

# **Data Analysis II**

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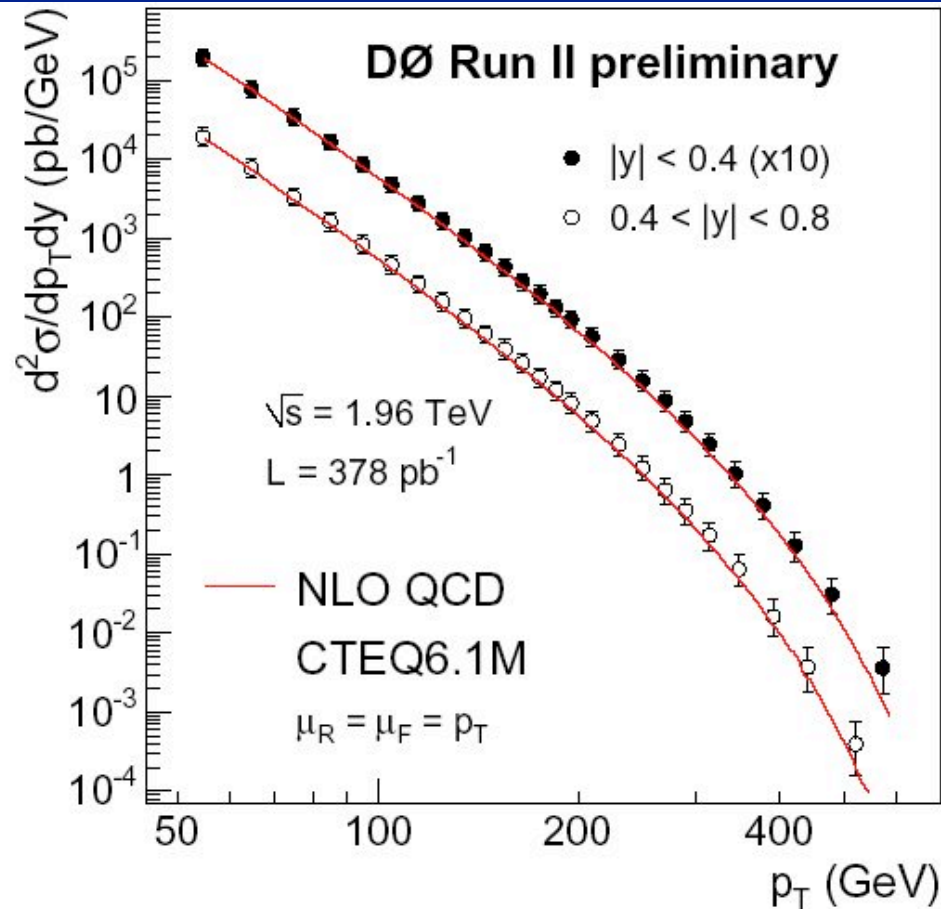
*UC Berkeley and Lawrence Berkeley National Laboratory*

Hadron Collider Physics Summer School, Fermilab, August 2008<sup>1</sup>

# Outline

- Lecture I:
  - Measuring a cross section
    - focus on acceptance
- Lecture II:
  - Measuring a property of a known particle
- Lecture III:
  - Searching for a new particle
    - focus on backgrounds

# Differential Cross Section



- Measure jet spectra differentially in  $E_T$  and  $\eta$
- Cross section in bin i: 
$$\sigma(i) = \frac{N_{\text{obs}}(i) - N_{\text{BG}}(i)}{\int L dt \epsilon(i)}$$

# Differential Cross Section: Unfolding

- “Unfolding” critical for jet cross sections
- Measure:
  - Cross section for calorimeter jets
- Want:
  - Cross section for hadron-jets

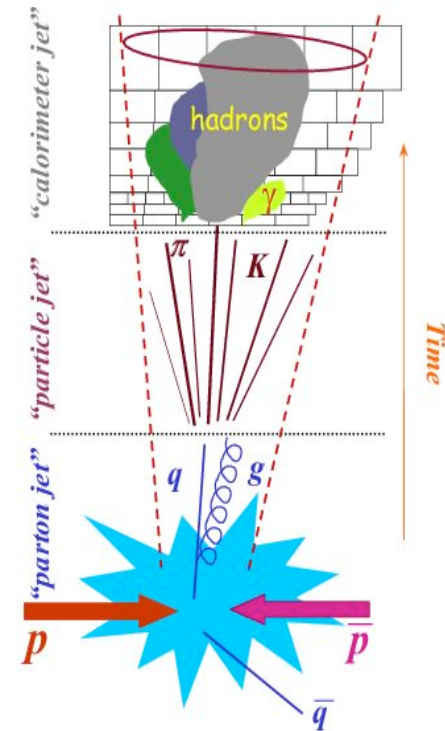
Unfolding factor (bin by bin):

$$C_i = \frac{N_{JET\ i}^{HAD}}{N_{JET\ i}^{CAL}}$$

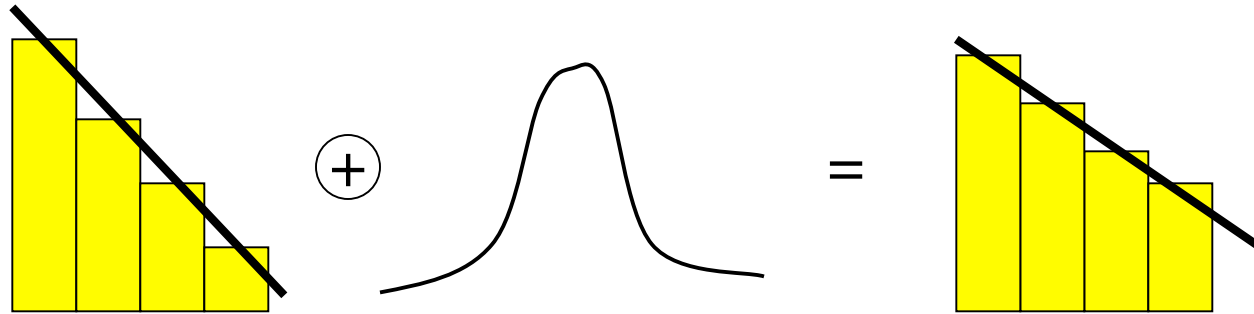
Then:

$$N_{JET\ i}^{DATA\ UNFOLDED} = C_i \cdot N_{JET\ i}^{DATA\ NOT\ UNFOLDED}$$

- But, unfolding factors depend on MC  $E_T$  spectrum

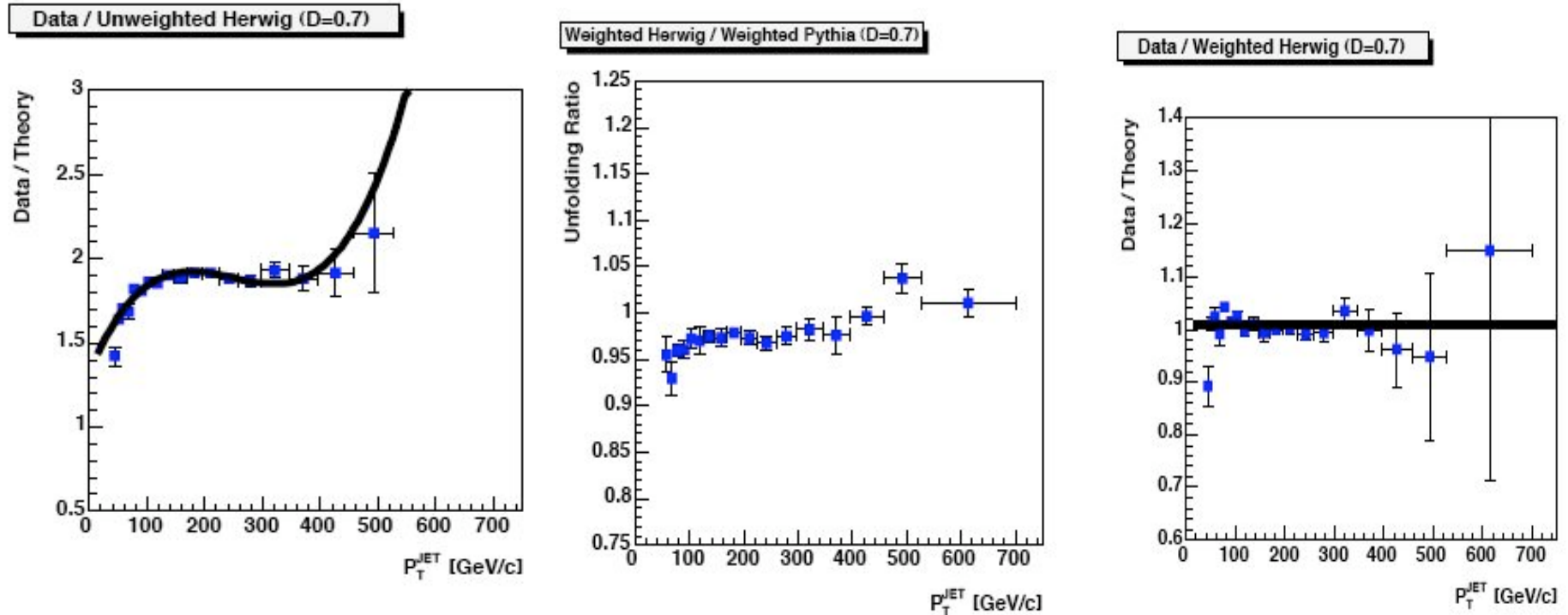


# Differential Cross Section: Unfolding



- **Problem:**
  - Steeply falling spectrum causes migrations to go from low to high  $p_T$ 
    - Measured spectrum “flatter” than true spectrum
  - Size of migration depends on input spectrum
- **Requires iterative procedure** (bin-by-bin unfolding):
  1. Measure using spectrum from MC
  2. Fit measurement
  3. Reweight MC to reflect data measurement => go back to 1.

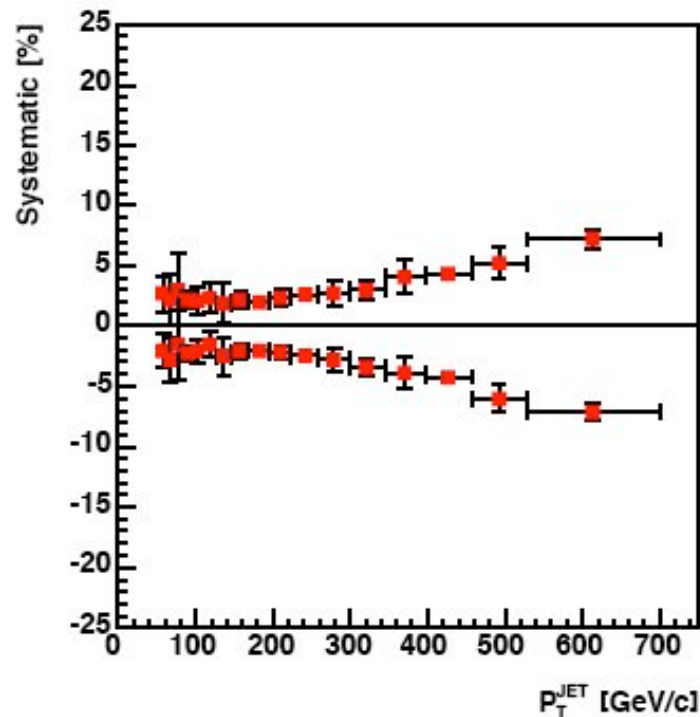
# Example for Bin-by-Bin Unfolding



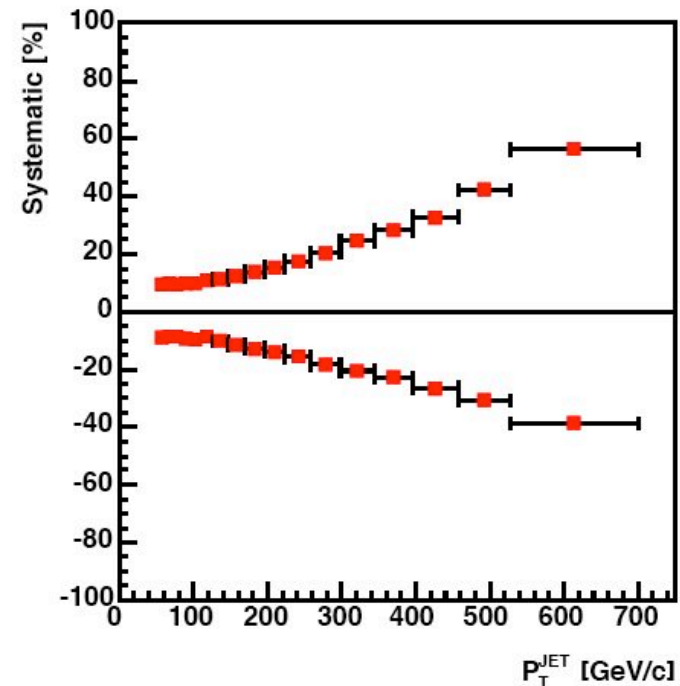
- Correction to unfolding factors <10%
  - One iteration sufficient in this example
  - Starting spectrum was already quite close to data

# Systematic Uncertainties: Jet Cross Section

8% uncertainty on resolution (D=0.5)

