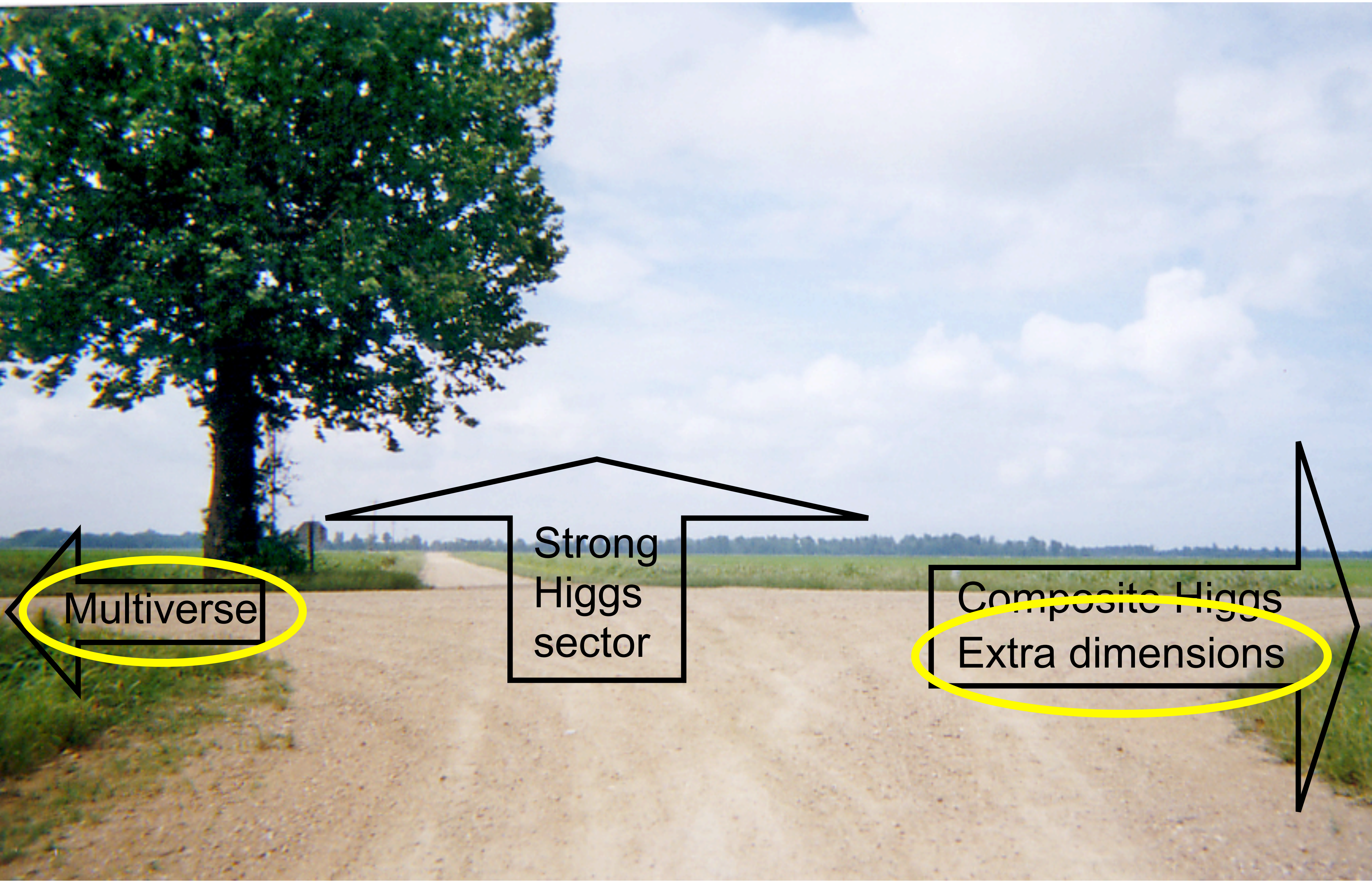

Multiverse
Strong
Higgs
sector
Composite Higgs
Extra dimensions

Extra Dimensions

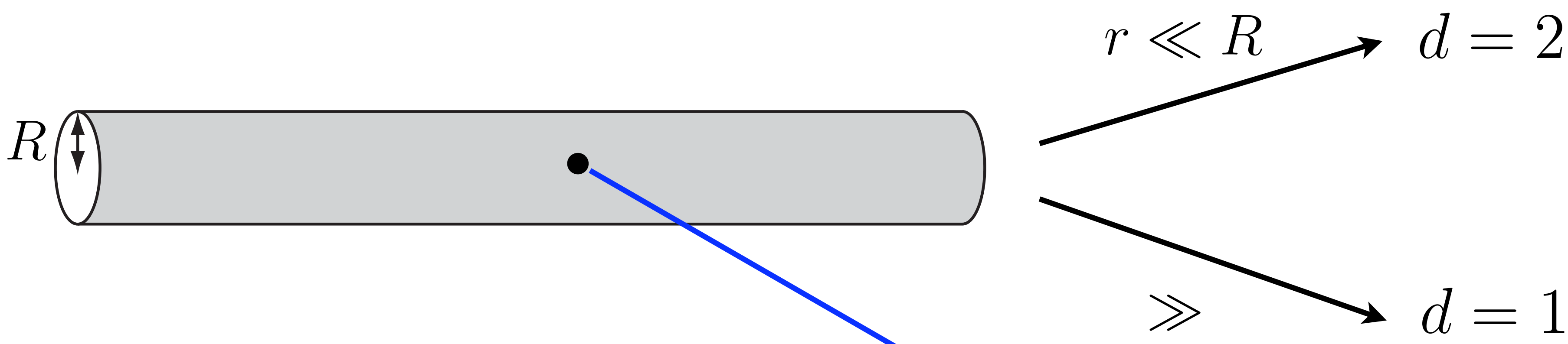
Motivation

- Non-trivial extension of spacetime symmetry (c.f. SUSY)
- Required by string theory

Leads in many interesting directions

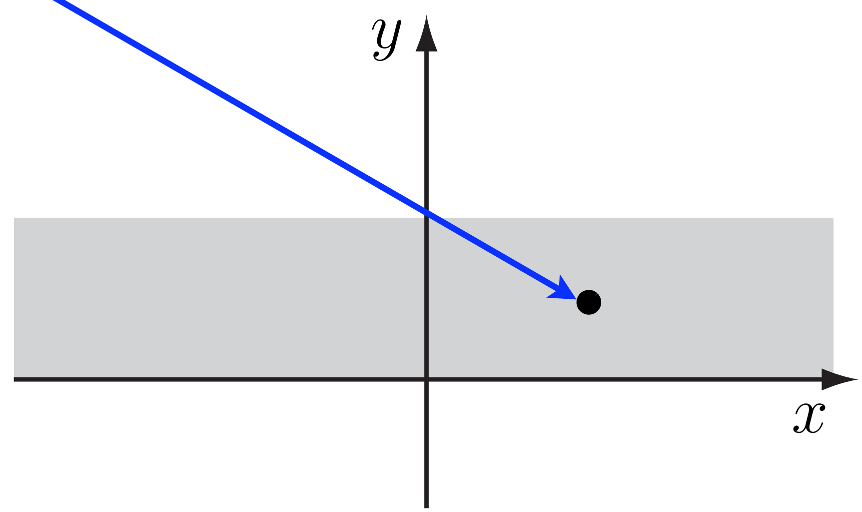
- Unification of spins (c.f. SUSY again)
  - Gauge-gravity unification (Kaluza 1919; Klein 1921)
  - Gauge-Higgs unification (Hosotani 1983)
- Quantum gravity at low scales (Arkani-Hamed, Dimopoulos, Dvali 1998)
- Hierarchy from gravitational warping (Randall, Sundrum 1999)
- 5D gravity $\leftrightarrow$ 4D gauge theory (Maldacena 1999)

Extra Dimensions 101



Field theory on a cylinder:

$$\Phi(x, y) = \Phi(x, y + 2\pi R)$$



4+1 dimensions just as easy:

$$\Phi(x^\mu, y) = \Phi(x^\mu, y + 2\pi R)$$

$$S = \int d^4x \int_0^{2\pi R} \left(\partial^M \Phi^\dagger \partial_M \Phi - m^2 \Phi^\dagger \Phi + \cdots \right)$$

