Lattice-QCD calculations for EDMs

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CP symmetry breaking

- Baryogenesis [Cirigliano's talk]
- SM & EW [Pospelov's talk]
  - It has been known the CP violation occurs by the phase of CKM matrix
  - K, D, B meson decay via direct and indirect CP violation
  - Contribution to EDM is very tiny, $\Rightarrow d_N^{\mathrm{KM}} \simeq 10^{-30\sim-33}\,\mathrm{e}\cdot\mathrm{cm}$

    about 6-orders magnitude below the exp. upper limit:
- QCD [Ritz, Mereghetti's talks] $|d_N^{\mathrm{exp}}| < 2.9 \times 10^{-26}\,\mathrm{e}\cdot\mathrm{cm}$
  - θ term in the QCD Lagrangian:

$$\boxed{\mathcal{L}_\theta = \bar{\theta}\frac{1}{64\pi^2}G\widetilde{G}, \quad \bar{\theta} = \theta + \arg\det M}$$

renormalizable and CP-violation comes due to topological charge density.

  - EDM experiment provides very strong constraint on $\bar{\theta} < 10^{-9}$

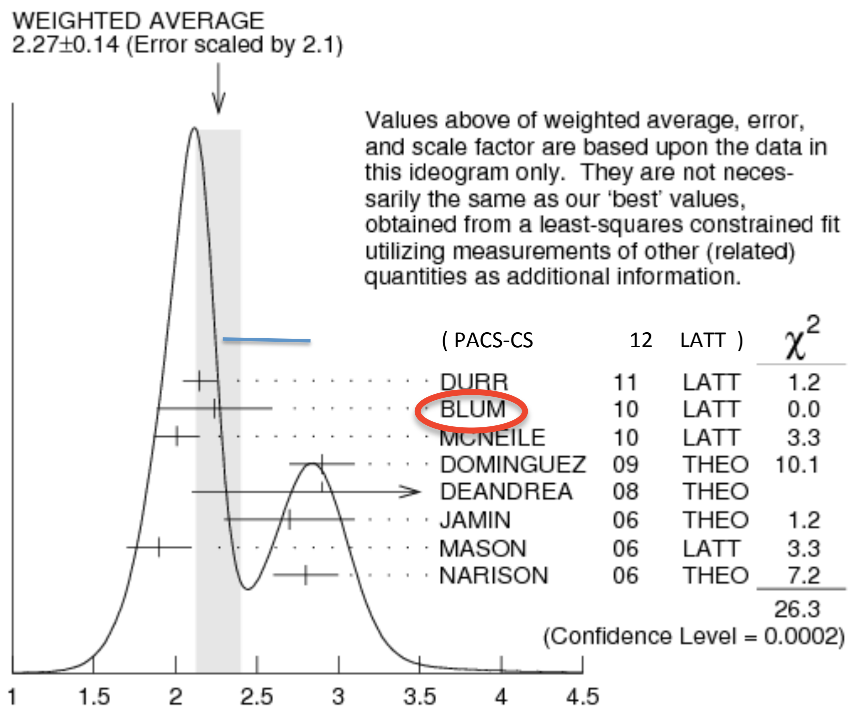
    ⇒ θ and $\arg\det$ M need to be unnaturally canceled !

    strong CP problem, unless massless quark(s)

m(up) = 0 ?

T. Blum et al. Phys.Rev. D82 (2010) 094508
T. Ishikawa et al. Phys.Rev.Lett. 109 (2012) 072002

- input : experimental masses on $\pi^{\pm}$, $K^{\pm}$, K^0
- Lattice QCD+QED simulation to solve for masses of up, down, strange quarks taking into account quark's electric charges



PDG 2012 average

m(up) = 2.27± 0.14 MeV

MSbar at 2 GeV

CP symmetry breaking beyond the SM

- Possible higher dimension operators
  [Ramsey-Musolf's talk]
  - Effective Hamiltonian with higher dimension than 4

$$H_{\mathrm{C}\not{P}} = \sum_k C_k(\mu)\,\mathcal{O}_k$$

$$\begin{aligned} \mathcal{O}_{\mathrm{qEDM}} &= d_q \bar{q}(\sigma\cdot F)\gamma_5 q && \text{: Quark-photon} \\ \mathcal{O}_{\mathrm{cEDM}} &= d_q^c \bar{q}(\sigma\cdot G)\gamma_5 q && \text{: Quark-gluon} \\ \mathcal{O}_{\mathrm{Weinberg}} &= d^G G G \widetilde{G} && \text{: Pure gluonic} \\ &\vdots \end{aligned}$$

Quark EDM contribution will play a significant role in neutron EDM.

Phenomenological estimate:

Hisano, Shimizu (04), Ellis, Lee, Pilaftsis (08), Hisano, Lee, Nagata, Shimizu (12)

$$\begin{aligned} d_N &= d_N^{\mathrm{QCD}}\bar{\theta} + d_N(d_q, d_q^c) + d_N(d^G) \\ &\sim 10^{-17}[\mathrm{e}\cdot\mathrm{cm}]\bar{\theta} + (1.4\text{--}0.47)d_d - (0.12\text{--}0.35)d_u + O(10^{-2})d_q^c \\ &\sim O(10^{-25}\text{--}10^{-27}) \end{aligned}$$

⇒ parameter space of (θ, d_q, d_q^c, …) using N, P, atom EDMs

Constraint on nEDM

- The present and future experiments are aiming to check/exclude of MSSM

[Kirch's talk]

pEDM @ BNL

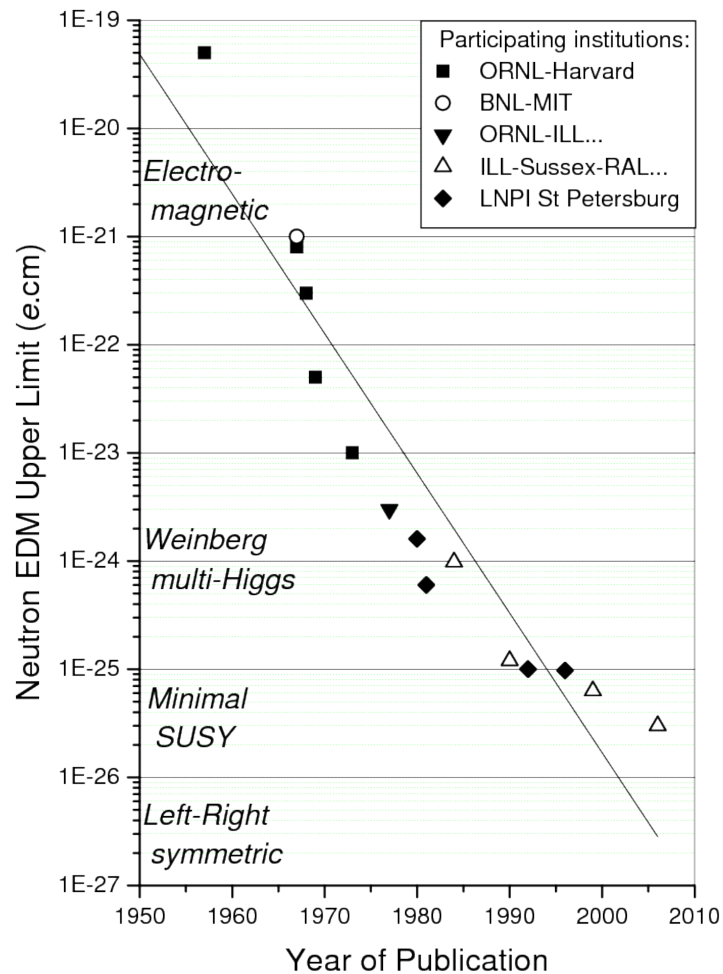
nEDM @ ONL, PSI, ILL, J-PARC, TRIUMF ,FNAL, FRM2, …

charged hadrons @ COSY

⇒ a sensitivity of $\underline{10^{-29}\ e\cdot cm}$!

- Current theoretical bound is based on quark model,

non-perturbative computations of EDM $d_n(\theta, d_q, d_q^c, \ldots)$ are necessary



Harris, 0709.3100

1. Introduction
2. EDM on Lattice (intro LGT)
3. source of CP violation on lattice
   - Reweighting
   - imaginary theta
4. Measurements of EDM
   - External Electric field method
   - Form Factor F3
5. Preliminary results from chiral Lattice (DWF)
6. Conclusion

What lattice QCD can do for EDMs

- In principle
  - Direct estimate of neutron and proton EDM from θ term, and BSM higher dim. CPv operators
  - CP violating π-N-N coupling
  - Matrix elements of higher dimension operators → SUSY constraint

$$\boxed{\langle N|\bar{q}\gamma_5\sigma\cdot Fq|N\rangle,\ \langle N|\bar{q}\gamma_5\sigma\cdot Gq|N\rangle, \langle N|GG\tilde{G}|N\rangle,\cdots}$$

- In practice there are some difficulties
  - <u>Statistical error</u>

    Source of CP violation comes from gauge background (topological charge, sea quark) which is intrinsically noisy.

    Disconnected diagram is necessary because of flavor singlet contraction.

  - <u>Systematic error</u>

    Chiral behavior is important, dN ~ O(mq).
    Volume effect may be significant

    ChPT may help

EDM from (dynamical) lattice QCD

- 1990 Aoki-Gocksch (BNL), quenched Wilson quark, Electric field

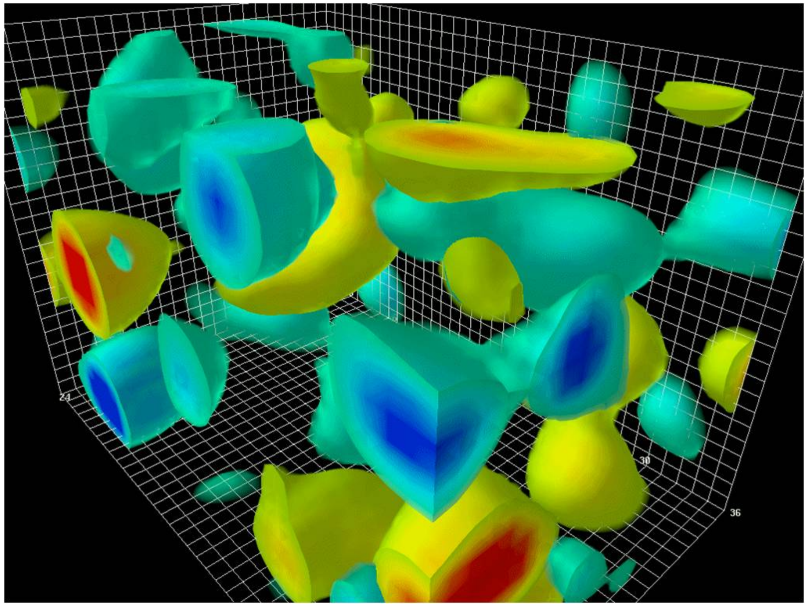
- 2006 Berruto et al. (RBC), Nf=2 chiral quark (DWF) F_3, Mπ~500 MeV
- 2006-08 Shintani et al (CP-PACS) Nf=2 Wilson-clover quark, F_3 & Electric field, Mπ~500 MeV
- 2008 TI + QCDSF Nf=2 Wilson-clover quark imaginary θ, F_3 & Electric field, Mπ~600 MeV

<u>on-going computations</u>

- 2009- QCDSF
- 2011- Shintani et al (RBC) Nf=2+1 chiral quarks (DWF) Mπ ~ 300 – 180 MeV
- 2012- Bhattacharya et al. BSM operators, clover-Wilson quark

EDM from lattice QCD

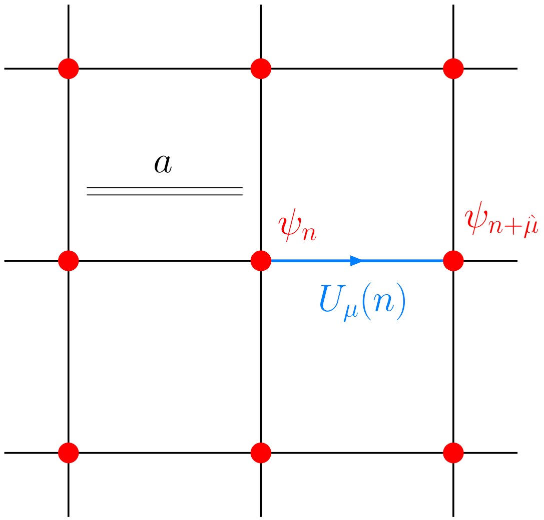
- Three ingredients
  - QCD vacuum samples
  - source of CP violation
    - *Reweighting from CP symmetric vacuum*
    - *Dynamical simulation*
  - Polarized Nucleons



- Two methods
  - Measure a spin splitting of energy
  - Form Factor

Lattice Gauge Theory

- Analysis of Quantum Field Theory such as Quantum Chromo Dynamics, needs non-perturbtive calculation
  ψ (x), A_μ(x), x : continuous
  quantum divergences needs
  regularization and renormalization



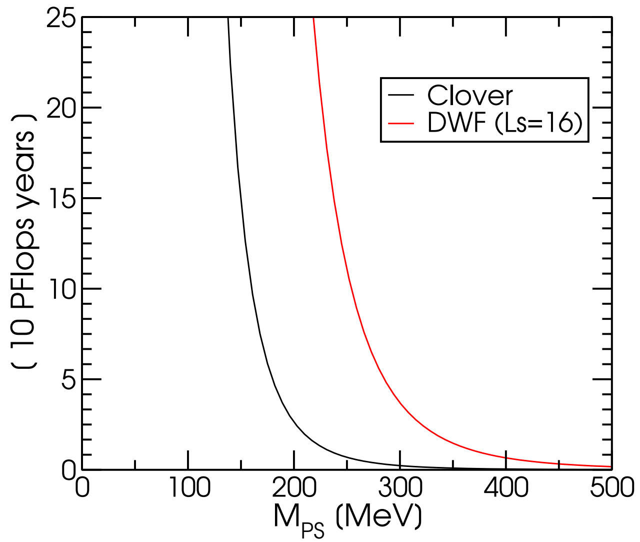
- Discretize Euclidean space-time
- lattice spacing : a ~ 0.15—0.04 fm
  (UV cut-off |p| < π / a ~ 1.3 GeV - 4 GeV)
- ψ (n) : Fermion field (Grassmann number)
- U_μ(n) : Gauge field (link variable)

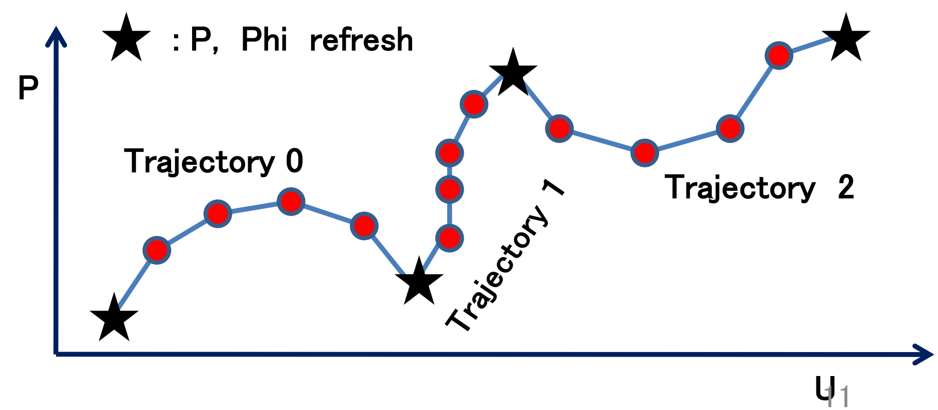
1. Accumulate samples of vacuum, typically O(10)—O(1,000) files of gauge configuration U_μ(n) on disk.
2. Then measure physical observables on the vacuum ensemble

$$\langle \mathcal{O} \rangle = \int \mathcal{D}U_\mu \mathrm{Prob}[U_\mu(x)] \times \mathcal{O}[U_\mu(x)]$$

