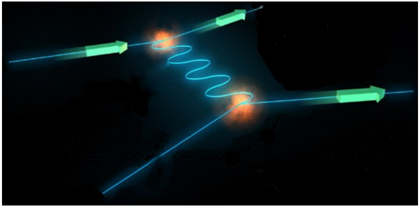


Quarkonium in an anisotropic plasma

The static potential from first principles

Marcus Tassler

in collaboration with

Owe Philipsen

McGill University,
University of Frankfurt

Fermilab, QWG10 Workshop

May 19, 2010

Overview

Topics of this talk

- The real-time static potential is introduced to generalize the QCD static potential to a thermal setting. The physical signature of quarkonium in an isotropic medium is discussed.
- A calculation of the $q\bar{q}$-potential in an anisotropic medium is presented.
- It will be shown that the potential at fixed density of the medium is insensitive to the degree of anisotropy.

References

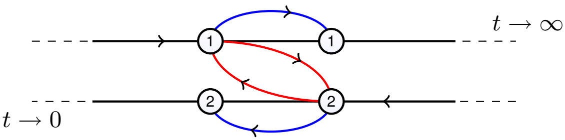
O. Philipsen and M. Tassler, *"On quarkonium in an anistropic quark gluon plasma,"* arXiv:0908.1746 [hep-ph]; M. Tassler, *"Heavy Quarkonia beyond Deconfinement and Real Time Lattice Simulations,"* arXiv:0812.3225 [hep-lat]; M. Laine, O. Philipsen and M. Tassler, *"Thermal imaginary part of a real-time static potential from classical lattice gauge theory simulations,"* JHEP 0709, 066; M. Laine, O. Philipsen, P. Romatschke and M. Tassler, *"Real-time static potential in hot QCD,"* JHEP 0703, 054;

Collaborators: M. Laine, O. Philipsen, P. Romatschke

Real-Time Static Potential

Definition of the static $q\bar{q}$-potential for thermal media and properties in thermal equilibrium

Schwinger-Keldysh Formalism



The real-time path integral is build along the *Schwinger-Keldysh time contour* consisting of a Euclidean patch of length β, as well as a real-time patch $\mathscr{C}$ running forward and returning in time.

A location on $\mathscr{C}$ is specified by a time $t \in \mathbb{R}$ and an index $i \in 1, 2$. The correlator of two operators $\hat{\varphi}, \hat{\psi}$ takes a matrix form:

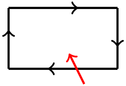
Real-Time Correlators

$$i\boldsymbol{G} = i \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} = \begin{pmatrix} \langle \mathcal{T} \hat{\psi}(t')\hat{\varphi}(t)\rangle & -\langle \hat{\varphi}(t)\hat{\psi}(t')\rangle \\ \langle \hat{\psi}(t')\hat{\varphi}(t)\rangle & \langle \tilde{\mathcal{T}} \hat{\psi}(t')\hat{\varphi}(t)\rangle \end{pmatrix}$$

Retarded, advanced and symmetric correlators are defined via:

$$\boldsymbol{R}^{-1} \cdot \boldsymbol{G} \cdot \boldsymbol{R} = \begin{pmatrix} 0 & G_A \\ G_R & G_S \end{pmatrix} \quad \text{where} \quad \mathbf{R} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$

Real Time Static Potential



$$[i\partial_t - V(t,r)]\, C_{21}(t,r) = 0$$

Static Potential (Laine,Philipsen,Romatschke,Tassler,JHEP 0703 (2007) 054)

The static $q\bar{q}$-potential in a thermal medium is obtained from a Schrödinger equation for the $q\bar{q}$-correlator C_{21} in the large time and static mass limit :

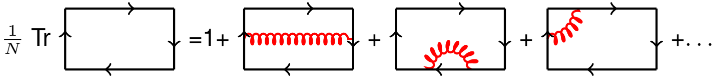
$$V(\mathbf{r}) = \lim_{t\to\infty} V(t,\mathbf{r})$$

The quarkonium resonance is subsequently estimated by solving the Schrödinger equation for physical quark masses:

$$\left(i\partial_t - [-\frac{\triangle_{\mathbf{r}}}{M} + V(\mathbf{r}) + 2M]\right) C_{21} = 0, \quad \mathrm{BC:}\ C_{21}(t=0) \sim \delta(\mathbf{r})$$