$\mathrm{dim}(\Phi) = \frac{3}{2}$ $\qquad M =$ 5D Lorentz index

Kaluza-Klein Decomposition

$$\Phi(x,y) = \sum_{n=0}^{\infty} \Phi^{(n)}(x) e^{iny/R}$$

$$S = \int d^4x \sum_{n=0}^{\infty} 2\pi R \left[\partial^M \Phi^{(n)\dagger} \partial_M \Phi^{(n)} - \left(m^2 + \frac{n^2}{R^2}\right) \Phi^{(n)\dagger} \Phi^{(n)} + \cdots \right]$$

from integral over extra dimension

$$\left|\partial_y e^{iny/R}\right|^2 = \frac{n^2}{R^2}$$

Define $\hat{\Phi}^{(n)} = \sqrt{2\pi R}\,\Phi^{(n)}$

$$S = \int d^4x \sum_{n=0}^{\infty} \left[\partial^M \hat{\Phi}^{(n)\dagger} \partial_M \hat{\Phi}^{(n)} - \left(m^2 + \frac{n^2}{R^2}\right) \hat{\Phi}^{(n)\dagger} \hat{\Phi}^{(n)} + \cdots \right]$$

Note $\dim(\Phi) = 1$

5D theory equivalent to infinite number of 4D fields with increasing mass $m_n^2 = m^2 + n^2/R^2$

Interactions

$$\int dy\, (\Phi^\dagger \Phi)^2 = \sum_{n_1,n_2,n_3,n_4} e^{-i(n_1-n_2+n_3-n_4)y/R}\, \Phi^{(n_1)\dagger}\Phi^{(n_2)}\Phi^{(n_3)\dagger}\Phi^{(n_4)}$$

$$= \sum_{n_1,n_2,n_3,n_4} \frac{1}{2\pi R} \underbrace{\delta_{n_1-n_2+n_3-n_4,0}}_{\text{5D momentum conservation}}\, \hat{\Phi}^{(n_1)\dagger}\hat{\Phi}^{(n_2)}\hat{\Phi}^{(n_3)\dagger}\hat{\Phi}^{(n_4)}$$

5D momentum conservation

$$\dim(\Phi^4) = 6 \qquad \Rightarrow \Delta\mathcal{L}_5 \sim \frac{1}{\Lambda}(\Phi^\dagger\Phi)^2$$

irrelevant in 5D

5D gauge interactions

$$D_M = \partial_M - i g_5 A_M$$

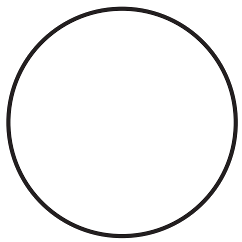
$$\dim(A_M) = \tfrac{3}{2} \qquad \Rightarrow g_5 \sim \frac{1}{\Lambda^{1/2}}$$

also irrelevant

4D $\rightarrow$ 5D $\Rightarrow$ give up renormalizability (UV completeness)

Shape of Extra Dimensions

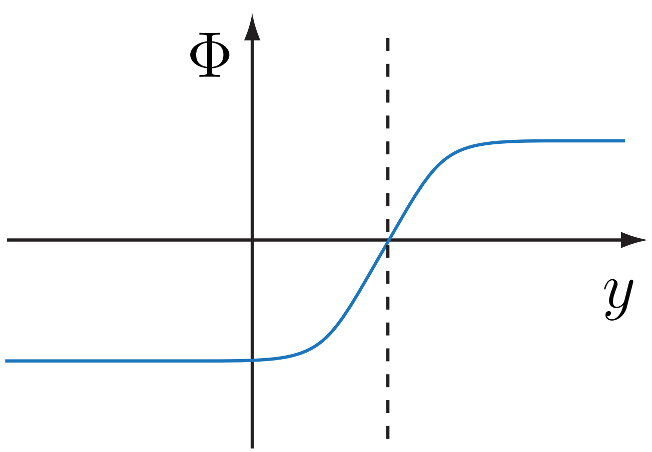
One extra dimension:

