Soft Gluon Effects in Four-Parton Hard-Scattering Processes

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outline

1. Motivation: Sudakov logs and Resummation
2. One-hadron inclusive cross section close to the **partonic threshold**
3. The production of a $t\bar{t}$ pair with **small** $\mathbf{q}_T$
4. Conclusions

soft-gluon effects

- Perturbative QCD predictions for **semi-inclusive** quantities are sensitive to soft-gluon effects.
- **Energy or momentum cuts** $\rightarrow$ logarithms of hard scales. Origin: the cancellation of real and virtual IR singularities.
- In the **threshold** regions where the real emission is kinematically inhibited (soft) these scales are strongly ordered.
  $\rightarrow$ The logarithmic terms are strongly **enhanced**.

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- $\alpha_{\mathrm{S}}\mathrm{L} \sim 1 \rightarrow$ perturbative expansion spoiled. Fixed-order predictions deviate significantly from the available experimental results.
- A reliable evaluation of any cross-section in the near-threshold region requires the **all-order resummation** of the large logarithms.

resummation

$$\sigma^{(\mathrm{res})} \sim \sigma^{(0)} C(\alpha_{\mathrm{S}})\ \exp\{\underbrace{\mathrm{L}\, g_1(\alpha_{\mathrm{S}}\mathrm{L})}_{\mathrm{LL}} + \underbrace{g_2(\alpha_{\mathrm{S}}\mathrm{L})}_{\mathrm{NLL}} + \underbrace{\alpha_{\mathrm{S}}\, g_3(\alpha_{\mathrm{S}}\mathrm{L})}_{\mathrm{NNLL}} + \ldots\}$$

- The logarithmic **structure** is, to some extent, universal.
- $C(\alpha_{\mathrm{S}})$: process-dependent constant terms,
  $\longrightarrow$ real corrections + (multi-loop) virtual corrections,
  $\longrightarrow$ fixed-order calculation: N^{x}LO to control N^{x}LL terms!

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- Resummed results are available in QCD and SCET, for hadron-hadron scattering processes with 2 hard partons at LO, both for $\mathbf{q}_T$ resummation [Parisi et al. '79, Curci et al. '79, Dokshitzer et al. '80, Bassetto et al. '80, Kodaira et al. '82, Davies et al. '84, Collins et al. '85, Catani et al. '88, de Florian et al. '01, Catani et al. '01, Bozzi et. al.'06, Catani et al. '11, Becher et al. '11] and for threshold resummation [Sterman '86, Catani et al. '89, Catani et al. '90, Vogt '00, Catani et al. '01, Catani et al. '03, Moch et al. '05, Laenen et al. '05].

- We focus on scatterings with **4 QCD partons** at the LO.
- We calculate the NLO (logs and non-logs) enhanced corrections.
- See also:

  [Kidonakis et al. '96, Bonciani et al. '98, Kidonakis et al. '98, Laenen et al. '98, Catani et al. '98, Bonciani et al. '03] [Manohar '03, Idilbi et al. '06, Becher et al. '06, Becher et al. '07, Ahrens et al. '08, Beneke et al. '09, Becher et al. '09]

  [Laenen et al. '98, Catani et al. '98, Sterman et al. '00, Bolzoni et al. '05, Becher et al. '09] [Kidonakis et al. '99, Becher et al. '11, de Florian et al. '05]

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- Eikonal approximation in color space ($\forall$ emitter a_i):

$$|\mathcal{M}_g(k)|^2 = -2\pi\alpha_{\rm S}\sum_{i,j=1}^{4}\frac{p_i p_j}{(p_i k)(p_j k)}\langle\mathcal{M}^{(0)}|\mathbf{T}_i\cdot\mathbf{T}_j|\mathcal{M}^{(0)}\rangle.$$

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! Non-soft collinear emission $\rightarrow$ Altarelli-Parisi splitting functions:

$$\frac{\mathbf{T}_i^2}{p_i k}\frac{1}{1-z_{ij}} \rightarrow \frac{\mathbf{T}_i^2}{p_i k}\left(\frac{1}{1-z_{ij}} + < \text{regular as } z_{ij}\rightarrow 1 >\right).$$

! The sum of virtual, real and collinear counterterms (PDFs) is finite.

