Lectures on Extra Dimensions
Part I

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Why Extra Dimensions (XD)?

- The idea that there are more than 3+1 dimensions has a long history...

  ...which I will not try to review... Sorry!

- Recent years have seen an outburst of activity in this field.

  In a sense, the driving force has been *experimental*, namely the real possibility to test these ideas at colliders (or even through cosmological observations, DM searches, etc.)

- XD should be considered a framework (even a set of frameworks), which can be realized through many, many models...

  Much as Quantum Field Theory relates to the Standard Model (SM)

  Hopefully, experimental data will tell us if/which XD are realized in nature

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- To proceed, I will simply assume that there is interest in learning about these ideas.

  Certainly, establishing experimentally the existence of XD would be revolutionary!

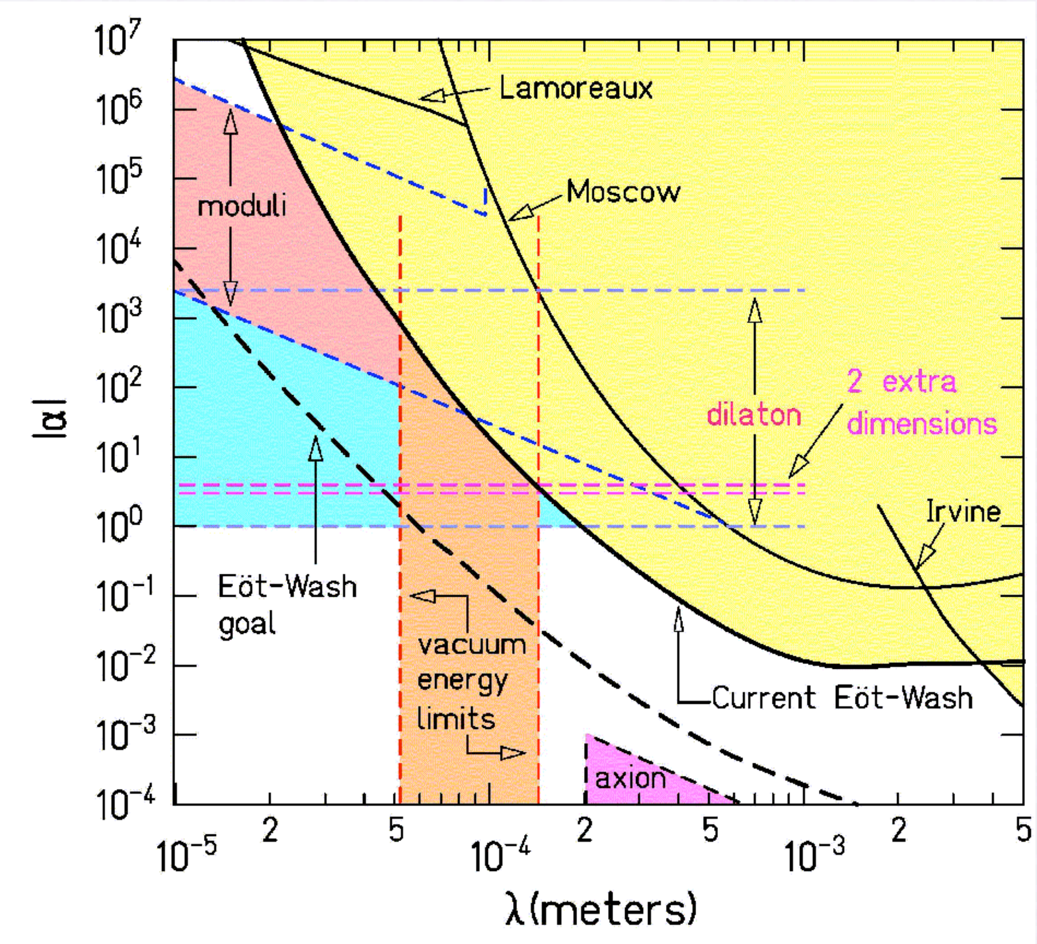
- I will not try to answer the question of this slide *now*, but rather as we go along...

Can we live in more than 4D?

- We perceive (either through our senses or experimentally) 3+1 dimensions
- Whether or not there could be more dimensions depends on how we can probe them

- *Gravitationally*: deviations from Newton's inverse square law tell us $R < 160\ \mu\text{m}$

  If these dimensions are cousins of the ones we see (geometric description via GR), we expect that gravity would always *see* them!



- If they can be probed by SM particles, constraints much tighter: we have probed distances of order 10^{-18} m $[\sim (100\ \text{GeV})^{-1}]$

  The SM is a 4D theory, and it works!

We would not have seen XD much smaller than this. Can they by lurking around?

My approach in these Lectures

- Can't be exhaustive... will have to leave many interesting topics out

Concentrate on XD at the TEV scale

(i.e. those that can in principle be probed in an environment like the LHC)

- Start by discussing basics, highlighting properties of general applicability
- Illustrate with the physics of a couple of examples (probably a biased exposition)
- Cover some phenomenological consequences (collider, radion, dark matter...)

Plan

PART I

- General theoretical remarks
  - The Kaluza-Klein decomposition
  - Boundary conditions
  - Localization in the extra dimensions

- Extra Dimensions at the TeV scale: two categories (examples)
  - Flat Extra Dimensions
  - Warped Extra Dimensions

PART II

- Dynamical breaking of the Electroweak (EW) symmetry
- The Radion
- Dark matter
- Collider Phenomenology

The Kaluza-Klein Decomposition

(or how compact dimensions are different)

- Compact dimensions involve a scale: size/volume of the extra dimension(s) $\rightarrow$ "R"

- Two equivalent descriptions are possible, and have different domains of usefulness:

1) At scales large or comparable to R

A 4D language is appropriate $\longrightarrow$ The concept of Kaluza-Klein (KK) modes

2) At scales small compared to R

Higher-D language better $\longrightarrow$
- Emphasis on higher-D spacetime structure
- Take into account effects of all KK modes at once
- E.g. useful to understand structure of divergences

- In most applications, we (would) be interested in the KK mode language
  - Easy to obtain low-energy description (it better describe physics as well as the SM does)
  - Relevant description of new physics at colliders