Compact:  or 

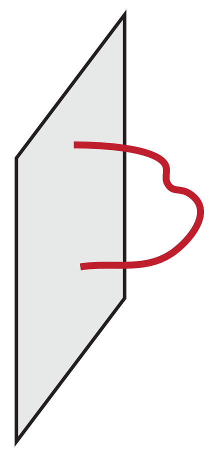
Non-compact:  or

“Brane” = special point in extra dimension

Field varying in 5D

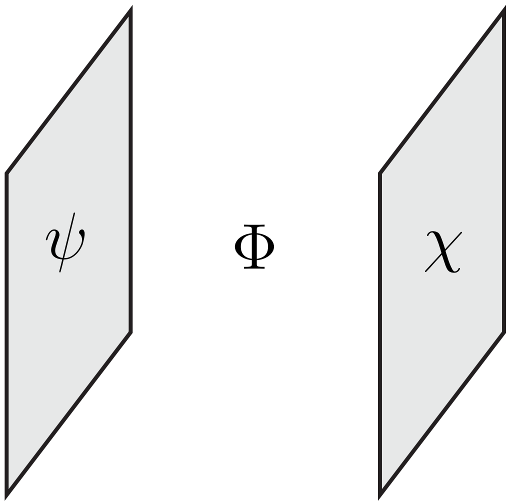


D-branes in string theory



Brane Localization

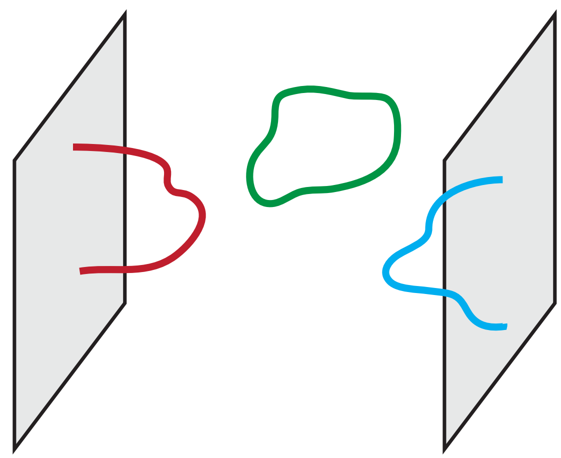
Particles can be localized on branes



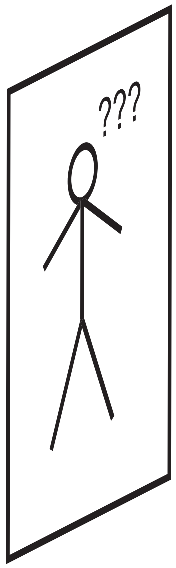
$$\mathcal{L}_{\text{eff}} = \int d^4x \Big[\int_0^L dy\, \mathcal{L}_{\text{bulk}}(\Phi(x,y)) + \mathcal{L}_0(\Phi(x,y=0),\psi(x)) + \mathcal{L}_L(\Phi(x,y=L),\chi(x)) \Big]$$

May occur in field theory

Required in string theory



Life on a Brane



Could we be living on a brane
in an infinite extra dimension?

a) No: gravity = spacetime curvature

$$V(r) \propto \frac{1}{r^{d-}} \qquad \Rightarrow d = 3 + 1$$

b) Yes: gravity can be localized on extra dimension

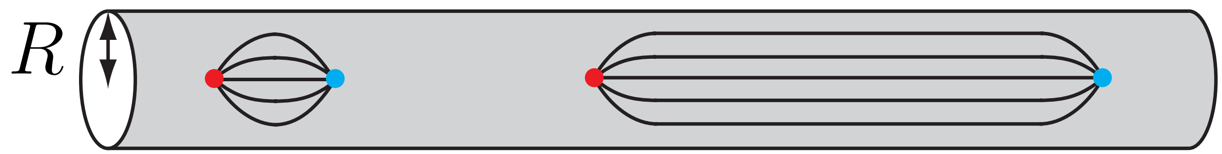
(Dvali, Gabadadze, Porati 2000)

⇒ interesting (dangerous) modification of gravity
at <u>long</u> distances

Large Extra Dimensions

(Arkani-Hamed, Dimopoulos, Dvali 1998)