1. single-hadron inclusive cross section

- At high $\mathbf{p}_T$: leading contributions at the **partonic threshold**.

$h_1 h_2 \to h_3 X$ $\qquad\qquad$ $a_1 a_2 \to a_3 X$

$$\frac{d\sigma_{h_1 h_2 h_3}}{d^3\boldsymbol{P}_3/E_3}(P_i) = \sum_{a_1,a_2,a_3} f^{(\mu_F)}_{h_1/a_1} \otimes f^{(\mu_F)}_{h_2/a_2} \otimes d^{(\mu_f)}_{h_3/a_3} \otimes \frac{d\hat{\sigma}_{a_1 a_2 a_3}}{d^3\boldsymbol{p}_3/p_3^0}(p_i, \mu_F, \mu_f)$$

$\to d\eta\, d^2\mathbf{p}_T$

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$\to d\eta\, d^2\mathbf{p}_T$

- Up to NLO:

$$\frac{d\hat{\sigma}^{\text{sing}}_{a_1 a_2 a_3}}{d^3\boldsymbol{p}_3/p_3^0} = \frac{\alpha_S^2(\mu_R^2)}{\pi\, s}\left[\frac{1}{v}\frac{d\hat{\sigma}^{(0)}}{dv}\,\delta(1-w) + \frac{\alpha_S(\mu_R^2)}{2\,\pi\, v\, s}\,\mathcal{C}^{(1)}(s,v;w) + \mathcal{O}(\alpha_S^2)\right]$$

at fixed

$$s, \quad v = 1 + \frac{t}{s}, \quad w = \frac{-u}{s+t} \qquad (s \leftrightarrow p_T, \quad v \leftrightarrow \eta, \quad x_{\text{th}} = 1 - w).$$

- Structure of the singular term at NLO: [Aversa, Chiappetta, Greco, Guillet '89]

$$\mathcal{C}^{(1)}(s,v;w) = \mathcal{C}_3 \left(\frac{\ln(1-w)}{1-w}\right)_+ + \mathcal{C}_2(s,v)\left(\frac{1}{1-w}\right)_+ + \mathcal{C}_1(s,v)\,\delta(1-w)\,.$$

- Our result at threshold, factorized in color space:

$$16\pi N^{(in)}\mathcal{C}^{(1)} = \langle \mathcal{M}^{(0)}|\,\mathbf{C}^{(1)}|\mathcal{M}^{(0)}\rangle + \Big(\langle \mathcal{M}^{(0)}|\mathcal{M}^{(1)\,\mathrm{fin}}\rangle + \mathrm{c.c.}\Big)\delta(1-w)\,.$$

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- Logarithmic terms: $\mathbf{C}^{(1)} = 2\left(\frac{\ln(1-w)}{1-w}\right)_+ \mathbf{C}^{(1)}_{\mathrm{LL}} - \left(\frac{1}{1-w}\right)_+ \mathbf{C}^{(1)}_{\mathrm{L}}\,,$

$$\mathbf{C}^{(1)}_{\mathrm{LL}} = 2\sum_{i=1}^{3} \mathbf{T}_i^2 - \mathbf{T}_4^2\,, \qquad \mathbf{C}^{(1)}_{\mathrm{L}} = 2\sum_{i=1}^{3} \mathbf{T}_i^2 \left(\ln\frac{1-v}{v} + \ln\frac{\mu_{Fi}^2}{s} \right) - 2\,\mathbf{T}_4^2 \ln(1-v)$$
$$+ 8\left(\mathbf{T}_1 \cdot \mathbf{T}_3 \ln(1-v) + \mathbf{T}_2 \cdot \mathbf{T}_3 \ln v \right) + \gamma_4\,.$$

