Neutron EDM from Lattice QCD

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CP symmetry breaking in the SM

- EW
  - It has been known the CP violation occurs by the phase of CKM matrix
  - K, D, B meson decay via direct and indirect CP violation
  - Contribution to EDM is very tiny, $\Rightarrow d_N^{\rm KM} \simeq 10^{-30\sim-33}\,{\rm e}\cdot{\rm cm}$

    6-orders magnitude below the exp. upper limit: $|d_N^{\rm exp}| < 2.9\times 10^{-26}\,{\rm e}\cdot{\rm cm}$
- QCD
  - θ term in the QCD Lagrangian:

$$\mathcal{L}_\theta = \bar{\theta}\frac{1}{64\pi^2}G\widetilde{G}, \quad \bar{\theta} = \theta + \arg\det M$$

    renormalizable and CP-violation comes due to topological charge density.
  - EDM experiment provides very strong constraint on $\bar{\theta} < 10^{-9}$

    ⇒ θ and $\arg\det$ M need to be unnaturally canceled ! (strong CP problem)

CP symmetry breaking beyond the SM

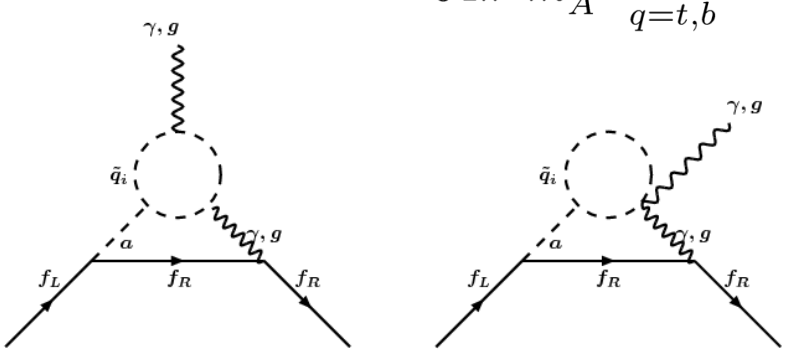
- Possible higher dimension operators
  - Effective Hamiltonian with higher dimension than 4

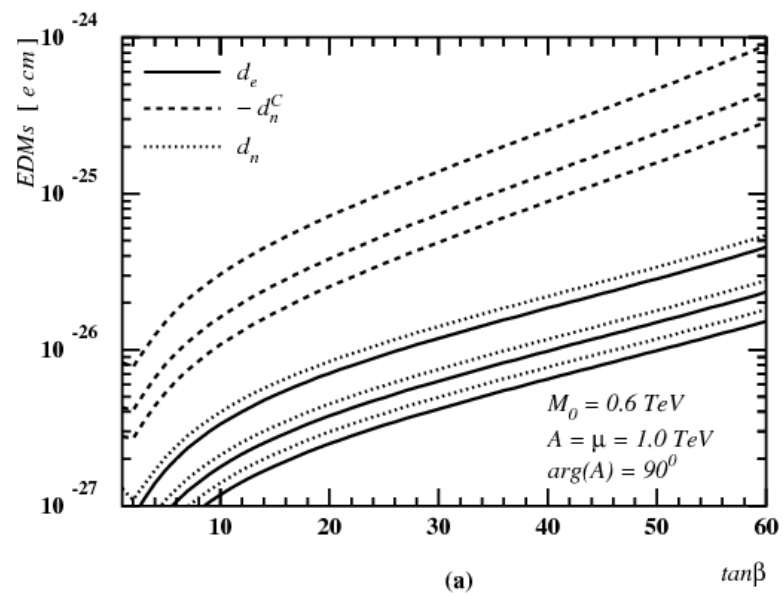
$$H_{\not{CP}} = \sum_k C_k(\mu)\,\mathcal{O}_k$$

$$\begin{aligned}
\mathcal{O}_{\rm qEDM} &= \bar{q}(\sigma\cdot F)\gamma_5 q && \text{: Quark-photon}\\
\mathcal{O}_{\rm cEDM} &= \bar{q}(\sigma\cdot G)\gamma_5 q && \text{: Quark-gluon}\\
\mathcal{O}_{\rm Weinberg} &= GG\widetilde{G} && \text{: Pure gluonic}\\
&\vdots
\end{aligned}$$

SUSY model

$$d_N \sim d_d^{\rm cEDM} \sim -\frac{\alpha m_d \tan\beta}{64\pi^3 m_A^2}\sum_{q=t,b}\xi_q Q_q^2 F$$

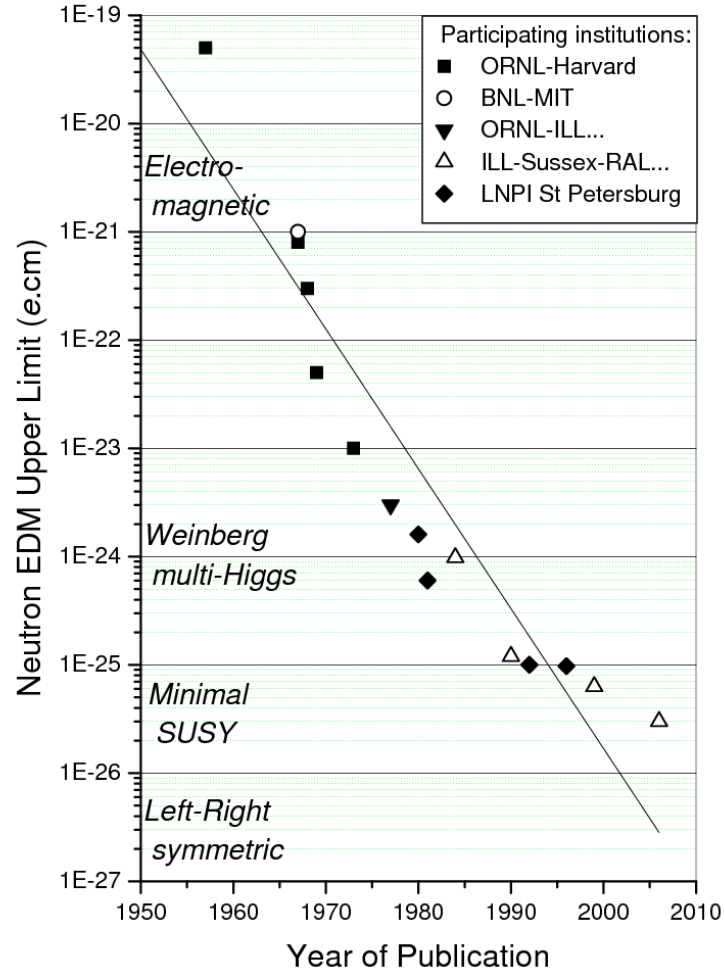




Chang, et al. (99), Ibrahim and Nath (08)

Constraint on nEDM

- The present and future experiment is close to "exclude" of MSSM
  pEDM experiment @ BNL,
  nEDM experiment @ J-PARC, …
  ⇒ reaching a sensitivity of 10^{-29} e・cm !
- Current theoretical bound is based on quark model.
- Non-perturbative computation is necessary to draw more reliable conclusion.



