Proton Decay Matrix Elements from Lattice QCD

T. Izubuchi on behalf of E. Shintani

Y. Aoki, A. Soni

for RBC/UKQCD collaboration

Proton decay

- Baryon asymmetric process
  - p $\longrightarrow$ $e^+ + \pi^0$, etc
  - Baryogenesis
  - GUTs scaling physics

    (in the SM, $\tau(t \to e^+ \nu_\mu \nu_\tau) \sim 10^{150}\,\mathrm{yr}$ by anomalies)
  - Mediated by colored and flavored heavy particle

- Low energy matrix element
  - p $\longrightarrow$ [PS + lep] lifetime, Br, …

    needs QCD matrix element

    $\Rightarrow$ model prediction

GUTs

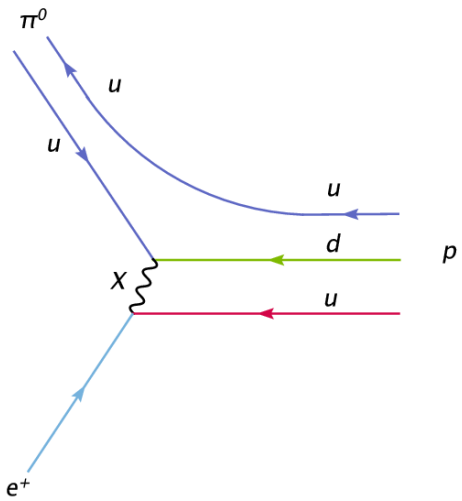
- Baryon number breakings
  - Mediated by X boson

    minimal SU(5), dim 6 Op

    predicts $\tau(p \to \pi^0 e^+) \sim 10^{30}\,\mathrm{yr}$

    ⇒ ruled out by recent SK results

$$O^i(\mu) = (qq)_\Gamma (ql)_{\Gamma'}$$

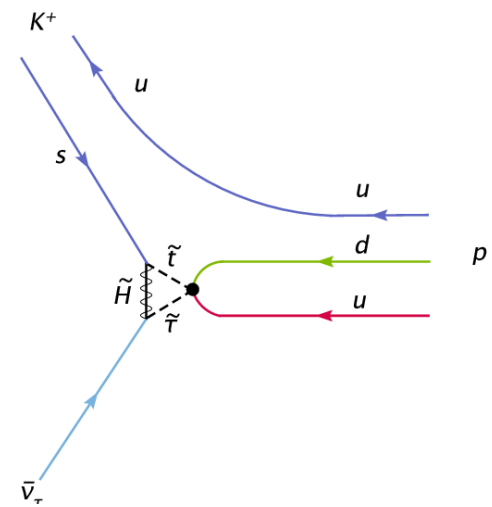


  - Mediated by colored sparticle, Higgsino (or wino)

    SUSY SU(5), dim 5 Op

    increase lifetime, dominate

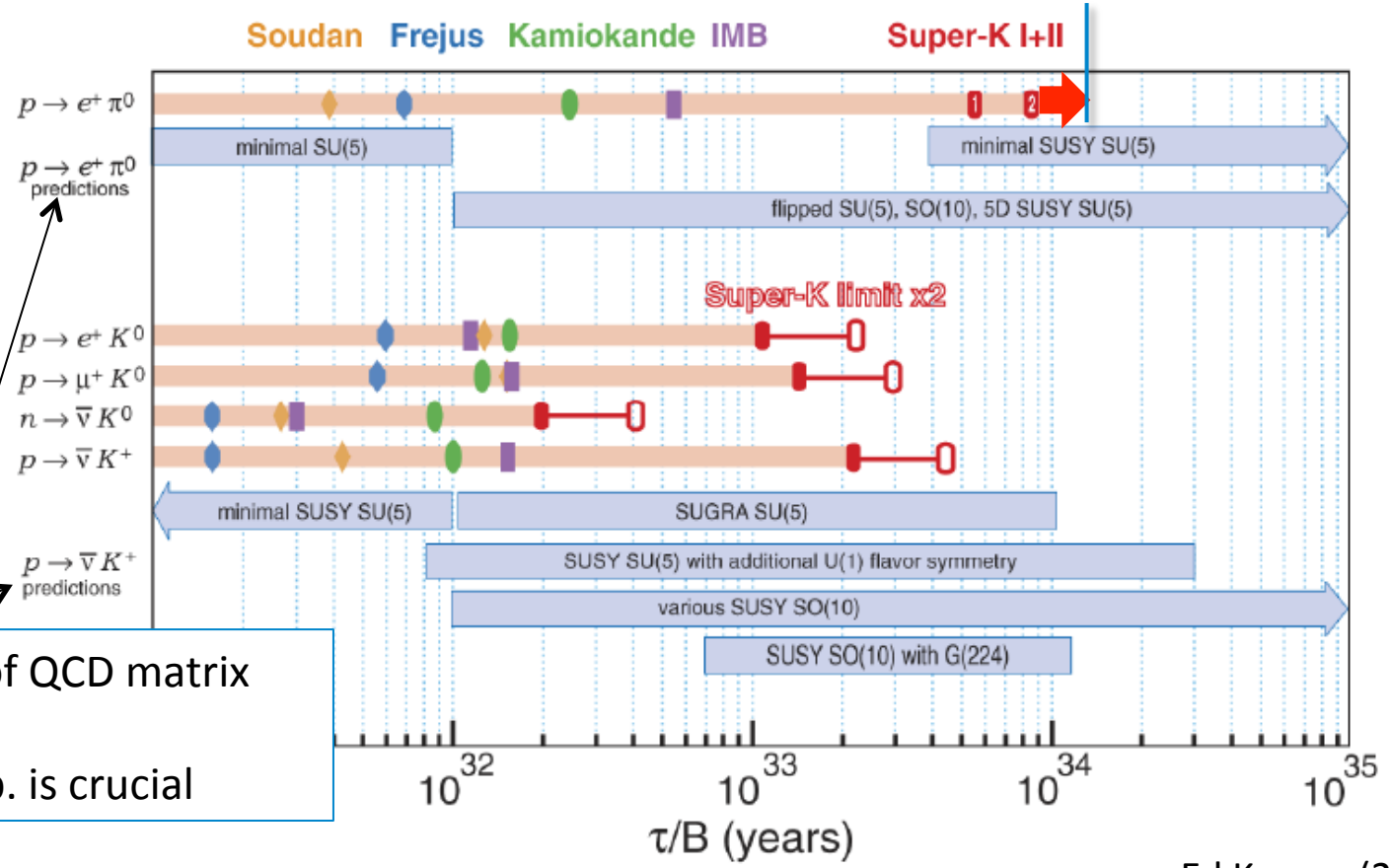
    $p \to K^+ + \bar{\nu}$ mode



Proton lifetime limit

- Recent Super-Kamiokande results:

