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EDMs and Baryogenesis

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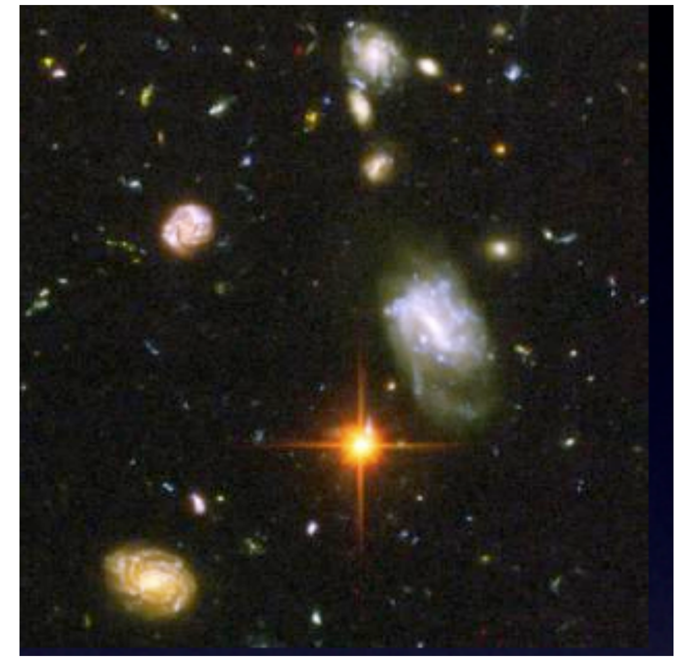
Outline

- Introduction: Baryogenesis
- EDMs and baryogenesis: high-level connection
- Examples:
  - Electroweak Baryogenesis
  - Leptogenesis
- Conclusions

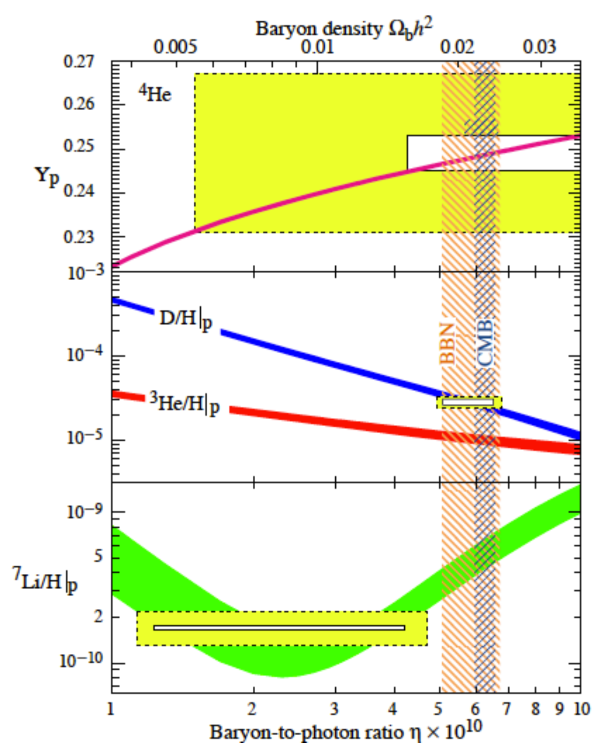
Baryogenesis

The Baryon Asymmetry

- What happened to the antimatter?
- Can we understand the origin and size of the baryon asymmetry of the Universe?



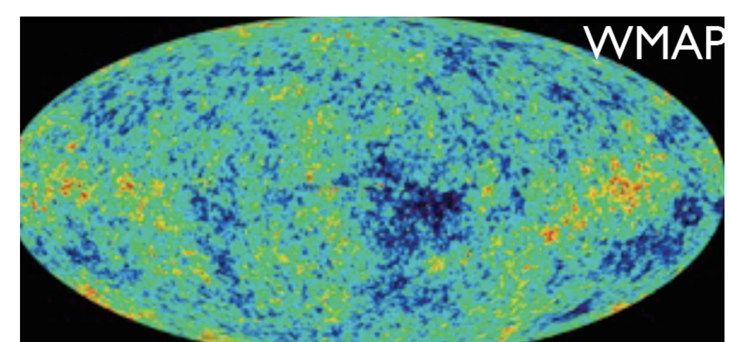
$$\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} = (6.3 \pm 0.3) \times 10^{-10}$$



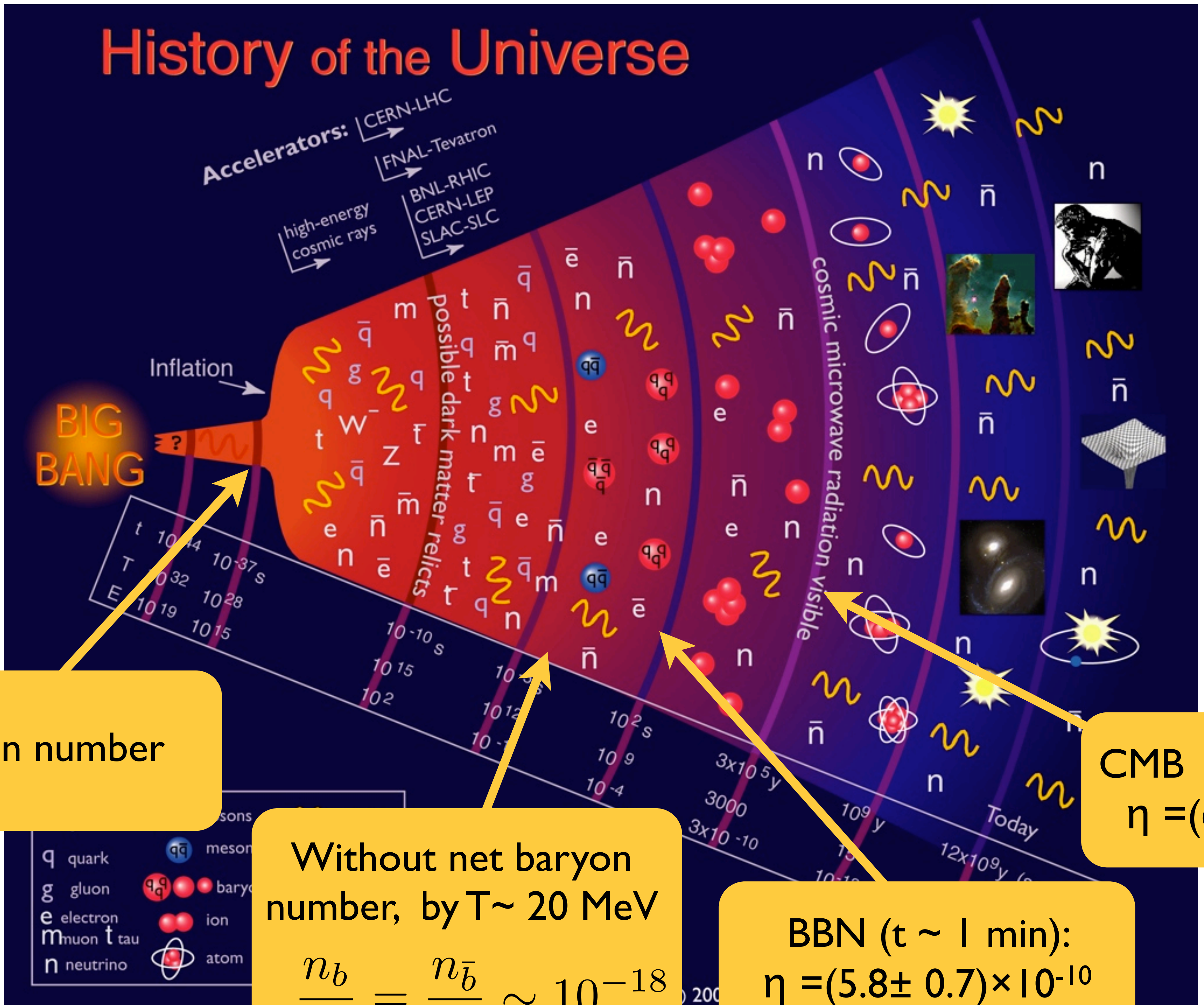
From Big Bang Nucleosynthesis

and

CMB anisotropies



Baryogenesis



No baryon number

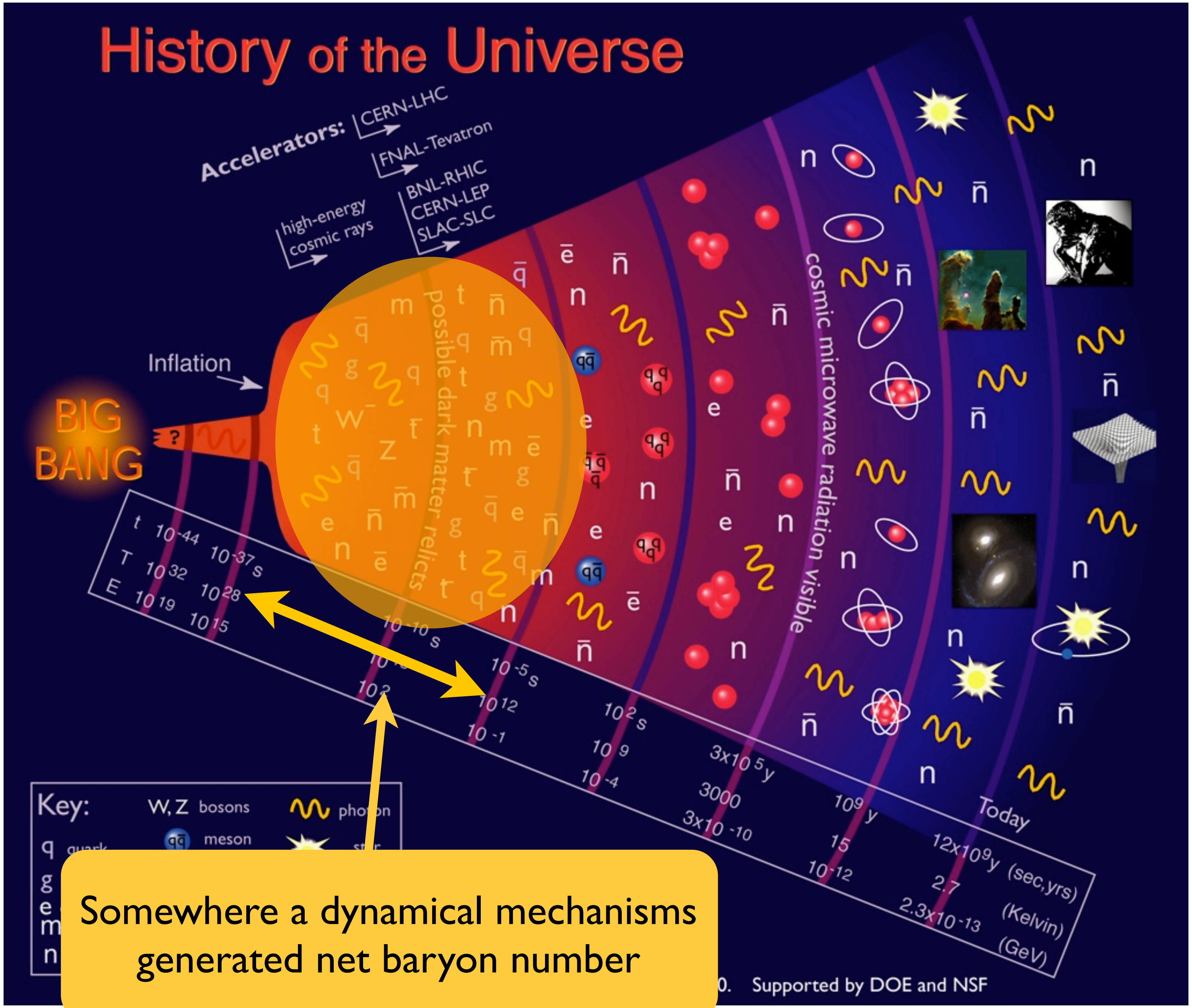
Without net baryon number, by T~ 20 MeV

$$\frac{n_b}{n_\gamma} = \frac{n_{\bar{b}}}{n_\gamma} \sim 10^{-18}$$

BBN (t ~ 1 min):
η =(5.8± 0.7)×10^{-10}

CMB (t ~ 300,000 yr):
η =(6.1± 0.3)×10^{-10}

Baryogenesis



Somewhere a dynamical mechanisms generated net baryon number

Conditions for Baryogenesis

- The dynamical generation of net baryon number during cosmic evolution requires the concurrence of three conditions:

Sakharov '67

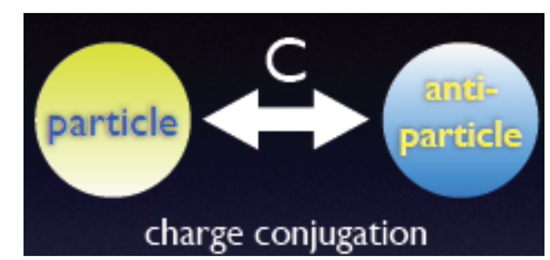
"According to our hypothesis, the occurrence of C asymmetry is the consequence of violation of CP invariance in the nonstationary expansion of the hot Universe during the superdense stage, as manifest in the difference between the partial probabilities of the charge-conjugate reactions."