Generation of LGT vacuum

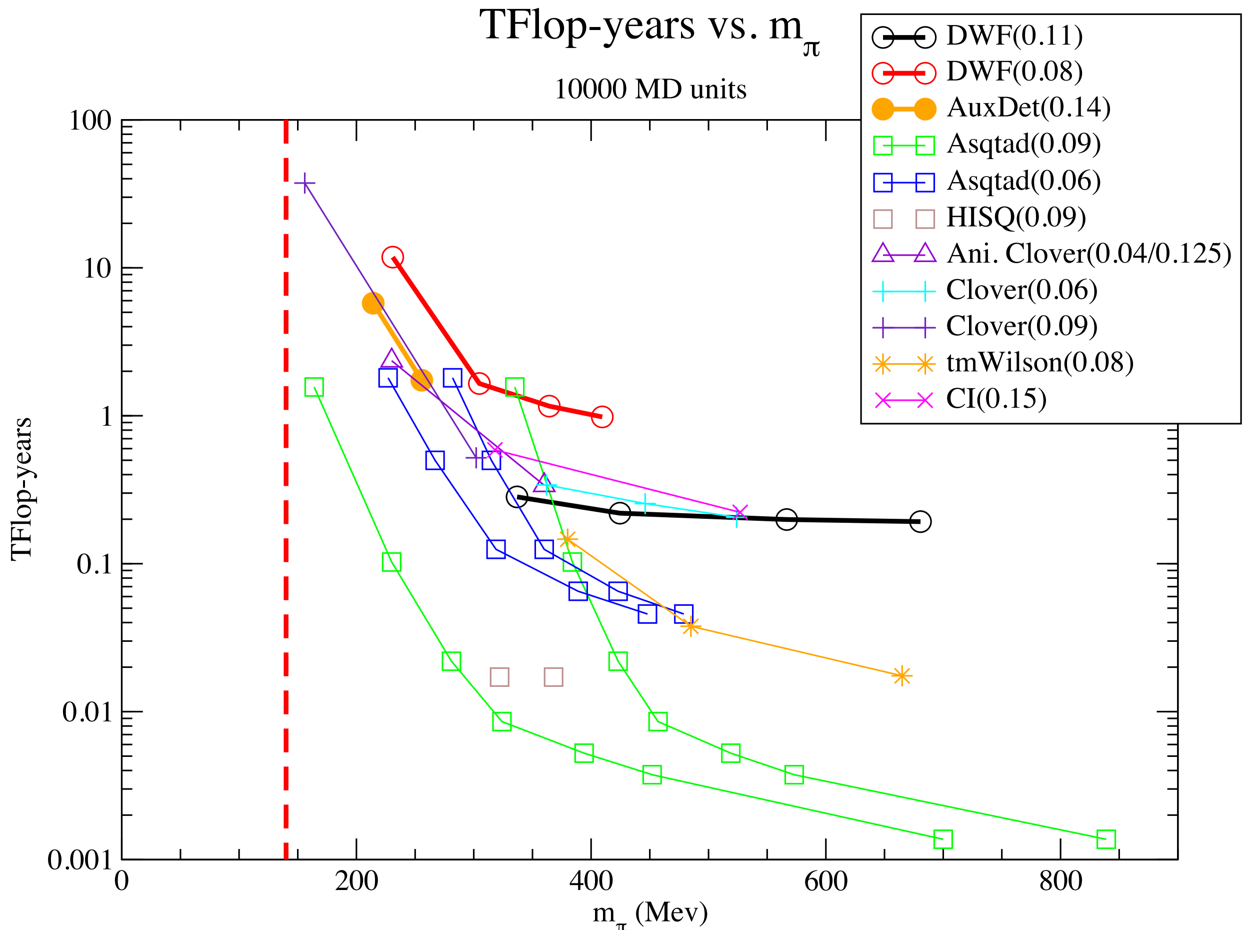
- Fermion (Quark) is not a floating point number (Grassmannian)→ Integrate out
- Dirac determinant of N_F quarks, $\det[D]^N$, is then stochastically evaluated by Hybrid Monte Carlo (HMC) algorithms (lighter quark, more FLOPS)





Berlin Wall torn down

- New developments for HMC [Hassenbusch, Lüscher, Clark-Kennedy....]
- TFLOPS*years vs M_π [MeV] 10K MD unit L = max(m_π/4, 2.5 fm)



CP violation on lattice : Reweighting

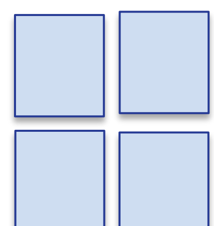
- Source of CP violation (Θ in our case)

$$S_\theta = i\frac{\theta}{32\pi^2}\int d^4x\ \mathrm{tr}[\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}G^{\mu\nu}]$$

$$= i\theta\ Q_{\mathrm{top}}$$

- Topological charge is measured either by gluonic observable GG^* or by counting zero mode of chiral fermions

$$Q \rightarrow \Sigma\ G_{12}\,G_{34}, \quad G_{\mu\nu} = \mathrm{Im}$$

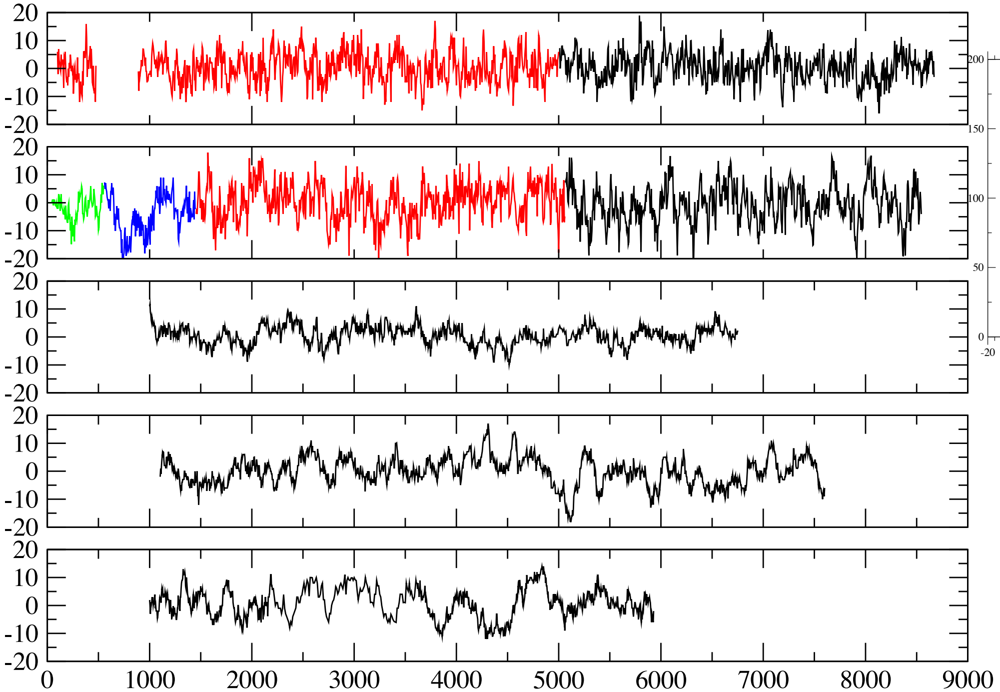


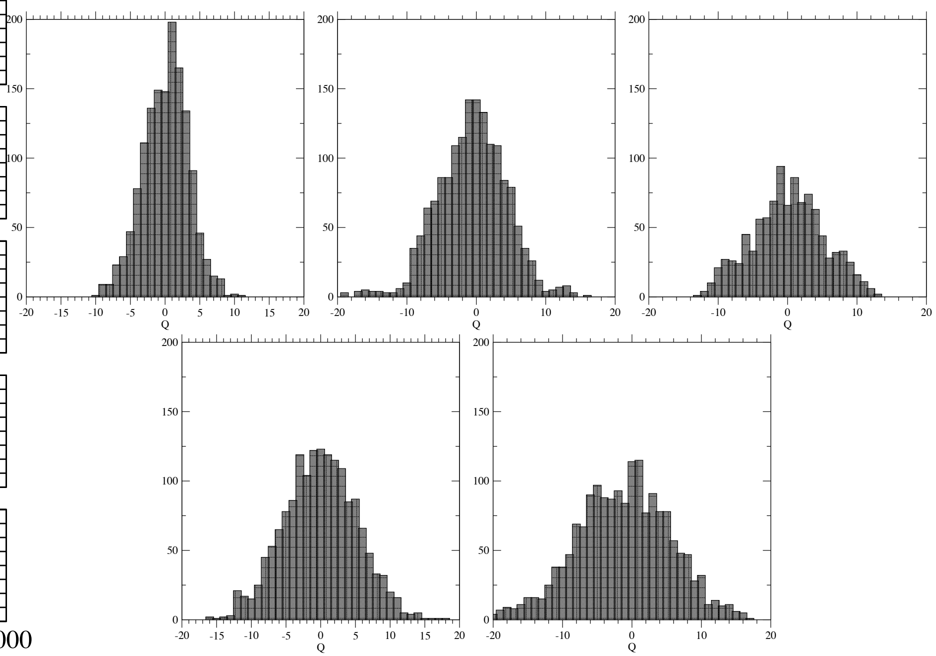
- Θ=0 lattice QCD ensemble is generated, then each sample of QCD vacuum are reweighted using topological charge

$$\langle \mathcal{O} \rangle_\theta = \langle \mathcal{O} e^{i\theta Q} \rangle_{\theta=0}$$

Qtop on lattice (Θ=0)

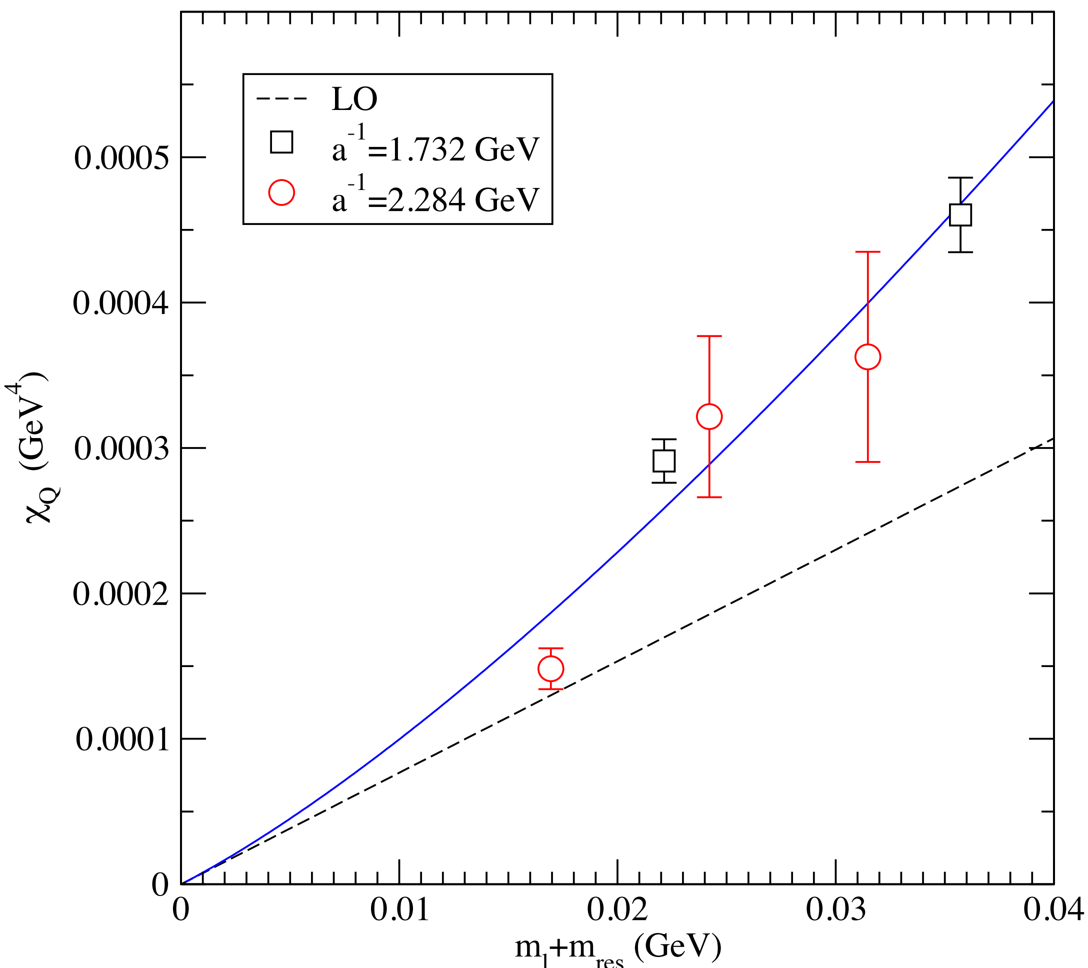
- Qtop history in simulation Nf=2+1 DWF, [RBC/UKQCD]
- 1/a= 1.73, 2.28 GeV
- mps =290 – 420 MeV





Qtop susceptibility

- NLO ChPT



Qtop on lattice (Imaginary θ)

TI(07), Horsley et al. (08)

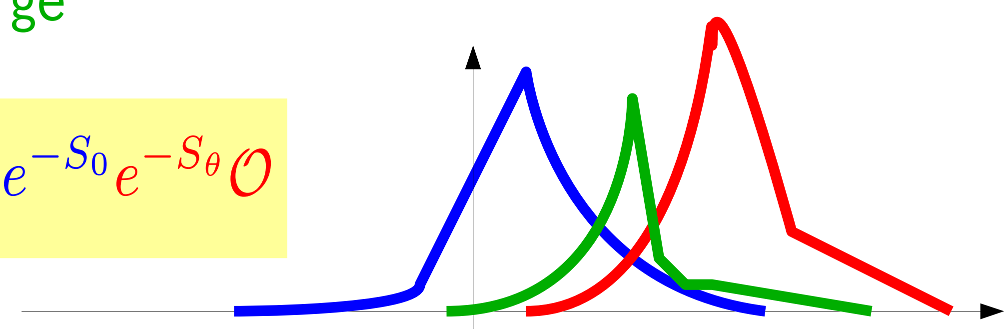
- S_θ = i Q_{top} : sign problem, Prob[Uμ] is not positive

  → Reweighting method $\boxed{\langle \mathcal{O} \rangle_\theta = \langle \mathcal{O} e^{i\theta Q} \rangle_{\theta=0}}$

- Alternatively, analytically continued to pure imaginary,
  $\theta \rightarrow i\theta^I$
  - Would be beneficial in terms of efficiency in Monte Carlo important sampling

  Concentrate on points (configuration) where the integrand is large

$$\langle \mathcal{O} \rangle = \int [DU] e^{-S_0} e^{-S_\theta} \mathcal{O}$$



Simulation with imaginary θ

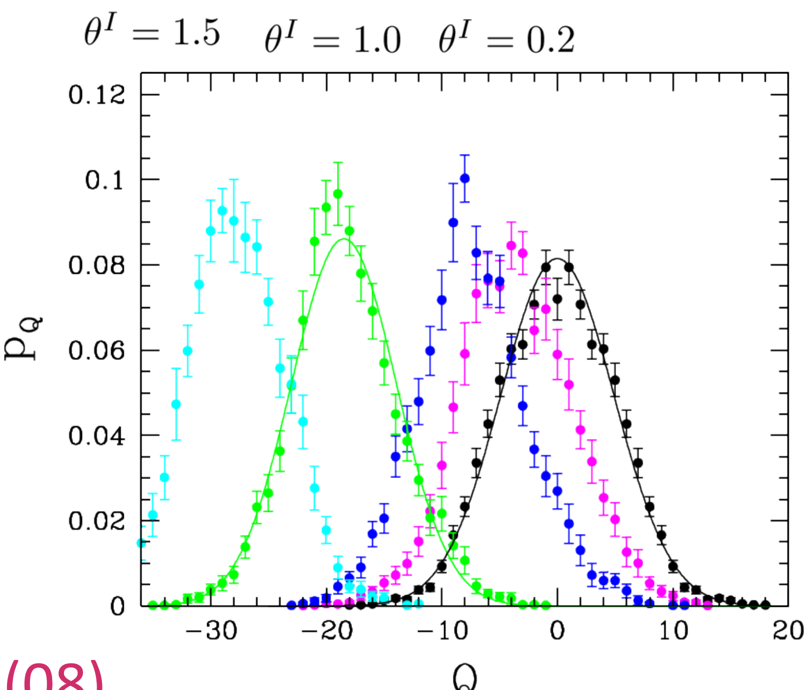
- Axial U(1) rotation to absorb into fermion's γ_5 mass term

$$\psi(x) \to e^{\theta\gamma_5}\psi(x)$$
$$\bar{\psi}(x) \to \bar{\psi}(x)e^{\theta\gamma_5}$$
$$\mathcal{D}\psi\mathcal{D}\bar{\psi} \to \mathcal{D}\psi\mathcal{D}\bar{\psi}S^{+S_\theta}$$

$$S_\theta = \frac{\theta}{32\pi^2}\sum G\tilde{G}(x)$$

$$\mathcal{L}_\theta = \frac{m\theta}{2}\bar{\psi}\gamma_5\psi$$

- Generate the QCD ensemble with imaginary θ
  - using Nf=2 clover ermion
  - Distribution of Q_top is shifted



