# Green Function Formalism and Nuclear Response

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# Outline

- ⋆ Intro: spectral representation of the Green function for noninteracting and interacting fermion systems
- ⋆ Connection between nuclear response and spectral functions: initial and final state effects
- ⋆ Dynamical models and approximations
- ⋆ Testing theoretical models against electron scattering data
- ⋆ Prospects and perspectives

- Definition of Green function

$$iG(x-x') = \langle 0|T[\psi(x)\psi^\dagger(x')]|0\rangle$$

After Fourier transformation $(\eta = 0^+)$

$$G(\mathbf{k},E) = \sum_n \left\{ \frac{|\langle n_{(N+1)}(\mathbf{k})|a^\dagger_{\mathbf{k}}|0_N\rangle|^2}{E-(E_n-E_0)+i\eta} + \frac{|\langle n_{(N-1)}(-\mathbf{k})|a_{\mathbf{k}}|0_N\rangle|^2}{E+(E_n-E_0)-i\eta} \right\}$$

$$= G_p(\mathbf{k},E) + G_h(\mathbf{k},E) = \int dE' \left[ \frac{P_p(\mathbf{k},E')}{E-E'+i\eta} + \frac{P_h(\mathbf{k},E')}{E+E'-i\eta} \right]$$

- Spectral functions of hole and particle states

$$P_h(\mathbf{k},E) = \sum_n |\langle n_{(N-1)}(\mathbf{k})|a_{\mathbf{k}}|0_N\rangle|^2 \delta(E-E_n+E_0) = \mathrm{Im}\, G_h(\mathbf{k},E)/\pi$$

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# Non interacting fermions

- Translationally invariant system at density $\rho = s k_F^3 / 6\pi^2$ (Fermi gas)

$$\sum_n |\langle n_{(N+1)}(\mathbf{k})| a^\dagger_{\mathbf{k}} | 0_N \rangle|^2 \longrightarrow \theta(|\mathbf{k}| - k_F) \ \ , \ \ E_0 - E_n = \epsilon^0_k = |\mathbf{k}^2|/2m$$

$$\sum_n |\langle n_{(N-1)}(\mathbf{k})| a_{\mathbf{k}} | 0_N \rangle|^2 \longrightarrow \theta(k_F - |\mathbf{k}|) \ \ , \ \ E_n - E_0 = -\epsilon^0_k$$

$$G(\mathbf{k}, E) = \frac{\theta(|\mathbf{k}| - k_F)}{E - \epsilon^0_k + i\eta} + \frac{\theta(k_F - |\mathbf{k}|)}{E - \epsilon^0_k - i\eta}$$

- Spectral functions

$$P_h(\mathbf{k}, E) = \theta(k_F - |\mathbf{k}|)\delta(E - \epsilon^0_k)$$

$$P_p(\mathbf{k}, E) = \theta(|\mathbf{k}| - k_F)\delta(E - \epsilon^0_k)$$

# Interacting fermions

- Bottom line: replace $\epsilon_k^0 \longrightarrow \epsilon_k^0 + \Sigma(\mathbf{k}, E)$

$$G_h(\mathbf{k}, E) = \frac{1}{E - \epsilon_k^0 - \Sigma(\mathbf{k}, E)}$$

- Quasiparticle picture: isolate contributions of $1h$ (i.e. *bound*) intermediate states, having strength

$$Z_k = |\langle -\mathbf{k}|a_{\mathbf{k}}|0\rangle|^2$$

and exhibiting poles at the quasiparticle energies $\epsilon_k = \epsilon_k^0 + \mathrm{Re}\,\Sigma(\mathbf{k}, \epsilon_k)$

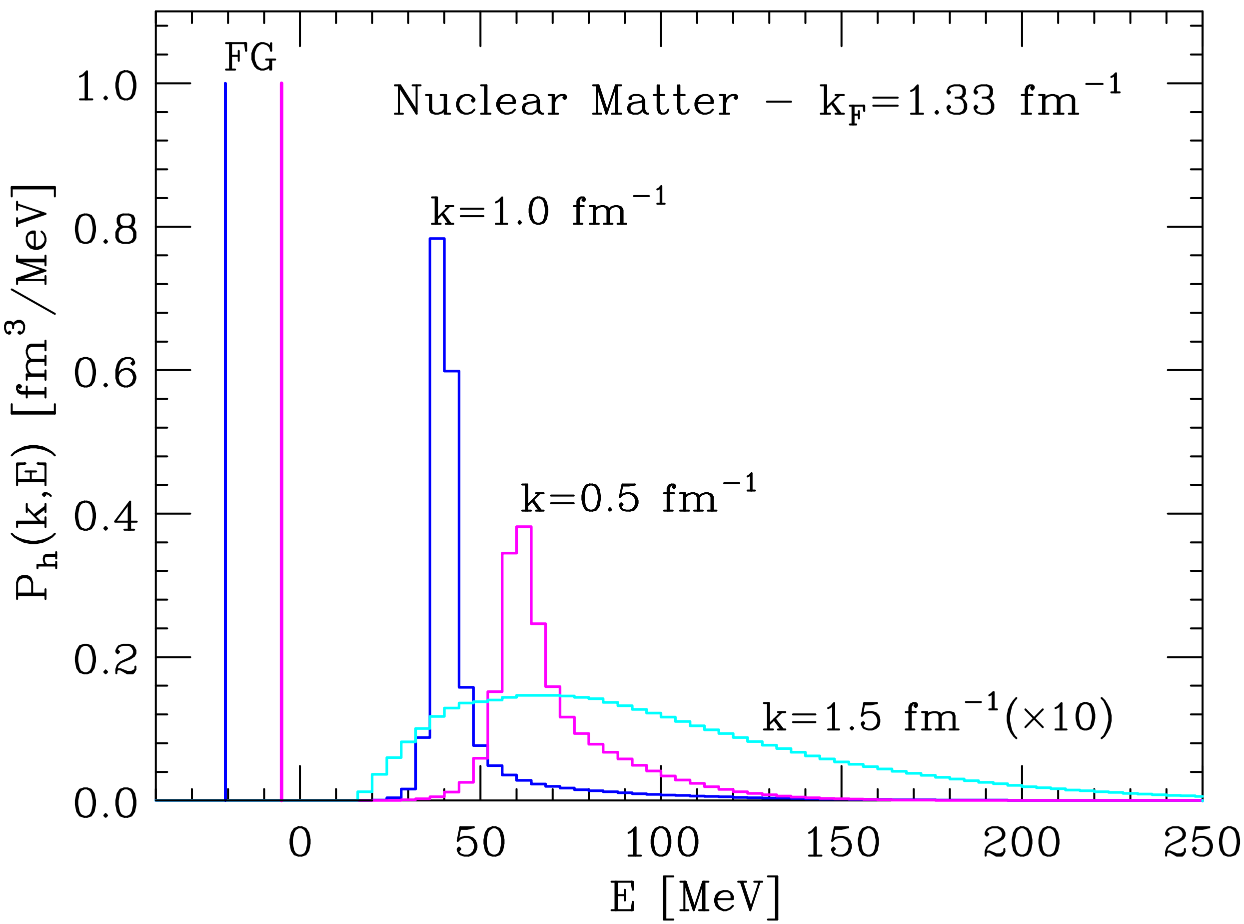
- Rewrite the Green function (e.g. for hole states) as