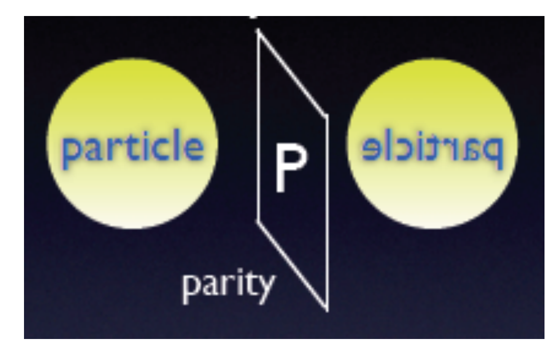
Conditions for Baryogenesis

- The dynamical generation of net baryon number during cosmic evolution requires the concurrence of three conditions:

Sakharov '67

1. B (baryon number) violation
   - To depart from initial B=0
2. C and CP violation
   - To distinguish baryon and anti-baryon production
3. Departure from thermal equilibrium
   - <B>=0 in equilibrium, if CPT is a good symmetry

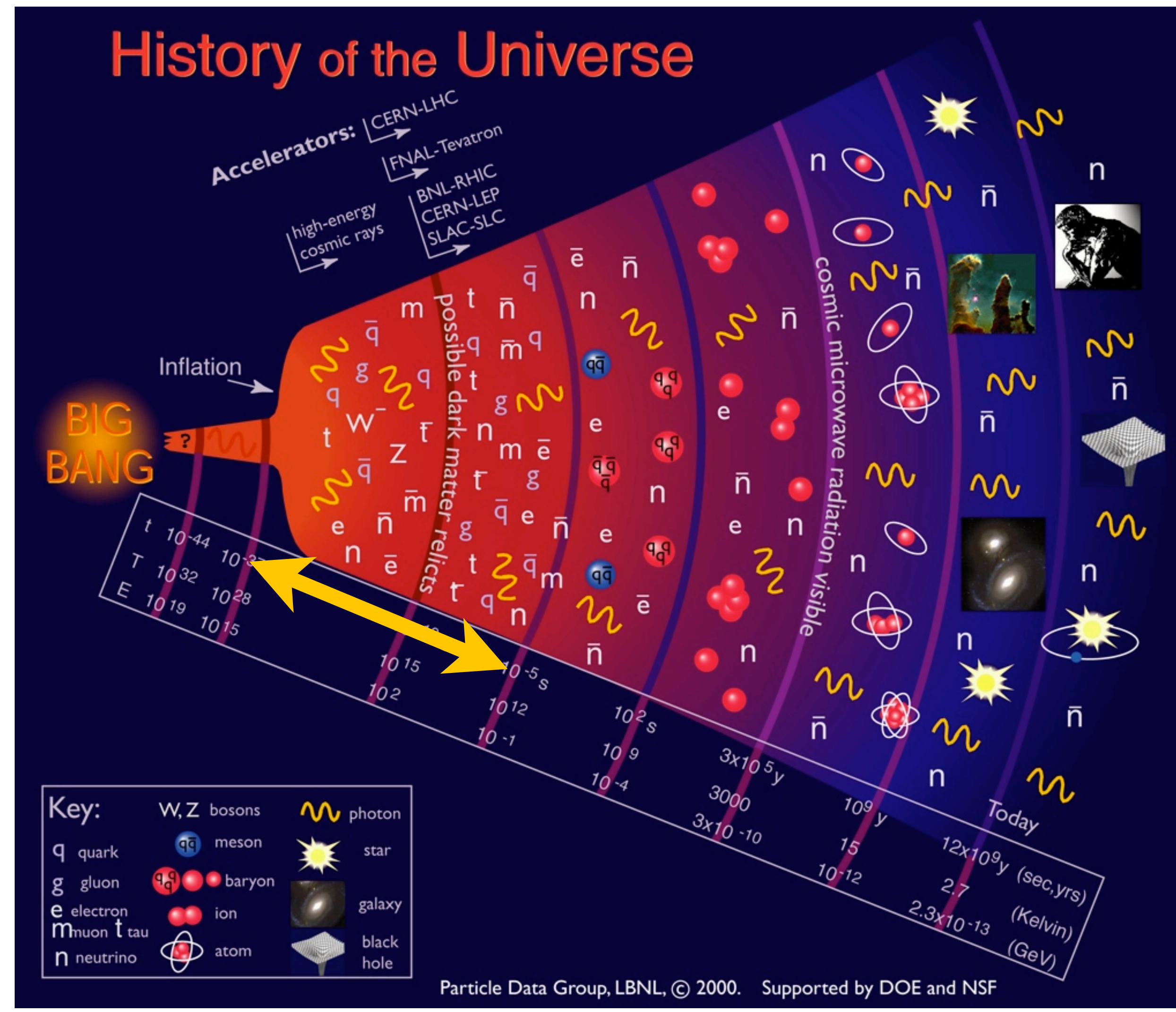




Baryogenesis Scenarios

- Many scenarios can realize these conditions, at different epochs in the Early Universe

- GUT baryogenesis: T ~ M_{GUT}
- Leptogenesis: T ~ M_{GUT} → TeV
- Electroweak baryogenesis: T ~ 100 GeV
- Affleck-Dine (coherent scalar fields)
- Hidden sector



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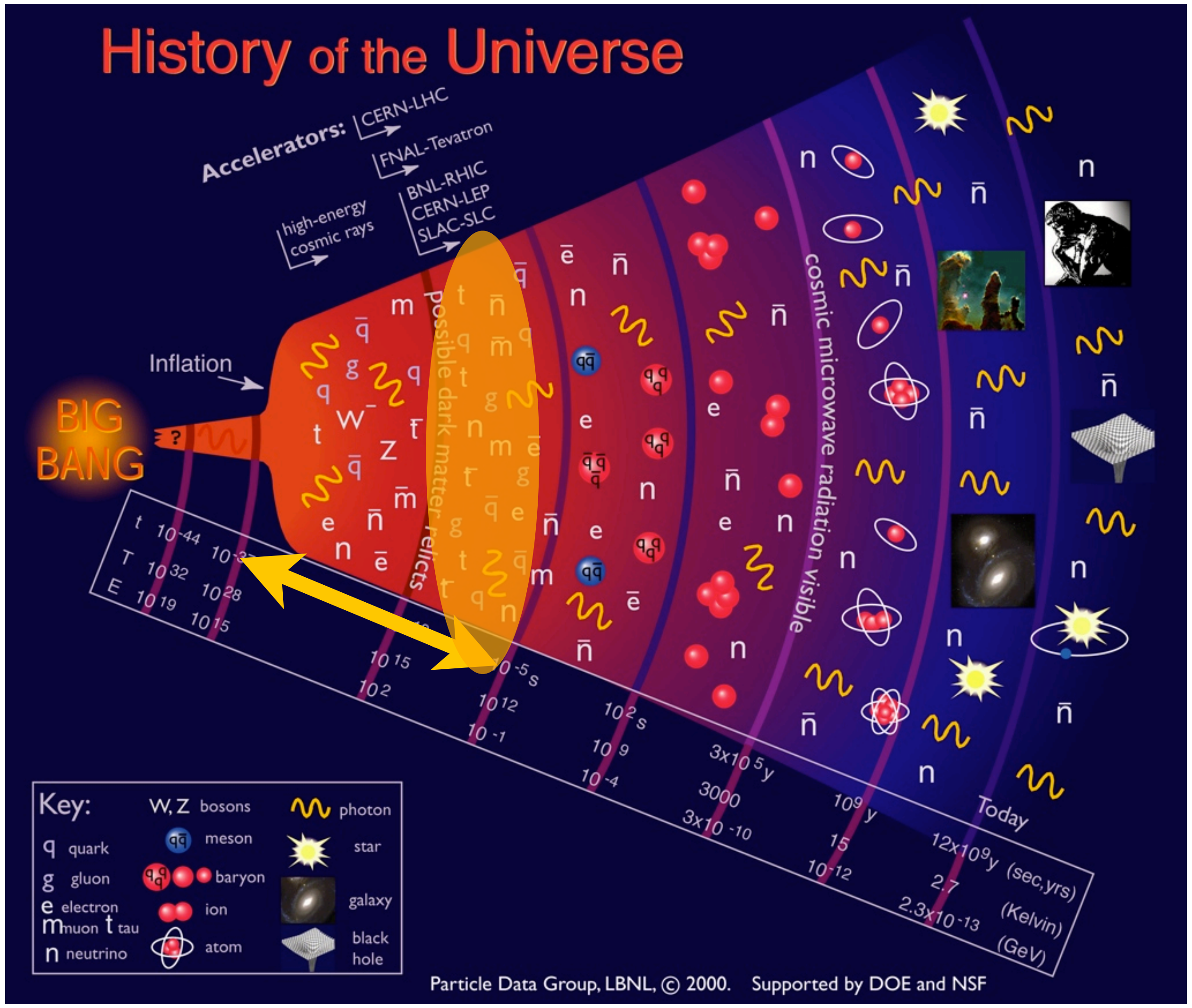
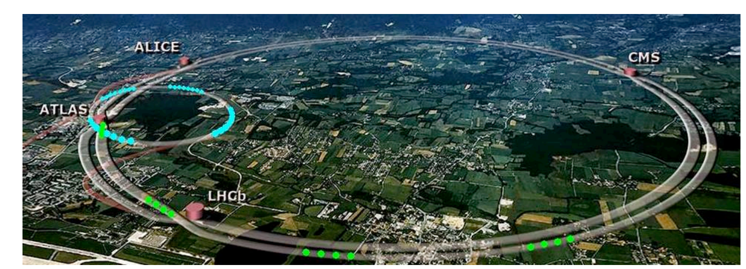


- "Low-scale" mechanisms could lead to experimental signatures (LHC, EDMs, flavor) -- falsifiable scenarios!

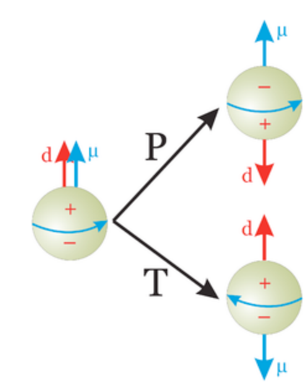
EDMs and baryogenesis: high-level connection

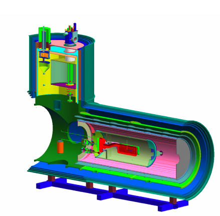
- Degrees of freedom involved in low-scale (T ~ TeV) scenarios can lead to observable signatures in:

- Collider phenomenology
    - New particles, EWSB sector, CP, ...



- Precision low-energy measurements
    - EDMs, flavor physics: directly probe one of Sakharov’s necessary ingredients (CP violation)





EDMs and baryogenesis: high-level connection

- Quantitative connection?

$$\eta = \sum_i F_i[m_\alpha, g_\alpha] \, \sin\phi_i$$

$$\sigma[m_\alpha, g_\alpha]$$

(collider)

$$d = \sum_i G_i[m_\alpha, g_\alpha] \, \sin\phi_i$$

(EDMs, flavor)

- Requires cosmology & non-eq. QFT (F_i), collider physics (σ), low-energy physics (G_i) (from QCD to atomic physics)

EDMs and baryogenesis: high-level connection

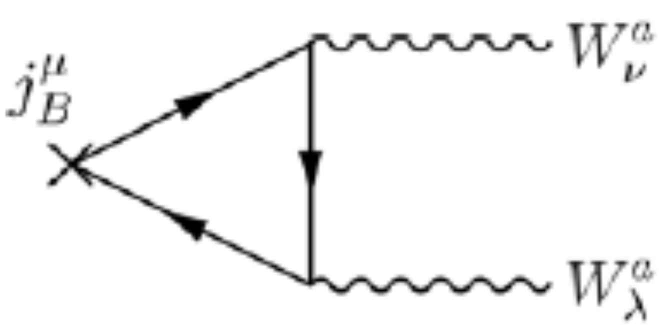
- Quantitative connection?
- Connection is model- and mechanism-dependent
- Two classes of models have received much attention:
    - Electroweak Baryogenesis -- poster child for EDM-BAU connection
    - Leptogenesis

Electroweak Baryogenesis

Sakharov conditions in EWB

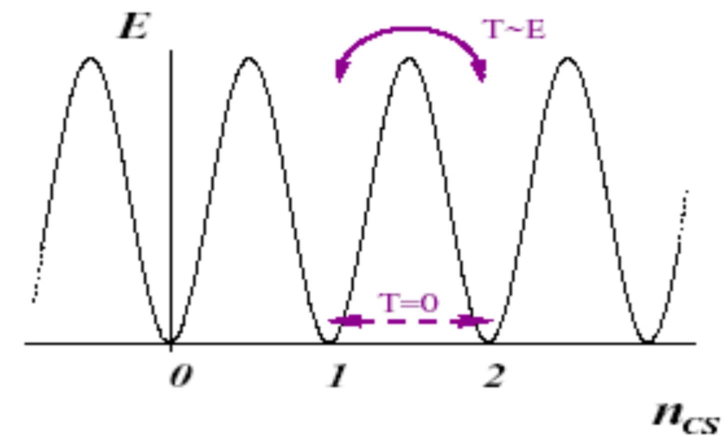
★ B violation: anomaly + sphaleron transitions