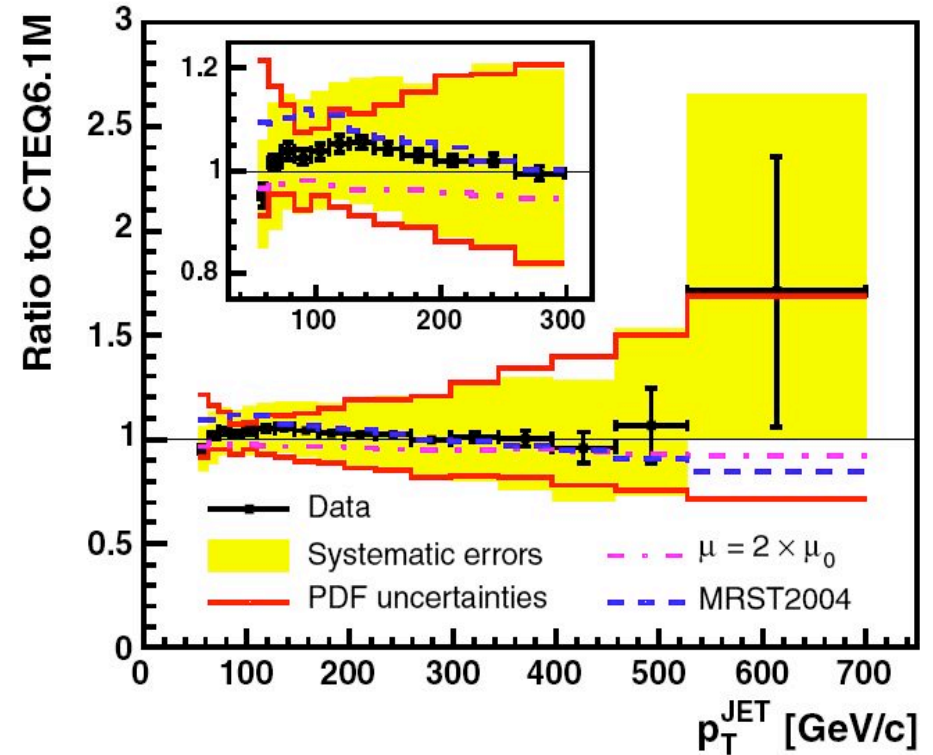
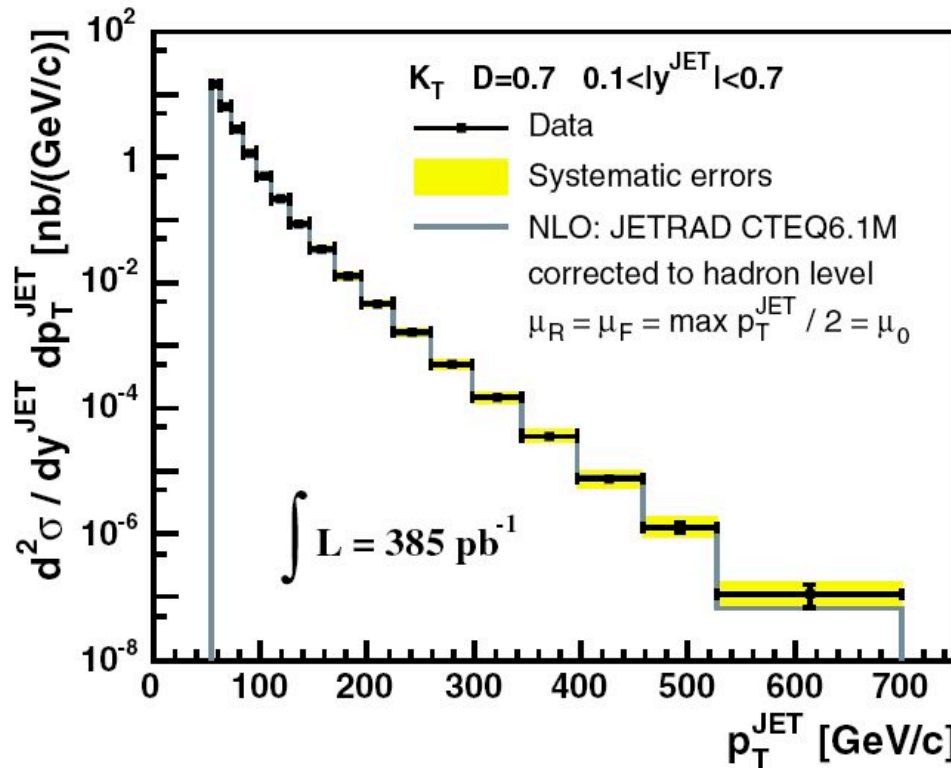


Jet Energy Scale Uncertainty (D=0.5)



- For Jet Cross Section the Jet Energy Scale (JES) uncertainty is dominant systematic error
  - 3% uncertainty on JES results in up to 60% uncertainty on cross section
  - 8% uncertainty on JE resolution causes <10% uncertainty on cross section

# Jet Cross Section Result



- Cross section falls by 8 orders of magnitude in measured  $E_T$  range
- Data in good agreement with QCD prediction
  - Experimental and theoretical errors comparable

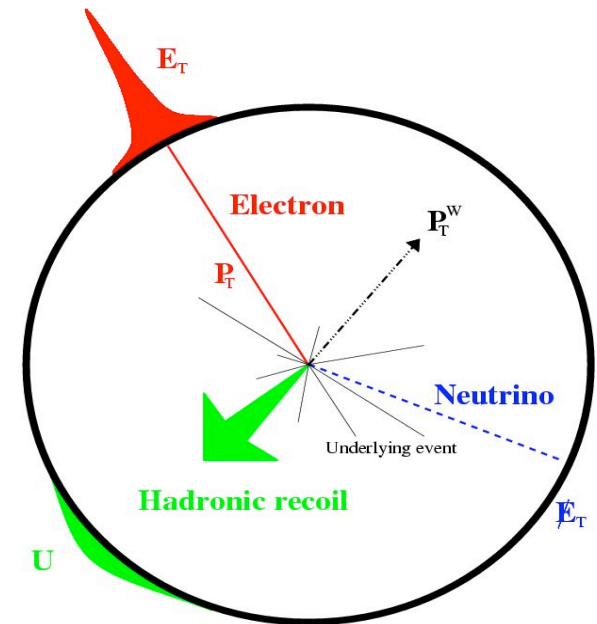


# **Measuring Properties of Particles**

# **The $W^\pm$ Boson Mass**

# W Boson mass

- Real **precision** measurement:
  - LEP:  $M_W = 80.367 \pm 0.033 \text{ GeV}/c^2$
  - Precision: 0.04%
    - => Very challenging!
- Main measurement ingredients:
  - **Lepton**  $p_T$
  - **Hadronic recoil** parallel to lepton:  $u_{||}$
- $Z \rightarrow \ell\ell$  superb calibration sample:
  - ♣ but statistically limited:
    - About a factor 10 less Z's than W's
    - Most systematic uncertainties are related to size of Z sample
      - Will scale with  $1/\sqrt{N_Z}$  ( $=1/\sqrt{L}$ )



$$m_T = \sqrt{2p_T^l \cancel{p}_T (1 - \cos \Delta\phi)},$$

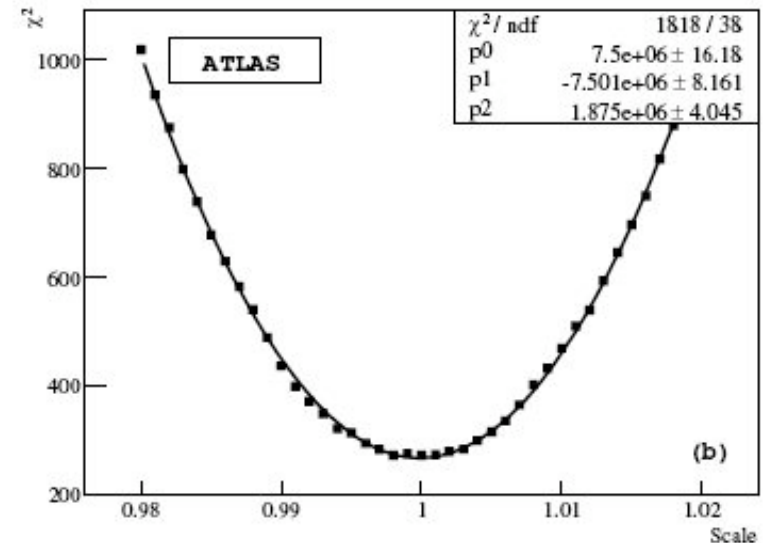
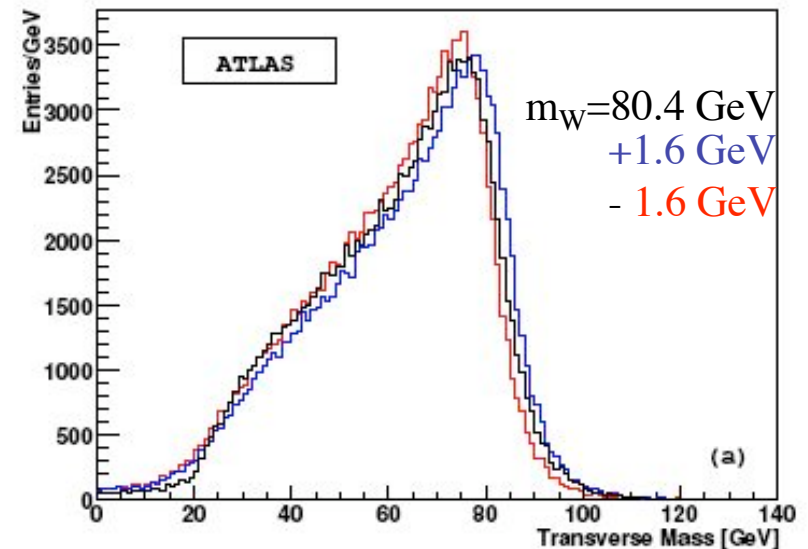
$$\cancel{p}_T \approx |p_T + u_{||}|$$

$$m_T \approx 2p_T \sqrt{1 + u_{||}/p_T} \approx 2p_T + u_{||}$$

# How to Extract the W Boson Mass

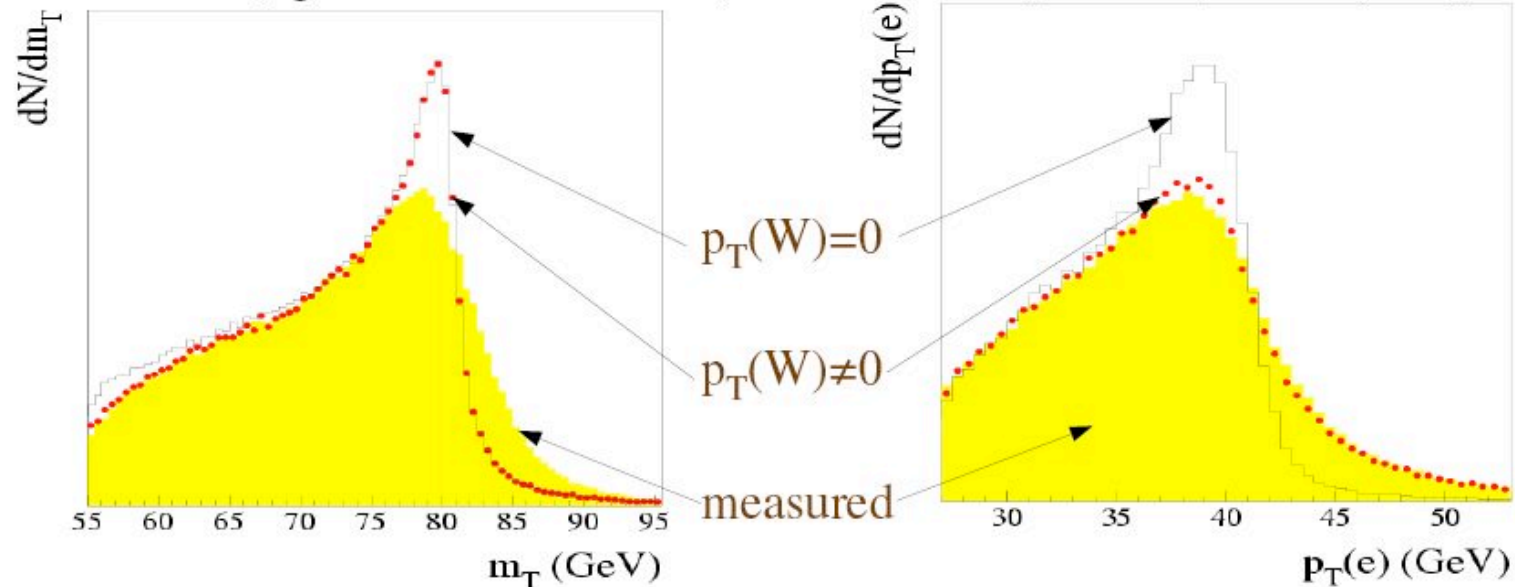
- Uses “Template Method”:
  - Templates created from MC simulation for different  $m_W$
  - Fit to determine which template fits best
  - Minimal  $\chi^2 \Rightarrow W$  mass!
- Transverse mass of lepton and Met

