High-energy atmospheric neutrinos

S. I. Sinegovsky[1], A. A. Kochanov[2], and T. S. Sinegovskaya[3]

[1]Institute of Applied Physics, Irkutsk State U., Gagarin Blvd. 20, Irkutsk
e-mail: sinegovsky@api.isu.ru
[2]Institute of Solar-Terrestrial Physics, SD RAS, Lermontov str. 126a, Irkutsk
[3]Department of Higher Mathematics, Irkutsk State Railway University, Chernyshevskii str. 15, Irkutsk

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Abstract

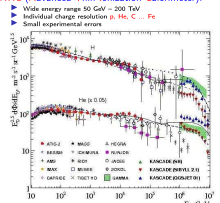
High-energy neutrinos, arising from decays of mesons produced in cosmic rays collisions with air nuclei, form the noise for the astrophysical neutrinos. The atmospheric neutrino flux above 1 PeV should be supposedly dominated by the contribution of charmed particle decays. The prompt neutrinos originated from decays of massive and shortlived particles, $D^{\pm}$, D^0, $\overline{D}^0$, $D_s^{\pm}$, Λ_c^+, form the most uncertain fraction of the high-energy atmospheric neutrino flux because of poor explored processes of the charm production. Besides, an ambiguity in high-energy behavior of pion and especially kaon production cross sections for nucleon-nucleus collisions may affect essentially the calculated neutrino flux. There is the energy region where above flux uncertainties superimpose. A new calculation presented here reveals sizable differences, up to the factor of 1.7 above 1 TeV, in muon neutrino flux predictions obtained with usage of known hadronic models, SIBYLL 2.1 and QGSJET-II. New calculation of the atmospheric neutrino flux in the energy range $10 - 10^7$ GeV is made within $1D$ approach to solve nuclear cascade equations in the atmosphere, which takes into account non-scaling behavior of the inclusive cross-sections for the particle production, the rise of total inelastic hadron-nucleus cross-sections and nonpower-law character of the primary cosmic ray spectrum. This approach was recently tested in the atmospheric muon flux calculations [Astropart. Phys. 30 (2008) 219]. The results of the neutrino flux calculations are compared with the data of Frejus, AMANDA-II and IceCube experiments.

Motivation

- The atmospheric neutrino flux in wide energy range is the problem of the great interest: the low energy AN flux is the research matter in neutrino oscillations studies, and the high energy atmospheric neutrino flux is the background for astrophysical neutrino experiments, NT200+, AMANDA, ANTARES, IceCube etc.
- We don't know whether the differences are considerable in high-energy AN flux resulted from various hadronic interaction models.
- Does the atmospheric muon (and/or neutrino) flux measurements ensure sufficient test of the hadronic cross sections?
- In spite of numerous calculations we do not know how strong is discrepancy in results due to uncertainties in primary cosmic ray spectra and composition.

Primary cosmic ray spectra

ATIC (Advanced Thin Ionization Calorimeter):
- Wide energy range 50 GeV – 200 TeV
- Individual charge resolution p, He, C ... Fe
- Small experimental errors



p, He spectra with balloon, satellite and ground-based measurements.

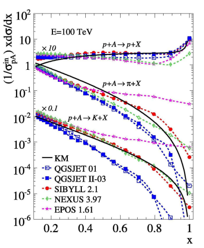
red curcles: ATIC-2, A.D. Panov et al., Bull. Russ. Acad. Sci. Phys. 71, 494 (2007); astro-ph/0612377

Solid curve: Zatsepin & Sokolskaya, A & A 458, 1 (2006); Astron. Lett. 33, 25 (2007) ;

dashed: the spectrum by Gaiser, Honda, Lipari, and Stanev, Proc. 27th ICRC, Hamburg, 2001, vol. 1, p. 1643; Gaisser & Honda, Annu. Rev. Nucl. Part. Sci. 52, 153 (2002);

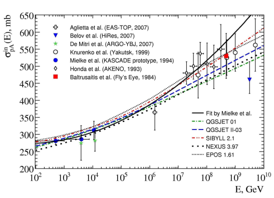
dotted: Berezhko & Völk, Astrophys. J. Lett. 661 (2007) L175

