# Simulations of Plasma Dynamics in HPRF Cavity

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# Introduction

- RF power loading is an important issue for HPRF cavity

- We present simulation studies of plasma dynamics relevant to RF power loading

- SPACE, a parallel EM-PIC code for self-consistent simulation of plasmas with atomic physics processes: **IPAC 2015 MOPMN012**

- Simulations studies related to HPRF experimental program at MTA: **IPAC 2014 MOPME043**, **IPAC 2015 MOPMN013**

- Simulations suggest ion-electron recombination rate, electron attachment time, and ion-ion recombination rate

# Code SPACE

- Parallel Electromagnetic PIC code **SPACE** for relativistic particles and ele ctromagnetic fields

- Main novelty:
  - Fully relativistic treatment of particles
  - Resolution of atomic physics processes / plasma chemistry
  - Interaction of plasma with neutral matter
  - Advanced numerical solutions
    - Approximations enabling long physical time simulations
    - Adaptive refinement by variable particle mass / charge
    - Data transfer algorithms between relativistic moving and laboratory frames, transforming particles to the same physical time
  - Implementation for modern multicore supercomputers

- **Support of BNL RHIC projects**
- Use of plasma for the mitigation of beam-beam effects
- Simulations of Coherent Electron Cooling

# Outline of Plasma Loading Simulation

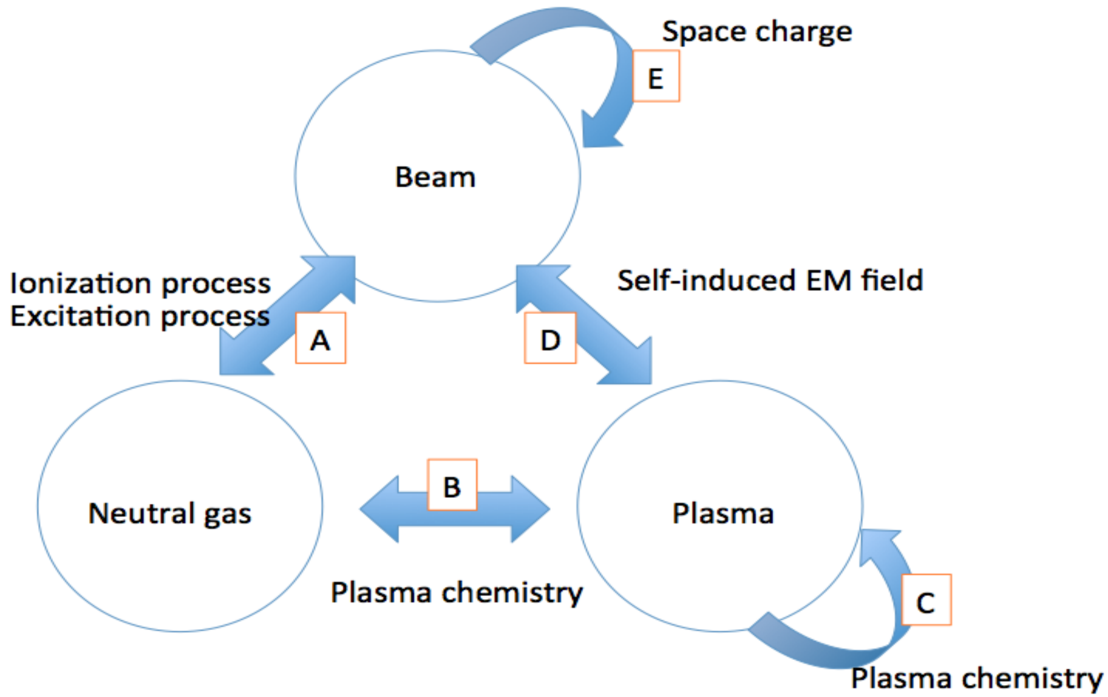



Figure. Interaction Chart. Courtesy K. Yonehara

Main processes

- A: Neutral gas ionization by beam (Bethe-Bloch Formula)
- B: Plasma dynamics in neutral gas (e.g. low mobility of plasma particles)
- C: Plasma chemistry: recombination and attachment processes
- External field gradient drop in cavity
- D, E: beam-plasma interaction

# Main Parameters
## in Simulations and Experiments

| Parameter | Units | Value |
|---|---|---|
| Kinetic Energy of Beam | MeV | 400 |
| Initial Velocity of Beam | m/s | 2.13728e+8 |
| $\beta$ | % | 71.292 |
| $H_2$ Gas Pressure | atm (psi) | 20.4 (300) |
| dE/dx | MeV $cm^2$ / g | 6.332 |
| W (Average Ionization Energy) | eV | 36.2 |
| Electric Field (Frequency) | MV/m (MHz) | 8.8 (808.45) |
| Bunch Population | # / bunch | 1.61e+8 ~ 2.09e+8 |
| Bunch Spacing | nanosecond | 5 |
| # of Bunches | # | 1500 ~ 2000 |

# Plasma Loading

$$\frac{d\,\mathrm{V(t)}}{dt} = \frac{V_0 - \mathrm{V(t)}}{RC} - \frac{P}{C\,\mathrm{V(t)}}$$

$V_0$ is peak voltage, $V(t)$ is RF amplitude at time $t$

$$P = \frac{dw}{T} \times n$$ is the power consumption in the cavity

$$dw = \int_0^T q\mu E^2 \sin^2(\omega t)dt = \frac{1}{2} q\mu E^2 T$$ is the average power dump by one ion–electron pair during one period of the external field