$$\tau_{\rm exp}(p \to \pi^0 e^+) > 1.01 \times 10^{34}\,{\rm yr}\,({\rm SK},\ 2009)$$



Precision of QCD matrix element
of GUTs op. is crucial

Ed Kearns (2007)

Matrix element

- Expansion to dim 6 coupling

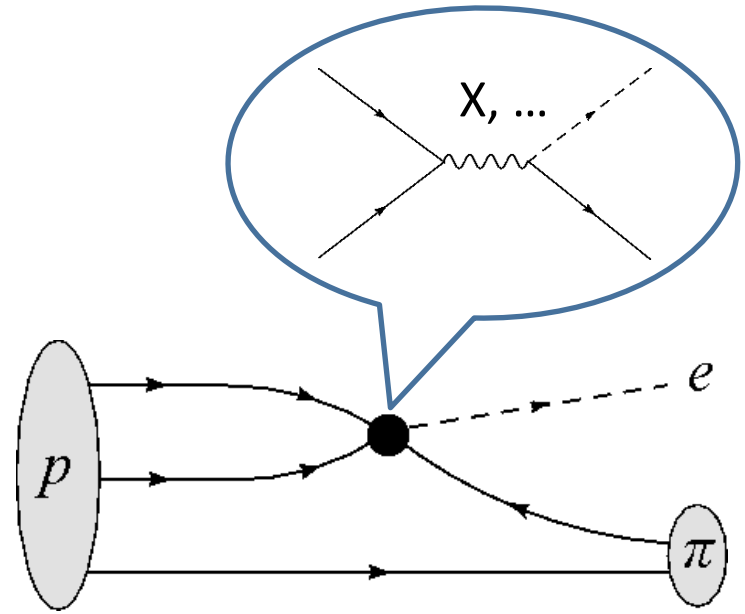
$$\mathcal{L}_{\rm GUT} = \mathcal{L}_{\rm SM} + \sum_i C_i(\mu) O_i(\mu)/M_{\rm GUT}^2$$

C_i depends on type of GUTs model

$$\langle \pi^0 e^+ | p \rangle_{\rm GUT} = \sum_{i=\Gamma,\Gamma'} C_i \langle \pi^0 | \mathcal{O}^{\Gamma\Gamma'}_{udu} | p \rangle \bar{v}^c_{e^+}$$

$\mathcal{O}^{\Gamma\Gamma'}_{abc} = (q_a q_b)_\Gamma q_{c\Gamma'}$: 3 quark operator

$C_i(\mu)O_i(\mu)/M_{GUT}^2$



$\langle PS; \vec{p} | \mathcal{O}^{\Gamma\Gamma'}_{abc} | N; \vec{k}, s \rangle \bar{v}^c_{e^+}$

- Decay width

$$\Gamma_{p\to\pi^0 e^+} = \frac{m_p}{32\pi^2}\left[1-\left(\frac{m_e}{m_p}\right)^2\right]^2 \left|\sum_i C_i W_0^i(p\to\pi^0)\right|^2$$

