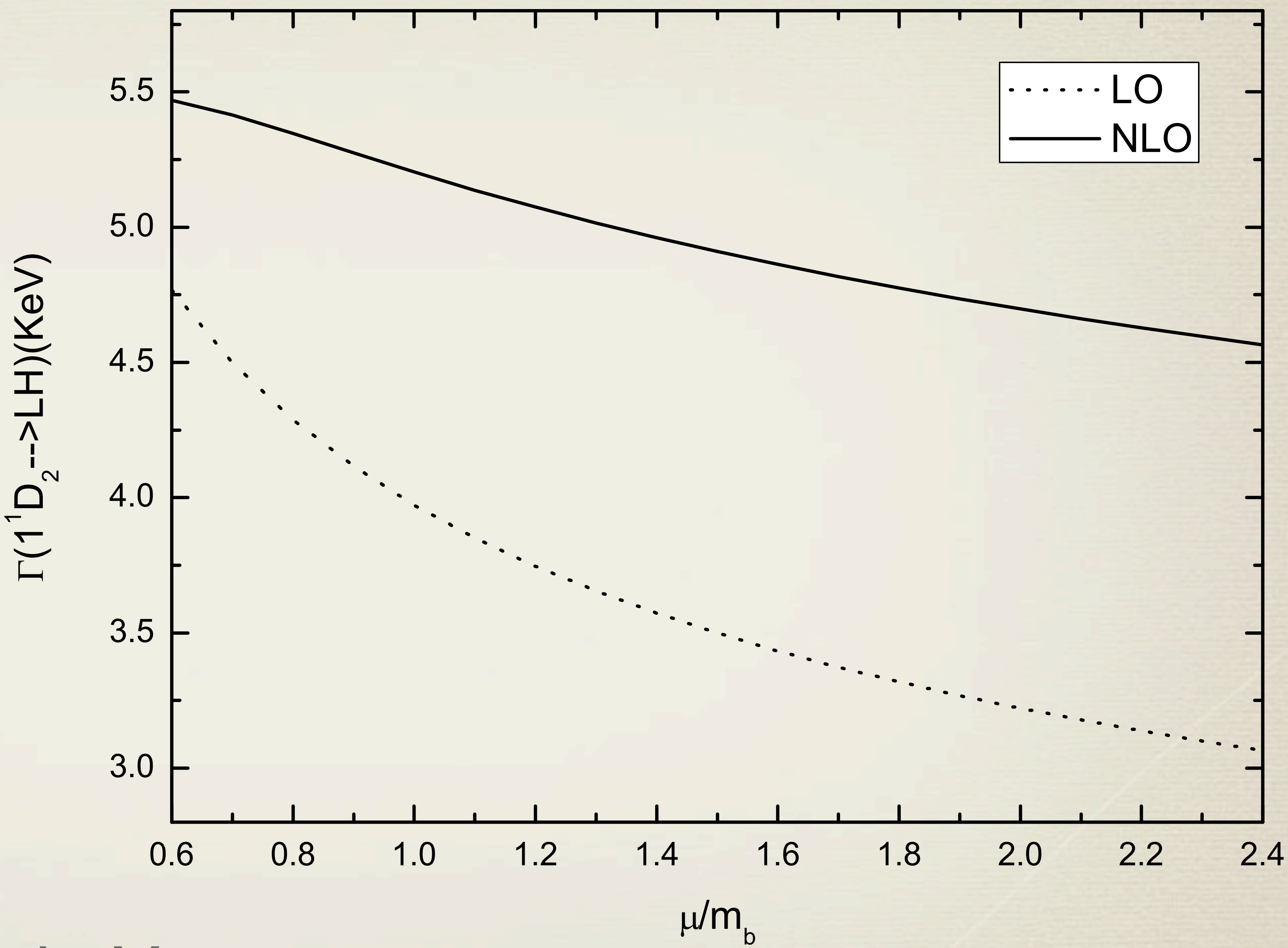




enhancement of factor 1.8

Results and Discussions

- m_b=4.6GeV, v^2=0.1, $\mu_{\Lambda 0}$=$m_b v$, μ_Λ=$2m_b$, $\alpha_s(2m_b)$ =0.180,N_f=4, Λ_{QCD}=340MeV