$\mu$ is the electron mobility

$n$ is the plasma density, $R = 1.41\ \mathrm{M\Omega}$, $C = 1.49$ pF

**The density of electrons and ions strongly affect plasma loading**

# Motion of plasma electrons in neutral gas

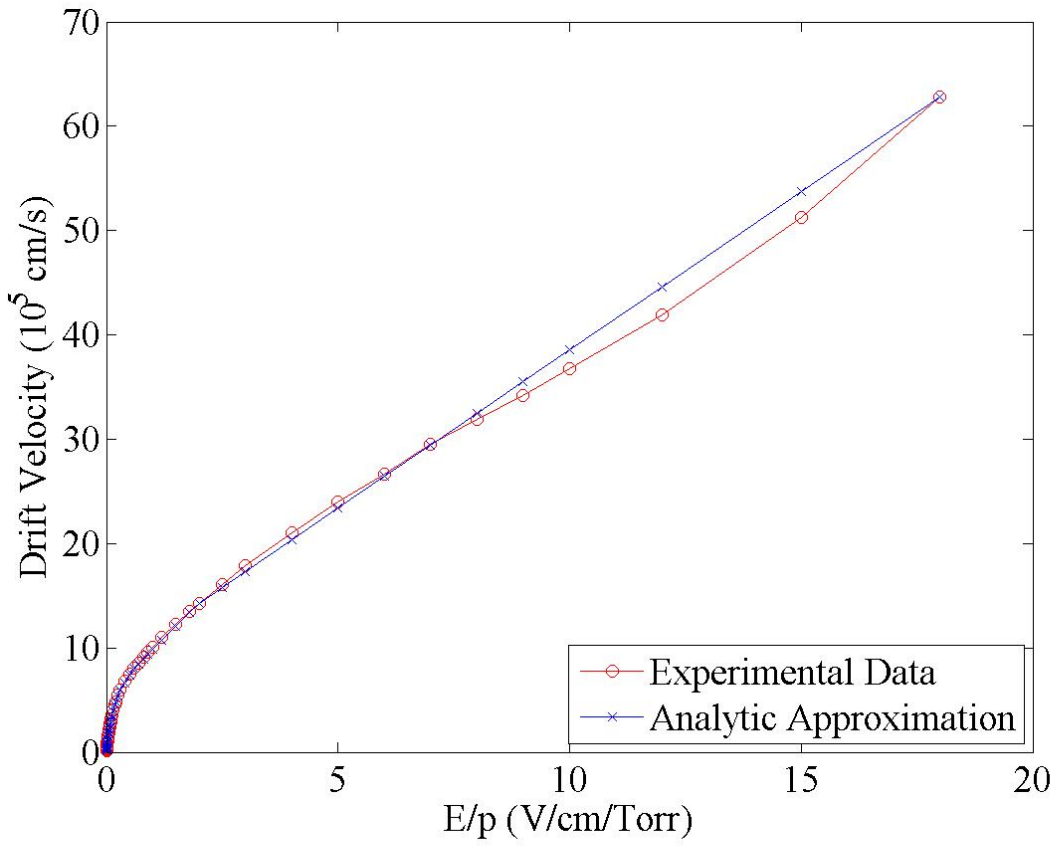



Figure : Drift Velocity of Electrons in $H_2$ gas at 293K.
J.J. Lowke. The Drift Velocity of Electrons in Hydrogen and Nitrogen. Aust. J. Phys., 16:115–135, 1962.

# External Field Profile

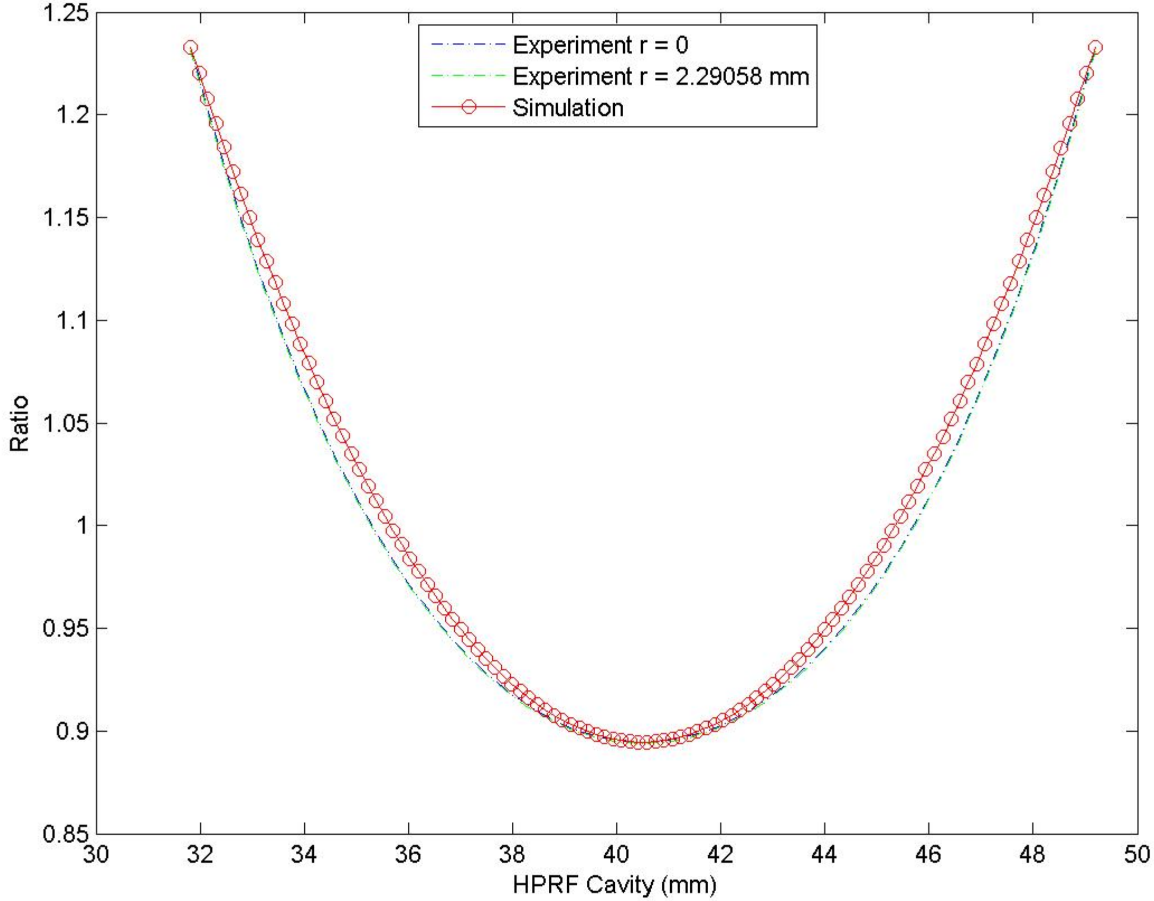



Fig. External Field Profile in HPRF cavity

Appr. Formula : Ratio = $0.00448361291\ z^2 - 0.3631830091\ z + 8.24927707$

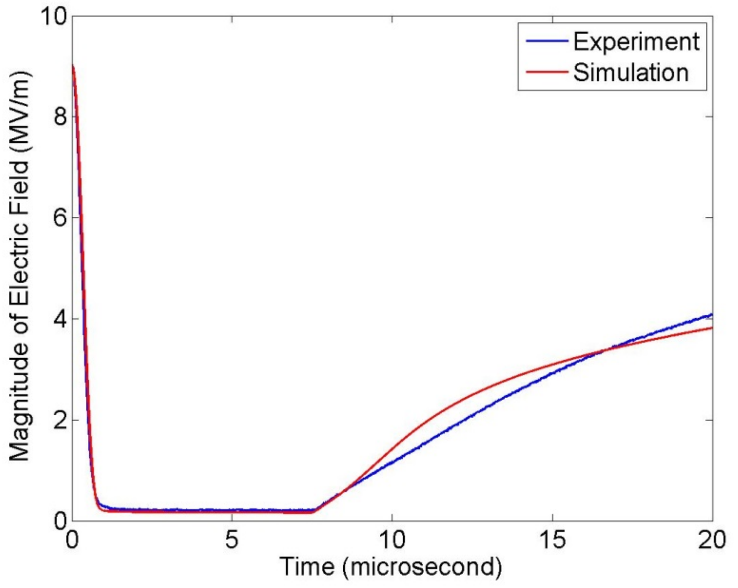
Max. Errors are less than 2% in both r=0 and r=2.3 mm

