Double charmonium production in exclusive bottomonia decays

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Outline:

- Introduction
- Leading twist bottomonia decays
- $\eta_b \to J/\Psi J/\Psi$ decay

Introduction

Hard exclusive processes

Exclusive processes:

1. Decays: $\Upsilon \to \rho\pi, \eta_c \to \rho\rho, \ldots$
2. Annihilation: $e^+e^- \to J/\Psi J/\Psi, \ldots$
3. Different formfactors: $F_\pi(Q^2), \ldots$

General property:

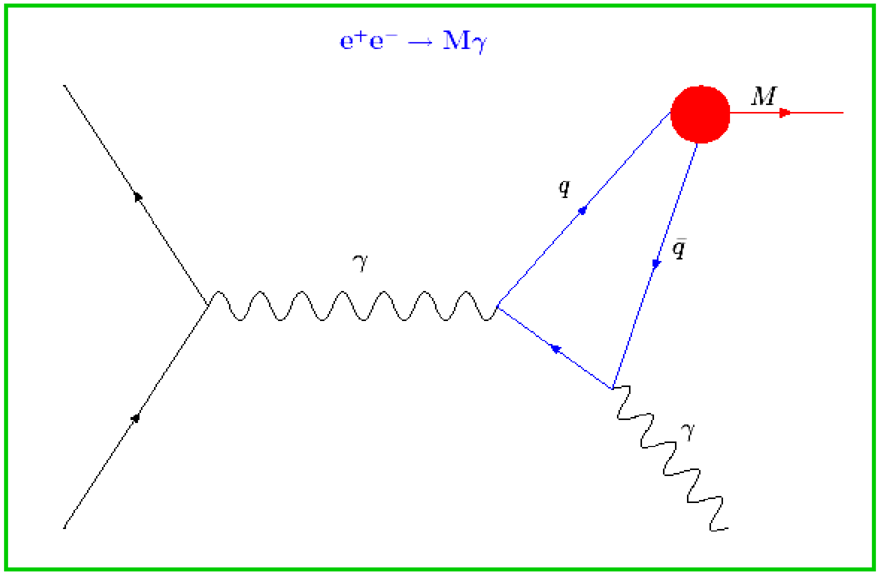
$$E_h \gg \Lambda_{QCD}, M_h$$

Expansion parameter: $\sim \dfrac{M_{cc}^2}{M_{bb}^2} \sim \dfrac{1}{10}$

$$\sigma = \underbrace{\frac{a_n(E_h = \infty)}{E_h^n}}_{Leading\ \ twist} + \underbrace{\frac{a_{n+1}(E_h = \infty)}{E_h^{n+1}}}_{Next-to-leading\ \ twist} + \ldots$$

Light cone expansion formalism

Factorization



Factorization formula :

$$T = \sum_n \overbrace{C_n}^{Short\ Dist.} \times \underbrace{< M \,|\, O_n(0) \,|\, 0 >}_{Large\ \ Dist.}$$

Different Contributions:

1. Short distance contribution : C_n (perturbative QCD)
2. Large distance contribution : $< M \,|\, O_n \,|\, 0 >$ (nonperturbative effects)

The leading twist distribution amplitude

Operators that contribute at the leading order approximation:

$$< \mathrm{M(p)}|\bar{\mathrm{q}}\, \gamma_\mu \gamma_5\ \mathrm{D}_{\mu_1}...\mathrm{D}_{\mu_n} \mathrm{q}|0 >\ \mathrm{z}^\mu \mathrm{z}^{\mu_1}...\mathrm{z}^{\mu_n} \sim\ (\mathrm{pz})^{\mathrm{n+1}} \int_{-1}^{1} d\xi\ \xi^{\mathrm{n}} \varphi(\xi), \quad \mathrm{z}^2 = 0, \quad \xi = x_1 - x_2$$

Distribution amplitude $\varphi(\xi)$ can be considered as a meson wave function.