$\Gamma_B(1^1D_2 \rightarrow LH)$=4.70 keV,
$\Gamma_B(2^1D_2 \rightarrow LH)$=8.78 keV.

enhancement of factor 1.5

Results and Discussions

$\Gamma(\eta_{c2} \to LH)$	274-392 keV
$\Gamma(\eta_{c2} \to \gamma\, h_c)$	339-375 keV
$\Gamma(\eta_{c2} \to \pi\pi\eta_c)$	≈ 45 keV
Γ_{total}	660-810 keV

- searching through $\eta_{c2} \to \gamma h_c \to \gamma\gamma\eta_c \to \gamma\gamma K\bar{K}\pi$

- Spin-singlet nature forbids η_{c2} to couple to a photon, or to be detected from the E1 transitions of higher spin-triplet charmonia

- Production rates are suppressed by the small values of the second derivative squared of the wave function at the origin

- The study for the inclusive light hadronic decay of η_{c2} in NRQCD will provide useful information on searching in high-energy $p\bar{p}$ collision, in B decay, in higher charmonium transitions, in e^+e^- process in BESIII at BEPC, and in the low-energy $p\bar{p}$ reaction in PANDA at FAIR.

Results and Discussions

$\Gamma(\eta_{c2} \to LH)$	274-392 keV
$\Gamma(\eta_{c2} \to \gamma h_c)$	339-375 keV
$\Gamma(\eta_{c2} \to \pi\pi\eta_c)$	≈ 45 keV
Γ_{total}	660-810 keV

- ☆searching through $\eta_{c2} \to \gamma h_c \to \gamma\gamma\eta_c \to \gamma\gamma K\bar{K}\pi$

- ☆Spin-singlet nature forbids η_{c2} to couple to a photon, or to be detected from the E1 transitions of higher spin-triplet charmonia

- ☆Production rates are suppressed by the small values of the second derivative squared of the wave function at the origin

- ☆The study for the inclusive light hadronic decay of η_{c2} in NRQCD will provide useful information on searching in high-energy $p\bar{p}$ collision, in B decay, in higher charmonium transitions, in e^+e^- process in BESIII at BEPC, and in the low-energy $p\bar{p}$ reaction in PANDA at FAIR.

Results and Discussions

$\Gamma(\eta_{c2} \to LH)$	274-392 keV
$\Gamma(\eta_{c2} \to \gamma\, h_c)$	339-375 keV
$\Gamma(\eta_{c2} \to \pi\,\pi\,\eta_c)$	≈ 45 keV
Γ_{total}	660-810 keV

$\Rightarrow$ $Br(\eta_{c2} \to \gamma\, h_c) = (44\text{-}54)\%$

- searching through $\eta_{c2} \to \gamma h_c \to \gamma\gamma\eta_c \to \gamma\gamma K\bar{K}\pi$

- Spin-singlet nature forbids η_{c2} to couple to a photon, or to be detected from the E1 transitions of higher spin-triplet charmonia

- Production rates are suppressed by the small values of the second derivative squared of the wave function at the origin

- The study for the inclusive light hadronic decay of η_{c2} in NRQCD will provide useful information on searching in high-energy $p\bar{p}$ collision, in B decay, in higher charmonium transitions, in e^+e^- process in BESIII at BEPC, and in the low-energy $p\bar{p}$ reaction in PANDA at FAIR.

