Refactorizing NRQCD short-distance coefficients in exclusive quarkonium production

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Based on arXiv:0812.1965, Y. J. and Deshan Yang;
Y. J., Jian-Xiong Wang, and Deshan Yang, to appear

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Outline

1. Motivation for the idea of **refactorization**;
Two influential approaches for exclusive quarkonium production:
NRQCD vs. light-cone approach

Key idea: *Both methods have strength and weakness;*
Most profitable to bridge them together to achieve the optimal predictions

2. Using light-cone approach to identify and resum leading logarithms appearing in NRQCD matching coefficients for $e^+e^- \to \eta_b + \gamma$

3. Bridging light-cone and NRQCD approaches for a class of double-quarkonium hard exclusive reactions: ***B_c* electromagnetic form factor**: $B_c + \gamma^* \to B_c$ at NLO in α_s expansion

Double charmonium production at B factories: $\gamma^* \rightarrow J/\psi + \eta_c$

- **Exlusive quarknium production at *B*-factories**

$$\text{Belle}\ :\sigma[e^+e^- \to J/\Psi\,\eta_c]\times B^{\eta_c}[\geq 2] = (25.6 \pm 2.8 \pm 3.4)\,fb$$

$$\text{BaBar}:\sigma[e^+e^- \to J/\Psi\,\eta_c]\times B^{\eta_c}[\geq 2] = (17.6 \pm 2.8 \pm^{1.5}_{2.1})\,fb$$

Abe et al. (Belle Collaboration), PRL 2002;
Aubert et al. (BaBar Collaboration), PRD 2005;

- **Surprises for theorists**

LO NRQCD: $\sigma[e^+e^- \to J/\Psi\,\eta_c] \approx (2.3 - 5.5)\,fb$

Braaten and Lee, PRD (2003);
Liu, He and Chao, PLB (2003) ;
Hagiwara, Kou and Qiao, PLB (2003).

Intensive studies of $\gamma^* \rightarrow J/\psi + \eta_c$ in both NRQCD and light-cone sides (I)

NRQCD side:

NLO perturbative Corrections: K~ 2

Zhang, Gao and Chao, PRL (2006);

Gong and Wang, PRD (2008)

Relativistic Corrections (resum a class): K~ 2

Braaten and Lee, PRD (2003);
He, Fan and Chao, PRD (2007);
Bodwin, Lee and Yu, PRD (2008)

Has the discrepancy between theory and data has been resolved? Perhaps not..., theoretical puzzle remains!

Intensive study of $\gamma^* \to J/\psi + \eta_c$ in both NRQCD and light-cone sides (II)

light-cone side:

Essentially same as $\gamma^* \to \rho\,\pi$ time-like form factor:

Lepage and Brodsky, PRD (1980);
Chernyak and Zhitnitsky, Phys. Rept. (1984)

Revisiting the problem using some model ansatz:

Ma and Si, PRD (2004); Bondar and Chernyak, PLB (2005);
Bodwin, Kang and Lee, PRD (2006);
Braguta, Likhoded and Luchinsky, PRD (2009); Braguta, PRD (2009)

Using phenomenologically-determined **Charmonium LC Distribution Amplitudes**
all the work is done at **LO level; NLO analysis** has never been done.
Some long-standing theoretical problems: **end-point singularity**

Strength and weakness of NRQCD factorization approach

Manifestly exploiting the non-relativistic characteristic of a heavy quarkonium

$v \ll 1$ is the typical velocity of quark
A hierarchy of dynamical scales: $m \gg m v \gg m v^2$

Organize the amplitude in v expansion:

$$\mathcal{M}[H] \sim \sum_n \mathcal{C}_n(Q,m)\langle H|\mathcal{O}_n|0\rangle,$$

Shortcoming: $C_n(Q; m)$ contains two widely-separated scales; large logarithms of type **ln (Q/m)** appears in each order in NRQCD short-distance coefficients; calls for further disentanglement of these two scales

Strength and weakness of standard **light-cone** approach

Based on collinear factorization theorem (light-cone OPE)

Rigorously 1/Q expansion, suitable for hard reactions for both light hadrons and heavy hadrons, as long as $\mathbf{m^2 << Q^2}$