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- Hard-virtual amplitude: $|\mathcal{M}^{(1)\,\mathrm{fin}}\rangle = |\mathcal{M}^{(1)}\rangle - \widetilde{\mathbf{I}}^{(1)}(\epsilon)\,|\mathcal{M}^{(0)}\rangle\,.$

$$
\begin{aligned}
2\widetilde{\mathbf{I}}^{(1)}(\epsilon) = \frac{1}{\Gamma(1-\epsilon)} & \left[\frac{1}{\epsilon^2} \sum_{(i\neq j)=1}^{4} \mathbf{T}_i \cdot \mathbf{T}_j \left(\frac{4\pi\mu_R^2\, e^{-i\lambda_{ij}\pi}}{2p_i p_j} \right)^{\epsilon} - \frac{1}{\epsilon} \sum_{i=1}^{4} \gamma_i \left(\frac{4\pi\mu_R^2\, s}{u\, t} \right)^{\epsilon} \right] \\
& + \frac{\pi^2}{2} \left(\mathbf{T}_1^2 + \mathbf{T}_2^2 + 3\mathbf{T}_3^2 - \frac{4}{3}\mathbf{T}_4^2 \right) - \sum_{i=1}^{3} \gamma_i \ln \frac{\mu_{Fi}^2}{s\, v(1-v)} + \gamma_4 \ln(1-v) \\
& - 2\mathbf{T}_3^2 \ln v \, \ln \frac{\mu_f^2}{s} + 2\mathbf{T}_2^2 \ln \frac{1-v}{v} \ln \frac{\mu_F^2}{s} + \ln v \ln(1-v) \left(\mathbf{T}_4^2 - \mathbf{T}_1^2 - \mathbf{T}_2^2 - \mathbf{T}_3^2 \right) \\
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✓ Consistent with (the dominant contribution of) known results:

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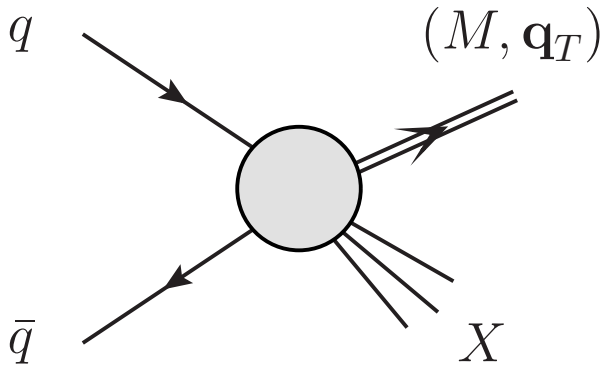
- ▸ Resummation formula: [S.Catani, M.Grazzini, A.T. '13].
- ✓ Consistent with [D.de Florian, W.Vogelsang '05] at $\eta = 0$ ($v = 1/2$).

2. $t\bar{t}$ pair production

- Similar to Drell-Yan: $h_1 + h_2 \to F(M, \mathbf{q}_T) + X$.



- Logarithmically enhanced terms as $\mathbf{q}_T^2 \ll M^2$.
- Simple color algebra ($\mathbf{T}_i^2 = C_i$) :
  LL+NLL from **soft-collinear** radiation,
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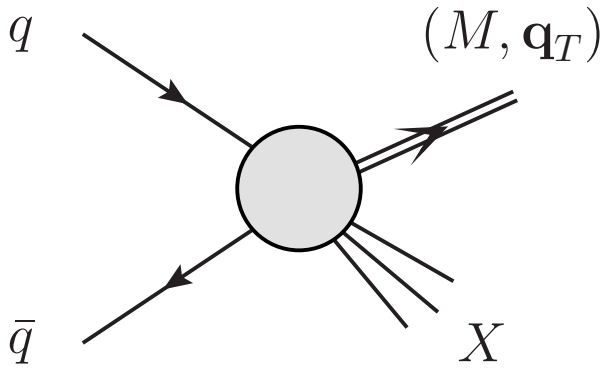

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  LL+NLL from **soft-collinear** radiation,
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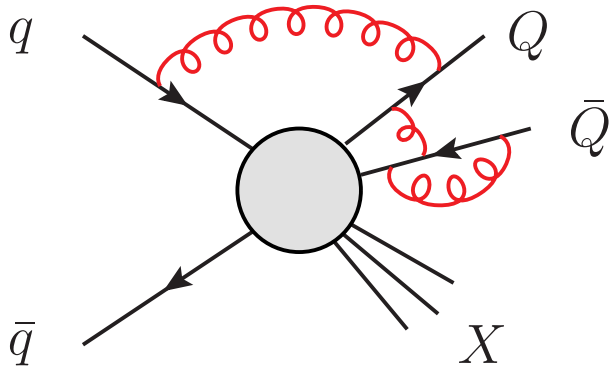
- Now F is replaced by $Q(p_3, \mathbf{T}_3) + \bar{Q}(p_4, \mathbf{T}_4)$.