The Kaluza-Klein Decomposition

(or how compact dimensions are different)

Quantum fields in 4+n dimensions:

$$\Phi(x^\mu, y^i) \qquad\qquad (\mu = 0, 1, 2, 3; \quad y^i \text{ parametrize compact space})$$

Go to ``Fourier" space, except *momentum* not necessarily a good quantum number in the XD

The point is: we can expand any function in any *complete* set of functions $\{f_n(y^i)\}$

$$\Phi(x^\mu, y^i) = \frac{1}{\sqrt{V}} \sum_n \phi^{(n)}(x^\mu) f_n(y^i)$$

``n-th KK mode"

Life is easier if the basis is orthonormal:

$$\langle f_n | f_m \rangle = \delta_{nm} \longrightarrow$$

Allows to think of the $\phi^{(n)}$ as "independent" d.o.f.

normally defined in terms of an integral

The Kaluza-Klein Decomposition

(or how compact dimensions are different)

How do we choose a convenient basis? $\longrightarrow$ Depends on the model in question

In general: ``perturbation theory philosophy"

- Understand free part of the theory, add interactions later...
- Free (quadratic) part defines a differential operator, e.g.

$$\int d^4x d^n y \, \frac{1}{2} \left(\partial_N \Phi \partial^N \Phi - M^2 \Phi^2 \right) = - \int d^4x d^n y \, \frac{1}{2} \Phi \left(\Box + M^2 \right) \Phi$$

- Use the *eigenfunctions* of the XD part of the free differential operator

$$\left(\partial_i \partial^i + M^2 \right) f_n = m_n^2 f_n$$

linear PDE (this we can solve)

impose *appropriate* boundary conditions
(should regard as part of the definition of the theory)

Mathematical upshot: define a ``self-adjoint" problem $\longrightarrow$ orthonormality, completeness

The Kaluza-Klein Decomposition

(or how compact dimensions are different)

Now replace $\Phi(x^\mu, y^i) = \frac{1}{\sqrt{V}} \sum_n \phi^{(n)}(x^\mu) f_n(y^i)$ back in the Lagrangian:

$$-\int d^4x d^n y \, \frac{1}{2} \Phi \left(\partial_\mu \partial^\mu + \partial_i \partial^i + M^2 \right) \Phi$$

The Kaluza-Klein Decomposition

(or how compact dimensions are different)

Now replace $\Phi(x^\mu, y^i) = \frac{1}{\sqrt{V}} \sum_n \phi^{(n)}(x^\mu) f_n(y^i)$ back in the Lagrangian:

$$-\int d^4x d^ny \, \frac{1}{2} \Phi \left(\partial_\mu \partial^\mu + \partial_i \partial^i + M^2 \right) \Phi$$

$$= -\frac{1}{V} \int d^4x d^ny \sum_{n,n'} \frac{1}{2} \phi^{(n)} f_n \left[f_{n'} \partial_\mu \partial^\mu \phi^{(n')} + \phi^{(n')} (\partial_i \partial^i + M^2) f_{n'} \right]$$

$$\left(\partial_i \partial^i + M^2 \right) f_n = m_n^2 f_n$$

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$$\left(\partial_i \partial^i + M^2 \right) f_n = m_n^2 f_n$$

The Kaluza-Klein Decomposition

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But $\left(\partial_i \partial^i + M^2 \right) f_n = m_n^2 f_n$, plus b.c.'s implies $\frac{1}{V} \int d^ny \, f_n f_{n'} = \delta_{n,n'}$

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***Physical* upshot**: the theory can be *rewritten* as $$-\sum_n \int d^4 x \, \frac{1}{2} \phi^{(n)} \left(\partial_\mu \partial^\mu + m_n^2 \right) \phi^{(n)}$$

or... a free High-D scalar is equivalent to infinite 4D scalars with masses m_n^2 *!*

Boundary Conditions

It has been remarked that specifying the theory (physics) requires a choice of b.c.'s

We implicitly used this before: integrate by parts and discard surface terms (how convenient!)

(in 4D, the analogous assumption is that fields vanish sufficiently fast at ``infinity")

The issue can be turned around to ask: given an XD dimensional space,

What are the b.c.'s that preserve the previous nice properties?

- freely integrate by parts (convenient)
- self-adjointness (completeness, orthonormality, transparent physical interpretation)

The question can be answered systematically by considering arbitrary variations of the action

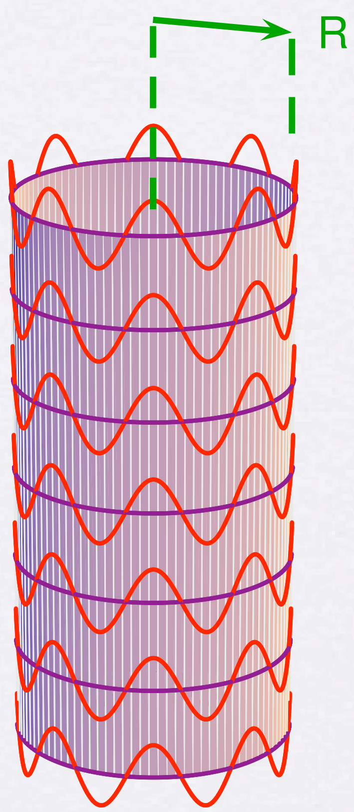
$$\text{Under } \delta\Phi \quad \longrightarrow \quad \delta S = \underbrace{\delta S_{\text{volume}}}_{\text{Eqs. of motion}} + \underbrace{\delta S_{\text{surface}}}_{\text{boundary cond.}}$$

Boundary Conditions (Examples)

Illustrate with a couple of relevant examples in one and two extra dimensions:

Compactification on a circle S^1

Periodic b.c.'s: $\Phi(y + 2\pi R) = \Phi(y)$



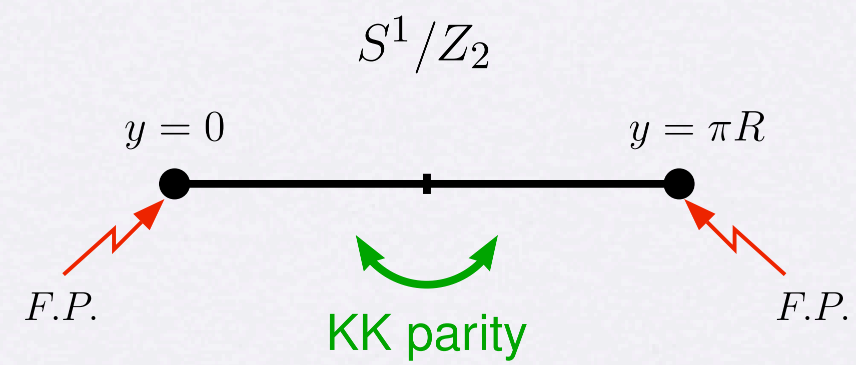
New quantum number is simply p_5

If 5D field is massless:
KK masses $m_n = \frac{n}{R}$

Compactification on the ``Interval"

(XD extends from $y = 0$ to $y = \pi R$)

B.c.'s at $y = 0$ and $y = \pi R$ are Dirichlet, Neumann ...
or linear combinations

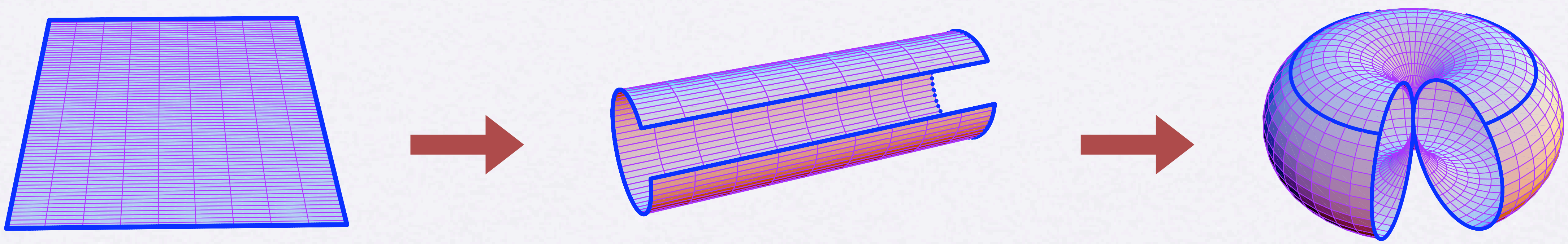


End points sometimes called ``Fixed points" or ``branes"

Boundary Conditions (Examples)

Torus compactification (periodic b.c.'s):

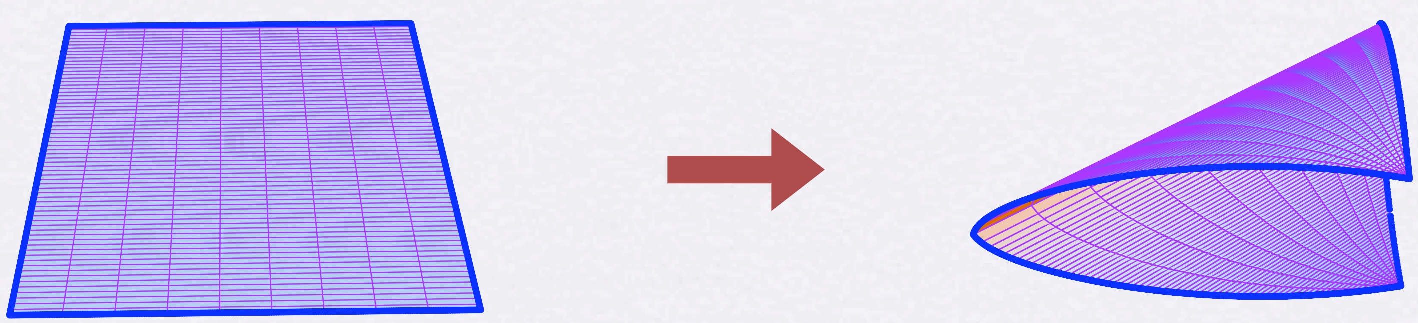
$$\Phi(x^4 + 2\pi R, x^5) = \Phi(x^4, x^5)$$
$$\Phi(x^4, x^5 + 2\pi R) = \Phi(x^4, x^5)$$



The ``Chiral Square":

$$\Phi(y, 0) = e^{in\pi/2}\Phi(0, y)$$
$$\Phi(y, \pi R) = e^{in\pi/2}\Phi(\pi R, y)$$

$$\partial_5\Phi_{(x^4,x^5)=(y,0)} = -e^{in\pi/2}\, \partial_4\Phi_{(x^4,x^5)=(0,y)}$$
$$\partial_5\Phi_{(x^4,x^5)=(y,\pi R)} = -e^{in\pi/2}\, \partial_4\Phi_{(x^4,x^5)=(\pi R,y)}$$



Zero-Modes

The KK decomposition can lead to 0-modes, i.e. solutions with $m_0 = 0$

- For 5D gauge fields (flat space), one finds:

$$f_n''(y) + 2f_n'(y) = -m_n^2 f_n(y) \qquad \text{which is solved by} \quad \begin{cases} m_0^2 = 0 \\ f_0(y) = 1 \end{cases}$$

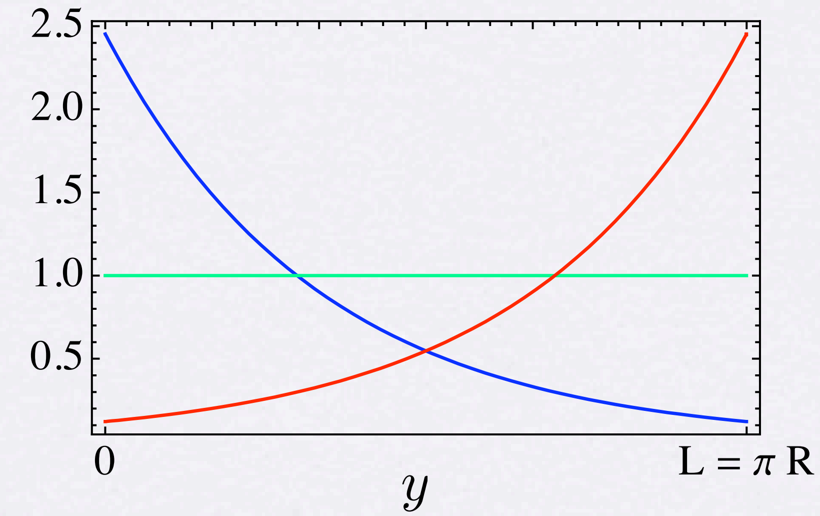
Flat wavefunction $\longrightarrow$ 4D gauge invariance