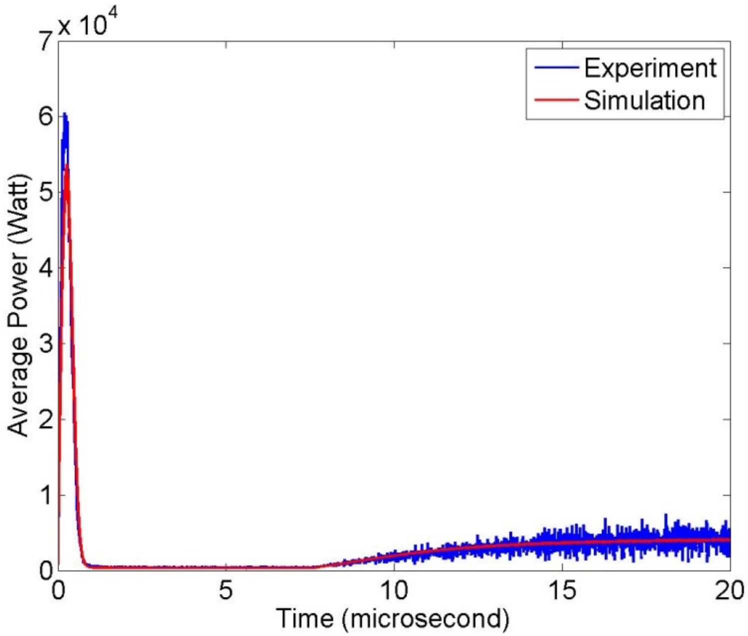
# HPRF Cavity Loading

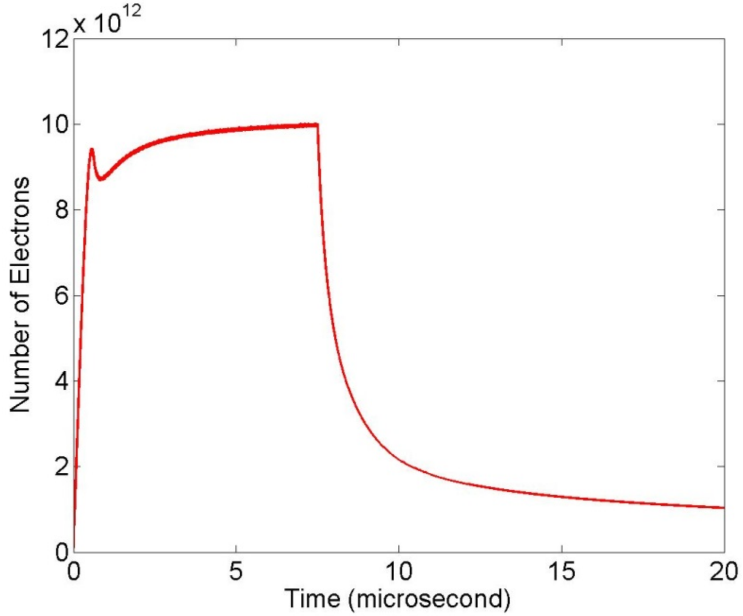
# I: Cavity Filled with Pure Hydrogen

# Simulation Results

- Various coefficients were tested.
- The most accurate fitting curve is
  **$\beta = 1.5\ 10^{-10}\ X^{-1.0}$**.
- Beam off at 7.5 .

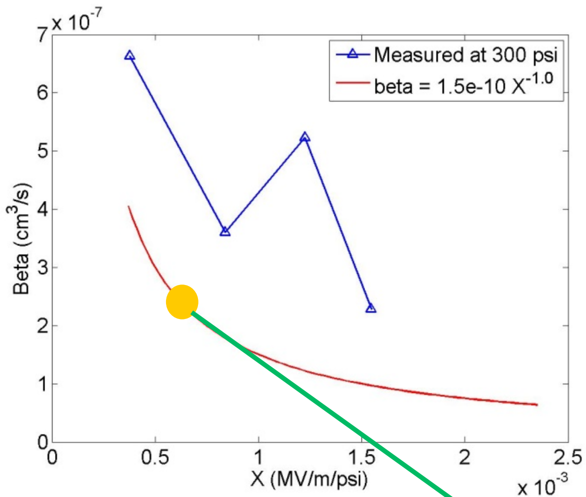


# Recombination Rate (β)

$\beta = a X^b$ where $X = E/P$ (MV/m/psi)




Fitting curve at 300 psi

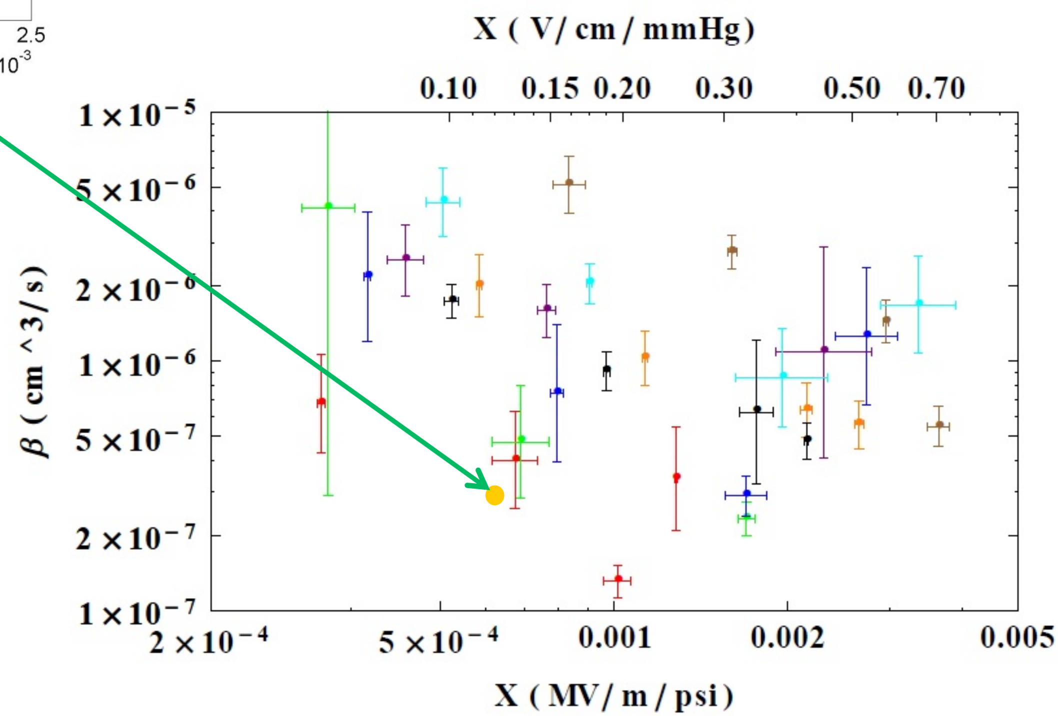



Figure from
B. Freemire's thesis (Fig. 3.24)