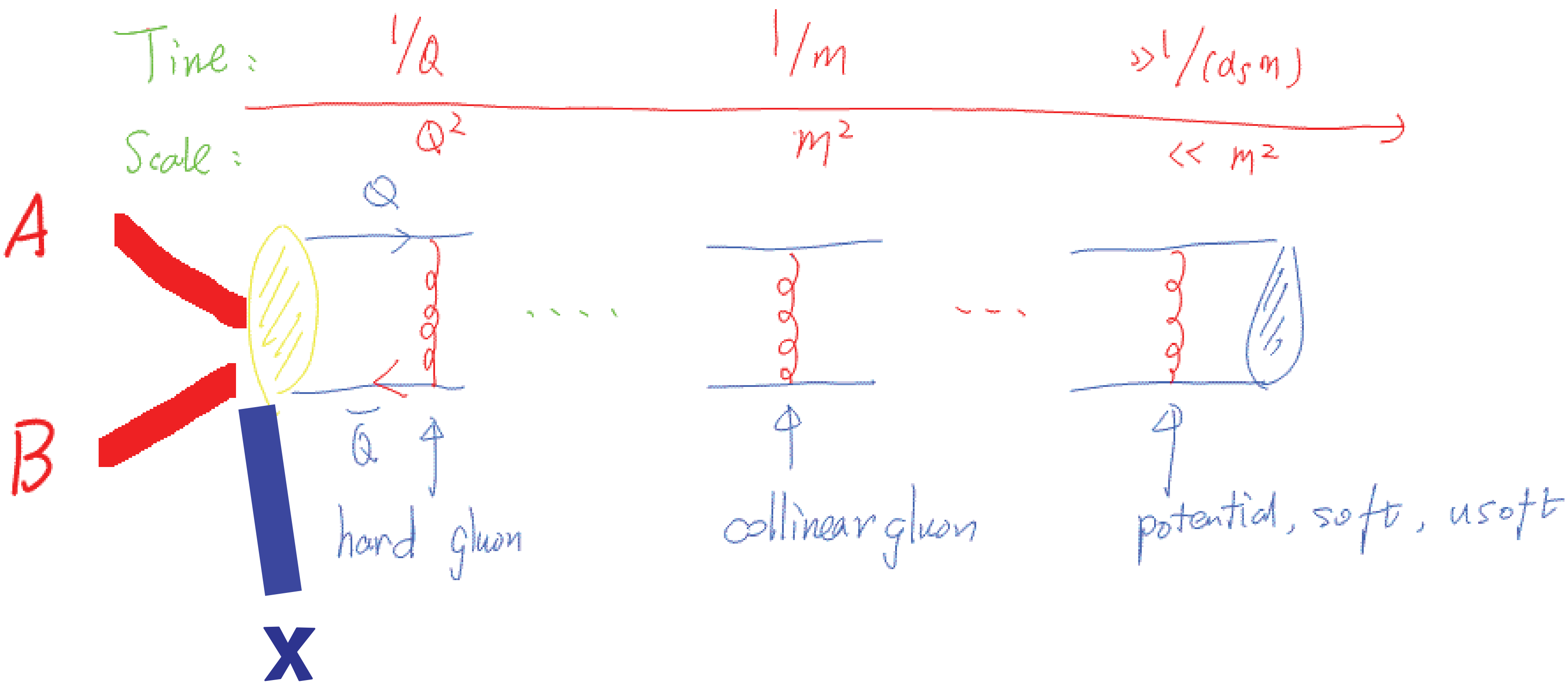
Shortcoming: it has not yet fully exploited the trait of quarkonium.

Treat quarkonium LCDA as intrinsically nonperturbative objects, and determine them by purely phenomenological means

η_c **LCDA:** Braguta, Likhoded and Luchinsky, PRD (2007);

J/ψ **LCDA:** Braguta, PRD (2007); Hwang, EPJC (2009)

space-time picture for exclusive single quarkonium production



Idea of refactorization: manipulation on NRQCD hard coefficient

NRQCD factorization for amplitude, at LO in v expansion

$$\mathcal{M}[H] \sim \sum_n \mathcal{C}_n(Q,m)\langle H|\mathcal{O}_n|0\rangle,$$

We can further factor $C_n(Q;m)$ as the following convolution:

$$\boxed{\mathcal{C}_1\left(\frac{m^2}{Q^2}\right) \sim T(x,Q,\mu)\otimes\hat{\phi}(x,m,\mu) + O\left(\frac{m^2}{Q^2}\right)}$$

The hard-scattering kernel is the same as the analogous process in which the **heavy** quark is replaced with a **massless** quark;

Since $m \gg \Lambda_{QCD}$, the LCDA of Q Qbar partonic state can be calculated reliably in perturbation theory.

Equivalent realization of refactoriztion: directly operating on LCDA of quarkonium

Inspired by further factoring the fragmentation function for a quarkonium into a jet function multiplying NRQCD matrix elements, one may refactor the LCDA of quarkonium, since it contains **collinear** degrees of freedom with vast different virtualities, ranging from order m^2 to order $(m v)^2$ or lower,

$$\Phi_{B_c}(x,\mu_F^2) = \frac{f_{B_c}}{2\sqrt{2N_c}}\hat{\phi}(x,\mu_F^2).$$

Ma, Si, PLB (2007)

All nonperturbative dynamics is encoded in decay constant f_{Bc}, and ϕ is perturbatively calculable:

$$\hat{\phi}(x,\mu_F^2) = \hat{\phi}^{(0)}(x) + \frac{\alpha_s(\mu_F^2)}{\pi}\hat{\phi}^{(1)}(x,\mu_F^2) + \cdots .$$

