Deeply Virtual Neutrino Scattering

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W. Melnitchouk, A.V. Radyushkin, and A. Psaker, Phys. Rev. D 75, 054001 (2007).

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Motivation

Compton scattering provides a ***unique tool*** for studying hadrons

Outline

- ❖ Introduction
- ❖ Handbag Factorization
- ❖ Generalized Parton Distributions
- ❖ Deeply Virtual Neutrino Scattering
- ❖ Summary
- ❖ Outlook

Introduction

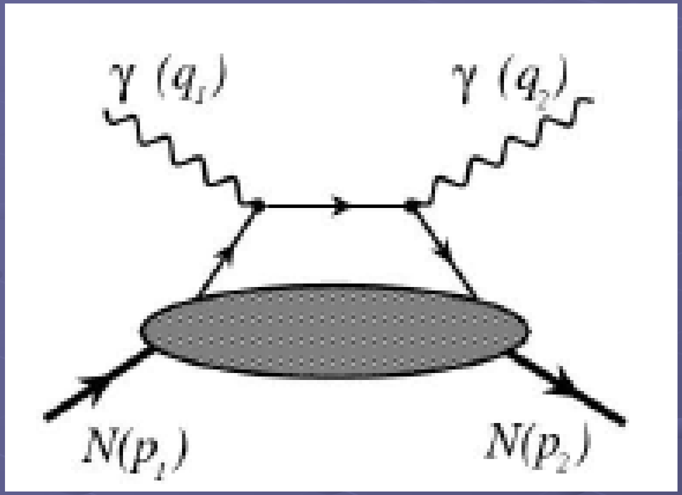
- QCD Lagrangian

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} + \sum_{\text{flavors}} \bar{\psi}_q(i\not{D} - m_q)\psi_q$$

$$G_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\mu A_\nu - ig[A_\mu, A_\nu]\ ,\ D_\mu \equiv \partial_\mu - igA_\mu$$

- In principle, QCD embraces all phenomena of hadronic physics
- Difficulty of QCD - formulation in terms of ***colored degrees of freedom***
- Projection of quark and gluon fields onto hadronic states - ***phenomenological functions*** (e.g. form factors, parton distribution functions, generalized parton distributions) describe hadronic matrix elements

Handbag Factorization



- Only one active parton
- Hard process: perturbation theory
- Soft, non-perturbative physics: PDFs, GPDs

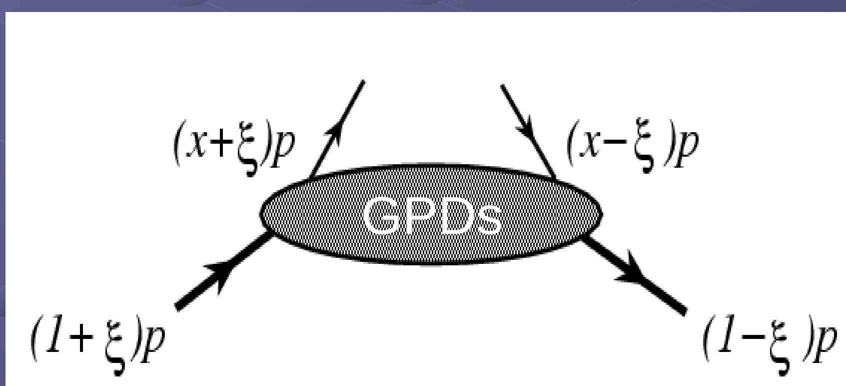
- Both photons highly virtual with equal space-like virtualities - ***forward Compton scattering amplitude***: DIS
- One of the photons highly virtual, small *t* - ***nonforward Compton scattering amplitude***: DVCS, DVMP
- Both photons are real, large *t*: WACS

Generalized Parton Distributions

Mueller at al (94), Ji (97), Radyushkin (97)

- ***Hybridization*** - encapsulating longitudinal momentum distributions and transverse coordinate distributions of hadron's constituents

- ***Universality*** - unified description of different hard processes



$0 \le \xi \equiv r^+/2p^+ \le 1,\ p \equiv (p_1 + p_2)/2,\ r = p_1 - p_2$

$\left.\begin{array}{l} \xi < x < 1 \\ -1 < x < -\xi \end{array}\right\}$ generalization of PDFs

$-\xi < x < \xi$ meson DAs

- Measuring coherence between two different parton momentum states in hadron, spin correlations, and studying flavor nondiagonal distributions through hadron transitions

- At leading ***twist-2*** level:

$$H_f(x,\xi,t), E_f(x,\xi,t) \text{ and } \tilde{H}_f(x,\xi,t), \tilde{E}_f(x,\xi,t)$$

- ***Reduction formulas*** in forward limit: $p_1 = p_2, r = 0, \xi = 0, t = 0$

$$H_f(x,0,0) = \begin{cases} f_N(x) & x > 0 \\ -\bar{f}_N(-x) & x < 0 \end{cases}$$

- ***Sum rules*** in local limit:

$$\int_{-1}^{1} dx H_f(x,\xi,t) = F_{1f}(t), \int_{-1}^{1} dx E_f(x,\xi,t) = F_{2f}(t)$$

$$\int_{-1}^{1} dx \tilde{H}_f(x,\xi,t) = g_{Af}(t), \int_{-1}^{1} dx \tilde{E}_f(x,\xi,t) = g_{Pf}(t)$$

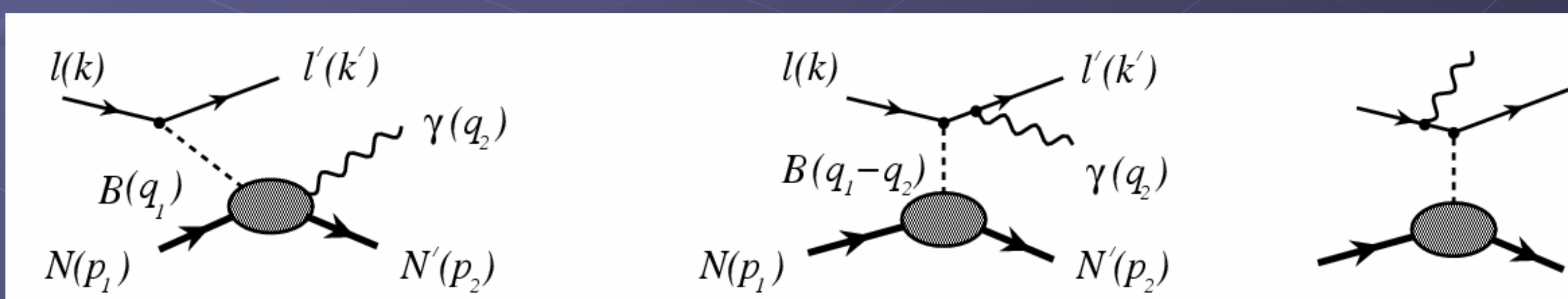
- Ji's sum rule: access to ***quark angular momentum*** and further extract ***gluon contribution***

$$J_q = \frac{1}{2} \sum_f \int_{-1}^{1} x dx \left[H_f(x,\xi,0) + E_f(x,\xi,0) \right] = \frac{1}{2} \Delta\Sigma_q + L_q \qquad \frac{1}{2} = J_q + J_g$$

Deeply Virtual Neutrino Scattering

- Why ***neutrino*** beam?
  1. ***Different set of GPDs*** due to axial part of *V-A* interaction (study C-odd and C-even combinations of GPDs, i.e. measuring valence and sea)
  2. ***Different flavor decomposition***, more sensitive to d-quarks
  3. ***Studying flavor non-diagonal distributions*** (e.g. neutron-to-proton transition)

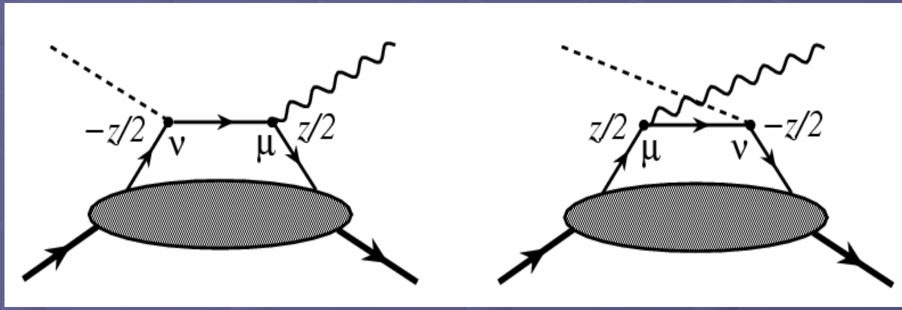
- DVCS-like process:



Compton　　　　　　　　Bethe-Heitler

Weak Virtual Compton Amplitude

- ***Handbag dominance*** in Bjorken regime: $-q_1^2, s \equiv (p_1 + q_1)^2 \to \infty, x_B \equiv -q_1^2 / 2(p_1 \cdot q_1)$



DVCS kinematics: $s > -q_1^2 >> -t$
(keeping *t*-dependence in soft part)

$$\mathcal{T}_W^{\mu\nu} = i \int d^4 z \, e^{iqz} \langle N' (p - r/2, s_2) | T \{ J_{EM}^{\mu} (z/2) \, J_W^{\nu} (-z/2) \} | N (p + r/2, s_1) \rangle$$

$$p \equiv (p_1 + p_2)/2, q \equiv (q_1 + q_2)/2 \ \& \ r = p_1 - p_2 = q_2 - q_1$$

- Two types of vertices:

  $Z^0 \to qq$ or $W^{\pm} \to qq$

- ***Nonlocal light-cone expansion of T-product*** in terms of QCD bilocal operators in coordinate space

I.I. Balitsky and V.M. Braun, Nucl. Phys. B311, 541 (1989).

- Weak neutral and electromagnetic currents:

$$iT\left\{J^{\mu}_{EM}(z/2)\,J^{\nu}_{WN}(-z/2)\right\} = -\frac{z_\rho}{4\pi^2 z^4}\sum_f Q_f\left\{c^f_V\left[s^{\mu\rho\nu\eta}\mathcal{O}^{f-}_{\eta}(z|0) - i\epsilon^{\mu\rho\nu\eta}\mathcal{O}^{f+}_{5\eta}(z|0)\right]\right.$$
$$\left. - c^f_A\left[s^{\mu\rho\nu\eta}\mathcal{O}^{f-}_{5\eta}(z|0) - i\epsilon^{\mu\rho\nu\eta}\mathcal{O}^{f+}_{\eta}(z|0)\right]\right\}$$

$$\mathcal{O}^{f\pm}_{\eta}(z|0) \equiv \left[\bar\psi_f(z/2)\gamma_\eta\psi_f(-z/2) \pm (z\to -z)\right]$$
$$\mathcal{O}^{f\pm}_{5\eta}(z|0) \equiv \left[\bar\psi_f(z/2)\gamma_\eta\gamma_5\psi_f(-z/2) \pm (z\to -z)\right]$$

$$c_V^{u,c,t} = 1/2 - 2Q_{u,c,t}\sin^2\theta_W$$
$$c_V^{d,s,b} = -1/2 - 2Q_{d,s,b}\sin^2\theta_W$$
$$c_A^{u,c,t} = 1/2$$
$$c_A^{d,s,b} = -1/2$$

- Weak charged and electromagnetic currents: ***different*** initial and final nucleons

$$Q_f - Q_{f'} = \pm 1$$

$$\left[Q_{f'}\bar\psi_{f'}(z/2)\,\not{z}\,\psi_f(-z/2) \pm Q_f(z\to -z)\right] = Q_\pm\mathcal{O}^{f'f+}(z|0) + Q_\mp\mathcal{O}^{f'f-}(z|0)$$
$$\left[Q_{f'}\bar\psi_{f'}(z/2)\,\not{z}\gamma_5\psi_f(-z/2) \pm Q_f(z\to -z)\right] = Q_\pm\mathcal{O}_5^{f'f+}(z|0) + Q_\mp\mathcal{O}_5^{f'f-}(z|0)$$

$$Q_\pm = (Q_{f'} \pm Q_f)/2$$

$$\mathcal{O}^{f'f\pm}(z|0) \equiv \left[\bar\psi_{f'}(z/2)\,\not{z}\,\psi_f(-z/2) \pm (z\to -z)\right]$$
$$\mathcal{O}_5^{f'f\pm}(z|0) \equiv \left[\bar\psi_{f'}(z/2)\,\not{z}\gamma_5\psi_f(-z/2) \pm (z\to -z)\right]$$

- ***Insolating twist-2 part***, and parametrizing its matrix elements on light-cone in terms of GPDs

$$\langle N(p_2,s_2)|\,\mathcal{O}^{f\pm}(z\,|0)\,|N(p_1,s_1)\rangle_{z^2=0} = \bar{u}(p_2,s_2)\,\not{z}\,u(p_1,s_1)\int_{-1}^{1}dx\;e^{ixp\cdot z}H_f^{\pm}(x,\xi,t)$$
$$+\bar{u}(p_2,s_2)\,\frac{(\not{z}\,\not{r}-\not{r}\,\not{z})}{4M}u(p_1,s_1)\int_{-1}^{1}dx\;e^{ixp\cdot z}E_f^{\pm}(x,\xi,t)$$
$$\langle N(p_2,s_2)|\,\mathcal{O}_5^{f\pm}(z\,|0)\,|N(p_1,s_1)\rangle_{z^2=0} = \bar{u}(p_2,s_2)\,\not{z}\gamma_5 u(p_1,s_1)\int_{-1}^{1}dx\;e^{ixp\cdot z}\widetilde{H}_f^{\mp}(x,\xi,t)$$
$$-\bar{u}(p_2,s_2)\,\frac{(r\cdot z)}{2M}\gamma_5 u(p_1,s_1)\int_{-1}^{1}dx\;e^{ixp\cdot z}\widetilde{E}_f^{\mp}(x,\xi,t)$$

$$\langle N'(p_2,s_2)|\,\mathcal{O}^{f'f\pm}(z\,|0)\,|N(p_1,s_1)\rangle_{z^2=0} = \bar{u}(p_2,s_2)\,\not{z}\,u(p_1,s_1)\int_{-1}^{1}dx\;e^{ixp\cdot z}H_{f'f}^{\pm}(x,\xi,t)$$
$$+\bar{u}(p_2,s_2)\,\frac{(\not{z}\,\not{r}-\not{r}\,\not{z})}{4M}u(p_1,s_1)\int_{-1}^{1}dx\;e^{ixp\cdot z}E_{f'f}^{\pm}(x,\xi,t)$$
$$\langle N'(p_2,s_2)|\,\mathcal{O}_5^{f'f\pm}(z\,|0)\,|N(p_1,s_1)\rangle_{z^2=0} = \bar{u}(p_2,s_2)\,\not{z}\gamma_5 u(p_1,s_1)\int_{-1}^{1}dx\;e^{ixp\cdot z}\widetilde{H}_{f'f}^{\mp}(x,\xi,t)$$
$$-\bar{u}(p_2,s_2)\,\frac{(r\cdot z)}{2M}\gamma_5 u(p_1,s_1)\int_{-1}^{1}dx\;e^{ixp\cdot z}\widetilde{E}_{f'f}^{\mp}(x,\xi,t)$$

Standard DVCS measures only plus GPDs, but here also accessing minus distributions (i.e. ***valence configuration***)

$$\mathcal{T}^{\mu\nu}_{\mathrm{WN}} = -\frac{1}{4(p\cdot q)}\bigg\{\bigg[\frac{1}{(p\cdot q_2)}(p^\mu q_2^\nu + p^\nu q_2^\mu) - g^{\mu\nu}\bigg]\bigg[\mathcal{H}^{+}_{\mathrm{WN}}(\xi,t)\bar{u}(p_2,s_2)\not{q}_2 u(p_1,s_1)$$
$$+\mathcal{E}^{+}_{\mathrm{WN}}(\xi,t)\bar{u}(p_2,s_2)\frac{(\not{q}_2\not{r}-\not{r}\not{q}_2)}{4M}u(p_1,s_1) - \tilde{H}^{-}_{\mathrm{WN}}(\xi,t)\bar{u}(p_2,s_2)\not{q}_2\gamma_5 u(p_1,s_1)$$
$$+\tilde{\mathcal{E}}^{-}_{\mathrm{WN}}(\xi,t)\frac{(q_2\cdot r)}{2M}\bar{u}(p_2,s_2)\gamma_5 u(p_1,s_1)\bigg] + \bigg[\frac{1}{(p\cdot q_2)}i\epsilon^{\mu\nu\rho\eta}q_{2\rho}p_\eta\bigg]\bigg[\tilde{H}^{+}_{\mathrm{WN}}(\xi,t)\bar{u}(p_2,s_2)\not{q}_2\gamma_5 u(p_1,s_1)$$
$$-\tilde{\mathcal{E}}^{+}_{\mathrm{WN}}(\xi,t)\frac{(q_2\cdot r)}{2M}\bar{u}(p_2,s_2)\gamma_5 u(p_1,s_1) - \mathcal{H}^{-}_{\mathrm{WN}}(\xi,t)\bar{u}(p_2,s_2)\not{q}_2 u(p_1,s_1)$$
$$-\mathcal{E}^{-}_{\mathrm{WN}}(\xi,t)\bar{u}(p_2,s_2)\frac{(\not{q}_2\not{r}-\not{r}\not{q}_2)}{4M}u(p_1,s_1)\bigg]\bigg\}$$