# Pure Hydrogen Test: Conclusions

- Electron – ion recombination is slow
- Large plasma (electron) density ($10^{13}$ 1/cc) remains in the cavity in the presence of the beam
- Significant (unacceptable) reduction of electric field
- To mitigate plasma loading, dry air dopant was used in the cavity
- We present simulations with 1% dry air dopant at 300 psi

# Plasma Chemistry in the Presence of Dopant

$$\frac{dn_e}{dt} = N - \beta_e\, n_e \sum n_{H_n{}^+} \qquad (2)$$

Previous equation (2) is replaced with the following system

$$\frac{dn_e}{dt} = N - \beta_e\, n_e n_{H^+} - \frac{n_e}{\boldsymbol{\tau}} \qquad (3)$$

$$\frac{dn_{H^+}}{dt} = N - \beta_e\, n_e n_{H^+} - \boldsymbol{\eta}\, n_{H^+} n_{O_2{}^-} \qquad (4)$$

$$\frac{dn_{O_2{}^-}}{dt} = \frac{n_e}{\boldsymbol{\tau}} - \boldsymbol{\eta}\, n_{H^+} n_{O_2{}^-} \qquad (5)$$

where $n_{H^+} = \sum n_{H_n{}^+}$ : the sum of all hydrogen ion cluster numbers,

$\boldsymbol{\eta}$: the effective recombination rate of hydrogen ion clusters and oxygen ion,

$\boldsymbol{\tau}$ is the attachment time of electrons on the dopant molecules.

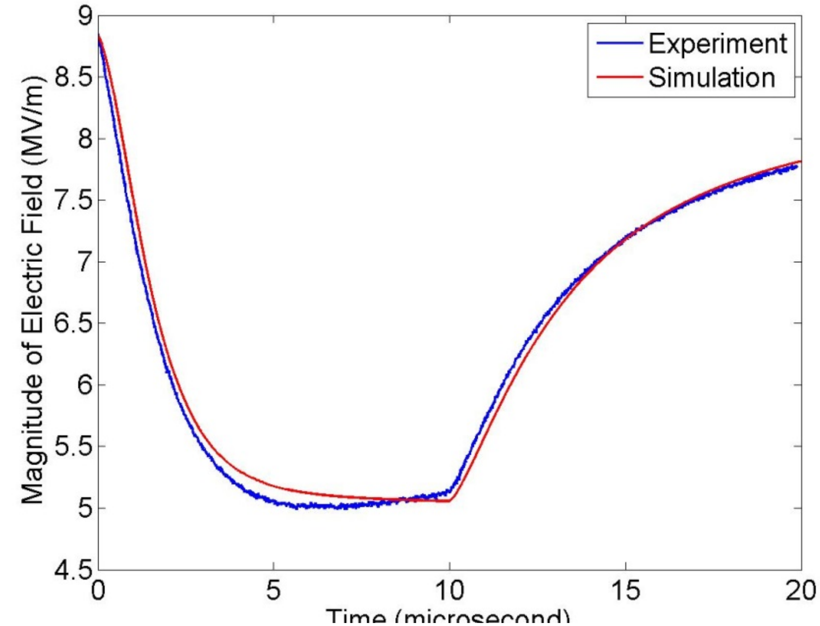
$\frac{1}{\boldsymbol{\tau}} = \sum_m k_m n_{O_2} n_m$ , m is one of $H_2, O_2, or\ N_2$,

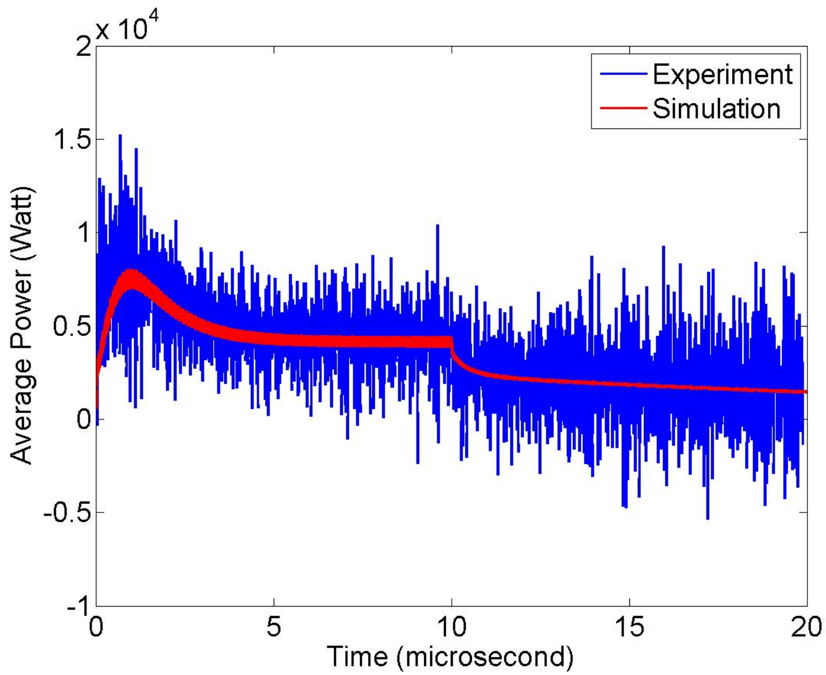
By the previous analysis, ion loading effect is significant after early time.