W_0^i : determine from QCD matrix element

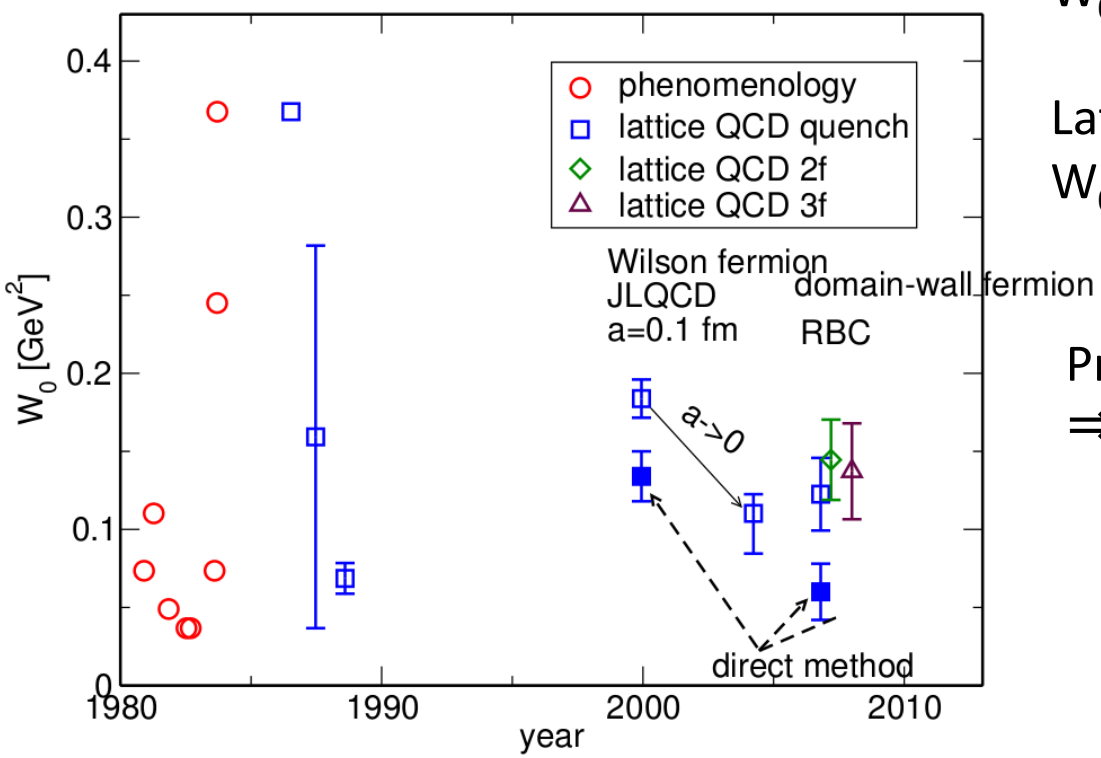
⇒ **lattice calculation is needed**

Lattice QCD

- The first principles of QCD

Determination of [p → PS] matrix element

$\langle\pi^0|(ud)_R u_L|p\rangle$



Phenomenological estimate:
$W_0\ (p\to\pi) \simeq 0.1$ -- $0.4\ \mathrm{GeV}^2$

Lattice results:
$W_0\ (p\to\pi) \simeq 0.1$ -- $0.2\ \mathrm{GeV}^2$

Proton lifetime ~ $O(m^3/W_0^2)$
⇒ factors of O(10) uncertainties

RBC/UKQCD efforts

- RBC(2007) Y. Aoki et al. PRD75 014507, hep-lat/0607002
  - Direct/indirect method in Quench DW
  - Dynamical effect (indirect method)
  - Non-perturbative renormalization (NPR)
- RBC/UKQCD(2008) Y. Aoki et al. PRD78 054505 arXiv:0806.1031
  - Indirect method in Nf=3 full QCD with dynamical DW
  - NPR
- RBC/UKQCD(2011) Y. Aoki et al. for RBC/UKQCD in prep.
  - Direct method in Nf=3 full QCD with dynamical DW
  - 2.4 fm^3 and pion mass: 315 -- 615 MeV
  - NPR

W_0 from lattice QCD

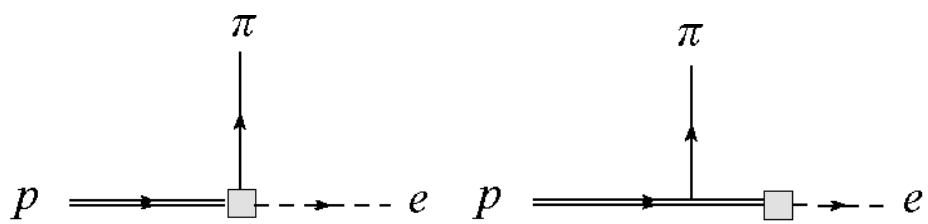
- Indirect method JLQCD(2000), Y. Aoki (2007)
  - Via soft-pion theorem,

$$\lim_{p\to 0}\langle \pi^k(p)|\mathcal{O}^{\Gamma\Gamma'}_{udu}|N\rangle = -\frac{i}{f}\langle 0|[Q_5^k, \mathcal{O}^{\Gamma\Gamma'}_{udu}]|N\rangle$$

$$\langle 0|(\bar{u}d)_L u|p\rangle = \alpha u_p, \quad \langle 0|(\bar{u}d)_R u|p\rangle = \beta u_p$$

  - Low-energy-constant is determined by 2pt function which easily and cheaply computes.
  - Through Chiral perturbation theory, matrix element

$$W_0^L(\pi^0 \to p) \simeq \frac{\alpha}{\sqrt{2}f}(1 + D + F)$$



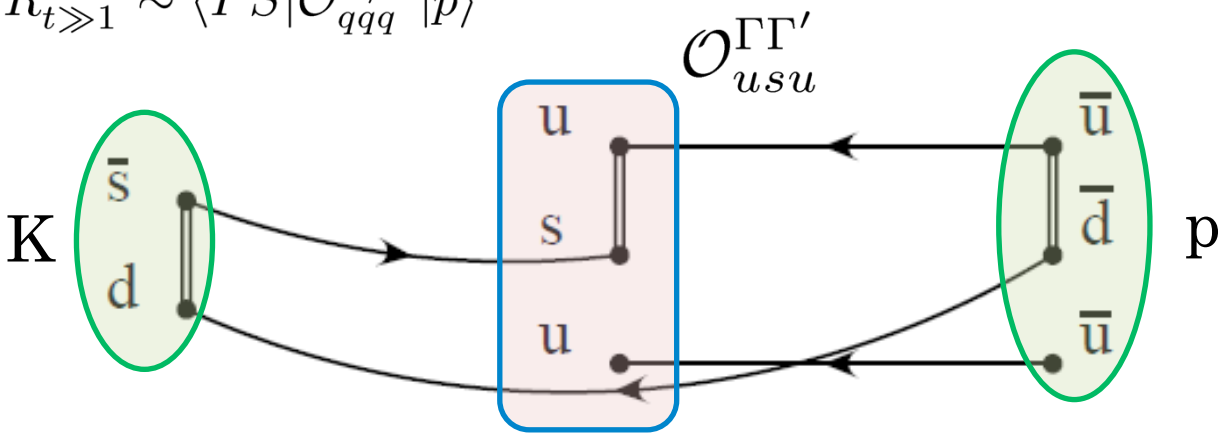
  - Uncertainties remain in soft-pion theorem from large mass and momentum

W_0 from lattice QCD

- Direct method CP-PACS(2000), Y. Aoki (2007)
  - Computation of 3pt function including B violating operator
  - Ratio provides matrix element and W_0

$$R^{L/R} = \frac{\langle 0|J_{PS}(\vec{p},t_{PS})\mathcal{O}^{L/R}_{qqq}(\vec{p},t)J_p(\vec{0},t_p)|0\rangle}{\langle 0|J_p(\vec{0},t_p)J_p(\vec{0},0)|0\rangle\langle 0|J_{PS}(\vec{p},t_{PS})J_{PS}(\vec{p},t)|0\rangle}\sqrt{Z_{PS}Z_p}V$$