$$\begin{aligned}
\mathcal{H}^{+(-)}_{WN}(\xi,t) &\equiv \sum_f Q_f c^f_{V(A)}\int_{-1}^{1}\frac{dx}{(x-\xi+i0)}H^{+(-)}_f(x,\xi,t)\\
&= \sum_f Q_f c^f_{V(A)}\int_{-1}^{1}dx\; H_f(x,\xi,t)\left(\frac{1}{x-\xi+i0}\pm\frac{1}{x+\xi-i0}\right)\\
\mathcal{E}^{+(-)}_{WN}(\xi,t) &\equiv \sum_f Q_f c^f_{V(A)}\int_{-1}^{1}\frac{dx}{(x-\xi+i0)}E^{+(-)}_f(x,\xi,t)\\
&= \sum_f Q_f c^f_{V(A)}\int_{-1}^{1}dx\; E_f(x,\xi,t)\left(\frac{1}{x-\xi+i0}\pm\frac{1}{x+\xi-i0}\right)\\
\widetilde{\mathcal{H}}^{+(-)}_{WN}(\xi,t) &\equiv \sum_f Q_f c^f_{V(A)}\int_{-1}^{1}\frac{dx}{(x-\xi+i0)}\widetilde{H}^{+(-)}_f(x,\xi,t)\\
&= \sum_f Q_f c^f_{V(A)}\int_{-1}^{1}dx\; \widetilde{H}_f(x,\xi,t)\left(\frac{1}{x-\xi+i0}\mp\frac{1}{x+\xi-i0}\right)\\
\widetilde{\mathcal{E}}^{+(-)}_{WN}(\xi,t) &\equiv \sum_f Q_f c^f_{V(A)}\int_{-1}^{1}\frac{dx}{(x-\xi+i0)}\widetilde{E}^{+(-)}_f(x,\xi,t)\\
&= \sum_f Q_f c^f_{V(A)}\int_{-1}^{1}dx\; \widetilde{E}_f(x,\xi,t)\left(\frac{1}{x-\xi+i0}\mp\frac{1}{x+\xi-i0}\right)
\end{aligned}$$

$$
\begin{aligned}
\mathcal{T}^{\mu\nu}_{\rm WC} = & -\frac{1}{4(p\cdot q)}\bigg\{\bigg[\frac{1}{(p\cdot q_2)}(p^\mu q_2^\nu + p^\nu q_2^\mu) - g^{\mu\nu}\bigg]\bigg[\mathcal{H}^{+}_{\rm WC}(\xi,t)\bar{u}(p_2,s_2)\not{q}_2 u(p_1,s_1) \\
& + \mathcal{E}^{+}_{\rm WC}(\xi,t)\bar{u}(p_2,s_2)\frac{(\not{q}_2\not{r}-\not{r}\not{q}_2)}{4M}u(p_1,s_1) - \tilde{H}^{-}_{\rm WC}(\xi,t)\bar{u}(p_2,s_2)\not{q}_2\gamma_5 u(p_1,s_1) \\
& + \tilde{\mathcal{E}}^{-}_{\rm WC}(\xi,t)\frac{(q_2\cdot r)}{2M}\bar{u}(p_2,s_2)\gamma_5 u(p_1,s_1)\bigg] + \bigg[\frac{1}{(p\cdot q_2)}i\epsilon^{\mu\nu\rho\eta}q_{2\rho}p_\eta\bigg]\bigg[\tilde{H}^{+}_{\rm WC}(\xi,t)\bar{u}(p_2,s_2)\not{q}_2\gamma_5 u(p_1,s_1) \\
& - \tilde{\mathcal{E}}^{+}_{\rm WC}(\xi,t)\frac{(q_2\cdot r)}{2M}\bar{u}(p_2,s_2)\gamma_5 u(p_1,s_1) - \mathcal{H}^{-}_{\rm WC}(\xi,t)\bar{u}(p_2,s_2)\not{q}_2 u(p_1,s_1) \\
& - \mathcal{E}^{-}_{\rm WC}(\xi,t)\bar{u}(p_2,s_2)\frac{(\not{q}_2\not{r}-\not{r}\not{q}_2)}{4M}u(p_1,s_1)\bigg] + \bigg[\frac{2}{(p\cdot q_2)}p^\mu p^\nu\bigg]\bigg[\mathcal{F}_1(t)\bar{u}(p_2,s_2)\not{q}_2 u(p_1,s_1) \\
& + \mathcal{F}_2(t)\bar{u}(p_2,s_2)\frac{(\not{q}_2\not{r}-\not{r}\not{q}_2)}{4M}u(p_1,s_1) - \mathcal{G}_A(t)\bar{u}(p_2,s_2)\not{q}_2\gamma_5 u(p_1,s_1) + \mathcal{G}_P(t)\frac{(q_2\cdot r)}{2M}\bar{u}(p_2,s_2)\gamma_5 u(p_1,s_1)\bigg]\bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{H}^{+(-)}_{\rm WC}(\xi,t) &\equiv \sum_{f,f'}\int_{-1}^{1}\frac{dx}{(x-\xi+i0)}[Q_{+(-)}H^{+}_{f'f}(x,\xi,t) + Q_{-(+)}H^{-}_{f'f}(x,\xi,t)] \\
&= \sum_{f,f'}\int_{-1}^{1}dx H_{f'f}(x,\xi,t)\bigg(\frac{Q_{f'}}{x-\xi+i0} \pm \frac{Q_f}{x+\xi-i0}\bigg)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{E}^{+(-)}_{\rm WC}(\xi,t) &\equiv \sum_{f,f'}\int_{-1}^{1}\frac{dx}{(x-\xi+i0)}[Q_{+(-)}E^{+}_{f'f}(x,\xi,t) + Q_{-(+)}E^{-}_{f'f}(x,\xi,t)] \\
&= \sum_{f,f'}\int_{-1}^{1}dx E_{f'f}(x,\xi,t)\bigg(\frac{Q_{f'}}{x-\xi+i0} \pm \frac{Q_f}{x+\xi-i0}\bigg)
\end{aligned}
$$

$$
\begin{aligned}
\tilde{H}^{+(-)}_{\rm WC}(\xi,t) &\equiv \sum_{f,f'}\int_{-1}^{1}\frac{dx}{(x-\xi+i0)}[Q_{+(-)}\tilde{H}^{+}_{f'f}(x,\xi,t) + Q_{-(+)}\tilde{H}^{-}_{f'f}(x,\xi,t)] \\
&= \sum_{f,f'}\int_{-1}^{1}dx \tilde{H}_{f'f}(x,\xi,t)\bigg(\frac{Q_{f'}}{x-\xi+i0} \mp \frac{Q_f}{x+\xi-i0}\bigg)
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\mathcal{E}}^{+(-)}_{\rm WC}(\xi,t) &\equiv \sum_{f,f'}\int_{-1}^{1}\frac{dx}{(x-\xi+i0)}[Q_{+(-)}\tilde{E}^{+}_{f'f}(x,\xi,t) + Q_{-(+)}\tilde{E}^{-}_{f'f}(x,\xi,t)] \\
&= \sum_{f,f'}\int_{-1}^{1}dx \tilde{E}_{f'f}(x,\xi,t)\bigg(\frac{Q_{f'}}{x-\xi+i0} \mp \frac{Q_f}{x+\xi-i0}\bigg)
\end{aligned}
$$

