The Polyakov loop and correlator of Polyakov loops at NNLO

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based on

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Outline

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4. An EFT interpretation

5. Discussion

Motivations

The Polyakov loop and the correlator of two Polyakov loops are two related and relevant quantities for the dynamics of one or two static sources in a thermal bath at temperature T.

- The Polyakov loop average in a thermal ensemble at a temperature T is
  $\langle L_R \rangle \equiv \frac{1}{d_R} \langle \mathrm{Tr\ L_R} \rangle$ (R $\equiv$ color representation)
  $d_A = N^2 - 1$, $d_F = N$ and $L_R(\mathbf{x}) = \mathrm{P} \exp\left(ig \int_0^{1/T} d\tau A^0(\mathbf{x}, \tau) \right)$.
- The (connected part of a) Polyakov loop correlator is
  $C_{\mathrm{PL}}(r, T) \equiv \frac{1}{N^2} \langle \mathrm{Tr} L_F^\dagger(\mathbf{0}) \mathrm{Tr} L_F(\mathbf{r}) \rangle - \langle L_F \rangle^2$.

Their relevance comes from the fact that they are gauge invariant quantities well known from lattice calculation.

○ `e.g. Petreczky EPJC 43(05)51`

Moreover, we have that

$$\langle \mathrm{Tr\ L_F^\dagger(\mathbf{0}) Tr\ L_F(\mathbf{r})} \rangle = \sum \mathrm{e}^{-\mathrm{E_n/T}}$$

○ `Luscher Weisz JHEP 0207(02)049, Jahn Philipsen PRD 70(04)074504`

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Despite the relevance, not much is known about the correlator in perturbation theory.
The correlator is known at LO since long.
∘ `McLerran Svetitsky PRD 24(81)450, Gross Pisarski Yaffe RMP 53(81)43`

Beyond leading order, it was computed only for $1/r \sim m_D$ (m_D is the Debye mass or electric screening length).
∘ `Nadkarni PRD 33(86)3738`

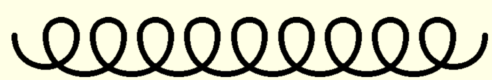
Static and non-static modes

It is convenient to perform the calculation in static gauge $\partial_0 A^0(x) = 0$:

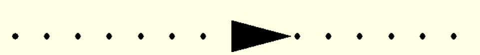
$$L(\mathbf{x}) = \exp\left(\frac{igA^0(\mathbf{x})}{T}\right)$$

Propagators may be split into a static and a non-static component:

$D_{00}(\omega_n, \mathbf{k}) =$ $= \dfrac{\delta_{n0}}{\mathbf{k}^2}, \qquad \omega_n \equiv 2\pi n T$

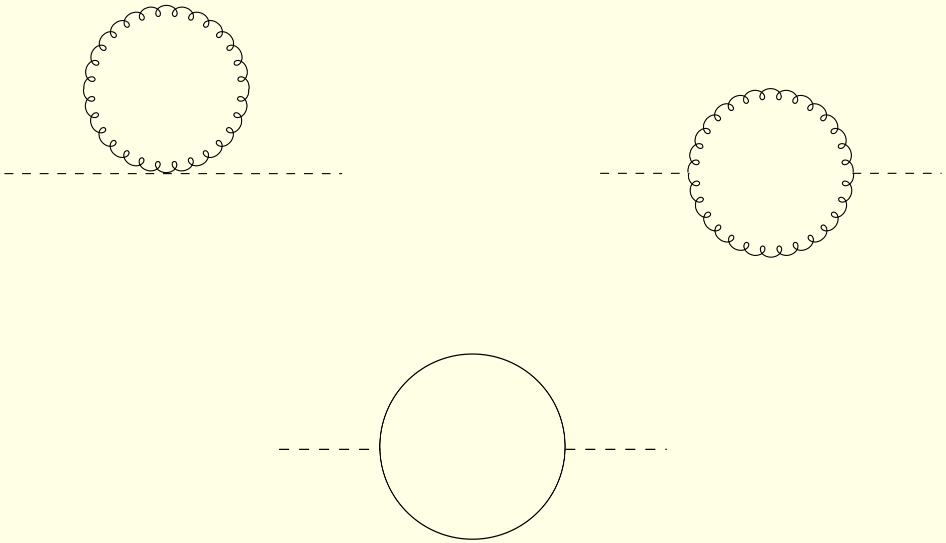
$D_{ij}(\omega_n \neq 0, \mathbf{k}) =$  $= \dfrac{1}{\omega_n^2 + \mathbf{k}^2}\left(\delta_{ij} + \dfrac{k_i k_j}{\omega_n^2}\right)(1 - \delta_{n0})$