- For 5D fermion fields (flat space), one finds:

$$\left.\begin{aligned} f_{n,L}' - M f_{n,L} &= m_n f_{n,R} \\ -f_{n,R}' - M f_{n,R} &= m_n^* f_{n,L} \end{aligned}\right\} \quad \text{which are solved by} \quad \begin{cases} m_0 = 0 \\ f_0^{L,R}(y) = \sqrt{\dfrac{1-e^{-2ML}}{2ML}}\, e^{\pm My} \end{cases}$$

These solutions may or may not be allowed by the b.c.'s

- 4D gauge symmetry can be (spontaneously) broken by b.c.'s
- Circle and Torus: allow both chiralities
- ``Interval" and ``Chiral Square": allow only one chirality



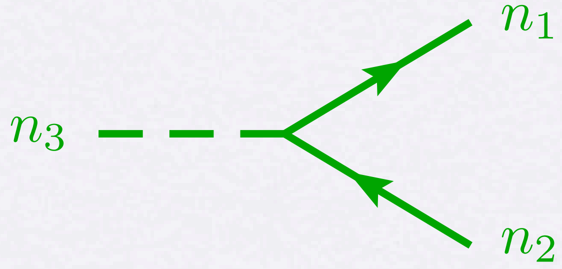
Interactions in KK Language

Having understood how to interpret a higher-D theory in 4D terms, we can consider interactions

- As long as these are perturbative, the physics can be understood in terms of KK modes
- In the free theory only the *spectrum* is observable. With interactions, the *wavefunctions* also become observable, since they determine the details of the interactions among KK modes, e.g.

$$\int d^4x\, d^ny\, \lambda_n \bar{\Psi}\Psi\Phi \to \sum_{n_1,n_2,n_3} \lambda_{n_1,n_2,n_3} \int d^4x\, \bar{\psi}^{(n_1)}\psi^{(n_2)}\phi^{(n_3)}$$

$$\lambda_{n_1,n_2,n_3} = \frac{\lambda_n}{V\sqrt{V}} \int d^ny\, f_{n_1} f_{n_2} f_{n_3}$$



Sometimes this integral obeys interesting selection rules, e.g. in 5D flat space on ``interval'':

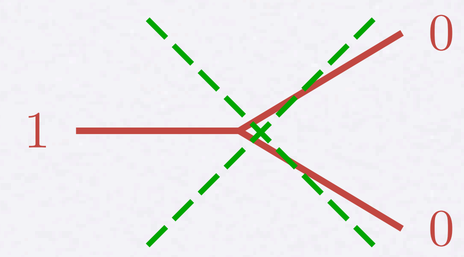
$$n_1 \pm n_2 \pm n_3 = 0$$

Hence, at tree-level, no KK mode can decay into 0-modes
(a similar selection rule holds on the chiral square and cousins)

KK parity (and new stable particles)

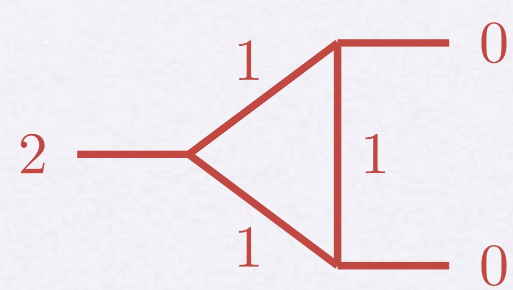
Compactification on flat spaces have a natural remnant of XD momentum conservation

At tree-level: all first-level KK modes are stable!



KK number *n* associated with the magnitude of XD momenta (conserved up to a sign)

But loops induce new interactions:

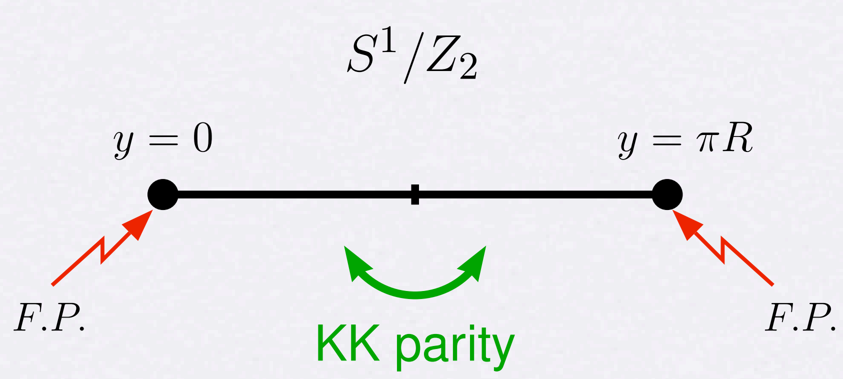


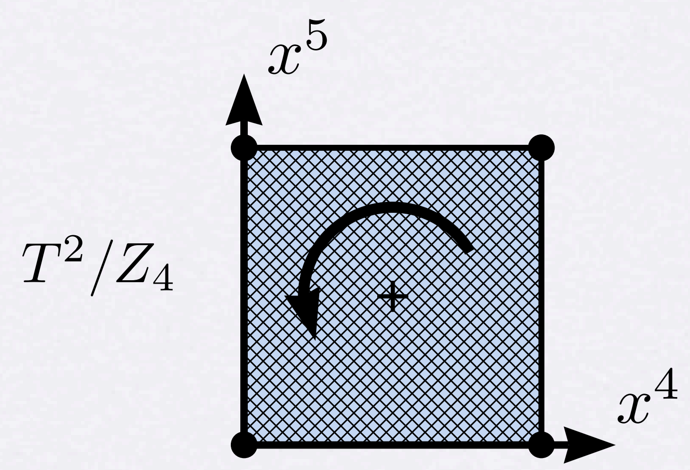
Can be interpreted as interactions localized at the fixed points