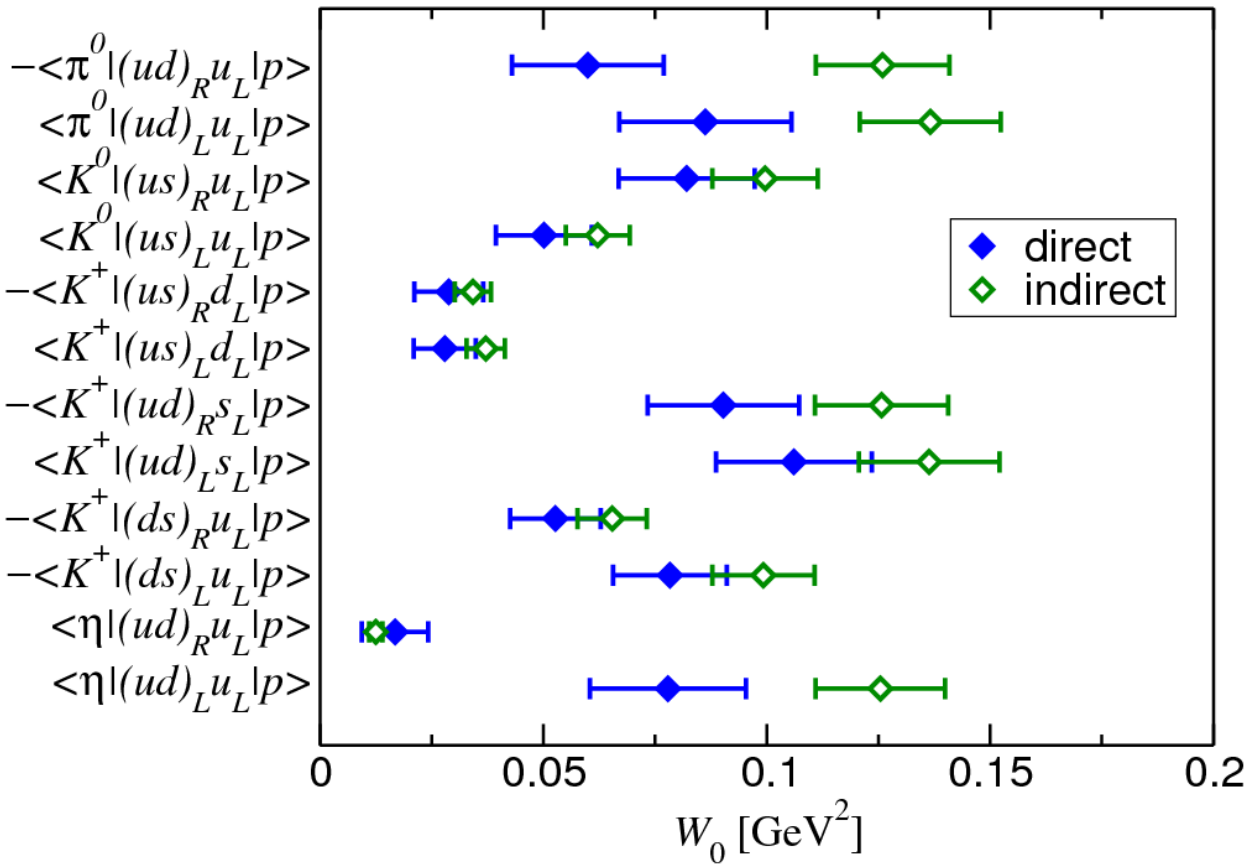
$$R^{L/R}_{t\gg 1} \sim \langle PS|\mathcal{O}^{L/R}_{qqq}|p\rangle$$



- No intermediate model including the above estimate
- Individually obtain W_0 which depends on finial meson state.