$$G_h(\mathbf{k}, E) = \frac{Z_k}{E - \epsilon_k - iZ_k\,\mathrm{Im}\,\Sigma(\mathbf{k}, e_k)} + G_h^B(\mathbf{k}, E)$$

$G_h^B(\mathbf{k}, E)$ is a smooth contribution, corresponding to $2h-1p, 3h-2p, \ldots$ intermediate states

## ⋆ Spectral function

$$P_h(\mathbf{k}, E) = \frac{1}{\pi} \frac{Z_k^2 \, \Sigma(\mathbf{k}, \epsilon_k)}{[E - \epsilon_k^0 - \Sigma(\mathbf{k}, E)]^2 + [Z_k \mathrm{Im}\, \Sigma(\mathbf{k}, \epsilon_k)]^2} + P_h^B(\mathbf{k}, E)$$




CBF calculation (O.B., A. Fabrocini & S. Fantoni, NPA 505(89)267)

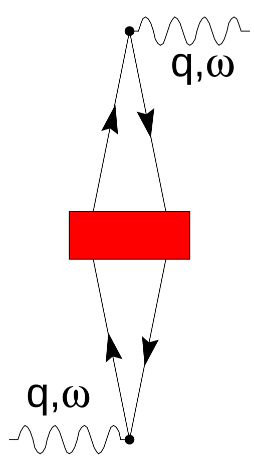
# Nuclear response

⋆ Consider scattering of a scalar probe, for simplicity

$$\frac{d\sigma}{d\omega d\Omega} \propto \sigma_{el}\, S(\mathbf{q},\omega)$$

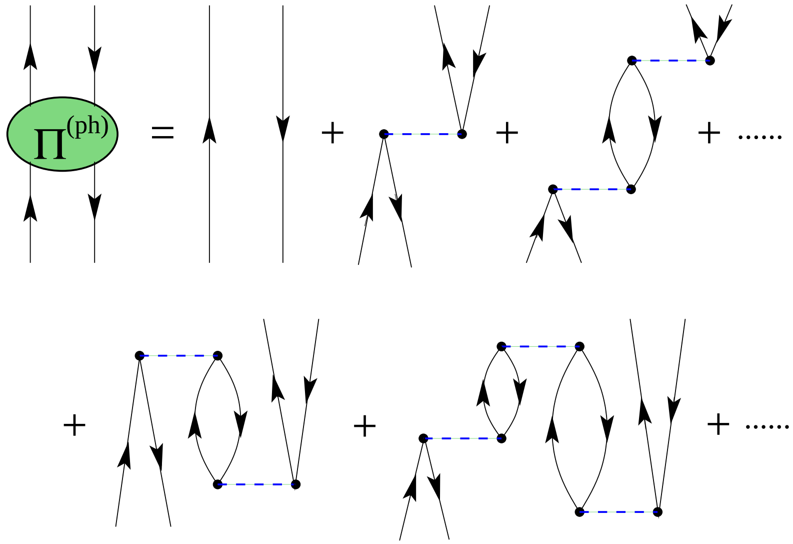
$$\begin{aligned} S(\mathbf{q},\omega) &= \sum_n \Big| \sum_k \langle n| a^\dagger_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}} |0\rangle \Big|^2 \delta(\omega + E_0 - E_n) \\ &= \int \frac{dt}{2\pi}\, \mathrm{e}^{i(\omega+E_0)t} \sum_{p,k} \langle 0| a_{\mathbf{p}+\mathbf{q}} a^\dagger_{\mathbf{p}}\, \mathrm{e}^{-iHt}\, a^\dagger_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}} |0\rangle \end{aligned}$$