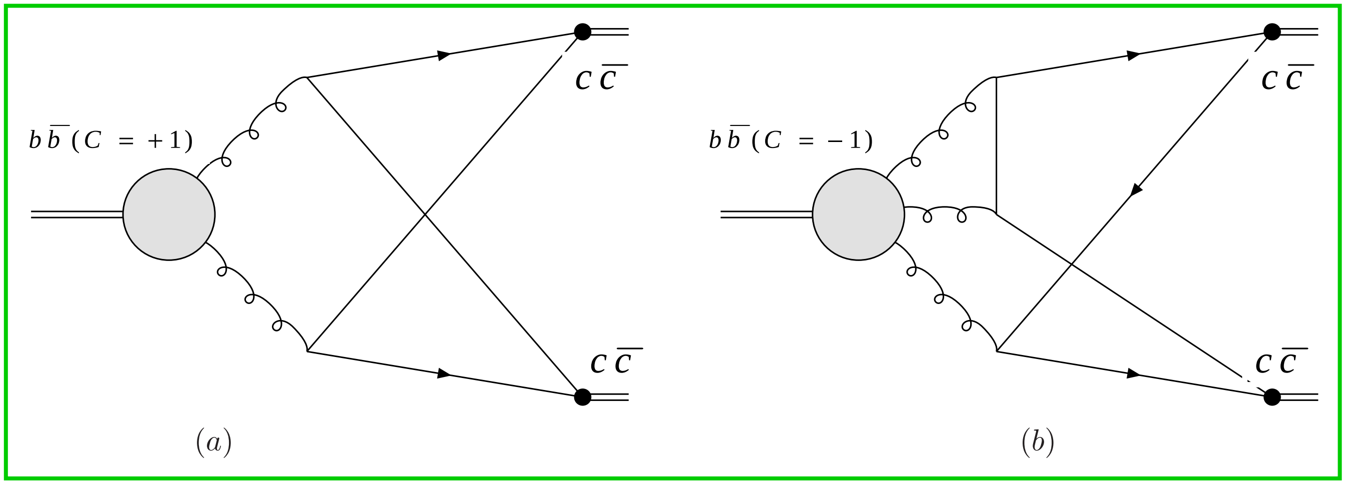
Distribution amplitudes

- Resume infinite series of operators
- Resume leading logarithmic radiative corrections $\left(\alpha_s(E_h) \times Log\left(\frac{E_h^2}{m_c^2}\right)\right)^n$

$$T = \int_{-1}^{1} d\xi\ \mathrm{H}(\xi, \mu)\, \varphi(\xi, \mu), \qquad \mu \sim \mathrm{E}_{\mathrm{hard}}$$

Leading twist bottomonia decays

Leading twist decays



- C=-1 bottomonia are suppressed
- Leading twist decays $J_{bb} = 0, 2$

The amplitudes of the decays

$$T = \int d\xi_1 d\xi_2 \; H(\xi_1, \xi_2) \times \varphi_1(\xi_1)\, \varphi_2(\xi_2)$$

Charmonia distribution amplitudes

- Distribution amplitudes of the S-wave charmonia
  3 DAs, Phys.Lett.B646,80,(2007), Phys.Rev.D75:094016,(2007), Phys.Rev.D77:034026,(2008)
- Distribution amplitudes of the P-wave charmonia
  7 DAs, Phys.Rev.D79:074004,(2009)

$$\langle v^2 \rangle_{1S} = 0.21 \pm 0.04 \quad \langle v^2 \rangle_{2S} = 0.54^{+0.33}_{-0.36} \quad \langle v^2 \rangle_{2P} = 0.30 \pm 0.10$$

Charmonia are unique mesons since one can build realistic models of charmonia distribution amplitudes