Some recent uses: A. Dumitru, Y. Guo and M. Strickland, *“The imaginary part of the static gluon propagator in an anisotropic (viscous) QCD plasma,”* arXiv:0903.4703 [hep-ph]; N. Brambilla, J. Ghiglieri, A. Vairo and P. Petreczky, *“Static quark-antiquark pairs at finite temperature,”* Phys. Rev. D **78** (2008); A. Beraudo, J. P. Blaizot and C. Ratti, *“Real and imaginary-time $Q\bar{Q}$ correlators in a thermal medium,”* Nucl. Phys. A **806** (2008) ; M. A. Escobedo and J. Soto, *“Non-relativistic bound states at finite temperature (I): the hydrogen atom,”* arXiv:0804.069 ; Y. Burnier, M. Laine and M. Vepsalainen, *“Heavy quarkonium in any channel in resummed hot QCD,”* JHEP **0801** (2008)

Expansion of the Wilson Loop



Diagrams contributing to the Wilson-Loop

The Real-Time static potential to $\mathcal{O}(g^2)$

$$V(\boldsymbol{r}) = g^2 \mathrm{C_F} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} (1 - \cos \boldsymbol{k} \cdot \boldsymbol{r}) \tilde{G}^{00}_{11}(\omega = 0, \boldsymbol{k})$$

Here $\tilde{G}^{00}_{11}$ is the longitudinal component of the time ordered gluon propagator which can be decomposed as: $\tilde{G}_{11} = Re\ \tilde{G}_R + \frac{1}{2}\tilde{G}_S$.

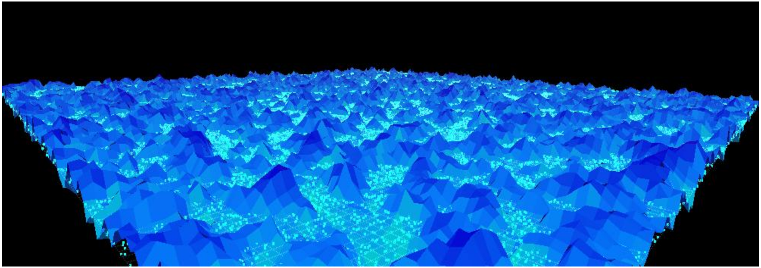
In the special case of ***thermal equilibrium*** the potential takes the form:

$$V(r) = \underbrace{-\frac{g^2 C_F}{4\pi}[m_D + \frac{exp(-m_D r)}{r}]}_{\text{Re(V): Retarded contribution}} \underbrace{-i\frac{g^2 T C_F}{2\pi}\phi(m_D r)}_{\text{Im(V): Symmetric contribution}}$$

where $\phi(x) = 2\int_0^\infty \frac{dzz}{(z^2+1)^2}\left[1 - \frac{sin(zx)}{zx}\right]$.

Note: The potential has an imaginary part originating from Landau damping. The real part is the usual Debye screened potential.

Imaginary Part from Classical Simulations



Magnetic energy density in classical Yang-Mills theory with quasiparticles.

Classical Approximation of Thermal Field Theories

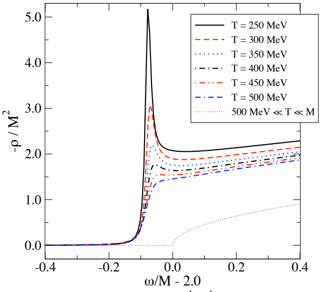
Bare symmetric propagator G_S for a bosonic field with coupling $g^2 \sim \hbar$:

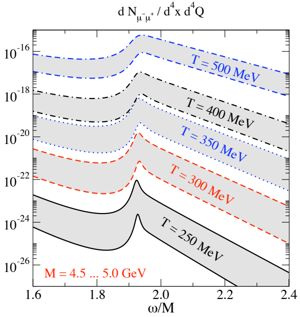
$$G_S \sim n_B(\omega) + \frac{1}{2} = \frac{T}{\hbar\omega} + \frac{1}{12}\frac{\hbar\omega}{T} + \dots$$

The classical limit $\hbar \to 0$ resums the IR contributions of all diagrams with a maximal number of symmetric insertions. They capture the long range physics of a plasma at large T.

Measurement: The existence of an imaginary part of the potential is confirmed and a significant non-perturbative IR enhancement is observed.

Physical signatures





Quarkonium signatures from the finite mass Schrödinger equation:

Spectral function Laine et al.,JHEP0801:043

- The spectral function is depicted for Bottomonium.
- The imaginary part induces a finite width to the resonance peak (melting of the resonance).

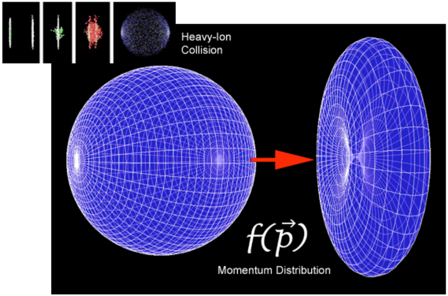
Potential Laine, JHEP 0705:028

- The Dilepton rate is shown for Charmonium and Bottomonium.
- A softening of the resonance is seen for increased temperature.