- DY-like collinear emission [Berger, Meng '94].
- Q massive: additional source of **soft** radiation
  $\to$ **extra NLL + color correlations** ($\mathbf{T}_i \cdot \mathbf{T}_j$).
- Resummed result obtained in SCET
  [H.T.Li, C.S.Li, D.Y.Shao, L.L.Yang, H.X.Zhu '13].

- Distribution at fixed $q^\mu = p_3^\mu + p_4^\mu$ and $\mathbf{\Omega} \sim \{y_3, \phi_3\}$:

$$\frac{d\sigma_{h_1 h_2 \to Q\bar{Q}X}}{d\mathbf{q}_T^2 dM^2 dy\, d\mathbf{\Omega}} = \frac{M^2}{s} \sum_{b_1, b_2} f_{h_1/b_1}^{(\mu_F)} \otimes f_{h_2/b_2}^{(\mu_F)} \otimes \frac{d\hat{\sigma}_{b_1 b_2 \to Q\bar{Q}X}}{dx_T^2\, dz_1\, dz_2\, d\mathbf{\Omega}} \,.$$

- Singular cross section $\left(x_T^2 = \mathbf{q}_T^2/M^2, \quad z_{1,2} = e^{\pm\hat{y}}\sqrt{M^2/\hat{s}}\right)$:

$$\frac{d\hat{\sigma}^{\text{sing}}_{b_1 b_2 \to Q\bar{Q}X}}{dx_T^2\, dz_1\, dz_2\, d\mathbf{\Omega}} = \frac{\pi}{M^2} \sum_{c=q,\bar{q},g} \langle \mathcal{M}^{\text{fin}}_{c\bar{c}\to Q\bar{Q}} | \, \mathbf{\Delta}_{c\bar{c};b_1 b_2}(x_T^2; z_1, z_2) \, | \mathcal{M}^{\text{fin}}_{c\bar{c}\to Q\bar{Q}} \rangle \,.$$

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- Hard-virtual amplitude: $|\mathcal{M}^{\text{fin}}_{c\bar{c}\to Q\bar{Q}}\rangle = \left[1 - \widetilde{\mathbf{I}}^{\text{DY}}_{c\bar{c}}(\epsilon) - \widetilde{\mathbf{I}}_{\text{HQ}}(\epsilon)\right] |\mathcal{M}_{c\bar{c}\to Q\bar{Q}}\rangle$.

- Distribution at fixed $q^\mu = p_3^\mu + p_4^\mu$ and $\mathbf{\Omega} \sim \{y_3, \phi_3\}$:

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- DY-like soft and collinear emission

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- Distribution at fixed $q^\mu = p_3^\mu + p_4^\mu$ and $\mathbf{\Omega} \sim \{y_3, \phi_3\}$:

$$\frac{d\sigma_{h_1 h_2 \to Q\bar{Q}X}}{d\mathbf{q}_T^2 dM^2 dy\, d\mathbf{\Omega}} = \frac{M^2}{s} \sum_{b_1, b_2} f_{h_1/b_1}^{(\mu_F)} \otimes f_{h_2/b_2}^{(\mu_F)} \otimes \frac{d\hat{\sigma}_{b_1 b_2 \to Q\bar{Q}X}}{dx_T^2\, dz_1\, dz_2\, d\mathbf{\Omega}} \,.$$

- Singular cross section $\left(x_T^2 = \mathbf{q}_T^2/M^2, \quad z_{1,2} = e^{\pm\hat{y}}\sqrt{M^2/\hat{s}}\right)$:

$$\frac{d\hat{\sigma}^{\text{sing}}_{b_1 b_2 \to Q\bar{Q}X}}{dx_T^2\, dz_1\, dz_2\, d\mathbf{\Omega}} = \frac{\pi}{M^2} \sum_{c=q,\bar{q},g} \langle \mathcal{M}^{\text{fin}}_{c\bar{c}\to Q\bar{Q}} |\, \mathbf{\Delta}_{c\bar{c};b_1 b_2}(x_T^2; z_1, z_2)\, | \mathcal{M}^{\text{fin}}_{c\bar{c}\to Q\bar{Q}} \rangle \,.$$

