

Echoes in accelerators

Echoes are a fascinating phenomenon:

- Acoustic echo, measure sound speed [?, <1000 B.C.]
- Spin echo in Nuclear Magnetic Resonance [Hahn, 1950]
- Echo in plasma [Baker, Ahern, Wong, 1968]
- There is also echo in accelerators [G. Stupakov, 1992]

Particle motion in an accelerator has 4 dimensions:

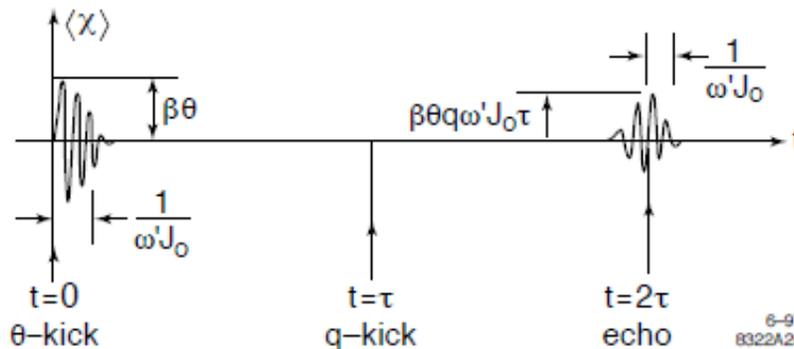
- Transverse dimensions
 - horizontal = x
 - vertical = y
- Longitudinal dimension = z
- Spin dimension = S

There is an echo effect in each of these 4 dimensions, each with its own special characteristics.

=> Echoes are everywhere in an accelerator!

1. Introduction

Consider this experiment in a proton storage ring:



- A beam is stored for a long time in a steady state. Its centroid signal reads $\langle x \rangle = 0$.
- The beam is kicked at $t = 0$ by a dipole kicker. The beam's centroid subsequently executes a free oscillation (“betatron oscillation”).
- Due to a spread in the betatron oscillation frequencies of the particles in the beam, the centroid signal decays (“decoheres”) in a short time ~ 1 ms.
- Long after the kick, say 1 s later, the centroid signal is of course dead zero. At this point, $t = \tau$, we give the beam a second, quadrupole kick. Such a kick does not affect the beam centroid, and thus $\langle x \rangle$ stays zero.
- Now we wait exactly 1 s after the quadrupole kick. The beam centroid will see a sudden and pronounced blip. The blip lasts only ~ 1 ms, and quiet afterwards.

This sudden blip is an echo. It occurs at exactly $t = 2\tau$. It is a result of the correlation and interplay between the two kicks and a long memory of the intricate beam dynamics in the phase space.

Above is transverse echo. There is also longitudinal echo, in which case an rf phase shift and an rf amplitude jump play the roles of the dipole and quadrupole kicks, respectively.

In addition to the transverse and longitudinal orbital motions, particle motion also has a 4-th dimension involving spin. There is also a spin echo.

[Chao, Courant, 2007]

Recently, echo has also been suggested as a clever way to generate microbunching in beam distribution for the purpose of generating X-ray free electron lasers!

[G. Stupakov 2008]

All echo effects involve intricate microstructures (minute particulars!) in phase space, and they come about because memory in phase space lasts real long times. The long memory comes about because of Liouville theorem, i.e. you cannot destroy phase space!

2. Transverse decoherence ($0 < t < \tau$)

Consider the setup for transverse echo mentioned earlier.

Echo has been described qualitatively, but now let us do some calculations.

First concentrate on the period from $t = 0$ to $t = \tau$, which is the decoherence period. Echo will be discussed later.

Calculation:

- Transform phase space $(x, p) \rightarrow (\phi, J)$
- Initial distribution in (x, p) is centered Gaussian $\psi_0(J)$
- dipole kick at $t = 0 \Rightarrow$ displaced Gaussian
- after dipole kick, distribution filaments because ω_β depends on J .
- Now calculate $\langle x \rangle(t)$. It decoheres after the dipole kick.

$$\langle x \rangle(t) \approx \frac{\beta\theta}{1 + \Theta^2} \sin(\omega_0 t + 2 \tan^{-1} \Theta)$$

$$\Theta \equiv \omega' J_0 t$$

$\omega_0 t$ is fast time variable, Θ is slow time variable.