ANISOTROPIC MEDIA

The static $q\bar{q}$-potential in a plasma with anisotropic momentum distribution

Anisotropic Media



In the following the potential will be discussed for an anisotropic plasma characterized by the ***static*** momentum distribution:

$$f(\vec{p}) = n_B(p\sqrt{1 + \xi(\vec{v}_p \cdot \vec{n})^2})$$

n_B: Thermal Bose Distribution, ξ: Anisotropy, $\vec{n}$: Collision axis

Normalization: We don't know the relation between the particle density and the parameters $\{\xi, \beta\}$. Instead we keep the density fixed.

Observation: *Any change of the potential is a density effect !*

See also the following talk by A.Mocsy and: A. Dumitru, Y. Guo and M. Strickland, *"The imaginary part of the static gluon propagator in an anisotropic QCD plasma,"* Phys. Rev. D **79**; Y. Burnier, M. Laine and M. Vepsalainen, *"Quarkonium dissociation in the presence of a small momentum space anisotropy,"* JHEP **1001**, 054.

Normalization

The medium is diluted once ξ is increased if $f(\vec{p})$ is not normalized. The physical relation between ξ and the particle density in heavy ion collisions is very hard to obtain from first principles.

How to keep the particle density fixed ?

The simplest approach is to multiply the momentum distribution function $f(T,\xi)$ by a normalization prefactor $N(\xi)$. Another approach is to rescale the temperature $T \to T(\xi)$.

- ***Multiplicative Normalization***

  To keep the particle density of the medium fixed the distribution function $f(\vec{p})$ is multiplied by the prefactor

$$N(\xi) = \sqrt{1+\xi}.$$

- ***Landau matching***

  This matching procedure is often used in the context of hydrodynamic simulations. The particle density is kept fixed by rescaling T:

$$T(\xi) = T R^{-\frac{1}{4}}(\xi) \quad \text{where} \quad R(\xi) = \frac{1}{2}\left(\frac{1}{1+\xi} + \frac{\arctan(\sqrt{\xi})}{\sqrt{\xi}}\right).$$

The results from both schemes are consistent for $\xi \ll 1$.

How to obtain the gluon propagator $\tilde{G}^{00}_{11}(\omega = 0)$?

The longitudinal part of $\tilde{G}_{11} = Re\ \tilde{G}_R + \frac{1}{2}\tilde{G}_S$ in the static limit is needed.
$\tilde{G}_R$ is known and $\tilde{G}_S$ is obtained from a Schwinger-Dyson relation.

- ***Retarded gluon propagator in covariant gauge*** Dumitru et al., Romatschke

$$\begin{aligned} \tilde{G}_R^{\mu\nu}(K) \quad &= \quad \Delta_G \left[(K^2 - \alpha - \gamma) \frac{\omega^4}{K^4} B^{\mu\nu} + (\omega^2 - \beta) C^{\mu\nu} + \delta \frac{\omega^2}{K^2} D^{\mu\nu} \right] \\ &\quad + \Delta_A [A^{\mu\nu} - C^{\mu\nu}] - \frac{\lambda}{K^4} K^\mu K^\nu \end{aligned}$$

with structure functions $\alpha(K) - \delta(K)$ and

$$\Delta_G^{-1} = (K^2 - \alpha - \gamma)(\omega^2 - \beta) - \delta^2[\boldsymbol{k}^2 - (n \cdot K)^2] \text{ and } \Delta_A^{-1} = K^2 - \alpha.$$

$A(K) - D(K)$ form a *tensor basis* for this system where Lorentz symmetry is broken by the plasma rest frame and the anisotropy vector.

- ***Schwinger-Dyson Relation*** Arnold, Moore, Yaffe

The needed Schwinger-Dyson relation [$\boldsymbol{\Pi}_{\mathrm{S}}$: symmetric self-energy] is:

$$\tilde{\boldsymbol{G}}_{\mathrm{S}} = \tilde{\boldsymbol{G}}_{\mathrm{R}} \cdot \boldsymbol{\Pi}_{\mathrm{S}} \cdot \tilde{\boldsymbol{G}}_{\mathrm{R}}^{*}.$$