$D_{ij}(\omega_n = 0, \mathbf{k}) =$  $= \dfrac{1}{\mathbf{k}^2}\left(\delta_{ij} - (1 - \xi)\dfrac{k_i k_j}{\mathbf{k}^2}\right)\delta_{n0}$

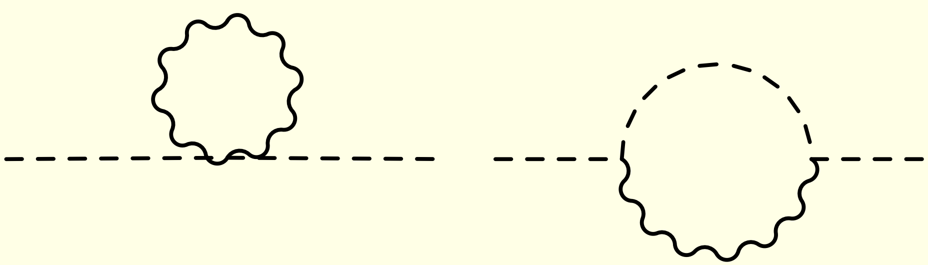
$D_{\text{ghost}}(\omega_n, \mathbf{k}) =$  $= \dfrac{\delta_{n0}}{\mathbf{k}^2}$

Π_{00} at one loop

The temporal component of the gluon self-energy gets non-static



and static contributions



- The calculation is performed in dimensional regularization: $d = 3 - 2\epsilon$.
- $\Pi_{00}(|\mathbf{k}| \ll T) = m_D^2 + \ldots$ where m_D is the Debye mass:

$$m_D^2 \equiv \frac{g^2 T^2}{3} \left(N + \frac{n_f}{2} \right) .$$

- We keep order ϵ corrections of the type

$$T |\mathbf{k}|^{1-2\epsilon} \epsilon$$

  because the Fourier transform of $|\mathbf{k}|^{1-2\epsilon}/|\mathbf{k}|^4$, coming from a self-energy insertion in a temporal-gluon propagator, is divergent.
- Static loops contribute only through the scale m_D.

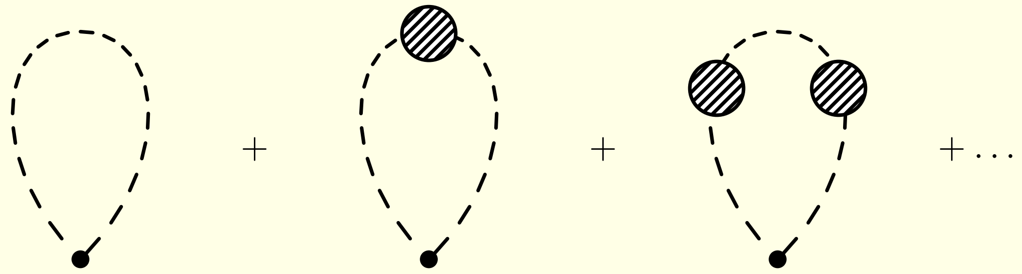
◦ Curci Menotti ZPC 21(84)281, Heinz Kajantie Toimela AP 176(87)218
Rebhan PRD 48(93)3967, NPB 430(94)319

The Polyakov loop at NNLO

We assume the following hierarchy of scales:

$$T \gg m_D$$

Up to NNLO the contributing diagrams are



giving

$$\langle L_R \rangle = 1 + \frac{C_R \alpha_s}{2} \frac{m_D}{T} + \frac{C_R \alpha_s^2}{2} \left[C_A \left(\ln \frac{m_D^2}{T^2} + \frac{1}{2} \right) - n_f \ln 2 \right] + \mathcal{O}(g^5)$$

- At the scale m_D, the gluon-self energies get resummed in the propagator

$$\frac{1}{\mathbf{k}^2 + m_D^2}$$

- The logarithm, $\ln m_D^2/T^2$, signals that an infrared divergence at the scale T has canceled against an ultraviolet divergence at the scale m_D.