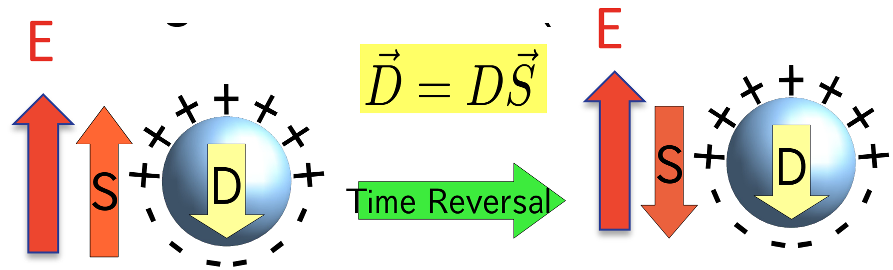
TI(07), Horsley et al. (08)

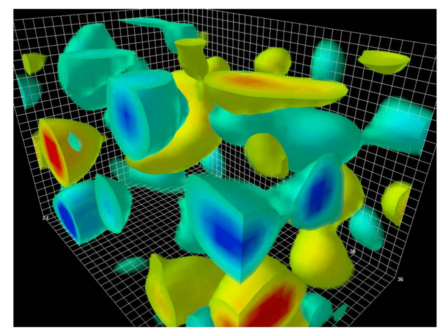
EDM Computations on Lattice

- Measure energies with external Electric field

$$\frac{\langle N_\uparrow(t)\bar{N}_\uparrow(t_0)\rangle}{\langle N_\downarrow(t)\bar{N}_\downarrow(t_0)\rangle} \to Ce^{\Delta M t}$$

$$\begin{aligned}\Delta M &= M_N(E,\uparrow) - M_N(E,\downarrow)\\ &= -2D_N(\theta)S\cdot E\end{aligned}$$

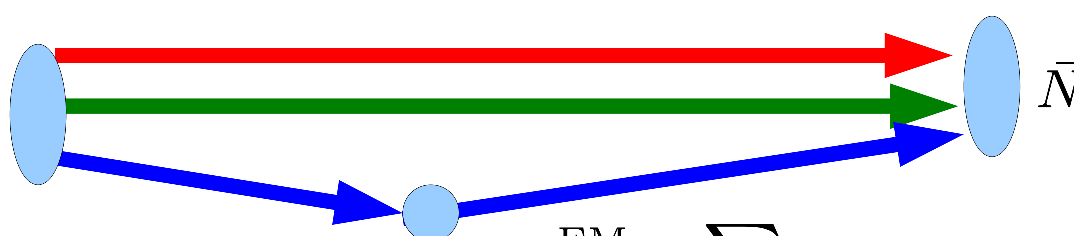




- Form factors

$$\langle N(p')|V_\mu^{\mathrm{EM}}(q)|N(p)\rangle = F_1(q^2)\gamma_\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N} + F_3(q^2)\frac{\sigma^{\mu\nu}q^\nu\gamma^5}{2m_N}$$

N



$$V_\mu^{\mathrm{EM}} = \sum_q e_q\bar{q}\gamma_\mu q$$

$$d_N = \lim_{Q^2\to 0} F_3(Q^2)/2m_N$$

External Electric field method

[E.Shintani et al. (06, 07)]

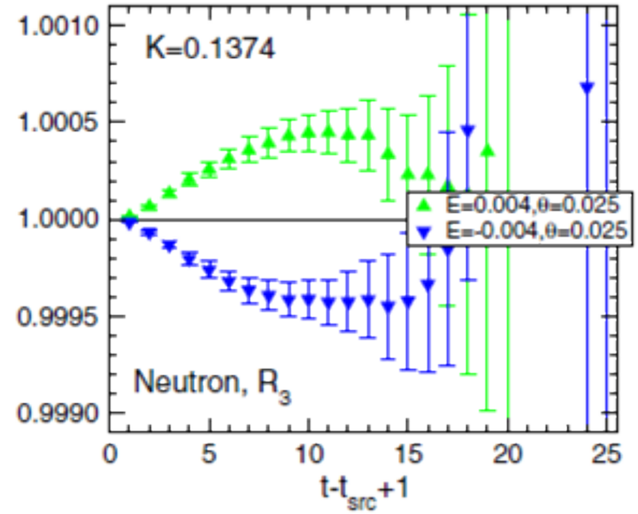
- Ratio of spin up and down

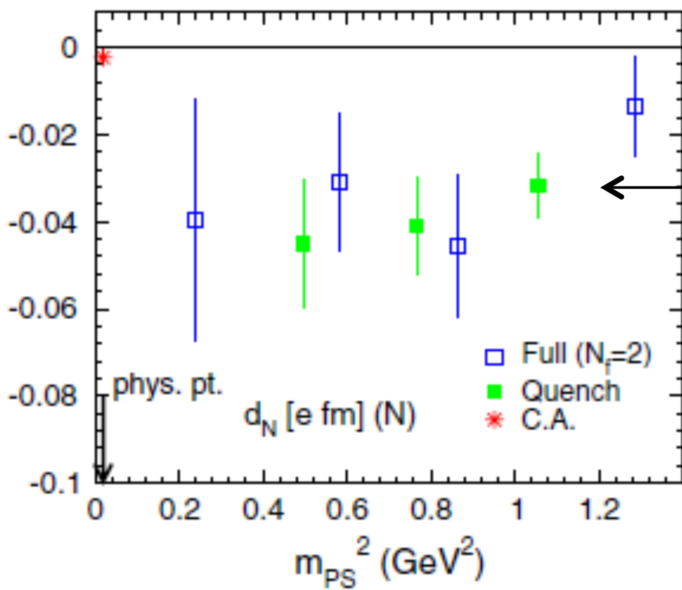
$$R_3 = \frac{\langle N(t)\bar{N}(0)\rangle^{\rm up}_{\theta,E}}{\langle N(t)\bar{N}(0)\rangle^{\rm down}_{\theta,E}} \simeq 1 + d_N E\theta t$$

Linear response, gradient is a signal of EDM.

- Reweighting works well for small real θ
- Temporal periodicity is broken by electric field.

  ⇒ additional systematic effects





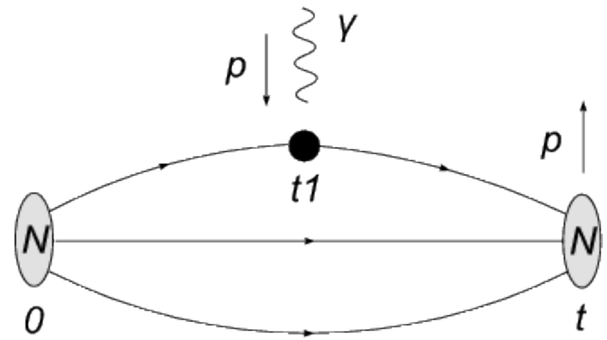
Full QCD with clover fermion:

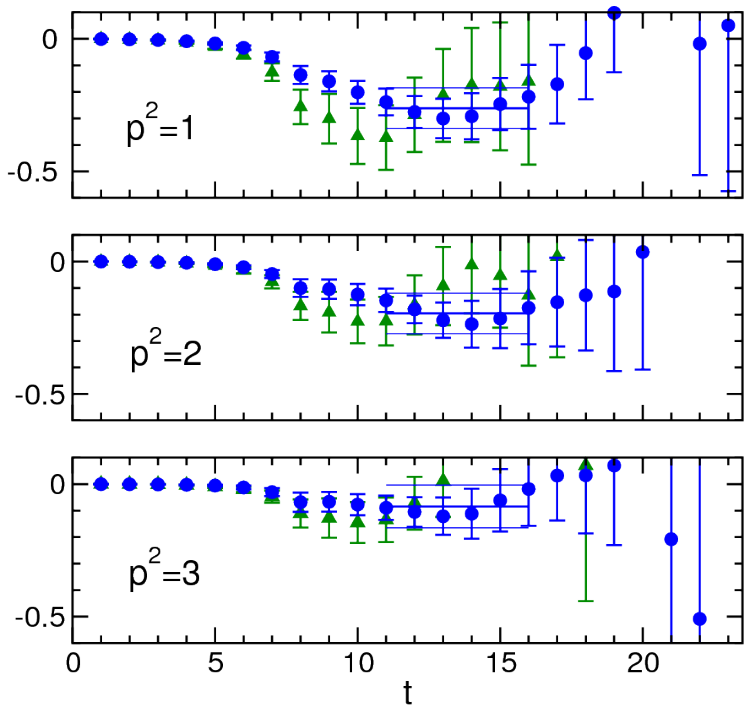
- There seems to be no significant difference between quench and full QCD for this heavy quark mass (pion mass >~ 500 MeV) .
- Statistical error is still large.
- Finite size effect from breaking of temporal periodicity is also significant

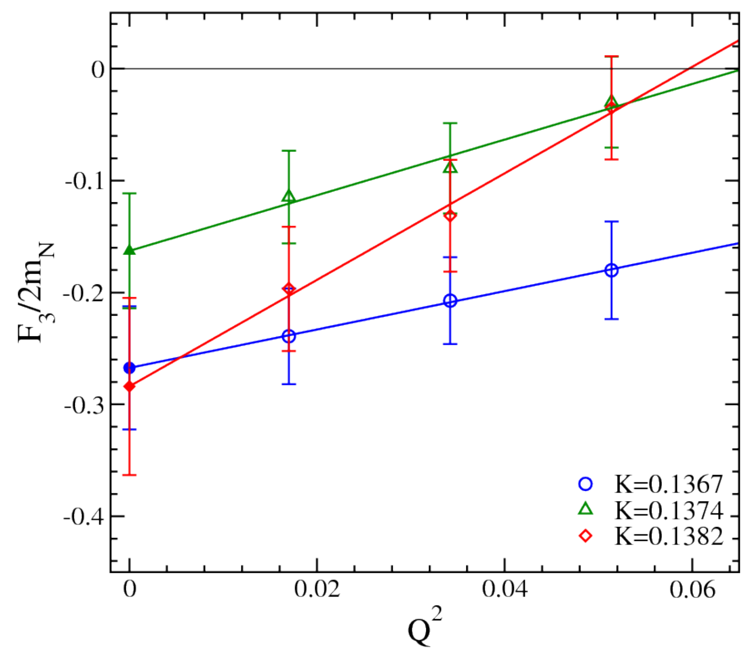
Form factor $F_3(q^2)$

F. Berruto et al. (05)
E.Shintani et al. (05, 08)

- Matrix element in θ vacuum
  - N_f=2 clover fermion
  - Size is $24^3 \times 48$ lattice (~2 fm^3)
  - Signal appears in 11 -- 16
  - $Q^2 \rightarrow 0$ limit is doable with linear func.







Imaginary θ

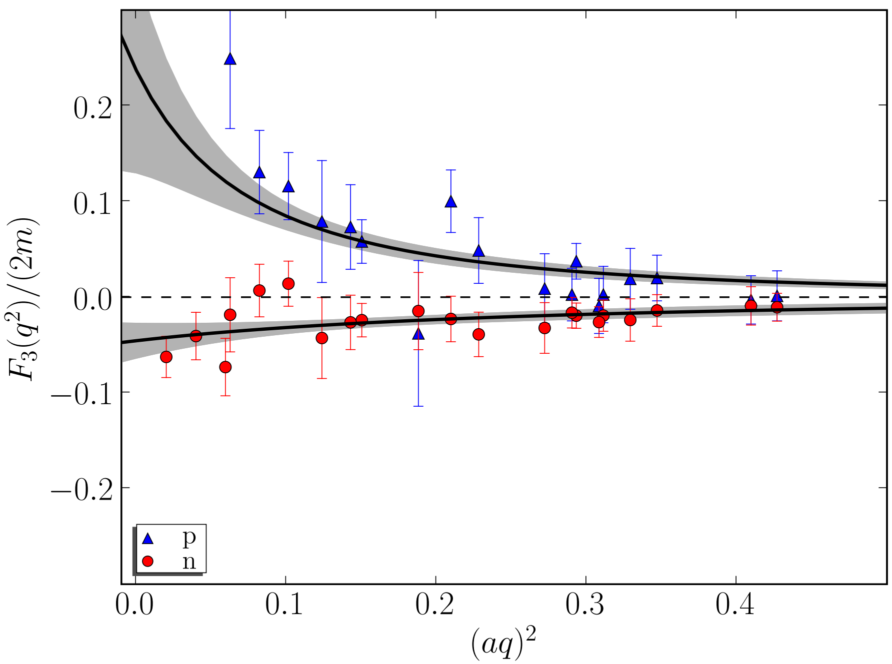
Izubuchi(07), Horsley et al. (08)

- Full QCD with Wilson fermion

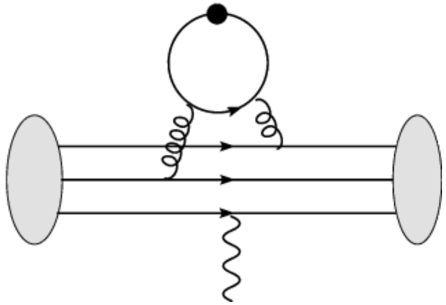
Nf=2, $16^3 \times 32$ lattice, m_π = 700 MeV

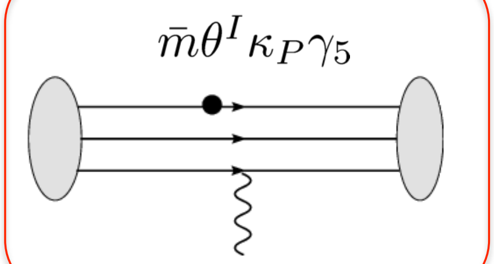
Fermionic insertion of imaginary theta:

$$\mathcal{L}_\theta = \bar{m}\theta^I \bar{q}\gamma_5 q/2$$



$\bar{m}\theta^I\gamma_5$





Effects from valence theta

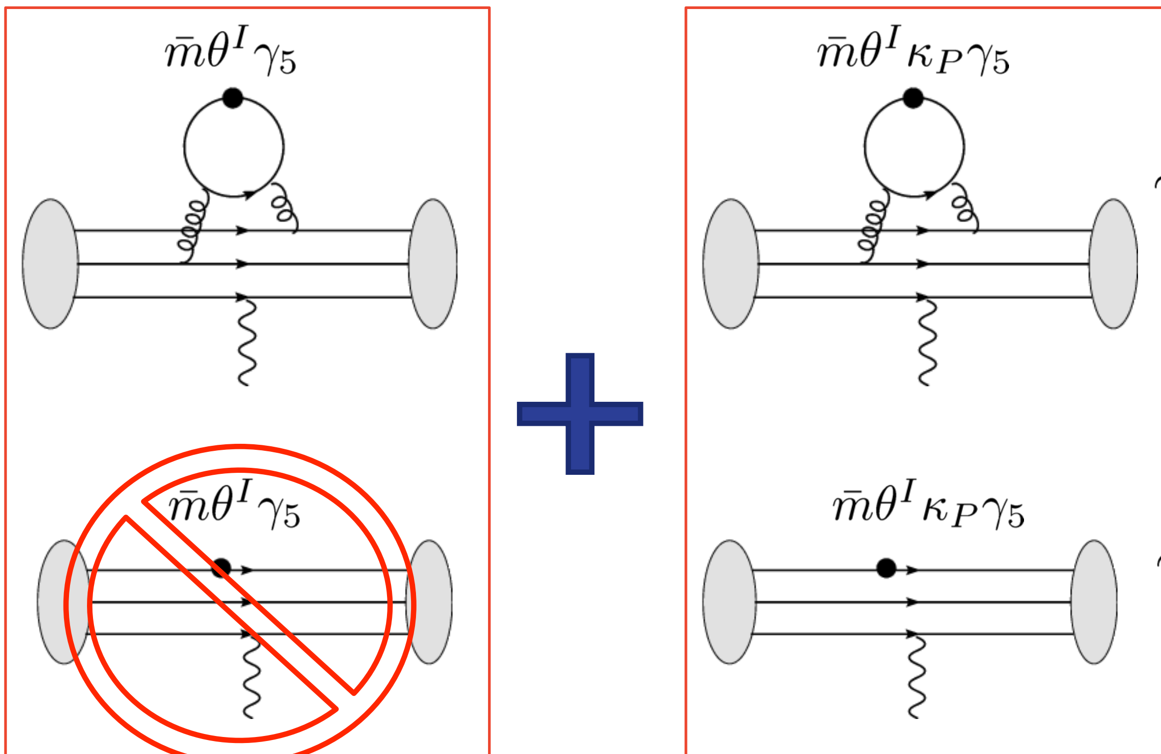
Quaenstion in Measurements in Imaginary θ