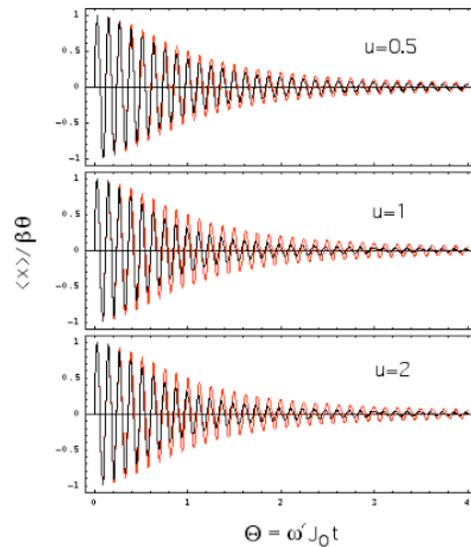
This signal has a slowly varying oscillation amplitude

$$\langle x \rangle^{\text{ampl}}(t) \approx \frac{\beta\theta}{1 + \Theta^2}$$

This amplitude starts with the value $\beta\theta$ at $t=0$, but slowly decreases with time. Decoherence time:

$$\tau_{\text{decoh}} \approx \frac{1}{\omega' J_0}$$

Typically frequency spread $\omega' J_0 \sim 10^{-3}$, meaning $\tau_{\text{decoh}} \sim 10^3$ turns, or ~ 1 ms, which is very short.



3. Transverse echo ($t > \tau$)

We now let the beam decohere for a time $\tau \gg \tau_{\text{decoh}}$, e.g. $\tau \sim 1$ s or even longer. The beam centroid signal completely vanishes due to decoherence.

At time $t = \tau$, we give the beam a quadrupole kick, $q = \beta/f$ with $f =$ quadrupole's focal length.

After the quadrupole kick ($t > \tau$), the beam distribution can be calculated to yield

$$\begin{aligned} \langle x \rangle (t) \approx & 2\beta\theta q\tau \int_0^\infty J^2 dJ \omega'(J) \psi'_0(J) \\ & \times \int_0^{2\pi} d\phi \cos \phi \sin[2\phi - 2\omega(J)t + 2\omega(J)\tau] \cos[\phi - \omega(J)t] \end{aligned}$$

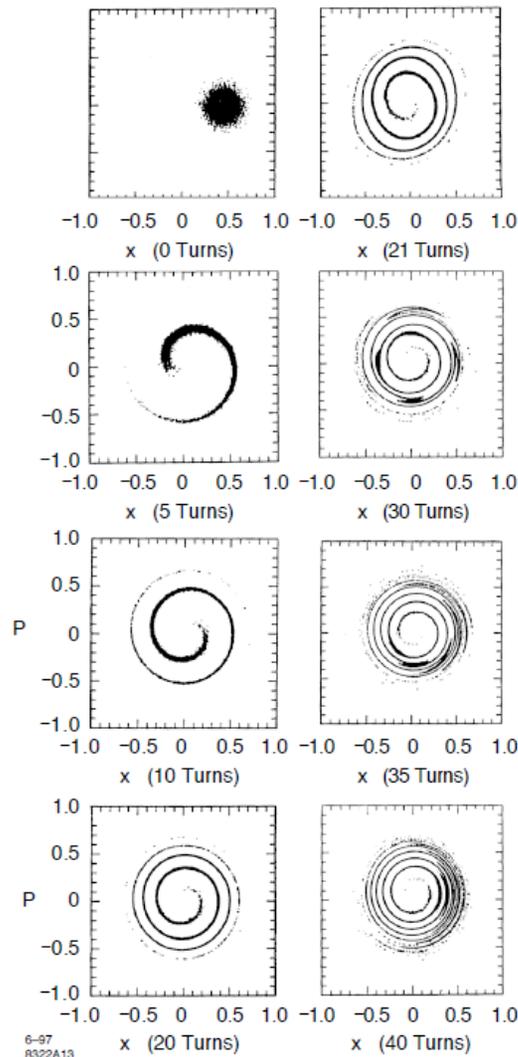
This integration over ϕ is the key to the echo phenomenon. Magically, the two kicks, the frequency spread, and the particle evolution in phase space conspire to produce a recoherence at time $t = 2\tau$!

$$\langle x \rangle (t) \approx 2\beta\theta q\tau \int_0^\infty J^2 dJ \omega'(J) \psi'_0(J) \left(-\frac{\pi}{2} \sin[\omega(J)(t - 2\tau)] \right)$$

The recoherence, and the fact that it occurs at $t = 2\tau$, do not depend on the exact form of $\psi_0(J)$ or $\omega_\beta(J)$!