Comparison with the literature

In 1981, Gava and Jengo obtained:

$$\langle L_R \rangle_{\rm GJ} = 1 + \frac{C_R \alpha_{\rm s}}{2} \frac{m_D}{T} + \frac{C_R C_A \alpha_{\rm s}^2}{2} \left(\ln \frac{m_D^2}{T^2} - 2 \ln 2 + \frac{3}{2} \right) + \mathcal{O}(g^5)\,.$$

This result disagrees with ours. The origin of the disagreement may be traced back to not having resummed the Debye mass in the temporal gluons contributing to the static gluon self energy.

- `Gava Jengo PLB 105(81)285`

Our result agrees with a recent determination of Burnier, Laine and Vepsalainen.

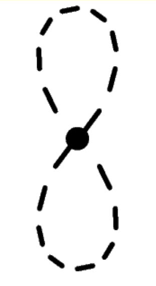
- `Burnier Laine Vepsalainen JHEP 1001(10)054`

The Polyakov loop: some higher order terms

- Non-static modes at the scale m_D:

$$\delta\langle L_R\rangle_{\mathrm{NS},\,m_D} = \frac{3g^4 C_R}{4(4\pi)^3}\frac{m_D}{T}\left[\beta_0 \ln\left(\frac{\mu}{4\pi T}\right)^2 + 2\beta_0\gamma_E + \frac{11}{3}C_A - \frac{2}{3}n_f\,(4\ln 2 - 1)\right].$$

This contribution fixes the renormalization scale of g^3 in the LO term to $\mu = 4\pi T$.

- 

$$\delta\langle L_{\mathrm{R}}\rangle = \left(3C_R^2 - \frac{C_R C_A}{2}\right)\frac{\alpha_{\mathrm{s}}^2}{24}\left(\frac{m_D}{T}\right)^2$$

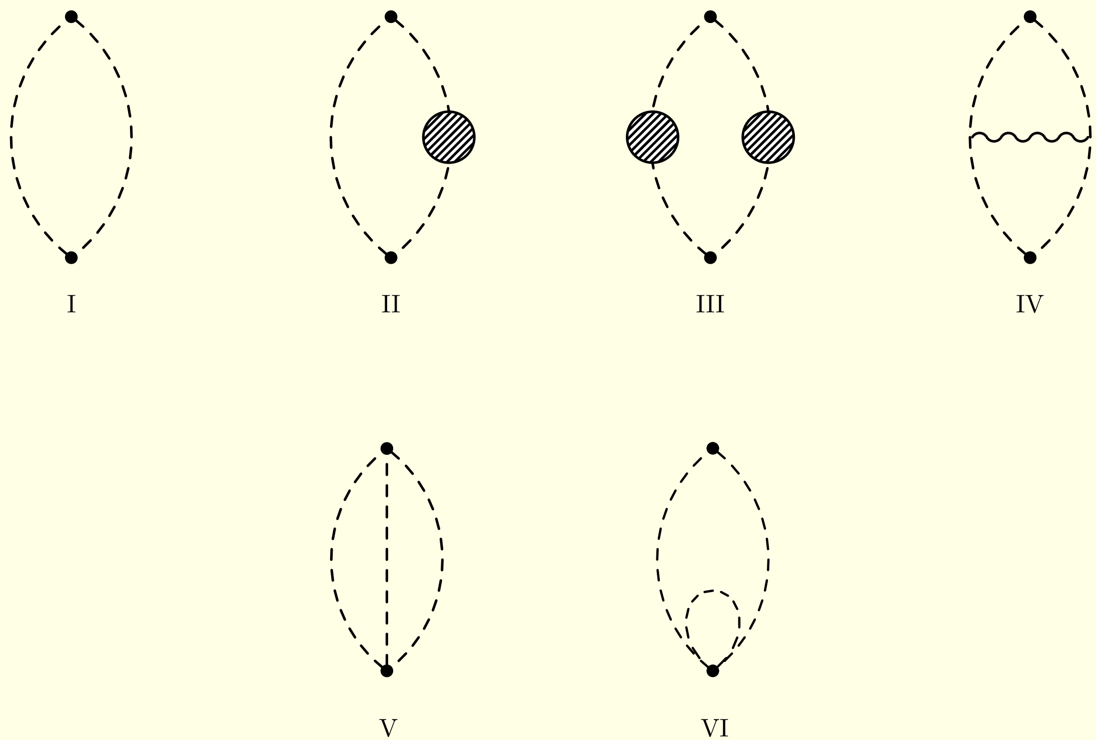
This contribution comes from the scale m_D: it is the leading contribution whose color structure is non linear in C_R