Izubuchi(07), Horsley et al. (08)

- Full QCD with Wilson fermion O(a) chiral breaking

Fermionic insertion of imaginary theta should be changed by Wilson term:

$$\mathcal{L}_\theta = \bar{m}\theta^I \bar{q}\gamma_5 q/2 \to \mathcal{L}_\theta^W = \bar{m}(1+\kappa_P)\theta^I \bar{q}\gamma_5 q,\ \kappa_P \sim \mathcal{O}(a) : \text{renom. const.}$$



$\sim \mathcal{O}(am_{\text{sea}}\alpha^2\theta^I)$

→ negligible

$\sim \mathcal{O}(am_{\text{val}}\theta^I)$

→ significant

Cf. discussion in Aoki, Gocksh, Manohar,Sharpe (1990)

Chiral symmetry & EDM

$$\boxed{\begin{aligned} q(x) &\to e^{i\gamma_5\theta} q(x) \\ \bar{q}(x) &\to \bar{q}(x) e^{-i\gamma_5\theta} \end{aligned}}$$

- Chiral symmetry is broken by lattice systematic error

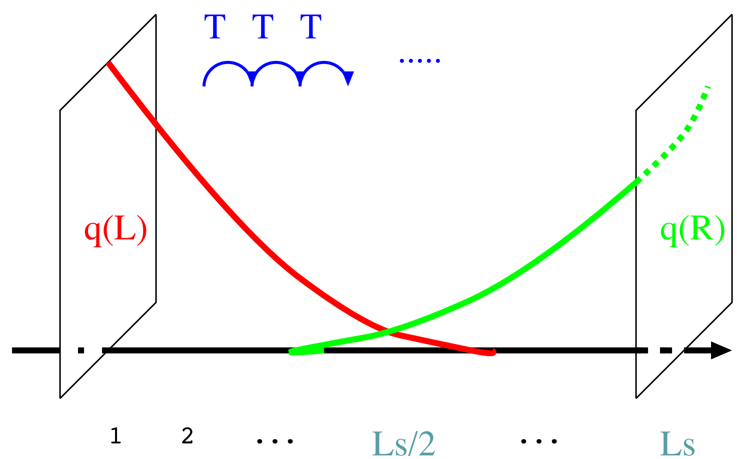
  for Wilson-type quarks, which has "wrong" Pauli term by O(a)

$$\boxed{\mathcal{L}_{\mathrm{Wilson}} = \mathcal{L}_{\mathrm{QCD}} + ca\bar{q}\sigma_{\mu\nu}\cdot F_{\mu\nu}q}$$

- CP violation from θ or other BSM operators introduce extra artificial CP violation in simulation.

- In fact, chiral rotation of valence quark is unobservable in continuum theory, and the EDM signal measured in Wilson quark due to valence quark's θ is unphysical, which should be carefully removed by taking continuum limit a → 0 [S. Aoki-Gockschu, Manohar, Sharpe et al. Phys.Rev.Lett. 65 (1990) 1092-1095 (1990)]

→ Our choice : chiral lattice quark called domain-wall fermions (DWF)
[97 Blum Soni, 99 CP-PACS, 00- RBC, 05 RBC/UKQCD...]



Ongoing and Near future plan

- Form factor in DWF configurations
  - Chiral symmetry on the lattice

    Reduction of systematic error coming from finite a
  - RBC/UKQCD collaboration

    Generate the ensembles including dynamical up-down, strange quarks
  - Large size and small mass

    Control the finite size and chiral extrapolation ($m_\pi \rightarrow m_\pi^{phys}$)

Lattice size	Physical size	Lattice spacing	L_s	Gauge action	Pion mass
$24^3 \times 64$	2.7 fm^3	0.114 fm	16	Iwasaki	315 -- 615 MeV
$32^3 \times 64$	2.7 fm^3	0.087 fm	16	Iwasaki	295 -- 397 MeV
$32^3 \times 64$	4.6 fm^3	0.135 fm	32	DSDR	171 -- 241 MeV
$48^3 \times 96$	5.5 fm^3	0.115 fm	8	Iwasaki	135 MeV

Difficulties in EDM calculations

- Very sensitive to chiral symmetry
- Intrinsically noisy, theta is only affecting to Nucleon observable indirectly, in contrast to the case of (Chromo) quark EDMs.
- To avoid the excited state contamination, one needs to separate Nucleons and the EM current.

But signal to noise degrates exponentially [Lepage]

$$S/N \sim C\exp[-t/\tau],$$

$$\tau = [m_N - 3/2m_\pi]^{-1} \approx 0.25\mathrm{fm}$$

[12 Blum, TI, Shintani]

AMA : Error reduction techniques

Covariant approximation averaging (CAA)

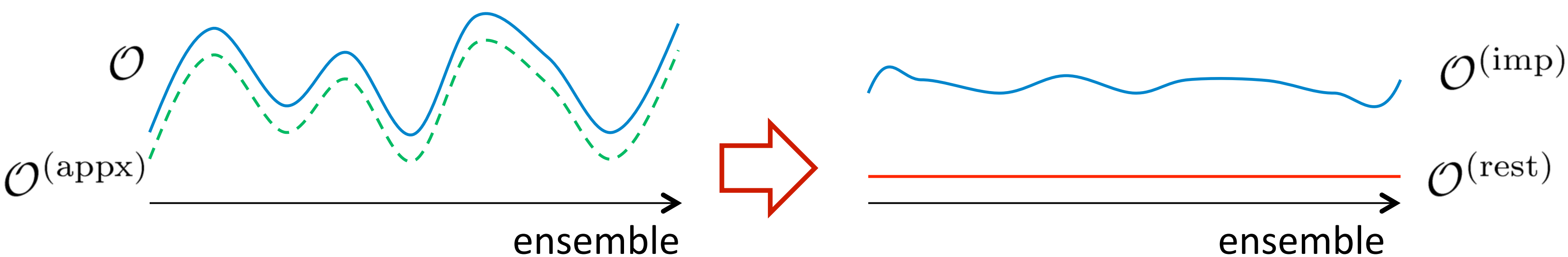
- For original observables $\mathcal{O}$, (unbiased) improved estimator

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G}\sum_{g\in G}\mathcal{O}^{(\mathrm{appx}),g}, \quad \mathcal{O}^{(\mathrm{rest})} = \mathcal{O} - \mathcal{O}^{(\mathrm{appx})}$$

which satisfies $\langle O \rangle = \langle O^{\mathrm{imp}} \rangle$ if approximation is covariant under lattice symmetry g, and error becomes

- Ideal approximation

$$\boxed{\mathrm{err}^{\mathrm{imp}} \simeq \mathrm{err}/\sqrt{N_G}}$$



- Ignoring the error from $O^{(\mathrm{rest})}$
- There may be many candidates of $O^{(\mathrm{appx})}$ e.g. LMA, heavy mass, …
- The cost of approximated observable need to be smaller than the original.

Examples of Covariant Approximations

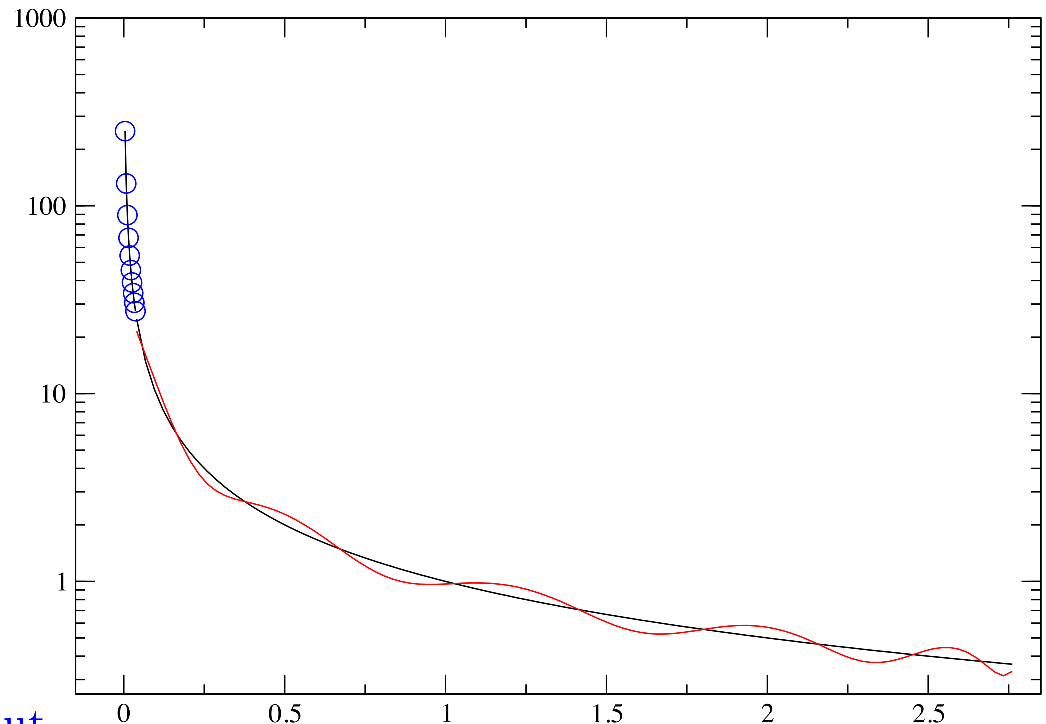
- **All Mode Averaging AMA**
  Sloppy CG or Polynomial approximations

$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],$$

$$S_l = \sum_{\lambda} v_\lambda f(\lambda) v_\lambda^\dagger,$$

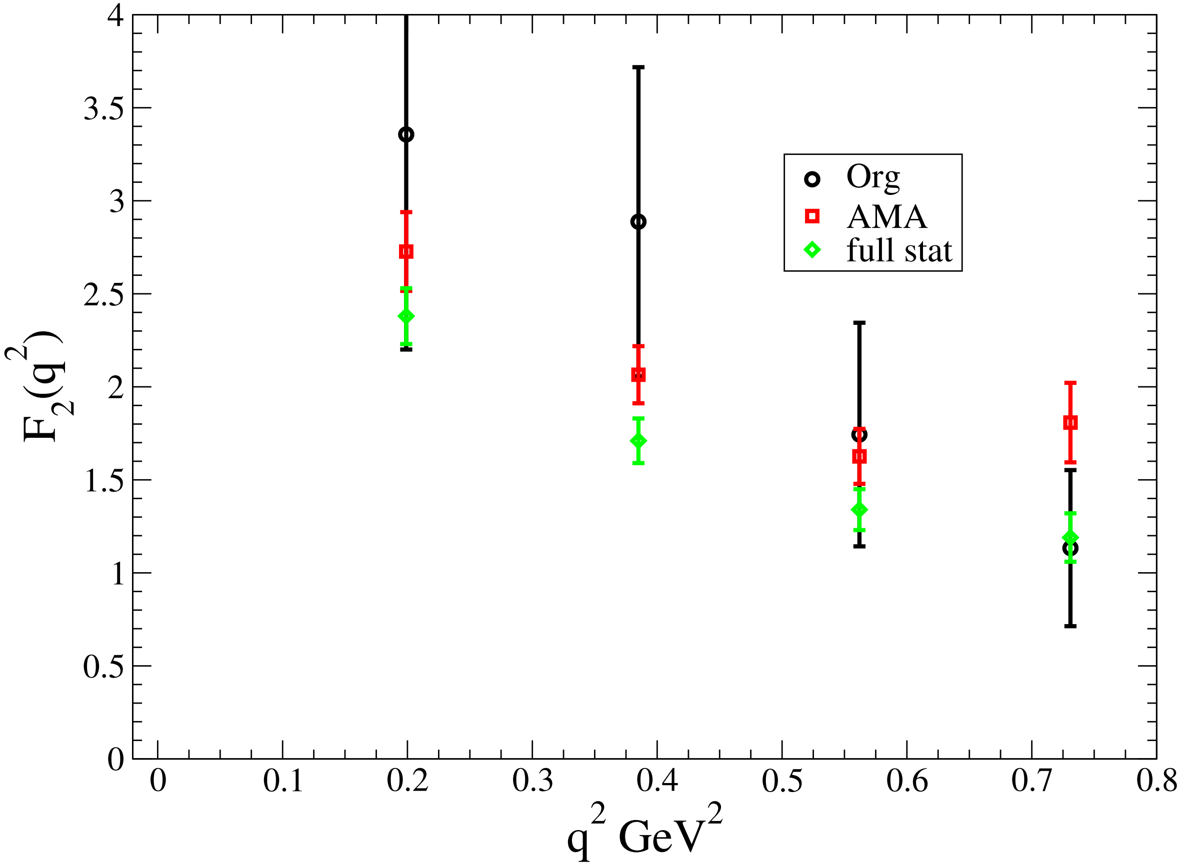
$$f(\lambda) = \begin{cases} \frac{1}{\lambda}, & |\lambda| < \lambda_{\mathrm{cut}} \\ P_n(\lambda) & |\lambda| > \lambda_{\mathrm{cut}} \end{cases}$$

$$P_n(\lambda) \approx \frac{1}{\lambda}$$



accuracy control :
- low mode part : # of eig-mode
- mid-high mode : degree of poly.

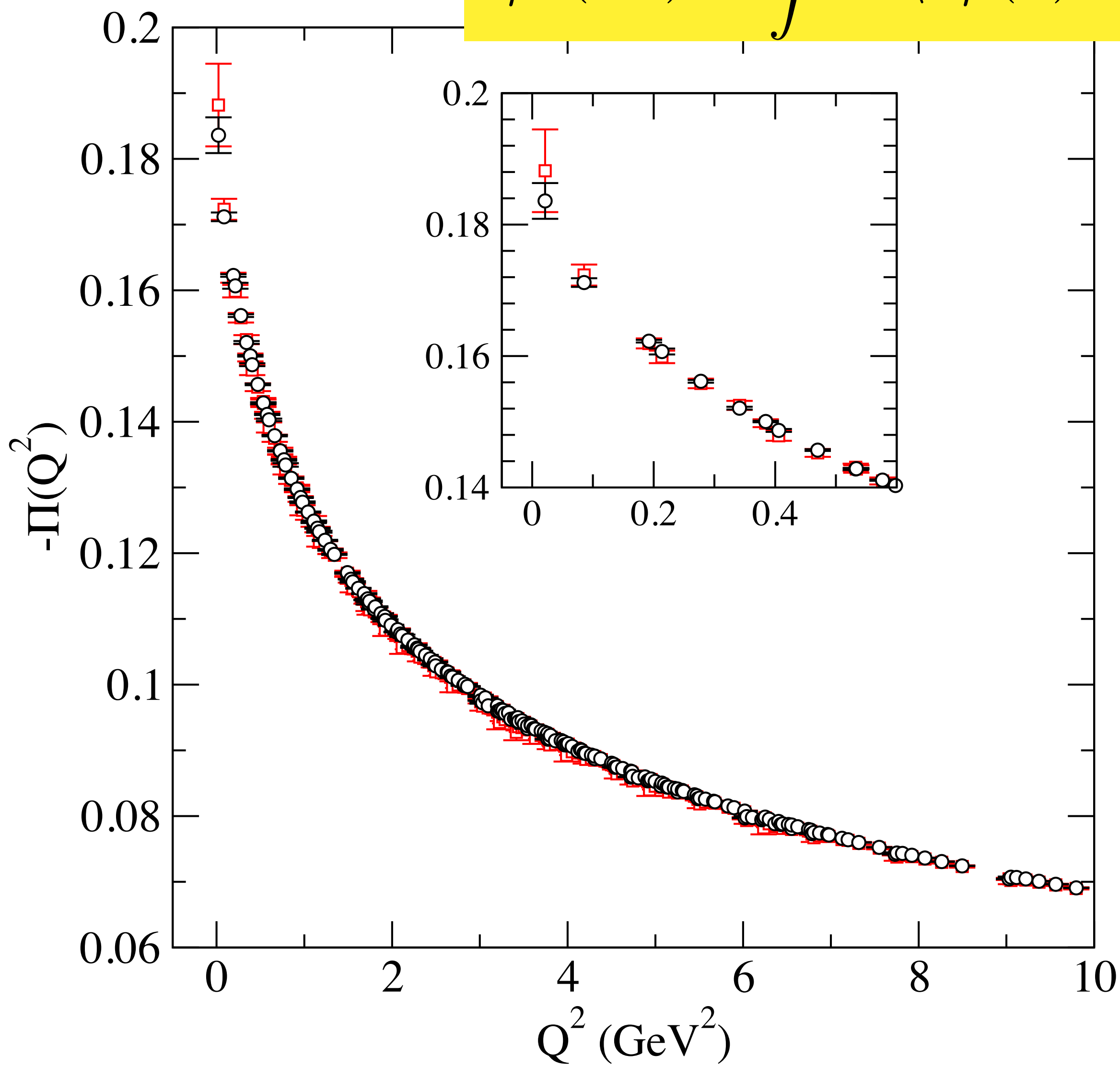
AMA for CP even form factors

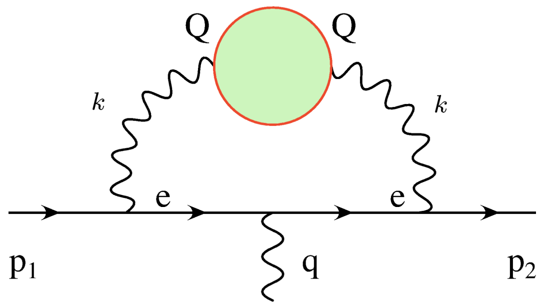


- a factor of **15** cost reduction, for 300 MeV pion, 3 fm box (a=0.11fm)
- a factor of more than **40** for 170 MeV pion, 5 fm box (a=0.14 fm)