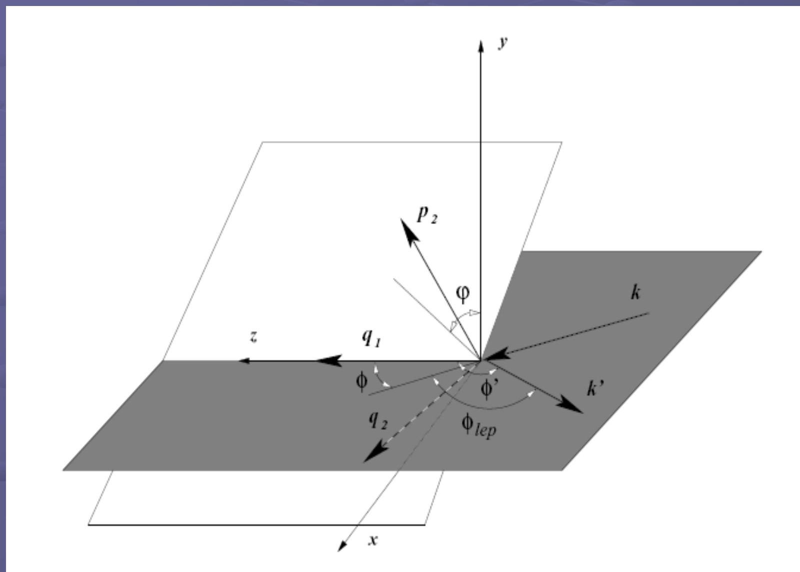
$$
\begin{aligned}
\mathcal{F}_1(t) &\equiv \sum_{f,f'}Q_-\int_{-1}^{1}dx\, H^{-}_{f'f}(x,\xi,t) = \sum_{f,f'}(Q_{f'}-Q_f)\int_{-1}^{1}dx\, H_{f'f}(x,\xi,t) \\
\mathcal{F}_2(t) &\equiv \sum_{f,f'}Q_-\int_{-1}^{1}dx\, E^{-}_{f'f}(x,\xi,t) = \sum_{f,f'}(Q_{f'}-Q_f)\int_{-1}^{1}dx\, E_{f'f}(x,\xi,t) \\
\mathcal{G}_A(t) &\equiv \sum_{f,f'}Q_-\int_{-1}^{1}dx\, \widetilde{H}^{+}_{f'f}(x,\xi,t) = \sum_{f,f'}(Q_{f'}-Q_f)\int_{-1}^{1}dx\, \widetilde{H}_{f'f}(x,\xi,t) \\
\mathcal{G}_P(t) &\equiv \sum_{f,f'}Q_-\int_{-1}^{1}dx\, \widetilde{E}^{+}_{f'f}(x,\xi,t) = \sum_{f,f'}(Q_{f'}-Q_f)\int_{-1}^{1}dx\, \widetilde{E}_{f'f}(x,\xi,t)
\end{aligned}
$$

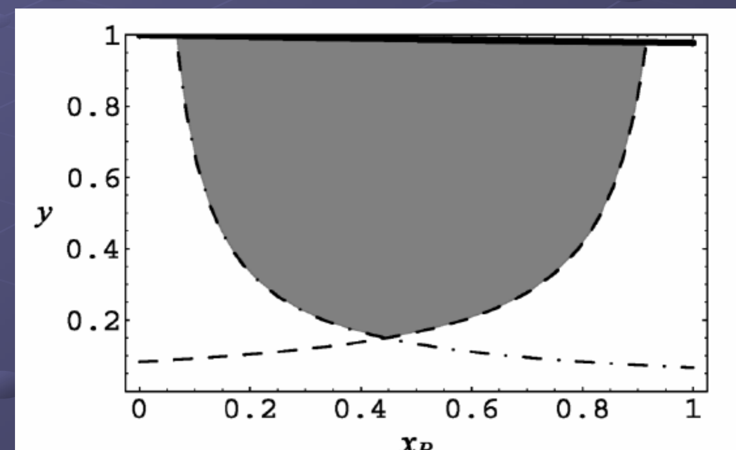
Kinematics

- Cross section in target rest frame:

$$\frac{d^4\sigma}{dx_B dQ_1^2 dt d\varphi} = \frac{1}{32}\frac{1}{(2\pi)^4}\frac{x_B y^2}{Q_1^4}\frac{1}{\sqrt{1+4x_B^2 M^2/Q_1^2}}|\mathrm{T}|^2$$

$$\mathrm{T} = \mathrm{T}_C + \mathrm{T}_{BH}$$



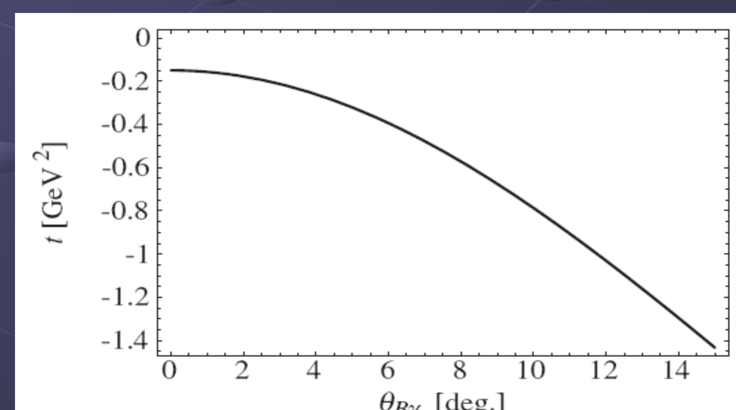


- Kinematically allowed region:

$$\omega = 20\,\mathrm{GeV}$$
$$\hat{s} \equiv (p_1 + q_1)^2 \geq 4\,\mathrm{GeV}^2$$
$$Q_1^2 \geq 2.5\,\mathrm{GeV}^2$$

$$\omega = 20\,\mathrm{GeV}, Q_1^2 = 2.5\,\mathrm{GeV}^2, x_B = 0.35, y = 0.19$$

(in-plane)
$$\varphi = 0$$
$$\phi = 20.2^0 \quad \text{and} \quad \phi' = 25.3^0$$



Model

- No contribution from sea quarks - ***valence nucleon GPDs***

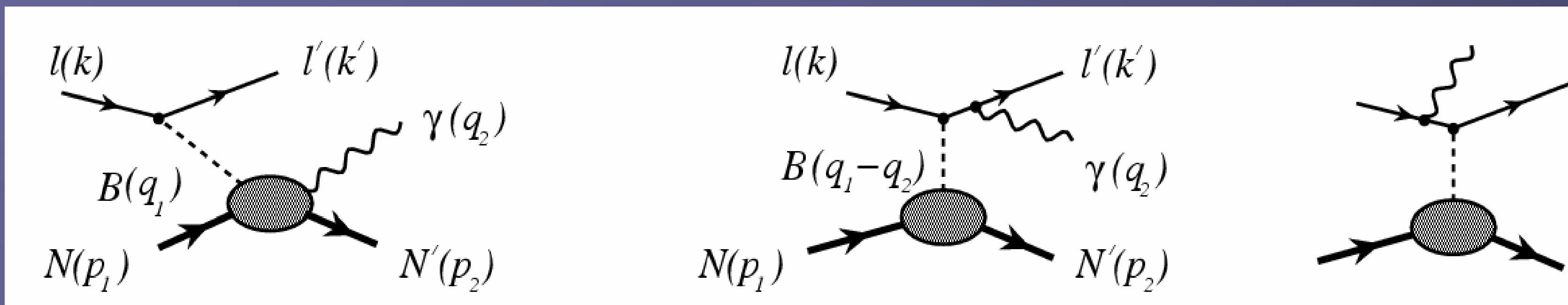
$$H_f^+ = H_f^- \equiv H_f^{val},\ \tilde{H}_f^+ = \tilde{H}_f^- \equiv \tilde{H}_f^{val},\ E_f^+ = E_f^- \equiv E_f^{val},\ \tilde{E}_f^+ = \tilde{E}_f^- \equiv \tilde{E}_f^{val}$$
$$f = u, d$$

- ***Factorized t-dependence*** driven by corresponding form factors

- Dependence on ***ξ only in*** $\tilde{E}_f$

A.V. Radyushkin, Phys. Rev. D58, 114008 (1998).
P.A.M. Guichon and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 41, 125 (1998).
A.V. Belitsky, D. Mueller, and A. Kirchner, Nucl. Phys. B629, 323 (2002).
R.D. Carlitz and J. Kaur, Phys. Rev. Lett. 38, 673 (1977); 38, 1102(E) (1997).
M. Goshtasbpour and G.P. Ramsey, Phys. Rev. D 55, 1244 (1997).