Harris, 0709.3100

What lattice QCD can do for nEDM

- In principle
  - Direct estimate of neutron and proton EDM from θ term, higher dim. $\not{C}\not{P}$ operators
  - Matrix elements of higher dimension operators

    $\langle N|\bar{q}\gamma_5\sigma\cdot Fq|N\rangle,\ \langle N|\bar{q}\gamma_5\sigma\cdot Gq|N\rangle, \langle N|GG\tilde{G}|N\rangle, \cdots$
- In practice there are some difficulties
  - Statistical error

    Source of CP violation comes from gauge background (topological charge, sea quark) which is intrinsically noisy.

    Disconnected diagram is necessary because of flavor singlet contraction.
  - Systematic error

    Volume effect may be significant.

    Chiral behavior is important, $d_N \sim O(m)$

Possible lattice methods

- Spectrum method

  Aoki-Gocksch(89), ES et al. CP-PACS(06, 07)

  - Spin splitting of nucleon energy in external electric field and θ term, which is given by 2-pt function: $m_{\uparrow} - m_{\downarrow} = 2d_N\theta E$
  - Computational cost is cheap, and directly obtain EDM.
- Form factor

  ES et al. CP-PACS(05), RBC(06)

$$\langle n(P_1)|J_\mu^{\rm EM}|n(P_2)\rangle_\theta = \bar{u}_N^\theta \Big[\underbrace{\frac{F_3^\theta(Q^2)}{2m_N}\gamma_5\sigma_{\mu\nu}Q_\nu}_{\text{P,T-odd}} + \underbrace{F_1\gamma_\mu + \frac{F_2}{2m_N}\sigma_{\mu\nu}Q_\nu}_{\text{P,T-even}} + \cdots \Big] u_N^\theta$$

  - F_3 in $Q^2 \to 0$ provides d_N
  - Subtraction of contribution of CP-odd phase in n propagator.
- Imaginary θ
  - Generate new configurations with imaginary θ term, which may enhance signal.

    Izubuchi (07), Horsley et al. (08)

Spectrum method

ES et al. (06, 07)

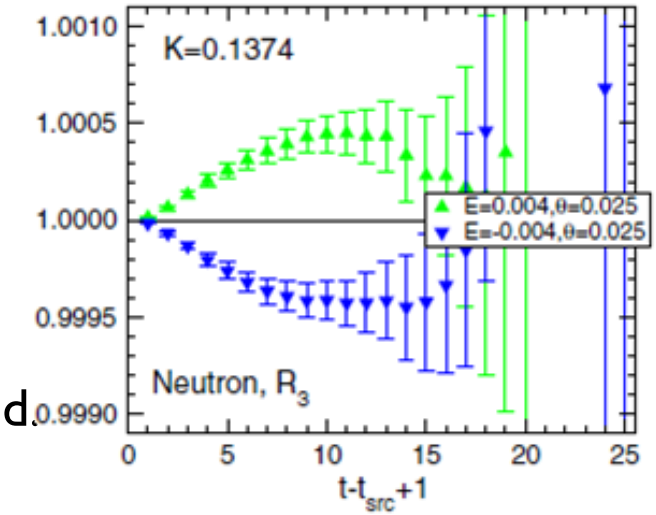
- Ratio of spin up and down

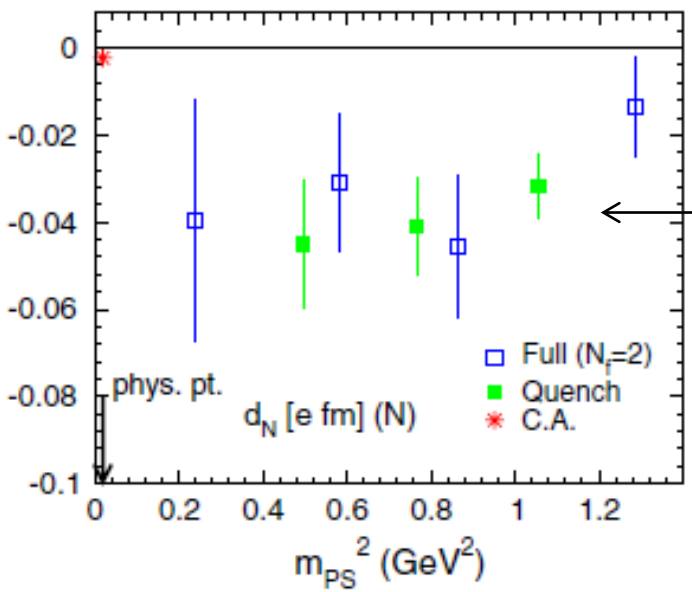
$$R_3 = \frac{\langle N(t)\bar{N}(0)\rangle^{\rm up}_{\theta,E}}{\langle N(t)\bar{N}(0)\rangle^{\rm down}_{\theta,E}} \simeq 1 + d_N E\theta t$$

Linear response, gradient is a signal of EDM.

- Reweighting works well for small real θ
- Temporal periodicity is broken by electric field.