=>

In spite of its phase space minute particulars, which implies sensitivity to minute details, echo is a very robust phenomenon!



[G. Stupakov, S. Kauffmann, 1992]

A Gaussian beam receives a dipole kick at 0-th turn and a quadrupole kick at 20-th turn. From 0-th turn to 20-th turn the beam decoheres. By about the 30-th turn, several "kinks" develop in the beam's phase space distribution. These kinks come from an interplay of the two kicks. Each of the kicks occurs at a specific amplitude, which are most conspiring in the sense that, with the amplitude-dependent frequency, they all come to coherence simultaneously at the 40-th turn as they migrate in the phase space relative to one another, thus yielding an echo.

Example If $\omega_\beta(J) = \omega_0 + \omega'J$, and $\psi_0(J) = \text{Gaussian}$, then

$$\langle x \rangle^{\text{echo ampl}}(t) = \beta\theta q \frac{\omega' J_0 \tau}{(1 + \xi^2)^{3/2}}$$

$$\xi = \omega' J_0 (t - 2\tau)$$

The maximum echo amplitude occurs at $t = 2\tau$, and the maximum value is

$$\langle x \rangle^{\text{echo ampl max}} = \beta\theta q \omega' J_0 \tau$$

Echo duration is $\sim \frac{1}{\omega' J_0} \sim \tau_{\text{decoh}}$.

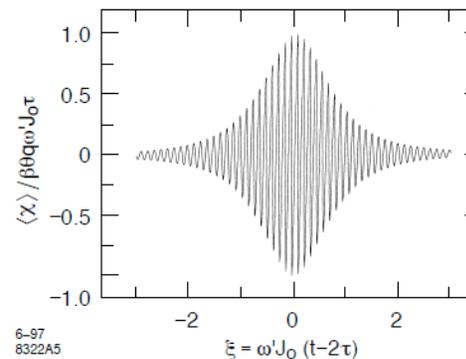


Figure 4: A close-up observation of the echo signal.

If we compare the maximum echo amplitude to the initial kick amplitude $\beta\theta$, we see that the echo amplitude is weaker by a factor of $q\omega'J_0\tau$ which we have assume to be $\ll 1$.

Simulation shows what happens when q is varied. As q is increased, the echo signal develops a double hump structure. The maximum echo amplitude is found to be about 40% of the initial kick amplitude (case 3).

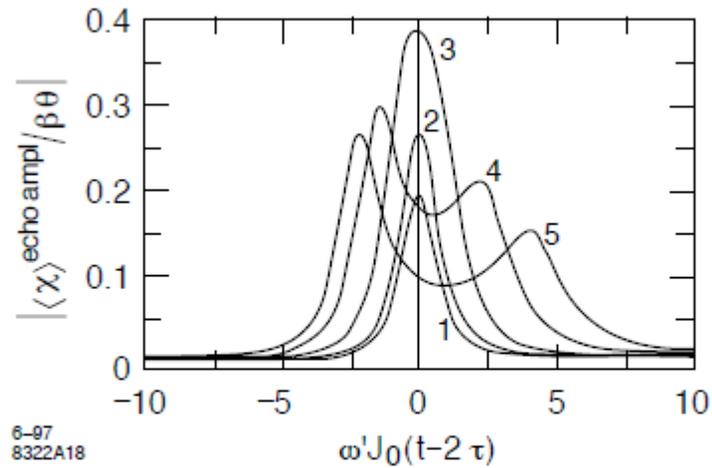


Figure 6: Echo amplitude signal obtained in a simulation. Curves 1 to 5 correspond to $q = 0.02, 0.03, 0.08, 0.2, 0.3$ respectively.