However, one can still have a discrete symmetry that makes the *lightest* 1st mode stable

$$\phi^{(n)} \to (-1)^n \phi^{(n)}$$

$$\phi^{(n_4,n_5)} \to (-1)^{n_4+n_5} \phi^{(n_4,n_5)}$$





KK Decompositions in Warped Spaces

For 5D theories preserving 4D Lorentz invariance:

$$ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

For scalars:

$$\Phi(x^\mu, y) = \frac{a(y)^{-1}}{\sqrt{L}} \sum_n \phi^{(n)}(x^\mu) f_n(y)$$

Eq. of motion:

$$f_n'' + 2\frac{a'}{a}f_n' - \left[\frac{a''}{a} + 2\,\frac{a'^2}{a^2} + M^2\right] f_n = -m_n^2 a^{-2} f_n$$

Solution for $a(y) = e^{-ky}$ and bulk mass $M^2 = \left[c_s^2 + c_s - \frac{15}{4}\right] k^2$:

$$f_n(y) = N_n e^{ky}\left[J_{|c_s+1/2|}(m_n e^{ky}/k) + b_n\, Y_{|c_s+1/2|}(m_n e^{ky}/k)\right]$$

For fermions:

$$\Psi_{L,R}(x^\mu, y) = \frac{a(y)^{-3/2}}{\sqrt{L}} \sum_n \psi^{(n)}_{L,R}(x^\mu) f_n^{L,R}(y)$$

Eqs. of motion:

$$f'_{n,L} - (c_f - 1/2)\,\frac{a'}{a} f_{n,L} = m_n a^{-1} f_{n,R}$$

$$-f'_{n,R} - (c_f - 1/2)\,\frac{a'}{a} f_{n,R} = m_n^* a^{-1} f_{n,L}$$

Solution for $a(y) = e^{-ky}$ and bulk mass $M = c_f k$:

$$f_n(y) = N_n e^{ky}\left[J_{|c_f+1/2|}(m_n e^{ky}/k) + b_n Y_{|c_f+1/2|}(m_n e^{ky}/k)\right]$$

KK Decompositions in Warped Spaces

For gauge fields with a gauge fixing term $\frac{1}{2\xi}\left\{\eta^{\mu\nu}\partial_\mu A_\nu - \xi\,\partial_y\left[a(y)^2 A_5\right]\right\}^2$:

$$A_\mu(x^\mu, y) = \frac{1}{\sqrt{L}}\sum_n A_\mu^{(n)}(x^\mu) f_n(y)$$

Eq. of motion:

$$f_n''(y) + 2\frac{a'}{a}f_n'(y) = -m_n^2 a^{-2} f_n(y)$$

Solution for $a(y) = e^{-ky}$:

$$f_n(y) = N_n e^{ky}\left[J_1(m_n e^{ky}/k) + b_n\, Y_1(m_n e^{ky}/k)\right]$$

All wavefunctions normalized according to:

$$\frac{1}{L}\int d^n y\, f_n f_{n'} = \delta_{n,n'}$$

These wavefunctions reflect the strength of the interactions at each point y

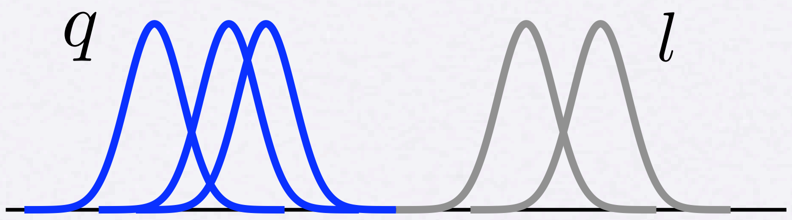
Boundary conditions fix the constants b_n and the spectrum m_n.

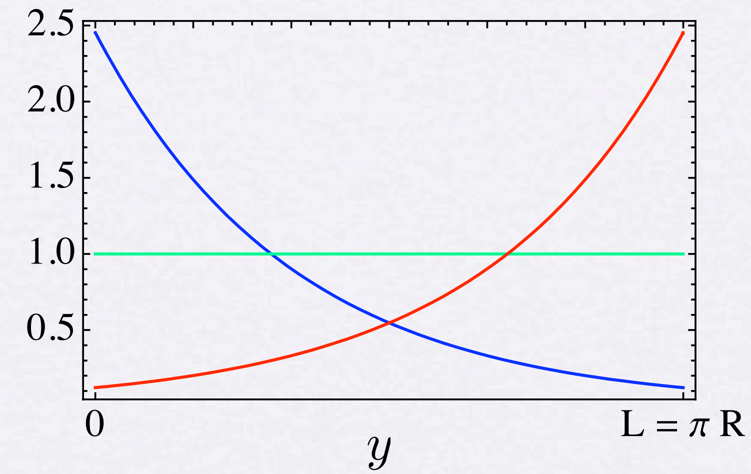
Using the Extra Real Estate

The low-energy physics (that of the ``0-modes") can be very sensitive to the XD

We already observed that:

- 5D fermion masses control localization
- Couplings are proportional to overlap integrals





Hence it is easy to explain exponentially small (dimensionless) numbers from the underlying (unseen) XD

Yukawa couplings:

$$y_t \sim 1 \qquad y_e \sim 10^{-5} \qquad (y_\nu \sim 10^{-12} \; ?)$$

In such scenarios one can argue that the emergence of exponential herarchies is the norm, thus making the observations of the SM far less ``puzzling"

Using the Extra Real Estate

- Scalars can also be localized in a manner similar to fermions.

Unfortunately, the existence of a (localized) scalar 0-mode, depends on the relation between the bulk mass and two ``brane-localized" masses

In general: tuning required to obtain a light mode (compared to the KK scale)

(The fact that the possibility exists, is tied to the SUSY limit of the XD framework)

- Nevertheless, there are other ways of getting *naturally* localized 4D scalars...