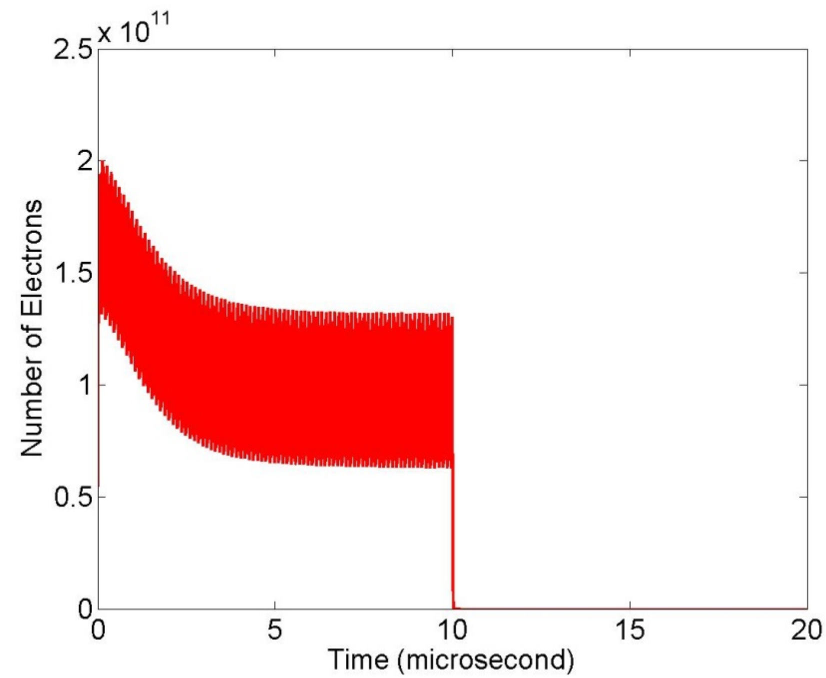
➡ **Ion loading effect is added.**

# Simulation Results

- Various coefficients were tested.
- The most accurate combination is
  $\tau = 4.0 * 10^{-7} X^{1.0}$
  $\eta = 1.6 * 10^{-10} X^{-1.0}$.
- Hydrogen ion mobility is **20.0** $cm^2/V/s$
- Beam off at 10 $\mu s$.

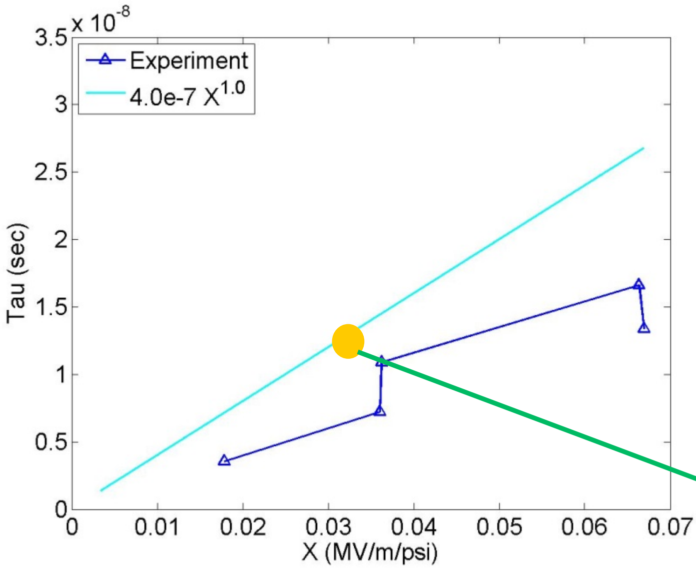


# Attachment Time (τ)

$\tau = a X^b$ where $X = E/P$ (MV/m/psi)




Fitting curve at 300 psi with 1% dry air dopant

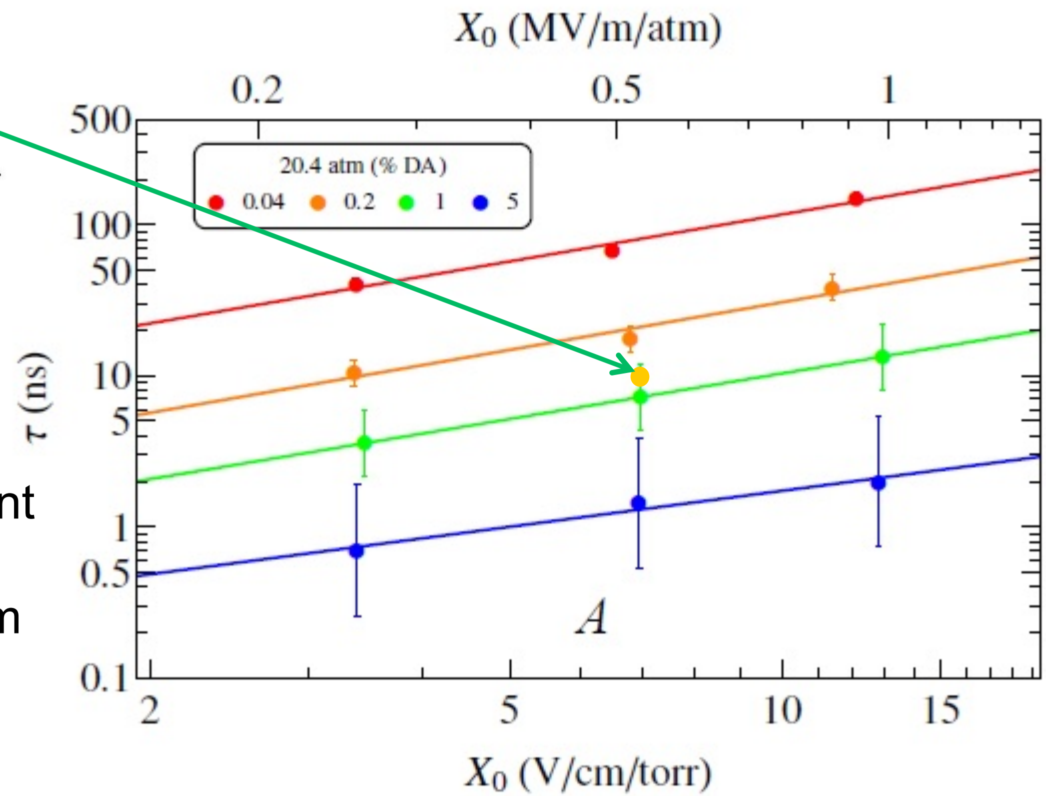



**Figure. IPAC 2014 THPRI064, Fig. 2**
Measurements of electron attachment time to oxygen in hydrogen (points) and fits to the data (lines) at 20.4 atm for various dopant concentrations

# Mobility of Plasma Ions

Table. Values of ion mobility (B. Freemire's thesis)

| Ion | Reduced Mobility ($\frac{cm^2}{V\,s}$) |
|---|---|
| $H_3^+$ | 11.2 |
| $H_5^+$ | 9.6 |
| $O_2^-$ | 11.4 |