$$m_T = \sqrt{|p_T^\ell|^2 + |p_T^\nu|^2 - (\vec{p}_T^\ell + \vec{p}_T^\nu)^2}$$



# How to Extract the W Boson Mass

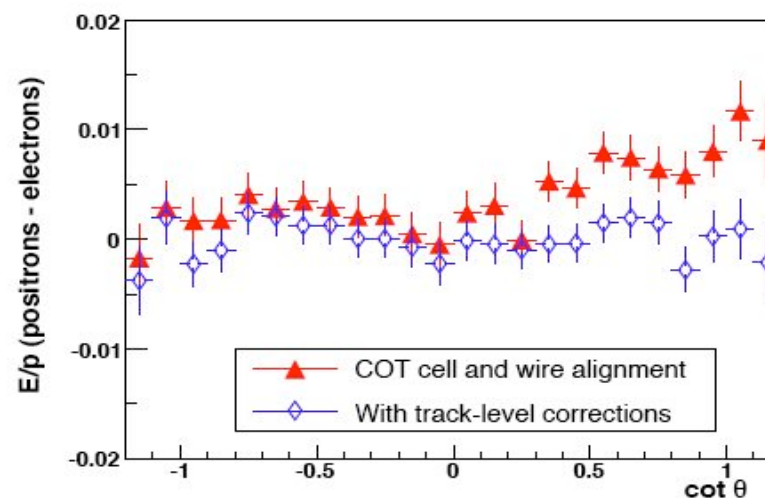
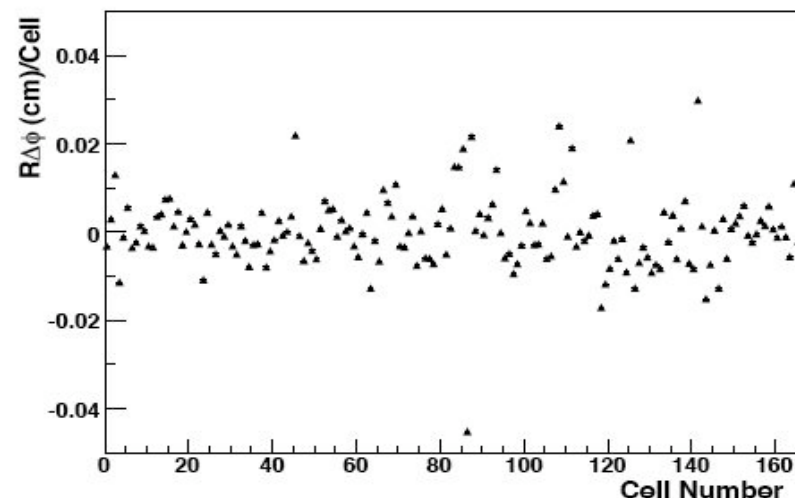
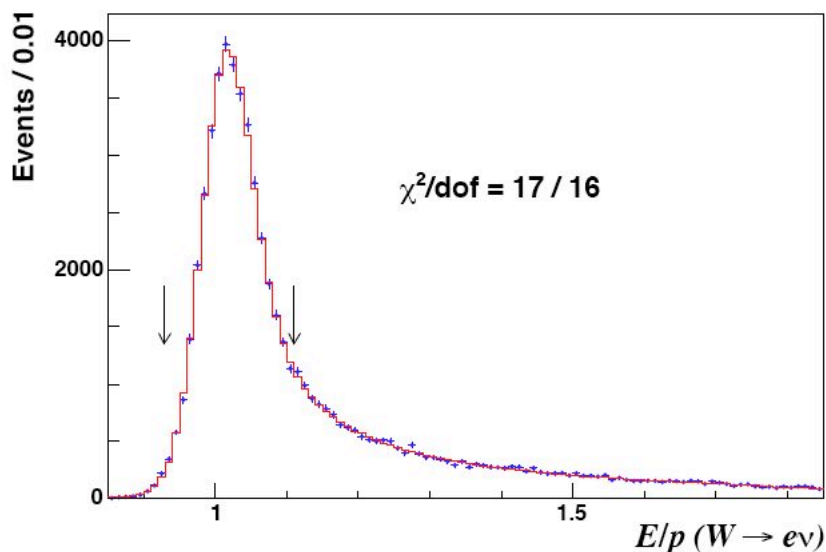
(figures from Abbott *et. al.* (D0 Collaboration), PRD 58, 092003 (1998))



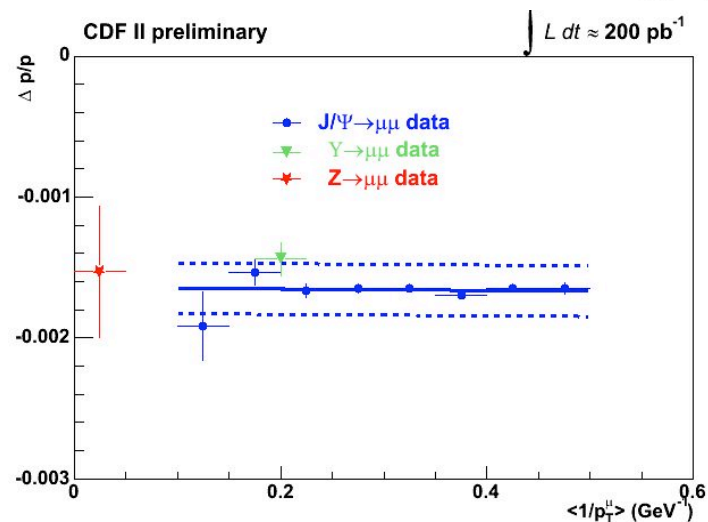
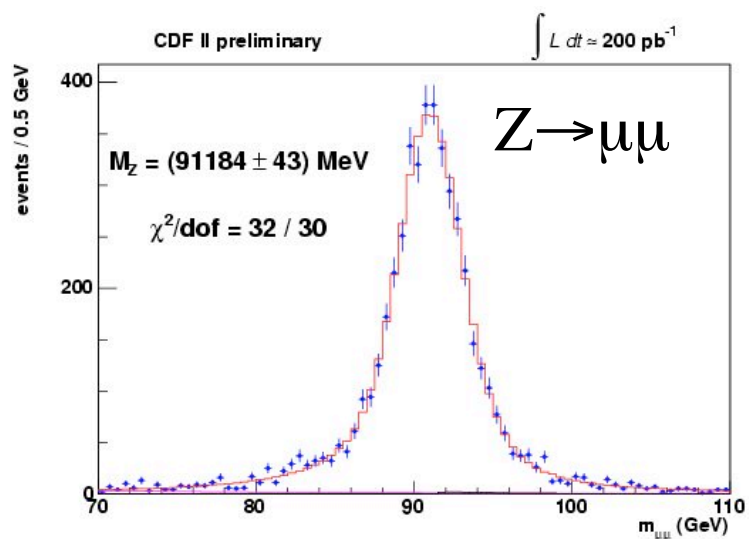
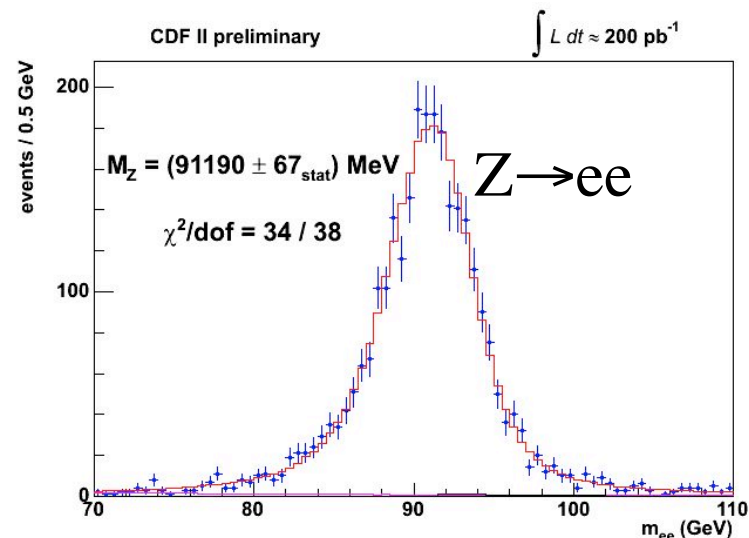
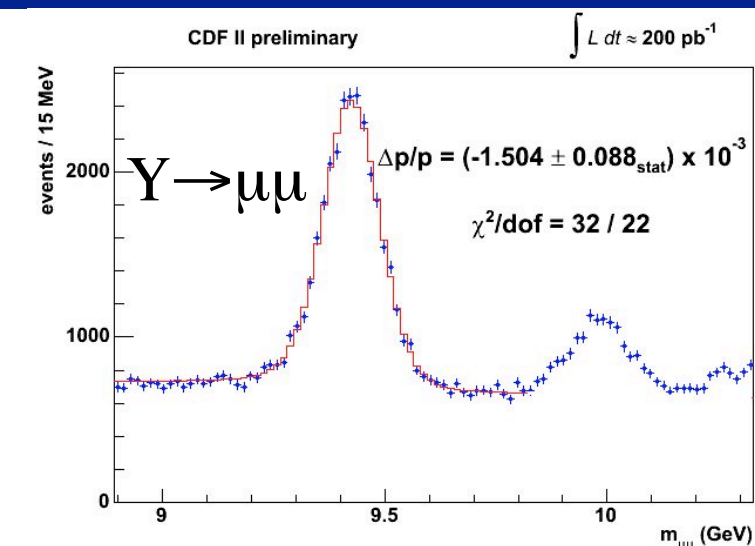
- Alternatively can fit to
  - Lepton  $p_T$  or missing  $E_T$
- Sensitivity different to different systematics
  - Very powerful checks in this analysis:
    - Electrons vs muons
    - Z mass
    - $m_T$  vs  $p_T$  vs  $ME_T$  fits
  - The redundancy is the strength of this difficult high precision analysis 13

# Lepton Momentum Scale

- Momentum scale:
  - Cosmic ray data used for detailed cell-by-cell calibration of CDF drift chamber
  - $E/p$  of  $e^+$  and  $e^-$  used to make further small corrections to  $p$  measurement
  - Peak position of overall  $E/p$  used to set electron energy scale
    - Tail sensitive to passive material



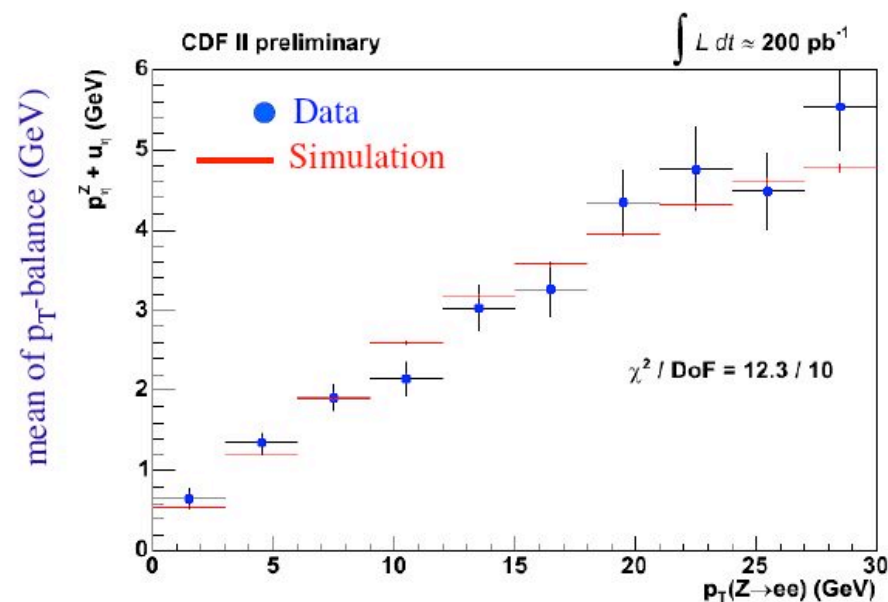
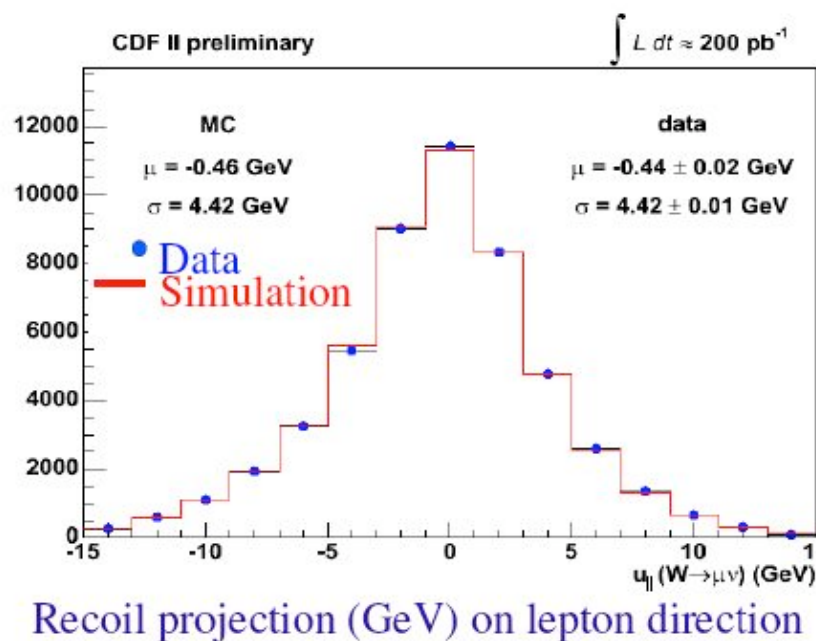
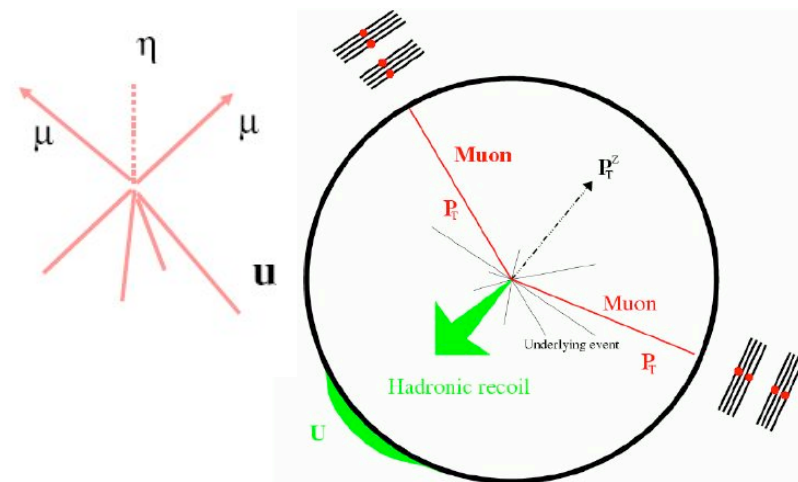
# Momentum/Energy Scale and Resolution