The Polyakov loop correlator at NNLO

We assume the following hierarchy of scales:

$$\frac{1}{r} \gg T \gg m_D \gg \frac{g^2}{r}.$$

We calculate the Polyakov loop correlator up to order $g^6 (rT)^0$:



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$$\begin{aligned}
C_{\rm PL}(r,T) \quad = \quad & \frac{N^2-1}{8N^2} \left\{ \frac{\alpha_{\rm s}(1/r)^2}{(rT)^2} - 2\frac{\alpha_{\rm s}^2}{rT}\frac{m_D}{T} \right. \\
& + \frac{\alpha_{\rm s}^3}{(rT)^3}\frac{N^2-2}{6N} + \frac{1}{2\pi}\frac{\alpha_{\rm s}^3}{(rT)^2}\left(\frac{31}{9}C_A - \frac{10}{9}n_f + 2\gamma_E\beta_0\right) \\
& + \frac{\alpha_{\rm s}^3}{rT}\left[C_A\left(-2\ln\frac{m_D^2}{T^2} + 2 - \frac{\pi^2}{4}\right) + 2n_f \ln 2\right] \\
& \left. + \alpha_{\rm s}^2 \frac{m_D^2}{T^2} - \frac{2}{9}\pi\alpha_{\rm s}^3 C_A \right\} + \mathcal{O}\left(g^6(rT), \frac{g^7}{(rT)^2}\right)
\end{aligned}$$

Comparison with the literature

In 1986, Nadkarni calculated the Polyakov loop correlator at NNLO assuming the hierarchy:

$$T \gg 1/r \sim m_D$$

Whenever the previous results do not involve the hierarchy $rT \ll 1$, they agree with Nadkarni's ones, expanded for $m_D r \ll 1$.

∘ `Nadkarni PRD 33(86)3738`

The Polyakov loop correlator in an EFT language

Integrating out $1/r$ from static QCD leads to pNRQCD:

$$
\begin{aligned}
\mathcal{S}_{\rm pNRQCD} \quad = \quad & \int_0^{1/T} d\tau \int d^3x \int d^3r \, {\rm Tr}\Big\{ {\rm S}^\dagger(\partial_0 + V_s){\rm S} + {\rm O}^\dagger(D_0 + V_o){\rm O} \\
& \qquad -iV_A \left({\rm S}^\dagger \mathbf{r}\cdot g\mathbf{E}{\rm O} + {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}{\rm S} \right) \\
& \qquad -\frac{i}{2}V_B \left({\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}{\rm O} + {\rm O}^\dagger {\rm O}\mathbf{r}\cdot g\mathbf{E} \right) \\
& \qquad +\frac{i}{8}V_C \left(r^i r^j {\rm O}^\dagger D^i g E^j {\rm O} - r^i r^j {\rm O}^\dagger {\rm O} D^i g E^j \right) + \delta\mathcal{L}_{\rm pNRQCD} \Big\} \\
& + \int_0^{1/T} d\tau \int d^3x \, \left(\frac{1}{4} F^a_{\mu\nu}F^a_{\mu\nu} + \sum_{l=1}^{n_f} \bar{q}_l \not{D} q_l \right)
\end{aligned}
$$

where S and O$= O^a T^a$ are the quark-antiquark color singlet and octet fields.

- Pineda Soto NPB PS 64(98)428
  Brambilla et al NPB 566 (00) 275, PRD 67(03)034018

pNRQCD potentials

$$V_s(r) = -C_F \frac{\alpha_{\rm s}(1/r)}{r} \left[1 + \left(\frac{31}{9}C_A - \frac{10}{9}n_f + 2\gamma_E\beta_0\right)\frac{\alpha_{\rm s}}{4\pi} + \mathcal{O}(\alpha_{\rm s}^2)\right],$$

$$V_o(r) = \frac{1}{2N} \frac{\alpha_{\rm s}(1/r)}{r} \left[1 + \left(\frac{31}{9}C_A - \frac{10}{9}n_f + 2\gamma_E\beta_0\right)\frac{\alpha_{\rm s}}{4\pi} + \mathcal{O}(\alpha_{\rm s}^2)\right],$$

$$(N^2-1)V_o(r) + V_s(r) = \frac{N(N^2-1)}{8}\frac{\alpha_{\rm s}^3}{r}\left(\frac{\pi^2}{4} - 3\right)\left[1 + \mathcal{O}(\alpha_{\rm s})\right].$$