  ⇒ additional systematic effects





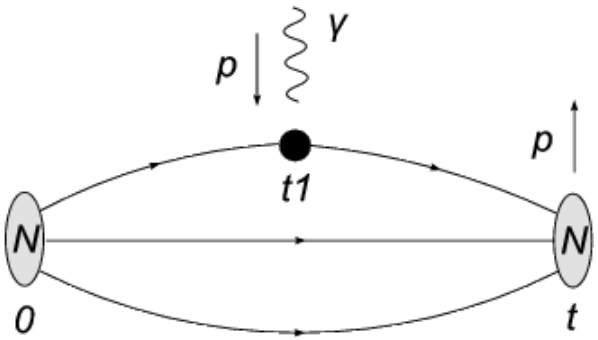
Full QCD with clover fermion:

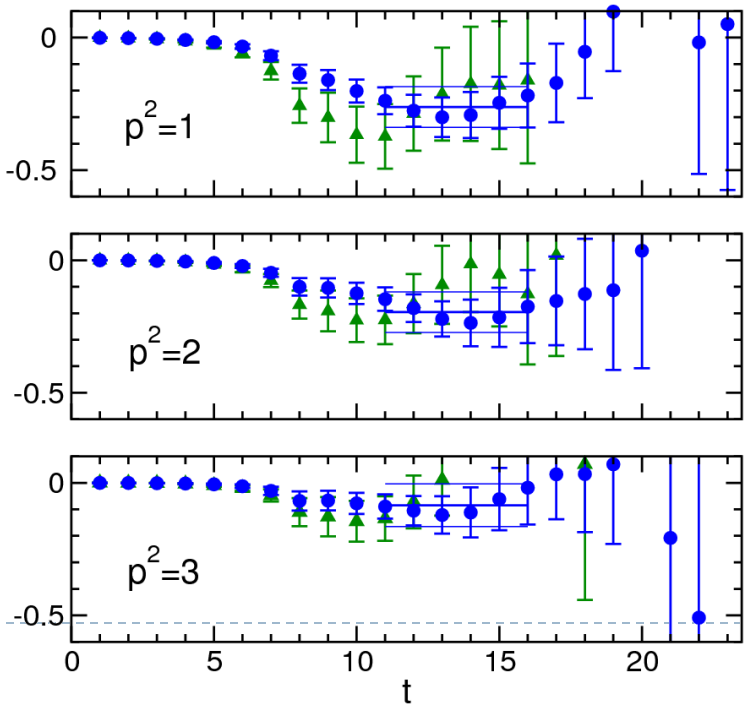
- There seems to be no significant difference between quench and full QCD.
- Statistical error is still large.
- Finite size effect from breaking of temporal periodicity is also significant

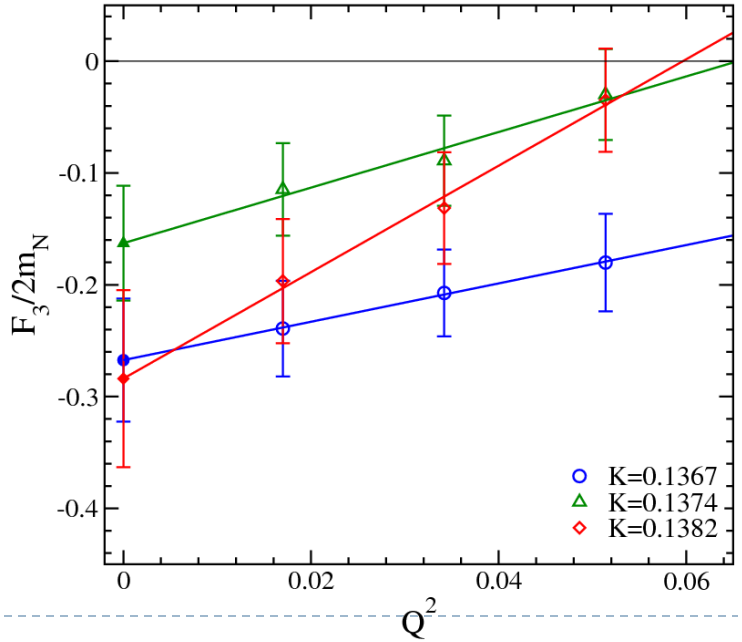
Form factor

ES(05, 08)

- Matrix element in θ vacuum
  - 3pt function
    - N_f=2 clover fermion
    - Size is $24^3 \times 48$ lattice (~2 fm^3)
    - Signal appears in 11 -- 16
    - $Q^2 \rightarrow 0$ limit is doable with linear func.