- Systematic uncertainty on momentum scale: 0.04%

# Hadronic Recoil Model

- Hadronic recoil modeling
  - Tune data based on Z's
  - Check on W's





# Systematic Uncertainties

$m_T$ Fit Uncertainties			
Source	$W \rightarrow \mu\nu$	$W \rightarrow e\nu$	Correlation
Tracker Momentum Scale	17	17	100%
Calorimeter Energy Scale	0	25	0%
Lepton Resolution	3	9	0%
Lepton Efficiency	1	3	0%
Lepton Tower Removal	5	8	100%
Recoil Scale	9	9	100%
Recoil Resolution	7	7	100%
Backgrounds	9	8	0%
PDFs	11	11	100%
$W$ Boson $p_T$	3	3	100%
Photon Radiation	12	11	100%
Statistical	54	48	0%
Total	60	62	-

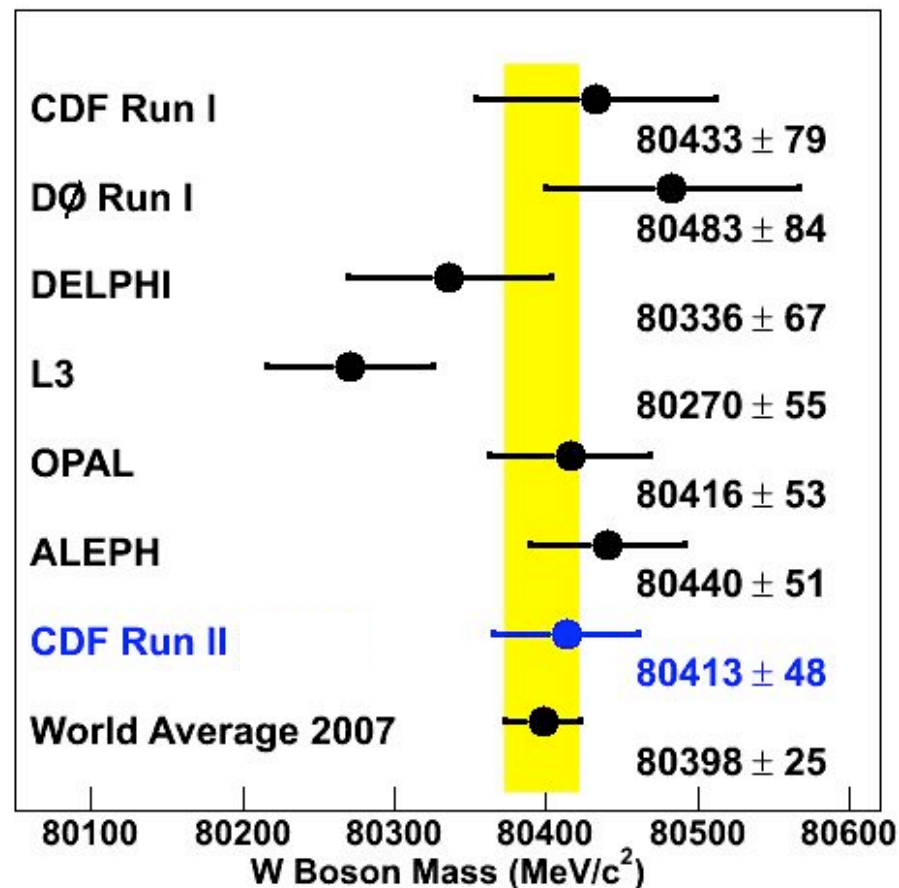
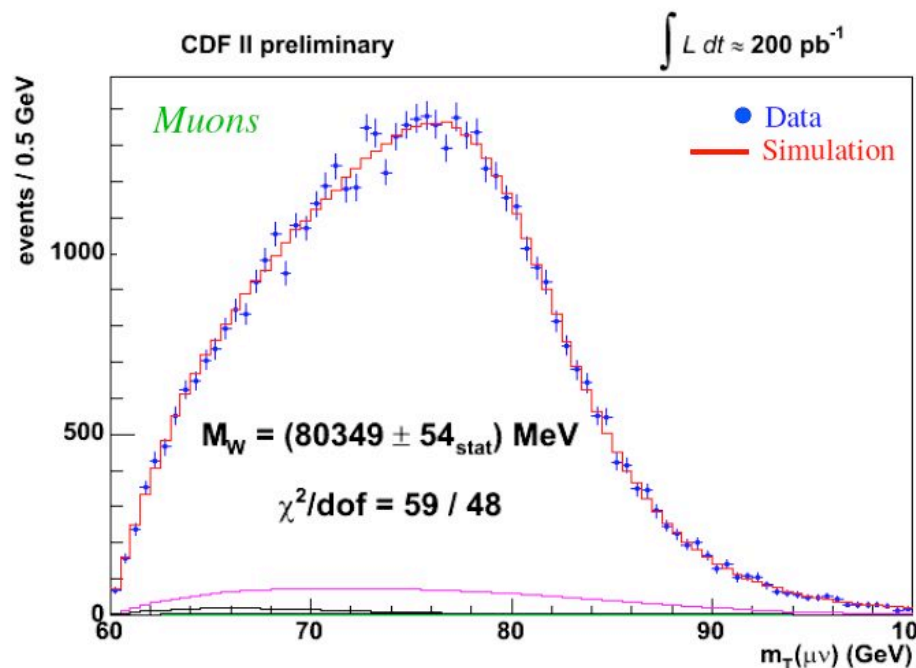
Limited by data statistics

Limited by data and theoretical understanding

TABLE IX: Uncertainties in units of MeV on the transverse mass fit for  $m_W$  in the  $W \rightarrow \mu\nu$  and  $W \rightarrow e\nu$  samples.

- Overall uncertainty 60 MeV for both analyses
  - Careful treatment of correlations between them
- Dominated by stat. error (50 MeV) vs syst. (33 MeV)

# W Boson Mass



- World average:  
 **$M_W = 80398 \pm 25 \text{ MeV}$**
- Ultimate Run 2 precision:  
 **$\sim 15 \text{ MeV}$**

# **The Top Quark**

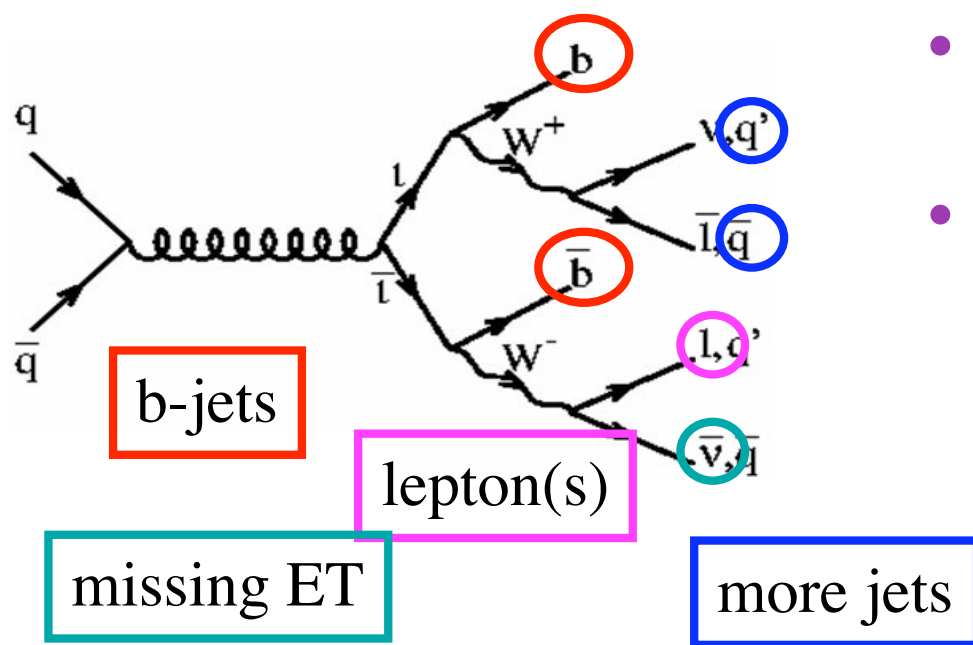
# Top Quark Cross Section

SM:  $t\bar{t}$  pair production,  $\text{Br}(t \rightarrow bW) = 100\%$ ,  $\text{Br}(W \rightarrow l\nu) = 1/9 = 11\%$

dilepton (4/81) 2 leptons + 2 jets + missing  $E_T$

lepton+jets (24/81) 1 lepton + 4 jets + missing  $E_T$

fully hadronic (36/81) 6 jets

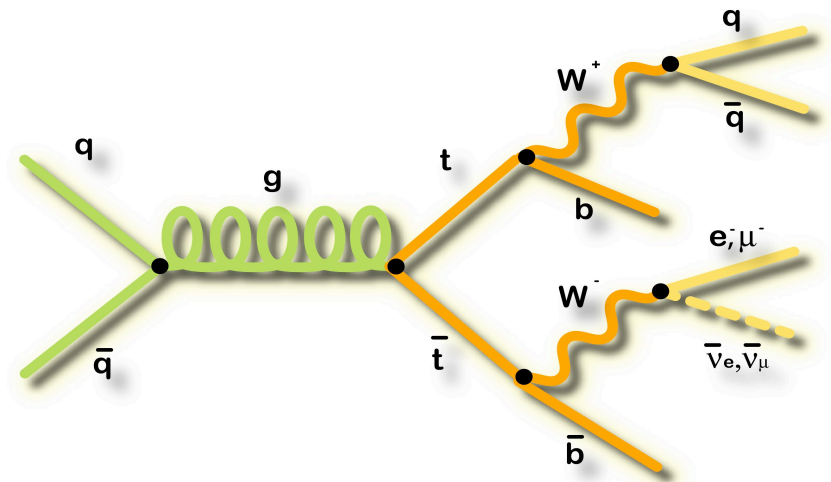
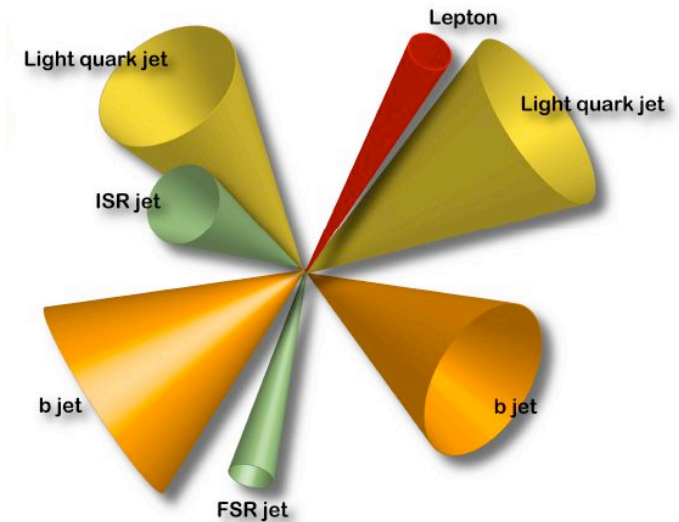


- Trigger on electron/muon
  - Like for Z's
- Analysis cuts:
  - Electron/muon  $p_T > 25$  GeV
  - Missing  $E_T > 25$  GeV
  - 3 or 4 jets with  $E_T > 20-40$  GeV