$M_1 M_2$	Γ_{NRQCD}, eV	Γ_{LC}, eV	$\mathrm{Br}_{\mathrm{LC}}$, %
$\eta_b \to h_c J/\psi$	$73.^{+15.}_{-11.} \pm 38. \pm 37.$	$520.^{+44.}_{-43.} \pm 59. \pm 140.$	0.0047
$\eta_b \to h_c \psi(2S)$	$39.^{+7.8}_{-6.} \pm 26. \pm 19.$	$300.^{+28.}_{-28.} \pm 33. \pm 78.$	0.0027
$\eta_b \to \eta_c \chi_{c0}$	$56.^{+18.}_{-14.} \pm 29. \pm 28.$	$49.^{+4.1}_{-4.1} \pm 25. \pm 71.$	4.5×10^{-4}
$\eta_b \to \eta_c(2S) \chi_{c0}$	$30.^{+9.3}_{-7.4} \pm 20. \pm 15.$	$26.^{+2.4}_{-2.4} \pm 18. \pm 37.$	2.3×10^{-4}
$\eta_b \to \eta_c \chi_{c2}$	$17.^{+5.2}_{-5.2} \pm 8.9 \pm 8.6$	$97.^{+8.2}_{-8.1} \pm 48. \pm 71.$	8.9×10^{-4}
$\eta_b \to \eta_c(2S) \chi_{c2}$	$9.1^{+2.8}_{-2.7} \pm 6.1 \pm 4.6$	$51.^{+4.8}_{-4.8} \pm 35. \pm 37.$	4.6×10^{-4}
$\eta_b \to \chi_{c0} \chi_{c1}$	$11.^{+2.}_{-2.} \pm 8.4 \pm 5.5$	$26.^{+2.2}_{-2.2} \pm 13. \pm 38.$	2.4×10^{-4}
$\eta_b \to \chi_{c1} \chi_{c2}$	$4.7^{+1.2}_{-1.1} \pm 3.6 \pm 2.4$	$51.^{+4.4}_{-4.3} \pm 26. \pm 37.$	4.7×10^{-4}
$\chi_{b0} \to \eta_c \chi_{c1}$	$12.^{+2.4}_{-2.4} \pm 6.1 \pm 5.8$	$53.^{+2.4}_{-2.3} \pm 26. \pm 17.$	0.0064
$\chi_{b0} \to \eta_c(2S) \chi_{c1}$	$6.2^{+1.3}_{-1.3} \pm 4.2 \pm 3.1$	$39.^{+5.2}_{-6.1} \pm 27. \pm 13.$	0.0048
$\chi_{b0} \to \chi_{c0} \chi_{c2}$	$(5.9^{+42}_{-4.1} \pm 4.5 \pm 2.9) \times 10^{-4}$	$0.81^{+0.19}_{-0.18} \pm 0.41 \pm 1.8$	10^{-4}
$\chi_{b0} \to \eta_c \eta_c$	$49.^{+11.}_{-8.8} \pm 35. \pm 25.$	$51.^{+2.1}_{-1.9} \pm 25. \pm 16.$	0.0062
$\chi_{b0} \to \eta_c \eta_c(2S)$	$26.^{+5.7}_{-4.8} \pm 17. \pm 13.$	$76.^{+9.7}_{-11.} \pm 52. \pm 24.$	0.0093
$\chi_{b0} \to \eta_c(2S) \eta_c(2S)$	$14.^{+3.}_{-2.5} \pm 15. \pm 6.9$	$30.^{+7.2}_{-8.4} \pm 25. \pm 9.6$	0.0036
$\chi_{b0} \to J/\psi J/\psi$	$27.^{+5.}_{-4.2} \pm 19. \pm 13.$	$79.^{+3.2}_{-2.9} \pm 2.7 \pm 25.$	0.0096
$\chi_{b0} \to J/\psi \psi(2S)$	$14.^{+2.8}_{-2.2} \pm 9.3 \pm 7.1$	$130.^{+16.}_{-19.} \pm 4. \pm 41.$	0.016
$\chi_{b0} \to \psi(2S) \psi(2S)$	$7.5^{+1.4}_{-1.2} \pm 8.3 \pm 3.8$	$54.^{+13.}_{-15.} \pm 1.5 \pm 17.$	0.0066
$\chi_{b0} \to h_c h_c$	$0.094^{+0.023}_{-0.029} \pm 0.072 \pm 0.047$	$15.^{+3.5}_{-3.3} \pm 2.3 \pm 3.9$	0.0018
$\chi_{b0} \to \chi_{c0} \chi_{c0}$	$0.048^{+0.061}_{-0.034} \pm 0.037 \pm 0.024$	$0.21^{+0.048}_{-0.048} \pm 0.11 \pm 1.1$	2.5×10^{-5}
$\chi_{b0} \to \chi_{c1} \chi_{c1}$	$0.56^{+0.26}_{-0.16} \pm 0.43 \pm 0.28$	$14.^{+0.88}_{-0.87} \pm 6.8 \pm 15.$	0.0017
$\chi_{b0} \to \chi_{c2} \chi_{c2}$	$0.026^{+0.034}_{-0.013} \pm 0.02 \pm 0.013$	$0.81^{+0.19}_{-0.18} \pm 0.4 \pm 1.8$	9.9×10^{-5}
$\chi_{b1} \to h_c J/\psi$	$1.2^{+0.22}_{-0.19} \pm 0.64 \pm 0.61$	$14.^{+1.7}_{-1.7} \pm 1.6 \pm 4.6$	8.9×10^{-4}
$\chi_{b1} \to h_c \psi(2S)$	$0.65^{+0.12}_{-0.1} \pm 0.44 \pm 0.32$	$15.^{+4.}_{-4.4} \pm 1.7 \pm 4.7$	9.2×10^{-4}