The scale separation between Q and m is simply achieved

By this refactorization procedure,

$$\boxed{\mathcal{C}_1\left(\frac{m^2}{Q^2}\right) \sim T(x, Q, \mu) \otimes \hat{\phi}(x, m, \mu) + O\left(\frac{m^2}{Q^2}\right)}$$

Hard kernel $T(x, Q, \mu_f)$ depends on Q but not *m*

LCDA $\phi(x; m, \mu_f)$ depends on *m* but not on Q

Two scales Q and *m* get fully disentangled

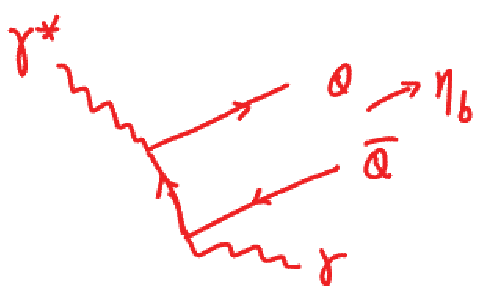
$e^+e^- \to \eta_b + \gamma$ as a concrete example

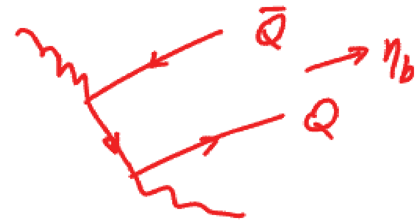
The amplitude at LO in v reads:

$$\mathcal{M}[\gamma^* \to \eta_b + \gamma] = \widehat{\mathcal{M}}[\gamma^* \to b\bar{b}({}^1S_0^{(1)}, P) + \gamma] \frac{\langle \eta_b | \psi^\dagger \chi | 0 \rangle}{\sqrt{2N_c m}},$$

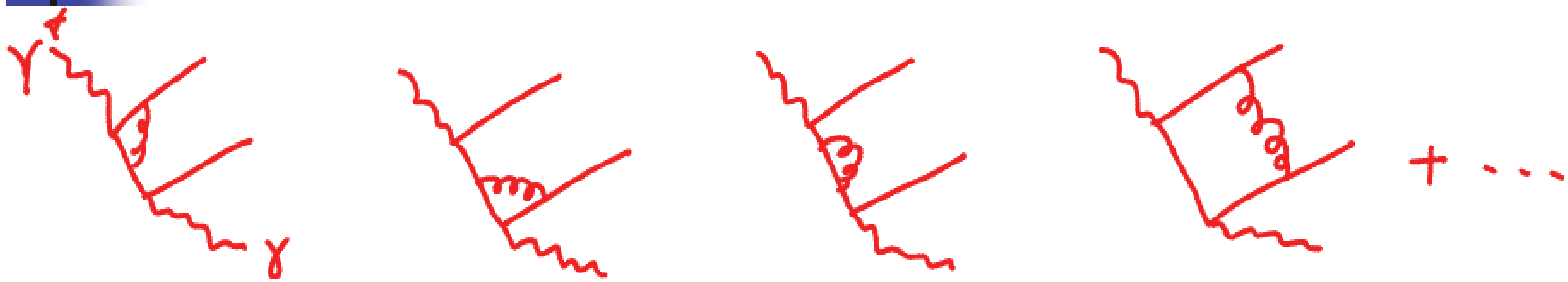
The short-distance coefficient can be parameterized as the form factor F (m²/Q²) times some kinematic factors:

$$\widehat{\mathcal{M}}[\gamma^* \to b\bar{b}({}^1S_0^{(1)}, P) + \gamma] = \sqrt{2N_c} \frac{e^2 e_b^2}{Q^2} \epsilon_{\mu\nu\alpha\beta} \varepsilon^{\mu}_{\gamma^*} \varepsilon^{*\nu}_{\gamma} Q^\alpha k^\beta F\left(\frac{m^2}{Q^2}\right)$$





NLO correction to $e^+e^- \rightarrow \eta_b + \gamma$



Kinematic setup

In the center mass frame:

$$\text{momentum of } \gamma^*\text{: } P^\mu = (\sqrt{s}, \vec{0}),$$
$$\text{momentum of } \gamma\text{: } p'^\mu = E_\gamma n_+^\mu = (E_\gamma, 0, 0, E_\gamma)$$
$$\text{momentum of } c-\text{quark: } p_1^\mu = (p+q)^\mu,$$
$$\text{momentum of } \bar{c}-\text{quark: } p_2^\mu = (p-q)^\mu\,,$$

Leading region

Beneke, Smirnov, NPB (1998)

hard region: $k^\mu \sim \sqrt{s}\,, k^2 \sim s$

collinear region: $(n_+k, k_\perp, n_-k) \sim \sqrt{s}(1, \lambda, \lambda^2)\,,\ k^2 \sim m_c^2\,,$

anti-collinear region: $(n_+k, k_\perp, n_-k) \sim \sqrt{s}(\lambda^2, \lambda, 1)\,,\ k^2 \sim m_c^2\,,$

soft region: $k^\mu \sim \lambda\sqrt{s}, \quad , k^2 \sim \lambda^2 s \sim m_c^2$

potential region: $p^0 \sim O(m\,v^2)$, $p^i \sim O(m\,v)$ **Coulomb gluon**

soft region $p^\mu \sim O(m\,v)$

ultra-soft region: $p^\mu \sim O(m\,v^2)$

All these three low-energy regions are intrinsic to quarkonium rest frame, encoded in the long-distance NRQCD matrix element