Kuzmin-Rubakov-Shaposhnikov 85



N_{CS} labels vacuum states of gauge theory

$$\Delta B = \Delta L = n_F \Delta N_{CS}$$



$$\partial_\mu J^\mu_B = \frac{n_F}{32\pi^2} g^2 W^a_{\mu\nu} \tilde{W}^{a\mu\nu}$$

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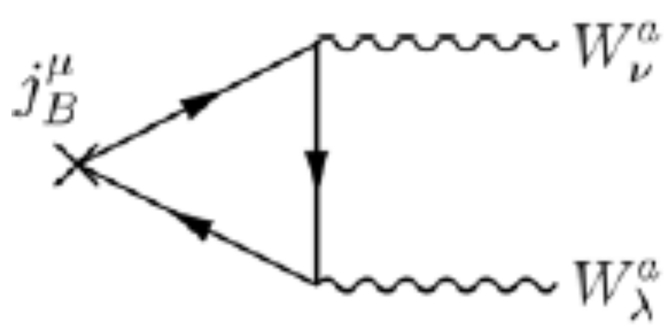

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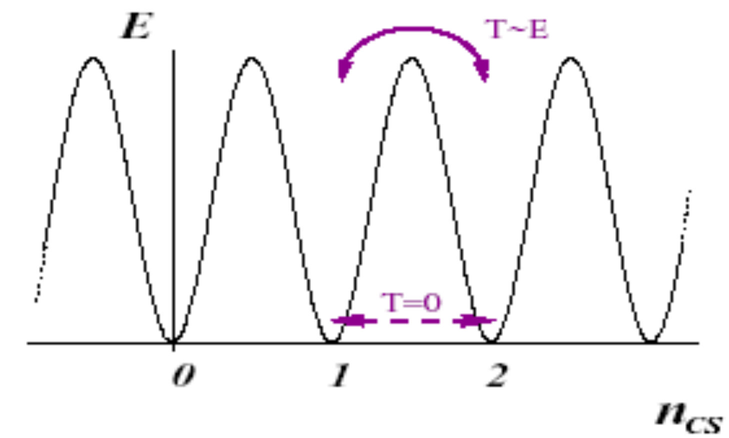
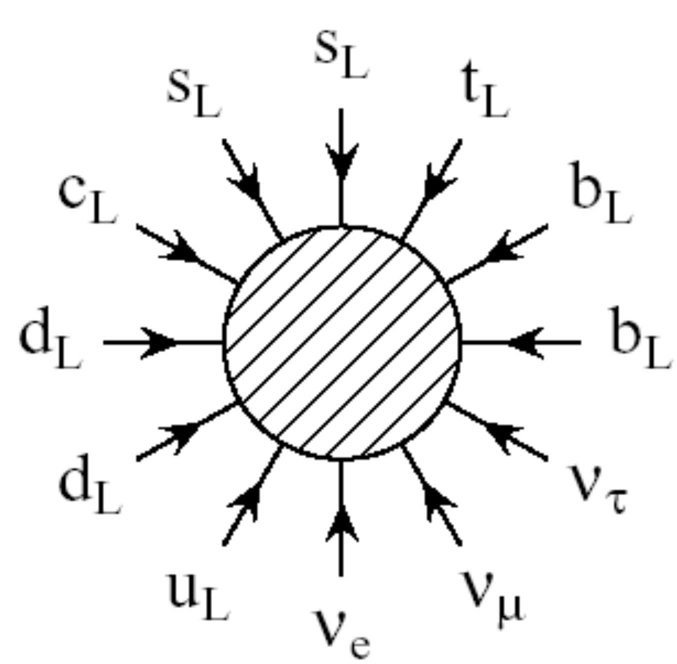


$$\partial_\mu J_B^\mu = \frac{n_F}{32\pi^2} g^2 W_{\mu\nu}^a \tilde{W}^{a\mu\nu}$$



Unbroken phase

$\Gamma_{ws} \sim \kappa_1 \alpha_w^5 T$

$\Gamma_{ws} \sim \kappa_2 \langle H \rangle e^{-\kappa \langle H \rangle / T}$

Broken phase

Sakharov conditions in EWB

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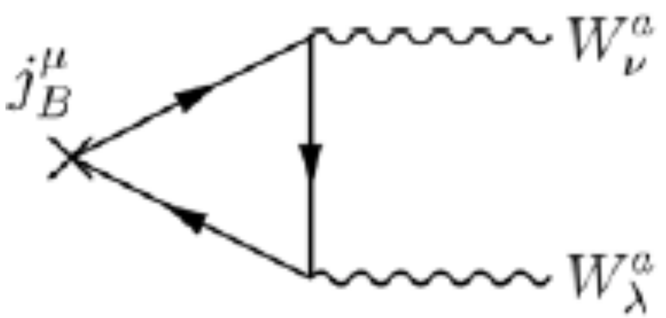

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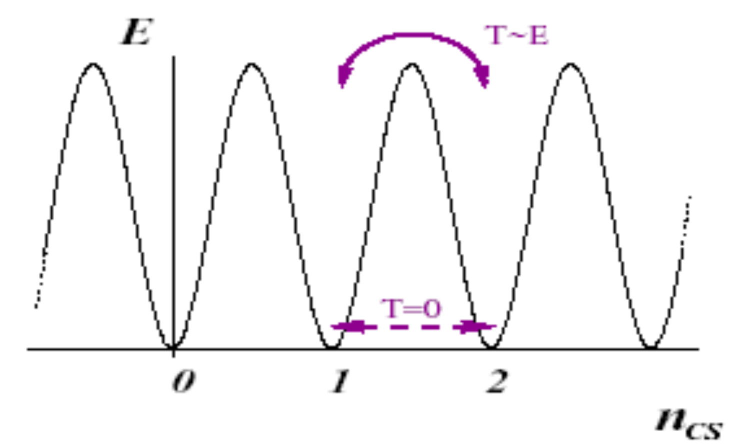
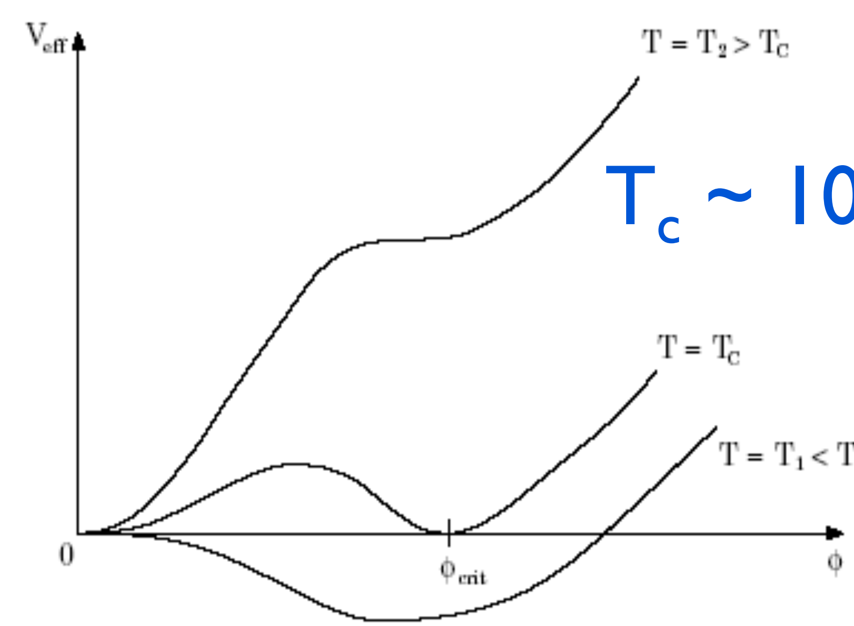


★ C & CP violation: CKM + potential BSM sources

★ Departure from equilibrium:

- Electroweak phase transition
- Need 1st order to avoid "washout" ($<\phi>/T > 1$)



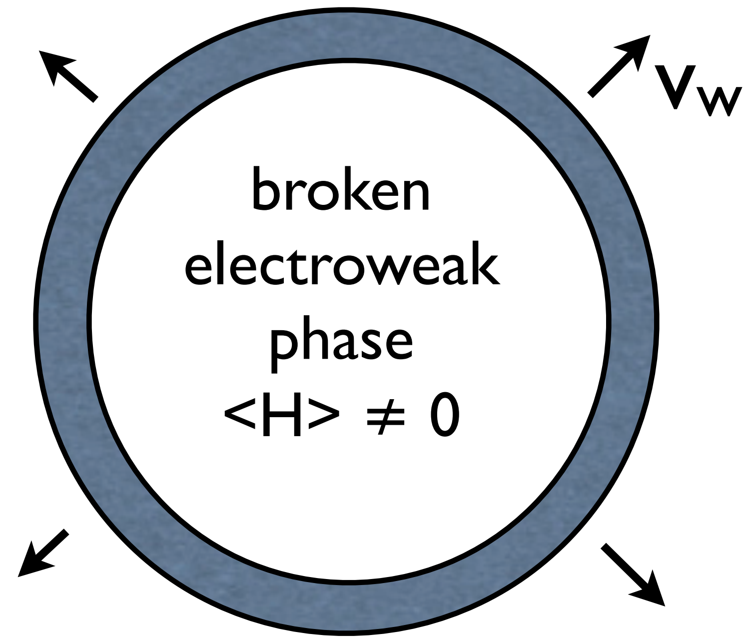
$T_c \sim 100$ GeV

How does it work?

Kuzmin-Rubakov-Shaposhnikov
Cohen-Kaplan-Nelson
....

$\not{C}P$ in the bubble wall (M_{ab})

$\not{B}$ in unbroken phase



1) Bubbles of broken electroweak phase nucleate and expand

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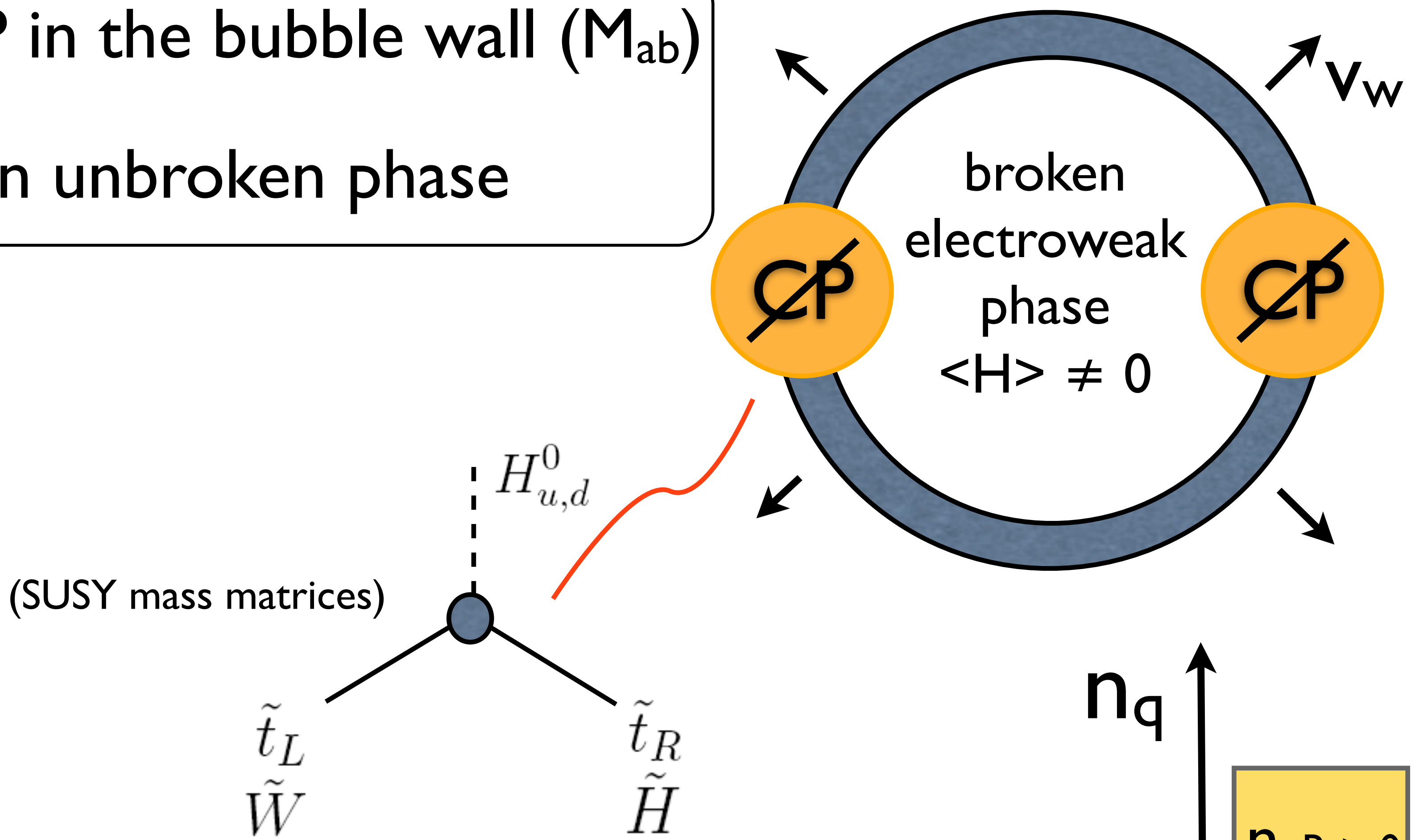