Previous work in NRQCD

- Removed the infrared divergences in inclusive P-wave charmonium
  - Production: Bodwin, Braaten,Yuan & Lepage (1992);
  - Decay : Bodwin, Braaten & Lepage(1995); Huang &Chao (1996,1997).
- Removed the infrared divergences in the Color Singlet Model calculation of inclusive light hadronic decays of ^3D_J charmonium states.
  - Belanger & Moxhay (1987): $\Gamma(^3D_1 \to 3g) = \frac{48640}{81\pi} \frac{\alpha_s^3 |R_D''(0)|^2}{M^6} \ln \frac{M}{\Delta}$
  - Bergstrom & Ernstrom (1991).

NON-$D\bar{D}$ ANNIHILATION DECAY OF $\Psi(3770)$

Motivation

Collaboration	$B_r(\Psi(3770)\rightarrow \text{non-}D\bar{D})$	References
BES	(14.5±1.7±5.8)%	PLB 641,145 (2006)
	(16.4±7.3±4.2)%	PRL 97,121801 (2006)
	(13.4±5.0±3.6)%	PRD 76,122002 (2007)
CLEO	$\cong 0$	PRL 96,092002 (2006)
	$(-3.3\pm1.4^{+6.6}_{-4.8})\%$, <9%	PRL 104,159901 (2010)

Very different results !

Light Hadronic Decay of Ψ(3770)

- Ψ(3770) viewed as 1^3D_1 dominated with small admixture of 2^3S_1 (Rosner,2001)

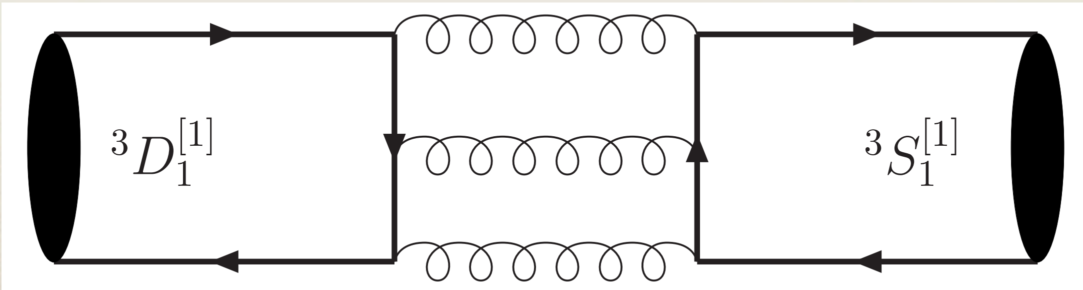
$$|\psi(3770)\rangle = \cos\theta\left|1^3D_1\right\rangle + \sin\theta\left|2^3S_1\right\rangle$$

$$|\psi(3686)\rangle = -\sin\theta\left|1^3D_1\right\rangle + \cos\theta\left|2^3S_1\right\rangle$$

- S-D mixing angle $\theta \cong (12\pm2)^o$, by fitting the leptonic decay widths of Ψ(3770) and Ψ(3686)

- Light hadronic decay width of Ψ(3770) is

$$\Gamma\left(\psi(3770)\to LH\right) = \cos^2\theta\Gamma\left(1^3D_1\to LH\right) + \sin^2\theta\Gamma\left(2^3S_1\to LH\right) + \text{interference term}$$



Results and Discussions

Subprocess	α_s^2(keV)	α_s^3(keV)
$(^3S_1)_1 \to LH$	0	0.24
$(^3S_1)_8 \to LH$	18	33
$(^3P_0)_8 \to LH$	184	410
$(^3P_1)_8 \to LH$	0	-5.8
$(^3P_2)_8 \to LH$	2.5	4.4
$(^3D_1)_1 \to LH$	0	-10
Total	205	436

- m_c=1.5±0.1GeV,

  $\Gamma(\Psi(3770) \to LH) = 467^{-187}_{+338}$ keV

- Total decay width(PDG06)