- Fischler NPB 129(77)157, Billoire PLB 92(80)343
  Kniehl et al PLB 607(05)96

The Polyakov loop correlator in pNRQCD

$$C_{\rm PL}(r,T) = \frac{1}{N^2}\Big[Z_s\langle S(\mathbf{r},\mathbf{0},1/T)S^\dagger(\mathbf{r},\mathbf{0},0)\rangle + Z_o\langle O^a(\mathbf{r},\mathbf{0},1/T)O^{a\,\dagger}(\mathbf{r},\mathbf{0},0)\rangle$$
$$+\mathcal{O}\left(\alpha_{\rm s}^3(rT)^4\right)\Big] - \langle L_F\rangle^2.$$

- Integrating out the scale $1/r$ and matching to the previous determination of $C_{\rm PL}(r,T)$, we get:

$$Z_s = Z_o = 1$$
$$\langle S(\mathbf{r},\mathbf{0},1/T)S^\dagger(\mathbf{r},\mathbf{0},0)\rangle|_{1/r} = e^{-V_s(r)/T}$$
$$\langle O^a(\mathbf{r},\mathbf{0},1/T)O^{a\,\dagger}(\mathbf{r},\mathbf{0},0)\rangle|_{1/r} = (N^2-1)e^{-V_o(r)/T}$$

  This is consistent with the spectral decomposition.

- If we assume instead the spectral decomposition, then the matching provides a non-trivial verification of the two-loop octet potential.

Integrating out T and m_D

- Integrating out T:

$$\langle S(\mathbf{r},\mathbf{0},1/T)S^\dagger(\mathbf{r},\mathbf{0},0)\rangle|_{1/r,T} = e^{-f_s(r,T)/T}$$
$$\langle O^a(\mathbf{r},\mathbf{0},1/T)O^{a\,\dagger}(\mathbf{r},\mathbf{0},0)\rangle|_{1/r,T} = (N^2-1)e^{-f_o(r,T)/T}$$

- Integrating out m_D:

$$\langle S(\mathbf{r},\mathbf{0},1/T)S^\dagger(\mathbf{r},\mathbf{0},0)\rangle|_{1/r,T,m_D} = e^{-f_s(r,T,m_D)/T}$$
$$\langle O^a(\mathbf{r},\mathbf{0},1/T)O^{a\,\dagger}(\mathbf{r},\mathbf{0},0)\rangle|_{1/r,T,m_D} = (N^2-1)e^{-f_o(r,T,m_D)/T}$$

where f_s and f_o may be interpreted as singlet and octet free energies in pNRQCD.

f_s and f_o

$$
\begin{aligned}
f_s(r,T,m_D) \quad &= \quad V_s(r) \\
&+\frac{2}{9}\pi N C_F \alpha_{\rm s}^2 r T^2 \left[1+\sum c_n^{\rm NS}(rT)^{2n+2}\right] - \frac{\pi}{36} N^2 C_F \alpha_{\rm s}^3 T \\
&-\left(\frac{3}{2}\zeta(3) C_F \frac{\alpha_{\rm s}}{\pi}(r m_D)^2 T - \frac{2}{3}\zeta(3) N C_F \alpha_{\rm s}^2 r^2 T^3\right)\left[1+\sum c_n^{\rm S}(rT)^{2n+2}\right] \\
&+C_F \frac{\alpha_{\rm s}}{6} r^2 m_D^3 + T\mathcal{O}\left(g^6(rT), \frac{g^8}{rT}\right)
\end{aligned}
$$

f_s and f_o

$$
\begin{aligned}
f_o(r,T,m_D) \quad = \quad & V_o(r) \\
& -\frac{C_A\alpha_{\rm s}}{2} m_D + \frac{1}{48} C_A^2 \alpha_{\rm s}^2 \frac{m_D^2}{T} \\
& -\frac{C_A\alpha_{\rm s}^2}{2} T \left[C_A \left(-\ln\frac{T^2}{m_D^2} + \frac{1}{2} \right) - n_f \ln 2 + b_1\, g + b_2\, g^2 + a\,\alpha_{\rm s} \right] \\
& -\frac{\pi}{9}\alpha_{\rm s}^2 r T^2 \left[1 + \sum c_n^{\rm NS} (rT)^{2n+2} \right] - \frac{\pi}{72} N \alpha_{\rm s}^3 T \\
& + \left(\frac{3}{4N}\zeta(3)\frac{\alpha_{\rm s}}{\pi}(r m_D)^2 T - \frac{1}{3}\zeta(3)\alpha_{\rm s}^2 r^2 T^3 \right) \left[1 + \sum c_n^{\rm S} (rT)^{2n+2} \right] \\
& -\frac{1}{N}\frac{\alpha_{\rm s}}{12} r^2 m_D^3 + T\mathcal{O}\left(g^6 (rT), \frac{g^8}{rT} \right)
\end{aligned}
$$