Scattering Processes



- ***Neutrino-proton***: measuring ***pure*** Compton contribution

$$\mathrm{T}_{\nu p} = \sqrt{2}\,|e|\,G_F\,\bar{u}\,(k')\,\gamma_\nu\,(1-\gamma_5)\,u(k)\epsilon^*_\mu\,(q_2)\,\mathcal{T}^{\mu\nu}_{WN}$$

- ***Neutrino-neutron***: Bethe-Heitler contamination

$$\mathrm{T}_{C\nu n} = \sqrt{2}\,|e|\,G_F\,\bar{u}\,(k')\,\gamma_\nu\,(1-\gamma_5)\,u\,(k)\,\epsilon^*_\mu\,(q_2)\,\mathcal{T}^{\mu\nu}_{WC}$$

$$\mathrm{T}_{BH\nu n} = \sqrt{2}\,|e|\,G_F\,\epsilon^*_\mu\,(q_2)\,\bar{u}\,(k')\left[\frac{\gamma^\mu\,(\not{k}'+\not{q}_2)\,\gamma^\nu\,(1-\gamma_5)}{(k'+q_2)^2}\right]u\,(k)\,\langle p\,(p_2,s_2)|\,J^{CC}_\nu\,(0)\,|n\,(p_1,s_1)\rangle$$

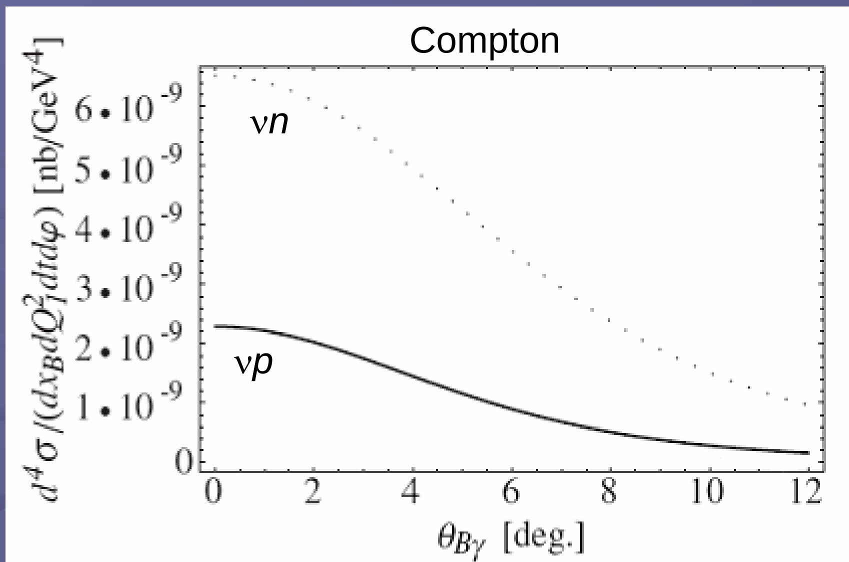
Isospin symmetry:

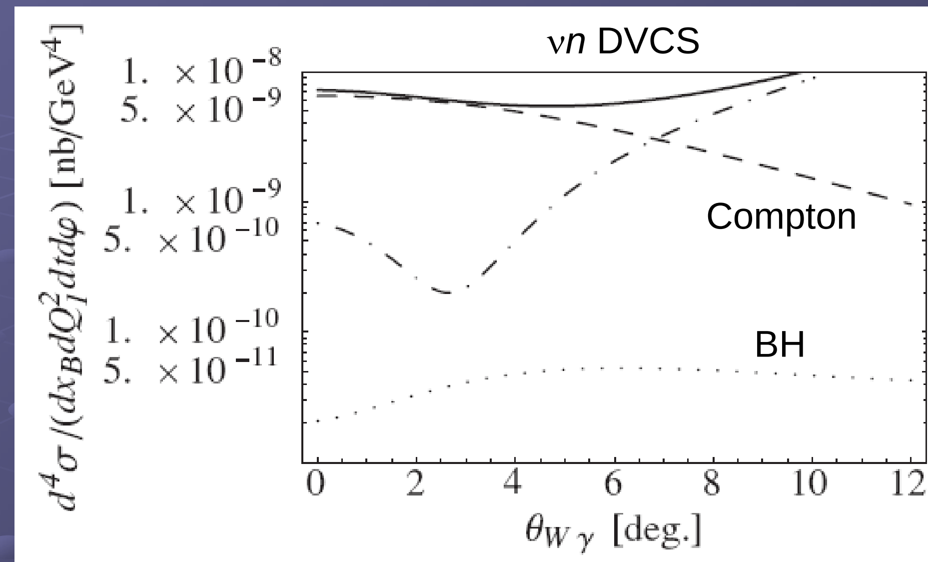
$$\langle p\,(p_2,s_2)|\,\mathcal{O}^{ud\pm}\,(z\,|0)\,|n\,(p_1,s_1)\rangle = \langle p\,(p_2,s_2)|\,\mathcal{O}^{u\pm}\,(z\,|0)\,|p\,(p_1,s_1)\rangle - \langle p\,(p_2,s_2)|\,\mathcal{O}^{d\pm}\,(z\,|0)\,|p\,(p_1,s_1)\rangle$$

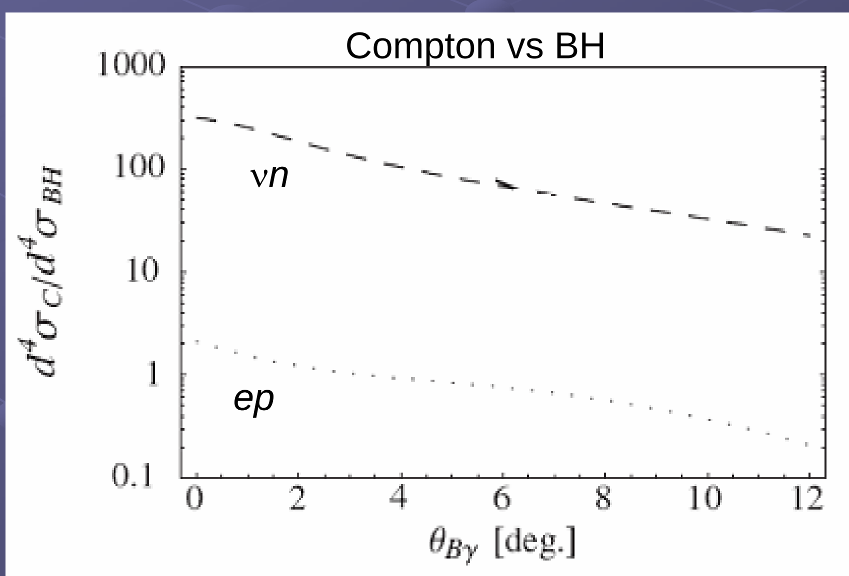
L. Mankiewicz, G. Piller, and T. Weigl, Phys. Rev. D 59, 017501 (1998).

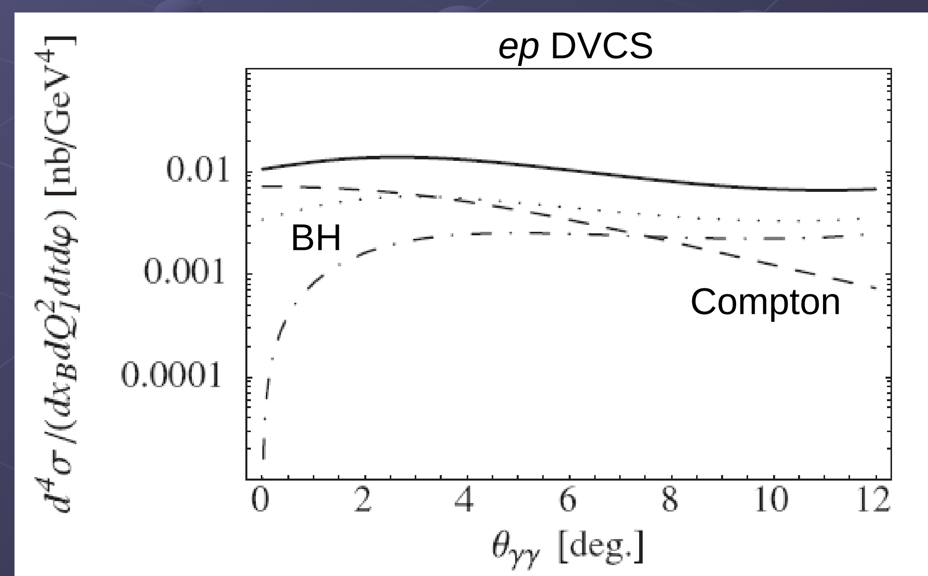
Results

$$\omega = 20\,\mathrm{GeV}, Q_1^2 = 2.5\,\mathrm{GeV}^2, x_B = 0.35, y = 0.19, \varphi = 0, |t| < 1\,\mathrm{GeV}^2$$









Summary

- GPDs provide the most complete and unified description of the internal quark-gluon structure of hadrons, which can be probed through wide variety of both inclusive and exclusive hard (i.e. light-cone dominated) processes.