# Top Mass Measurement:

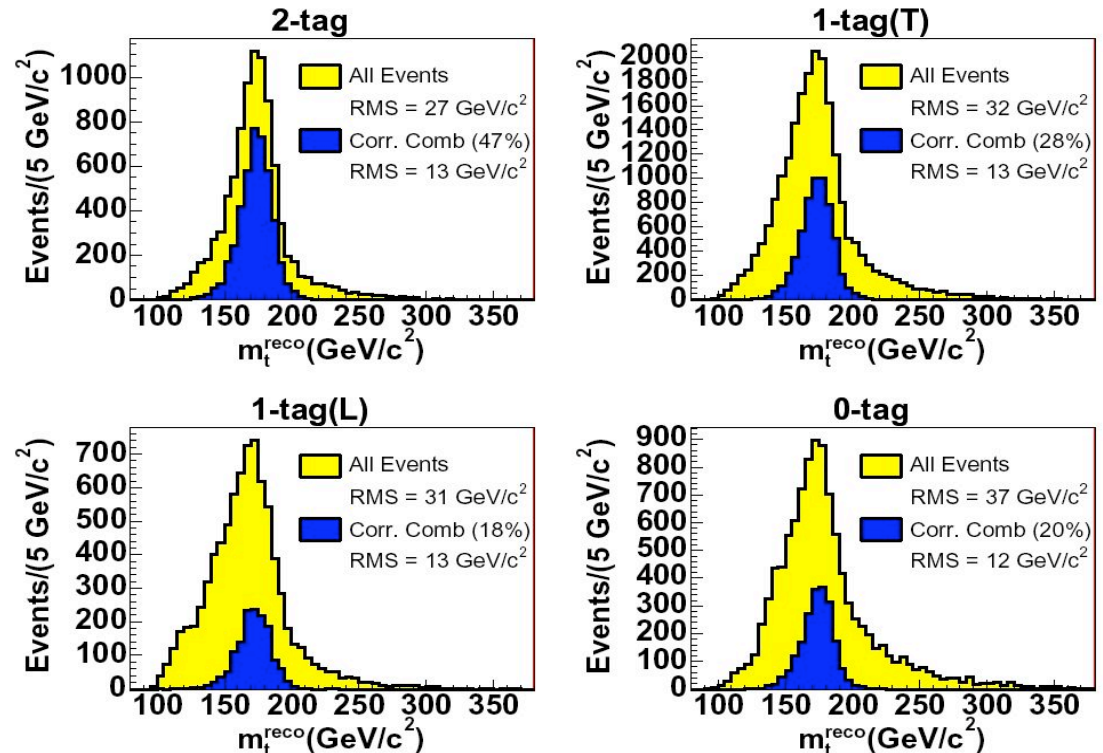
## $t\bar{t} \rightarrow (b\ell\nu)(bqq)$

- 4 jets, 1 lepton and missing  $E_T$ 
  - Which jet belongs to what?
  - Combinatorics!
- B-tagging helps:
  - 2 b-tags  $\Rightarrow$  2 combinations
  - 1 b-tag  $\Rightarrow$  6 combinations
  - 0 b-tags  $\Rightarrow$  12 combinations
- Two Strategies:
  - Template method:
    - Uses “best” combination
    - Chi2 fit requires  $m(t) = m(\bar{t})$
  - Matrix Element method:
    - Uses all combinations
    - Assign probability depending on kinematic consistency with top



# Top Mass Determination

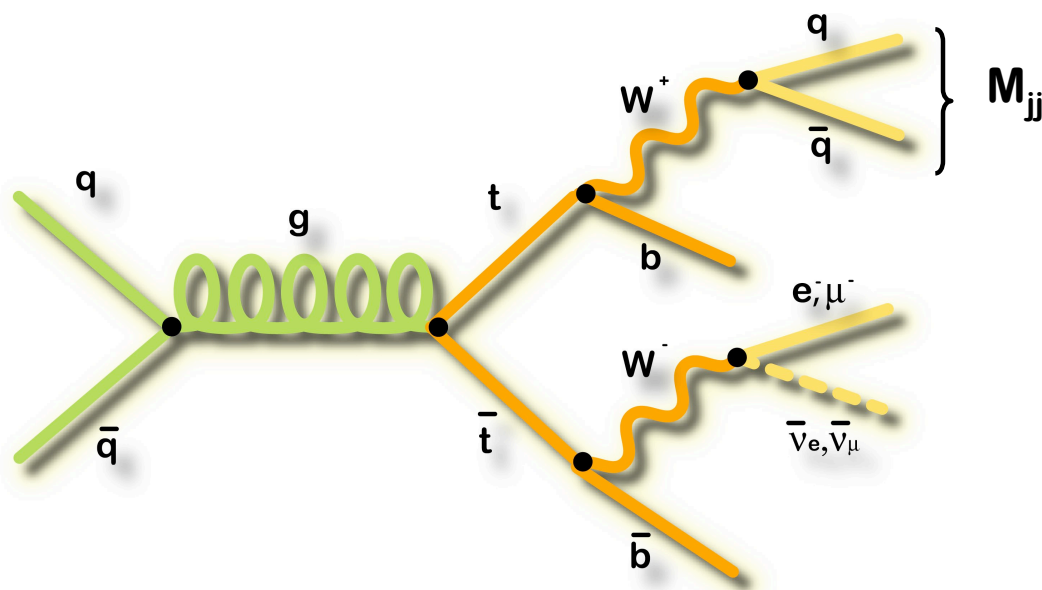
- Inputs:
  - Jet 4-vectors
  - Lepton 4-vector
  - Remaining transverse energy,  $p_{T,UE}$ :
    - $p_{T,v} = -(p_{T,l} + p_{T,UE} + \sum p_{T,jet})$
- Constraints:
  - $M(l\nu) = M_W$
  - $M(qq) = M_W$
  - $M(t) = M(\bar{t})$
- Unknown:
  - Neutrino  $p_z$
- 1 unknown, 3 constraints:
  - Overconstrained
  - Can measure  $M(t)$  for each event:  $m_t^{reco}$



Selecting correct combination  
20-50% of the time

# *In-situ* Measurement of JES

- Additionally, use  $W \rightarrow jj$  mass resonance ( $M_{jj}$ ) to measure the jet energy scale (JES) uncertainty



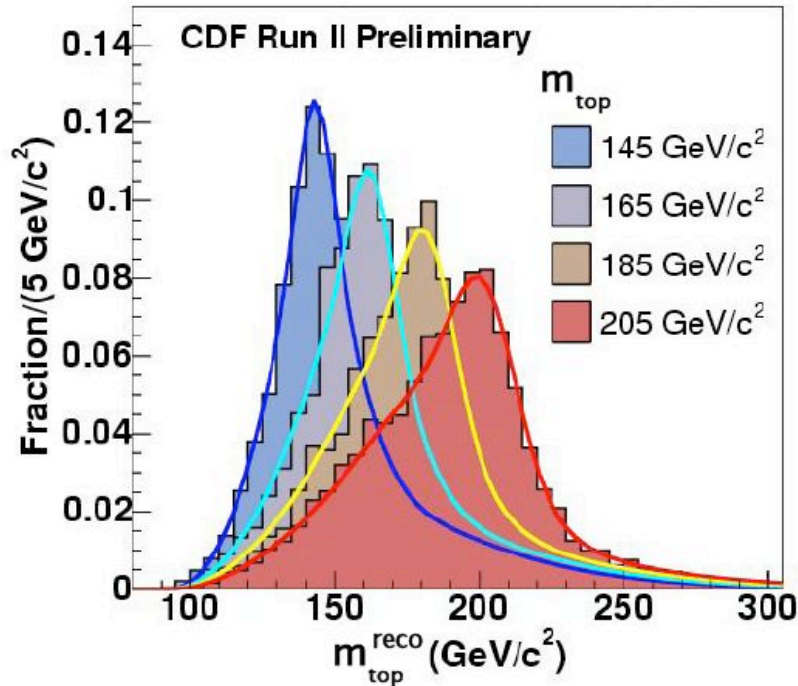
2D fit of the invariant mass of the non-b-jets and the top mass:

$$\text{JES} \propto M(jj) - 80.4 \text{ GeV}/c^2$$

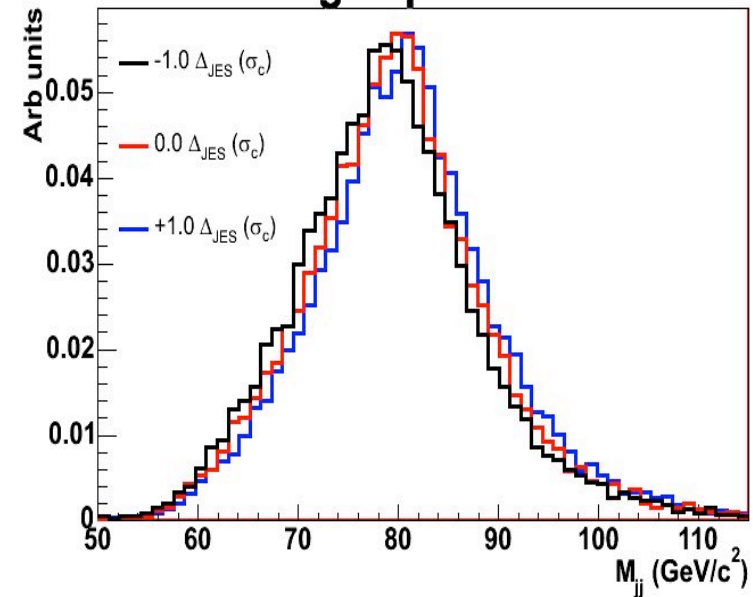
Measurement of JES scales directly with data statistics



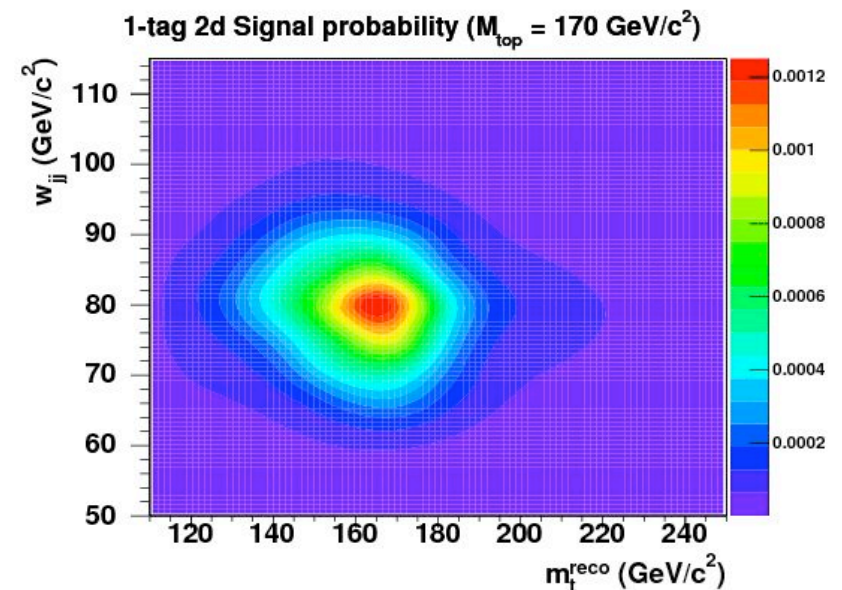
# Top Mass Templates



## 1-tag Lepton+Jets



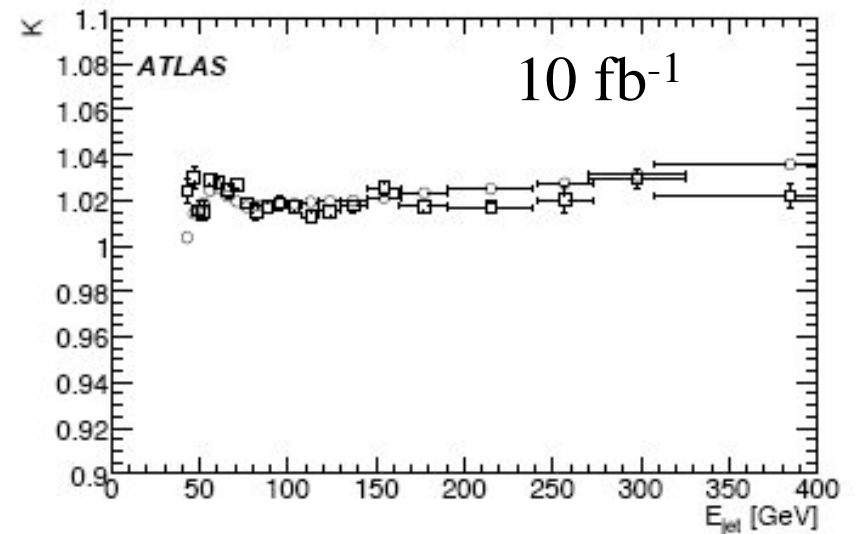
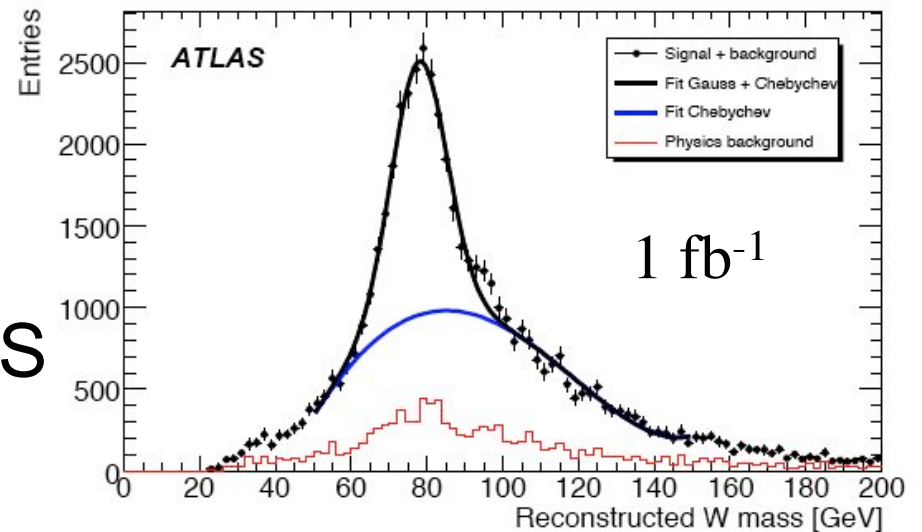
- Fit to those templates for
  - Top mass
  - Jet Energy Scale