4. Longitudinal echo (unbunched beam)

We need two kicks separated by a long time τ . The first kick at time $t = 0$ is to impose an rf energy kick

$$\Delta\delta(z) = \frac{eV_1}{E_0} \sin\left(h_1 \frac{z}{R} + \phi_1\right)$$

where h_1 is the harmonic number of the rf system.

After the first kick, the beam begins to bunch up into h_1 bunches, but due to the energy spread of the beam, this bunching signal decoheres quickly.

Long after the signal has decohered, at $t = \tau$, a second kick is applied,

$$\Delta\delta(z) = \frac{eV_2}{E_0} \sin\left(h_2 \frac{z}{R} + \phi_2\right)$$

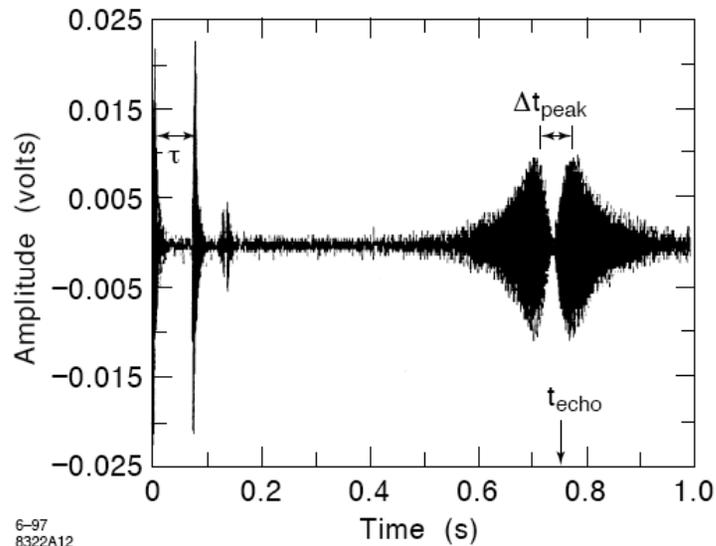
with harmonic number h_2 .

At a much later time, $t = t^{echo}$, one observes a sudden echo with harmonic number $h_2 - h_1$.

$$I^{echo\ ampl}(t) = -\frac{1}{2} \text{sgn}(h_2) I_0 \frac{eV_1}{E_0} \frac{eV_2}{E_0} \frac{h_1 \eta c \tau}{R \sigma_\delta} \xi e^{-\xi^2/2}$$

$$t^{echo} = \frac{|h_2| \tau}{|h_2| - |h_1|}$$

$$\xi = \frac{\eta c \sigma_\delta (|h_2| - |h_1|)}{R} (t - t^{echo})$$



A beautiful experiment at CERN AA
[L. Spenzouris, J.-F. Ostiguy, P. Colestock, 1996]

First kick $t = 0$, $h_1 = 9$

Second kick $t = \tau$, $h_2 = 10$

\Rightarrow

echo at $t = h_2 \tau / (h_2 - h_1) = 10\tau$

The shape of the echo also comes out right!

5. Longitudinal echo with diffusion

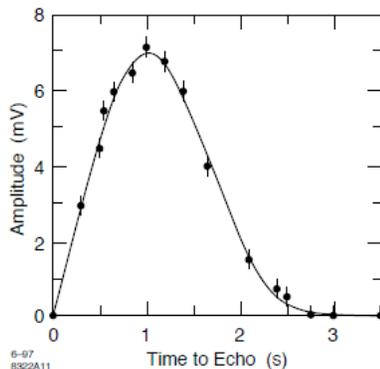
It is easy to imagine that the echo signal will be very susceptible to diffusion effects. Any small diffusion can mess up the intricate phase space that echoes rely on.

This is why echoes occur only in proton storage rings. Electrons don't echo because of their very strong diffusion due to synchrotron radiation. They lose their memory way too quickly.