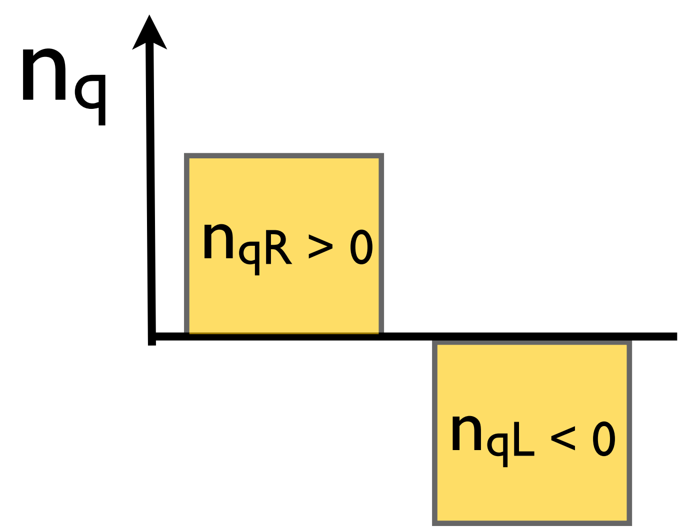

1) Bubbles of broken electroweak phase nucleate and expand

2) *Charge asymmetries (i) develop through CPV flavor mixing and interactions,*
(ii) diffuse in unbroken phase and get converted into L-handed fermionic charge (n_L)

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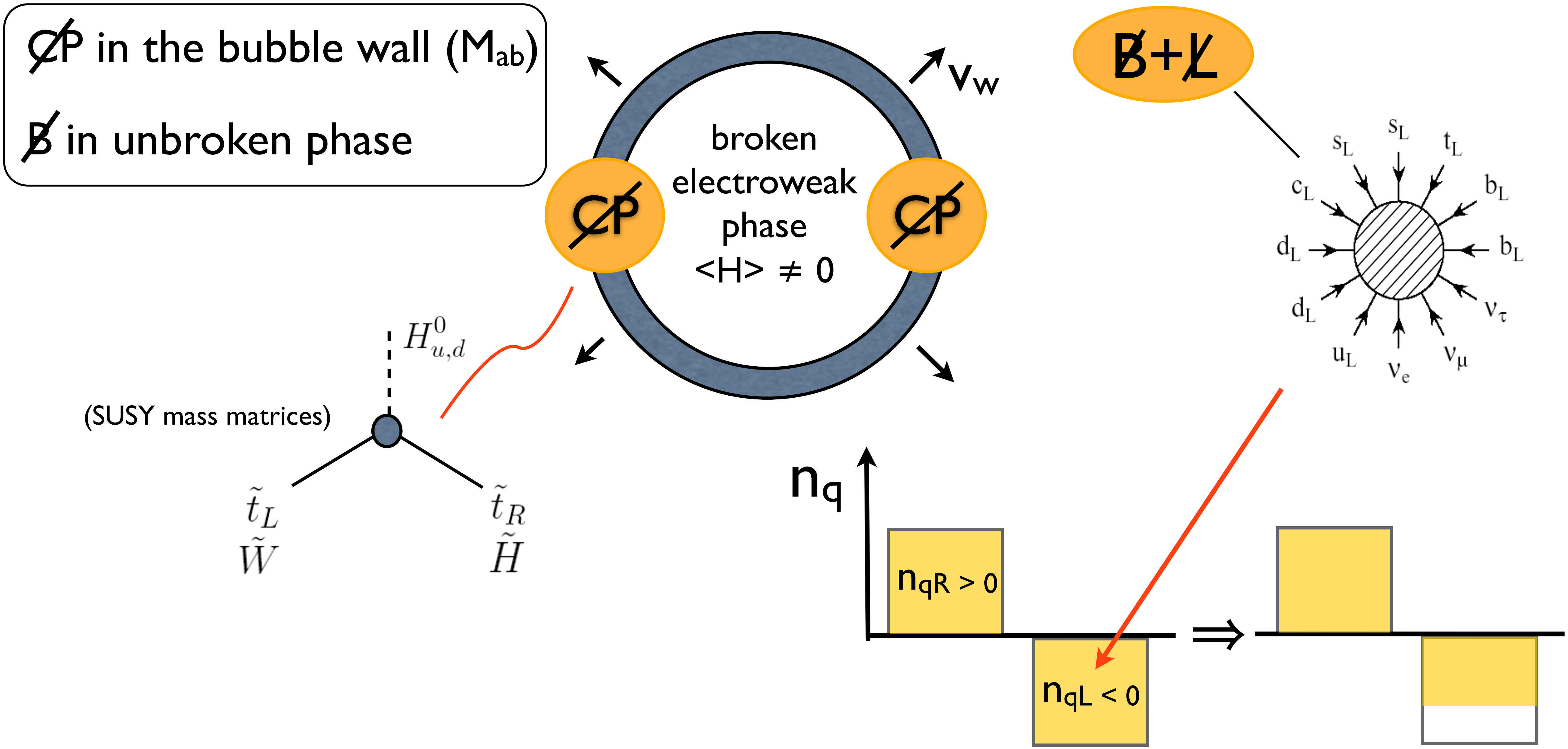

$n_B > 0$

1) Bubbles of broken electroweak phase nucleate and expand

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3) Sphalerons convert excess of n_L into net baryon number

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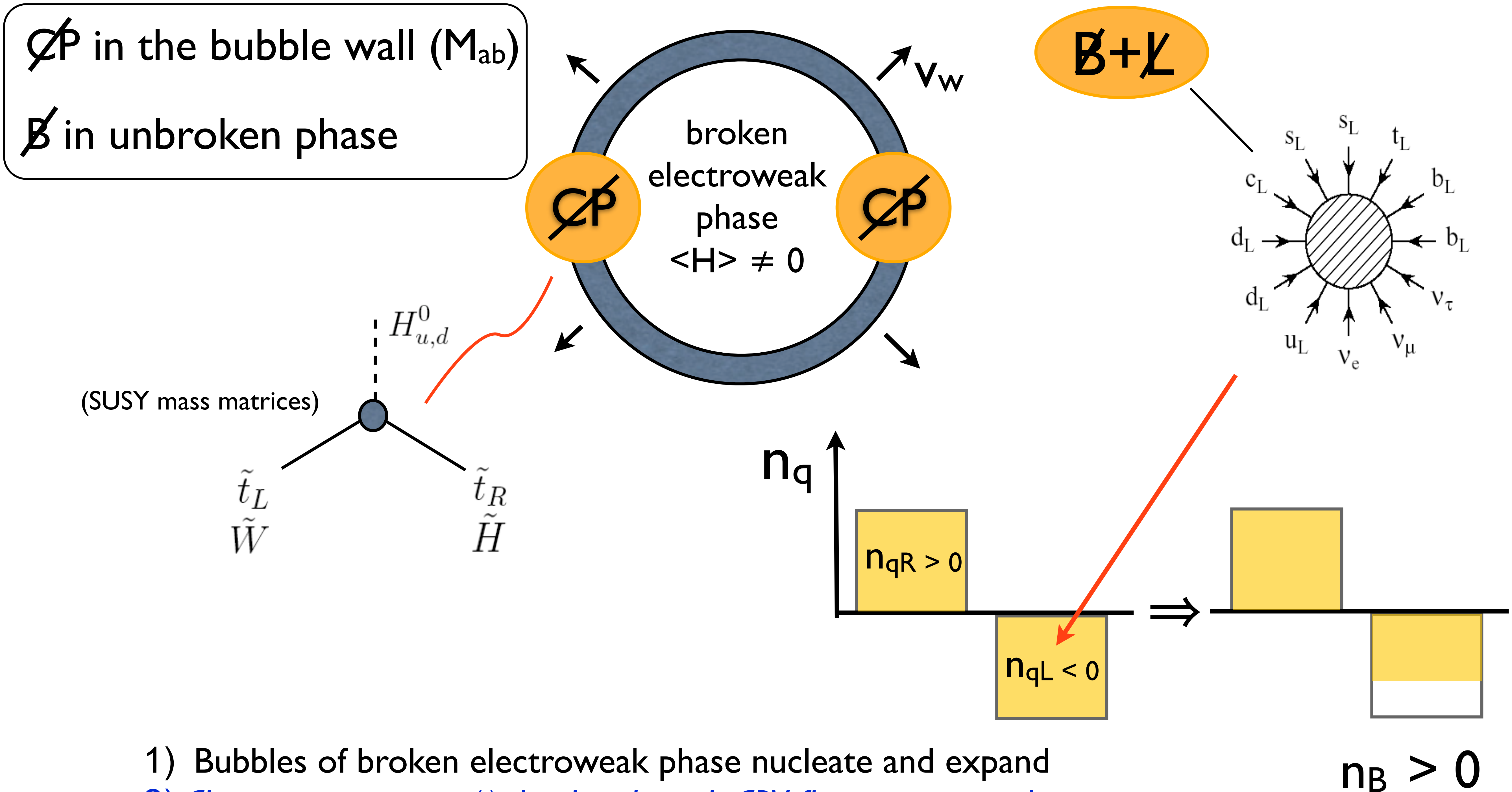

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2) *Charge asymmetries (i) develop through CPV flavor mixing and interactions,*
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3) Sphalerons convert excess of n_L into net baryon number

4) Baryon asymmetry is captured by expanding bubble wall and "freezes in"

What does it take to calculate n_B?

- Phase transition dynamics

  - First order?   Bubble profiles $\Leftarrow$ $V_{eff}(\varphi, T)$

  - Bubble propagation

- Rate of non-perturbative B+L violating transitions

- Transport: generation and diffusion of CPV densities

$$\dot{n}_i - D_i \nabla^2 n_i + \Gamma_{ij} n_j = S_i$$

$$\downarrow$$

$$n_L \{n_i\}$$

$$\downarrow$$

$$\dot{n}_B - \bar{D}_q \nabla^2 n_B + \mathcal{R}\Gamma_{\rm ws} n_B = -\frac{n_F \Gamma_{\rm ws}}{2} n_L$$

EWB and new physics

- Does not work in the SM (phase transition, CP violation)
- Phase transition: in the SM it is 1st order only for $m_H < 70$ GeV

Gurtler et al ’97 Kajantie et al ’98 Csikor et al ’98 Aoki et al ’99

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- CP violation: CKM phase is O(1) but re-phasing invariant is suppressed by quark mixing angles and mass differences**

$$S_{CP} \sim \frac{1}{T_c^{12}} \prod_{i>j u,c,t} \left(m_i^2 - m_j^2\right) \prod_{i>j d,s,b} \left(m_i^2 - m_j^2\right) \, \mathrm{Im}\left(V_{ud} V_{cb} V_{ub}^* V_{cd}^*\right) \sim 10^{-19}$$

**Based on treating quark masses as perturbations, OK for $p \sim T \gg m_q$, but not correct for, e.g.,, $m_d < p < m_s$

Farrar-Shaposhnikov '94

- Detailed calculations confirm this scaling: non-trivial effects of Farrar-Shaposhnikov suppressed by thermal damping $\Gamma \gg m_s - m_d$

Gavela et al '94

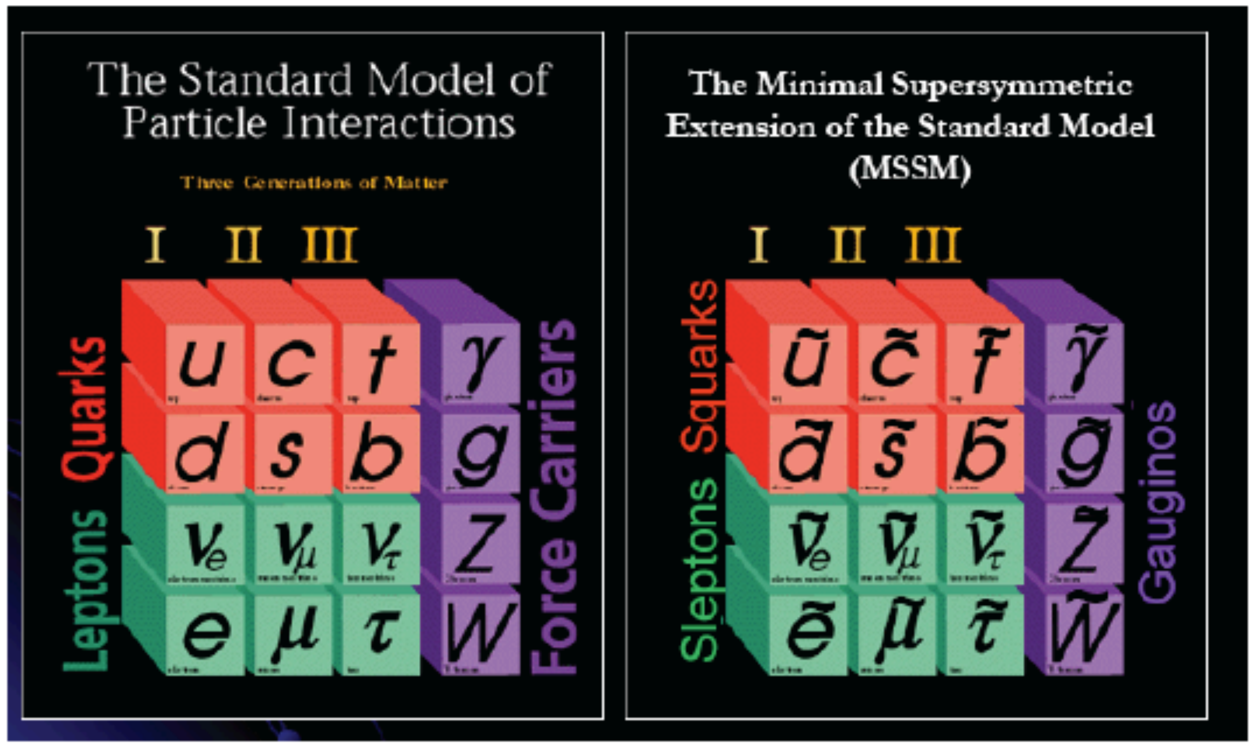
EWB and new physics

- Does not work in the SM (phase transition, CP violation) $\Rightarrow$ EWB tied to BSM physics at the weak scale (2HDM, supersymmetry, ...)