=



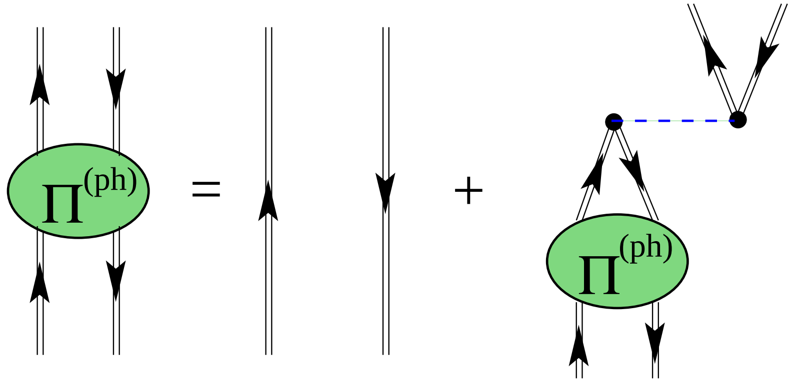
⋆ The nuclear response can be expressed in terms of Green functions, i.e. spectral functions, in a variety of approximation schemes

RPA




*dressing* all particle and hole lines leads to

DRPA



# Impulse (or ladder) approximation (IA)

- ⋆ Consider the first contribution to the DRPA series. The corresponding response reads

$$S(\mathbf{q},\omega) = \int d^3k\, dE\; P_h(\mathbf{k},E) P_p(\mathbf{k}+\mathbf{q},\omega - E)$$

- ⋆ the particle and hole spectral functions describe initial and final state effects, respectively
- ⋆ Neglecting all interactions (Fermi gas model)

$$S(\mathbf{q},\omega) = \int d^3k\; \theta(|\mathbf{k}| - k_F)\theta(k_F - |\mathbf{k}+\mathbf{q}|)\delta(\omega + e^0_k - e^0_{|\mathbf{k}+\mathbf{q}|})$$

- ⋆ Neglecting final state interactions (FSI)

$$S(\mathbf{q},\omega) = \int d^3k\, dE\; P_h(\mathbf{k},E)\theta(k_F - |\mathbf{k}+\mathbf{q}|)\delta(\omega - e^0_{|\mathbf{k}+\mathbf{q}|} - E)$$

# Nuclear Many-Body Theory

- ⋆ Bottom line: all theoretical descriptions of the nuclear response involve spectral functions.
- ⋆ Main issue: need both a *dynamical model* and *approximation schemes* suitable to describe the physics relevant to the different kinematical regimes
  - ▷ Theoretical bias can be minimized using dynamical model that can be tested in exactly solvable systems (two- and few-nucleon systems)
  - ▷ The resulting nuclear hamiltonian can be used to calculate the spectral functions of any nucleus, without introducing any additional *adjustable parameters*
  - ▷ Both the quasiparticle and background part of the theoretical spectral functions can be compared to electron scatterting data.

# Comparison to $(e, e'p)$ data

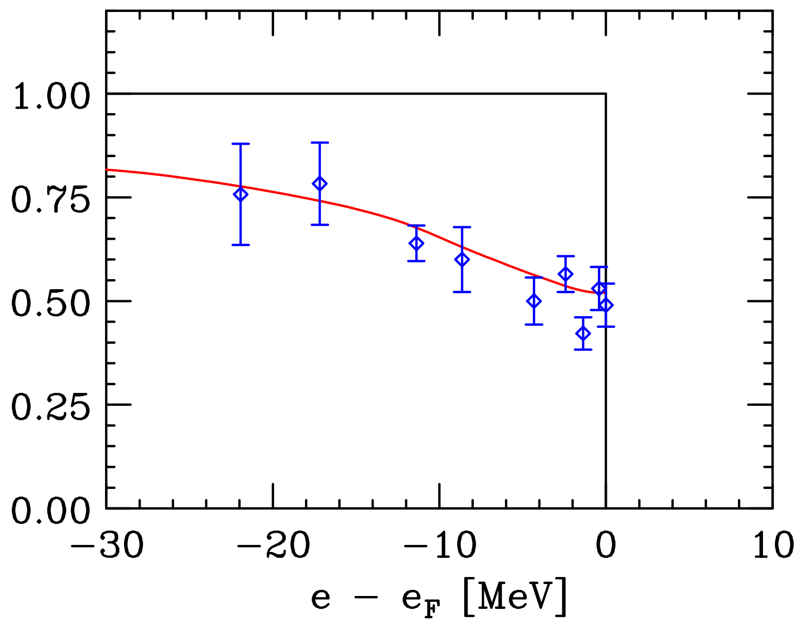
- ⋆ Within the IA and neglecting FSI, the x-setion of the process

$$e + A \rightarrow e' + (A-1)^* + p$$

can be written

$$\frac{d\sigma_{eA}}{d\omega d\Omega_{e'} dE_p d\Omega_p} = K \frac{d\sigma_{eN}}{d\omega d\Omega_{e'} dE_p d\Omega_p} P_h(\mathbf{p}-\mathbf{q}, \omega - E_p)$$

- ⋆ The quasiparticle part of $P_h(\mathbf{k}, E)$ has been extracted from $(e, e'p)$ data, mostly at Saclay and NIKHEF-K, for a variety of targets, ranging from $^3$He to $^{208}$Pb
- ⋆ The background contribution has been measured at Saclay for light nuclei $^3$He and $^4$He and more recently at Jlab for complex nuclei $^{12}$C, $^{56}$Fe and $^{197}$Au