  $\Gamma(\Psi(3770) \to \text{anything}) = 23.0 \pm 2.7$ MeV

- Branching ratio

  $B_r(\Psi(3770) \to LH) = (2.0^{-0.80}_{+1.50})\%$

- E1 transition

  $\Gamma(\Psi(3770) \to \Upsilon + \chi_{CJ}) = 250 \pm 50$ keV

- Hadronic transition

  $\Gamma(\Psi(3770) \to J/\psi + LH) = 100 \sim 150$ keV

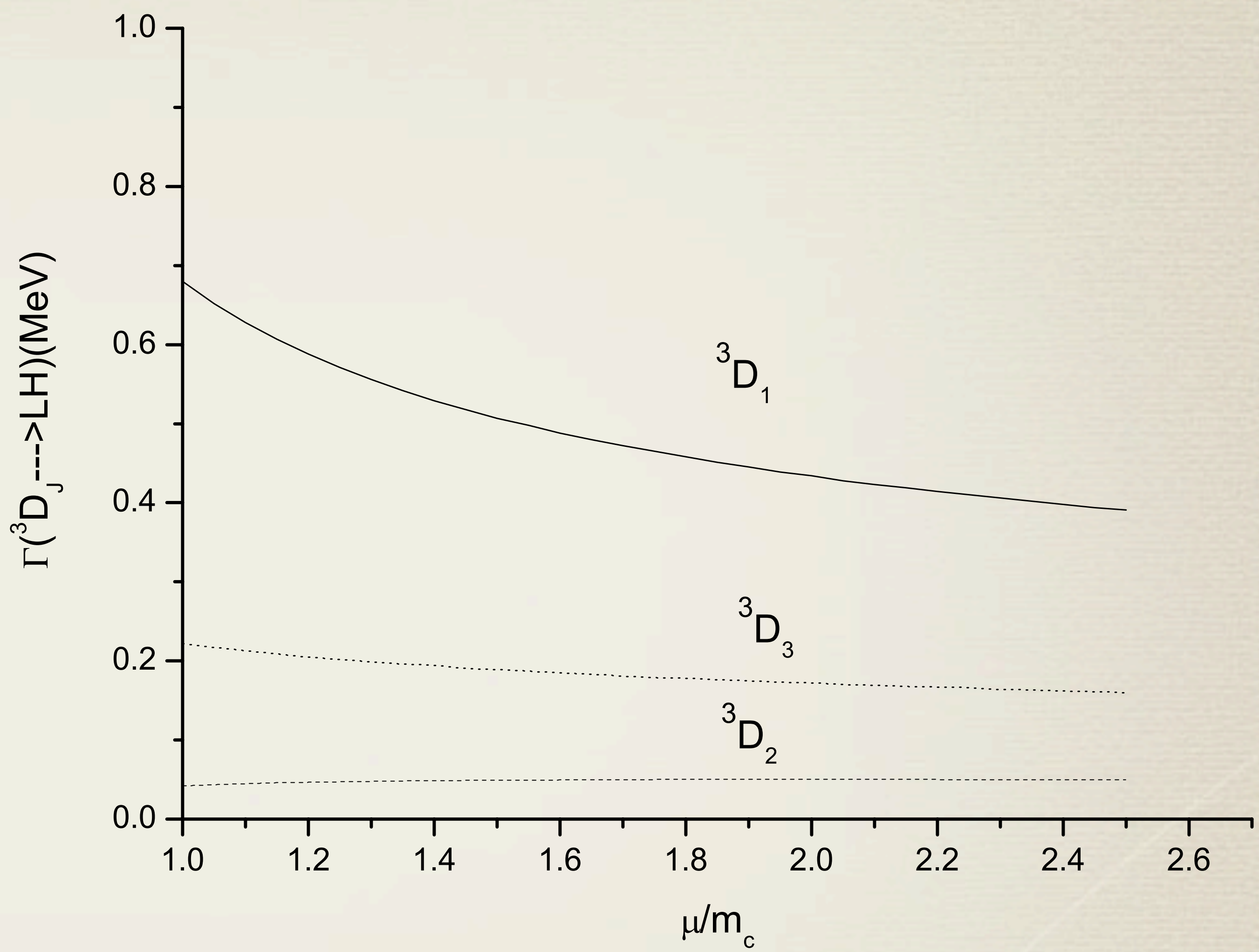
- Final result