Hadronic vacuum polarization(AsqTad)

$$\Pi_{\mu\nu}(Q^2) = \int dx \langle V_\mu(x) V_\nu(0) \rangle e^{-q(x-y)}$$

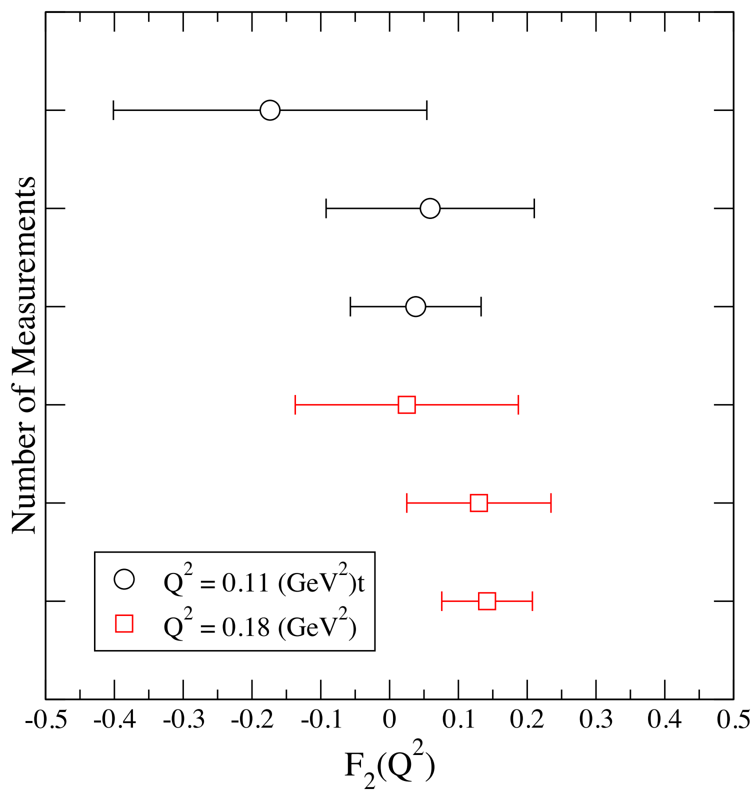


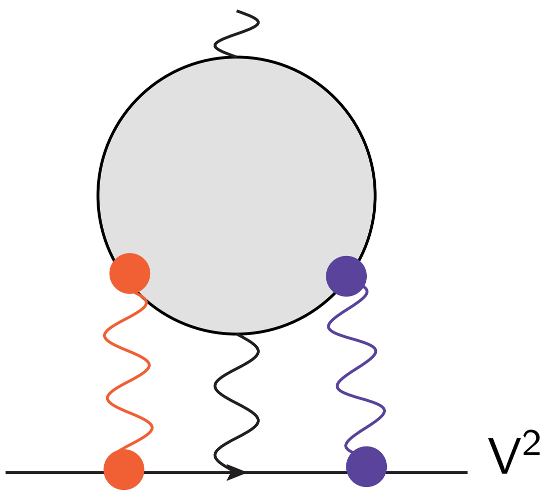


muon g-2 light-by-light
[T. Blum et al. LATTICE 2012]

a_μ(HLbL) in 2+1f lattice QCD+QED (PRELIMINARY)

$F_2(Q^2)$ stable with additional measurements (20 $\to$ 40 $\to$ 80 configs)





$m_\pi \approx 329$ MeV

$m_\mu \approx 190$ MeV

Recent results (preliminary)

- Nf=2+1 DWF QCD

  Θ=0, lattice (3 fm3), m_π = 0.3, 0.4 GeV, reweighting

- Matrix element

$$\langle n(P_1)|J_\mu^{\rm EM}|n(P_2)\rangle_\theta = \bar{u}_N^\theta \Big[\underbrace{\frac{F_3^\theta(Q^2)}{2m_N}\gamma_5\sigma_{\mu\nu}Q_\nu}_{\text{P,T-odd}} + \underbrace{F_1\gamma_\mu + \frac{F_2}{2m_N}\sigma_{\mu\nu}Q_\nu}_{\text{P,T-even}} + \cdots \Big] u_N^\theta$$

- Reweight for theta vacuum

$$\langle\theta|\eta_N J_\mu^{\rm EM}\bar{\eta}_N|\theta\rangle \quad = \quad \langle 0|\eta_N J_\mu^{\rm EM}\bar{\eta}_N|0\rangle + i\theta\langle 0|\eta_N J_\mu^{\rm EM} Q\bar{\eta}_N|0\rangle$$

- Subtract CP-even from factor contribution from the mixing, α , of Parity-odd states into Nucleon interpolation field

$$\begin{aligned}&\langle 0|\eta_N(t_1)J_\mu^{\rm EM}(t)Q\bar{\eta}_N(t_0)|0\rangle \\ &= \boxed{\frac{\alpha_N}{2}\gamma_5\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{ip\cdot\gamma + m_N}{2E_N}} + \boxed{\frac{1+\gamma_4}{2}\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{\alpha_N}{2}\gamma_5} \\ &+ \frac{1+\gamma_4}{2}\left[F_3\frac{q_\nu\gamma_5\sigma_{\mu\nu}}{2m_N} + F_A(iq^2\gamma_\mu\gamma_5 - 2m_N q_\mu\gamma_5)\right]\frac{ip\cdot\gamma + m_N}{2E_N}\end{aligned}$$

mixing of parity-odd states α

- Nucleon interpolation field in CP-even work.

$$\chi_N = \epsilon_{abc}[d_a^T C\gamma_5 u_b]d_c\ , \qquad \langle 0|\chi_N^\dagger|N(p,s)\rangle = Z_N v_s(\vec{p})$$

$$\sum_s u_s(\vec{p})\bar{u}_s(\vec{p}) = -ip\cdot\gamma + m_N$$

- When CP is violated, N and P-odd states (N*, N+π,...) mix [99 Pospelov Ritz]

$$\langle 0|\chi_N^\dagger|N(p,s)\rangle = Z_N \exp(i\alpha\gamma_5/2)v_s(\vec{p})$$

- Nucleon 2pt function with θ reweighting

$$\langle \chi_N(t)\chi_N^\dagger(0)\rangle_\theta = Z_N \frac{ip\cdot\gamma + me^{i\alpha\gamma_5}}{2E}e^{-Et}$$

$$\longrightarrow \mathrm{tr}\frac{1+\gamma_t}{2}\gamma_5\langle\chi_N\chi_N^\dagger\rangle_\theta = iZ_N\alpha e^{-m_N t}$$

  - α which is CP-odd phase is necessary to extract EDM form factor.
  - It is good check of applicability of LMA/AMA to CP-odd sector.
  - Effective mass plot shows the consistency of the above formula

Nucle on 2pt with CPV

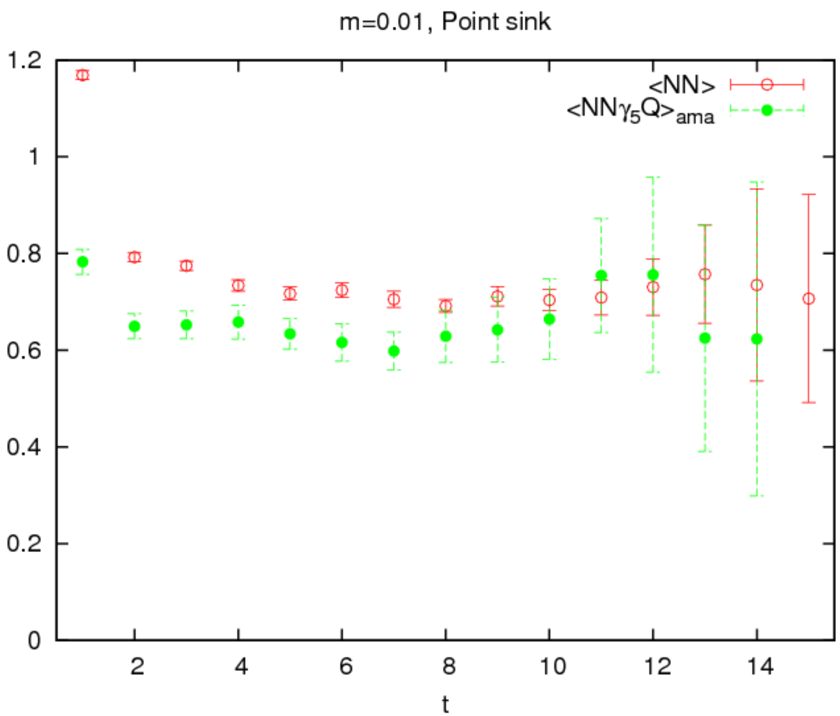


Table 1: Fitting result of orignal and AMA ($m = 0.005$)

m=0.005				
fit-range	[7, 12]	[6, 12]	[5, 10]	[5, 10]
$\vec{p}^2(\mathrm{GeV}^2)$	$N_{\mathrm{pt}}(\mathrm{GeV})$	$N_{\mathrm{gs}}(\mathrm{GeV})$	$\alpha_{\mathrm{pt}}(\mathrm{GeV})$	$\alpha_{\mathrm{gs}}(\mathrm{GeV})$
0.000	1.1322(158)	1.1307(214)	-0.246(301)	-0.219(334)
0.205	1.2072(172)	1.1998(216)	-0.171(187)	-0.224(266)
0.410	1.3050(256)	1.2821(279)	-0.215(391)	-0.215(263)
0.615	1.3869(410)	1.3475(462)	-0.125(280)	-0.240(411)
0.821	1.4736(372)	1.4643(685)	-0.094(140)	-0.158(210)
m=0.005				
fit-range	[7, 12]	[6, 12]	[5, 10]	[5, 10]
$\vec{p}^2(\mathrm{GeV}^2)$	$N_{\mathrm{pt}}(\mathrm{GeV})$	$N_{\mathrm{gs}}(\mathrm{GeV})$	$\alpha_{\mathrm{pt}}(\mathrm{GeV})$	$\alpha_{\mathrm{gs}}(\mathrm{GeV})$
0.000	1.1519(27)	1.1404(34)	-0.282(40)	-0.295(41)
0.205	1.2393(30)	1.2244(39)	-0.262(40)	-0.319(49)
0.410	1.3229(39)	1.3063(49)	-0.238(41)	-0.321(57)
0.615	1.4010(56)	1.3887(76)	-0.236(52)	-0.284(71)
0.821	1.4728(88)	1.4705(188)	-0.317(130)	-0.373(131)

- Effective mass from Nucleon 2pt
- CP-even and CP-odd part has (roughly) consistent mass
- From ratio 2pt amplitudes CPV induced mixing, α, is obtained with ~ 15 % error

Neutron F3

P-even : α F1j, F2

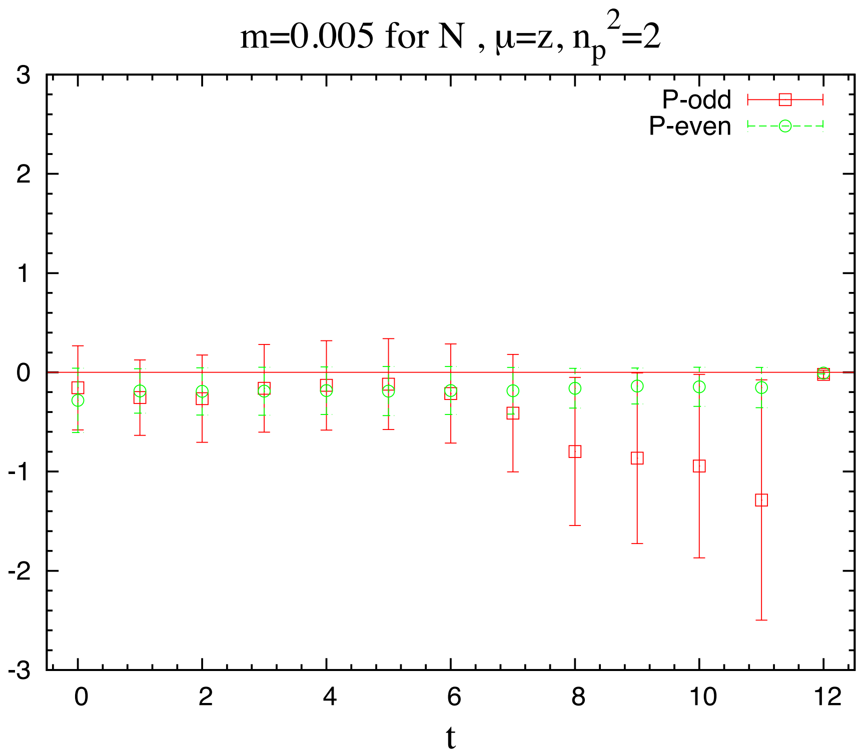
P-odd : 3pt function

$$\langle 0|\eta_N(t_1)J_\mu^{\rm EM}(t)Q\bar{\eta}_N(t_0)|0\rangle$$

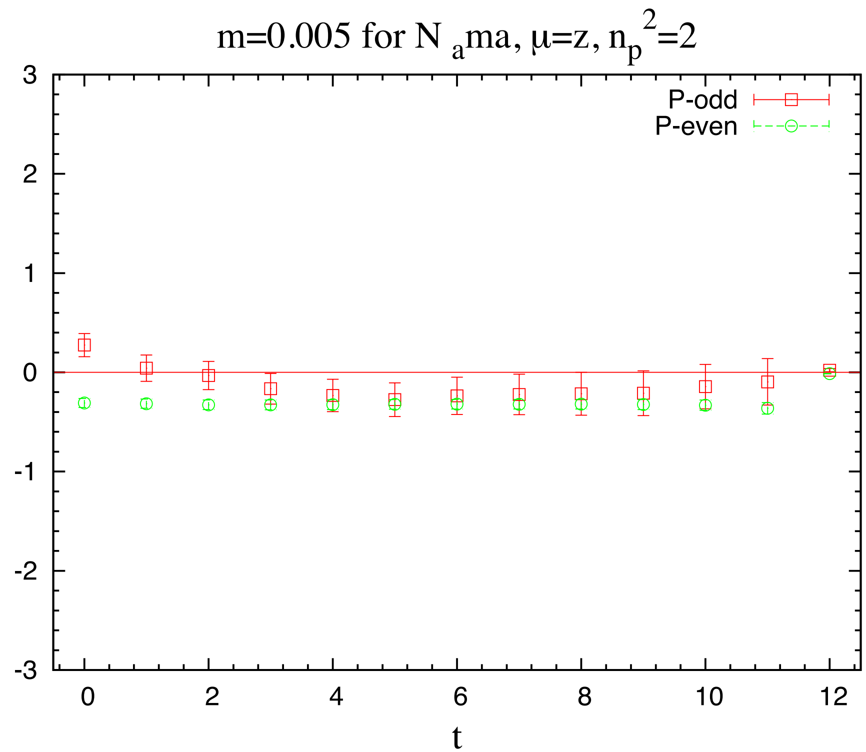
$$= \boxed{\frac{\alpha_N}{2}\gamma_5\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{ip\cdot\gamma + m_N}{2E_N} + \frac{1+\gamma_4}{2}\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{\alpha_N}{2}\gamma_5}$$

$$\boxed{+\frac{1+\gamma_4}{2}\left[F_3\frac{q_\nu\gamma_5\sigma_{\mu\nu}}{2m_N} + F_A(iq^2\gamma_\mu\gamma_5 - 2m_N q_\mu\gamma_5)\right]\frac{ip\cdot\gamma + m_N}{2E_N}}$$