$\chi_{b1} \to \eta_c \chi_{c0}$	$0.24^{+0.081}_{-0.05} \pm 0.13 \pm 0.12$	$1.3^{+0.18}_{-0.18} \pm 0.67 \pm 0.43$	8.3×10^{-5}
$\chi_{b1} \to \eta_c(2S) \chi_{c0}$	$0.13^{+0.027}_{-0.027} \pm 0.087 \pm 0.064$	$1.3^{+0.35}_{-0.38} \pm 0.89 \pm 0.41$	$8. \times 10^{-5}$
$\chi_{b1} \to \eta_c \chi_{c2}$	$0.74^{+0.13}_{-0.12} \pm 0.38 \pm 0.37$	$2.7^{+0.33}_{-0.31} \pm 1.3 \pm 0.42$	1.7×10^{-4}
$\chi_{b1} \to \eta_c(2S) \chi_{c2}$	$0.39^{+0.069}_{-0.063} \pm 0.26 \pm 0.19$	$2.5^{+0.69}_{-0.75} \pm 1.8 \pm 0.41$	1.6×10^{-4}
$\chi_{b1} \to \chi_{c0} \chi_{c1}$	$0.6^{+0.21}_{-0.18} \pm 0.45 \pm 0.3$	$0.68^{+0.088}_{-0.088} \pm 0.35 \pm 3.4$	4.3×10^{-5}
$\chi_{b1} \to \chi_{c1} \chi_{c2}$	$0.13^{+0.024}_{-0.019} \pm 0.098 \pm 0.064$	$1.4^{+0.17}_{-0.17} \pm 0.68 \pm 0.43$	8.4×10^{-5}
$\chi_{b2} \to \eta_c \chi_{c1}$	$1.8^{+0.31}_{-0.29} \pm 0.92 \pm 0.88$	$17.^{+0.53}_{-0.52} \pm 8.2 \pm 2.7$	0.0069
$\chi_{b2} \to \eta_c(2S) \chi_{c1}$	$0.93^{+0.17}_{-0.15} \pm 0.63 \pm 0.47$	$11.^{+1.1}_{-1.3} \pm 7.6 \pm 1.8$	0.0046
$\chi_{b2} \to \chi_{c0} \chi_{c2}$	$0.58^{+0.21}_{-0.17} \pm 0.44 \pm 0.29$	$1.4^{+0.21}_{-0.2} \pm 0.72 \pm 6.1$	5.8×10^{-4}
$\chi_{b2} \to \eta_c \eta_c$	$1.6^{+0.5}_{-0.49} \pm 1.1 \pm 0.78$	$16.^{+0.47}_{-0.43} \pm 7.8 \pm 10.$	0.0066
$\chi_{b2} \to \eta_c(2S) \eta_c$	$0.83^{+0.26}_{-0.26} \pm 0.54 \pm 0.41$	$21.^{+2.1}_{-2.5} \pm 15. \pm 14.$	0.0088
$\chi_{b2} \to \eta_c(2S) \eta_c(2S)$	$0.44^{+0.14}_{-0.14} \pm 0.49 \pm 0.22$	$8.^{+1.7}_{-2.} \pm 6.7 \pm 5.1$	0.0033
$\chi_{b2} \to J/\psi J/\psi$	$65.^{+14.}_{-12.} \pm 46. \pm 32.$	$270.^{+10.}_{-9.1} \pm 41. \pm 93.$	0.11
$\chi_{b2} \to \psi(2S) J/\psi$	$34.^{+7.8}_{-6.2} \pm 23. \pm 17.$	$380.^{+45.}_{-83.} \pm 120. \pm 130.$	0.16
$\chi_{b2} \to \psi(2S) \psi(2S)$	$18.^{+4.}_{-3.3} \pm 20. \pm 9.1$	$140.^{+34.}_{-39.} \pm 59. \pm 50.$	0.059
$\chi_{b2} \to h_c h_c$	$0.42^{+0.11}_{-0.1} \pm 0.32 \pm 0.21$	$37.^{+4.4}_{-4.2} \pm 8. \pm 24.$	0.015
$\chi_{b2} \to \chi_{c0} \chi_{c0}$	$0.014^{+0.0035}_{-0.0036} \pm 0.011 \pm 0.0072$	$0.36^{+0.083}_{-0.051} \pm 0.18 \pm 0.11$	1.5×10^{-4}
$\chi_{b2} \to \chi_{c1} \chi_{c1}$	$0.18^{+0.057}_{-0.059} \pm 0.14 \pm 0.089$	$8.^{+0.88}_{-0.83} \pm 4.7 \pm 4.$	0.0033
$\chi_{b2} \to \chi_{c2} \chi_{c2}$	$0.2^{+0.043}_{-0.049} \pm 0.15 \pm 0.098$	$9.8^{+1.9}_{-1.8} \pm 7.7 \pm 0.8$	0.004
$\chi_{b2} \to h_c J/\psi$	$7.6^{+1.5}_{-1.5} \pm 4. \pm 3.8$	$100.^{+4.4}_{-4.2} \pm 42. \pm 24.$	0.042
$\chi_{b2} \to h_c \psi(2S)$	$4.^{+0.79}_{-0.78} \pm 2.7 \pm 2.$	$71.^{+8.8}_{-10.} \pm 36. \pm 17.$	0.029
$\chi_{b2} \to \chi_{c1} \chi_{c2}$	$0.32^{+0.087}_{-0.049} \pm 0.24 \pm 0.16$	$11.^{+2.2}_{-2.1} \pm 9.5 \pm 0.047$	0.0045