NRQCD factorization guarantees no overlap/double counting of regions

Asymptotic behavior of form factor F (m^2/Q^2) can be reproduced by light-cone approach

NRQCD

$$F\left(\frac{m^2}{Q^2}\right)=\sum_{n=0}^{\infty}\sum_{l=0}^{n}C_{nl}\left(\frac{\alpha_s}{\pi}\right)^n \ln^l\left(\frac{Q^2}{m^2}\right)+O\left(\frac{m^2}{Q^2}\right),$$

Collinear factorization:

$$F\left(\frac{m^2}{Q^2}\right)=\int_0^1 dx\, T(x,Q,\mu)\hat{\phi}(x,m,\mu)+O\left(\frac{m^2}{Q^2}\right),$$

Hard kernel $\quad T=T^{(0)}+\frac{\alpha_s}{\pi}T^{(1)}+\cdots$

tree-level $\quad T^{(0)}(x,Q)=\frac{1}{x}+\frac{1}{1-x}$

Perturbatively calculable LCDA

$$\hat{\phi}(x,m,\mu)=-\frac{1}{\sqrt{2N_c}}\int\frac{dw^-}{2\pi}e^{-ixP^+w^-}\langle b\bar{b}(^1S_0^{(1)},P)|\bar{b}(0,w^-,\mathbf{0}_\perp)\gamma^+\gamma_5 b(0)|0\rangle,$$

tree-level $\quad \hat{\phi}^{(0)}(x,\mu\sim m)\equiv\hat{\phi}^{(0)}(x)=\delta(x-\frac{1}{2})$

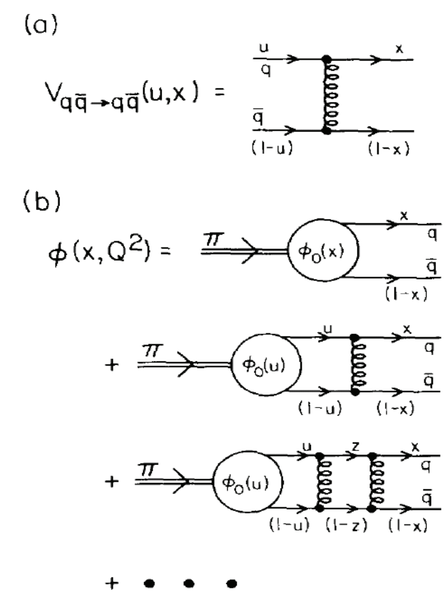
RGE for LCDA (evolution equation)

$$\mu^2 \frac{\partial}{\partial \mu^2} \hat{\phi}(x,\mu) = \frac{\alpha_s(\mu^2)}{\pi} \int_0^1 dy \, \frac{C_F}{2} V_0(x,y)\, \hat{\phi}(y,\mu),$$

Efremov-Radyushkin-Brodsky-Lepage (ERBL) kernel

$$V_0(x,y) = \left[\frac{1-x}{1-y}\left(1+\frac{1}{x-y}\right)\theta(x-y) + \frac{x}{y}\left(1+\frac{1}{y-x}\right)\theta(y-x) \right]_+$$

Physics of building up these large collinear logs can be easily understood



Identifying the leading collinear logarithms (LL) to any orders in α_s using ERBL equation

Substituting formal solution of BL equation into factorization formula:

$$F\left(\frac{m^2}{Q^2}\right)_{\text{LL}} = \int_0^1 dx\, T^{(0)}(x)\hat{\phi}^{(0)}(x, Q) = \int_0^1 dx\, T^{(0)}(x)\exp[\kappa C_F V_0 \star]\hat{\phi}^{(0)}(x),$$

where

$$\kappa \equiv \frac{2}{\beta_0}\ln\left(\frac{\alpha_s(m^2)}{\alpha_s(Q^2)}\right) = \frac{\alpha_s(Q^2)}{2\pi}\ln\left(\frac{Q^2}{m^2}\right) + \beta_0\frac{\alpha_s^2(Q^2)}{(4\pi)^2}\ln^2\left(\frac{Q^2}{m^2}\right) + \cdots.$$

Identifying the leading collinear logarithms (LL) to any orders in α_s using ERBL equation (cont')

Expanding it recursively,

$$F\left(\frac{m^2}{Q^2}\right)_{\rm LL} = \int_0^1 dx\, T^{(0)}(x)\hat{\phi}^{(0)}(x) + \kappa C_F \int_0^1 dx \int_0^1 dy\, T^{(0)}(x) V_0(x,y)\hat{\phi}^{(0)}(y) + \frac{\kappa^2 C_F^2}{2!}\int_0^1 dx \int_0^1 dy \int_0^1 dz\, T^{(0)}(x) V_0(x,y) V_0(y,z)\hat{\phi}^{(0)}(z) + \cdots .$$