Hadronic interaction models



- SIBYLL 2.1: R.S. Fletcher, T.K. Gaisser, P. Lipari, T. Stanev, Phys. Rev. D 50 (1994) 5710; R. Engel, T. K. Gaisser, P. Lipari, T. Stanev, Proc. 26th ICRC, 1999, vol. 1, p. 415
- QGSJET-II: N.N.Kalmykov, S.S.Ostapchenko, A. I. Pavlov, Nucl. Phys. B (Proc. Suppl.) 52 (1997) 17; S. S. Ostapchenko, Nucl. Phys. B (Proc. Suppl.) 151 (2006) 143; Phys. Rev. D 74 (2006) 014026
- Kimel & Mokhov model (KM): A.N.Kalinovsky, N.V.Mokhov, Yu.P. Nikitin, Passage of high-energy particles through matter, AIP, NY, 1989
- NEXUS: H.J.Drescher et al., Phys. Rep. 350 (2001) 93; T.Pierog et al., Nucl. Phys. A 715 (2003) 895; K. Werner et al., J. Phys. G 30 (2003) S211
- EPOS: K.Werner, T.Pierog, AIP Conf. Proc. 928 (2007) 111. T.Pierog, K.Werner, Proc. 30th ICRC, Merida, HE.1.6-905, 2007; K.Werner, Nucl.Phys. B (Proc. Suppl.) 175-176 (2008) 81

Inelastic cross-section of p-air collisions



Cosmic-ray spectrum weighted moments

$$z_{pc}(E_0) = \int_0^1 x^{\gamma} \frac{1}{\sigma_{pA}^{in}} \frac{d\sigma_{pc}}{dx} dx, \quad \gamma = 1.7$$

Model	E_0, GeV	z_{pp}	z_{pn}	$z_{p\pi^+}$	$z_{p\pi^-}$	z_{pK^+}	z_{pK^-}
QGSJET-II	10^2	0.174	0.088	0.043	0.035	0.0036	0.0030
	10^3	0.198	0.094	0.036	0.029	0.0036	0.0028
	10^4	0.205	0.090	0.033	0.028	0.0034	0.0027
SIBYLL 2.1	10^2	0.211	0.059	0.036	0.026	0.0134	0.0014
	10^3	0.209	0.045	0.038	0.029	0.0120	0.0022
	10^4	0.203	0.043	0.037	0.029	0.0097	0.0026
KM	10^2	0.178	0.060	0.044	0.027	0.0051	0.0015
	10^3	0.190	0.060	0.046	0.028	0.0052	0.0015
	10^4	0.182	0.052	0.046	0.029	0.0052	0.0015

The method: *Nucleon cascade equations*

The neutrino flux calculation is based on the method to solve the atmospheric hadron cascade equations, Naumov & Sinegovskaya, Phys. Atom. Nucl. 63, 1927 (2000); Proc. of 27 ICRC, Hamburg, 2001, Vol. 1, p. 4173; hep-ph/0106015.
Neglecting the processes $\pi + A \to N\bar{N} + X$ one can reduce the system of equations for the differential energy spectra of nucleons $N_{\pm}(E,h)$ at the depth h to that of:

$$\frac{\partial N_{\pm}(E,h)}{\partial h} = -\frac{N_{\pm}(E,h)}{\lambda_N(E)} + \frac{1}{\lambda_N(E)} \int_0^1 \Phi_{NN}^{\pm}(E,x) N_{\pm}(E/x,h) \frac{dx}{x^2},$$
$$N_{\pm}(E,h) = p(E,h) \pm n(E,h),$$

$$\Phi_{NN}^{\pm}(E,x) = \frac{E}{\sigma_{pA}^{in}(E)} \left[\frac{d\sigma_{pp}(E_0,E)}{dE} \pm \frac{d\sigma_{pn}(E_0,E)}{dE}\right]_{E_0=E/x},$$

with the boundary condition: $N_{\pm}(E,0) = p_0(E) \pm n_0(E)$.

Here $\lambda_N(E) = 1/\left[N_0 \sigma_{pA}^{in}(E)\right]$ - nucleon interaction length; $d\sigma_{ab}(E_0,E)/dE$ - differential cross section for reaction $a + A \to b + X$; E_0 and E – the energies of the projectile and final nucleons, respectively.