Θ = 1



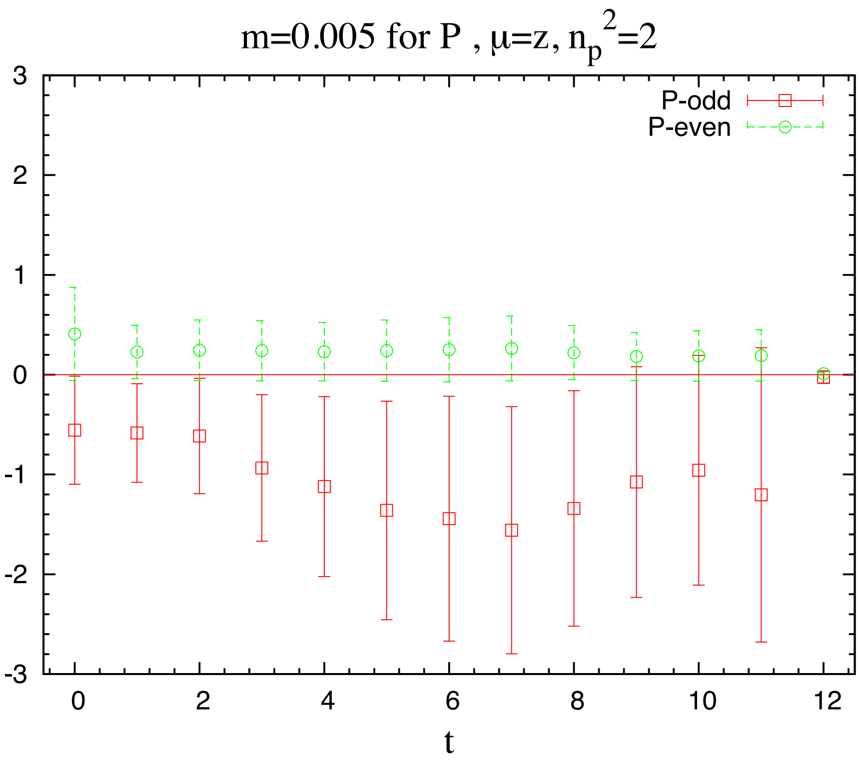
without AMA

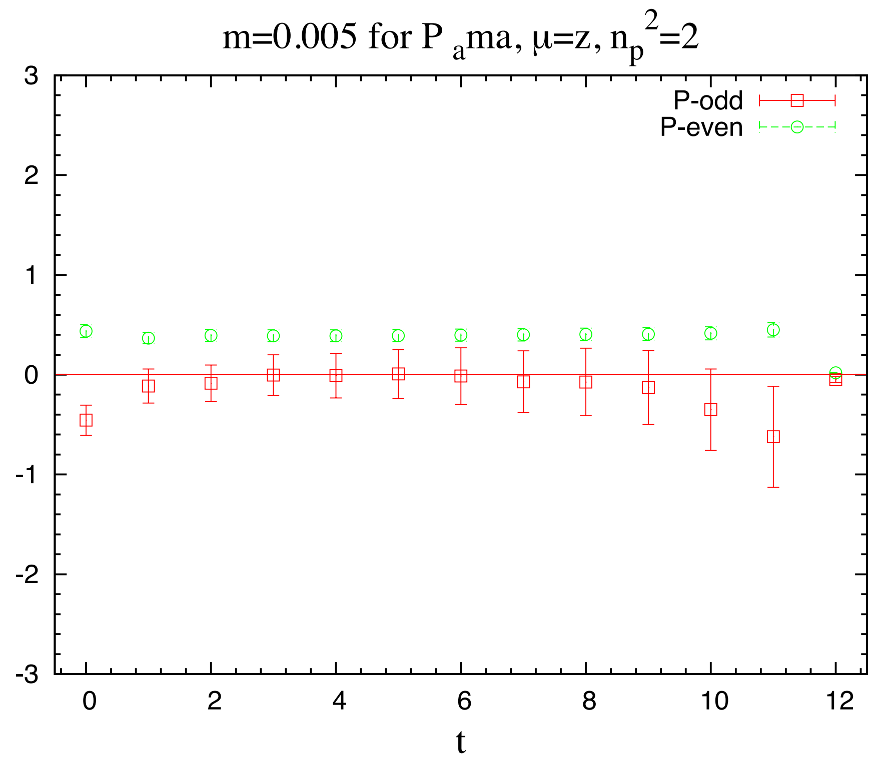


with AMA

Proton F3

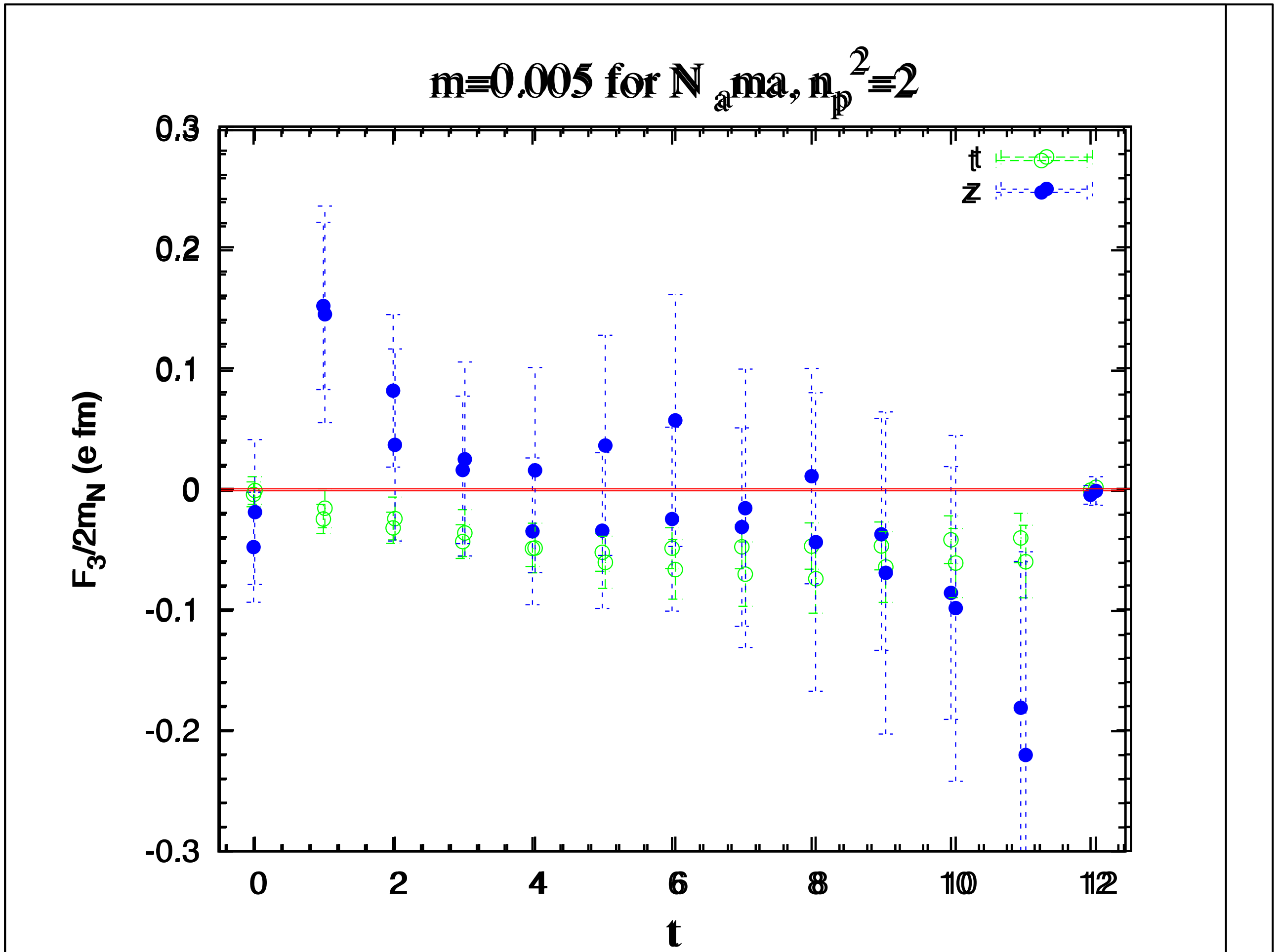
- Noisier than Neutron (stronger α, but weaker CPV 3pt)



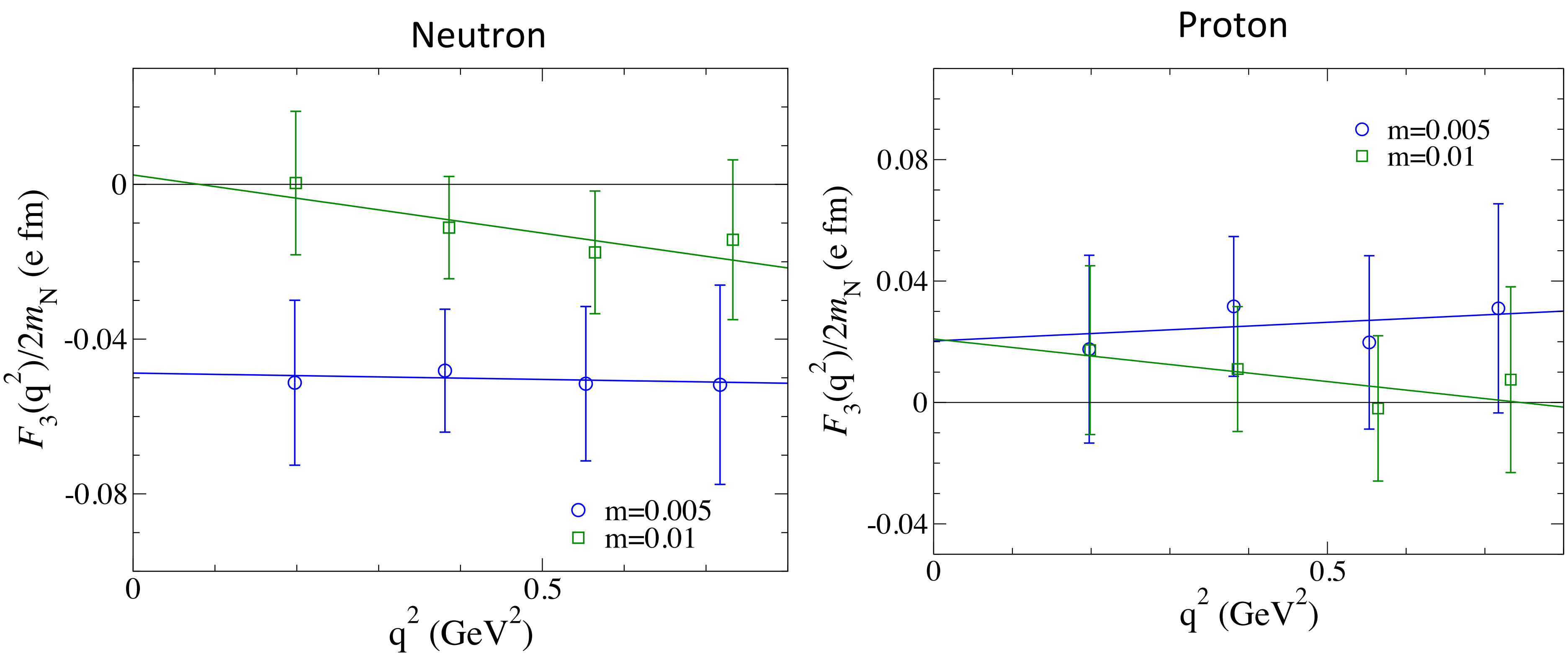


Statistical signal check

- Mpi = 330 MeV, a=0.11 fm, V=(3fm)^3
- 191 config * 32 measurements vs 380 config * 32 measurements
- green: from Vt, blue: from Vz



$F_3(q^2)$ vs q^2



- (iso-vector) CP violating g(π-N-N) is related to the slope of F_3
  [11 Vries, Timmermans, Mereghetti, van Kolck]

Comparison of results

Full QCD

• Lattice results are consistent within 1σ.
(not include systematic error)
• larger than the results of QCD Sum Rules or ChPT.

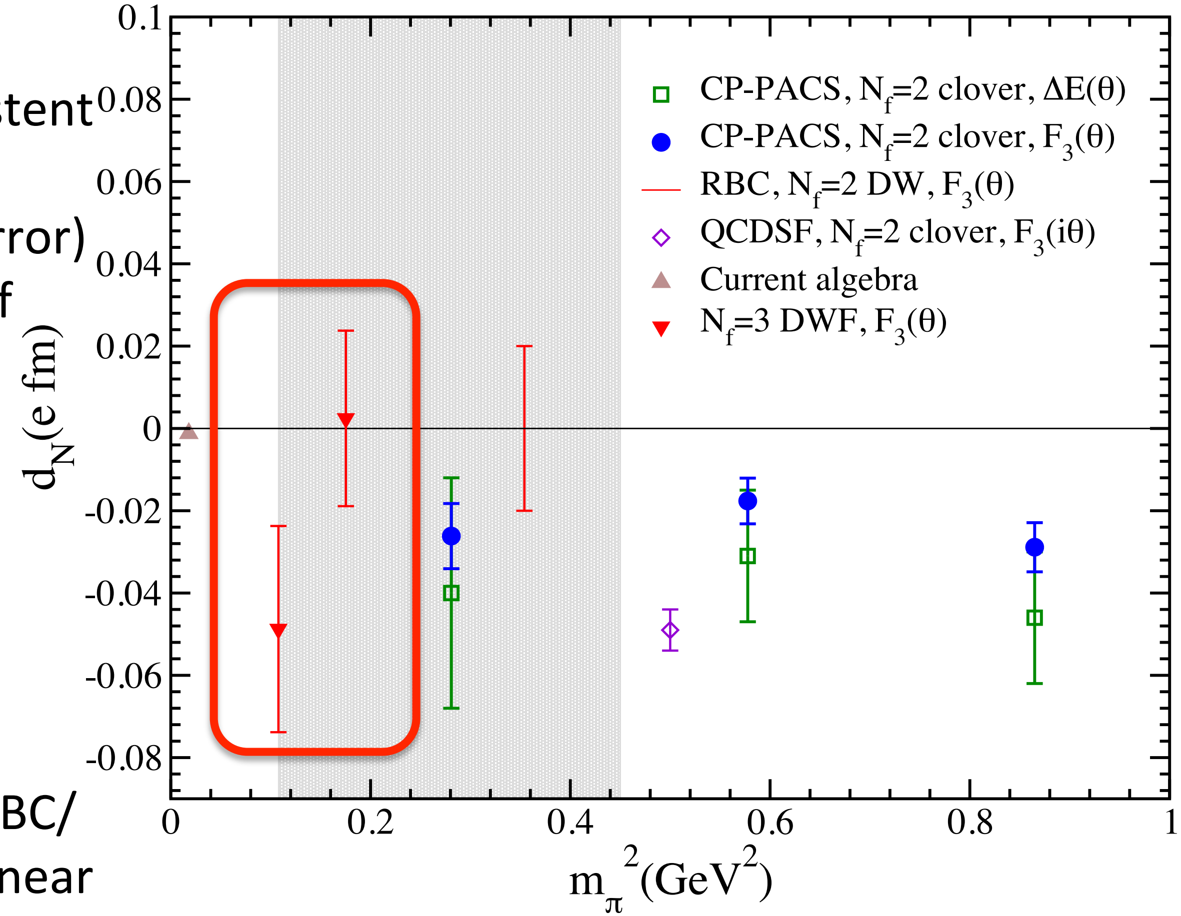
•no obvious quark mass dependence yet

statistics ?