We then obtain

$$F\left(\frac{m^2}{Q^2}\right)_{\rm LL} = C_{00}\left\{1 + \frac{C_F \alpha_s(Q^2)}{4\pi}\ln\left(\frac{Q^2}{m^2}\right)(3 - 2\ln 2) + C_F \frac{\alpha_s^2(Q^2)}{(4\pi)^2}\ln^2\left(\frac{Q^2}{m^2}\right)\left[\beta_0\left(\frac{3}{2} - \ln 2\right) + C_F\left(\frac{9}{2} - \frac{\pi^2}{6} - 8\ln 2 + \ln^2 2\right)\right] + \cdots\right\},$$

agrees with explicit NLO calculation

Sang, Chen, PRD(2010)

Summing the leading collinear logarithms (LL) to any orders in α_s in moment space

Starting from $\hat{\phi}^{(0)}(x,\mu\sim m) \equiv \hat{\phi}^{(0)}(x) = \delta(x-\frac{1}{2})$.

Decomposing it in terms of Gegenbauer polynomials

$$F\left(\frac{m^2}{Q^2}\right)_{\rm LL} = \sum_{n=0}^{\infty} \hat{\phi}^{(0)}_{2n} \left(\frac{\alpha_s(Q^2)}{\alpha_s(m^2)}\right)^{d_{2n}},$$

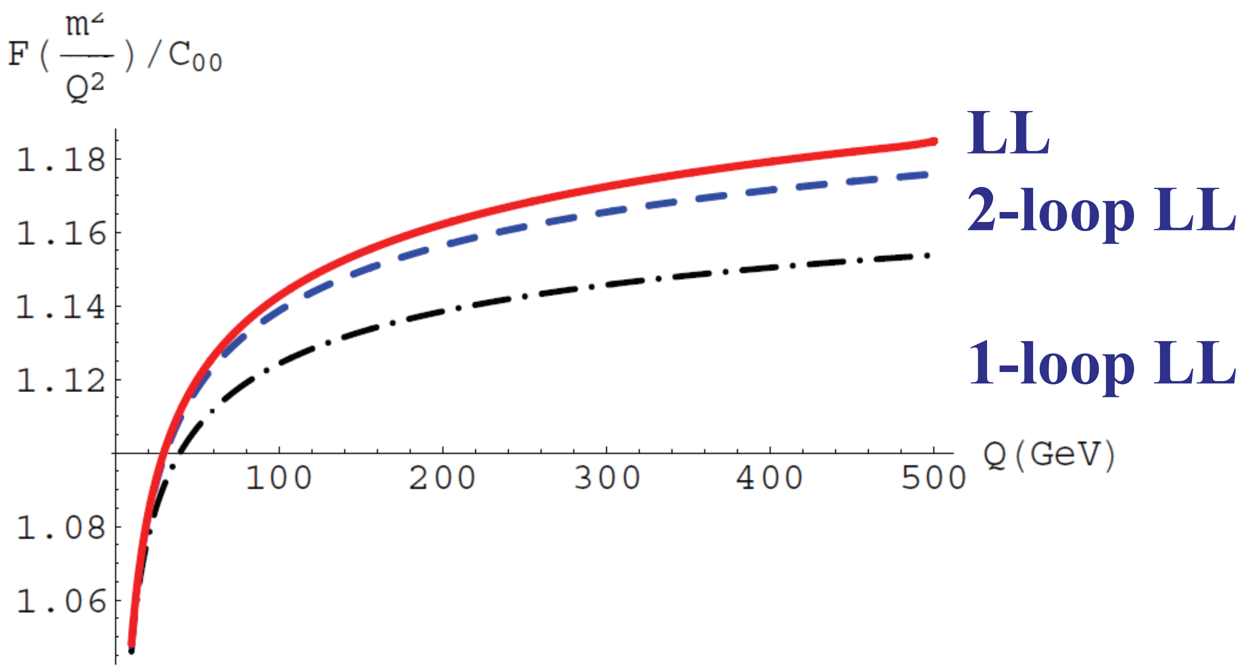
$$\hat{\phi}^{(0)}_{2n} = 4(-1)^n(4n+3)\frac{(2n-1)!!}{(2n+2)!!}.$$

Anomalous dimension for each moment is

$$d_n = 2C_F\gamma_n/\beta_0 \qquad \gamma_n = \frac{1}{2} + 2\sum_{j=2}^{n+1}\frac{1}{j} - \frac{1}{(n+1)(n+2)}.$$

Impact of LL resummation

Numerical impact seems not important



Similar LL resummation was also done in an old-fashioned light-OPE context **Shifman, Vysotsky, NPB (1981)**

Another advantage of **refactorzation**: efficient and systematic means to reproduce fixed-order NRQCD factorization prediction

In addition to LL resummation, light-cone + refactorization is much more advantageous than a brute-force NRQCD matching calculation.