Table. Ion mobility from Physics Review Volume 114 Num. 2, Apr. 15, 1959

TABLE I. Calculated mobilities at standard gas density (0°C, 1 atmos).

| $T$ (°K) | $K_0$ (cm² volt⁻¹ sec⁻¹) $H_3^+$ in $H_2$ | $H_2^+$ in $H_2$ | $H^+$ in $H_2$ |
|---|---|---|---|
| 0 | 14.2 | 15.6 | 19.3 |
| 50 | 15.1 | 14.8 | 19.0 |
| 100 | 16.2 | 14.6 | 18.8 |
| 150 | 17.9 | 14.4 | 18.7 |
| 200 | 19.5 | 14.2 | 18.6 |
| 300 | 22.0 | 13.9 | 18.3 |
| 400 | 23.4 | 13.6 | 18.0 |
| 500 | 24.4 | 13.3 | 17.8 |

# Ion – Ion Recombination Rate (η)

$\eta = a\,X^{b}$ where $X = E/P$ (MV/m/psi)

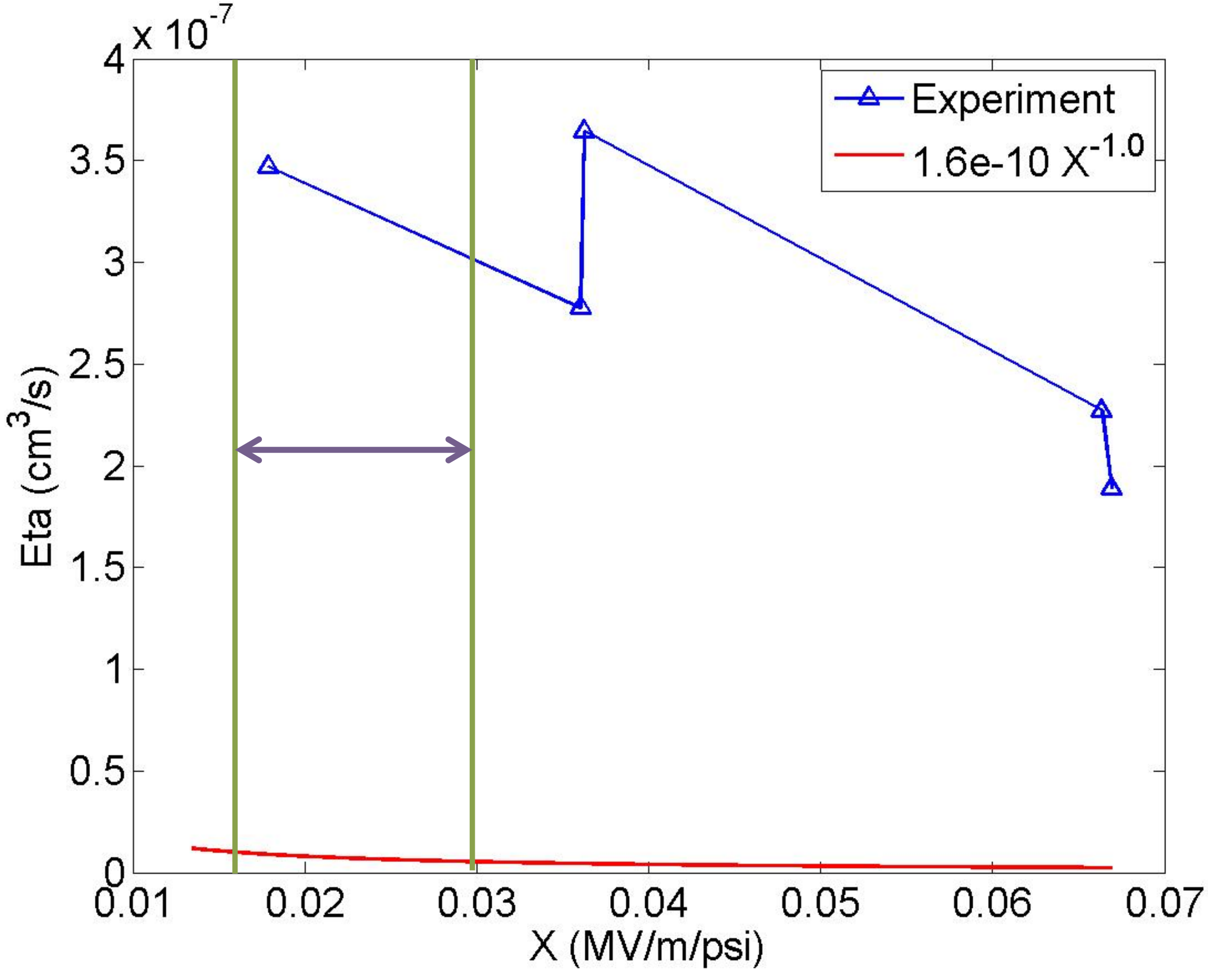



Figure. Ion – Ion Recombination Rate at 300 psi with 1% dry air dopant
The right green vertical bar denotes X at the initial time. (8.8 MV/m / 300 psi => 0.029)