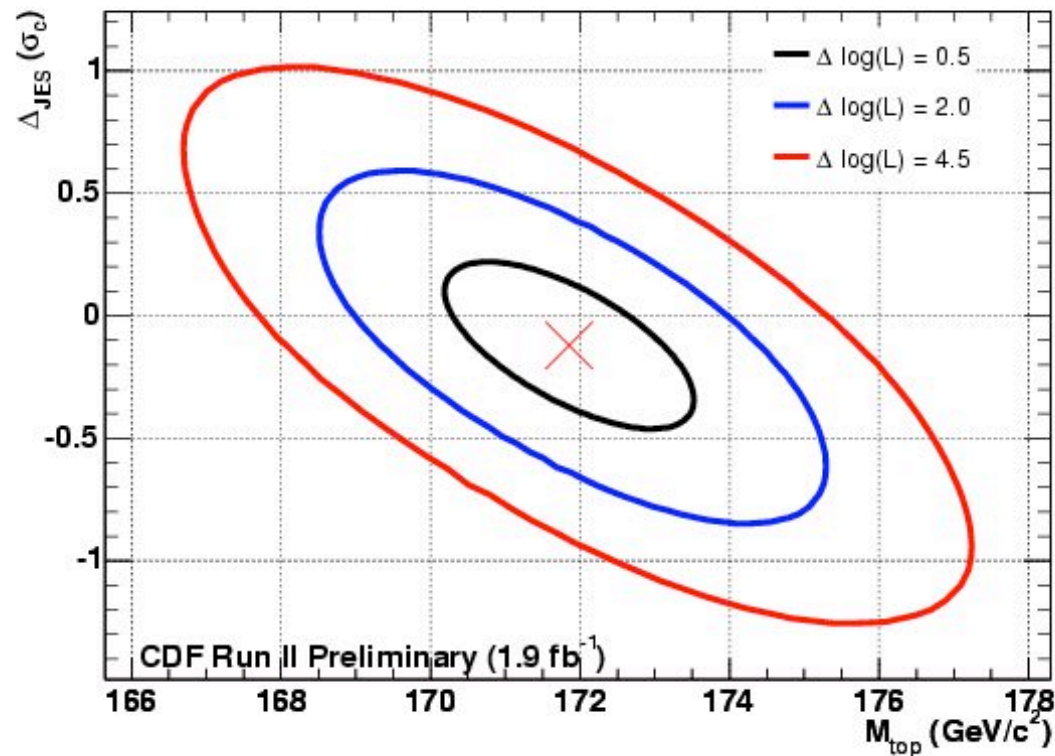
# Measurement of JES at LHC

- Large top samples
  - Clean W mass peak
- Allow measurement of JES as function of Jet Energy
- Can achieve 1% precision with  $10 \text{ fb}^{-1}$



# Template Analysis Results on $m_{\text{top}}$

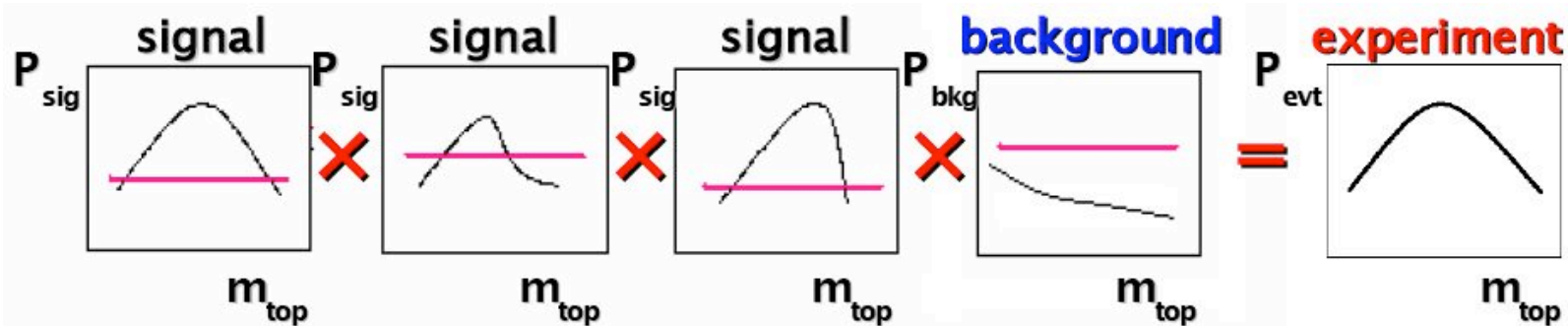
- Using 344 lepton+jets and 144 dilepton candidate events in  $1.9 \text{ fb}^{-1}$
- Using in-situ JES calibration results in factor four improvement on JES



$$m_{\text{top}} = 171.9 \pm 1.7 \text{ (stat.+JES)} \pm 1.0 = 171.6 \pm 2.0 \text{ GeV}/c^2$$

# “Matrix Element Method”

- Construct probability density function as function of  $m_{\text{top}}$  for each event
- Multiply those probabilities of all events

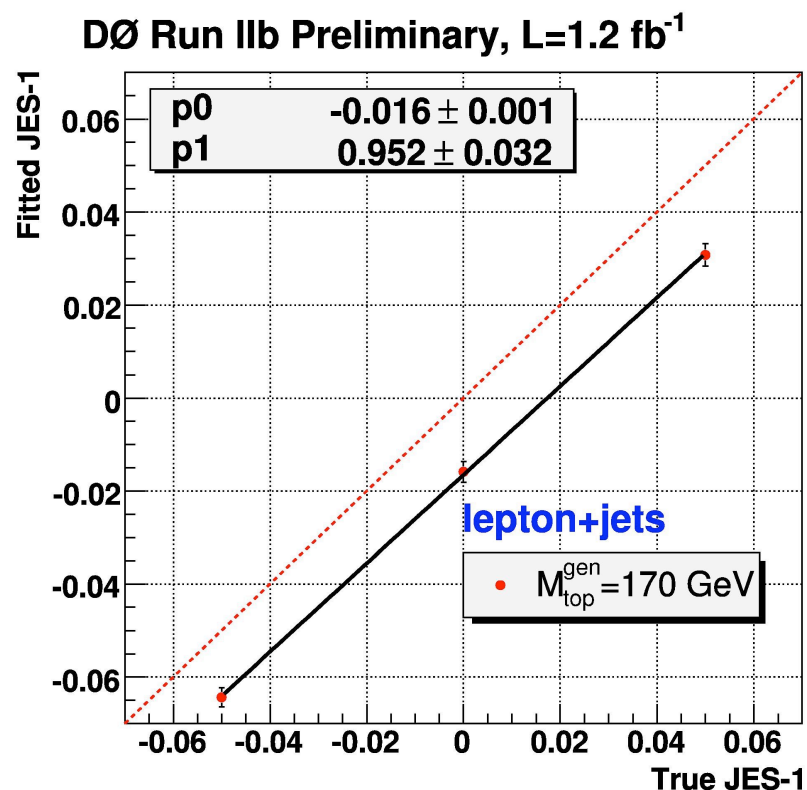
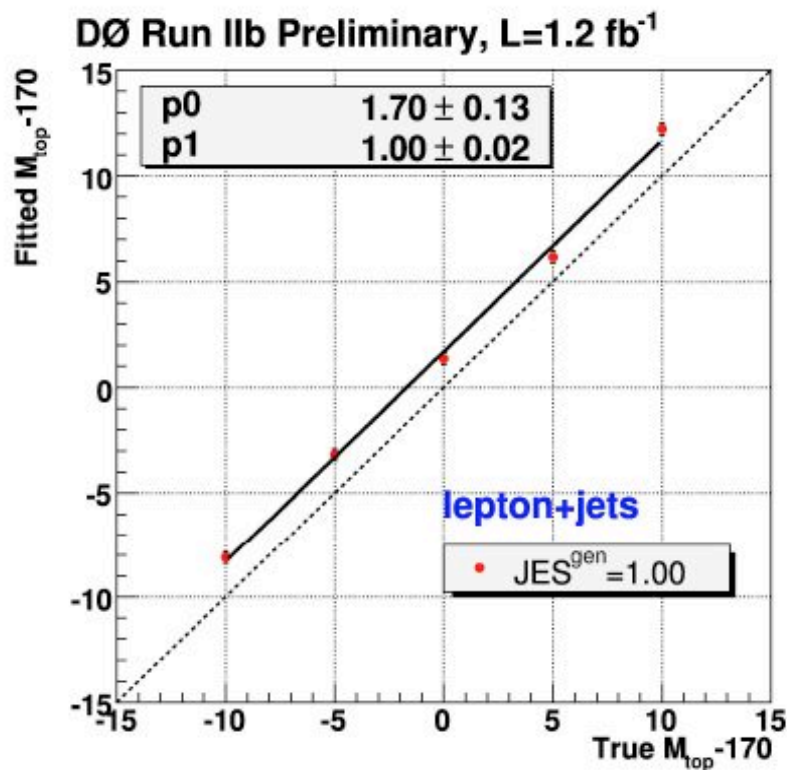


$$P_{\text{sig}}(x; m_{\text{top}}, JES) = \underbrace{\text{Acc}(x)}_{\text{Acceptance (selection, trigger,...)}} \times \frac{1}{\sigma} \int d^n \underbrace{\sigma(y; m_{\text{top}})}_{\text{LO-Matrix element } \times \text{ phase space}} \underbrace{dq_1 dq_2 f(q_1) f(q_2)}_{\text{PDF's}} \underbrace{W(x, y; JES)}_{\text{Transfer Functions (Probability to measure } x \text{ when } y \text{ was produced)}}$$

- maximum Likelihood fit:**

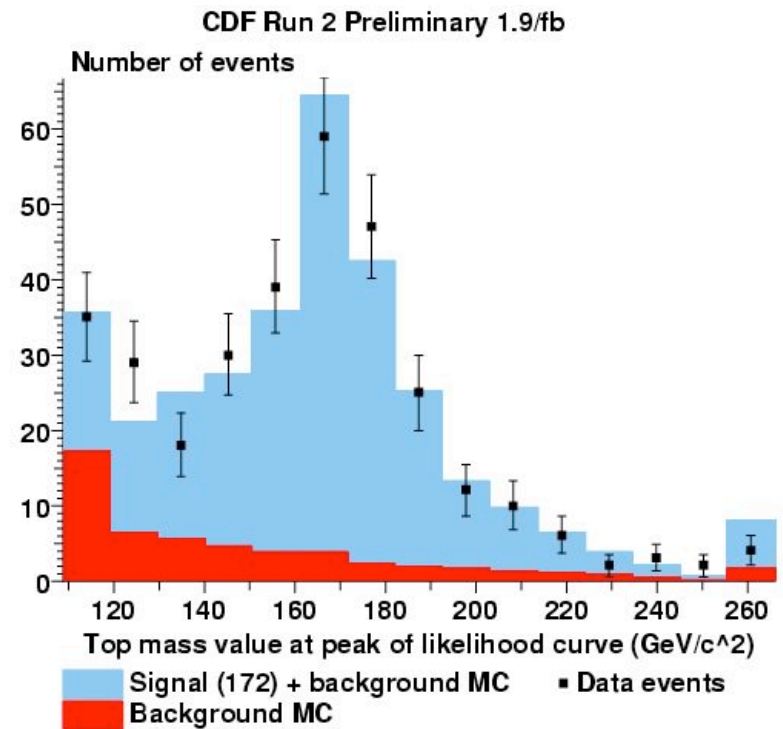
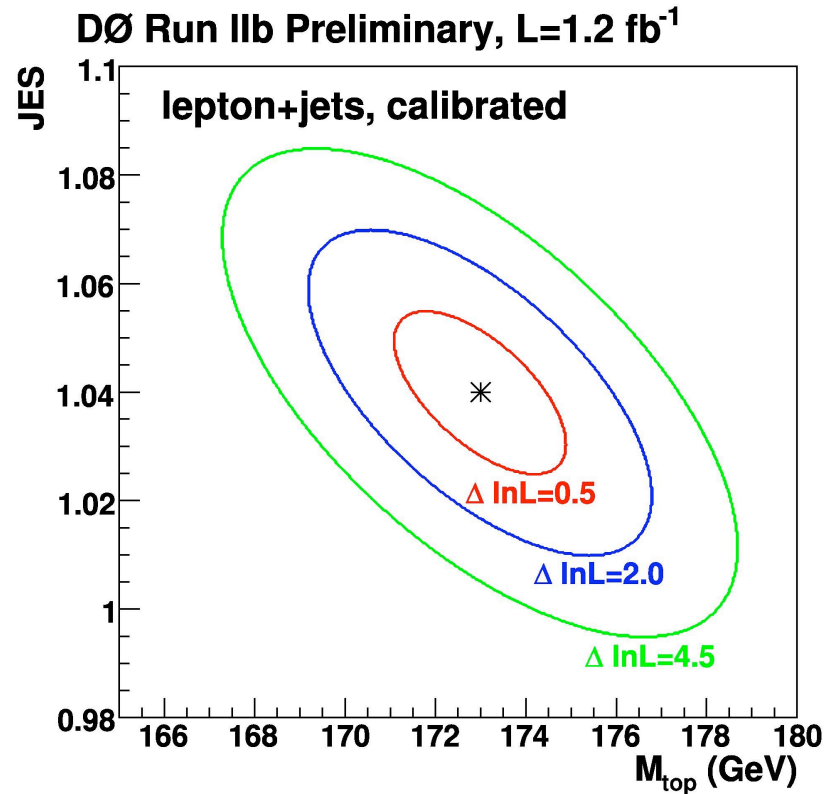
$$L(x_1, \dots, x_n; m_{\text{top}}, JES, f_{\text{top}}) = \prod_{i=1}^n P_{\text{evt}}(x_i; m_{\text{top}}, JES, f_{\text{top}})$$