Using the Extra Real Estate

1) Consider the 5th polarization of a 5D gauge field

$$\text{If } A_\mu \text{ obeys } (-,-) \text{ b.c.'s} \qquad (\text{Dirichlet at both } y = 0, L)$$

(4D gauge symmetry broken by b.c.'s)

$$\text{Then } A_5 \text{ obeys } (+,+) \text{ b.c.'s} \qquad (\text{Neumann at both } y = 0, L)$$

In a warped background, the EOM for A_5 is:

$$\partial_y^2 \left(a^2 f_0\right) = 0 \quad \longrightarrow \quad f_0(y) = N_0\, a^{-2}(y) \xrightarrow{a = e^{-ky}} \sqrt{\frac{2kL}{e^{2kL} - 1}}\, e^{2ky}$$

(additive constant forbidden by b.c.'s)

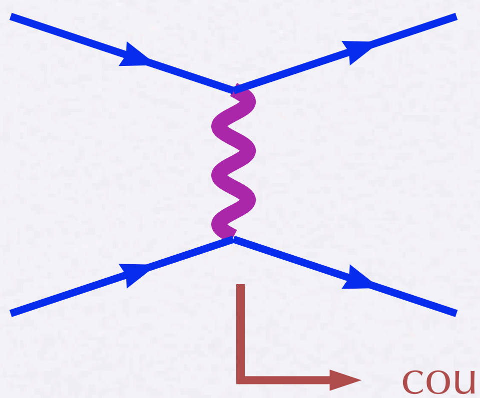
$\longrightarrow$ Localization at $y = L$ (near the ``IR brane'', or where warp factor smallest)

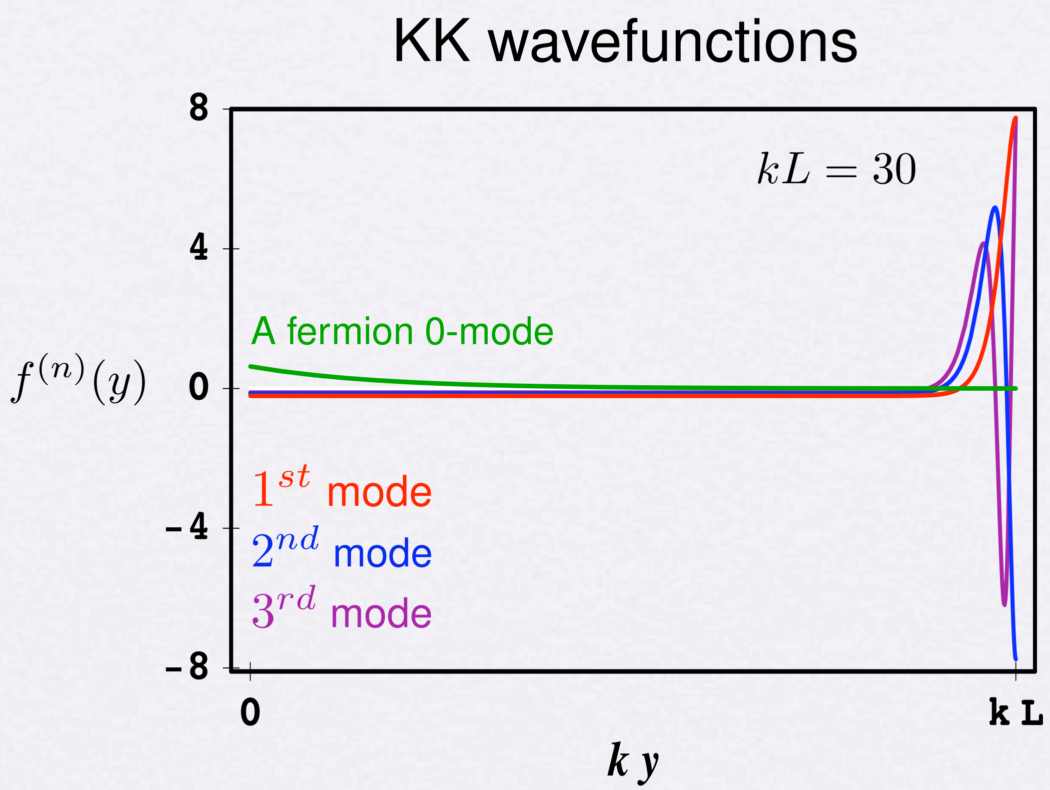
- 4D scalar from A_5 can be light and have non-trivial couplings to other light fields
- Notice there are no adjustable parameters, localization happens dynamically!

Using the Extra Real Estate

2) Strongly interacting fermions can form scalar bound states

- Attractive channels from KK gluon exchange
- KK gluons localized near $y = L$





Upshot:

- Fermion localization triggers formation of bound state (also a condensate)
- Resulting scalar bound state is effectively localized on IR brane (because fermion constituents are!)
- Scalar mass set dynamically well below KK scale

Models: Examples

Universal Extra Dimensions

Warped Extra Dimensions (Randall-Sundrum)

Universal Extra Dimensions (UED)

Assumption: maybe the SM lives in 4+n flat dimensions

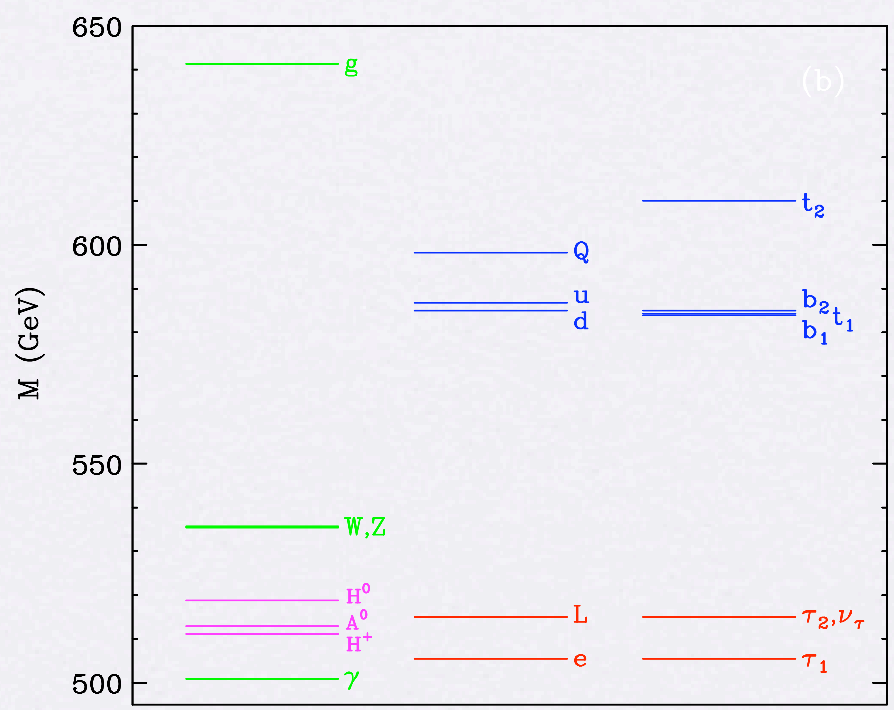
$\longrightarrow$ All SM particles have KK excitations that can be studied at colliders

Models in 5D and 6D have been studied...