- It can be tested through
  - Energy frontier (LHC): EWSB mechanism, phase transition
  - Intensity frontier: CPV in meson decays and EDMs as probes of weak-scale BSM CP violation

- Use MSSM as representative example

Supersymmetric EWB



Interactions dictated by Supersymmetry

Bosons ⟺ Fermions

and gauge invariance

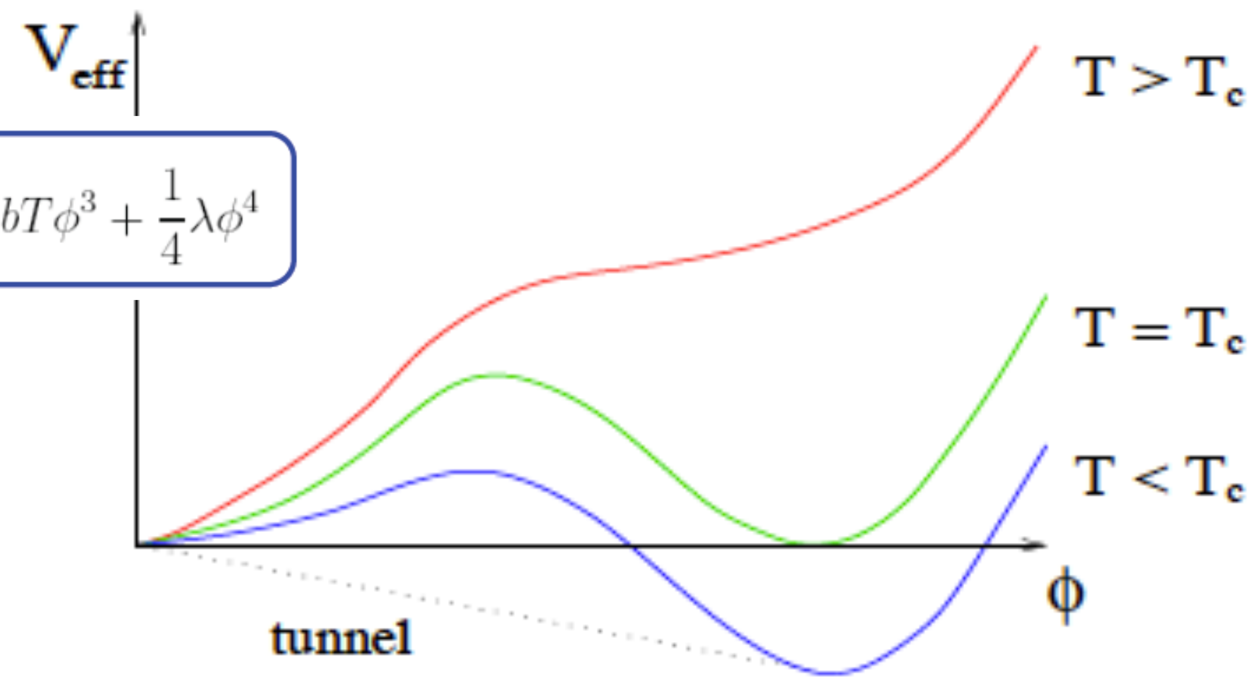
Two Higgs doublets: H_u , H_d ←→ $\tilde{H}_u$, $\tilde{H}_d$ higgsinos (spin 1/2)

- MSSM has sufficient new degrees of freedom to
  - Make phase transition 1st order ←
  - Generate η via new sources of CP violation ↙

Probed by nearly independent phenomenological handles

Phase transition and the Higgs

$$V_{eff}(\phi, T) = \frac{1}{2}a(T^2 - T_1^2)\phi^2 - \frac{1}{3}bT\phi^3 + \frac{1}{4}\lambda\phi^4$$



$$\phi_{crit} = \frac{2b}{3\lambda}T_c$$

- Cubic term that drives 1st order transition can be induced by light scalars (X) couplings to the Higgs (light stop in the MSSM)

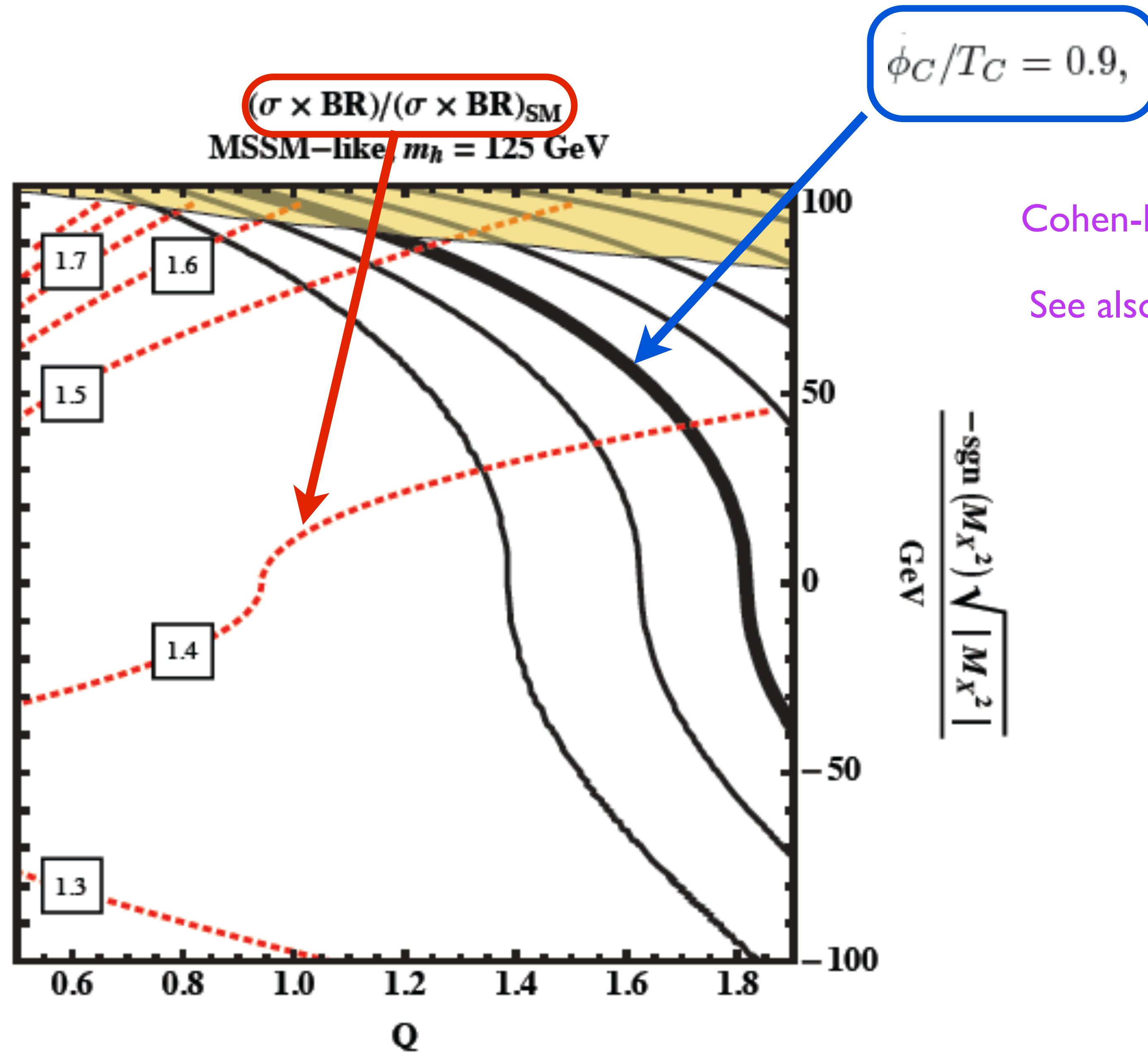
$$-\mathcal{L} \supset M_X^2|X|^2 + \frac{K}{6}|X|^4 + Q\,|X|^2|H|^2$$

$$\downarrow$$

$$\Delta V_{eff}(\phi, T) \supset -\frac{n_X T}{12\pi}\left[\Pi_X(T) + M_X^2 + Q\,\phi^2/2\right]^{3/2}$$