In the Polyakov loop correlator, $C_{\rm PL}(r,T)$, strong cancellations occur between the singlet energy, octet energy and Polyakov loop

$$\begin{aligned}\langle L_R \rangle &= 1 + \frac{C_R \alpha_{\rm s}}{2}\frac{m_D}{T} + \frac{C_R \alpha_{\rm s}^2}{2}\left[C_A\left(\ln\frac{m_D^2}{T^2}+\frac{1}{2}\right) - n_f \ln 2 + a\,\alpha_{\rm s} + b_1\,g + b_2\,g^2\right] \\ &\quad + \left(3C_R^2 - \frac{C_R C_A}{2}\right)\frac{\alpha_{\rm s}^2}{24}\left(\frac{m_D}{T}\right)^2 + \mathcal{O}\left(g^7\right).\end{aligned}$$

They lead, up to order $g^6(rT)^0$, to the previous result.

Comparison with the literature and discussion I

EFT approaches for the calculation of the correlator of Polyakov loops for the situation $m_D \gtrsim 1/r$ and $T \gg 1/r$ were developed in the past. In that situation, the scale $1/r$ was not integrated out, and the Polyakov-loop correlator was described in terms of EQCD and MQCD, while the complexity of the bound-state dynamics remained implicit in the correlator.

Those descriptions are valid for largely separated Polyakov loops when the correlator is either screened by the Debye mass, for $m_D r \sim 1$, or the mass of the lowest-lying glueball in MQCD, for $m_D r \gg 1$.

- `Braaten Nieto PRL 74(95)3530`
  `Nadkarni PRD 33(86)3738`

Comparison with the literature and discussion II

Up to order $g^6(rT)^0$, the obtained expression for the Polyakov loop correlator agrees with the spectral decomposition

$$C_{\rm PL}(r,T) = e^{-{\rm Re}\,V_s(r)/T} + (N^2-1)e^{-{\rm Re}\,V_o(r)/T}$$

where ${\rm Re}\,V_s(r)$ and ${\rm Re}\,V_o(r)$ are the real parts of the color singlet and color octet potentials as calculated in the real-time formalism.

- `Brambilla, Ghiglieri, Petreczky Vairo PRD 78(08)014017`

Comparison with the literature and discussion III

Jahn and Philipsen have analyzed the gauge structure of the allowed intermediate states in the correlator of Polyakov loops: intermediate states φ, describing a quark located in $\mathbf{x}_1$ and an antiquark located in $\mathbf{x}_2$, should transform as

$$\varphi(\mathbf{x}_1, \mathbf{x}_2) \to g(\mathbf{x}_1)\varphi(\mathbf{x}_1, \mathbf{x}_2)g^\dagger(\mathbf{x}_2)$$

under a gauge transformation g.

• For pNRQCD in the weak-coupling regime, these states include quark-antiquark states both in a color-singlet or in a color-octet configuration.
• For pNRQCD in the strong-coupling regime instead, the spectrum is made of the quark-antiquark color singlet state and its excitations (hybrids, glueballs).

○ Jahn Philipsen PRD 70(04)074504

Comparison with the literature and discussion IV

Burnier Laine Vepsalainen have recently performed a weak-coupling calculation of the untraced Polyakov-loop correlator in Coulomb gauge and of the cyclic Wilson loop up to order g^4.

Both these objects may be seen as contributing to the correlator of two Polyakov loops. It is expected that large cancellations occur between these correlators and their octet counterparts in order to reproduce the Polyakov-loop correlator. Such large cancellations should occur at the level of the scales $1/r$, T and m_D. In the case of the untraced Polyakov-loop correlator, the octet contribution should restore gauge invariance and, in the case of the cyclic Wilson loop, the octet contribution should cancel the divergences observed at order g^4.

○ `Burnier Laine Vepsalainen JHEP 1001(10)054`