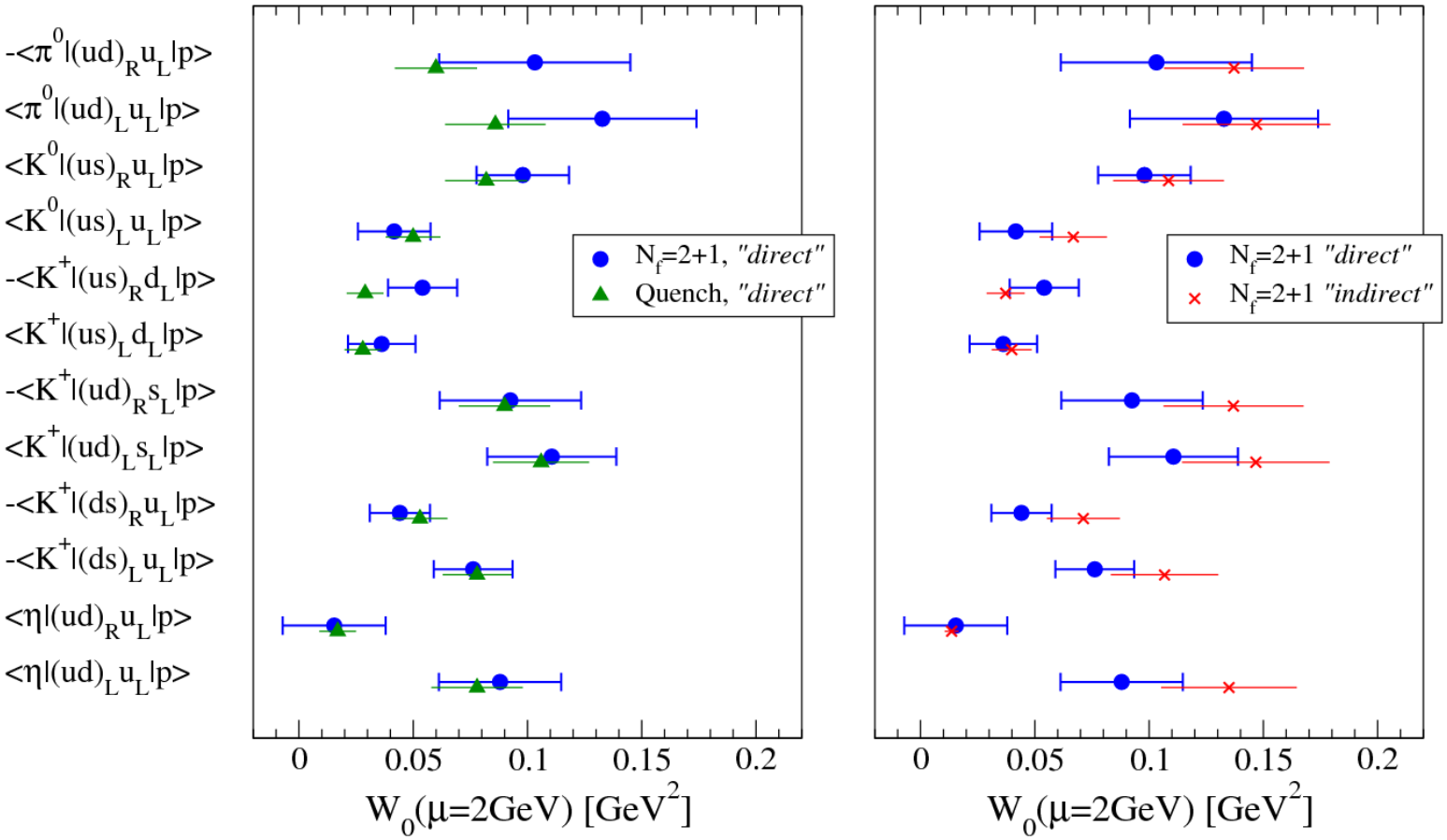
In quench QCD

Y. Aoki (2007)



In full QCD

RBC/UKQCD in prep.



- There is no significant discrepancy between each results.
- Statistical and systematic errors are still large.

Error budget

	$W_0(\mu = 2\text{GeV})$ GeV^2	Extrapolation	$\mathcal{O}(a^2)$	ΔZ	Δa^{-1}
$-\langle\pi^0\|(ud)_R u_L\|p\rangle$	0.103(23)(34)	0.033	0.005	0.008	0.004
$\langle\pi^0\|(ud)_L u_L\|p\rangle$	0.133(29)(28)	0.026	0.007	0.011	0.005
$\langle K^0\|(us)_R u_L\|p\rangle$	0.098(15)(12)	0.007	0.005	0.008	0.003
$\langle K^0\|(us)_L u_L\|p\rangle$	0.042(13)(8)	0.007	0.002	0.003	0.001
$-\langle K^+\|(us)_R d_L\|p\rangle$	0.054(11)(9)	0.008	0.003	0.004	0.002
$\langle K^+\|(us)_L d_L\|p\rangle$	0.036(12)(7)	0.007	0.002	0.003	0.001
$-\langle K^+\|(ud)_R s_L\|p\rangle$	0.093(24)(18)	0.016	0.005	0.008	0.003
$\langle K^+\|(ud)_L s_L\|p\rangle$	0.111(22)(16)	0.012	0.006	0.009	0.004
$-\langle K^+\|(ds)_R u_L\|p\rangle$	0.044(12)(5)	0.003	0.002	0.004	0.002
$-\langle K^+\|(ds)_L u_L\|p\rangle$	0.076(14)(9)	0.006	0.004	0.006	0.003
$\langle\eta\|(ud)_R u_L\|p\rangle$	0.015(14)(17)	0.017	0.001	0.001	0.001
$\langle\eta\|(ud)_L u_L\|p\rangle$	0.088(21)(16)	0.014	0.004	0.007	0.003

← uncertainty of extrapolation to physical pion mass is 30 % error, and statistical error is also 20 %.

Future plan

- Improvement of statistical errors Izubuchi and Shintani in prep.
  - New algorithm, LMA/AMA in which computational cost can be reduced by factor 1/10 -- 1/100.
- Improvement of systematic errors
  - Computation in more realistic lattice parameters
  - Reduction of chiral extrapolation uncertaity using small quark mass

  ⇒ ~10% accuracy

Lattice size	Physical size	Lattice spacing	L_s	Gauge action	Pion mass
$24^3 \times 64$	2.7 fm^3	0.114 fm	16	Iwasaki	315 -- 615 MeV
$32^3 \times 64$	2.7 fm^3	0.087 fm	16	Iwasaki	295 -- 397 MeV
$32^3 \times 64$	4.6 fm^3	0.135 fm	32	DSDR	171 -- 241 MeV
$48^3 \times 96$	5.5 fm^3	0.115 fm	8	Iwasaki	135 MeV

Backup

non-perturbative renormalization

- *To avoid lattice perturbation theory: expansion poorly converges*
- *RI/MOM scheme constructed:* [YA et al., PRD75(07)014507] → *NLO matching to MSbar*
- *Exact chiral symmetry → multiplicative renormalization*
- *DWF has negligible chiral symmetry breaking*
- *In general, mixing occurs*

$$
\begin{aligned}
O_{RL} &= (\overline{u^c} P_R d) \cdot P_L s \\
O_{LL} &= (\overline{u^c} P_L d) \cdot P_L s \\
O_{A(LV)} &= (\overline{u^c} \gamma_\mu \gamma_5 d) \cdot P_L \gamma_\mu s
\end{aligned}
$$