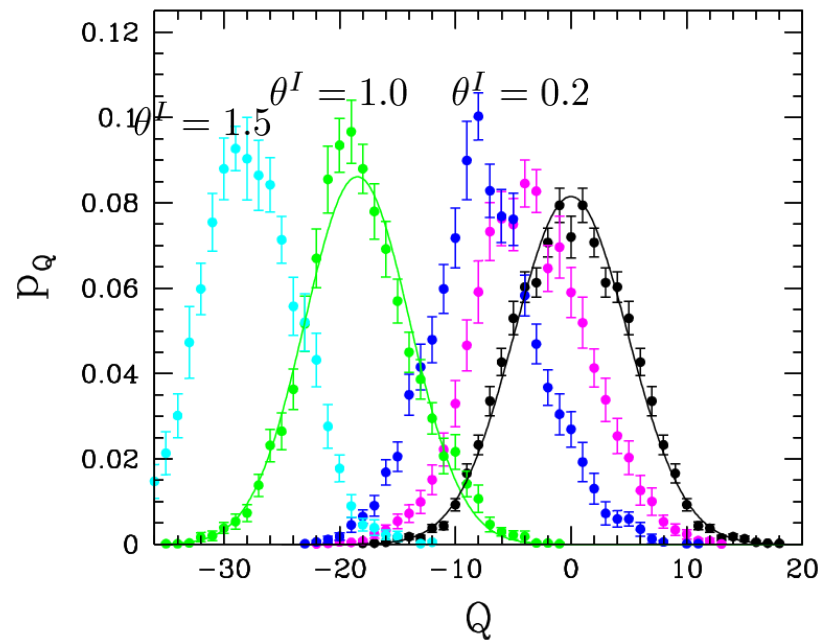
Imaginary θ

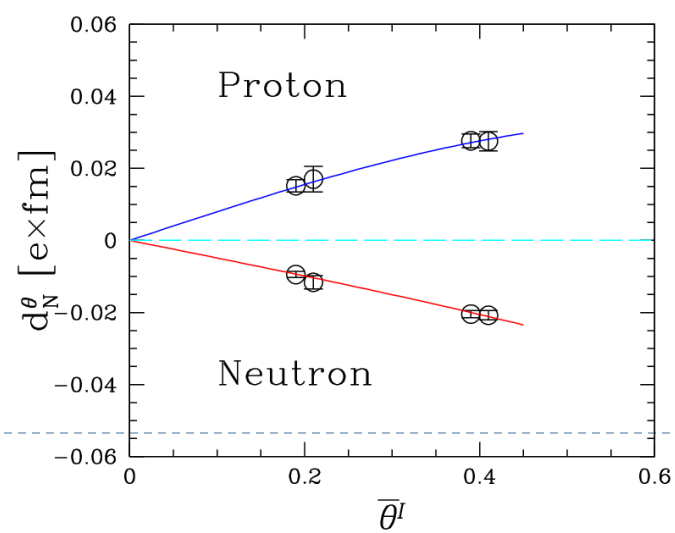
Izubuchi(07), Horsley et al. (08)

- Analytically continued to pure imaginary, $\theta \to i\theta^I$

$$\langle O e^{i\theta Q} \rangle \to \langle O e^{-\theta^I Q} \rangle$$

  - There is no sign problem, expect better signal.
  - Generate the QCD ensemble with θ^I: distribution of topological charge is shifted by θ^I





Full QCD with clover fermion

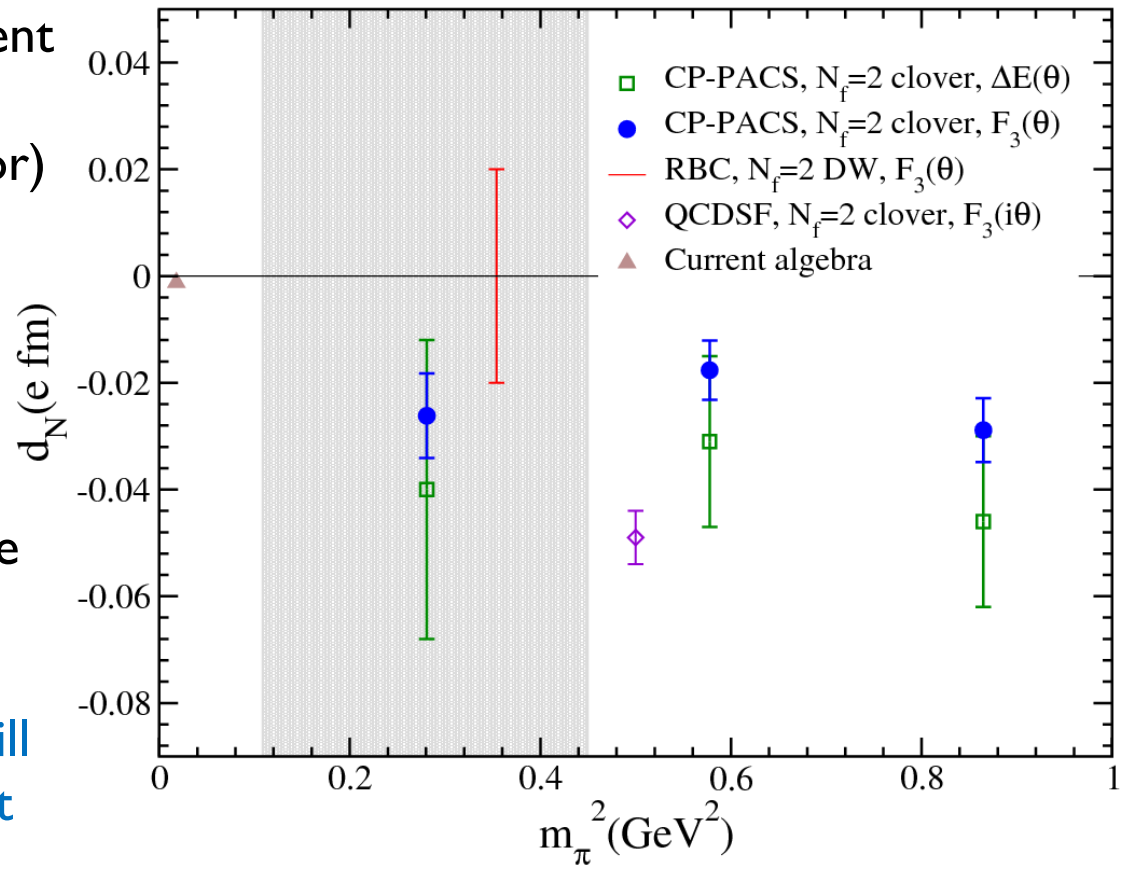
- EDM is given by the slope.
- Clear signal, but systematic error due to chiral symmetry breaking of clover fermion has not been taken into account.

Comparison of results

- Full QCD
  - Lattice results are consistent within 1σ.
    (not include systematic error)
  - An order of magnitude larger than the results of current algebra.

  - N_f = 2+1 DWF configs. (RBC/UKQCD) are available for near physical pion mass.

  - Large statistical error is still problem. (O(100) stat. is not enough)



Near future plan

- Form factor in DWF configurations
  - Chiral symmetry on the lattice

    Reduction of systematic error coming from finite *a*
  - RBC/UKQCD collaboration

    Generate the ensembles including dynamical up-down, strange quarks
  - Large size and small mass

    Control the finite size and chiral extrapolation ($m_\pi \rightarrow m_\pi^{phys}$)

	Lattice size	Physical size	Lattice spacing	L_s	Gauge action	Pion mass
	$24^3 \times 64$	2.7 fm^3	0.114 fm	16	Iwasaki	315 -- 615 MeV
	$32^3 \times 64$	2.7 fm^3	0.087 fm	16	Iwasaki	295 -- 397 MeV
	$32^3 \times 64$	4.6 fm^3	0.135 fm	32	DSDR	171 -- 241 MeV
Not available yet →	$48^3 \times 96$	5.5 fm^3	0.115 fm	8	Iwasaki	135 MeV

Near future plan

- Improvement of statistical error
  - Development of algorithm

    Low-mode-averageing (LMA), all-mode-averaging (AMA) Izubuchi, ES in prep.