Gravity in compactified extra dimensions



$r \ll R$

$$V(r) \sim \frac{1}{M_*^{d-2}} \frac{1}{r^{d-3}}$$

$r \gg R$

$$V(r) \sim \frac{1}{M_{\mathrm{P}}^2} \frac{1}{r}$$

Match at $r \sim R \;\Rightarrow M_{\mathrm{P}}^2 \sim M_*^{d-2} R^{d-4}$

Experiment ⇒ $R \lesssim \mu\mathrm{m} \sim 5 \times 10^{11}\ \mathrm{GeV}^{-1}$

Scale of quantum gravity $\sim M_* \sim \begin{cases} 10\ \ \mathrm{GeV} & d = 4+1 \\ 5\ \mathrm{TeV} & d = 5+1 \\ 5\ \mathrm{GeV} & d = 6+1 \end{cases}$

No hierarchy problem!

Must still explain origin of $R \gg M_*^{-1}$...

Quantum Gravity at TeV

Exciting, but also dangerous

- KK gravitons $m_{\rm KK} \sim \frac{1}{R} \sim 10^{-3}\ {\rm eV}$

  $\Rightarrow$ *many* accessible states

- Cosmological "normalcy bound"

  $T_{\rm max} \lesssim\ {\rm GeV}$

- Supernova emission, neutron star heating

  $\Rightarrow M_* \gtrsim 1000\ {\rm TeV} \quad (d = 6)$

Asymmetric extra dimensions still allow quantum gravity at TeV

Signals

Black holes at LHC!

- Production

  Black hole formed for impact parameter

  $b \lesssim R$ = Schwartzschild radius $(R_{\rm S})^{d-3} \sim \dfrac{\sqrt{\hat{s}}}{M_*^{d-2}}$

  $\sqrt{\hat{s}} \sim \text{TeV} \Rightarrow \sigma_{\rm BH} \sim \text{TeV}^{\,2} \sim \text{nb}$

- Decay

  Hawking radiation ⇒ democratic decay

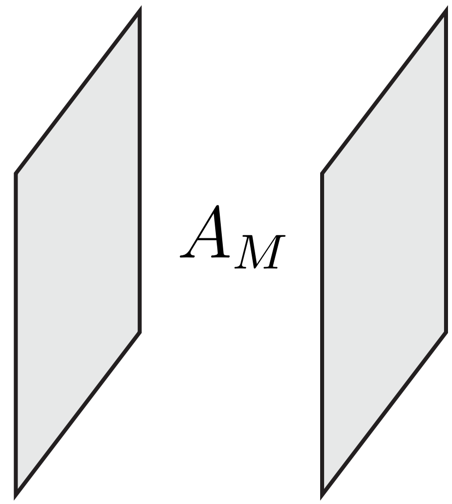
  $\langle n \rangle \simeq 6$ particles in final state

Higgsless Models

(Csaki, Grojean, Murayama, Pilo, Terning, 2003)

Can we get W, Z masses from KK mechanism?

Example: $U(1)$ gauge field in 5D



$A_M = (A_\mu, A\ \ )$ gauge fix $A_5 \equiv$

$$A_\mu(x,y) = \sum_n A_\mu^{(n)}(x) f_n(y)$$

$$A_\mu^{(n)}(x) \propto e^{ip\cdot x} \;\; \Rightarrow (p^2 + \partial_y^2) f_n(y) = 0$$

Eigenvalues determine masses $\ ^2 = \ _n^2$

Boundary conditions fixed by boundary terms

$$\Delta\mathcal{L}_{\rm bdy} = -\tfrac{1}{2} m A_\mu^2$$

$$\Rightarrow mA_\mu \pm F_{5\mu}| \qquad y \qquad , L$$

Higgsless Models II

$$mA_\mu \pm F_{5\mu}| = 0$$

- $m \to \infty \Rightarrow \partial_5 A_\mu| = 0$

$$f_n(y) = \cos\left(\frac{n\pi y}{L}\right) \qquad n = 0, 1, \ldots$$

$$m_n = \frac{n\pi}{L} \qquad n = 0$$ mode massless

- $m \to \infty \Rightarrow A_\mu| = 0$

$$f_n(y) = \sin\left(\frac{n\pi y}{L}\right) \qquad n = 1, 2, \ldots$$

$$m_n = \frac{n\pi}{L}$$ no massless 0 mode

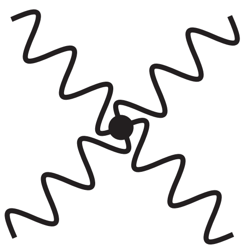
Gauge symmetry broken by boundary conditions