Formal solution of the nucleon equations:

$$N_{\pm}(E,h) = N_{\pm}(E,0) \exp\left[-\frac{h}{\Lambda_{\pm}(E,h)}\right], \quad \frac{1}{\Lambda_{\pm}(E,h)} = \frac{1 - Z_{NN}^{\pm}(E,h)}{\lambda_N(E)}.$$

The functions $\Lambda_{\pm}(E,h)$ are generalized attenuation lengths (γ-dependent).

Equations for the nucleon $Z(E,h)$ functions

Z-factors obey the integral equation:

$$hZ_{NN}^{\pm}(E,h) = \int_0^h dt \int_0^1 \eta_{NN}^{\pm}(E,x) \Phi_{NN}^{\pm}(E,x) \times \exp\left[-tD_{NN}^{\pm}(E,x,t)\right] dx,$$

$$D_{NN}^{\pm}(E,x,t) = \frac{1 - Z_{NN}^{\pm}(E/x,t)}{\lambda_N(E/x)} - \frac{1 - Z_{NN}^{\pm}(E,t)}{\lambda_N(E)}, \quad \eta_{NN}^{\pm}(E,x) = \frac{N_{\pm}(E/x,0)}{x^2 N_{\pm}(E,0)}$$

Zero approximation:

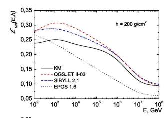
$$Z_{NN}^{\pm(0)}(E,h) = 0, \quad D_{NN}^{\pm(0)}(E,x,h) = 1/\lambda_N(E/x) - 1/\lambda_N(E).$$

n-th step:

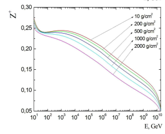
$$hZ_{NN}^{\pm(n)}(E,h) = \int_0^h dt \int_0^1 \eta_{NN}^{\pm}(E,x) \Phi_{NN}^{\pm} \exp\left[-t\, D_{NN}^{\pm(n-1)}(E,x,t)\right] dx$$

$$D_{NN}^{\pm(n)}(E,x,h) = \frac{1 - Z_{NN}^{\pm(n)}(E/x,h)}{\lambda_N(E/x)} - \frac{1 - Z_{NN}^{\pm(n)}(E,h)}{\lambda_N(E)}.$$

$Z(E,h)$ functions: 4 hadronic models +2 PCR spectra



$Z_{NN}^{+}(E,h)$ for ATIC-2 spectra (combined with Zatsepin & Sokolskaya model)



$Z_{NN}^{+}(E,h)$, KM + GH

π, K-meson cascade equations

A Kochanov, T.Sinegovskaya, S.Sinegovsky, Astropart. Phys. 30, 219 (2008), arXiv:0803.2943

The system of transport equations for charged pions is given by:

$$\frac{\partial \pi(E,h,\vartheta)}{\partial h} = -\frac{\pi(E,h,\vartheta)}{\lambda_\pi(E)} - \frac{m_\pi \pi(E,h,\vartheta)}{p_T \pi \rho(h,\vartheta)} + \sum_i G_{i\pi}^{int}(E,h,\vartheta) + \sum_K G_{K\pi}^{dec}(E,h,\vartheta) + \frac{1}{\lambda_\pi(E)} \int_{E_{\pi\pi}^{min}}^{\infty} \frac{1}{\sigma_{\pi A}^{in}(E)} \frac{d\sigma_{\pi\pi}(E_0,E)}{dE} \pi(E_0,h,\vartheta) dE_0,$$

where $\pi = (\pi^+, \pi^-)$ – flux of charged pions; $\lambda_\pi(E) = 1/N_0 \sigma_{\pi A}^{in}$ – pion interaction length ($\sigma_{\pi A}^{in}$ – pion total inelastic cross section, N_0 – number of nuclei A in 1 gm of air); $\rho(h,\vartheta)$ – air density; $G_{i\pi^{\pm}}^{int}$, $G_{K\pi^{\pm}}^{dec}$ – source functions; $i = p, n, K^{\pm}, K^0, \bar{K}^0$.