Braguta, Likhoded, Luchinsky, Phys.Rev.D80:094008,2009

$\eta_b \rightarrow J/\Psi J/\Psi$ decay

$\eta_b \to J/\Psi J/\Psi$ decay

The amplitude of the process

$$T = F\, e_{\mu\nu\rho\sigma}\, p_1^{\mu} \varepsilon_1^{\nu}\, p_2^{\mu} \varepsilon_2^{\nu}$$

F is the only formfactor of the process

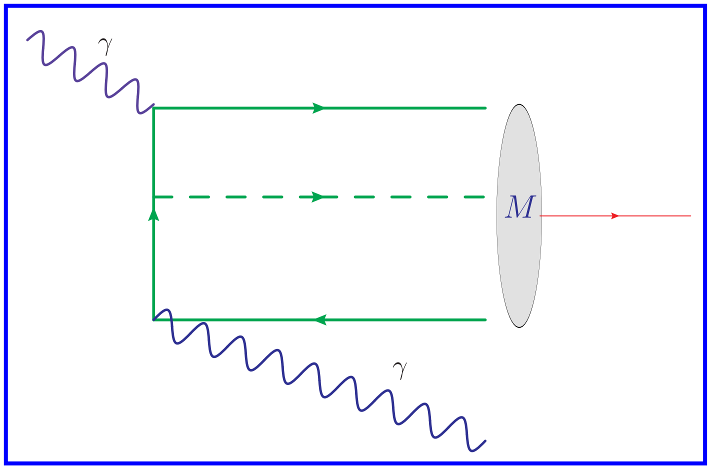
- Helicity conservation + angular momentum conservation => $\Lambda_{J/\Psi}=0$
- Helicity flip in gluon-quark-quark vertex leads to the suppression of the amplitude