    ⇒ reduction of computational cost to 1/10 -- 1/100

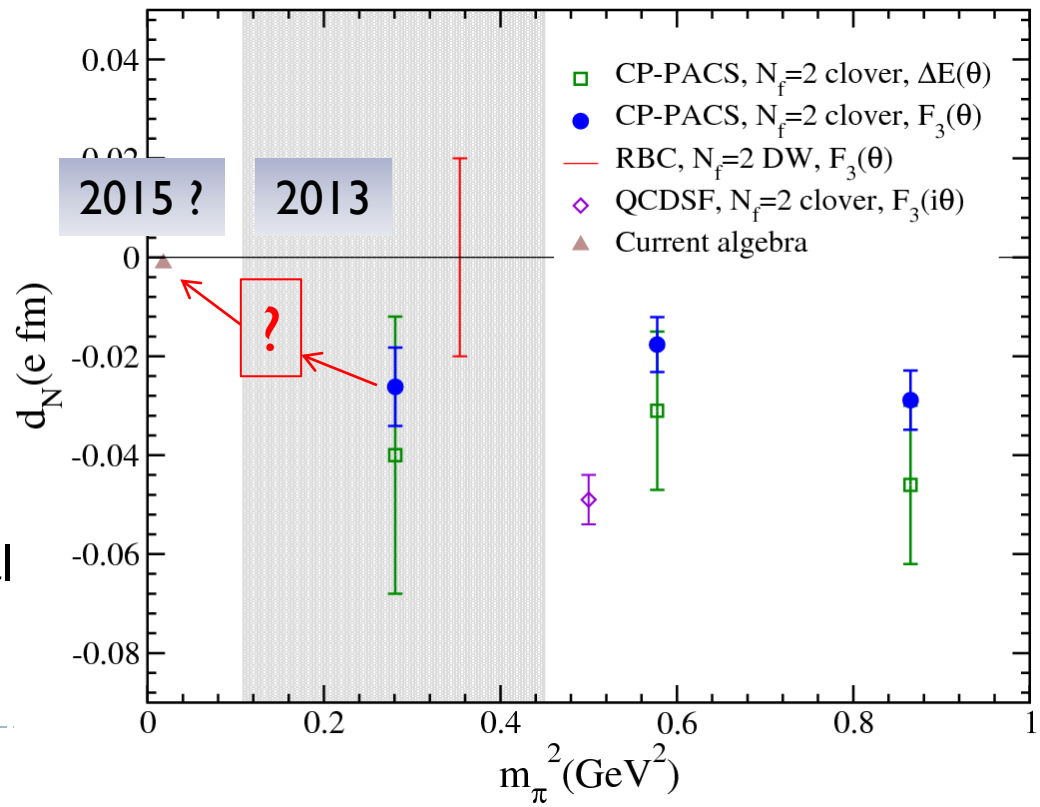
Prospects:

- 2012 -- 2013

  Continuum limit using
  24 cube Iwasaki,
  32 cube Iwasaki,
  estimate of error budget

- 2014 -- 2015 ??

  10 % accuracy ? at physical point



Summary

- Nucleon (neutron and proton) EDM in lattice QCD
  - Large statistical error is still problem.
  - Need the new algorithm (LMA/AMA).
  - Aim for less than 10% statistical error.
  - Systematic study of finite size effect, chiral behavior, …
  - Other source $\not{CP}$ effect

Thank you.

Backup

Error reduction techniques

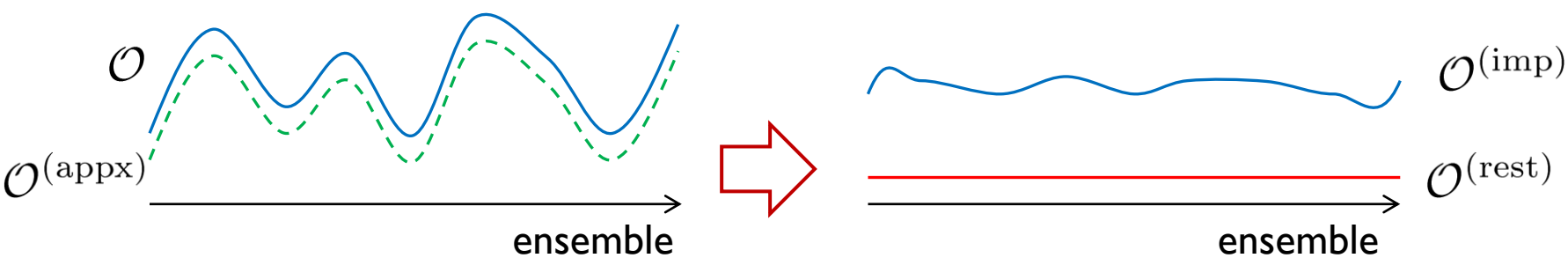
RBC in prep.

- Covariant approximation averaging (CAA)
  - For original observables O, (unbiased) improved estimator

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G}\sum_{g\in G}\mathcal{O}^{(\mathrm{appx}),g}, \quad \mathcal{O}^{(\mathrm{rest})} = \mathcal{O} - \mathcal{O}^{(\mathrm{appx})}$$

which satisfies <O> = <O^{imp}> if approximation is covariant under lattice symmetry g, and error becomes $\boxed{\mathrm{err}^{\mathrm{imp}} \simeq \mathrm{err}/\sqrt{N_G}}$

  - Ideal approximation



- Ignoring the error from O$^{(\mathrm{rest})}$
- There may be many candidates of O$^{(\mathrm{appx})}$ e.g. LMA, heavy mass, …
- The cost of approximated observable need to be smaller than the original.

Examples of CAA

- Lowmode averaging (LMA)

Guisti et al.(04), Neff et al.(01), DeGrand et al. (04)

  - Using lowlying eigenmode of Dirac operator to approximate propagator:

$$\mathcal{O}^{(\mathrm{appx})} = \sum_{\lambda}^{N_\lambda} \mathcal{O}_\lambda^{\mathrm{low}}$$

where N_λ is number of lowmode computed by Lanczos.

Except for computational cost of eigenmode, Cost(LMA) $\simeq$ 0, but approximation is only lowmode part (long distance contribution).