With a diffusion coefficient D_0 ,

$$I^{\text{echo}} \text{ ampl} = \frac{1}{2} I_0 \frac{eV_1}{E_0} \frac{eV_2}{E_0} \frac{h_1 \eta c \tau}{R \sigma \delta} \xi e^{-\xi^2/2} \exp \left[-\frac{1}{3} \frac{\eta^2 c^2}{R^2} D_0 \frac{h_1^2 h_2}{h_2 - h_1} \tau^3 \right]$$

$$\xi = \frac{\eta c \sigma \delta (|h_2| - |h_1|)}{R} (t - t^{\text{echo}})$$



Fitting the AA data above yields $D_0 = 1.3 \times 10^{-10}/\text{s}$.
(Would be a straight line in case of no diffusion.)

Figure 11: Measured maximum echo amplitude as a function of t^{echo} in the AA ring. Diffusion is playing an important role.

6. Echo for X-ray free electron laser

Echo can be used to generate microbunching in a free electron laser!

[G. Stupakov, 2008]

- FEL echo is analogous to the longitudinal echo except that it is now applied to a **linac** instead of a storage ring. → It applies to electrons as well as protons (no synchrotron radiation in linacs).
- Our longitudinal echo analysis calculates only the linear effects to first order in the modulation strengths V_1 and V_2 . As a result, echo occurs only at low harmonic h_2-h_1 . For X-ray FEL echo, we wish to generate echo frequency at high harmonics $n_1h_2+n_2h_1$. This requires terms nonlinear in V_1 and V_2 . → FEL echo is therefore intrinsically a nonlinear effect.
- Being a nonlinear effect, however, it opens up another possibility of a “steady state echo” back in a storage ring but this time including electrons. → It is possible to use echo to generate microbunching in an **electron** storage ring, which has the advantage of a very high repetition rate. [Ratner, Chao, 2009]

7. Spin motion of a single particle

Consider a single particle near a depolarization resonance

$$\text{spin tune } G\gamma = \kappa$$

(= integer, or integer $\pm \nu_y$, etc)

where $G = (g-2)/2$ is anomalous magnetic moment, γ is Lorentz energy factor.

Let the resonance strength = ε (a complex dimensionless quantity, Fourier component of perturbing magnetic fields)

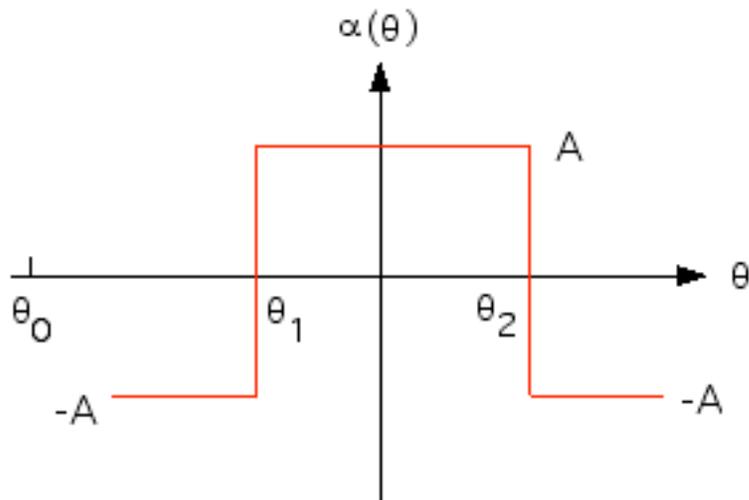
Let the spin tune of the particle be near the resonance according to

$$G\gamma = \kappa + \alpha(\theta)$$

$\alpha(\theta)$ is deviation from resonance, and is function of time $\theta = (\text{number of turns}) \times 2\pi$

The simplest crossing of a resonance is a sudden spin tune jump. Successively jumping across a depolarization resonance twice produces interesting spin dynamics effects.

$\alpha(\theta)$ is piecewise constant.



By adjusting these parameters, the beam polarization after the second jump exhibits a wealth of effects of constructive interference, destructive interferences and spin echo.