- These new scalars modify Higgs properties

H →γγ

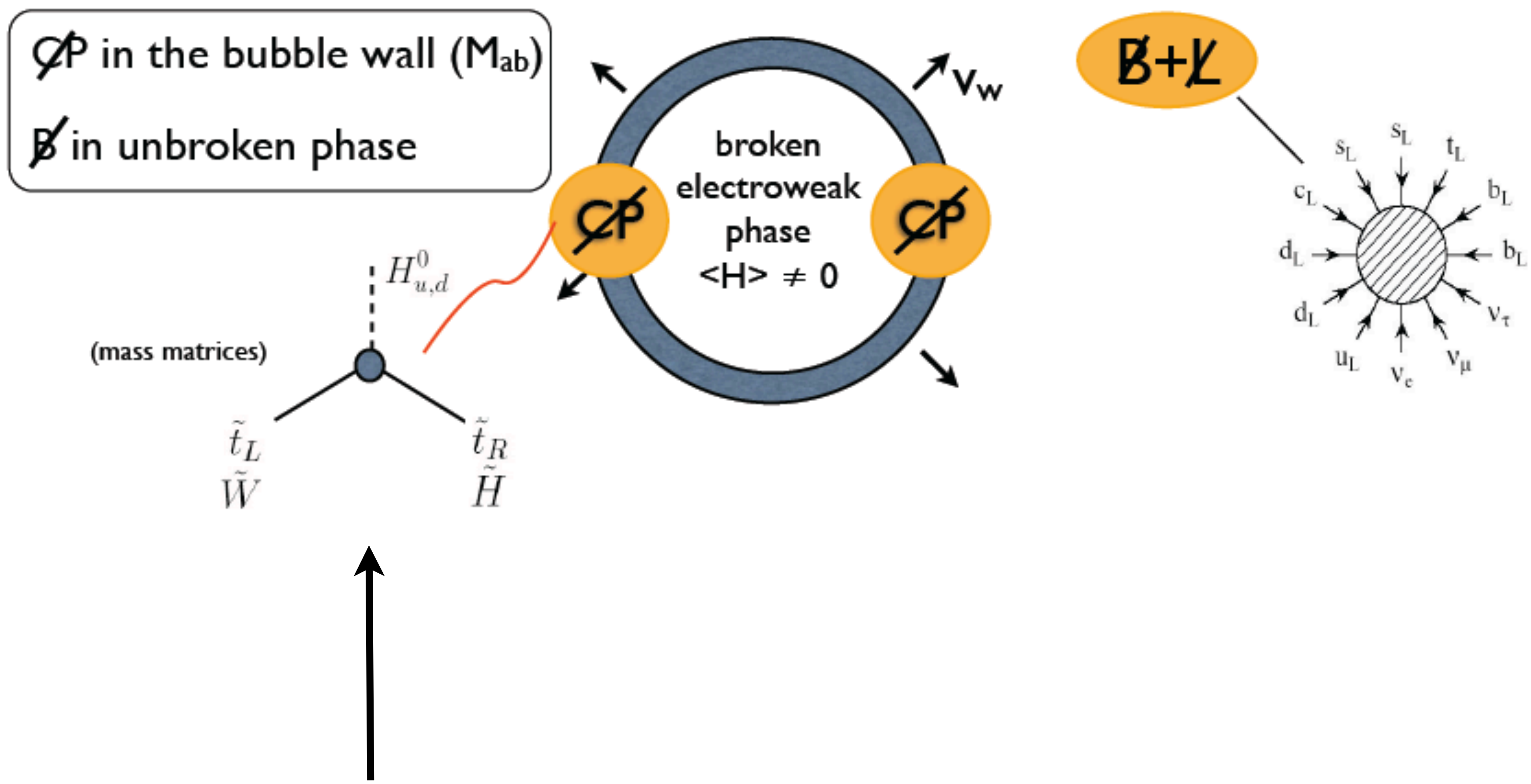


Cohen-Morrissey-Pierce 2012

See also Carena et. al 2012

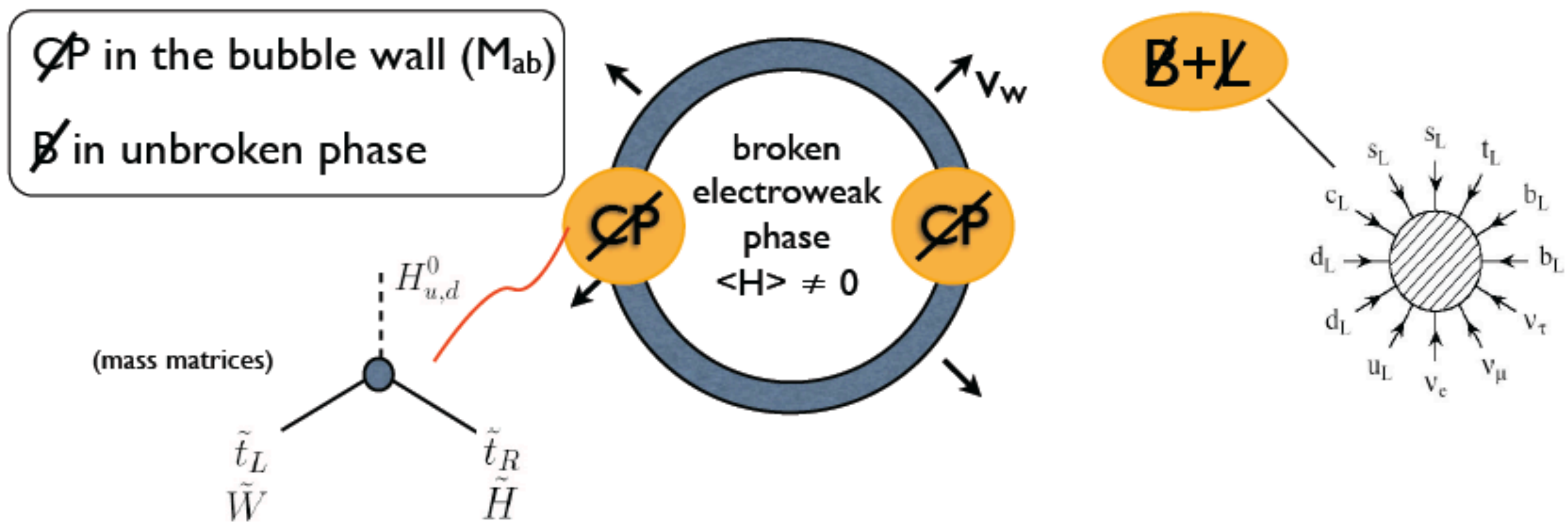
- Region consistent with 1st order EWPT (light stop) implies sizable deviations from SM rates -- MSSM EWB nearly ruled out!

CP violation and EDMs

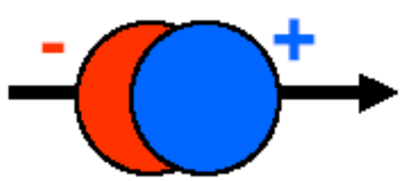


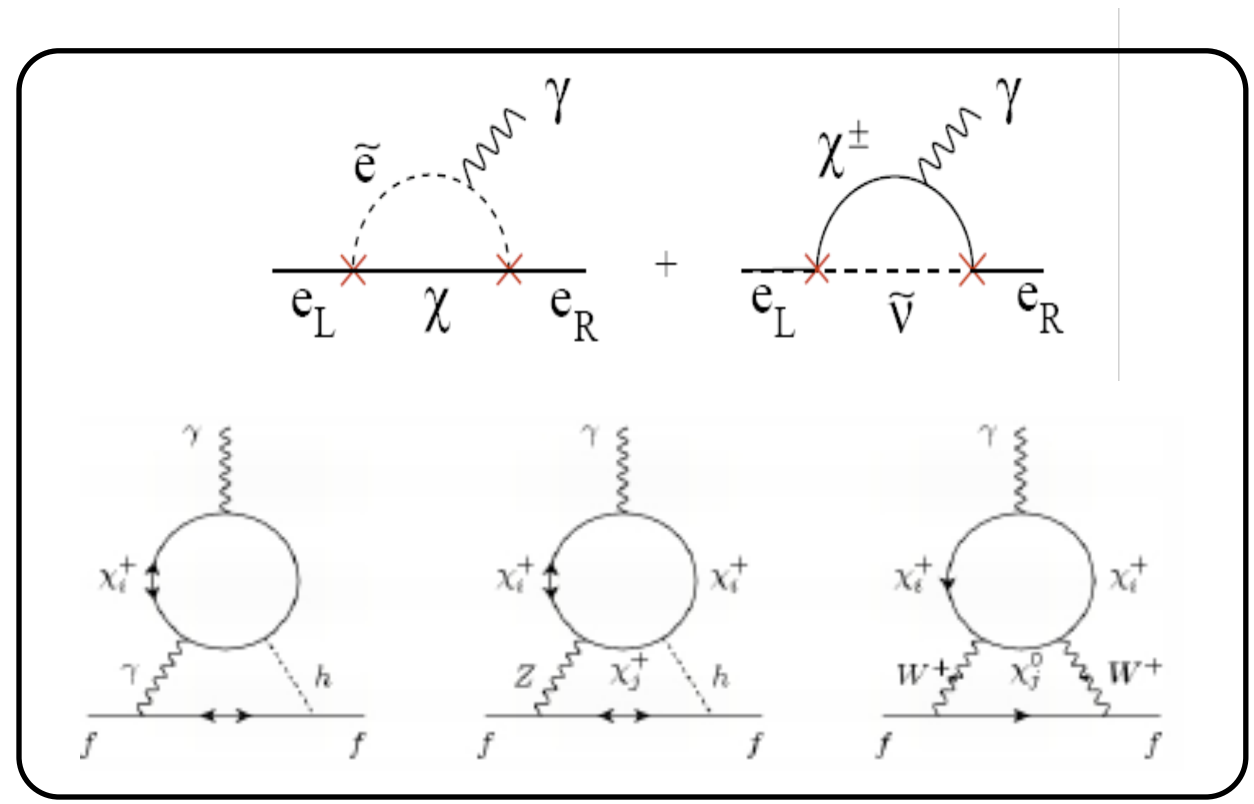
$$\overline{\psi}_R M_\chi \psi_L = (\overline{\widetilde{w}^+},\ \overline{\widetilde{h}_2^+})_R \begin{pmatrix} m_2 & gH_2(x) \\ gH_1(x) & \mu \end{pmatrix} \begin{pmatrix} \widetilde{w}^+ \\ \widetilde{h}_1^+ \end{pmatrix}_L + \text{h.c.},$$

CP violation and EDMs



CPV phases appearing in the mass matrices of sfermions, charginos, neutralinos contribute to both baryogenesis and EDMs!





Chang-Chang-Keung. Giudice-Romanino, Pilaftsis, Li, Profumo, Ramsey-Musolf

CP violation and EDMs

- The region of parameter space in which EWB is viable is determined by consistency with: (i) 1st order phase transition, (ii) precision EW tests, (iii) EDMs constraint.

- Stops: one light (m~ 125 GeV), one heavy (>TeV) and ~ left-handed

- Other sfermions: heavy (m > few TeV)

- Free parameters: gaugino masses ($M_{1,2}$), complex higgsino mass (μ), heavy Higgs mass (m_A)

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$$d_f \sim e\left(\frac{m_f}{M^2}\right)\frac{\alpha_k}{4\pi}\sin\phi$$

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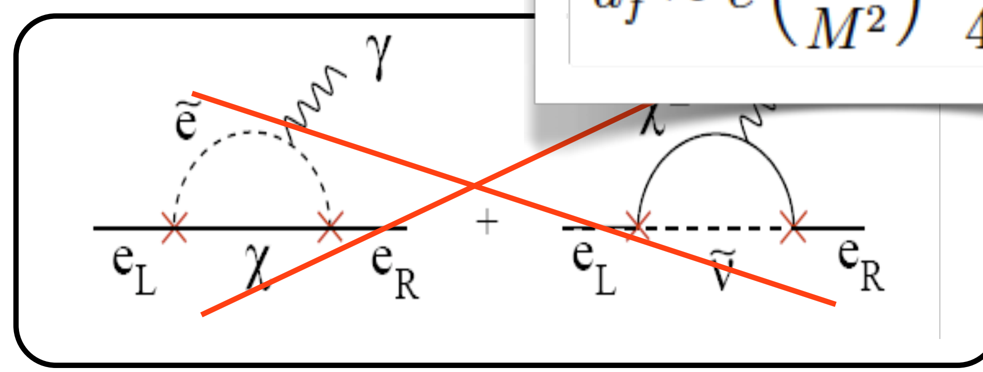

One-loop EDM contributions decouple

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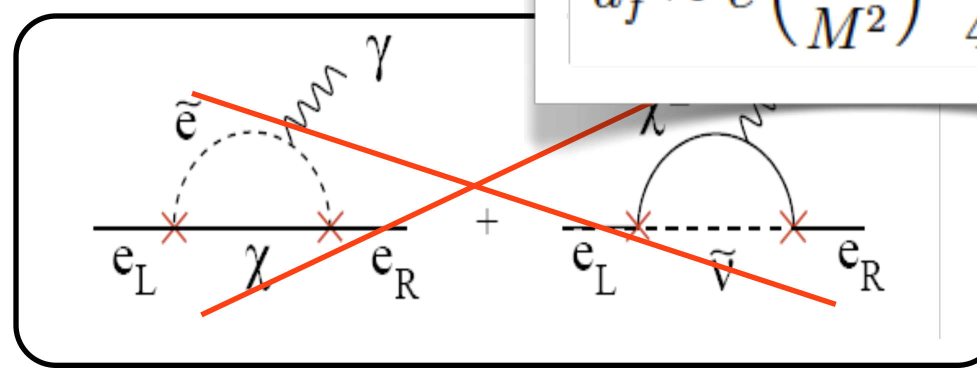

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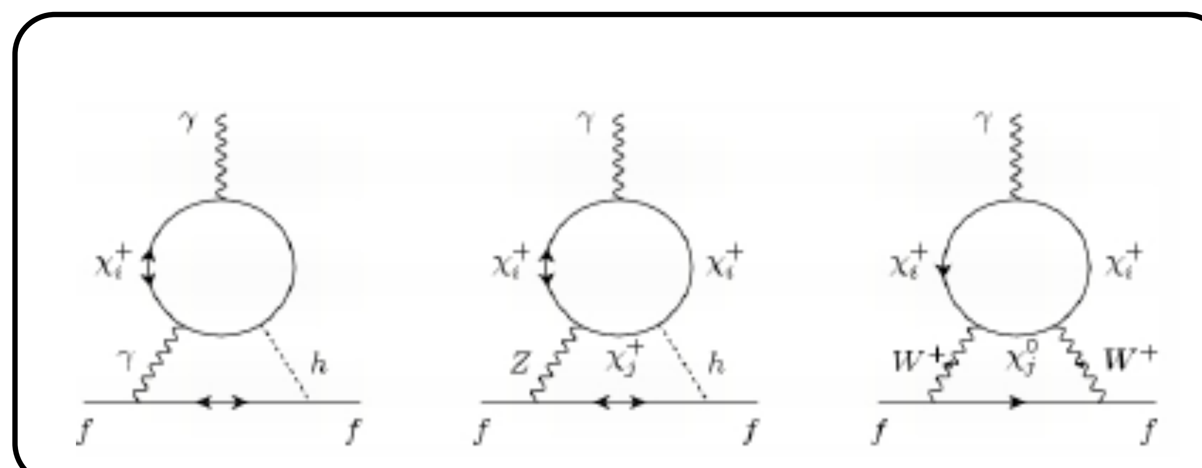