  $\Gamma(\Psi(3770) \to \text{non-}D\bar{D}) = 1.15 \sim 1.20$ MeV

  $B_r(\Psi(3770) \to \text{non-}D\bar{D}) \cong 5\%$

Results and Discussions

LH decay width	$\psi(1^3D_1)$ (keV)	$\psi(1^3D_2)$ (keV)	$\psi(1^3D_3)$ (keV)
$\mu=2m_c$	435	50	172
$\mu=m_c$	683	42	223
Potential model	240	18	102



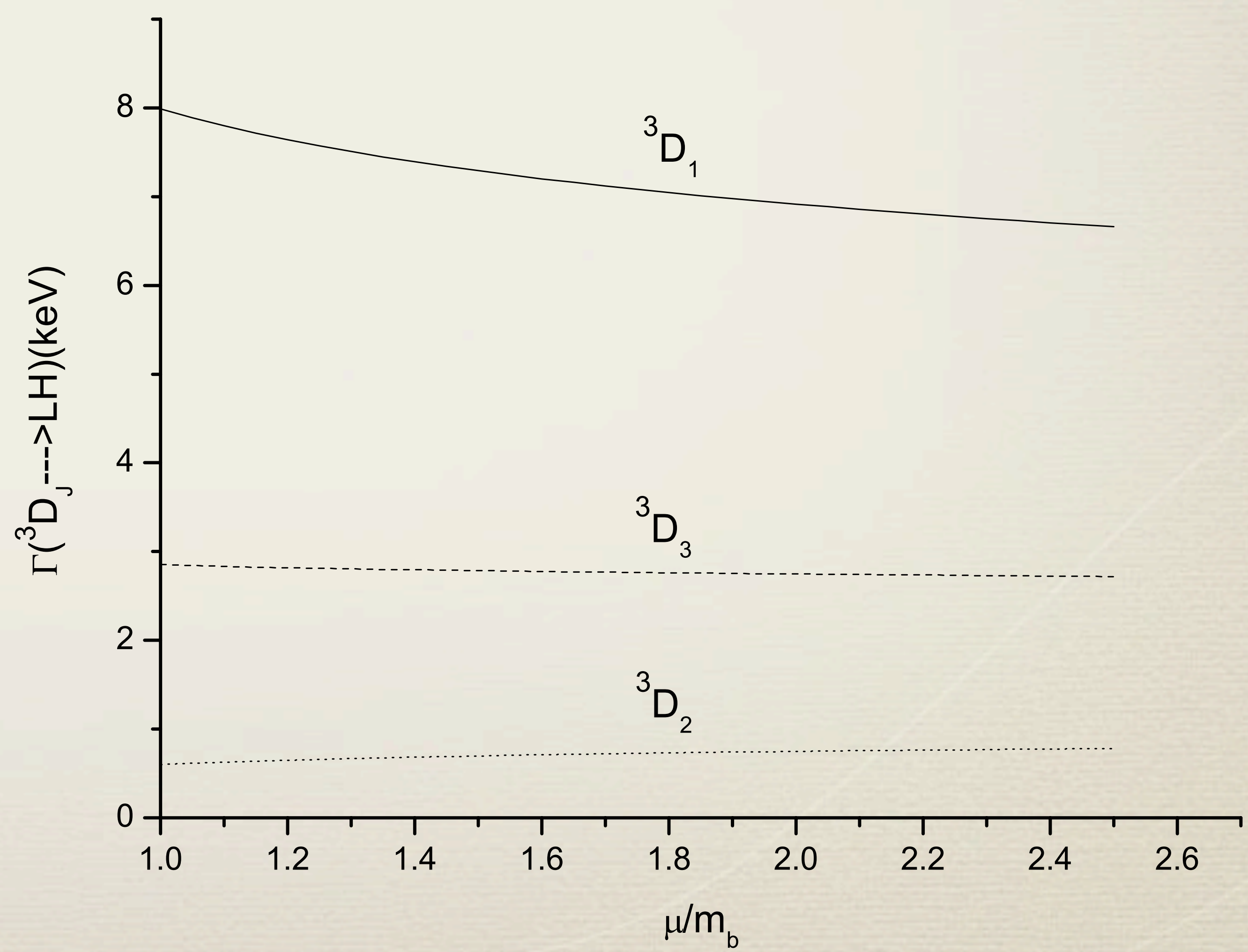
NRQCD predictions about 2~3 times larger than potential model results