⋆ $Z_k$ of the hole states of $^{208}$Pb

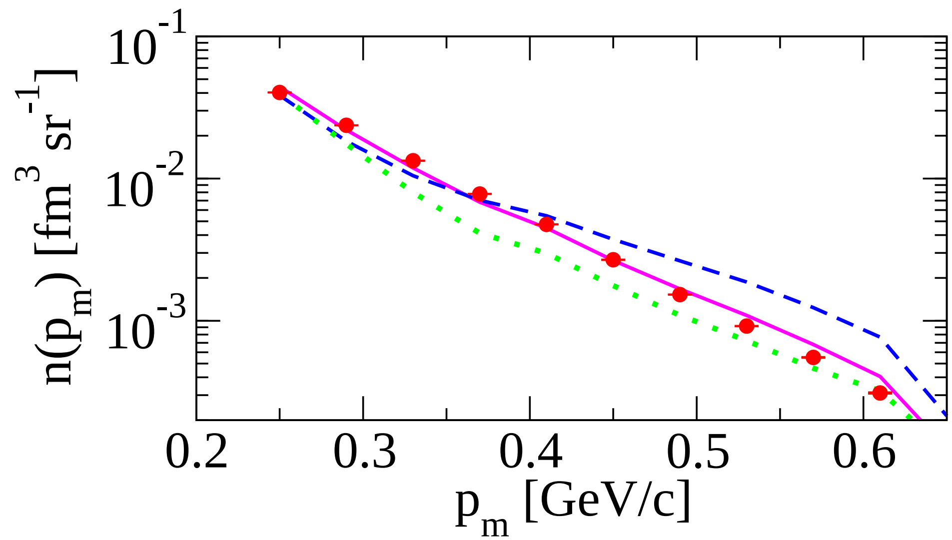
Data: NIKHEF-K, E.N.M. Quint, Ph.D. Thesis (1988)

Theory: CBF calculation of O.B., A. Fabrocini and S. Fantoni, PRC 41(1990)R24

⋆ Momentum distribution at $|\mathbf{k}| > k_F$

Data: JLab E97-006, Daniela Rohe's talk @ NUINT05

Theory: G-matrix (H. Müther, G. Knehr, and A. Polls, PRC 52(1995)2955; T. Frick and H. Müther, PRC 68(2003)034310) and CBF (O.B., A. Fabrocini, S. Fantoni, and I. Sick, NPA 579(1994)493. )



# Comparison to $(e, e')$ data

⋆ Within the IA and neglecting FSI, the x-setion of the process

$$e + A \to e' + X$$

can be written

$$\frac{d\sigma_{eA}}{d\omega d\Omega_{e'}} = \int d^3k dE \left( \frac{d\sigma_{eN}}{d\omega d\Omega_{e'}} \right) P_h(\mathbf{k}, E)$$

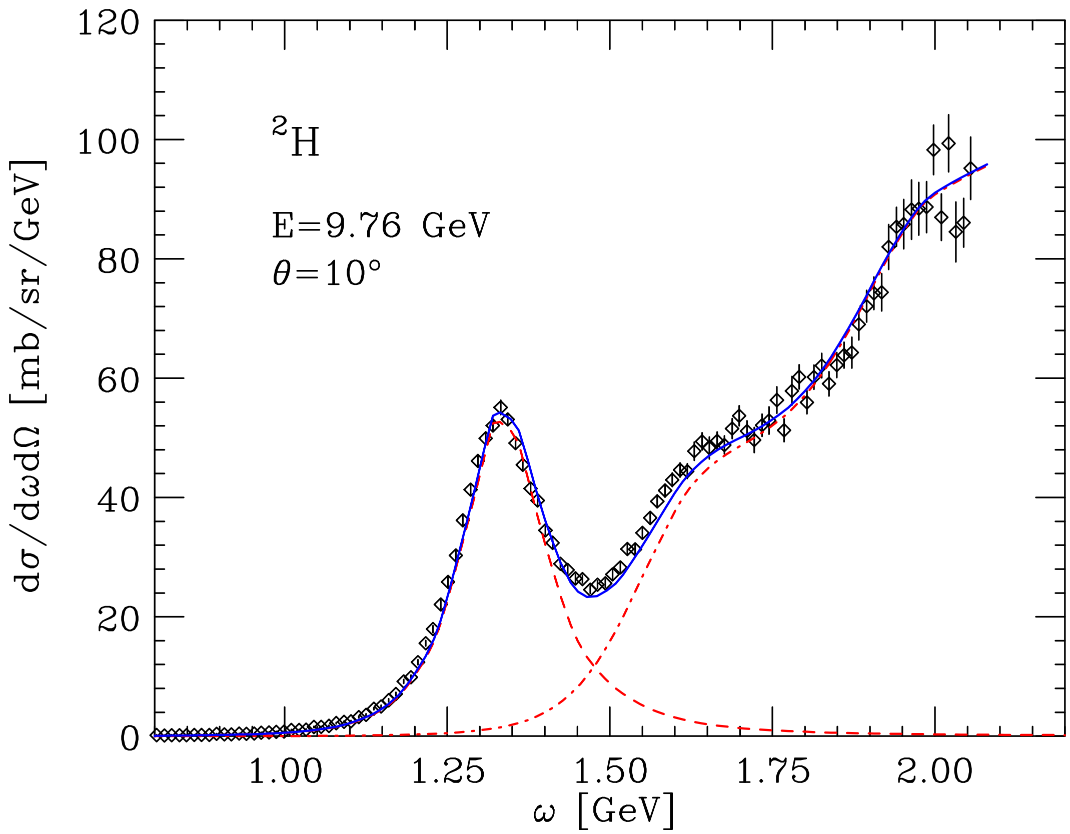
with

$$\frac{d\sigma_{eN}}{d\omega d\Omega_{e'}} = \frac{\alpha^2}{Q^4} \frac{E'_e}{E_e} \frac{m}{E_k} L_{\mu\nu} W_N^{\mu\nu}$$

$$W_N^{\mu\nu} = W_1^N \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + \frac{W_2^N}{m^2} \left( k^\mu - \frac{(pk)}{q^2} q^\mu \right) \left( k^\nu - \frac{(pk)}{q^2} q^\nu \right)$$