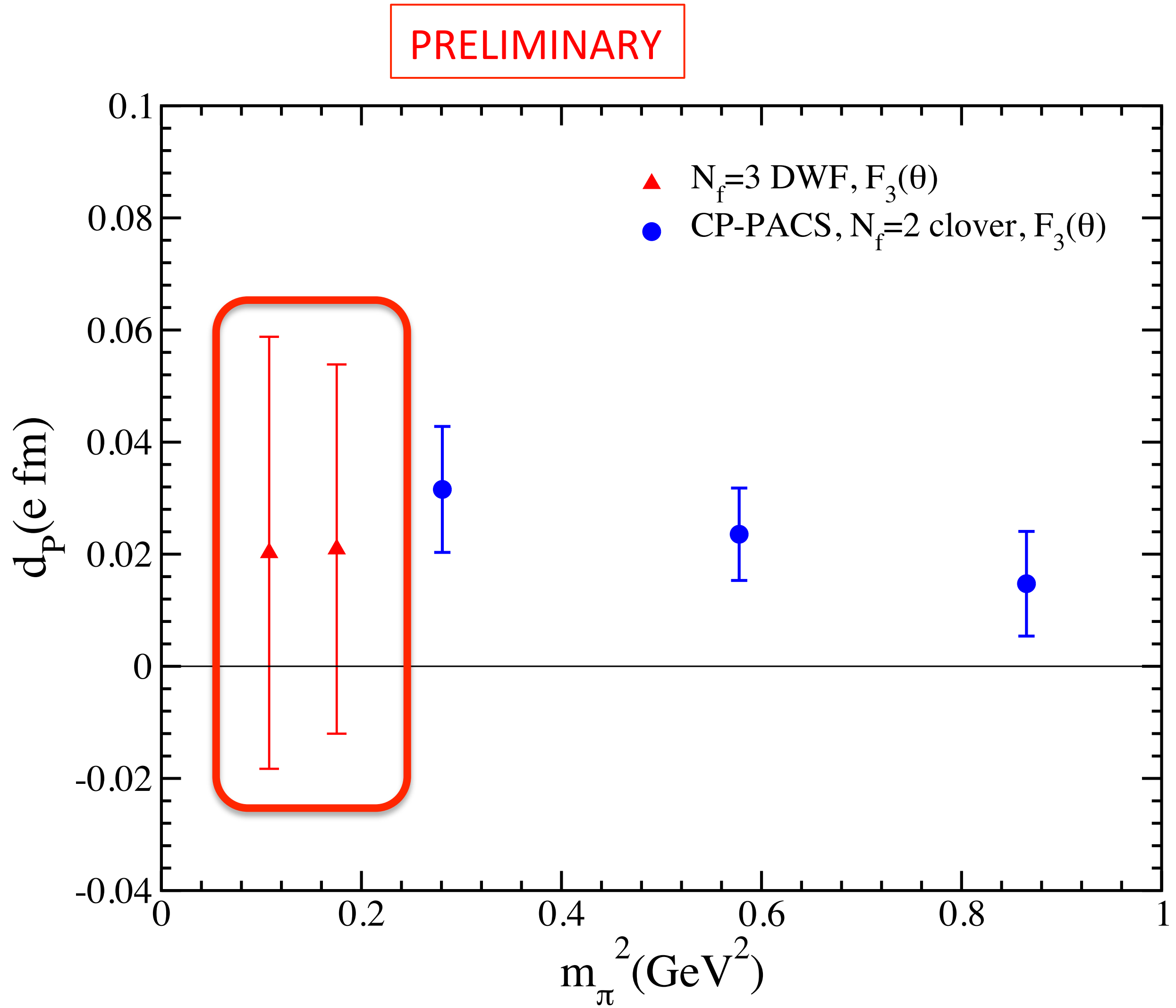
•Nf = 2+1 DWF configs. (RBC/UKQCD) are available for near physical pion mass (170 MeV) is examined now.

PRELIMINARY

Θ = 1



Proton EDM results



qEDM terms into QCD action

Bhattacharya, Cirigliano, Gupta Lattice 2012

- Plan to do extension to BSM action
  - Matrix element including BSM operators
    quark QED EDM and quark chromo EDM

$$\langle n|J_\mu^{\rm EM}|n\rangle_{\rm qEDM} \quad : \quad \frac{d_u+d_d}{2}\langle n|\bar{u}q_\nu\sigma^{\mu\nu}\gamma_5 u+\bar{d}q_\nu\sigma^{\mu\nu}\gamma_5 d|n\rangle + \frac{d_u-d_d}{2}\langle n|\bar{u}q_\nu\sigma^{\mu\nu}\gamma_5 u-\bar{d}q_\nu\sigma^{\mu\nu}\gamma_5 d|n\rangle$$

$$\langle n|J_\mu^{\rm EM}|n\rangle_{\rm cEDM} \quad : \quad \frac{\partial}{\partial A_\mu}\left\langle n\middle|\int(d_u^c\bar{u}\sigma_{\nu\kappa}u+d_d^c\bar{d}\sigma_{\nu\kappa}d)\widetilde{G}_{\nu\kappa}\middle|n\right\rangle_E$$

$$\langle n|J_\mu^{\rm EM}|n\rangle \sim \frac{F_3(q^2)}{2m_N}\bar{n}q_\nu\sigma^{\mu\nu}\gamma_5 n$$

  - Iso-vector tensor charge is related to the CPV form factors
  - Perhaps easier statistical signal than θ angle
  - The mixing, $\sim\alpha$, from P-odd in the nucleon interpolation field ?
  - complication could be the operator mixing and/or chiral symmetry breaking of clover-Wilson ?

Summary

- Ab initio computations for EDMs on Lattice
- Current difficulties
  - Statistical error
    All Mode Averaging (AMA)
    Faster Computers (QCDCQ)
  - Systematic errors
    EDM is a chiral-sensitive quantities
    chiral lattice quarks (DWF)
    or careful a→0 (Wilson-clover)

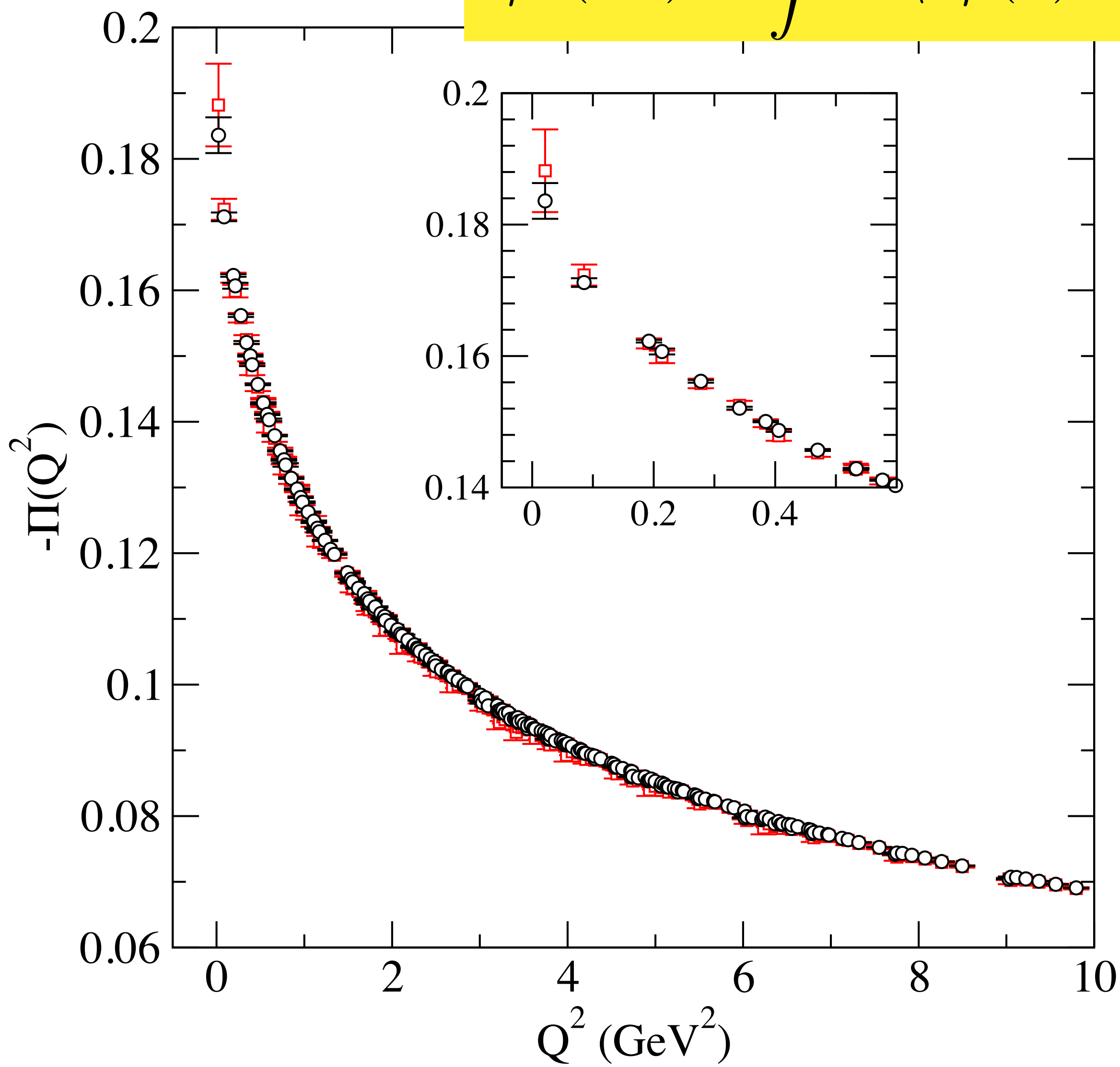
3.5 racks of QCDCQ
@BNL BNL, 700 TFLOPS peak

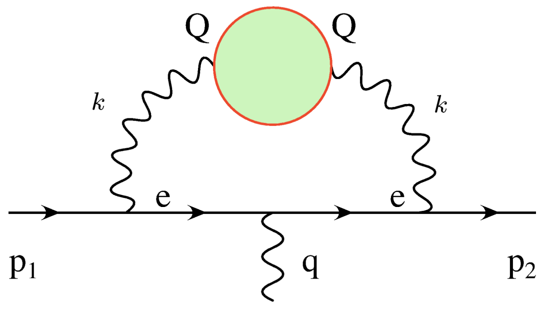
- Future plans
  - Increase statistics
    with smaller N-N separation
  - Lighter quark mass Mpi = 170 MeV
  - Higher dimension operators

BACK UP SLIDES

Hadronic vacuum polarization(AsqTad)

$$\Pi_{\mu\nu}(Q^2) = \int dx \langle V_\mu(x) V_\nu(0) \rangle e^{-q(x-y)}$$





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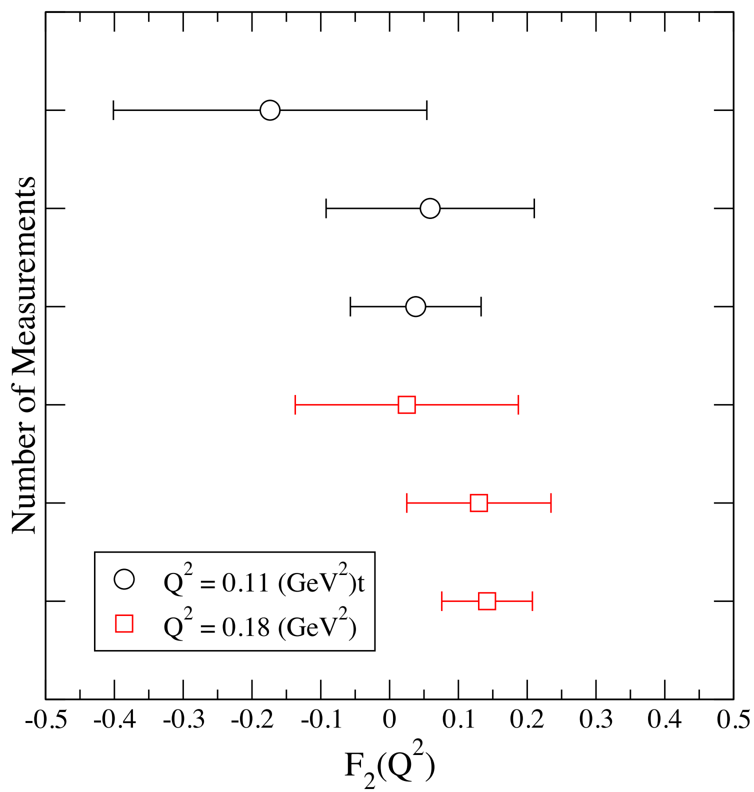

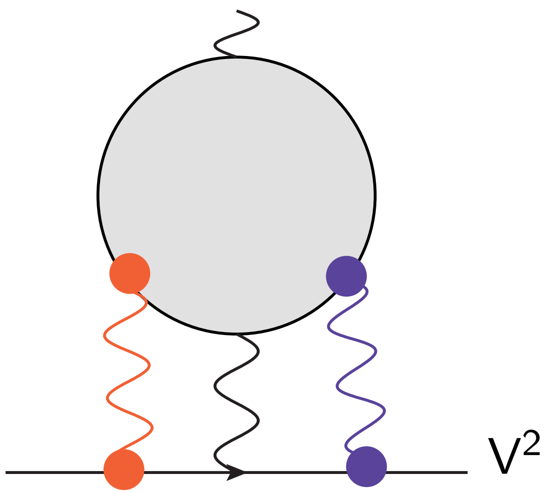

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$m_\mu \approx 190$ MeV

Cost comparison for test cases

- x 16 for DWF Nucleon mass (M_{PS}=330MeV, 3fm)
- x 2- 20 for AsqTad HVP (MPS=470 MeV, 5 fm)
- should be better for lighter mass & larger volume ?

	$N_{\rm conf}$	$N_{\rm meas}$	LM	$\mathcal{O}$	$\mathcal{O}_G^{(\rm appx)}$	Tot.	scaled cost	
m_N			$m = 0.005$, 400 LM				gauss	pt
AMA	110	1	213	18	91+23	350	0.063	0.065
LMA	110	1	213	18	23	254	0.279	0.265
Ref. [2]	932	4	-	3728	-	3728[a]	1	1
			$m = 0.01$, 180 LM					
AMA	158	1	297	74	300+22	693	0.203	0.214
LMA	158	1	297	74	22	393	0.699	0.937
Ref. [2]	356	4	-	1424	-	1424	1	1
HVP			$m = 0.0036$, 1400 LM				max	min
AMA	20	1	96	11	504+420	1031	0.387	0.050
LMA	20	1	96	11	420	527	10.3	3.56
Ref. [1]	292	2	-	584	-	584	1	1

AMA : Error reduction techniques

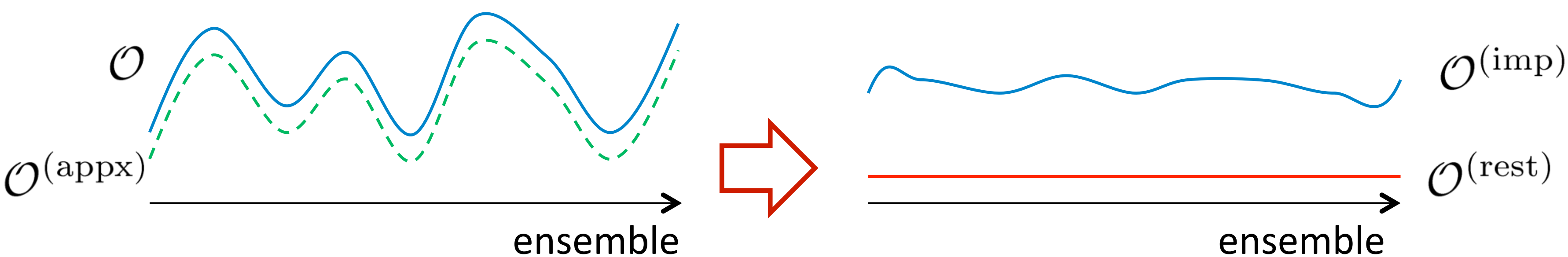
- Covariant approximation averaging (CAA)
  - For original observables O, (unbiased) improved estimator

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G}\sum_{g\in G}\mathcal{O}^{(\mathrm{appx}),g}, \quad \mathcal{O}^{(\mathrm{rest})} = \mathcal{O} - \mathcal{O}^{(\mathrm{appx})}$$

which satisfies $\langle O\rangle = \langle O^{\mathrm{imp}}\rangle$ if approximation is covariant under lattice symmetry g, and error becomes