Results and Discussions

LH decay width	$\Upsilon(1^3D_1)$ (keV)	$\Upsilon(1^3D_2)$ (keV)	$\Upsilon(1^3D_3)$ (keV)
$\mu=2m_b$	6.91	0.75	2.75
$\mu=m_b$	7.99	0.60	2.85
Potential model	5.4	0.51	2.3

LH decay width	$\Upsilon(2^3D_1)$ (keV)	$\Upsilon(2^3D_2)$ (keV)	$\Upsilon(2^3D_3)$ (keV)
$\mu=2m_b$	12.9	1.40	5.14
$\mu=m_b$	14.9	1.21	5.33

To some extent, potential model estimations are in agreement with our NRQCD numerical predictions with $\mu=2m_b$.



SUMMARY

Summary

- Prediction of $^1D_2$ state in NRQCD will help find this missing state in experiment
- Light hadronic decay width of $\Psi(3770)$ was also determined in NRQCD, and the non-$D\bar{D}$ annihilation decay branching ratio does not favor BES or CLEO results
- ^3D_J decay widths have also been predicted, and compared with potential model results too.
- More experiment analyses are needed to test theoretical predictions

THANK YOU !

BACK UP

$^3D_1$ inclusive decay in CSM

color factor[4,5] 5/18 (for color-triplet quarks). The choice of the infrared cutoff is ambiguous in the case of a quark–antiquark bound state; a procedure which works reasonably well in the P-wave case[1,2,4,5] is to assume an infrared cutoff $\Delta \sim \langle r \rangle^{-1}$, where $\langle r \rangle$ is the radius of the bound state. Then, defining the bound-state mass $M \approx 2m$, we find for three-gluon annihilation of quarkonium:

$$\Gamma(^3D_1 \to 3g) \quad = \quad \frac{48640}{81\pi} \frac{\alpha_s^3 |R_D''(0)|^2}{M^6} \ln \frac{M}{\Delta} \quad ,$$

γ⁵ scheme & projection operator

$$\gamma^5 = -\frac{i}{4!}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma$$

D dimensions; equivalent to the naive γ^5 scheme for processes where each closed fermion chain contains at most one γ^5

$$\Pi^0 = \frac{1}{2\sqrt{2}(E_Q+m_Q)}(\frac{\not{P}}{2}+\not{q}+m_Q)\frac{[(\not{P}+M)\gamma^5+\gamma^5(-\not{P}+M)]}{2M}(\frac{\not{P}}{2}-\not{q}-m_Q)$$

$$\Pi^0 = \frac{1}{2\sqrt{2}(E_Q+m_Q)}(\frac{\not{P}}{2}+\not{q}+m_Q)\frac{(\not{P}+M)\gamma^5(-\not{P}+M)}{2M^2}(\frac{\not{P}}{2}-\not{q}-m_Q)$$

the above two projection operators both give correct results and keep **C** parity conservation

Feynman Diagrams