# Comparison to $^2H(e,e')$ data

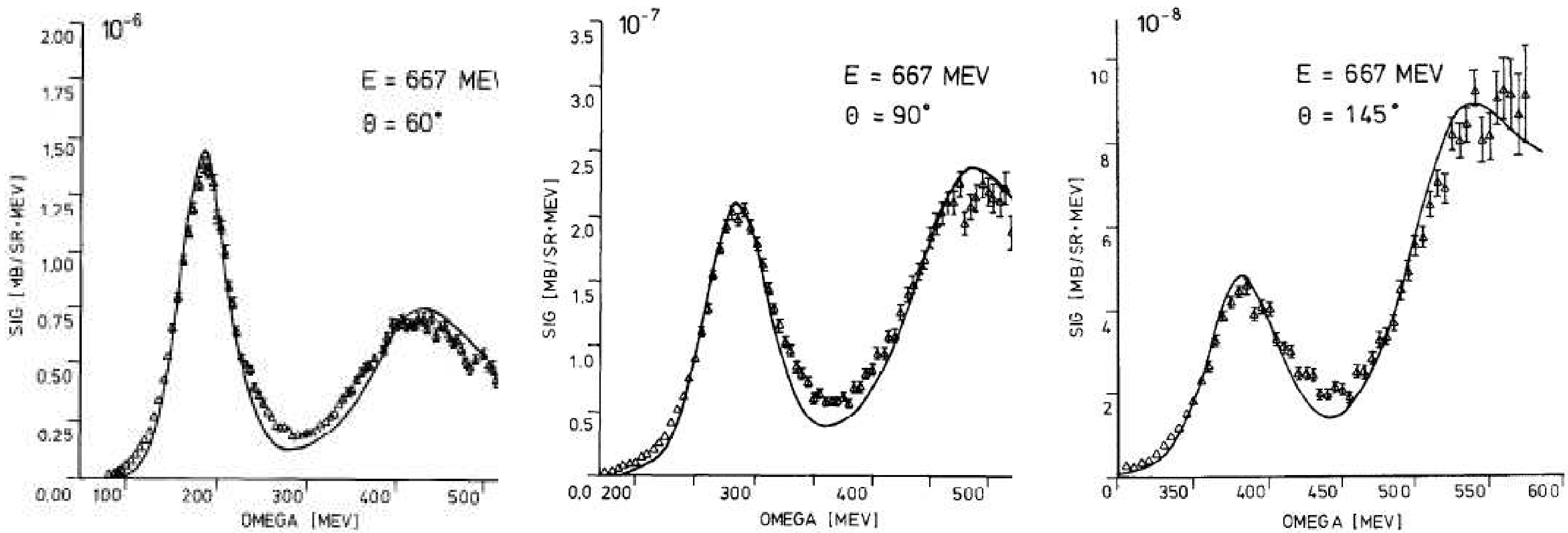
⋆ Testing $d\sigma_{eN}/d\omega d\Omega_{e'}$




▷ Calculations of O.B. and V.R. Pandharipande PRC 47(93)2218; SLAC data from S. Rock et al PRL 49(82)1139