Spinor equation of motion

$$\frac{d\psi}{d\theta} = -\frac{i}{2} \begin{bmatrix} -G\gamma & \epsilon e^{i\kappa\theta} \\ \epsilon^* e^{-i\kappa\theta} & G\gamma \end{bmatrix} \psi$$

Define

$$\psi = e^{\frac{i}{2}[\kappa\theta + \beta(\theta)]\sigma_y} \begin{bmatrix} f(\theta) \\ g(\theta)e^{i\beta(\theta)} \end{bmatrix}$$

$$\beta(\theta) = \int_{\theta_0}^{\theta} d\theta' \alpha(\theta') \quad \sigma_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

The particle's polarization along the vertical y -axis is

$$P_y(\theta) = \psi^\dagger \sigma_y \psi = |f(\theta)|^2 - |g(\theta)|^2$$

8. Spin echo

For a single particle, an interference occurs between any two resonance crossings even if they are far apart. Interference can be constructive or destructive.

For a beam of particles, its behavior is more complicated because it has an energy spread σ_δ .

Averaged over the energy spread, polarization of the beam is found to be

$$\begin{aligned}
 P_y(\theta < \theta_1) &\approx \frac{A}{\Omega} \\
 P_y(\theta_2 > \theta > \theta_1) &\approx \frac{A}{\Omega^3} \left\{ A^2 - |\epsilon_0|^2 + 2|\epsilon_0|^2 e^{-\frac{A^2 \kappa^2 \sigma_\delta^2}{2\Omega^2} (\theta - \theta_1)^2} \cos \Omega(\theta - \theta_1) \right\} \\
 P_y(\theta > \theta_2) &\approx \frac{A}{\Omega^5} \left\{ (A^2 - |\epsilon_0|^2)^2 + 2|\epsilon_0|^4 e^{-\frac{A^2 \kappa^2 \sigma_\delta^2}{2\Omega^2} (2\theta_2 - \theta - \theta_1)^2} \cos \Omega(\theta - \theta_1) \right. \\
 &\quad - 2A^2 |\epsilon_0|^2 e^{-\frac{A^2 \kappa^2 \sigma_\delta^2}{2\Omega^2} (\theta - \theta_1)^2} \cos \Omega(\theta + \theta_1 - 2\theta_2) \\
 &\quad + 2|\epsilon_0|^2 (A^2 - |\epsilon_0|^2) e^{-\frac{A^2 \kappa^2 \sigma_\delta^2}{2\Omega^2} (\theta - \theta_2)^2} \cos \Omega(\theta - \theta_2) \\
 &\quad \left. + 4A^2 |\epsilon_0|^2 e^{-\frac{A^2 \kappa^2 \sigma_\delta^2}{2\Omega^2} (\theta_2 - \theta_1)^2} \cos \Omega(\theta_2 - \theta_1) \right\}
 \end{aligned}$$

shock response to first jump (pointing to the first exponential term in the second equation)

shock response to second jump (pointing to the first exponential term in the third equation)

interference term (pointing to the second exponential term in the third equation)

echo terms (pointing to the last two terms in the third equation)

where $\Omega = \sqrt{A^2 + |\varepsilon|^2}$.

- The echo term occurs at a time τ after the second jump ($\tau =$ time between the two jumps).
- There is only one echo. Waiting longer does not give more echoes.
- The echo signal maximizes when $A = |\varepsilon_0|/2$. The maximum value is $P_y(\text{echo, max}) = (4/5)^{5/2} = 57\%$.
- Echo signal oscillates with frequency Ω .

9. Proposed spin experiments

No spin echo or spin interference experiments have been done so far. Here are proposals:

COSY synchrotron (Germany)

2.1 GeV/c polarized protons

$\sigma_\delta = 10^{-4}$ (with electron cooling)

$\kappa = 4.4$

$f_c = 1.5$ MHz

$N_{\text{jump}} < 100$

Polarized protons initially 100% polarized, brought adiabatically to launching position $-A$ below the resonance $G\gamma=\kappa$, jump across to $+A$, wait for time τ , jump back to $-A$, then measure final polarization.

Adjustable parameters: A , τ , $|\epsilon_0|$.