- DVNS is an important tool to complement the study of GPDs in the more familiar electron-induced DVCS. In addition to measuring different combinations and flavor decomposition of GPDs, providing a direct separation of the their valence and sea contents, the process enables one to access the distributions that are nondiagonal in quark flavor, such as those associated with neutron-to-proton transition.

- We have derived the twist-2 Compton amplitudes for both weak neutral and weak charged current interactions by means of the light-cone expansion of the current product in coordinate space. Using a simple model for the nucleon GPDs, which only includes the valence quark contribution, we gave prediction for cross sections in the kinematics relevant to future high-intensity neutrino experiments.

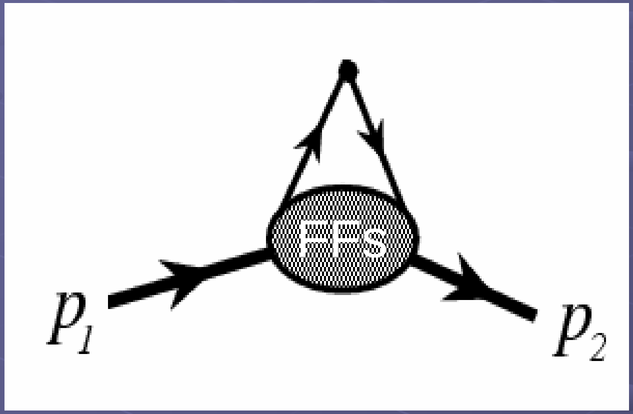
- Unlike the standard electromagnetic DVCS, we find that at small scattering angles the Compton contribution is enhanced relative to the corresponding BH contribution, and hence should make a contamination from BH background less of a problem when extracting the weak DVCS signal.

Outlook

- To use more realistic models for nucleon GPDs, including sea quark effects
- To elaborate separately contributions from the plus and minus distributions
- To include twist-3 terms in order to apply the formalism at moderate energies

Appendix

- Form factors are defined through matrix elements of ***electromagnetic*** and ***weak (neutral and charged) currents*** between hadronic states.



Dirac and Pauli electromagnetic form factors:

$$\langle N(p_1,s_1)|J^{\mu}_{EM}(0)|N(p_2,s_2)\rangle = \bar{u}(p_2,s_2)\left[\gamma^{\mu}F_1(t) - i\sigma^{\mu\nu}\frac{r_\nu}{2M}F_2(t)\right]u(p_1,s_1)$$

$$r \equiv p_1 - p_2,\ t = r^2$$

Dependence on *t* is clear evidence for ***extended structure*** of nucleon.

Flavor decomposition of form factors and their limiting values:

$$\sum_f Q_f \overline{\psi}_f(0)\gamma^\mu \psi_f(0) \rightarrow F_{1,2}(t) = \sum_f Q_f F_{1,2\,f}(t)$$

$$F_1(t=0) = Q_N \qquad F_2(t=0) = \kappa_N$$

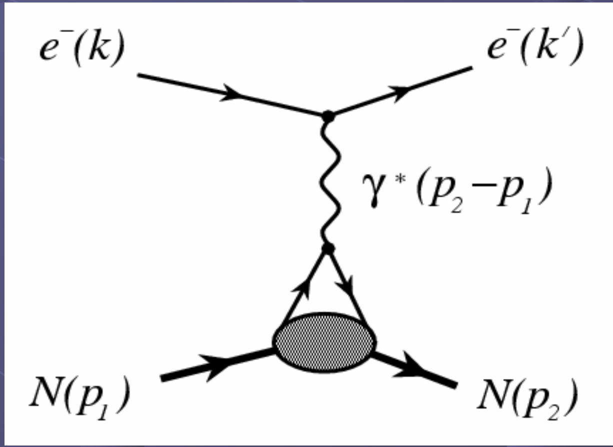
$$F_{1p}(t=0)=1,\ F_{1n}(t=0)=0$$
$$F_{2p}(t=0)=1.793,\ F_{2n}(t=0)=-1.913$$

Sachs electric and magnetic form factors:

$$G_E(t) = F_1(t) + \frac{t}{4M^2}F_2(t) \text{ and } G_M(t) = F_1(t) + F_2(t)$$

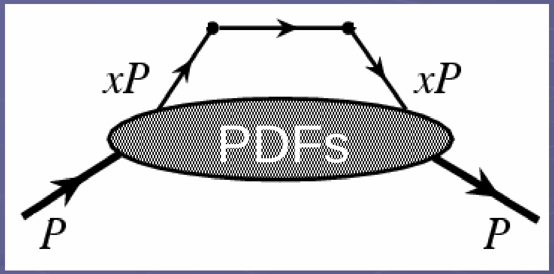
$$G_{Ep}(t=0)=1 \text{ and } G_{Mp}(t=0)=\mu_p = 2.793$$
$$G_{En}(t=0)=0 \text{ and } G_{Mn}(t=0)=\mu_n = -1.913$$

Nucleon electromagnetic form factors are measured through elastic electron-nucleon scattering.



(one-photon exchange approximation)

- Parton distribution functions are defined through ***forward matrix elements of light-like correlation functions*** (i.e. quark and gluon fields separated by light-like distances).



Unpolarized and polarized parton distribution functions:

$$\langle N(P,S)|\overline{\psi}_f(-z/2)\gamma^\mu\psi_f(z/2)|N(P,S)\rangle_{z^2=0} = \overline{u}(P,S)\gamma^\mu u(P,S)\int_0^1 dx\left[e^{ix(P\cdot z)}f_N(x) - e^{-ix(P\cdot z)}\overline{f}_N(x)\right]$$

$$\langle N(P,S)|\overline{\psi}_f(-z/2)\gamma^\mu\gamma_5\psi_f(z/2)|N(P,S)\rangle_{z^2=0} = \overline{u}(P,S)\gamma^\mu\gamma_5 u(P,S)\int_0^1 dx\left[e^{ix(P\cdot z)}\Delta f_N(x) + e^{-ix(P\cdot z)}\Delta\overline{f}_N(x)\right]$$

Definition of PDFs has form of ***plane wave decomposition***

Alternatively:

$$\int_0^1 dx\,[\ldots] \rightarrow \int_{-1}^1 dx\, e^{ix(P\cdot z)}\tilde{f}_N(x) \;\text{ and }\; \int_{-1}^1 dx\, e^{ix(P\cdot z)}\Delta\tilde{f}_N(x) \;\text{ with}$$

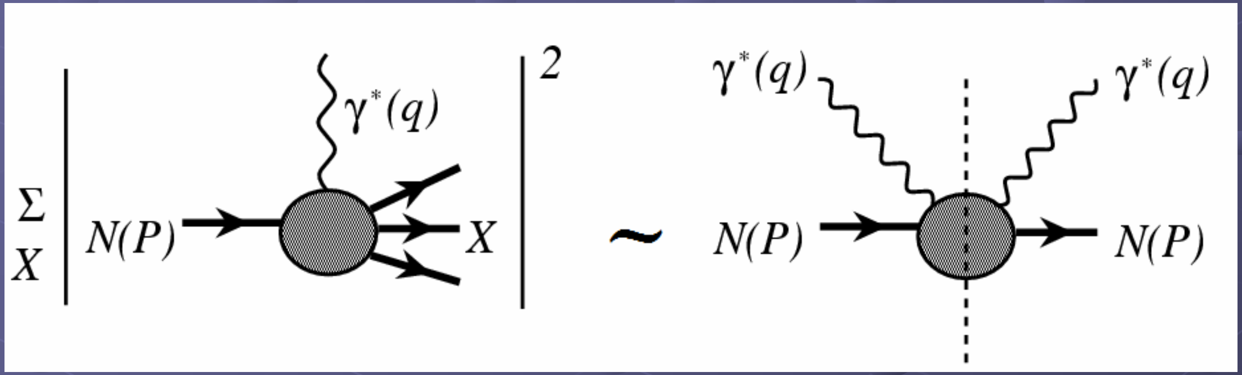
$$\tilde{f}_N(x) = \begin{cases} f_N(x) & x>0 \\ -\bar{f}_N(-x) & x<0 \end{cases}$$

$$\Delta\tilde{f}_N(x) = \begin{cases} \Delta f_N(x) & x>0 \\ \Delta\bar{f}_N(-x) & x<0 \end{cases}$$

Intensive study of PDFs in ***hard inclusive processes*** for last three decades.