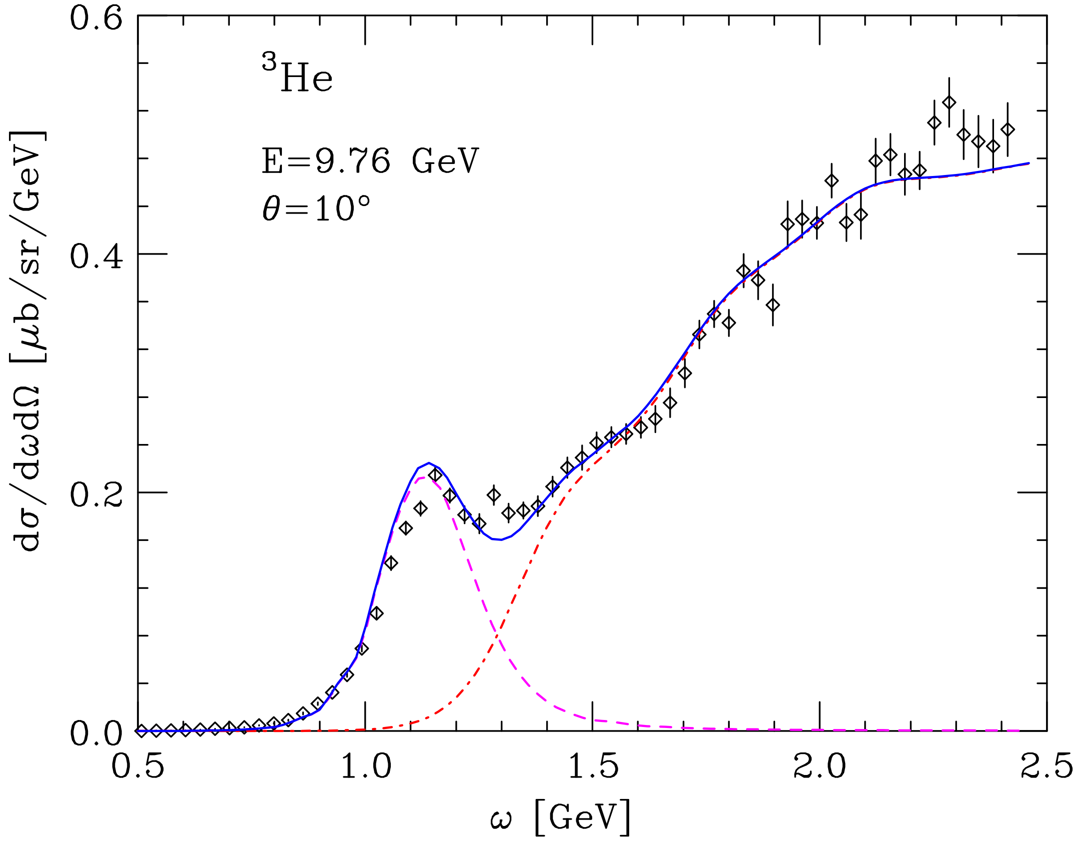
# Comparison to $^3He(e,e')$ data

- Calculations of H.Meier-Hadjuk, U. Oelfke and P. Sauer, NPA 499(89)637; Saclay data from C. Marchand et al PLB 153(85)29



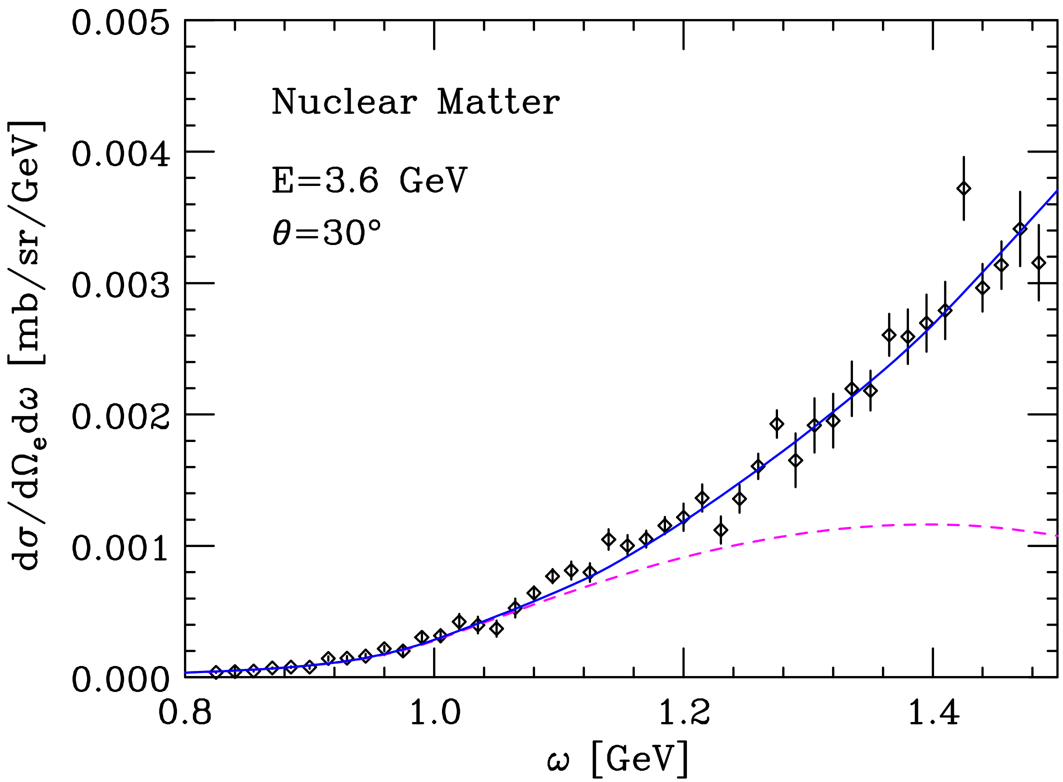
# Comparison to $^3He(e,e')$ data (continued)

⋆ Calculations by O.B. and V.R. Pandharipande PRC 47(93)2218; SLAC data by D. Day et al PRL 43(79)1143



# Comparison to data extrapolated to $A \longrightarrow \infty$

⋆ Calculations by O.B. et al PRC 44(91)2328; Extrapolation of SLAC data by D. Day et al PRC 40(89)1011



# Including FSI

- ⋆ Rewrite

$$S(\mathbf{q},\omega) = \int d\omega' S_0(\mathbf{q},\omega') f_{|\mathbf{k}+\mathbf{q}|}(\omega - \omega')$$

$$S_0(\mathbf{q},\omega) = \int d^3k dE \; P_h(\mathbf{k},E)\theta(k_F - |\mathbf{k}+\mathbf{q}|)\delta(\omega - e^0_{|\mathbf{k}+\mathbf{q}|} - E)$$

$$P_p(\mathbf{k}+\mathbf{q},\omega - E) = \theta(k_F - |\mathbf{k}+\mathbf{q}|) \; f_{|\mathbf{k}+\mathbf{q}|}(\omega - E - e^0_{|\mathbf{k}+\mathbf{q}|})$$

- ⋆ The *folding function* $f_p(\omega)$ accounts for the finite width of the state describing the struck particle

$$f_p(\omega) = \int \frac{dt}{2\pi} \, \mathrm{e}^{i[\omega + i\Gamma_p(t)]t}$$

- ⋆ assuming $\Gamma_p$ to be time independent: $\Gamma_p = v\rho\sigma_{NN}/2 = \Gamma^0_p$ one recovers $P_p(\mathbf{k}+\mathbf{q},\omega - E)$ in the quasiparticle approximation