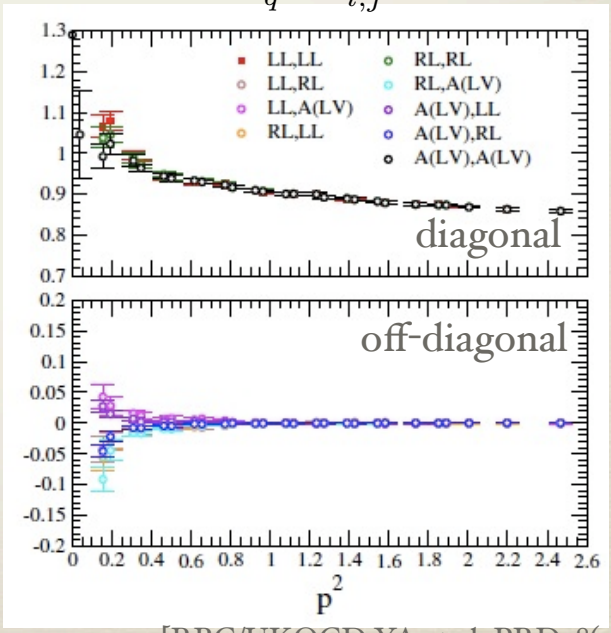
- *Z: diagonal matrix: practically no mixing!*

$$U^{\overline{\mathrm{MS}}\leftarrow\mathrm{latt}}(\mu = 2\ \mathrm{GeV})_{LL} = 0.662 \pm 0.010$$

$$U^{\overline{\mathrm{MS}}\leftarrow\mathrm{latt}}(\mu = 2\ \mathrm{GeV})_{RL} = 0.665 \pm 0.008,$$

+8% $O(\alpha_s^2)$ error

$Z_q^{3/2} Z_{i,j}^{-1}$



[RBC/UKQCD YA et al. PRD78(08)04505]

P decay operator

- Weinberg rep.

$$
\begin{aligned}
\mathcal{O}^1_{abcd} &= (D^i_a, U^j_b)_R (q^{k\,\alpha}_c, l^\beta_d)_L \varepsilon^{ijk} \varepsilon^{\alpha\beta}, \\
\mathcal{O}^2_{abcd} &= (q^{i\,\alpha}_a, q^{j\,\beta}_b)_L (U^k_c, l_d)_R \varepsilon^{ijk} \varepsilon^{\alpha\beta}, \\
\widetilde{\mathcal{O}}^4_{abcd} &= (q^{i\,\alpha}_a, q^{j\,\beta}_b)_L (q^{k\,\gamma}_c, l^\delta_d)_L \varepsilon^{ijk} \varepsilon^{\alpha\delta} \varepsilon^{\beta\gamma}, \\
\mathcal{O}^5_{abcd} &= (D^i_a, U^j_b)_R (U^k_c, l_d)_R \varepsilon^{ijk}
\end{aligned}
$$

a,b,c,d : generation, α,β,γ,δ: SU(2) indices

i,j,k: color indices

General form of three quark:

$$\mathcal{O}^{\Gamma\Gamma'}_{abc} = (q_a q_b)_\Gamma q_{c\Gamma'} = (q^{Ti}_a C P_\Gamma q^j_b) P_{\Gamma'} q^k_c \varepsilon^{ijk}$$

and matrix element

$$\langle PS; \vec{p} | \mathcal{O}^{\Gamma\Gamma'}_{abc} | N; \vec{k}, s \rangle$$ s: nucleon spin

P decay operator

- Parity invariance

$$\langle PS;\vec{p}|\mathcal{O}^{LL}|N;\vec{k},s\rangle = \gamma_4\langle PS;-\vec{p}|\mathcal{O}^{RR}|N;-\vec{k},s\rangle,$$
$$\langle PS;\vec{p}|\mathcal{O}^{LR}|N;\vec{k},s\rangle = \gamma_4\langle PS;-\vec{p}|\mathcal{O}^{RL}|N;-\vec{k},s\rangle$$

$\Rightarrow$

$$W_0^{LL} = W_0^{RR},$$
$$W_0^{LR} = W_0^{RL}$$

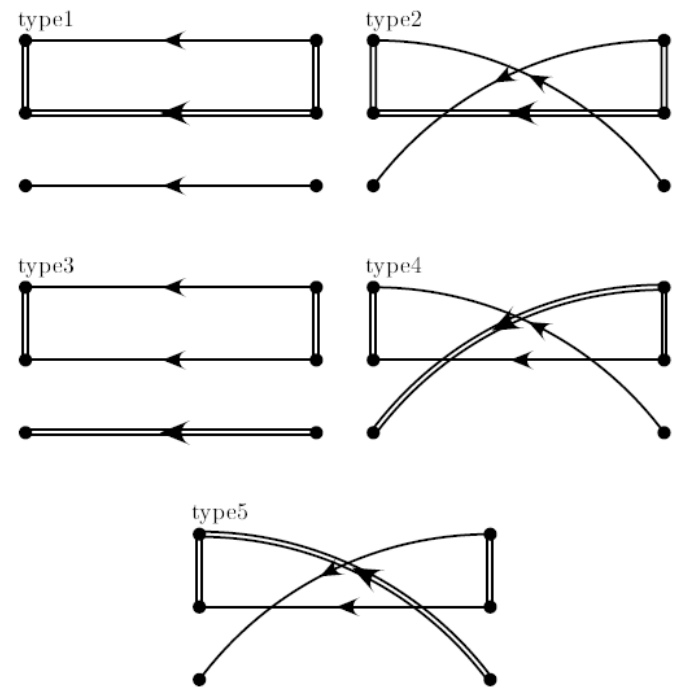
- Relation on [p,n]→PS decay mode

$$\begin{aligned}
\langle\pi^0|\mathcal{O}^{L/R}_{udu}|p\rangle &= \langle\pi^0|\mathcal{O}^{L/R}_{dud}|n\rangle,\\
\langle\pi^+|\mathcal{O}^{L/R}_{udd}|p\rangle &= -\langle\pi^-|\mathcal{O}^{L/R}_{duu}|n\rangle,\\
\langle K^0|\mathcal{O}^{L/R}_{usu}|p\rangle &= -\langle K^+|\mathcal{O}^{L/R}_{dsd}|n\rangle,\\
\langle K^+|\mathcal{O}^{L/R}_{usd}|p\rangle &= -\langle K^0|\mathcal{O}^{L/R}_{dsu}|n\rangle,\\
\langle K^+|\mathcal{O}^{L/R}_{uds}|p\rangle &= -\langle K^0|\mathcal{O}^{L/R}_{dus}|n\rangle,\\
\langle K^+|\mathcal{O}^{L/R}_{dsu}|p\rangle &= -\langle K^0|\mathcal{O}^{L/R}_{usd}|n\rangle,\\
\langle\eta|\mathcal{O}^{L/R}_{udu}|p\rangle &= -\langle\eta|\mathcal{O}^{L/R}_{dud}|n\rangle
\end{aligned}$$