Deeply inelastic lepton-nucleon scattering (DIS) played key role in revealing quark structure of nucleon.

Optical theorem - forward virtual Compton scattering amplitude (VCA).



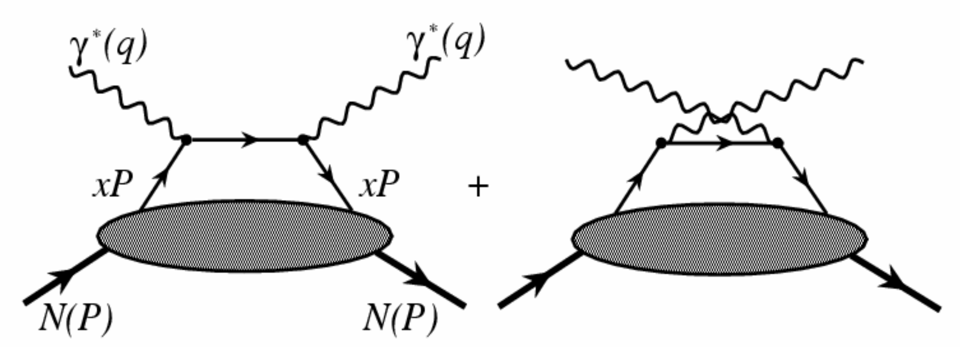
DIS structure functions are expressed by PDFs.

In ***Bjorken regime***: behavior of forward VCA is dominated by light-like distances.

Amplitude factorizes into convolution of perturbatively calculable hard scattering process and process independent matrix elements containing soft nonperturbative information about nucleon structure.

Leading contribution: ***s- and u-channel handbag diagrams***

$$Q^2 \equiv -q^2 \text{ and } s \equiv (P+q)^2 \to \infty \text{ with } x_B \equiv Q^2 / 2(P \cdot q) \text{ finite}$$

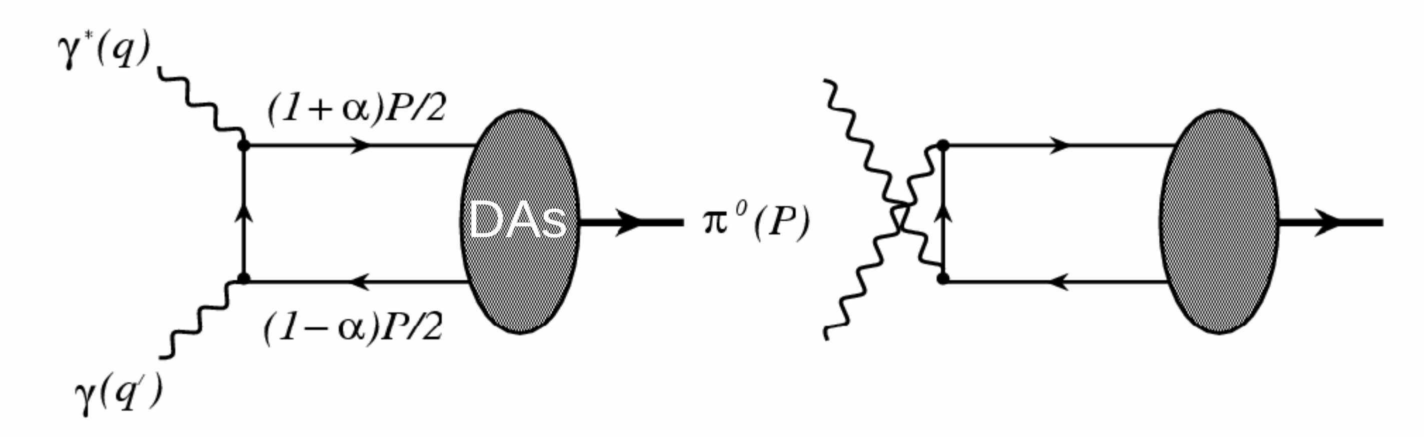


$$\frac{1}{(xP \pm q)^2 + i\varepsilon} \xrightarrow{\text{Im}} \frac{1}{2(P \cdot q)} \delta(x \mp x_B)$$

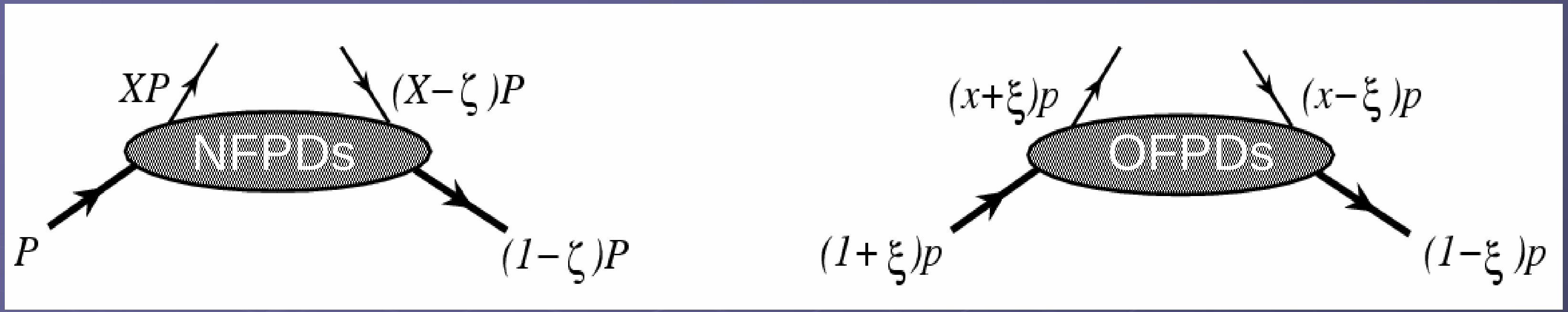
- Distribution amplitudes are defined through ***vacuum-to-hadron matrix elements of light-cone operators***, and describe hadrons in hard exclusive processes.

$$\left\langle 0 \left| \overline{\psi}_d \left(-z/2\right) \gamma^\mu \gamma_5 \psi_u \left(z/2\right) \right| \pi^+ (P) \right\rangle_{z^2=0} = iP^\mu f_\pi \int_{-1}^{1} d\alpha\, e^{i\alpha (P\cdot z)/2} \varphi_{\pi^+}(\alpha)$$

Transition: $\gamma^* + \gamma \to \pi^0$



- Generalized parton distributions: two implementation of the formalism.



$F_{\zeta}^{f}(X,t)$ where $0 \le \zeta \equiv r^{+}/P^{+} \le 1$

$H_{f}(x,\xi,t)$ where $0 \le \xi \equiv r^{+}/2p^{+} \le 1$

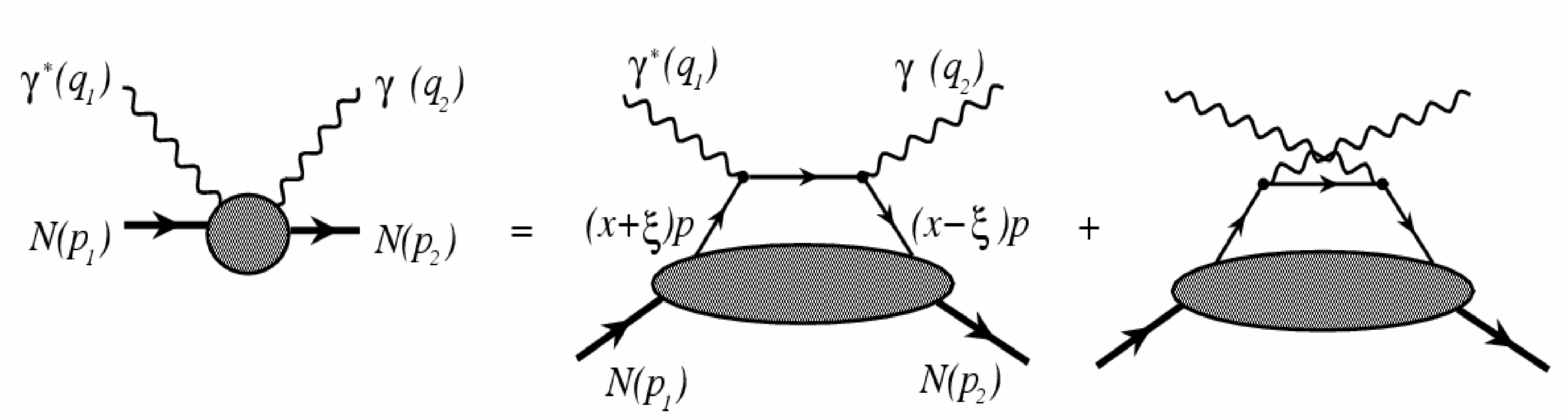
Variables (X, ζ) and (x, ξ) characterize longitudinal components of partons.

$$X = \frac{x+\xi}{1+\xi} \text{ and } \zeta = \frac{2\xi}{1+\xi}$$

High energy and high luminosity lepton accelerators combined with large acceptance spectrometers give unique opportunity to perform precision studies.