# Check you get the right answer



- Run “Pseudo-Experiments” on Monte Carlo to see if you get out the mass that was put in
  - Pretend MC is data and run analysis on it N times
- Non-trivial cross check given the complexity of the method
  - If not: derive “calibration curve” from slope and offset

# Matrix Element Top Mass Results



DØ:  $2.2 \text{ fb}^{-1}$

$$m_{\text{top}} = 172.2 \pm 1.0 \text{ (stat)} \pm 1.4 \text{ (syst)} \text{ GeV}$$

$\pm 1.0\%$

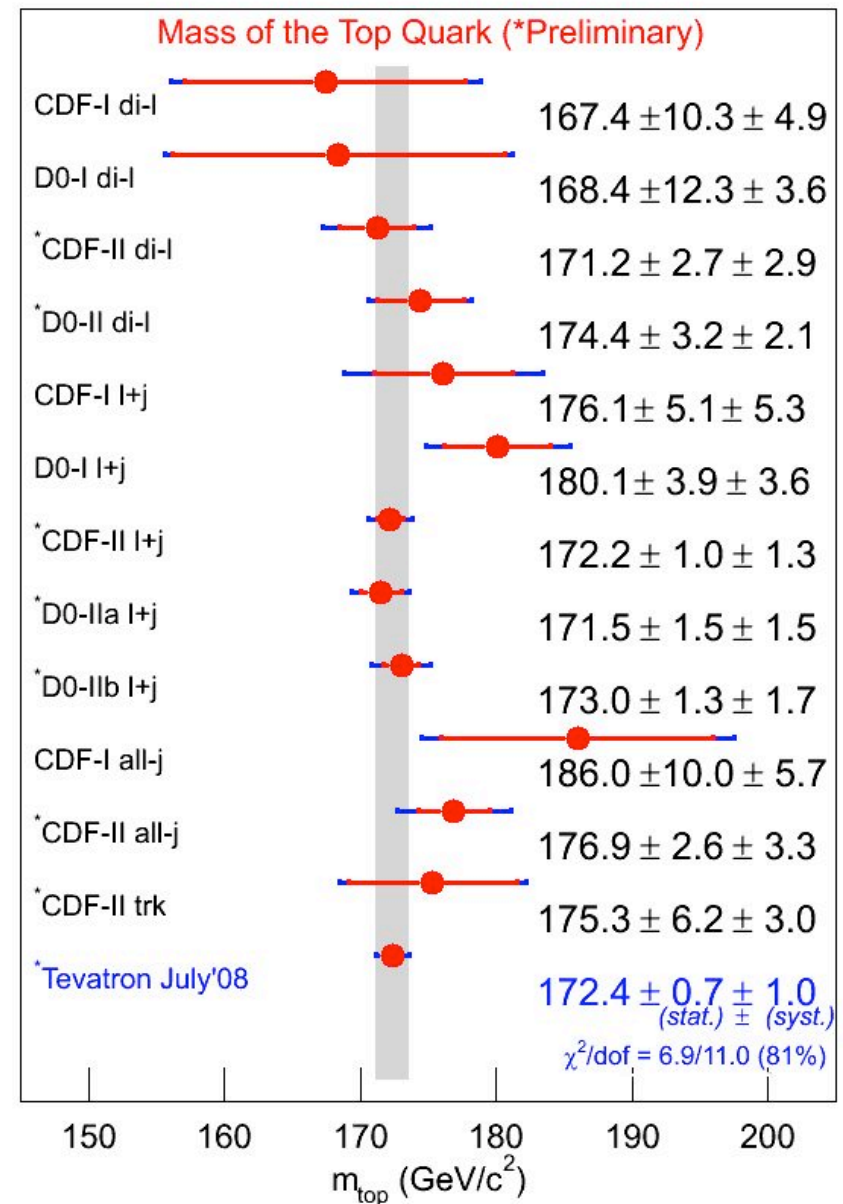
CDF:  $2.9 \text{ fb}^{-1}$

$$m_{\text{top}} = 172.2 \pm 1.0 \text{ (stat)} \pm 1.3 \text{ (syst)} \text{ GeV}$$

$\pm 1.0\%$

# Combining $M_{\text{top}}$ Results

- Excellent results in each channel
  - Dilepton
  - Lepton+jets
  - All-hadronic
- Combine them to improve precision
  - Include Run-I results
  - Account for correlations
- Uncertainty: **1.2 GeV**
  - Dominated by systematic uncertainties

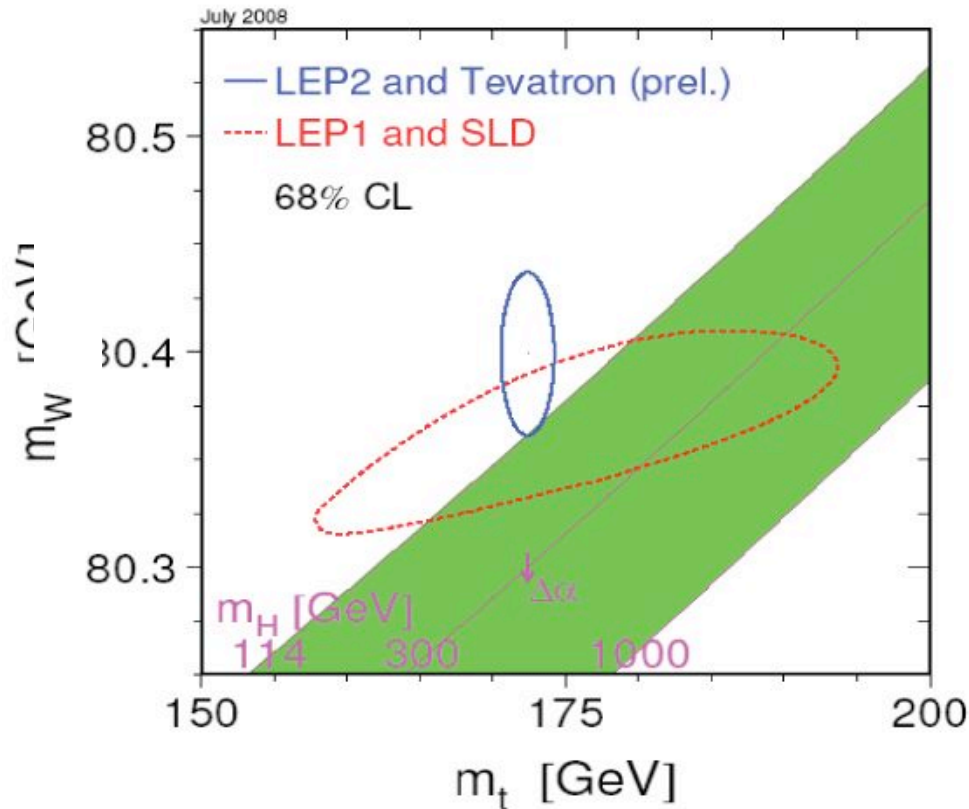




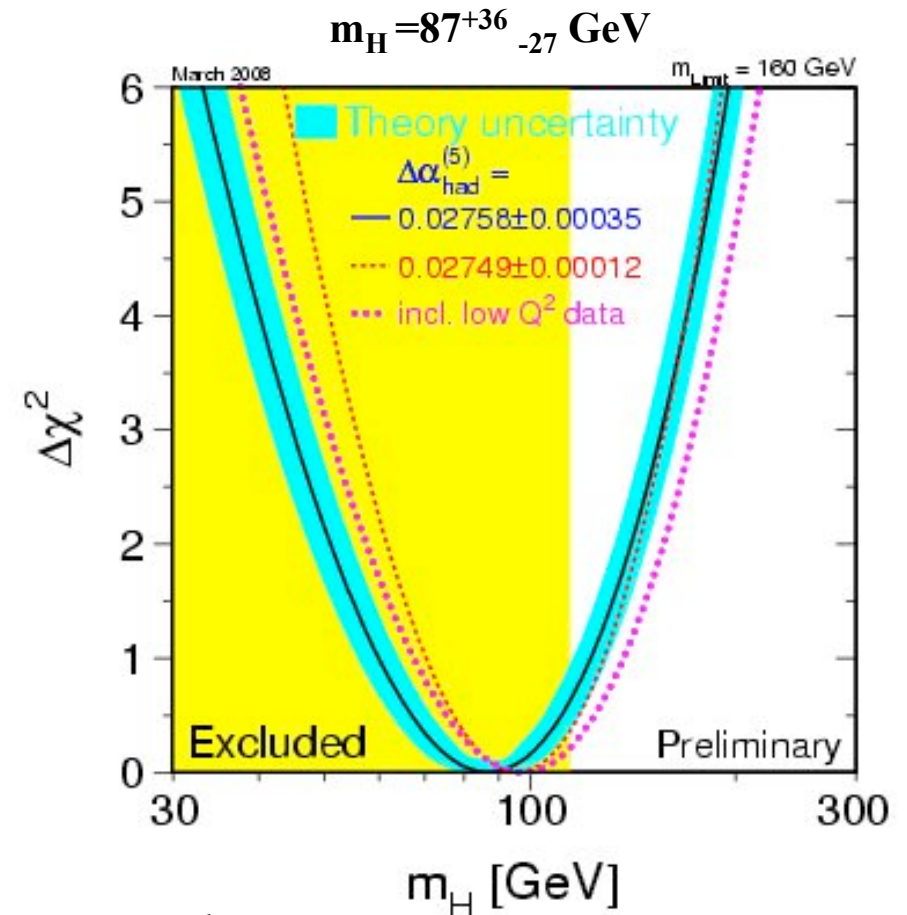
# Implications for Higgs Boson

$m_H$  constrained in the Standard Model

LEPEWWG 07/08



Direct searches at LEP2:  
 $m_H > 114.4 \text{ GeV}$  @95%CL



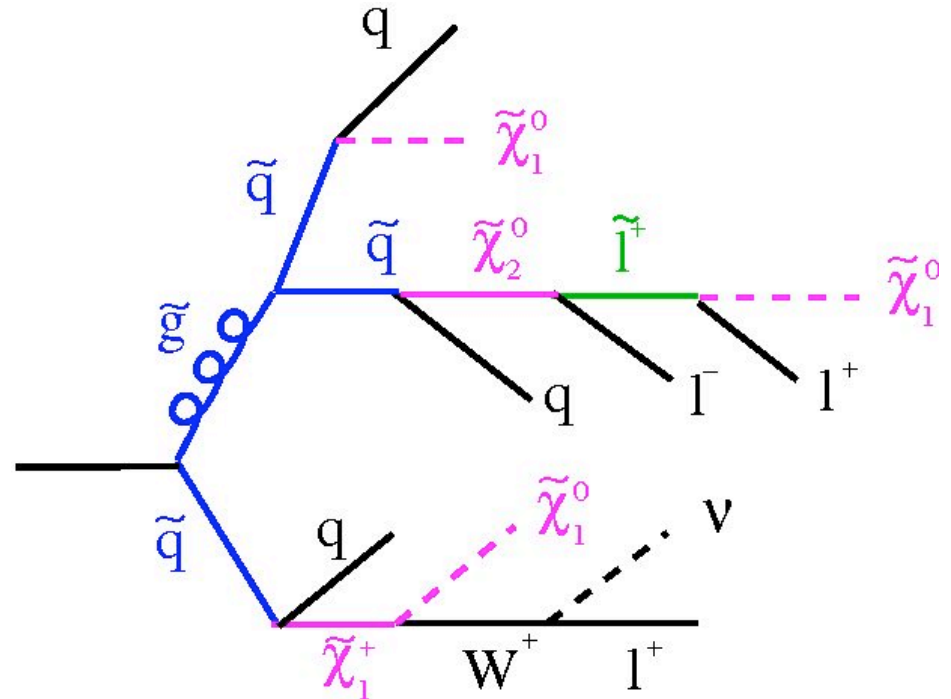
Indirect constraints:  
 $m_H < 160 \text{ GeV}$  @95%CL