- ***Retarded Propagator***

  It is straightforward to obtain the retarded propagator in the static limit:

$$\tilde{G}^{00}_{\mathrm{R}}(\omega = 0, \boldsymbol{k}) = \frac{k^2 + m_\alpha^2 + m_\gamma^2}{(k^2 + m_\alpha^2 + m_\gamma^2)(k^2 + m_\beta^2) - m_\delta^4}$$

  Effective masses [$\theta_k = \angle(\boldsymbol{n}, \boldsymbol{k})$]:

$$\hat{m}_\alpha^2 = -m_D^2 \frac{\xi}{3} \cos^2\theta_k \qquad \hat{m}_\beta^2 = 1 + \xi(\cos^2\theta_k - \frac{1}{6})$$

$$\hat{m}_\gamma^2 = \frac{\xi}{3} \sin^2\theta_k, \qquad \hat{m}_\delta^2 = -\xi \frac{\pi}{4} \sin\theta_k \cos\theta_k$$

  Note that $\hat{m_x} = m_x / m_D$ where m_D is the isotropic Debye mass.

- ***Schwinger-Dyson Relation***

  The calculation of the symmetric propagator is much more difficult. It can be shown however that only one contraction of $\tilde{\mathbf{G}}_{\mathrm{R}}$ and $\mathbf{\Pi}_{\mathrm{S}}$ is relevant:

$$\tilde{\mathbf{G}}^{00}_{\mathrm{S}} = \tilde{\mathbf{G}}^{00}_{\mathrm{R}} \cdot \mathbf{\Pi}^{00}_{\mathrm{S}} \cdot \tilde{\mathbf{G}}^{00*}_{\mathrm{R}}$$

The symmetric self energy has the following form in the static limit:

$$\mathrm{i}\Pi_{\mathrm{S}}^{\mu\nu} = 8\pi g^2 N \frac{1}{k} \int \frac{d^3p}{(2\pi)^3} v_p^\mu v_p^\nu f(\mathbf{p})(1 + f(\mathbf{p} + \boldsymbol{k}))\delta(\mathbf{v}_p \cdot \mathbf{v}_k),$$

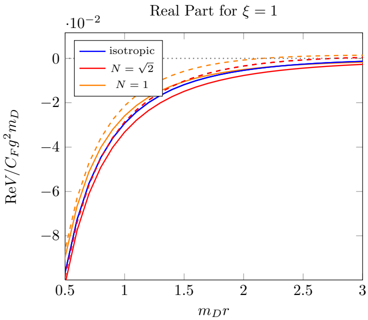
where $\mathbf{v}_p = \mathbf{p}/p$. Using the static limit of the retarded propagator and the Schwinger-Dyson relation the symmetric propagator is obtained.

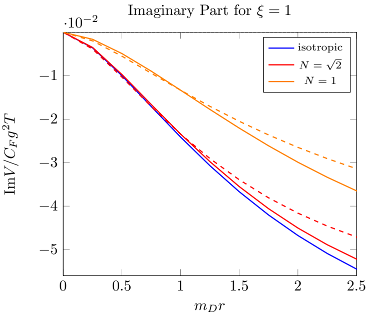
Symmetric Propagator in the static limit

$$\begin{aligned}\tilde{G}_{\mathrm{S}}^{00} \quad = \quad & -\mathrm{i}\frac{2\pi}{k(k^2+m_D^2)^2}\frac{m_D^2}{\beta}\left(1+\xi\left[\frac{\pi^2-3\zeta(3)}{\pi^2} - \frac{3}{4}\sin^2\theta_k\right]\right) \\ & +i\xi 4\pi \frac{m_D^4}{\beta}\frac{1}{k(k^2+m_D^2)^3}(\cos^2\theta_k - \frac{1}{6})\end{aligned}$$

Having obtained all needed parts of the gluon propagator the $q\bar{q}$-potential can finally be calculated. *The symmetric propagator contributes the imaginary part of the potential again.*

Results for $\xi = 1$





- ***No Normalization***

  This case implies a specific relation between the particle density and the anisotropy. The medium is diluted quickly with increased anisotropy and the (perturbative) vacuum potential is approached.

- ***Fixed particle density***

  The change in the potential is very small compared to the isotropic result. The change observed in the upper case is a density effect.

- ***Thermal Potential***
  The static $q\bar{q}$-potential in a thermal medium and the ensuing quarkonium resonance in equilibrium were discussed.
- ***Quarkonium in an Anisotropic Plasma***
  The static potential was calculated for an anisotropic plasma using a non-equilibrium Schwinger-Dyson relation.
- ***Role of Particle Density and Anisotropy***
  The $q\bar{q}$-potential is sensitve to the density of the medium. The anisotropy of the medium does not play a role as long as the density is kept fixed.

Thanks for your attention !