Remain as irreducible contributions to EDMs

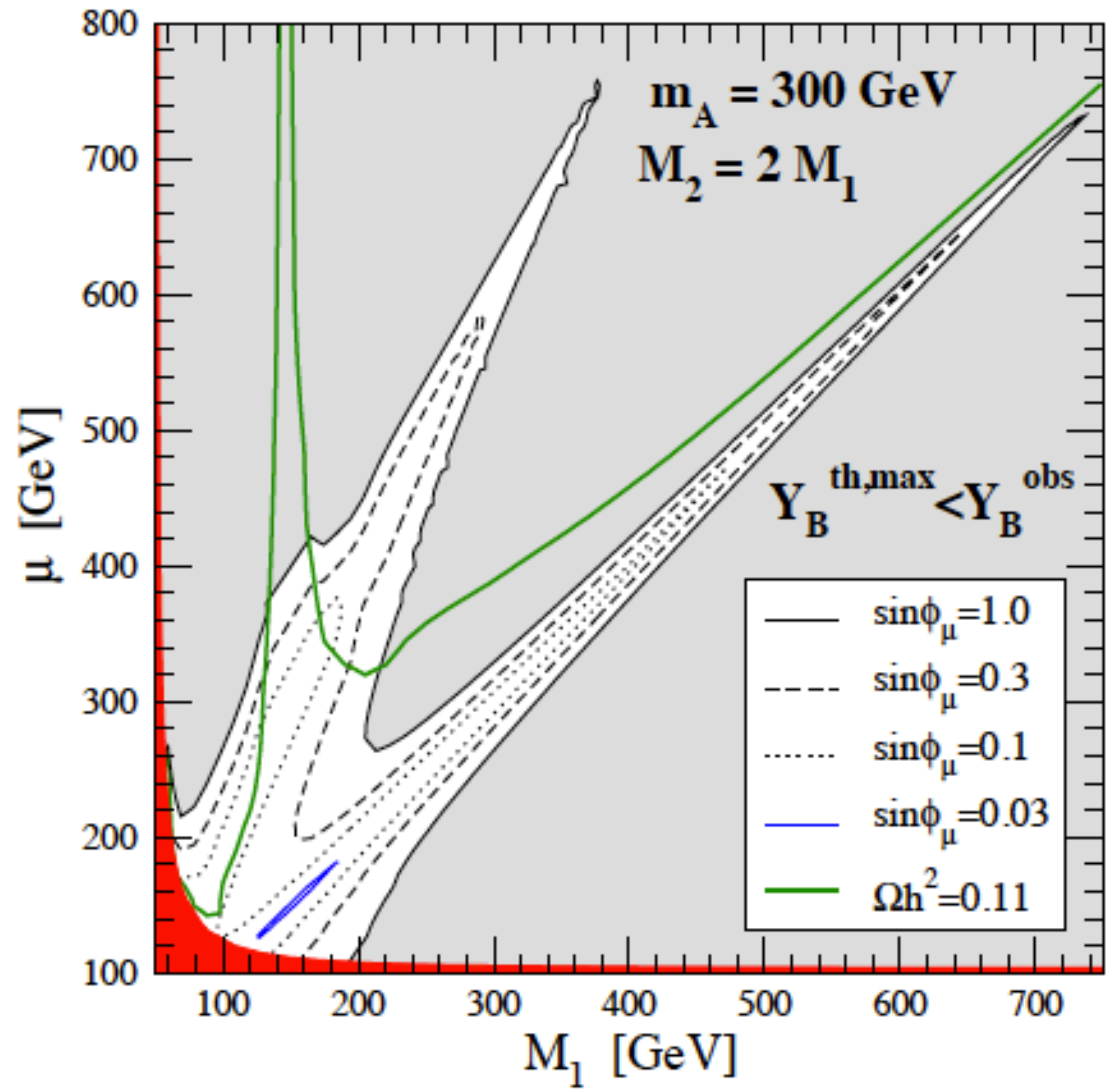
$\phi_1 = \mathrm{Arg}(\mu M_1 b^*)$ $\qquad \phi_2 = \mathrm{Arg}(\mu M_2 b^*)$

CP violation and EDMs

- The region of parameter space in which EWB is viable is determined by consistency with: (i) 1[st] order phase transition, (ii) precision EW tests, (iii) EDMs constraint.

VC, Li, Profumo, Ramsey-Musolf 10

- When projected on the higgsino-gaugino mass plane, the region of successful EWB has a two-funnel structure corresponding to resonant mixing of charginos (neutralinos): largest CP asymmetry for fixed phase



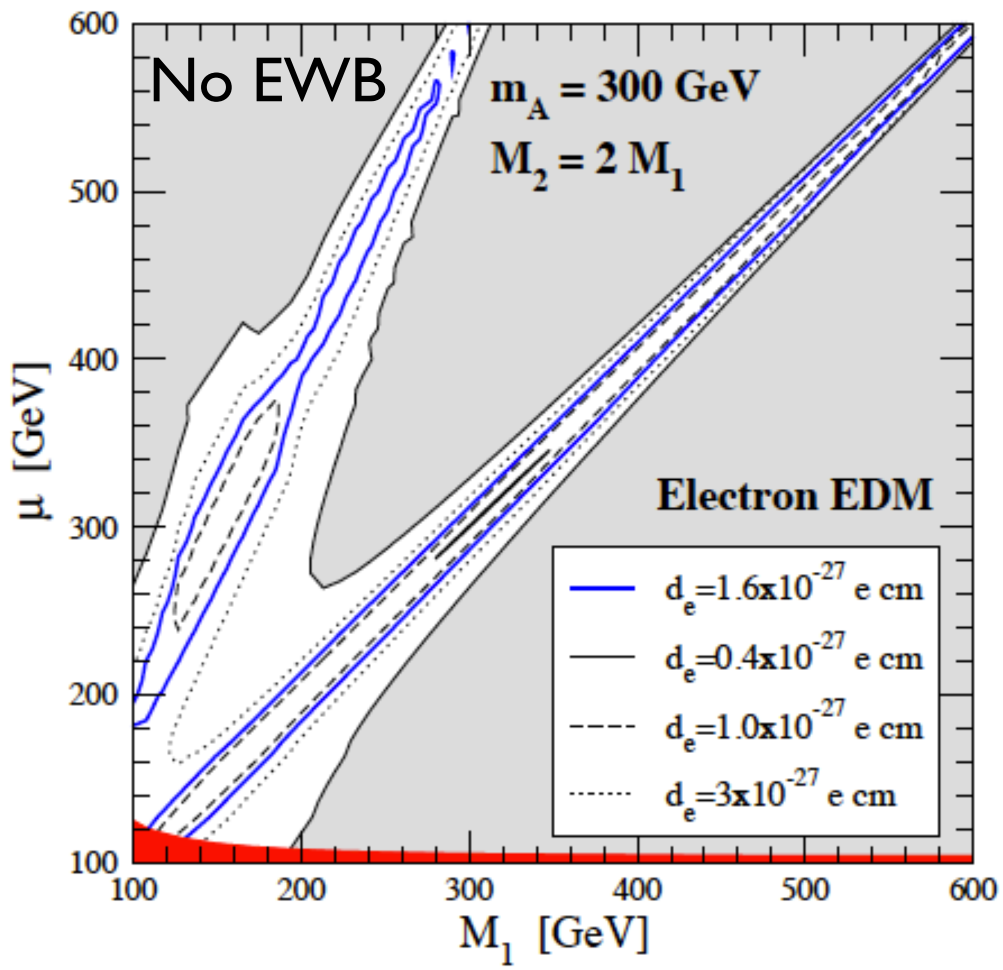
Universal phases

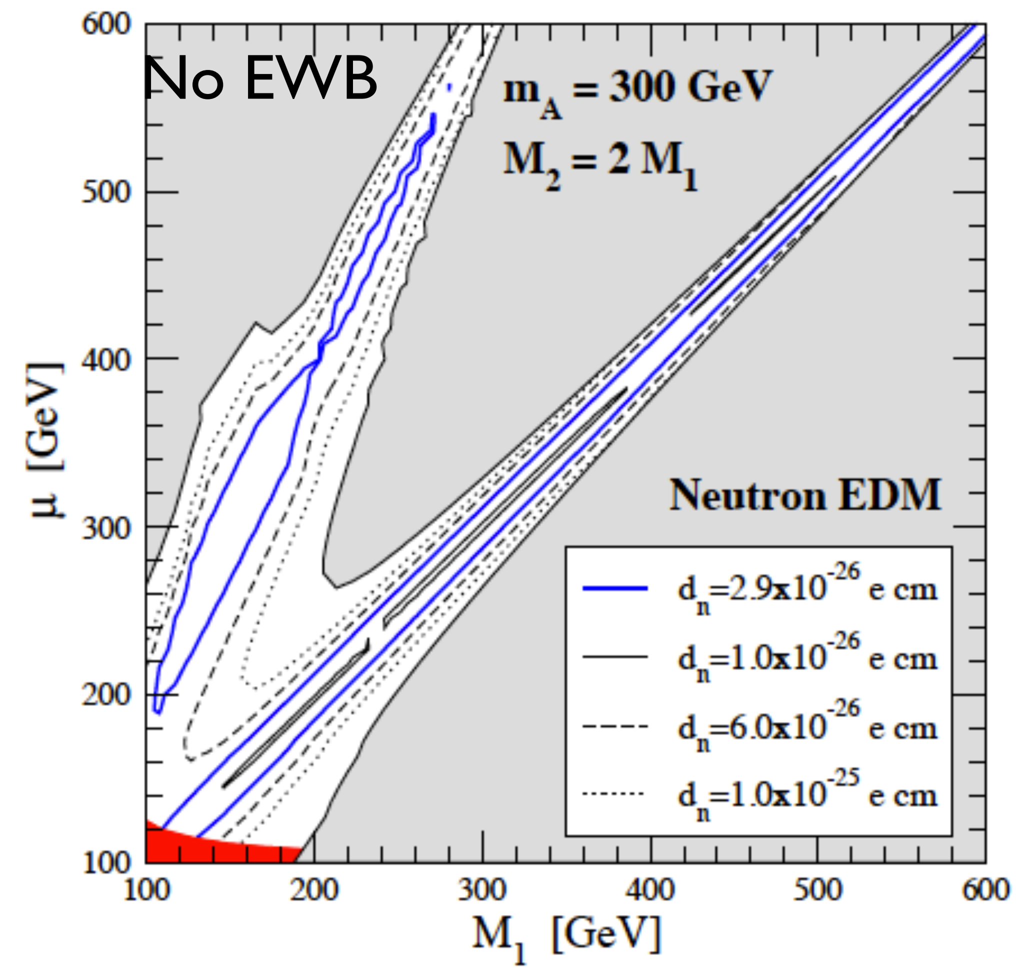
- For a given point on the μ-M₁ plane, determine CPV phase ϕ_μ by enforcing successful baryogenesis: then calculate EDMs

$$d_{e,n} \propto \sin\phi_\mu \qquad \eta \propto \sin\phi_\mu$$

$$\phi_1 = \phi_2 \equiv \phi_\mu$$

VC, Li, Profumo, Ramsey-Musolf 10





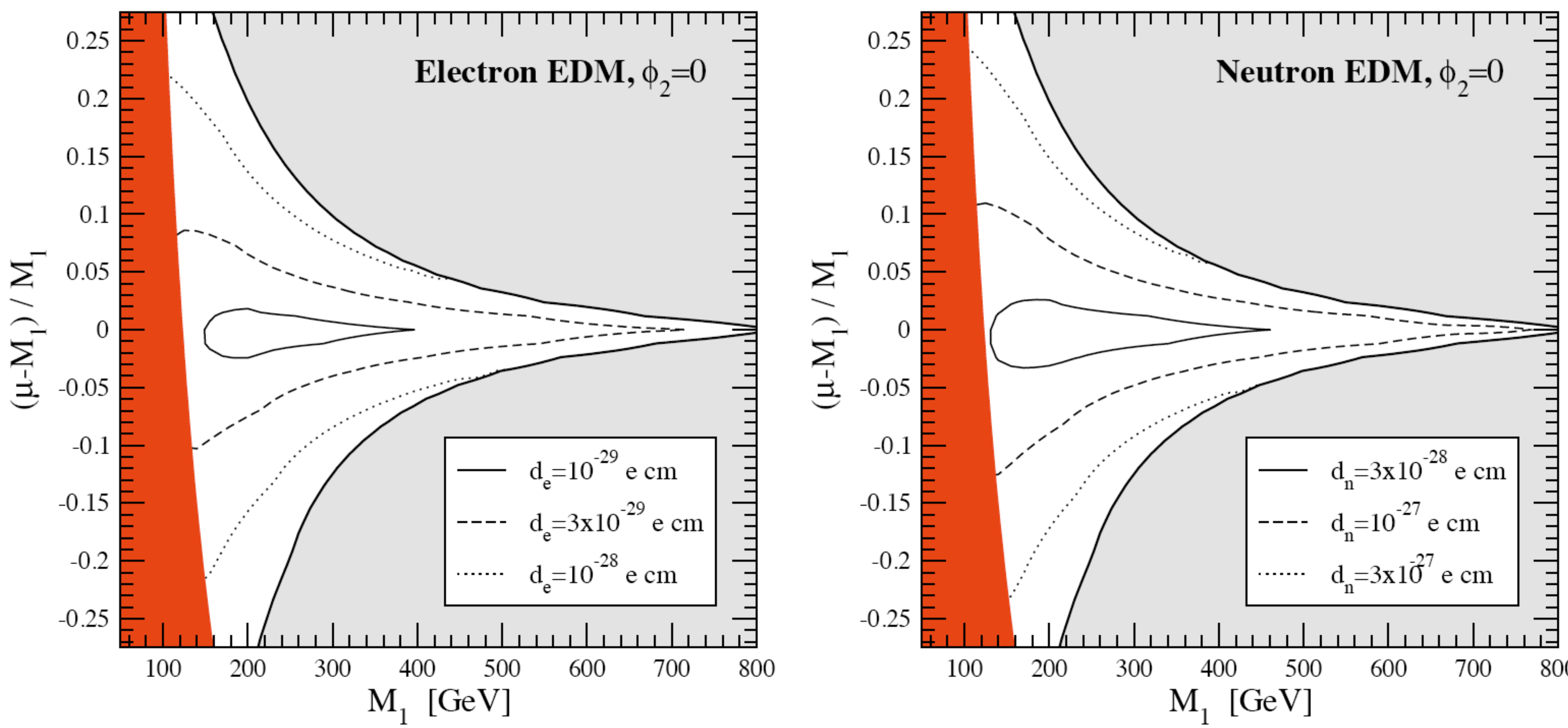
- Lower bounds on EDMs close to current experimental limits !