The system of transport equations for K-mesons ($K^{\pm}, K^0, \bar{K}^0$) has the same form:

$$\frac{\partial K(E,h,\vartheta)}{\partial h} = -\frac{K(E,h,\vartheta)}{\lambda_K(E)} - \frac{m_K K(E,h,\vartheta)}{p_T K \rho(h,\vartheta)} + G_{NK}(E,h) + G_{\pi K}(E,h,\vartheta) + \frac{1}{\lambda_K(E)} \sum_K \int_{E_{KK}^{min}}^{\infty} \frac{1}{\sigma_{KA}^{in}(E)} \frac{d\sigma_{KK}(E,E_0)}{dE} K(E_0,h,\vartheta) dE_0,$$

Solution for the meson cascade

Formal solution for pion cascade:

$$\Pi^{\pm}(E,h,\vartheta) = \int_0^h dt\, G_{N\pi}^{\pm}(E,t,\vartheta) \exp\left[-\int_t^h dz \left(\frac{1 - \mathcal{Z}_{\pi\pi}^{\pm}(E,z,\vartheta)}{\lambda_\pi(E)} + \frac{m_\pi}{p \tau_\pi \rho(z,\vartheta)}\right)\right],$$

where

$$\mathcal{Z}_{\pi\pi}^{\pm}(E,h,\vartheta) = \int_0^h \Phi_{\pi\pi}^{\pm}(E,x) \frac{\Pi^{\pm}(E/x,h,\vartheta)}{\Pi^{\pm}(E,h,\vartheta)} \frac{dx}{x^2}, \quad \Pi^{\pm}(E,h,\vartheta) = \pi^+(E,h,\vartheta) \pm \pi^-(E,h,\vartheta),$$

$$\Phi_{\pi\pi}^{\pm}(E,x) = \frac{E}{\sigma_{\pi A}^{in}(E)} \left[\frac{d\sigma_{\pi^+\pi^+}(E_0,E)}{dE} \pm \frac{d\sigma_{\pi^+\pi^-}(E_0,E)}{dE}\right]_{E_0=E/x}.$$

Zero approximation: $\mathcal{Z}_{\pi\pi}^{\pm(0)}(E,h,\vartheta) = 0$,

$$\Pi^{\pm(0)}(E,h,\vartheta) = \int_0^h dt\, G_{N\pi}^{\pm}(E,t,\vartheta) \exp\left[-\int_t^h dz \left(\frac{1}{\lambda_\pi(E)} + \frac{m_\pi}{p \tau_\pi \rho(z,\vartheta)}\right)\right].$$

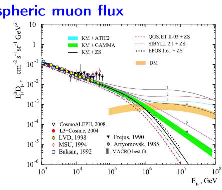
n-th approximation:

$$\Pi^{\pm(n)}(E,h,\vartheta) = \int_0^h dt\, G_{N\pi}^{\pm}(E,t,\vartheta) \exp\left[-\int_t^h dz \left(\frac{1 - \mathcal{Z}_{\pi\pi}^{\pm(n)}(E,z,\vartheta)}{\lambda_\pi(E)} + \frac{m_\pi}{p \tau_\pi \rho(z,\vartheta)}\right)\right],$$

where ($n = 0, 1, \ldots$): $\quad \mathcal{Z}_{\pi\pi}^{\pm(n+1)}(E,h,\vartheta) = \int_0^1 \Phi_{\pi\pi}^{\pm}(E,x) \frac{\Pi^{\pm(n)}(E/x,h,\vartheta)}{x^2 \Pi^{\pm(n)}(E,h,\vartheta)} dx.$

The solution for kaons can be obtained in the same way as for pion cascade.

Atmospheric muon flux



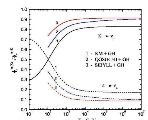
High-energy plot of the ground level muon spectrum Conventional muons: solid curves and bands show the calculations for ATIC-2, GAMMA Astropart. Phys. 28, 169 (2007), and ZS primary spectra with usage of KM model; dash-dotted line – SIBYLL; dashed – QGSJET II ; bold dots – EPOS. Prompt muons: 1 – RQPM, 2, 3 – two versions of PRS model Phys. Rev. D 59, 034020 (1999), 4 – QGSM, shaded band (DM) below the line 4 presents the dipole model calculation Phys. Rev. D 78, 043005 (2008).