- Hard-virtual amplitude: $|\mathcal{M}^{\text{fin}}_{c\bar{c}\to Q\bar{Q}}\rangle = \left[1 - \widetilde{\mathbf{I}}^{\text{DY}}_{c\bar{c}}(\epsilon) - \widetilde{\mathbf{I}}_{\text{HQ}}(\epsilon)\right] |\mathcal{M}_{c\bar{c}\to Q\bar{Q}}\rangle$.
- DY-like soft and collinear emission + emission from $Q\bar{Q}$:

$$\mathbf{\Delta}_{c\bar{c};b_1 b_2}(x_T^2; z_1, z_2) = \mathbf{\Delta}^{\text{DY}}_{c\bar{c};b_1 b_2}(x_T^2; z_1, z_2) + \mathbf{\Delta}_{\text{HQ}}(x_T^2)\, \delta_{c b_1}^{(1-z_1)}\, \delta_{\bar{c} b_2}^{(1-z_2)} + c.c.$$

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- @ NLO, with $\left(\frac{\alpha_S}{4\pi}\right)$ normalization:

$$\mathbf{\Delta}^{(1)}_{\text{HQ}}(x_T^2) - \widetilde{\mathbf{I}}^{(1)}_{\text{HQ}}(\epsilon)\, \delta(x_T^2) = \frac{1}{\epsilon}\left(\frac{M^2}{\mu_R^2}\right)^{-\epsilon} \delta(x^2)\, \mathbf{\Gamma}_T^{(1)} - \left(\frac{1}{x_T^2}\right)_+ \mathbf{\Gamma}_T^{(1)} + \delta(x^2)\, \mathbf{F}_T^{(1)} .$$

$$\mathbf{\Gamma}_T^{(1)} = \sum_{j=3,4} \mathbf{T}_j^2 \,(1 - i\pi) + \sum_{\substack{i=1,2\\ j=3,4}} \mathbf{T}_i \cdot \mathbf{T}_j \,\ln \frac{(2p_i p_j)^2}{M^2 m_j^2} + 2\mathbf{T}_3 \cdot \mathbf{T}_4 \left[\frac{1}{2v_{34}} \ln \frac{1+v_{34}}{1-v_{34}} - i\pi \left(\frac{1}{v_{34}} + 1 \right) \right],$$

$$\mathbf{F}_T^{(1)} = \sum_{j=3,4} \mathbf{T}_j^2 \,\ln \left(1 + \frac{\mathbf{p}_{jT}^2}{m_j^2} \right) - \sum_{\substack{i=1,2\\ j=3,4}} \mathbf{T}_i \cdot \mathbf{T}_j \,\mathrm{Li}_2 \left(-\frac{\mathbf{p}_{jT}^2}{m_j^2} \right) + 2\mathbf{T}_3 \cdot \mathbf{T}_4 \,\frac{1}{2v_{34}} \,L_{34}\,,$$

with

$$L_{34} = \frac{1}{2} \ln \frac{1+v_{34}}{1-v_{34}} \ln \left[\left(1 + \frac{\mathbf{p}_{3T}^2}{m_3^2} \right) \left(1 + \frac{\mathbf{p}_{4T}^2}{m_4^2} \right) \right] - 2\,\mathrm{Li}_2 \frac{2v_{34}}{1+v_{34}} - \frac{1}{4} \ln^2 \frac{1+v_{34}}{1-v_{34}}$$
$$+ \sum_{i=1,2} \left[\mathrm{Li}_2 \left(1 - \sqrt{\frac{1-v_{34}}{1+v_{34}}}\, r_{34,i} \right) + \mathrm{Li}_2 \left(1 - \sqrt{\frac{1-v_{34}}{1+v_{34}}} \frac{1}{r_{34,i}} \right) + \frac{1}{2} \ln^2 r_{34,i} \right],$$

and

$$v_{34} = \sqrt{1 - \frac{m_3^2 m_4^2}{(p_3 p_4)^2}}\,, \qquad r_{34,i} = \frac{p_i\, p_3}{p_i\, p_4} \sqrt{\frac{m_4^2}{m_3^2}}\,.$$