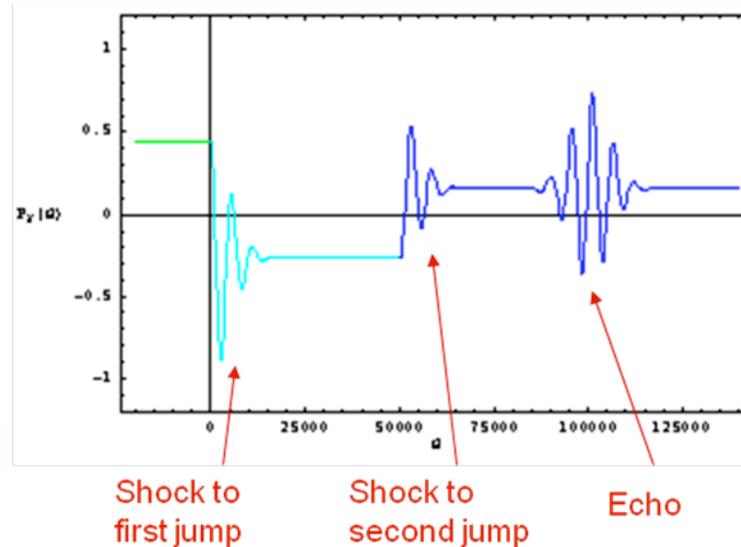
With rf dipole, the jumps are done by switching rf frequency, $|\epsilon_0|$ controlled by rf dipole strength. The rf dipole is turned on during the entire process.

Echo experiment

$$|\epsilon_0| = 10^{-3}$$
$$A = 0.5 \times 10^{-3}$$

$$\frac{1}{f_c^2} \frac{df}{dt} > 10^{-5}$$

$$\tau = \theta_2 - \theta_1 = 2\pi \times 8000$$



- To dramatize the echo, choose a large τ .
- Echo oscillates with Ω .
- Echo signal maximized by $A = |\epsilon_0|/2 \Rightarrow 57\%$.
- Polarimeter needs to be gated with 0.5 ms window. Statistics is problem. With 120 up- and 120 down-cycles, may be able to obtain $P_y = (57 \pm 10)\%$.

Interference experiment

Interference between the two spin tune jumps is destructive if $\Omega\tau = 2k\pi$ and constructive if $\Omega\tau = (2k+1)\pi$.

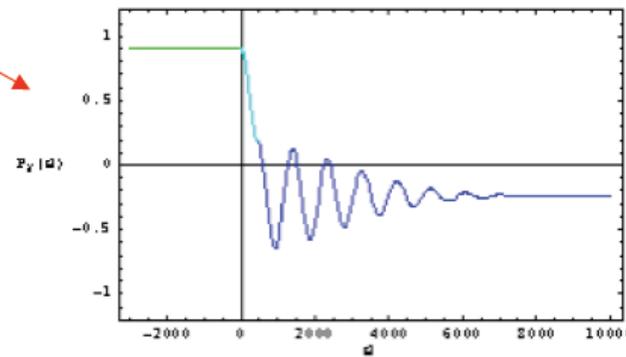
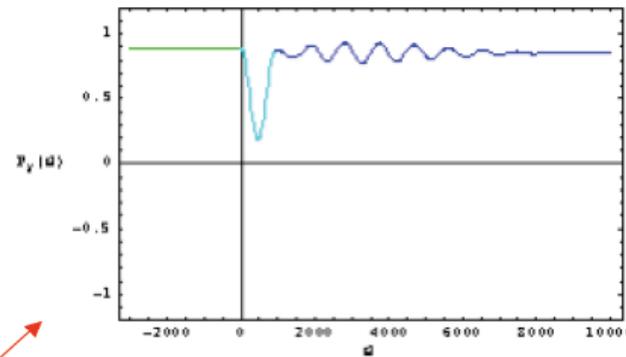
$$\tau = \begin{cases} \frac{2\pi}{\Omega} = 9.4 \times 10^3 = 1500 \text{ turns} & \text{for destructive interference} \\ \frac{\pi}{\Omega} = 4.7 \times 10^3 = 750 \text{ turns} & \text{for constructive interference} \end{cases}$$

$$|\epsilon_0| = 3 \times 10^{-4}$$

$$A = 6 \times 10^{-4}$$

- Final polarization is expected to depend sensitively on τ , indicating strong interference.

- Small echo signals are present.



Echo experiment proposal at RHIC

[M. Bai et al, 2008]

Computer simulation of the RHIC experiment:

Left: $\tau = 2\pi \times 40,000$.

Right: $\tau = 2\pi \times 300,000$, the amplitude of the echo signal is still present but reduced.