- All-mode averaging (AMA)
  - Using sloppy CG (loose stopping condition),

$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}^{\mathrm{sloppy}}$$

If stopping cond. is 0.003, Cost(AMA) $\simeq$ Cost(CG)/50(without deflation).

Approximation becomes better than LMA for other than lowmode dominanted observables (nucleon, finite momentum hadron, …).

Examples of Covariant Approximations

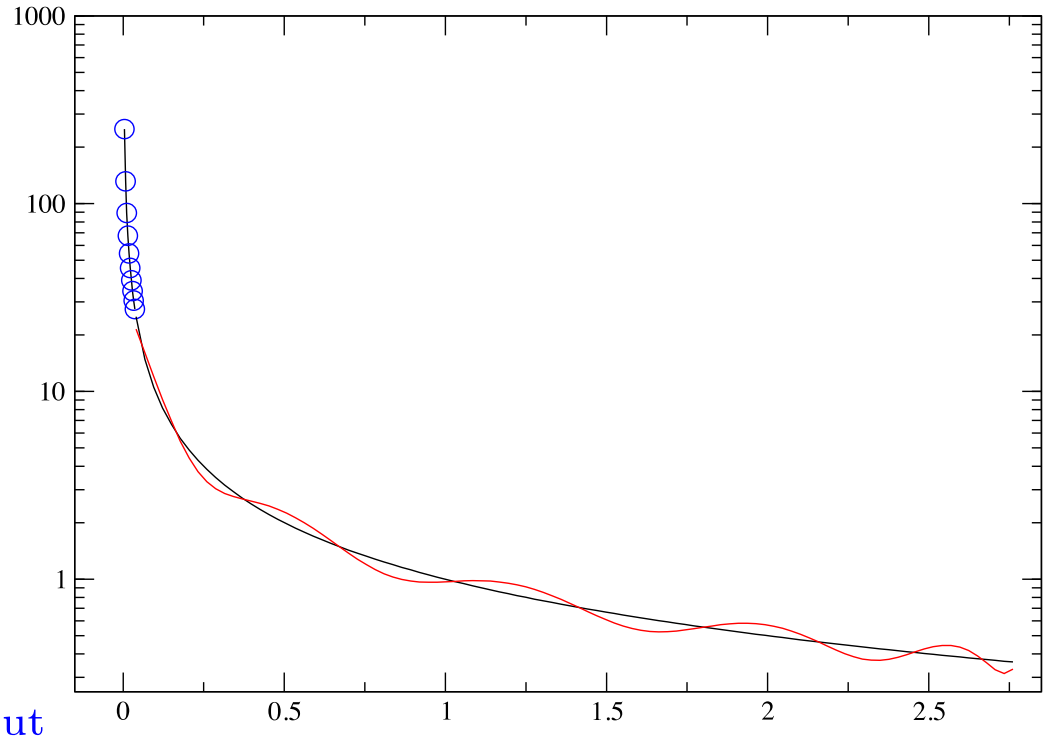
- All Mode Averaging
  AMA
  Sloppy CG or Polynomial approximations

$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],$$

$$S_l = \sum_\lambda v_\lambda f(\lambda) v_\lambda^\dagger,$$

$$f(\lambda) = \begin{cases} \frac{1}{\lambda}, & |\lambda| < \lambda_{\mathrm{cut}} \\ P_n(\lambda) & |\lambda| > \lambda_{\mathrm{cut}} \end{cases}$$

$$P_n(\lambda) \approx \frac{1}{\lambda}$$



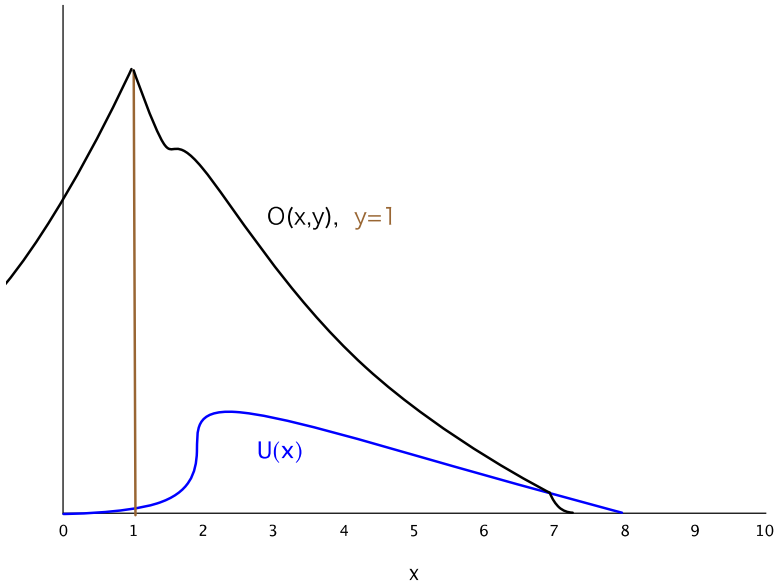
accuracy control :

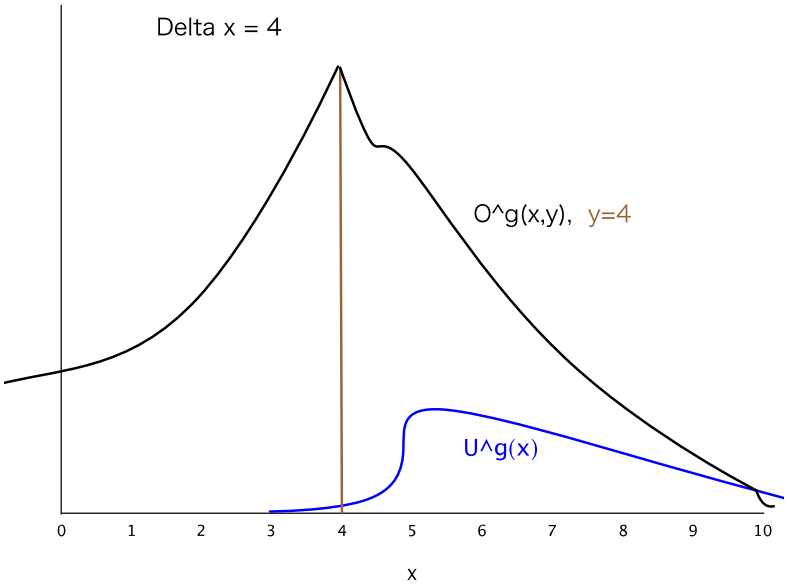
- low mode part : # of eig-mode
- mid-high mode : degree of poly.