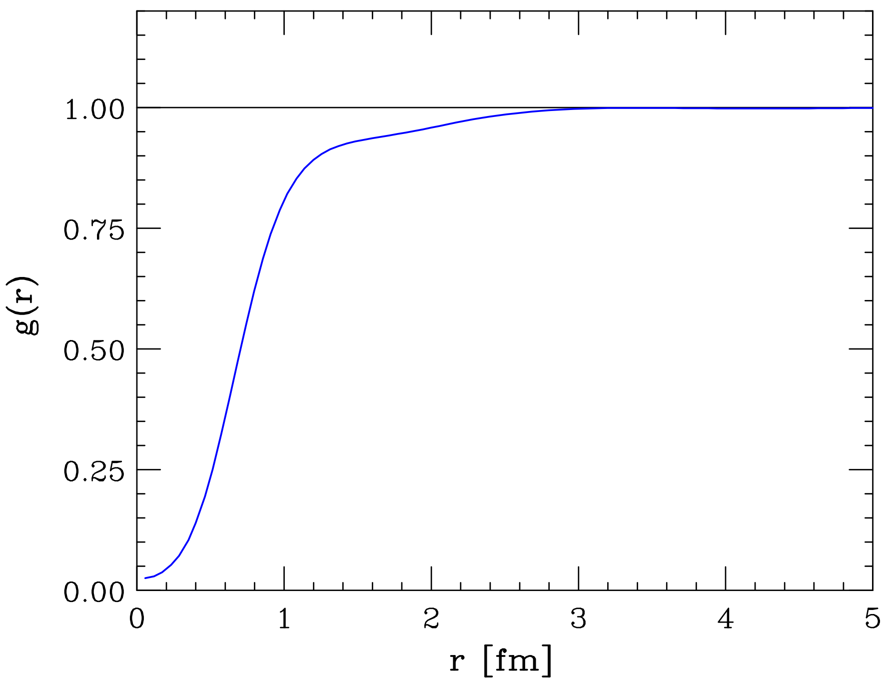
$$f_p(\omega - E - e^0_{|\mathbf{k}+\mathbf{q}|}) = \frac{1}{\pi} \frac{\Gamma^0_p}{[\omega - E - e^0_{|\mathbf{k}+\mathbf{q}|}]^2 + [\Gamma^0_p]^2}$$

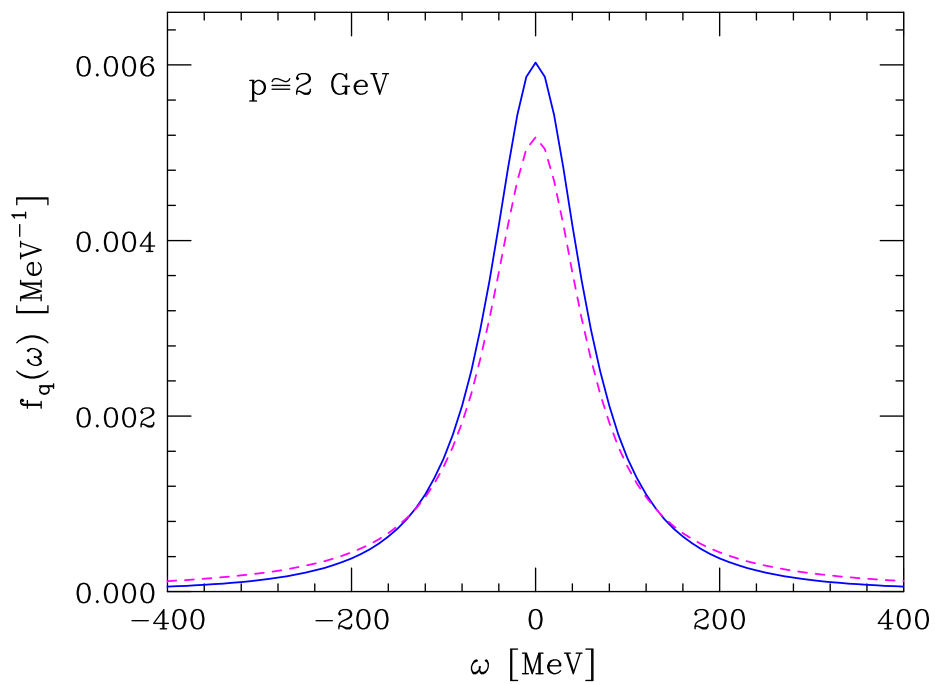
⋆ In presence of NN correlations inducing strong density fluctuations

$$\rho(\mathbf{r_1}, \mathbf{r_2}) = \rho^2 g(|\mathbf{r_1} - \mathbf{r_2}|)$$

and (use the eikonal approximation)