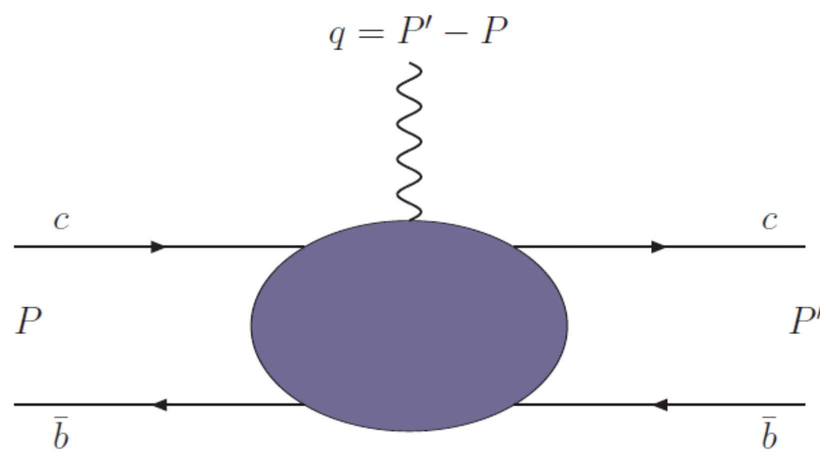
Because a literal NRQCD factorization calculation is already very difficult at NLO, not even mention NNLO … (never possible?)

$$F\left(\frac{m^2}{Q^2}\right) = \sum_{n=0}^{\infty}\sum_{l=0}^{n} C_{nl}\left(\frac{\alpha_s}{\pi}\right)^n \ln^l\left(\frac{Q^2}{m^2}\right) + O\left(\frac{m^2}{Q^2}\right)$$

We expect asymptote of NRQCD prediction will be exactly reproduced by the LC approach at LO in 1/Q and v expansions, but to any orders in α_s

Thus, refactorization serves as a simple and elegant tool to reproduce fixed-order NRQCD results

More complicated exclusive process involving double quarkonia: ***B_c* electromagnetic form factor** at NLO in α_s



Definition:

$$\langle B_c^+(P')|J^\mu_{\rm em}|B_c^+(P)\rangle = F(Q^2)(P+P')^\mu$$

Leading-twist factorization theorem

- We start form the factorization theorem

$$F_{\mathrm{LC}}(Q^2) = \int_0^1 dx \int_0^1 dy\, \Phi^*_{B_c}(y,\mu_F^2)\, T_H(x,y,Q^2,\mu_R^2,\mu_F^2)\, \Phi_{B_c}(x,\mu_F^2)$$

- To NLO accuracy, we need to consider

$$T_H(x,y,Q^2,\mu_R^2,\mu_F^2) = T_H^{(0)}(x,y,Q^2) + \frac{\alpha_s(\mu_R)}{\pi} T_H^{(1)}(x,y,Q^2,\mu_R^2,\mu_F^2) + \cdots$$

$$\Phi_{B_c}(x,\mu_F^2) = \frac{f_{B_c}}{2\sqrt{2N_c}}\hat{\phi}(x,\mu_F^2),$$

$$f_{B_c} = f_{B_c}^{(0)}\left(1 + \frac{\alpha_s(M_{B_c}^2)}{\pi}\mathfrak{f}_{B_c}^{(1)} + \cdots\right) \qquad \hat{\phi}(x,\mu_F^2) = \hat{\phi}^{(0)}(x) + \frac{\alpha_s(\mu_F^2)}{\pi}\hat{\phi}^{(1)}(x,\mu_F^2) + \cdots$$

NLO calculation of B_c electromagnetic form factor in light-cone approach

To NLO accuracy, we need following components

$$\hat{\phi}^{(1)} \otimes T_H^{(0)} \otimes \hat{\phi}^{(0)}, \qquad \hat{\phi}^{(0)} \otimes T_H^{(0)} \otimes \hat{\phi}^{(1)},$$

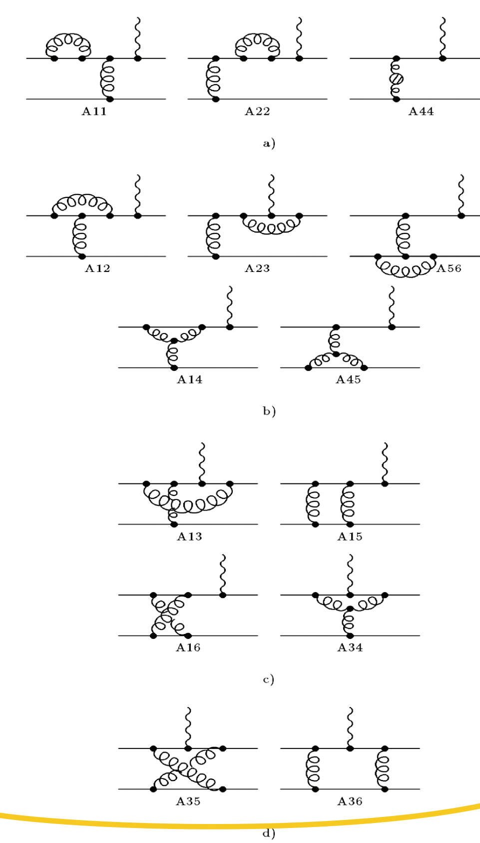
$$\hat{\phi}^{(0)} \otimes T_H^{(1)} \otimes \hat{\phi}^{(0)}, \qquad \mathfrak{f}_{B_c}^{(1)}\, \hat{\phi}^{(0)} \otimes T_H^{(0)} \otimes \hat{\phi}^{(0)}.$$