In the isospin limit $\langle\pi^+|\mathcal{O}^{L/R}_{udu}|p\rangle = \sqrt{2}\langle\pi^0|\mathcal{O}^{L/R}_{udu}|p\rangle$

$\Rightarrow$ total number is **12**

W_0 in direct method

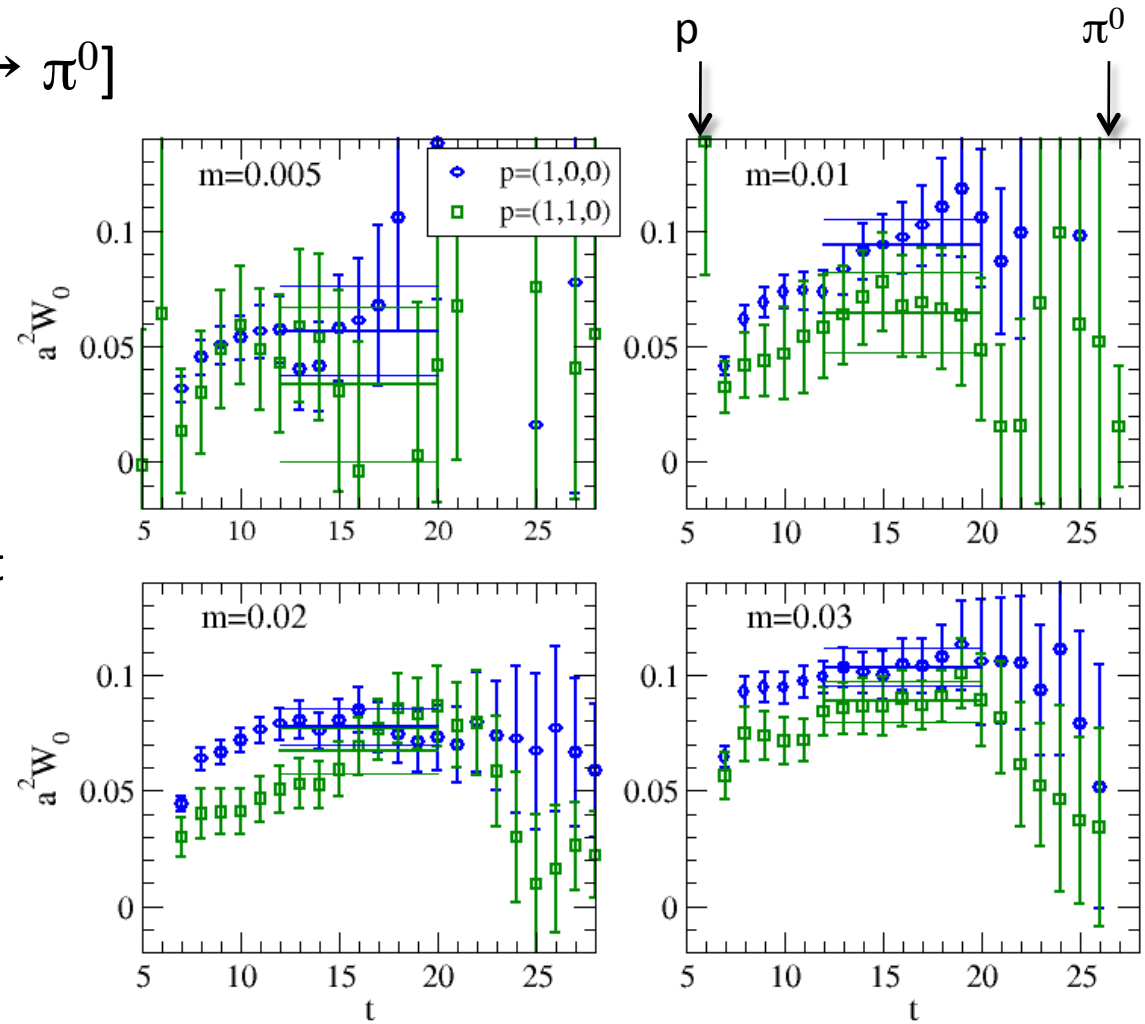


$$
\begin{aligned}
\langle \pi^0 | \mathcal{O}_{udu} | p \rangle &= \frac{1}{\sqrt{2}}(-T^2 + T^3 + T^4 + T^5), \\
\langle K^0 | \mathcal{O}_{usu} | p \rangle &= T^1 + T^2, \\
\langle K^+ | \mathcal{O}_{usd} | p \rangle &= -T^2 + T^5, \\
\langle K^+ | \mathcal{O}_{uds} | p \rangle &= T^3 + T^4, \\
\langle K^+ | \mathcal{O}_{dsu} | p \rangle &= -T^4 - T^5, \\
\langle \eta | \mathcal{O}_{udu} | p \rangle &= \frac{1}{\sqrt{6}}(2T^1 + T^2 + T^3 + T^4 + T^5),
\end{aligned}
$$