Higgsless Models III

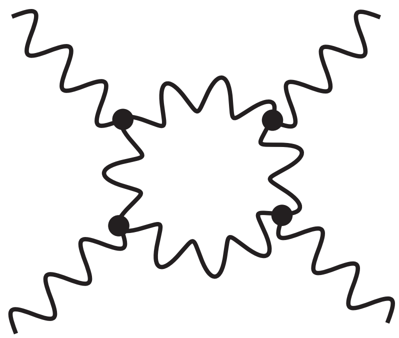
Generalize to $SU(N)$

KK modes have gauge coupling $g_4 \sim \frac{g_5}{\sqrt{L}}$

Where does 5D theory break down?



$\sim g$



$$\sim \frac{g_5^4 E}{24\pi^3} \quad \Rightarrow \Lambda_5 \sim \frac{24\pi^3}{g_5^2}$$

$$\Rightarrow \frac{m_W}{\Lambda_5} \sim \frac{L^{-1}}{24\pi^3/g_5^2} \sim \frac{g_4^2}{24\pi^3} \sim 100\ \mathrm{TeV}$$

Compare effective standard model: $\frac{m_W}{\Lambda_5} \sim \frac{g_4}{4\pi} \sim \mathrm{TeV}$

Partial UV completion from extra dimensions!

Higgsless Models IV

Realistic models?

- $SU(2) \quad \times U(1) \quad \to U(1)_{\rm EM}$
- Custodial symmetry
- Avoid $m_W = m_Z = \frac{1}{L}$
- Avoid $m_W = 2m_W$, etc.

Requires generalized boundary conditions

$SU(2) \times SU(2)$ bulk gauge fields

- $S \sim 10 + \cdots$

Requires some tuning

Signals

- No light Higgs
- Strong WW scattering
- Many TeV resonances

Haven't we seen this before?

QCD? Technicolor?

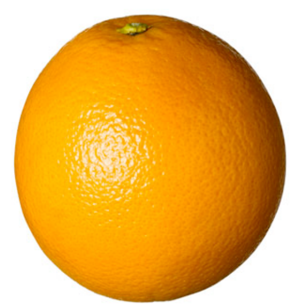
Duality

Similarity between 5D and 4D theories goes deep

Precise correspondence in string theory:
(Maldacena 1999)

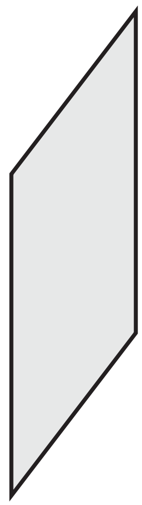
$\mathcal{N} = 4, \; d = 4$ super Yang Mills ⟷ String theory on $AdS_5 \times S_5$

?



Randall-Sundrum Model

(Randall, Sundrum 1999)



A slice of AdS_5

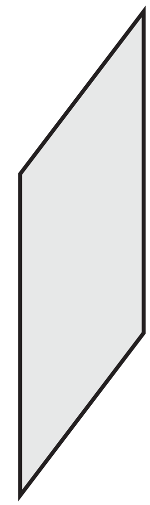
$$^{2} = {}^{-2ky}\eta_{\mu\nu} \;\square^{\mu}\; \square^{\nu} + \;{}^{2}$$

4D metric

on $y =$ constant slice

$y = 0$

UV



$y = L$

IR

All fundamental parameters $\sim M_*$

Higgs localized on IR brane $\Rightarrow m_H \sim M_* e^{-kL} \ll M_*$

$$k, M_* \sim M_{\mathrm{P}}, \;\; kL \sim 10 \;\; \Rightarrow m_H \sim \mathrm{TeV}$$

Hierarchy problem solved by gravitational “warping”