NLO correction to hard kernel in LC approach

Totally 62 diagrams, calculated by several groups over 25 years

Field, Gupta, Otto, Chang, NPB (1981);
Dittes, Radyushkin, Yad. Fiz. (1981);
Sarmardi, Ph.D. thesis (1982);
Braaten, Tse, PRD (1987);
Melic, Nizic, Passek, PRD (1998)

We take the result of $T^{(1)}_H$ from



New ingredient

- To accomplish a complete NLO accuracy, we need know the NLO correction to the LCDA:

Bell, Feldmann, JHEP(2008)

$$\hat{\phi}^{(1)}(x,\mu_F^2) = \frac{C_F}{2}\left\{\left(\ln\frac{\mu_F^2}{M_{B_c}^2(x_0-x)^2}-1\right)\left[\frac{x_0+\bar{x}}{x_0-x}\frac{x}{x_0}\theta(x_0-x)+\begin{pmatrix} x\leftrightarrow\bar{x} \\ x_0\leftrightarrow\bar{x}_0\end{pmatrix}\right]\right\}_+$$
$$+\ C_F\left\{\left(\frac{x\bar{x}}{(x_0-x)^2}\right)_{++}+\frac{1}{2}\delta'(x-x_0)\left(2x_0\bar{x}_0\ln\frac{x_0}{\bar{x}_0}+x_0-\bar{x}_0\right)\right\}.$$

NLO calculation of B_c electromagnetic form factor in NRQCD factorization approach

A very **tough** calculation: contains 3 different scales in loop integrals: **Q, m_b, m_c**

We employ one of the world-leading automated one-loop calculation package:

Feynman Diagram Calculation (FDC)

J. - X. Wang 1993,
B. Gong, J.-X. Wang, 2008

The calculation is done analytically, the resulting FDC output is pathologically complicated. It seems impossible by hand to deduce its asymptotic behavior analytically.

Numerical comparison of predictions made by NRQCD and light-cone approaches

$$F_{\rm NRQCD} = F^{(0)}_{\rm NRQCD} + \frac{\alpha_s(\mu_R^2)}{\pi} F^{(1)}_{\rm NRQCD} + \cdots$$

$$F_{\rm LC} = F^{(0)}_{\rm LC} + \frac{\alpha_s}{\pi} F^{(1)}_{\rm LC} + \cdots$$