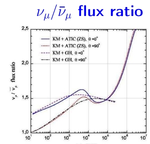
Sources of the atmospheric muon neutrinos

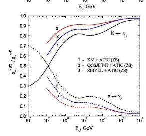
$$\phi_\nu(E,h,\vartheta) = \int_0^h dt \left[G_\nu^\mu(E,t,\vartheta) + G_\nu^{\pi,K}(E,t,\vartheta) + G_\nu^{D,\Lambda_c}(E,t,\vartheta)\right]$$

conventional ν_μ's	Decay modes	Fraction	prompt ν_μ's	Decay modes	Fraction
$\mu^{\pm}$	$e^{\pm} + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu)$	$\simeq 100\,\%$	$D^{\pm}$	$\mu^{\pm} + \nu_\mu(\bar{\nu}_\mu) + X$	17.2 %
$\pi^{\pm}$	$\mu^{\pm} + \nu_\mu(\bar{\nu}_\mu)$	$\simeq 100\,\%$	$D^0, \bar{D}^0$	$\mu^{\pm} + \nu_\mu(\bar{\nu}_\mu) + X$	7.31 %
$K^{\pm}$	$\mu^{\pm} + \nu_\mu(\bar{\nu}_\mu)$ $\pi^{\pm} + \pi^0$ $\pi^0 + \mu^{\pm} + \nu_\mu(\bar{\nu}_\mu)$	63.44 % 20.92 % 3.32 %	$D_s^{\pm}$	$\mu^{\pm} + \nu_\mu(\bar{\nu}_\mu) + X$	6.6 %
K_L^0 K_S^0	$\pi^{\pm} + \mu^{\mp} + \bar{\nu}_\mu(\nu_\mu)$ $\pi^+ + \pi^-$ $\pi^{\pm} + \mu^{\mp} + \bar{\nu}_\mu(\nu_\mu)$	27.02 % 69.20 % $4.66 \cdot 10^{-4}$	Λ_c^+	$\Lambda + \mu^+ + \nu_\mu$	2.0 %

π, K fraction of ν_μ flux; $\nu_\mu/\bar{\nu}_\mu$ flux ratio

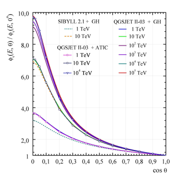






Curves, presented the relative π, K contributions, hint that AN flux serves as a magnifier of the kaon abundance in the hadronic models.

Zenith-angle enhancement of the $\nu_\mu + \bar{\nu}_\mu$ flux



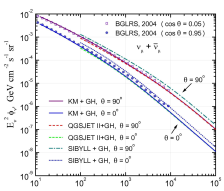
The angle enhancement weakly depends (close to horizontal at $E < 10$ TeV) on the hadronic model (SIBYLL 2.1, QGSJET-II) as well as on the primary spectrum (GH, ATIC-2)

Neutrino flux depending on hadronic model

Model-to-model ratio of the $\nu_\mu + \bar{\nu}_\mu$ flux at $\theta = 0°$ (90°)

E_ν, GeV	SIBYLL/KM	QGSJET/KM	SIBYLL/QGSJET
	PCR spectrum by GH		
10^2	1.65 (1.22)	0.97 (0.85)	1.65 (1.36)
10^3	1.71 (1.46)	0.96 (0.92)	1.73 (1.50)
10^4	1.60 (1.57)	0.96 (0.96)	1.58 (1.55
10^5	1.54 (1.49)	0.99 (0.96)	1.46 (1.46)
	PCR spectrum by ZS		
10^2	1.58 (1.26)	1.00 (0.91)	1.58 (1.38)
10^3	1.64 (1.39)	0.95 (0.92)	1.73 (1.51)
10^4	1.55 (1.46)	0.96 (0.95)	1.61 (1.54)
10^5	1.37 (1.23)	0.91 (0.83)	1.51 (1.48)
10^6	1.10 (0.95)	0.61 (0.55)	1.80 (1.73)
10^7	0.89 (0.75)	0.48 (0.43)	1.85 (1.74)