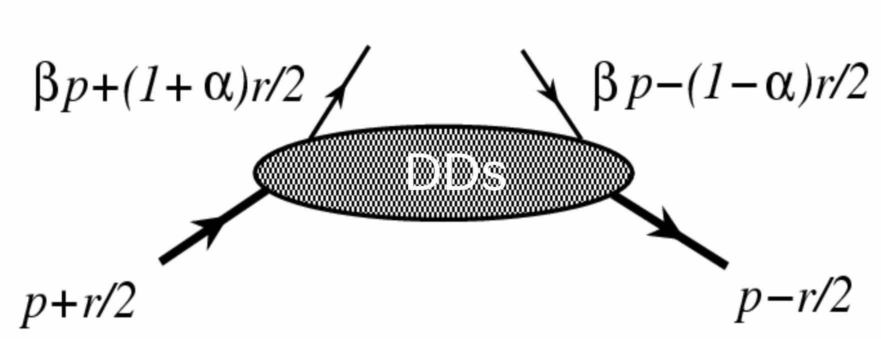
QCD factorization is more general - ***nonforward VCA***

$$-q_1^2, (p_1 + q_1)^2 \to \infty, q_2^2 = 0$$



GPDs appear at amplitude level whereas in inclusive processes, amplitude described by PDFs enter through optical theorem at level of cross section.

- Double distributions:



Active parton momentum is represented as sum of two components, specifying momentum flow in *s*- and *t*-channels.

Superposition of momentum fluxes in two different channels. Double distributions are hybrids between PDFs (with respect to β) and distribution amplitudes (with respect to α).

Connection between (α, β) and (x, ξ) is established via $r^+ = 2\xi p^+$

$$x = \beta + \xi\alpha$$

- Weak virtual Compton amplitude:

$$T_W^{\mu\nu} = i \int d^4x \int d^4y \, e^{-iq_1\cdot x + iq_2\cdot y} \langle N'(p_2, s_2)| T\{J^\mu_{EM}(y) J^\nu_W(x)\} |N(p_1, s_1)\rangle$$

$$X \equiv (x+y)/2 \text{ and } z \equiv y - x$$

$$\langle p_2| J^\mu(X) |p_1\rangle = \langle p_2| J^\mu(0) |p_1\rangle e^{-i(p_1-p_2)\cdot X}$$

$$T_W^{\mu\nu} = (2\pi)^4 \delta^{(4)}(p_1 + q_1 - p_2 - q_2) \mathcal{T}_W^{\mu\nu}$$

Totally symmetric traceless parts:

$$\left[\mathcal{O}_\eta^{f\pm}(z|0)\right]_{sym} = \frac{\partial}{\partial z^\eta} \int_0^1 d\beta \left[\bar{\psi}_f(\beta z/2) \not{z} \psi_f(-\beta z/2) \mp (z \to -z)\right]$$

$$\left[\mathcal{O}_{5\eta}^{f\pm}(z|0)\right]_{sym} = \frac{\partial}{\partial z^\eta} \int_0^1 d\beta \left[\bar{\psi}_f(\beta z/2) \not{z}\gamma_5 \psi_f(-\beta z/2) \mp (z \to -z)\right]$$

$$\mathcal{O}^{f\pm}(z|0) \equiv \left[\bar{\psi}_f(z/2) \not{z} \psi_f(-z/2) \pm (z \to -z)\right]$$

$$\mathcal{O}_5^{f\pm}(z|0) \equiv \left[\bar{\psi}_f(z/2) \not{z}\gamma_5 \psi_f(-z/2) \pm (z \to -z)\right]$$

$$\partial_z^2 \left[\mathcal{O}^{f\pm}(z|0)\right]_{twist-2} = 0$$

Sudakov (light-cone) decomposition of γ-matrix:

$$\gamma^\mu = a^\mu \not{n}_1 + b^\mu \not{n}_2 + \gamma_\perp^\mu$$

$$\gamma^\mu = \frac{1}{(p\cdot q_2)} \left(q_2^\mu \not{p} + p^\mu \not{q}_2\right)$$

Symmetry properties:

$$\begin{aligned} H_f^{\pm}(x) &= \mp H_f^{\pm}(-x) \\ E_f^{\pm}(x) &= \mp E_f^{\pm}(-x) \\ \tilde{H}_f^{\pm}(x) &= \pm \tilde{H}_f^{\pm}(-x) \\ \tilde{E}_f^{\pm}(x) &= \pm \tilde{E}_f^{\pm}(-x) \end{aligned}$$

- Model:

$$\begin{aligned} H_u^{val}(x,\xi,t) &= u_N^{val}(x)\, F_{1u}(t)/2 \\ H_d^{val}(x,\xi,t) &= d_N^{val}(x)\, F_{1d}(t) \\ E_u^{val}(x,\xi,t) &= u_N^{val}(x)\, F_{2u}(t)/2 \\ E_d^{val}(x,\xi,t) &= d_N^{val}(x)\, F_{2d}(t) \end{aligned}$$

$$\begin{aligned} u_p^{val}(x) &= 1.89x^{-0.4}(1-x)^{3.5}(1+6x) \\ d_p^{val}(x) &= 0.54x^{-0.6}(1-x)^{4.2}(1+8x) \end{aligned}$$

$$G_{Ep}(t) = \frac{G_{Mp}(t)}{1+\kappa_p} = \frac{G_{Mn}(t)}{\kappa_n} = \left(1-\frac{t}{\Lambda^2}\right)^{-2} \quad \text{and} \quad G_{En}(t) = 0$$

$$\Lambda^2 = 0.71\mathrm{GeV}^2$$

$$\begin{aligned} \tilde{H}_u^{val}(x,\xi,t) &= \Delta u_p^{val}(x)\left(1-\frac{t}{m_A^2}\right)^{-2} \\ \tilde{H}_d^{val}(x,\xi,t) &= \Delta d_p^{val}(x)\left(1-\frac{t}{m_A^2}\right)^{-2} \end{aligned}$$

$$\begin{aligned} \Delta u_p^{val} &= \cos\theta_D\left(u_p^{val}-\frac{2}{3}d_p^{val}\right) \\ \Delta d_p^{val} &= \cos\theta_D\left(-\frac{1}{3}d_p^{val}\right) \end{aligned}$$

$$\cos\theta_D = \left[1+\mathrm{H}_0\left(1-x^2\right)/\sqrt{x}\,\right]^{-1}$$

$$H_0 = 0.06$$

$$m_A = 1.03\mathrm{GeV}$$

$$\begin{aligned} \tilde{E}_u^{val}(x,\xi,t) &= \frac{1}{2}F_\pi(t)\,\frac{\theta(|x|<\xi)}{2\xi}\,\phi_\pi\left(\frac{x+\xi}{2\xi}\right) \\ \tilde{E}_d^{val}(x,\xi,t) &= -\tilde{E}_u^{val}(x,\xi,t) \end{aligned}$$

$$\phi_\pi(u) = 6u(1-u)$$

$$F_\pi(t) = 4g_A(t=0)\,M^2\left[\frac{1}{(m_\pi^2-t)/\mathrm{GeV}^2} - \frac{1.7}{(1-t/2\,\mathrm{GeV}^2)^2}\right]$$