# Electron Density in Simulations

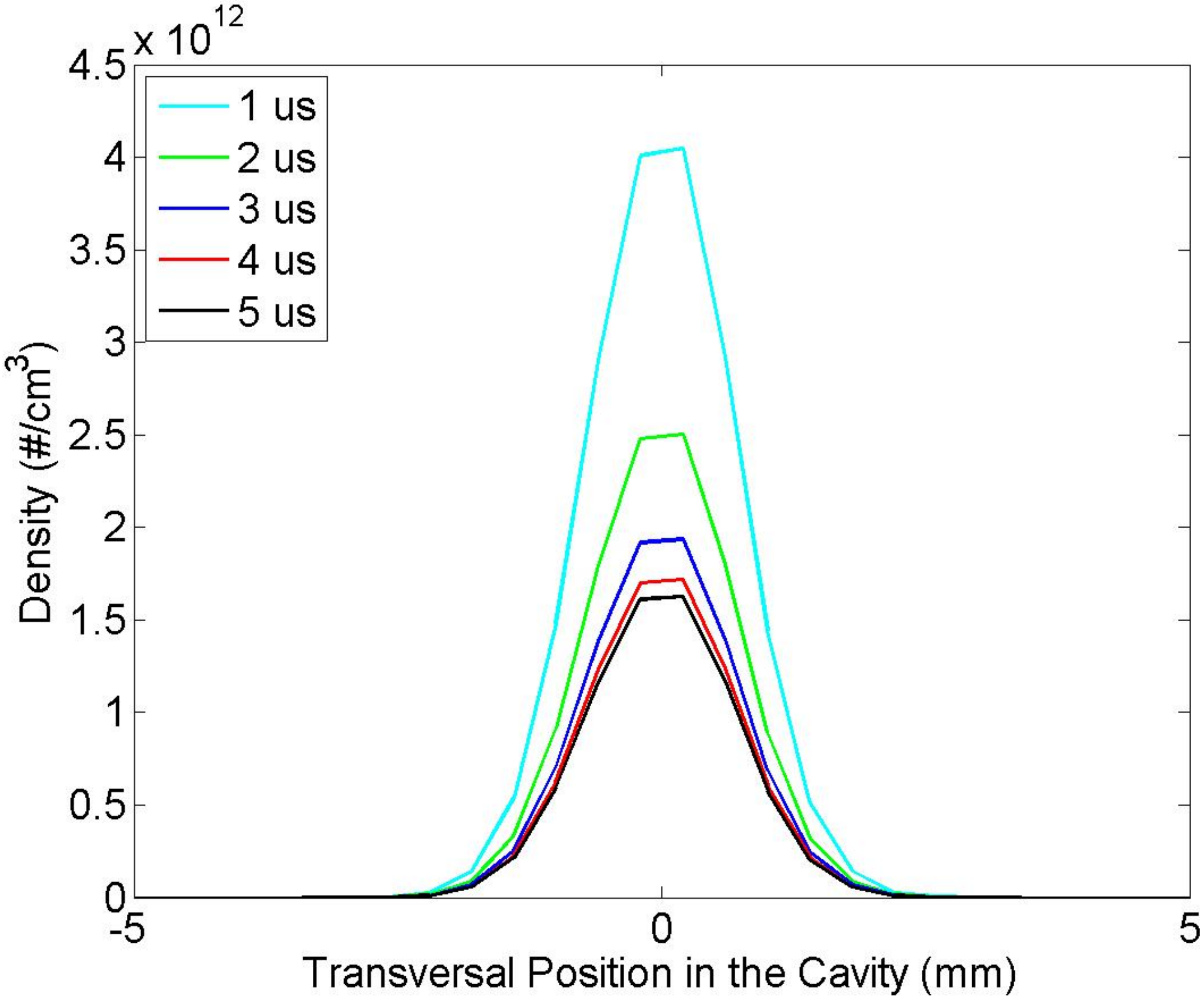
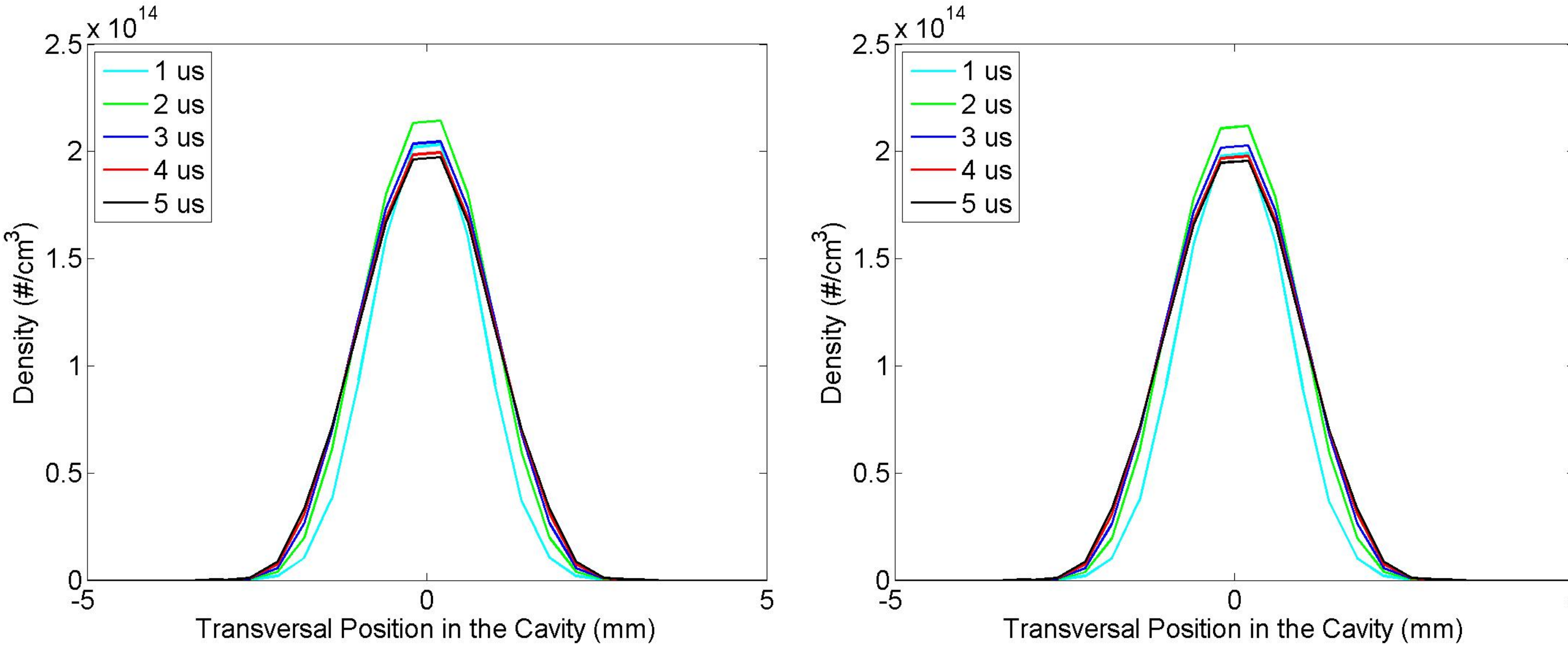



Figure. Electron density evolution in the center of the cavity.

# Ion Densities in Simulations




$H_n^+$density $O_2^-$ density

Figure. Evolution of ion densities in the center of the cavity.