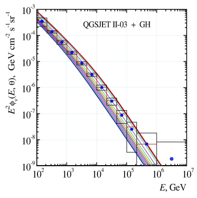
Two calculations for the GH spectrum



1) Lines display this work computation performed with three hadronic models, KM, SIBYLL 2.1, QGSJET-II

2) Crossed circles and open squares show the flux prediction by Barr, Gaisser, Lipari, Robbins and Stanev, Phys. Rev. D70, 023006 (2004)

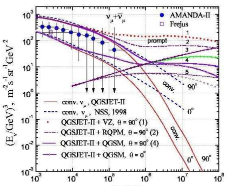
Comparison of the calculated $\nu_\mu + \bar{\nu}_\mu$ fluxes with the IceCube measurement



Conventional $\nu_\mu + \bar{\nu}_\mu$ flux at different zenith angles. Curves ($\cos\theta = 0.0 \div 1.0$ from top to bottom) display this work calcilations made for the Gaisser & Honda primary spectra an composition with usage of QGSJET-II model. Blue points: IceCube preliminary muon neutrino spectrum averaged over zenith angle D. Chirkin, Proc. 31st ICRC Lodz, Poland, 2009, HE.2.2-1418 (see also T. Montaruli, arXiv: 0910.4364v1).

Prompt atmospheric muon neutrinos

AMANDA-II measurements Phys. Rev. D 76, 042008 (2007) and prompt neutrino models



Prompt atmospheric neutrinos:

1 – Volkova & G.T.Zatsepin Phys. Lett. B 462, 211 (1999)

2 – recombination quark-parton model (RQPM), Nuovo Cim. C 12, 41 (1989); Phys. Rev. D 58, 054001 (1998)

3 – GGV ($\lambda = 0.5$) G.Gelmini, P.Gondolo, G.Varieschi, Phys. Rev. D 61, 056011 (2000)

4 – quark-qluon string model (QGSM), Nuovo Cim. C 12, 41 (1989); Phys. Rev. D 58, 054001 (1998)

5 – GGV ($\lambda = 0.1$)

Dashed lines: the calculation of the conventional neutrino flux by V.A.Naumov, T.S.Sinegovskaya, S.I.Sinegovsky, Nuovo Cim. A 111, 129 (1998); hep-ph/9802410

ν_μ flux calculation vs the AMANDA-II limit

AMANDA-II upper limit on the diffuse ν_μ flux and the prompt neutrinos

Model	$E_\nu^2 \phi_\nu$, GeV (cm^2 s sr)$^{-1}$, $E_\nu = 100$ TeV	
conventional $\nu_\mu + \bar{\nu}_\mu$:	0°	90°
QGSJET-II + ZS	1.20×10^{-8}	10.5×10^{-8}
QGSJET-II + GH	1.11×10^{-8}	9.89×10^{-8}
prompt $\nu_\mu + \bar{\nu}_\mu$:	90°	
Volkova & Zatsepin Phys. Lett. B 462, 211 (1999)	8.12×10^{-8}	
RQPM Nuovo Cim. C 12, 41 (1989)	4.61×10^{-8}	
QGSM Nuovo Cim. C 12, 41 (1989)	1.22×10^{-8}	
AMANDA-II upper limit	7.4×10^{-8}	

Summary

- Considerable flux differences originate from HE hadronic interaction models. As it can be seen by the example QGSJET-II and SIBYLL 2.1, the major factor of the conventional flux discrepancy is the kaon production in nucleon-nucleus collisions.
- A hope that atmospheric muon fluxes may serve as the tool to discriminate between the hadron production models seems to be illusive because the key differences in the π, K production impact variously on the AN flux and AM one. For the high-energy neutrino production at the atmosphere the kaon yield in nucleon-nucleus interactions is more strong factor in comparison with that for production of the atmospheric muons, despite on their common to neutrinos origin.
- The RQPM and QGSM prompt neutrino flux predictions do not contradict recent preliminary data of IceCube experiment as well as the the AMANDA-II upper limit for the muon neutrino diffuse flux from astrophysical sources.

Acknowledgements

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