KK decompositions rather simple, tree-level spectra given by $\begin{cases} \frac{n}{R} & 5D \\ \frac{\sqrt{j^2+k^2}}{R} & 6D \end{cases}$

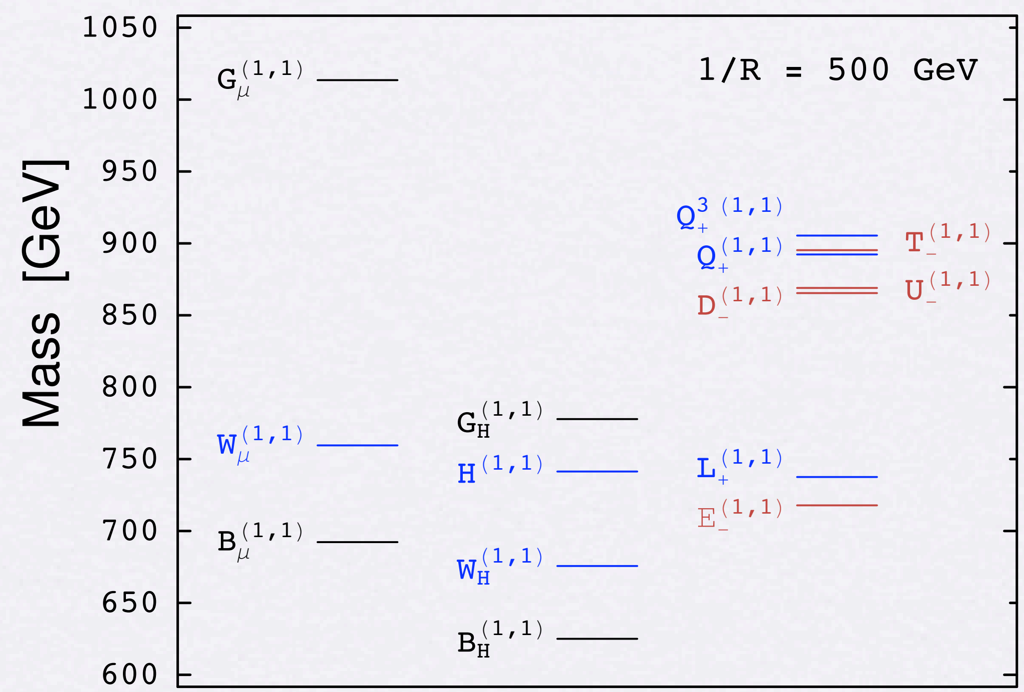
Loop effects play a central role:

5D UED: first level



(from Cheng, Matchev & Schmaltz)

6D UED: (1,1) level states



(from Pontón & Wang)

Some Interesting Features

- KK states can be relatively light (few hundres GeV)
- KK parity: natural dark matter candidates (more later)
- In 6D:
  - An understanding for number of generations based on anomaly cancellation
  - Higher-dimensional spacetime symmetries lead to discrete symmetries that:

    Can explain matter stability (even if baryon number violated near the weak scale)

    Predict three right-handed neutrinos

    Predict that neutrinos should be Dirac particles
- Phenomenology of first KK level similar to SUSY (missing energy signals)
- Phenomenology of second KK level can lead to well-defined resonances

The Randall-Sundrum Scenario

The magic of curvature (warping)

Assumptions:

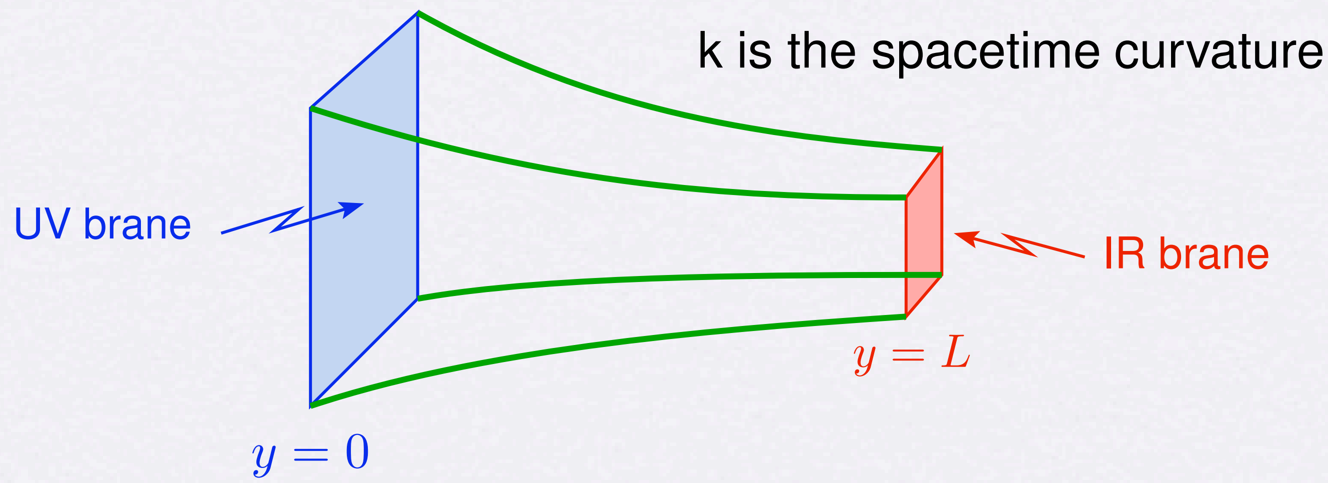
- 5D spacetime, with 5D cosmological constant
- Compactification on the ``Interval''