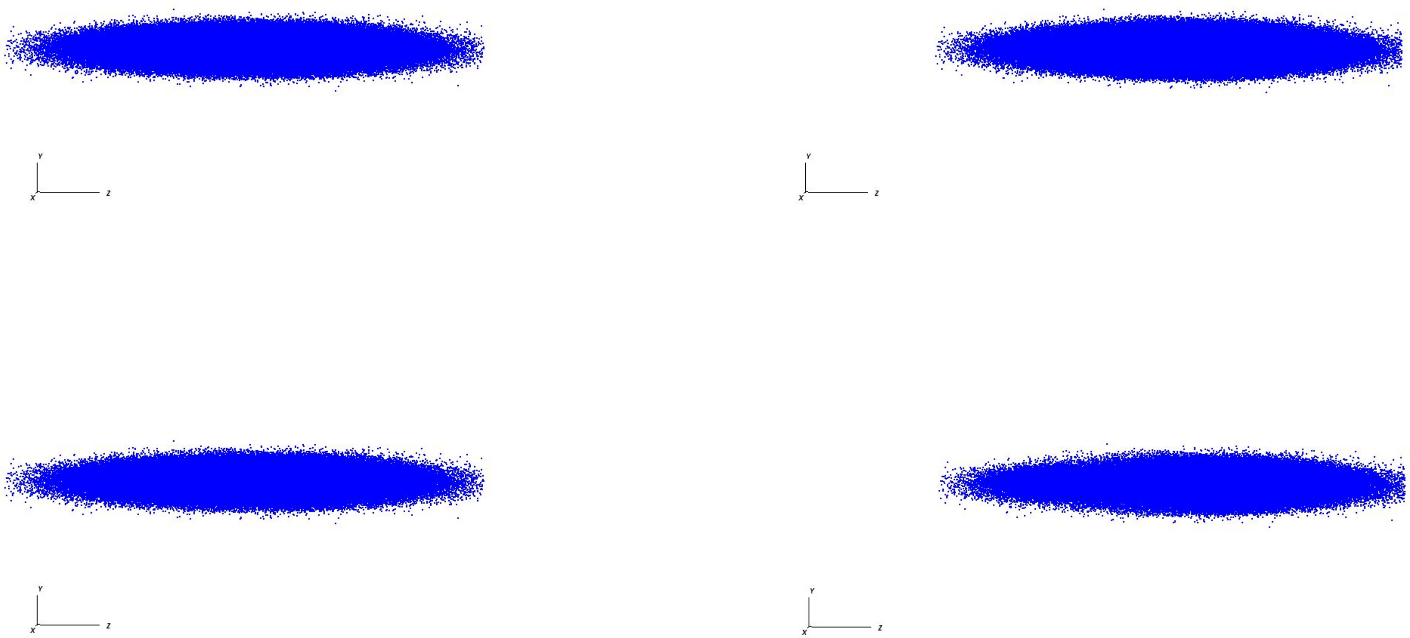
# APPENDIX
# Muon Beam-Plasma Simulation: Physical Parameters

| Parameter | Units | Value |
|---|---|---|
| Kinetic Energy of Beam | MeV | 121 |
| Initial Velocity of Beam | m/s | 2.65e+8 |
| β | % | 88.42 |
| $H_2$ Gas Pressure | atm (psi) | 160 (2351) |
| dE/dx | MeV $cm^2$ / g | 4.494 |
| W (Average Ionization Energy) | eV | 36.2 |
| Bunch Population | # / bunch | 5.0e+12 |
| Beam size (Length / Radius) | cm / mm | 3 / 2 |
| Cavity Length | cm | 5 |

# Muon Beam – Plasma interaction

In Vacuum

In $H_2$ Gas



150 picosecond

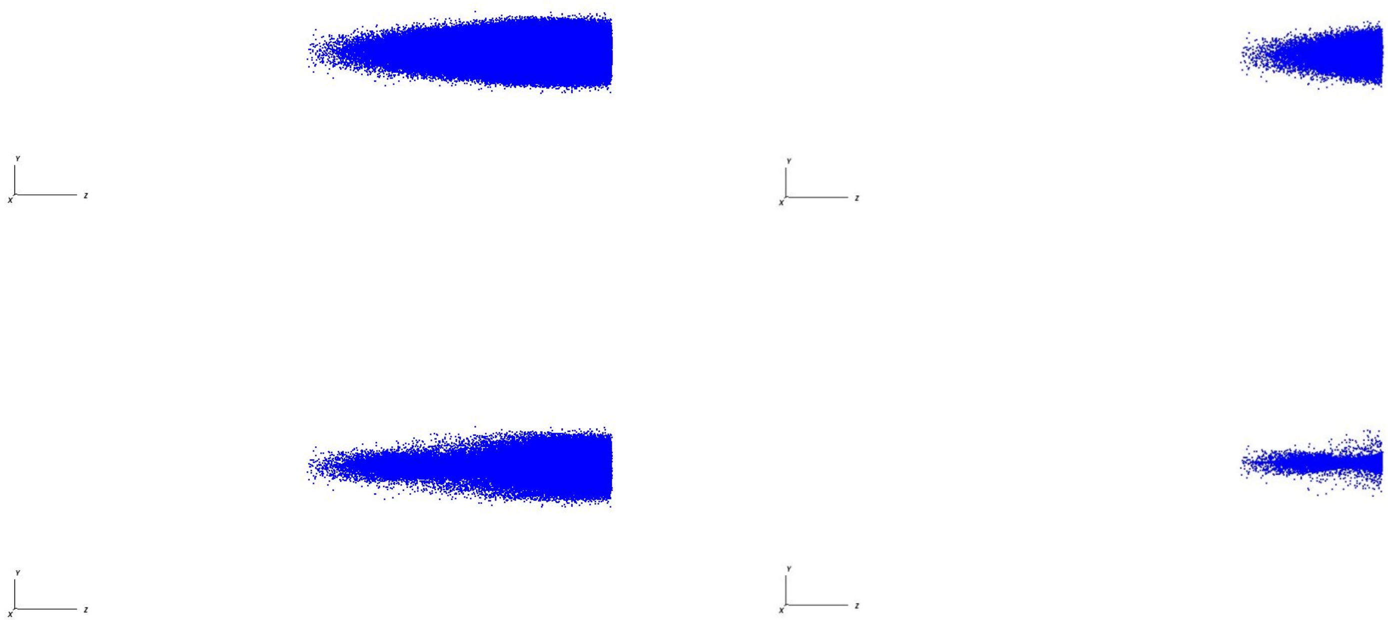
200 picosecond

Figure. 3 cm beam (2 mm radius) in 5 cm cavity, Plasma is invisible

# Muon Beam – Plasma interaction

In Vacuum

In $H_2$ Gas



250 picosecond

300 picosecond

Figure. 3 cm beam (2 mm radius) in 5 cm cavity, Plasma is invisible

# Main conclusions

- Simulations suggest accurate fitting function for β ($1.5*10^{-10}\ X^{-1.0}$) in pure hydrogen case (300 psi)
- In the case of dry air dopant, accurate results were obtained with τ ($4.0 * 10^{-7}\ X^{1.0}$) and η ($1.6 * 10^{-10}\ X^{-1.0}$)
  - τ (attachment time) is consistent with measured values
  - Ion mobility is close to measured / calculated values
  - η is far from measured values
  - τ is dominant at the early time; η is dominant after beam off
- Simulations help to reduce uncertainties / adjust values of measured quantities. After validation, simulations can be used for predictions
- Preliminary simulations show strong influence of plasma on intense muon beams.