**Measuring Properties of  
Supersymmetric Particles  
(in case they exist)**



## Spectacular SUSY Events (?)

- Long cascade decays via several SUSY particles
  - In classic models quite possible
    - Would be a wonderful experimental challenge!
  - But of course very possible also that it does not happen
- If Nature is like this:
  - Need to try to reconstruct masses of all those particles
- Main method:
  - Measure “edges”



## Spectacular SUSY Events (?)

- Long cascade decays via several SUSY particles, e.g.

$$\tilde{q}_L \rightarrow \tilde{\chi}_2^0 q (\rightarrow \tilde{\ell}^\pm \ell^\mp q) \rightarrow \tilde{\chi}_1^0 \ell^+ \ell^- q$$

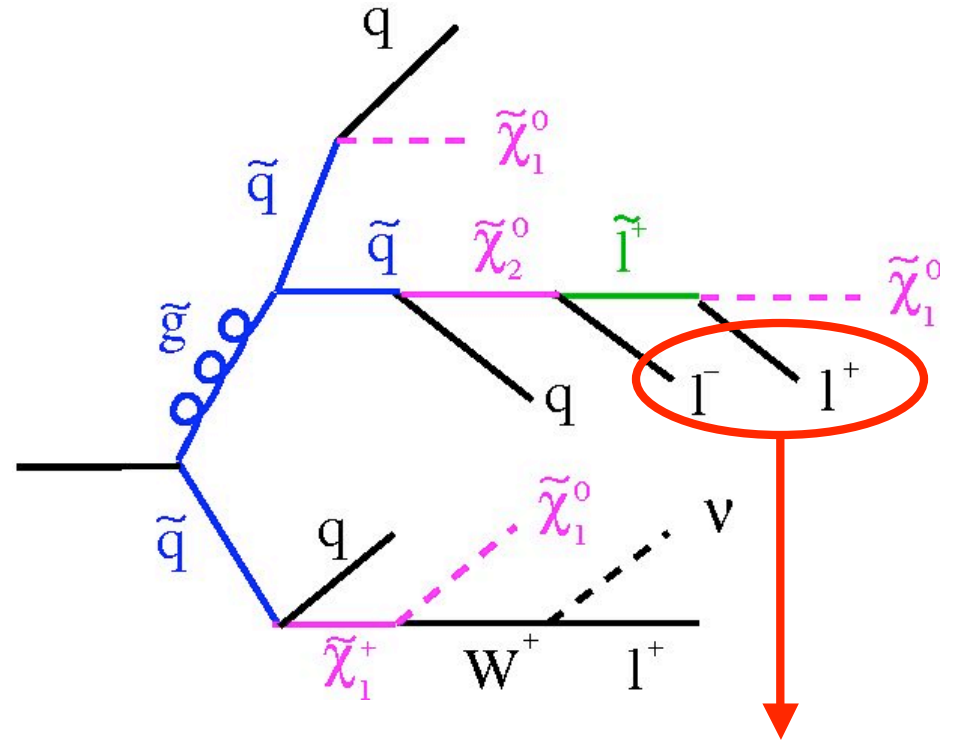
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- If Nature is like this:

- Need to try to reconstruct masses of all those particles

- Main method:

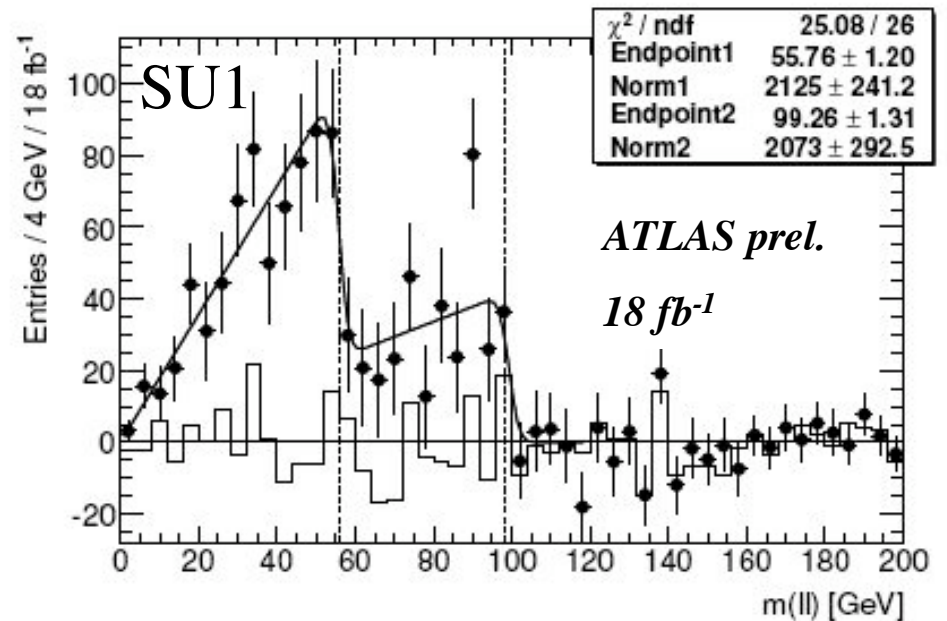
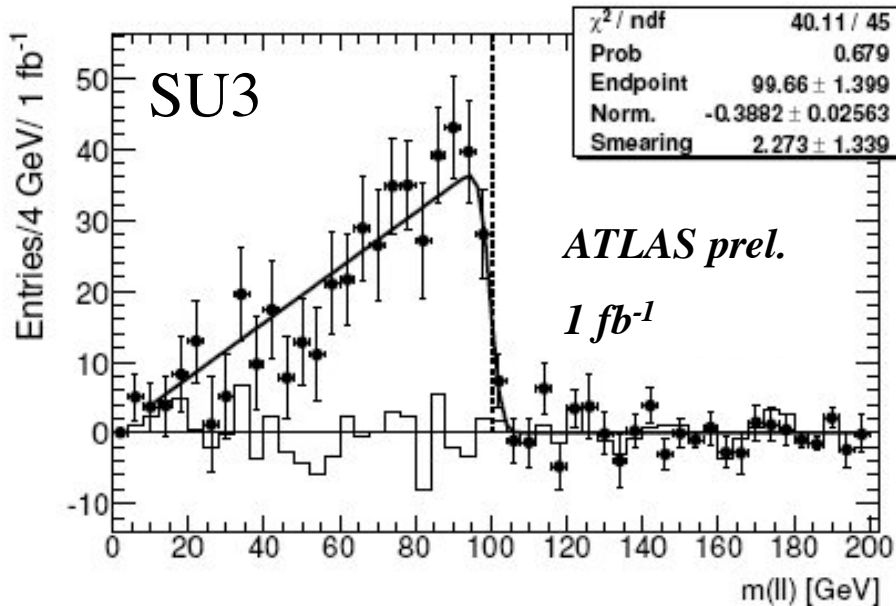
- Measure “edges”



$$m_{\ell\ell}^{\text{edge}} = m_{\tilde{\chi}_2^0} \sqrt{1 - \left(\frac{m_{\tilde{\ell}}}{m_{\tilde{\chi}_2^0}}\right)^2} \sqrt{1 - \left(\frac{m_{\tilde{\chi}_1^0}}{m_{\tilde{\ell}}}\right)^2}.$$

Only for opposite sign  
same-flavor (OSDF)  
leptons 34

# Dilepton Edge Fit



- Background from different flavors subtracted  $\Sigma e^+e^- + \mu^+\mu^- - e^+\mu^- - \mu^+e^-$ 
  - Removes random SUSY backgrounds, top backgrounds,...
- Fit for dilepton edge
  - With many such edges one can maybe get a beginning of an understanding what is happening!
  - Different models look differently

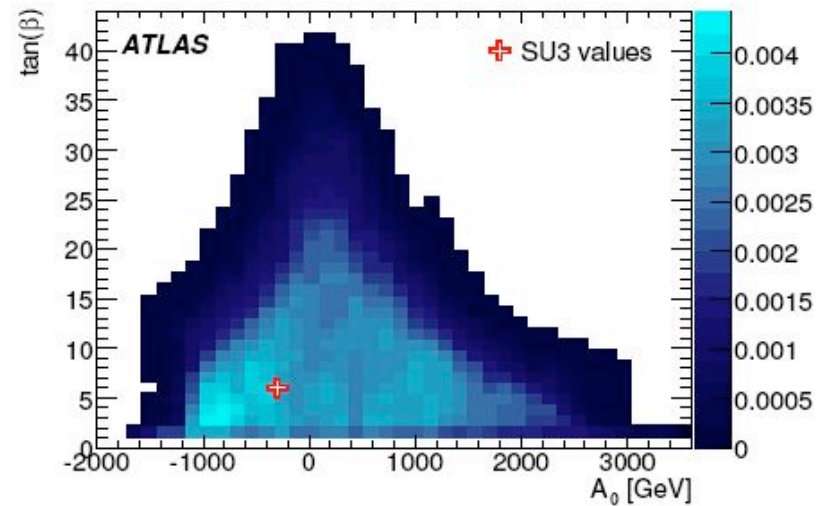
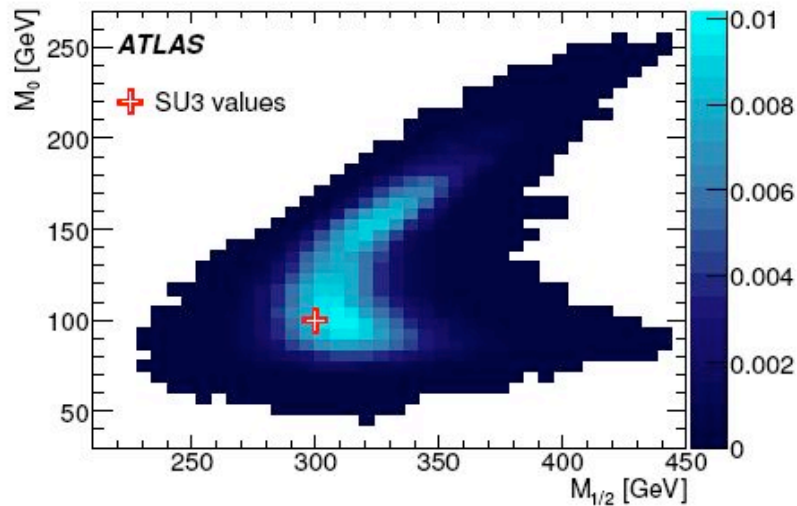
$$m_{\ell\ell}^{\text{edge}} = m_{\tilde{\chi}_2^0} \sqrt{1 - \left(\frac{m_{\tilde{\ell}}}{m_{\tilde{\chi}_2^0}}\right)^2} \sqrt{1 - \left(\frac{m_{\tilde{\chi}_1^0}}{m_{\tilde{\ell}}}\right)^2}.$$

# How well does this work?

Endpoint	SU3 truth	SU3 measured	SU4 truth	SU4 measured
$m_{\ell q}^{\text{edge}}$	501	$517 \pm 30 \pm 10 \pm 13$	340	$343 \pm 12 \pm 3 \pm 9$
$m_{\ell q}^{\text{thr}}$	249	$265 \pm 17 \pm 15 \pm 7$	168	$161 \pm 36 \pm 20 \pm 4$
$m_{lq(\text{low})}^{\text{max}}$	325	$333 \pm 6 \pm 6 \pm 8$	240	$201 \pm 9 \pm 3 \pm 5$
$m_{lq(\text{high})}^{\text{max}}$	418	$445 \pm 11 \pm 11 \pm 11$	340	$320 \pm 8 \pm 3 \pm 8$

- Works reasonably well...
- Can even try to extract high-level theory parameters

# SUSY Parameters at GUT scale!?!



Parameter	SU3 value	fitted value	exp. unc.
$\text{sign}(\mu) = +1$			
$\tan\beta$	6	7.4	4.6
$M_0$	100 GeV	98.5 GeV	$\pm 9.3$ GeV
$M_{1/2}$	300 GeV	317.7 GeV	$\pm 6.9$ GeV
$A_0$	-300 GeV	445 GeV	$\pm 408$ GeV
$\text{sign}(\mu) = -1$			
$\tan\beta$		13.9	$\pm 2.8$
$M_0$		104 GeV	$\pm 18$ GeV
$M_{1/2}$		309.6 GeV	$\pm 5.9$ GeV
$A_0$		489 GeV	$\pm 189$ GeV

- Depends if we understand our model well enough
- Personally I am very skeptical that we can do this
  - But would be great to have that problem!

# Conclusions

- Several methods of extracting property of particle
  - Template method is widely used
  - Matrix Element technique extracts more information
  - For known shapes simple fits can also be done
- Examples:
  - W boson mass (precision  $\sim 0.06\%$ )
  - Top quark mass (precision  $\sim 0.7\%$ )
  - SUSY particles
- I hope we will be able to measure properties of many new particles!
  - Let's see how to find them first in the next lecture