}

- Soln. to Einstein's Eqns.
- Slicing with 4D Lorentz invariant sections

Spacetime described by the line element

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \qquad y \in [0, L]$$



Fields can either propagate in the bulk, or be stuck to one of the ``branes''

The Randall-Sundrum Scenario

The magic of curvature (warping)

The point is that scales at different points in the XD are measured differently

To illustrate, consider a field ϕ_{IR} localized on the IR brane:

$$S \supset \int d^4x dy \sqrt{-G} \left\{ \delta(y-L) \left[\frac{1}{2} G^{\mu\nu} \partial_\mu \phi_{IR} \partial_\nu \phi_{IR} - \lambda_{IR} \left(\phi_{IR}^2 - v_{IR}^2 \right)^2 \right] \right\}$$

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$$\stackrel{\tilde{\phi}_{IR} = e^{-kL}\phi_{IR}}{=} \int d^4x \left\{ \frac{1}{2} \partial_\mu \tilde{\phi}_{IR} \partial^\mu \tilde{\phi}_{IR} - \lambda_{IR} \left(\tilde{\phi}_{IR}^2 - v_{IR}^2 e^{-2kL} \right)^2 \right\}$$

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$\tilde{\phi}_{IR} = e^{-kL} \phi_{IR}$

$$= \int d^4x \left\{ \frac{1}{2} \partial_\mu \tilde{\phi}_{IR} \partial^\mu \tilde{\phi}_{IR} - \lambda_{IR} \left(\tilde{\phi}_{IR}^2 - v_{IR}^2 e^{-2kL} \right)^2 \right\}$$

If instead the field was localized on the UV brane, ϕ_{UV} : all warp factors are unity

$$V = \lambda_{UV} \left(\tilde{\phi}_{UV}^2 - v_{UV}^2 \right)^2 + \lambda_{IR} \left(\tilde{\phi}_{IR}^2 - v_{IR}^2 e^{-2kL} \right)^2$$

(Hierarchically different vev's)

The Randall-Sundrum Scenario

The magic of curvature (warping)

Rule of thumb: all mass parameters on IR brane are warped down by e^{-kL}

$$\mathcal{L}_5^{\text{kinetic}} = \bar{\Psi}\partial\!\!\!/\Psi \qquad \longrightarrow \qquad [\Psi] = 2$$

Mass dimension in natural units

$$\mathcal{L}_4^{\text{kinetic}} = \bar{\psi}\partial\!\!\!/\psi \qquad \longrightarrow \qquad [\psi] = 3/2$$

Consider a 4-fermion operator (relevant for e.g. flavor):

$$\mathcal{L}_5 = \frac{\alpha}{\Lambda^3}(\bar{\Psi}_1\Psi_2)(\bar{\Psi}_3\Psi_4) \quad \Rightarrow \quad \mathcal{L}_4 = \frac{\alpha'}{\tilde{\Lambda}^2(\Lambda L)}(\bar{\psi}_1\psi_2)(\bar{\psi}_3\psi_4)$$

``warped down" scale $\tilde{\Lambda} = e^{-kL}\Lambda$

volume suppression for bulk fields

The Randall-Sundrum Scenario

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From mass dimension of 4D operator

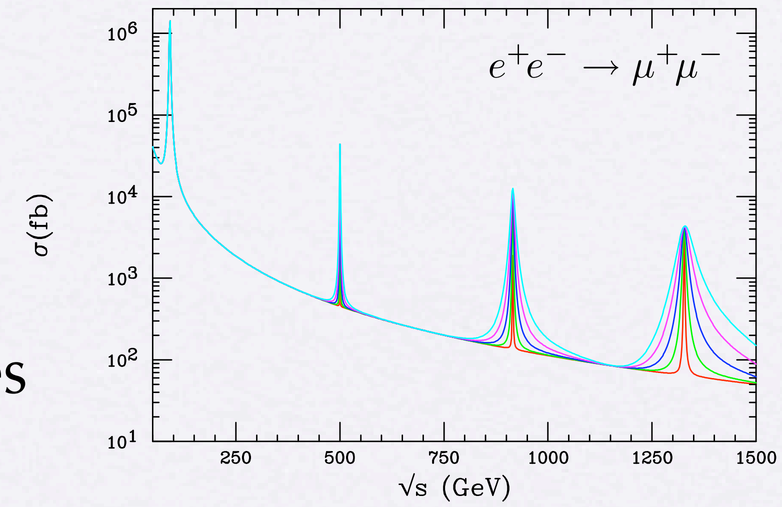
$$\alpha' = \frac{\alpha}{L} \int_0^L dy \, e^{-2k(L-y)} f_1^* f_2 f_3^* f_4$$

- Each KK mode $\rightarrow \sqrt{2kL}$
- 0-mode near IR $\rightarrow \sqrt{(1-2c)kL}$
- 0-mode flat $\rightarrow 1$
- 0-mode near UV $\rightarrow$ exp. suppression
- Eff. volume for integral $\rightarrow 1/kL$

The Randall-Sundrum Scenario

The warp factor can naturally accommodate the EW and Planck (say) scales, provided the Higgs (or the source of EWSB) is localized near the IR brane

(from Davoudiasl, Hewett & Rizzo)



Model building:

- Original RS scenario had all SM fields on the brane

  Only gravitons propagate in the bulk and have KK modes

- But only Higgs needs to be on IR brane. Bulk fields buy you interesting physics:
  - Understand exponential fermion mass hierarchies
  - Suppress dangerous FCNC's from higher-dimension operators
  - Other calculable FCNC's also suppressed

$$\left\{ \frac{\alpha'}{\tilde{\Lambda}^2(\Lambda L)} (\bar{\psi}_1\psi_2)(\bar{\psi}_3\psi_4) \right. \quad \text{with } \alpha' \text{ exp. suppr.}$$

$\longrightarrow$ Essentially a theory of flavor with physics at the TeV scale!

- Accommodates gauge coupling unification, and more...

End of Part I