  - Ideal approximation

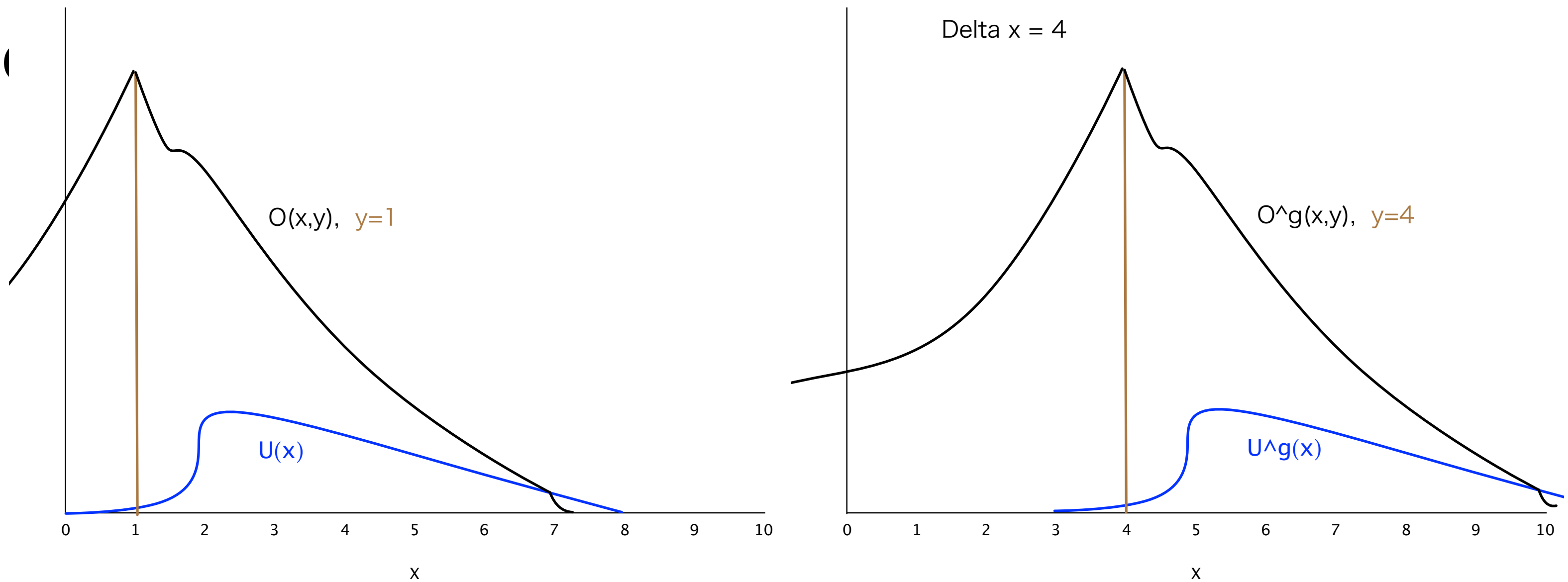
$$\boxed{\mathrm{err}^{\mathrm{imp}} \simeq \mathrm{err}/\sqrt{N_G}}$$



- Ignoring the error from $O^{(\mathrm{rest})}$
- There may be many candidates of $O^{(\mathrm{appx})}$ e.g. LMA, heavy mass, …
- The cost of approximated observable need to be smaller than the original.

Covariant approximation

- $O^{(appx)}$ needs to be precisely (to the numerical accuracy required) covariant under the symmetry of lattice action to avoid systematic (



One should check in the code using explicitly shifted gauge configuration

Examples of Covariant Approximations

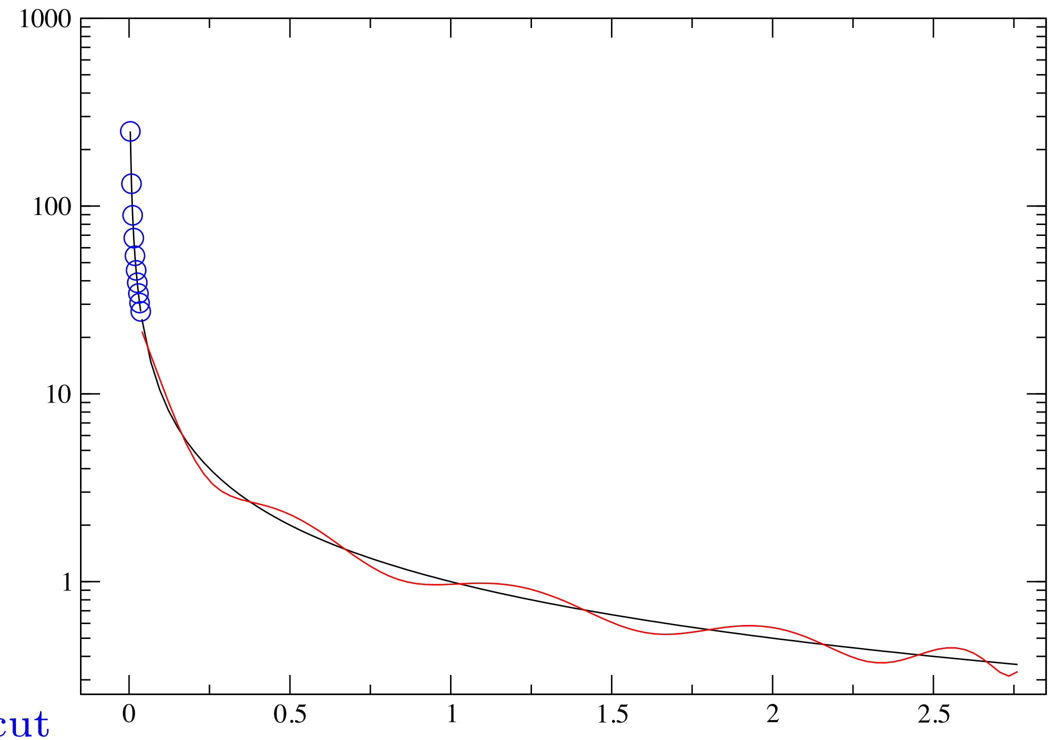
- All Mode Averaging **AMA**
  Sloppy CG or Polynomial approximations

$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],$$

$$S_l = \sum_\lambda v_\lambda f(\lambda) v_\lambda^\dagger,$$

$$f(\lambda) = \begin{cases} \frac{1}{\lambda}, & |\lambda| < \lambda_{\mathrm{cut}} \\ P_n(\lambda) & |\lambda| > \lambda_{\mathrm{cut}} \end{cases}$$

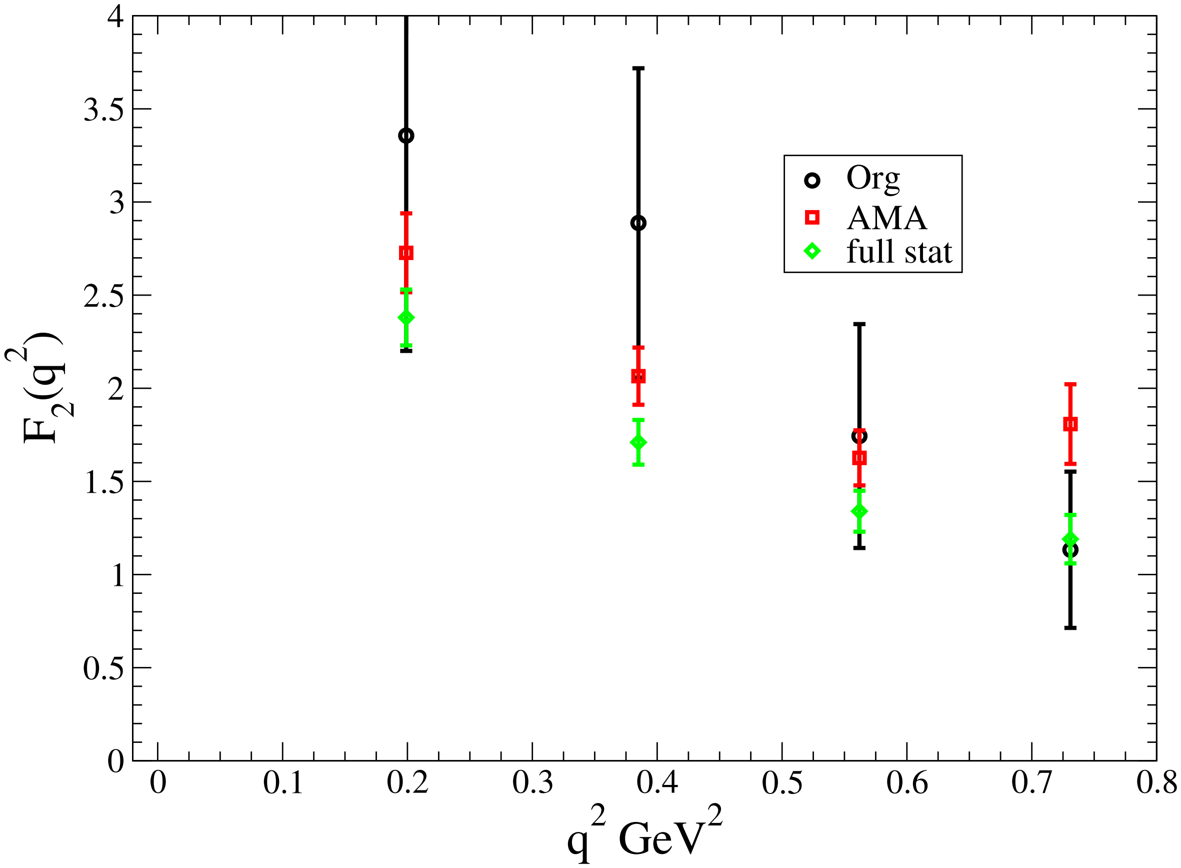
$$P_n(\lambda) \approx \frac{1}{\lambda}$$



accuracy control :

- low mode part : # of eig-mode
- mid-high mode : degree of poly.

AMA for CP even form factors



- a factor of <u>**15**</u> cost reduction, for 300 MeV pion, 3 fm box (a=0.11fm)
- a factor of more than <u>**40**</u> for 170 MeV pion, 5 fm box (a=0.14 fm)

Examples of CAA

- Lowmode averaging (LMA)

Guisti et al.(04), Neff et al.(01), DeGrand et al. (04)

  - Using lowlying eigenmode of Dirac operator to approximate propagator:

$$\mathcal{O}^{(\mathrm{appx})} = \sum_{\lambda}^{N_\lambda} \mathcal{O}_\lambda^{\mathrm{low}}$$

where N_λ is number of lowmode computed by Lanczos.

Except for computational cost of eigenmode, Cost(LMA) $\simeq$ 0, but approximation is only lowmode part (long distance contribution).

- All-mode averaging (AMA)
  - Using sloppy CG (loose stopping condition),

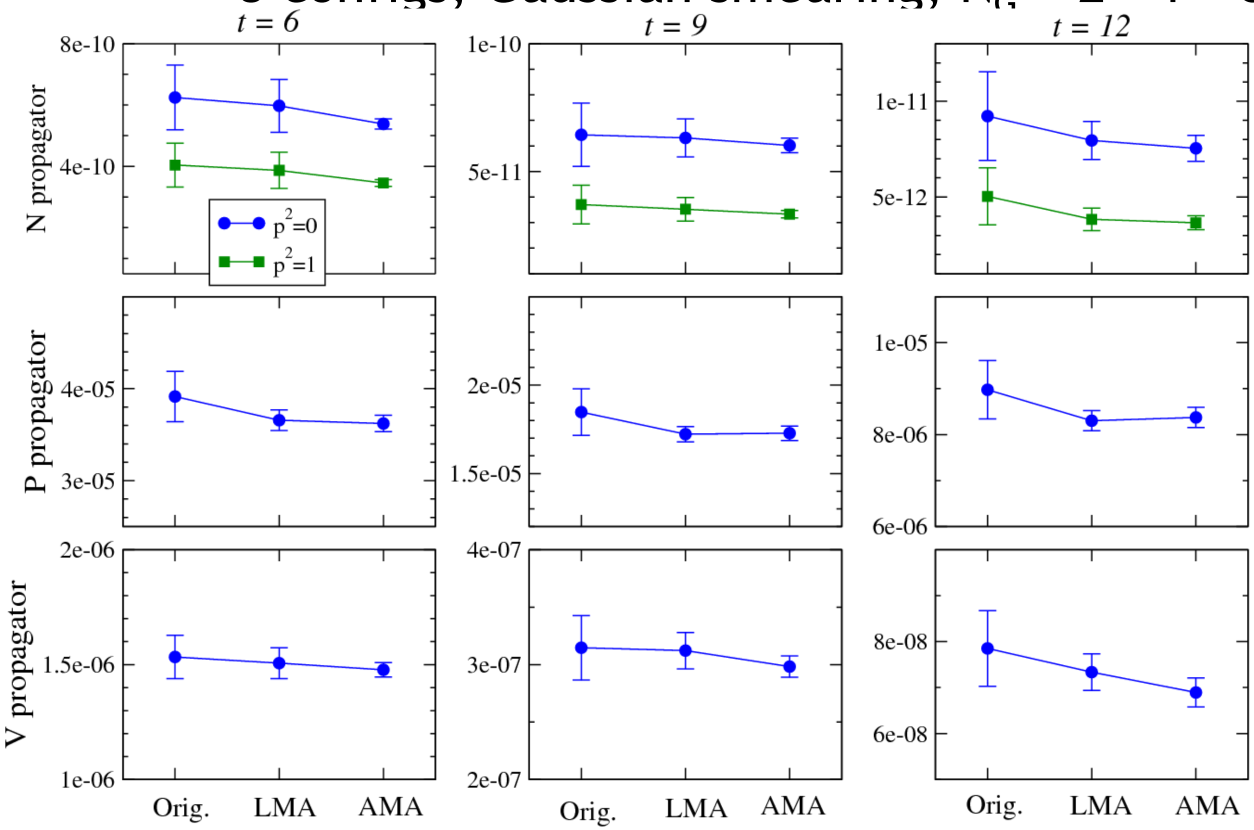
$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}^{\mathrm{sloppy}}$$

If stopping cond. is 0.003, Cost(AMA) $\simeq$ Cost(CG)/50(without deflation).

Approximation becomes better than LMA for other than lowmode dominanted observables (nucleon, finite momentum hadron, …).

Comparison between LMA/AMA

- Preliminary result
  - 8 configs, Gaussian smearing, $N_G = 2^3 \times 4 = 32$ sources, $24^3 64 \times 16$



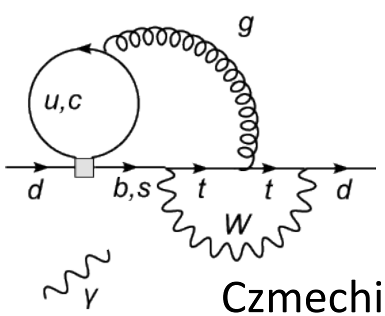
- t = 6:
  Error in AMA is actually reduced by factor 5 compared with orig. and LMA.

- t = 12
  Error in AMA/LMA is reduced by factor 3--4 compared with original.

CP symmetry breaking in the SM

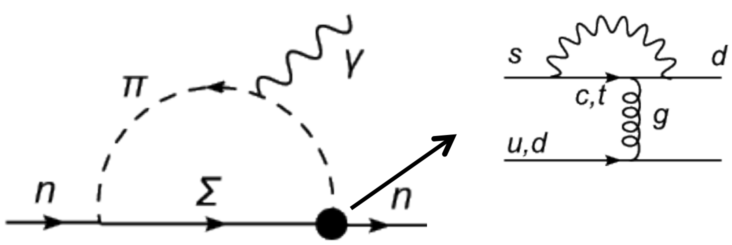
- Contribution to EDM from weak interaction is very small
  - Vanishing 1-loop (no Im part), 2-loop diagram
  - Three-loop order(short) and pion loop correction (long):

Short distance



Czmechi, Krause (1997)

Long distance



Khriplovich, Zhitnitsky (1982)

$$d_N^{\mathrm{KM\,short}} \sim \mathcal{O}(\alpha_s G^2) \sim -10^{-34}\,\mathrm{e\cdot cm} \qquad d_N^{\mathrm{KM\,long}} \sim 10^{-30\sim-33}\,\mathrm{e\cdot cm}$$

$$\Rightarrow d_N^{\mathrm{KM}} = d_N^{\mathrm{KM\,short}} + d_N^{\mathrm{KM\,long}} \simeq 10^{-30\sim-33}\,\mathrm{e\cdot cm}$$

which is the 6-order magnitude below the exp.
upper limit: $|d_N^{\mathrm{exp}}| < 2.9 \times 10^{-26}\,\mathrm{e\cdot cm}$