$\eta_b \to J/\Psi J/\Psi$ is a next-to-next-to-leading twist process

Higher twist distribution amplitudes

Amplitude :

$$T = \underbrace{\frac{a_n}{E_h^n}}_{\text{twist}-2} + \underbrace{\frac{a_{n+1}}{E_h^{n+1}}}_{\substack{\text{twist}-2,\\ \text{twist}-3}} + \underbrace{\frac{a_{n+2}}{E_h^{n+2}}}_{\substack{\text{twist}-2,\\ \text{twist}-3,\\ \text{twist}-4}} + \ldots$$



Expansion in Fock states

$$\left|Meson\right\rangle = \underbrace{\left|\mathrm{Q}\overline{\mathrm{Q}}\right\rangle}_{\substack{twist-2,\\ twist-3,\\ \ldots}} + \underbrace{\left|\mathrm{Q}\overline{\mathrm{Q}}g\right\rangle}_{\substack{twist-3,\\ twist-4,\\ \ldots}} + \underbrace{\left|\mathrm{Q}\overline{\mathrm{Q}}gg\right\rangle}_{\substack{twist-4,\\ twist-5,\\ \ldots}} + \underbrace{\left|\mathrm{Q}\overline{\mathrm{Q}}q\overline{q}\right\rangle}_{\substack{twist-4,\\ twist-5,\\ \ldots}} + \ldots$$

Higher twist decays as a probe of charmonia structures

Distribution amplitudes of J/Ψ up to twist-4

$$
\begin{aligned}
\langle J/\psi(p,\epsilon)|\bar{c}(x)\gamma_\rho[x,-x]c(-x)|0\rangle &= f_V M_V \left[\frac{(\epsilon x)}{(px)}p_\rho \int_{-1}^{1} d\xi e^{i\xi(px)}\left(\varphi_1(\xi,\mu)+\frac{M_V^2 x^2}{4}\varphi_2(\xi,\mu)\right)\right.\\
&+ \left(\epsilon_\rho - p_\rho\frac{(\epsilon x)}{(px)}\right)\int_{-1}^{1} d\xi e^{i\xi(px)}\varphi_3(\xi,\mu)\\
&- \left.\frac{1}{2}x_\rho\frac{(\epsilon x)}{(px)^2}M_V^2 \int_{-1}^{1} d\xi e^{i\xi(px)}\varphi_4(\xi,\mu)\right],\\
\langle J/\psi(p,\epsilon)|\bar{c}(x)\sigma_{\rho\lambda}[x,-x]c(-x)|0\rangle &= f_T(\mu)\left[\left(\epsilon_\rho p_\lambda - \epsilon_\lambda p_\rho\right)\int_{-1}^{1} d\xi e^{i\xi(px)}\left(\chi_1(\xi,\mu)+\frac{M_V^2 x^2}{4}\chi_2(\xi,\mu)\right)\right.\\
&+ \left(p_\rho x_\lambda - p_\lambda x_\rho\right)\frac{(\epsilon x)}{(px)^2}M_V^2 \int_{-1}^{1} d\xi e^{i\xi(px)}\chi_3(\xi,\mu)\\
&+ \left.\frac{1}{2}\left(\epsilon_\rho x_\lambda - \epsilon_\lambda x_\rho\right)\frac{M_V^2}{(px)}\int_{-1}^{1} d\xi e^{i\xi(px)}\chi_4(\xi,\mu)\right],\\
\langle J/\psi(p,\epsilon)|\bar{c}(x)\gamma_\rho\gamma_5[x,-x]c(-x)|0\rangle &= f_A(\mu)e_{\rho\lambda\alpha\beta}\epsilon^\lambda p^\alpha x^\beta \int_{-1}^{1} d\xi e^{i\xi(px)}\Phi_1(\xi,\mu),\\
\langle J/\psi(p,\epsilon)|\bar{c}(x)[x,-x]c(-x)|0\rangle &= -if_S(\mu)(\epsilon x)\int_{-1}^{1} d\xi e^{i\xi(px)}\Phi_2(\xi,\mu),
\end{aligned}
$$