Covariant approximation

Izubuchi, USQCD meeting 2012

- $O^{(appx)}$ needs to be precisely (to the numerical accuracy required) covariant under the symmetry of lattice action to avoid systematic errors.



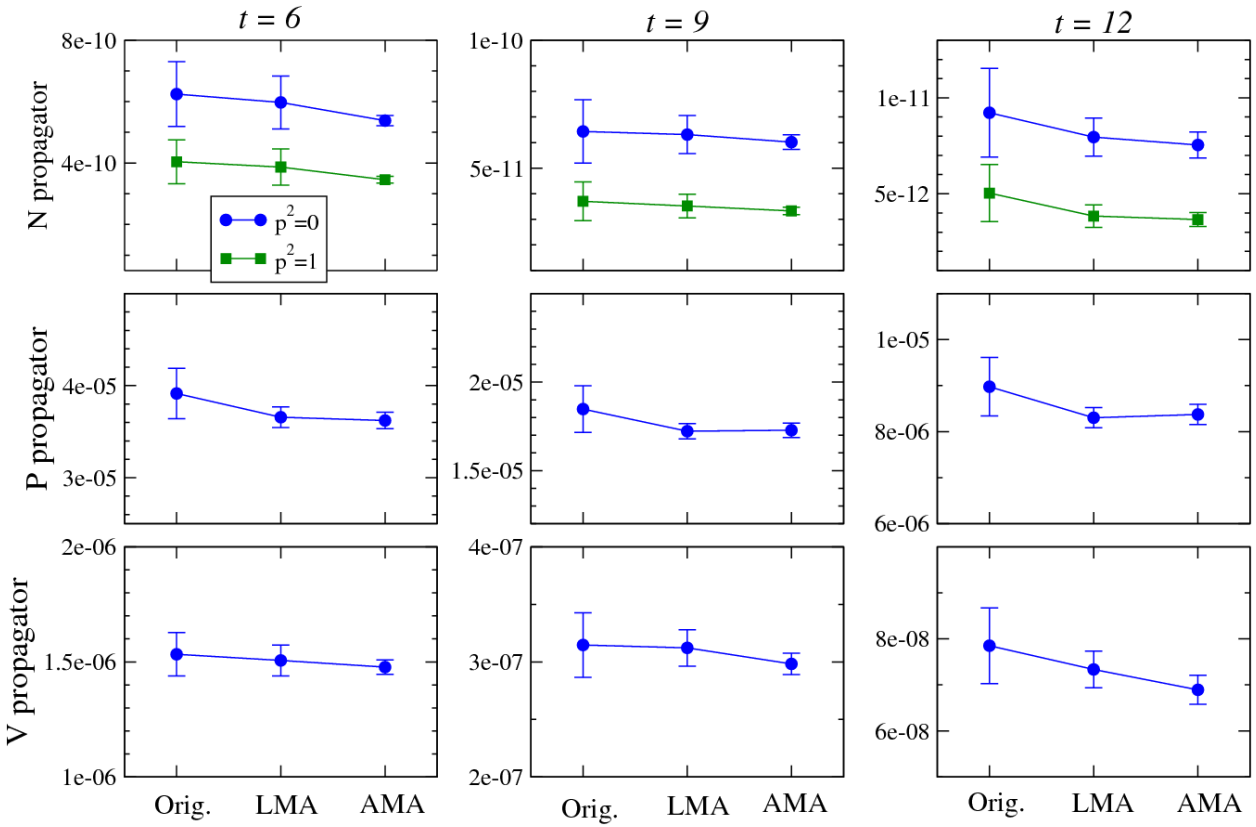


One should check in the code using explicitly shifted gauge configuration

Comparison between LMA/AMA

RBC in prep.

- Preliminary result
  - 8 configs, Gaussian smearing, $N_G = 2^3 \times 4 = 32$ sources, $24^3 64 \times 16$ DWF

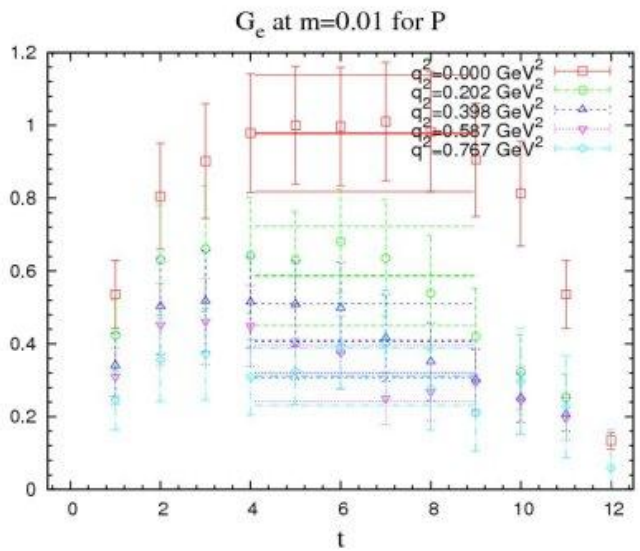


- t = 6:
  Error in AMA is actually reduced by factor 5 compared with orig. and LMA.

- t = 12
  Error in AMA/LMA is reduced by factor 3--4 compared with original.