RS with Bulk SM Fields

Several nice features

- Precision gauge coupling unification
- Fermion mass hierarchies natural

Also some problems

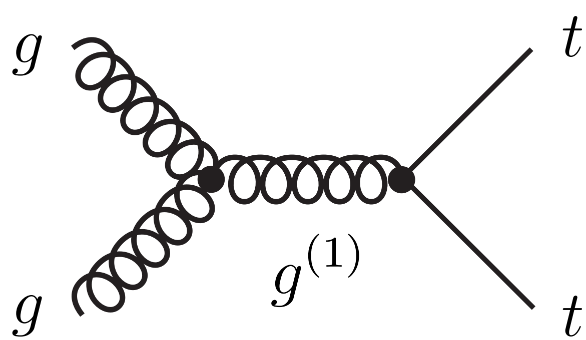
- Precision electroweak and flavor require some tuning

$$\sim 10\% \;\Rightarrow m_{\rm KK} \gtrsim 5 \text{ TeV}$$

Signals

- Top quark localized near IR brane

  ⇒ "composite"   (couples strongly to KK states)

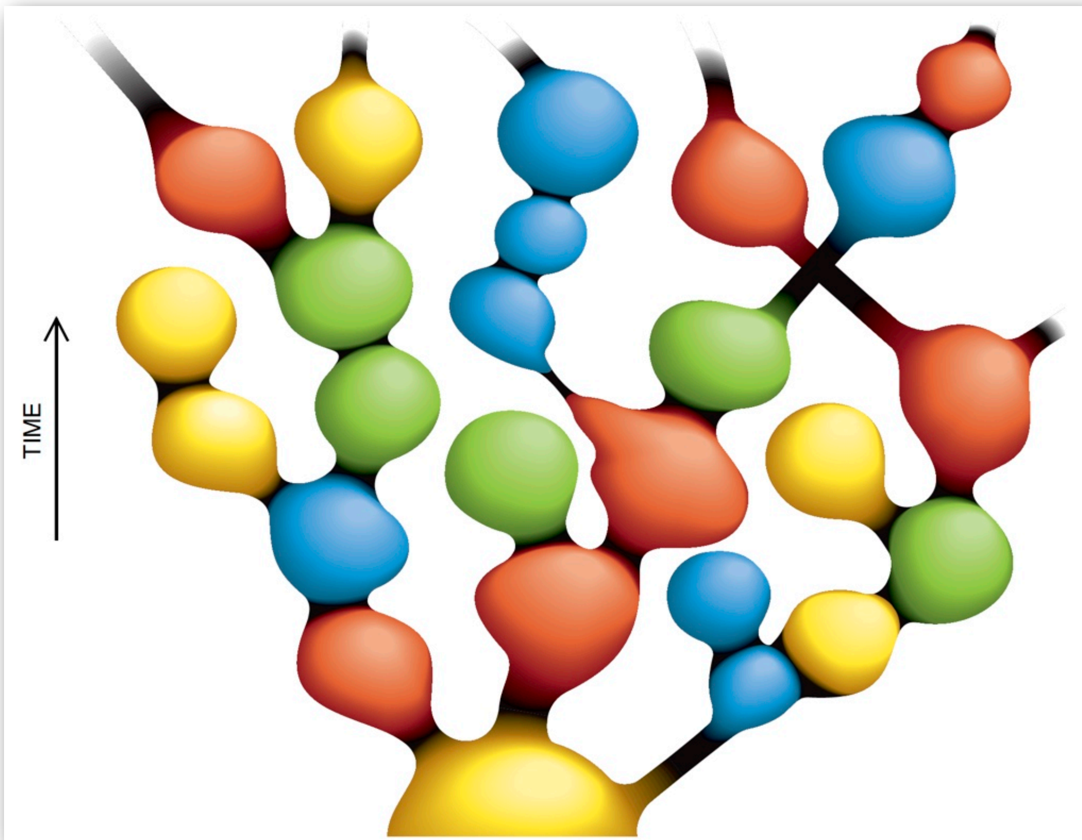


$\bar{t}t$ resonance

- Other KK resonances

  W, Z, graviton

Multiverse



To the blackboard...