$$\Gamma_p(t) = \Gamma_p^0 \, \frac{1}{t} \int_0^t dt' g(vt')$$

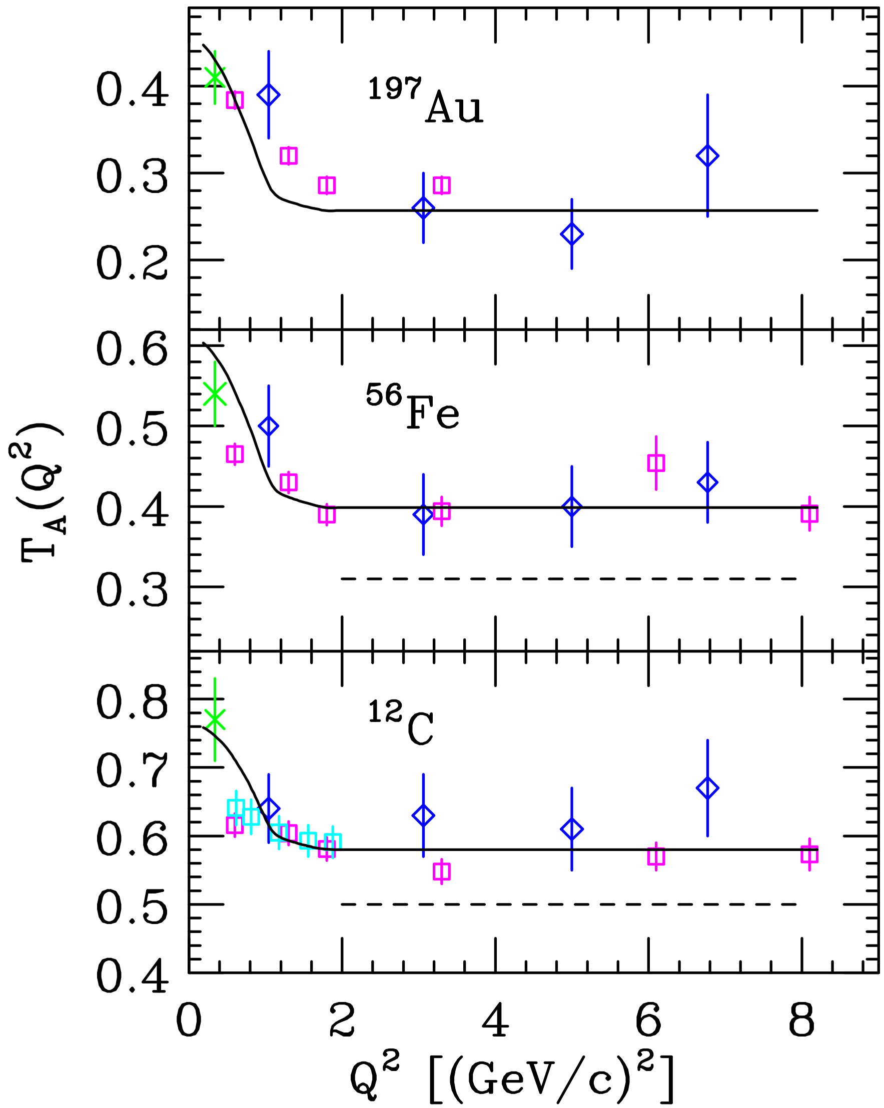



Calculations by O.B. et al PRC 44(91)2328

# Nuclear transparency measured in $(e, e'p)$

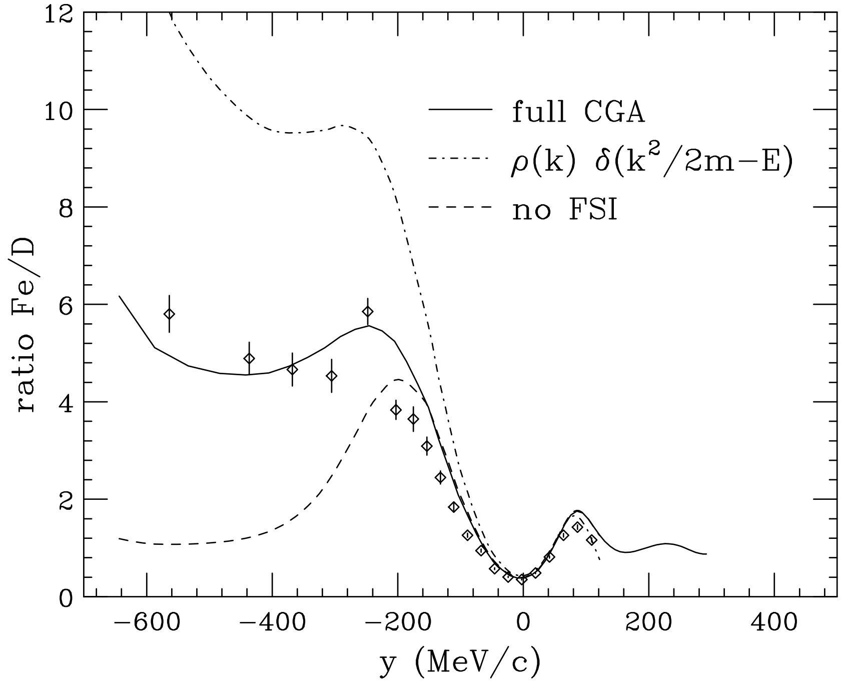
- ⋆ recall: no FSI $\rightarrow T_A \equiv 1$




D. Rohe et al PRC 72(05)054602

# Effect of FSI in inclusive processes

- ⋆ FSI lead to a quenching of the quasifree peak and an enhancement of the tails
- ⋆ At large momentum transfer the most visible effect is the enhancement of the cross section at low $\omega$ (or $x_{bj} \gg 1$, or large negative $y$)




Calculations by O.B. A. Fabrocini, S. Fantoni and I. Sick PLB 343(95)47; SLAC data

# Conclusions and prospects

- ⋆ Spectral functions are fundamental quantities needed for the theoretical description of the response of many-body system
- ⋆ Hole-state spectral functions obtained from NMBT provide a quantitative account of $(e, e'p)$ data
- ⋆ Inclusive $(e, e')$ x-sections in the IA regime are also accounted for in a variety of kinematical conditions, ranging from quasielastic to deep inelastic scattering
- ⋆ Inclusion of FSI effects requires modeling of the particle-state spectral function. Not feasible within NMBT for large $\mathbf{q}$.
- ⋆ Schemes based on the eikonal approximation work well for the $(e, e'p)$ transparency and the $(e, e')$ x-sections. A more complete description of rescattering processes is needed for event recontruction
- ⋆ Model spectral functions of particles other than proton and neutrons (e.g. $\pi$ mesons) are also needed