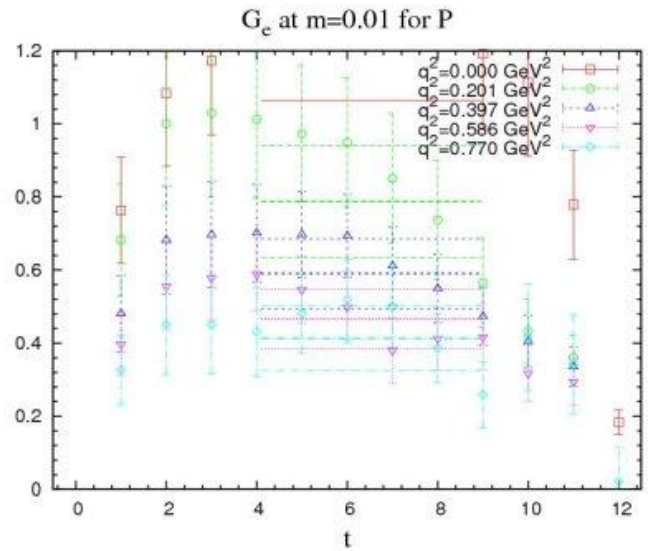
Comparison between LMA/AMA

- Very preliminary

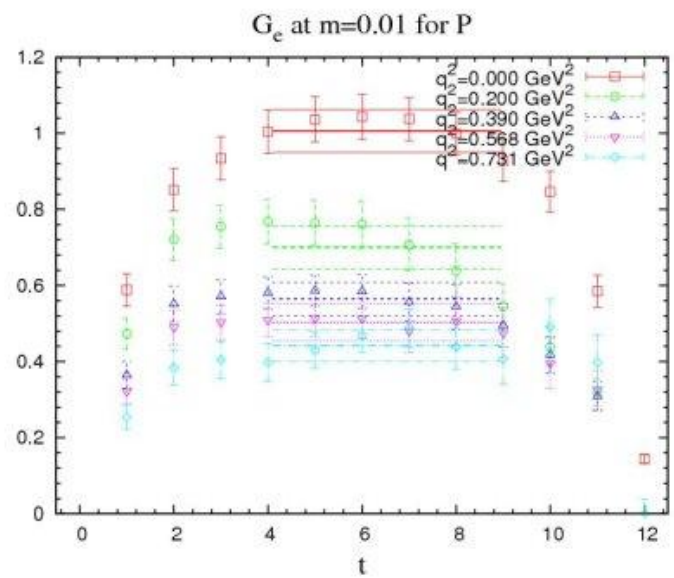


Proton Ge (Original)

Proton Ge (LMA)



Proton Ge (AMA)



Electric dipole moment (EDM)

- P(arity), T(ime reversal)[=CP] symmetry breaking

EDM:

$$\vec{D} = d\vec{S}, \quad \Delta H = \vec{D} \cdot \vec{E}$$

under discrete symmetries, spin and E have different behavior

$$\begin{aligned} \mathrm{P}: &\quad \vec{E} \to -\vec{E} \quad \vec{S} \to \vec{S} \\ \mathrm{T}: &\quad \vec{E} \to \vec{E} \quad \vec{S} \to -\vec{S} \end{aligned}$$

If $d \neq 0$,

$$\mathrm{P}, \mathrm{T} : \Delta H \to -\Delta H$$

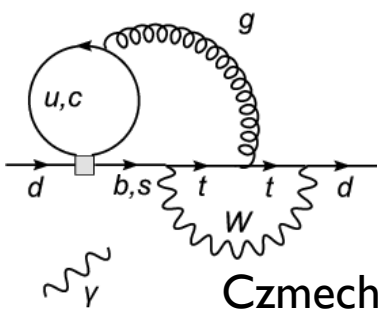
Non-vanishing EDM is a signal of the P, CP violation.

- In EW P, CP violation following Kobayashi-Maskawa mechanism.
- In QCD, it is natural to exist but there has been no signal the breaking would be also.

CP symmetry breaking in the SM

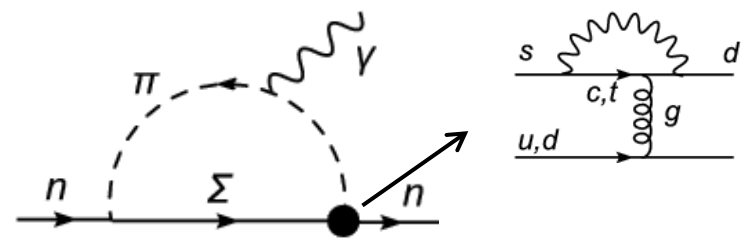
- Contribution to EDM from weak interaction is very small
  - Vanishing 1-loop (no Im part), 2-loop diagram
  - Three-loop order(short) and pion loop correction (long):

Short distance



Czmechi, Krause (1997)

Long distance



Khriplovich, Zhitnitsky (1982)

$$d_N^{\mathrm{KM\,short}} \sim \mathcal{O}(\alpha_s G^2) \sim -10^{-34}\,\mathrm{e\cdot cm} \qquad d_N^{\mathrm{KM\,long}} \sim 10^{-30\sim-33}\,\mathrm{e\cdot cm}$$

$$\Rightarrow d_N^{\mathrm{KM}} = d_N^{\mathrm{KM\,short}} + d_N^{\mathrm{KM\,long}} \simeq 10^{-30\sim-33}\,\mathrm{e\cdot cm}$$

which is the 6-order magnitude below the exp. upper limit: $|d_N^{\mathrm{exp}}| < 2.9\times 10^{-26}\,\mathrm{e\cdot cm}$

Spectrum method

Aoki-Gocksch(89), ES(06,07), Horsley(07), QCD-SF(08)

- Energy difference between nucleon spin
  - Energy eigenvalue if θ exist in the background

$$\mathcal{E}_{+S_3}(\vec{E}) - \mathcal{E}_{-S_3}(\vec{E}) = 2d_N\bar{\theta}S_3E_3 + \mathcal{O}(E^2, \bar{\theta}^2, E\bar{\theta})$$

in the case of $\vec{E}(t) = (0, 0, E_3)$

  - 2-pt function provides EDM as exponents

$$\langle N(t)\bar{N}(0)\rangle_{\theta,E}^{\text{upper}} \simeq Z(1 + A_N(E^2)\theta E_3)\exp(-m_N t - d_N\theta E_3 t/2)$$

$$\langle N(t)\bar{N}(0)\rangle_{\theta,E}^{\text{lower}} \simeq Z(1 - A_N(E^2)\theta E_3)\exp(-m_N t + d_N\theta E_3 t/2)$$

d_N is given by fitting with the above asymptotic function.

  - Two reweighting method

$$\langle N\bar{N}\rangle_{\theta,E} = \langle N\bar{N}(Ue^{q_e E})e^{i\theta Q}\rangle$$ Eucleadian E, real θ

$$= \langle N\bar{N}(Ue^{iq_e E})e^{\theta m_q\bar{q}\gamma_5 q}\rangle$$ Minkowski E, imaginay θ