There are 10 distribution amplitudes needed in the calculation

The result of the calculation

$$F = \int d\xi_1 d\xi_2 H(\xi_1,\xi_2,\mu)\Big(f_V f_A(\mu) M_{J/\psi}\varphi_1(\xi_1,\mu)\Phi_1(\xi_2,\mu) + f_V f_A(\mu) M_{J/\psi}\varphi_1(\xi_2,\mu)\Phi_1(\xi_1,\mu)$$
$$+ f_S(\mu) f_T(\mu)\chi_1(\xi_2,\mu)\Phi_2(\xi_1,\mu) + f_S(\mu) f_T(\mu)\chi_1(\xi_1,\mu)\Phi_2(\xi_2,\mu)\Big).$$
$$H(\xi_1,\xi_2,\mu) = \frac{1024\pi^2\alpha_s^2(\mu)}{27} f_{\eta_b}\frac{1}{M_{\eta_b}^6}\frac{1}{(1-\xi_1^2)(1-\xi_2^2)(1+\xi_1\xi_2)},$$

$$\langle J/\psi(p,\epsilon)|\bar c(0)\gamma_\mu c(0)|0\rangle = f_V M_{J/\psi}\epsilon_\mu, \quad \langle J/\psi(p,\epsilon)|\bar c(0)\sigma_{\mu\nu} c(0)|0\rangle = f_T(\mu)\big(\epsilon_\mu p_\nu - \epsilon_\nu p_\mu\big)$$
$$f_A(\mu) = \frac{1}{2}\left(f_V - f_T(\mu)\frac{2m_c(\mu)}{M_{J/\psi}}\right)M_{J/\psi}, \quad f_S(\mu) = \left(f_T(\mu) - f_V\frac{2m_c(\mu)}{M_{J/\psi}}\right)M_{J/\psi}^2$$

<u>**Fine tuning between parameters at the leading order approximation in NRQCD**</u>

$$\frac{f_T}{f_V} = 1 - \frac{\langle v^2\rangle_{J/\psi}}{3}, \quad \frac{M_{J/\psi}}{2m_c} = 1 + \frac{\langle v^2\rangle_{J/\psi}}{2}, \quad \langle v^2\rangle_{J/\psi} \sim 0.2$$
$$F = \frac{256\pi^2\alpha_s^2}{81}\frac{1}{m_b^6} f_{\eta_b} f_V^2 m_c^2 \langle v^2\rangle$$

Fine tuning is broken due to relativistic and radiative corrections what leads to the dramatic enhancement of the branching ratio

Numerical results

$$Br(\eta_b \to J/\psi J/\psi) = (6.2 \pm 3.5) \times 10^{-7},$$
$$Br(\eta_b \to J/\psi \psi') = (10 \pm 6) \times 10^{-7},$$
$$Br(\eta_b \to \psi' \psi') = (3.7 \pm 2.8) \times 10^{-7}.$$

Braguta, Kartvelishvili, Phys.Rev.D81:014012,2010

$Br(\eta_b \to J/\Psi J/\Psi) = (0.5\times 10^{-8} - 1.2\times 10^{-5})$ Santorelli, Phys.Rev.D77:074012,2008

$Br(\eta_b \to J/\Psi J/\Psi) = 2.4^{+4.2}_{-1.9}\times 10^{-8}$ Jia, Phys.Rev.D78:054003,2008

$Br(\eta_b \to J/\Psi J/\Psi) = (2.1 - 18.6)\times 10^{-8}$ Gong et al., Phys.Lett.B670:350,2009

Approximately 100 events of the $\eta_b \to J/\Psi J/\Psi$ decay at LHC per year

Thank you