Results of W_0

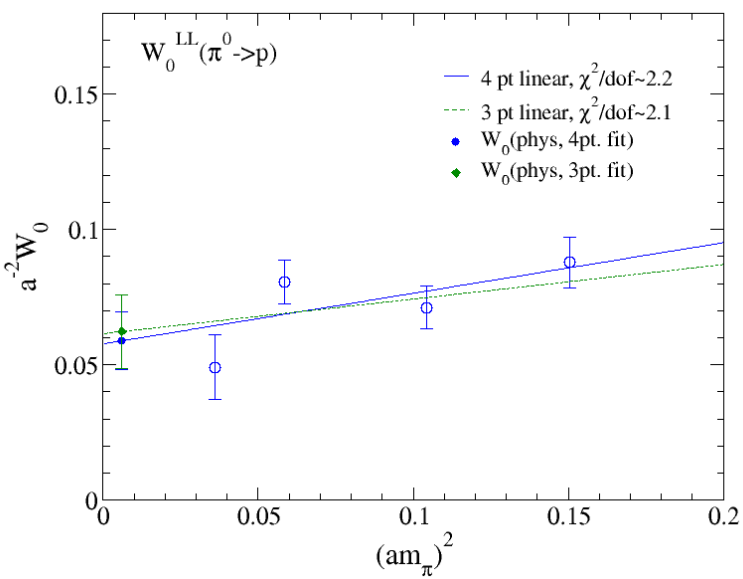
- For $W_0^{LL}[p \rightarrow \pi^0]$

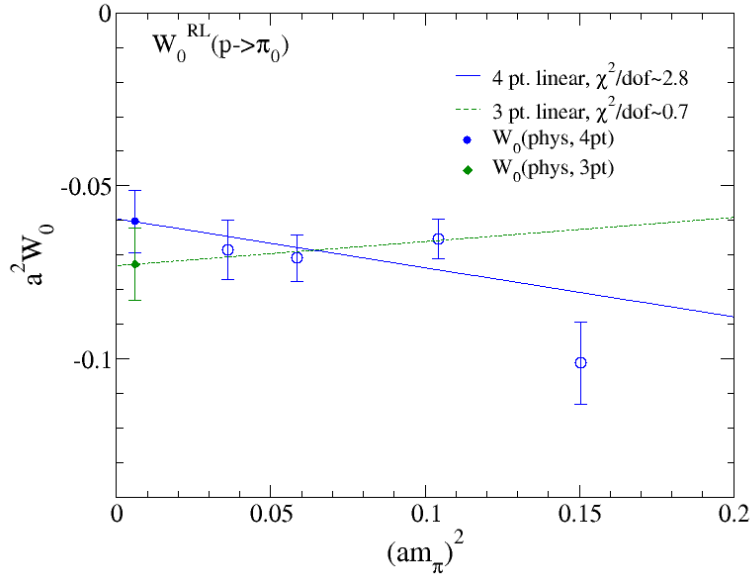
- Constant fit for plateau region
- Fit range [12,20]
- Large fluctuation at the lightest mass



Chiral extrapolation

- $W_0[p \to \pi^0]$ at $q^2 = 0$ point



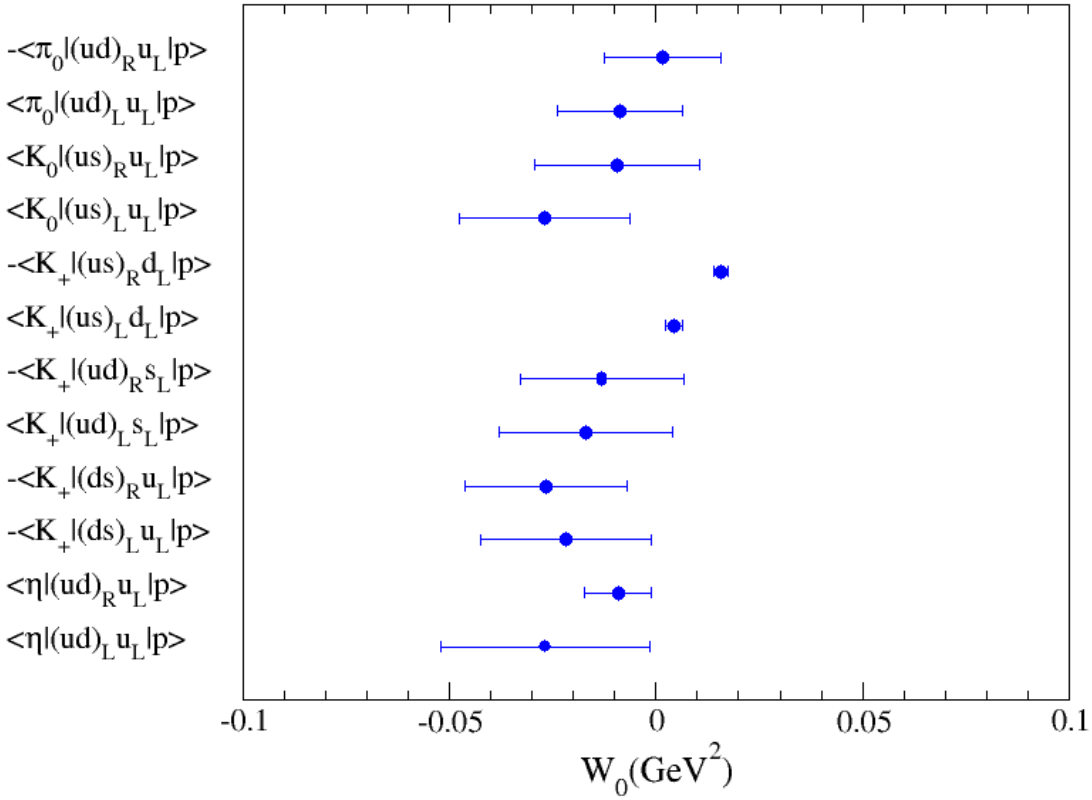


- Linear extrapolation for 2 momentum
  $\to$ deterministic
- After that chiral extrapolaton:

$$f(m_\pi^2, m_s) = W_0 + x_1 m_\pi^2 + x_2(m_s - m_s^{\rm phy})$$

RBC/UKQCD efforts

- Lattice size
  - 24^3 in spacial, 64 in temporal direction
  - a^{-1} = 1.73 GeV,
  - 1.8^3 fm^3 size
  - Iwasaki-type gauge action + dynamical DW
- Domain-wall fermion
  - Chiral improved fermion by L_s = 16
  - Suppressed O(a) error from χ breaking and unphysical operator mixing
  - Z factor from NPR Y. Aoki (2008)
  - m=0.005, 0.01, 0.02, 0.03, m_s=0.03
  - m_π= 0.3 GeV ～ 0.8 GeV

In the soft-pion limit

- Difference between W_0 [$p_\mu = 0$] and ChPT