$\mathbf{q}_T$-subtraction

- Total cross section:

$$d\sigma^{\mathrm{NLO}}_{t\bar{t}X} = \mathcal{H} \otimes d\sigma_0 + \int d\mathbf{q}_T \underbrace{\left[\left(\frac{d\sigma_{t\bar{t}J}}{d\mathbf{q}_T}\right) - \left(\frac{d\sigma_{t\bar{t}}}{d\mathbf{q}_T}\right)_{\mathrm{CT}}\right]}_{\text{regular as } \mathbf{q}_T \to 0},$$

where $\mathcal{H} \leftrightarrow \langle \mathcal{M}^{\mathrm{fin}} | \mathcal{M}^{\mathrm{fin}} \rangle$ and $\left(\frac{d\sigma_{t\bar{t}}}{d\mathbf{q}_T}\right)_{\mathrm{CT}} \leftrightarrow \langle \mathcal{M}^{(0)} | \mathbf{\Delta} | \mathcal{M}^{(0)} \rangle$.

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✓ $\mathbf{q}_T$-subtraction for $t\bar{t}$ + Jet implemented numerically [with H.Sargsyan] and consistent with know results.

✓ $\mathbf{\Gamma}^{(1)}_T$ consistent with ϵ-poles in [Catani, Dittmaier, Trócsányi '00] and with $\mathbf{\Gamma}$ in [A.Ferroglia, M.Neubert, B.D.Pecjak, L.L.Yang '09].

✓ $\mathbf{F}^{(1)}_T$ consistent with [H.T.Li, C.S.Li, D.Y.Shao, L.L.Yang, H.X.Zhu '13] in the $q\bar{q}$ channel (gg channel: check in progress).

Conclusions

- We have considered a general method to calculate QCD corrections in soft and collinear limits and we have applied it to hard-scattering processes involving 4 hard QCD partons at LO.
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1. Single-hadron inclusive cross section at the partonic threshold.

✓ NLO in agreement with known results.

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2. Production of a $t\bar{t}$ pair with small $\mathbf{q}_T$.

✓ Agreement with known results at NLO.

✓ $\mathbf{q}_T$-subtraction at NLO implemented.

→ Work in progress: $\mathbf{q}_T$-resummation.

Thank you!

Backup: all-order soft-gluon resummation

- In terms of $x_\omega = -\dfrac{t+u}{s}, \quad r = \dfrac{u}{t}, \quad p_T^2 = \dfrac{t\,u}{s},$

$$\frac{d\hat{\sigma}_{a_1 a_2 a_3}}{d^3\boldsymbol{p}_3/p_3^0} = \frac{\overline{|\mathcal{M}^{(0)}_{a_1 a_2 a_3 a_4}(r, p_T^2)|^2}}{(4\pi s)^2}\, \Sigma_{a_1 a_2 \to a_3}(x_\omega, r; p_T^2, \mu_F, \mu_f).$$

- In Mellin space (N conjugated to x_ω): [Bonciani, Catani, Mangano, Nason '03]

$$\Sigma^{\rm res}_{a_1 a_2 a_3, N} = \prod_{i=1,2,3} \Delta_{a_i, N_i}(Q_i^2, \mu_i^2)\, J_{a_4, N_4}(Q_4^2)\, \frac{\langle \mathcal{M}_{\rm H} | \boldsymbol{\Delta}_N^{(\rm int)} | \mathcal{M}_{\rm H} \rangle}{|\mathcal{M}^{(0)}|^2},$$

where [1305.3870]

$$\boldsymbol{\Delta}_N^{(\rm int)} = \boldsymbol{V}_N^\dagger \boldsymbol{V}_N\,, \qquad \ln \boldsymbol{V}_N = \sum_{i \neq j} \int_0^1 \frac{z^{N-1}-1}{1-z}\, \boldsymbol{\Gamma}\left(\alpha_{\rm S}\left((1-z)^2 p_T^2\right), r\right),$$

$$|\mathcal{M}_{\rm H}\rangle = [1 - \boldsymbol{I}_{\rm H}]\, |\mathcal{M}\rangle\,.$$