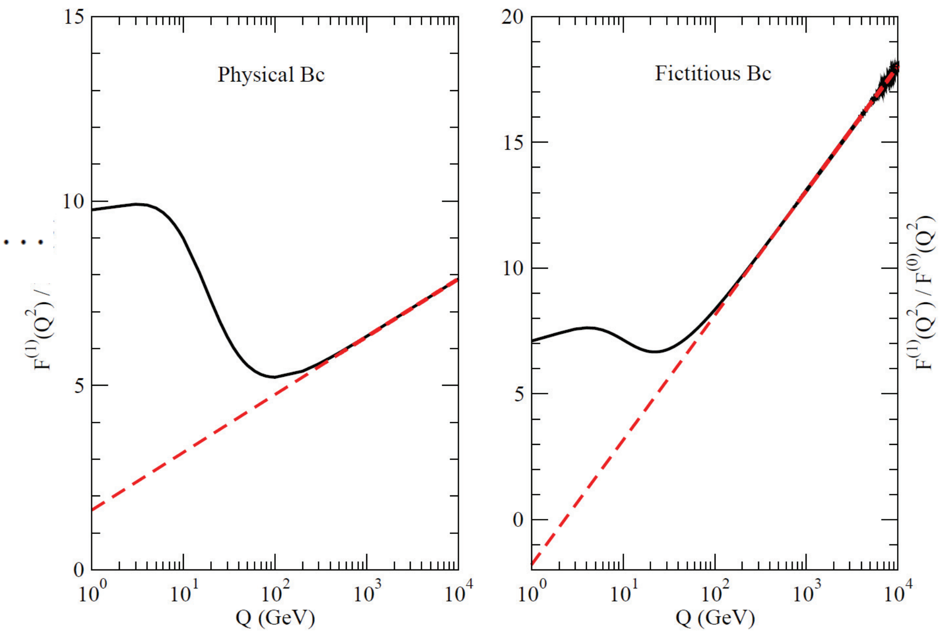


FIG. 2: The ratio $F^{(1)}_{B_c}(Q^2)/F^{(0)}_{B_c}(Q^2)$ as a function of Q with $M_{B_c} = 6.3$ GeV, n_f=5 ($\beta_0 = \frac{23}{3}$), and $\mu_R = Q$. Both the NRQCD and light-cone predictions are shown, where the former is represented by solid line, and the latter by the dashed line. The left panel is for the EM form factor of the physical B_c state with $m_c = 1.5$ GeV and $m_b = 4.8$ GeV, while the right panel for a fictitious B_c state with $m_c = m_b = 3.15$ GeV. Numerically, the dashed line in the left panel can be parameterized by $0.680 \ln Q + 1.624$, and that in the right panel by $2.152 \ln Q - 1.795$, where Q is in the unit of GeV.

Asymptotic behavior of NRQCD prediction for **a fictitious B_c meson**

We explicitly checked that, for a fake Bc meson ($m_b = m_c$), light-cone prediction implementing refactorization exactly agrees with the asymptote of NRQCD predictions

$$\frac{F^{(1)}_{\rm LC}(Q^2)}{F^{(0)}_{\rm LC}(Q^2)} = \frac{\beta_0}{4}\left(\frac{5}{3} + 2\ln 2 + \ln\frac{\mu_R^2}{Q^2}\right) + \boxed{\frac{C_F}{2}(3 - 2\ln 2)\ln\frac{Q^2}{M_{B_c}^2}}$$
$$- \frac{4}{3}\ln^2 2 + \frac{25}{9}\ln 2 - \frac{25}{9} - \frac{2\pi^2}{9}\,.$$

Single collinear logarithm;
Can be handled by ERBL equation

Summary and Outlook

1. Refactorization achieves the optimal scale disentanglement, by combining NRQCD and light-cone approach effectively.

2. It works perfectly for single or double quarkonium production processes **at leading twist**

3*. We do not know how refactorization works for the helicity-suppressed (**higher twist**) reaction like $\gamma^* \to J/\psi + \eta_c$, $\eta_b \to J/\psi + J/\psi$, $\Upsilon \to J/\psi + \eta_c$, where double logarithm first appear at one-loop order.

Backup slide I

We face dilemma in $\gamma^* \to J/\psi + \eta_c$ channel

At NLO, the leading log is the double log.

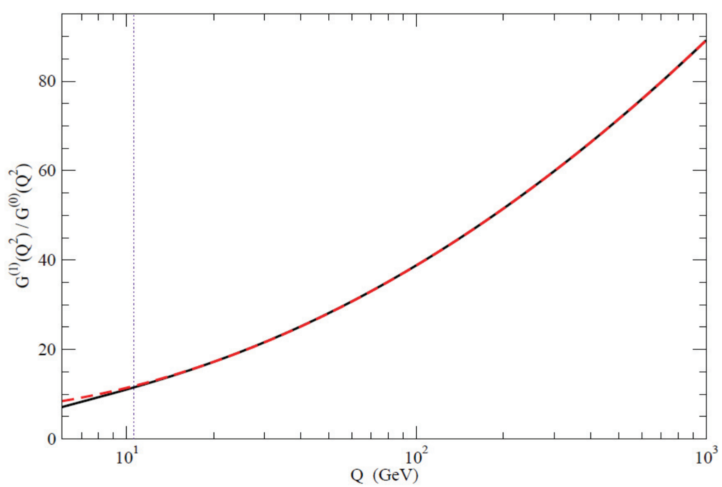


FIG. 3: The ratio of time-like EM form factor $F^{(1)}(Q^2)/F^{(0)}(Q^2)$ as a function of Q for $\gamma^* \to J/\psi + \eta_c$ with $m_c = 1.5$ GeV, n_f=4 ($\beta_0 = \frac{25}{3}$), and $\mu_R = Q \equiv \sqrt{s}$. Both the exact NLO expression and its asymptotic expression in NRQCD framework are shown, where the former is represented by solid line, and the latter by the dot-dashed line. The vertical line marks $\sqrt{s} = 10.58$ GeV of B factory energy.

Gong, Wang, PRD 2008

Backup slide II

Asymptotic behavior of $\gamma^* \rightarrow J/\psi + \eta_c$ channel deduced from

Gong, Wang, PRD 2008

$$\frac{\mathrm{Re}[C^{(1)}_{\mathrm{asym}}(Q)]}{C^{(0)}_{\mathrm{asym}}(Q)} = \boxed{\frac{13}{24}\ln^2\frac{Q^2}{m_c^2}} - \frac{41}{24}(2\ln 2 - 1)\ln\frac{Q^2}{m_c^2} + \frac{\beta_0}{4}\ln\frac{\mu_R^2}{Q^2} + \frac{71}{8}\ln 2 + \frac{59}{24}\ln^2 2 - \frac{17}{18} - \frac{\pi^2}{36}.$$

power-suppressed Sudakov logarithm

cannot be handled by ERBL equation

Thanks for your attention!