Non-universal phases

- Non universal phases: charginos give largest contributions to EDMs. Neutralinos can generate BAU. Correlation to EDMs is weakened

Li, Profumo, Ramsey-Musolf 08 VC, Li, Profumo, Ramsey-Musolf 10

$$d_{e,n} \propto C_1 \sin\phi_1 + C_2 \sin\phi_2 \qquad \eta \propto K_1 \sin\phi_1 + K_2 \sin\phi_2$$



- Lower bounds borderline with sensitivity of next gen. experiments

Non-universal phases

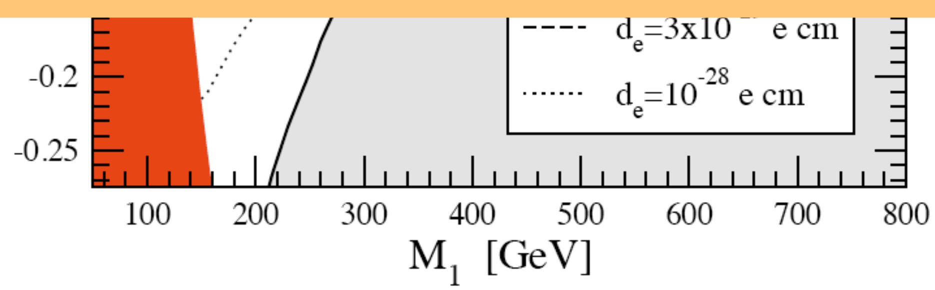
- Successful SUSY baryogenesis implies "guaranteed signals" for e and n EDMs, within reach of the next generation experiments

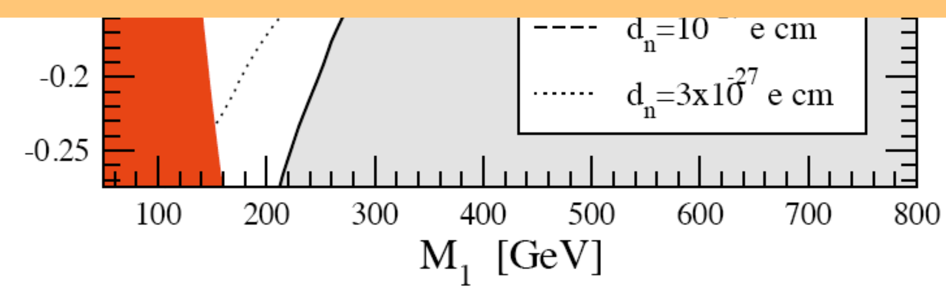
- CAVEAT: orders-of-magnitude spread among existing calculations, due to different transport treatments for flavor-mixing particles

Carena-Moreno-Quiros-Wagner-Seco '00-'02-'04 VC-Lee-RamseyMusolf-Tulin '06 Konstandin-Prokopec-Schmidt-Seco '05-'06

- Area of active investigation

VC-Lee-RamseyMusolf-Tulin '10
VC-Lee-Tulin '11
Herranen, Kainulainen et al '08 -'11





- Lower bounds borderline with sensitivity of next gen. experiments

Beyond MSSM

- Many extensions studied:
    - NMSSM
    - MSSM + higher dim. operators
    - U(1)' extensions
    - ...

Huber-Konstandin-Prokopec-Schmidt 2006-7

Carena et al 2007

Cheung-Hou-Lee-Senaha 2012

Blum-Delaunay-Losada-Nir-Tulin 2010

Kang-Langacker-Li-Liu 2004

- In some scenarios EDMs provide strong constraints on the viability of baryogenesis, in others one can evade EDM bounds

Beyond flavor diagonal CPV

- Recent attention to models with CPV in flavor-mixing vertices (e.g. 2HDM)

$$\mathcal{L} = \lambda^u_{ij}\bar{Q}^i(\epsilon H_d^\dagger)u_R^j - \lambda^d_{ij}\bar{Q}^i H_d d_R^j - y^u_{ij}\bar{Q}^i H_u u_R^j + y^d_{ij}\bar{Q}^i(\epsilon H_u^\dagger)d_R^j + h.c.$$

Tulin-Winslow 2011
Cline-Kainulainen-Trott 2011
Liu- Ramsey-Musolf- Shu 2011

- EDMs relatively insensitive to these sources of CP violation
- Best constraints from flavor physics (B, D mixing and decays)

$$\frac{\zeta_{bs}^2}{\Lambda_{bs}^2}(\bar{b}_L s_R)(\bar{b}_L s_R),$$

$\Lambda_{bs} \sim m^2_{H_{bs}}/v$ $\quad\quad$ $\zeta_{bs} \propto |\lambda_{bs}|(1 \pm \exp(i\theta_{\lambda_{bs}}))|$

Leptogenesis

Connecting BAU, EDMs, LFV

See-saw mechanism for m_ν

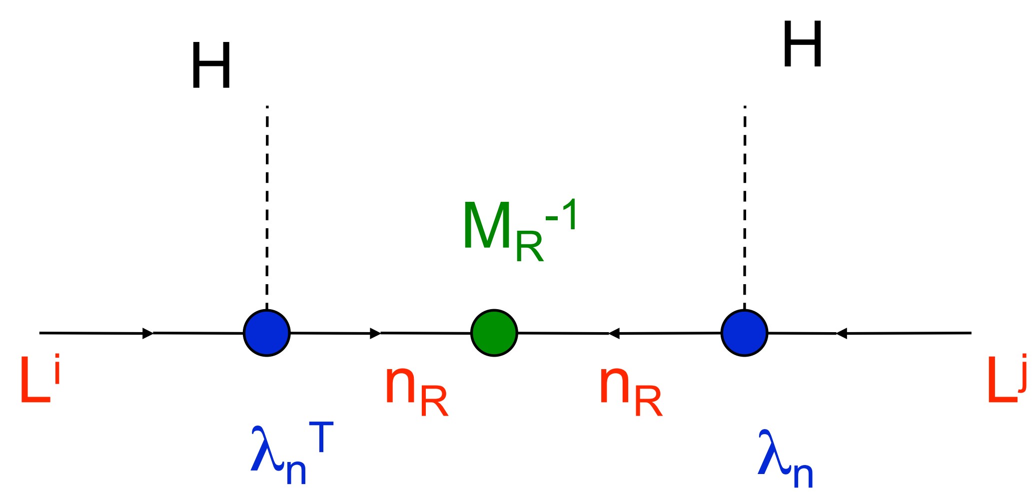
Type I for illustration

$$\mathcal{L} \supset \frac{1}{2} (M_R)_{ij} \, \nu_R^{Ti} \, C \, \nu_R^j \; - \; \lambda_\nu^{ij} \, \bar{\nu}_R^i \, (H_c^\dagger \, L_L^j) + \text{h.c.}$$

Heavy ν_R

M_R : L violation

λ_ν : CP and L_i violation



$m_n \sim v_{ew}^2 \; \lambda_n^T \, M_R^{-l} \, \lambda_n$

Connecting BAU, EDMs, LFV

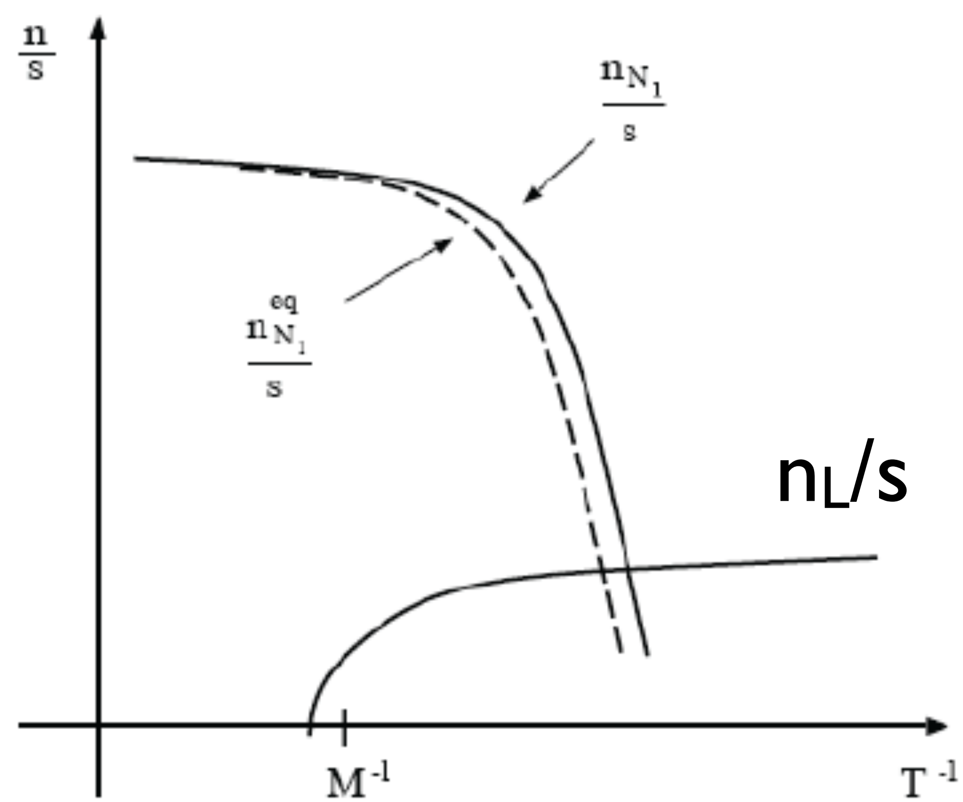
See-saw mechanism for m_ν

$$\mathcal{L} \supset \frac{1}{2}(M_R)_{ij}\, \nu_R^{Ti}\, C\, \nu_R^j \;-\; \lambda_\nu^{ij}\, \bar{\nu}_R^i\, (H_c^\dagger\, L_L^j) + \mathrm{h.c.}$$

M_R : L violation
λ_ν : CP and L_i violation

1) $\not{CP}$ and $\not{L}$ out-of-equilibrium decays of N_i ($T \sim M_R$) $\Rightarrow n_L$

$$\Gamma(N_i \to l_k\, H^*) \neq \Gamma(N_i \to \bar{l}_k\, H)$$



Connecting BAU, EDMs, LFV

See-saw mechanism for m_ν

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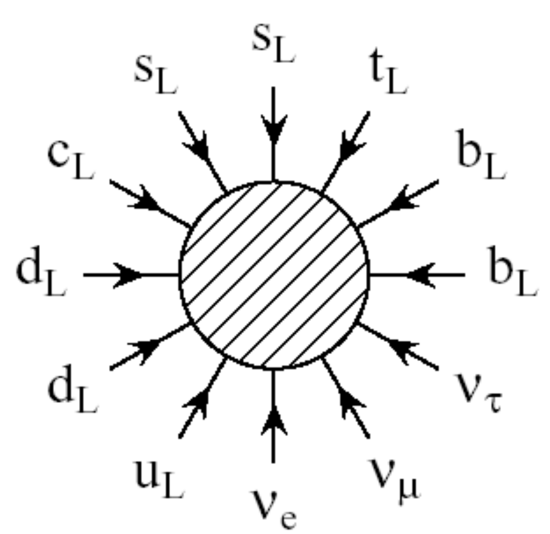
2) EW sphalerons $\Rightarrow n_B = - k\, n_L$

$$\eta_B \equiv \frac{n_B}{n_\gamma} \neq 0$$



Connecting BAU, EDMs, LFV

See-saw mechanism for m_ν

$$\mathcal{L} \supset \frac{1}{2}(M_R)_{ij}\, \nu_R^{Ti}\, C\, \nu_R^j \;-\; \lambda_\nu^{ij}\, \bar{\nu}_R^i\, (H_c^\dagger\, L_L^j) + \text{h.c.}$$

M_R : L violation

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1) $\not{CP}$ and $\not{L}$ out-of-equilibrium decays of N_i ($T \sim M_R$) ⇒ n_L

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2) EW sphalerons ⇒ $n_B = - k\, n_L$

$$\eta_B \equiv \frac{n_B}{n_\gamma} \neq 0$$

If CP & L_i violation is communicated to particles with mass $\Lambda \sim$ TeV

Observable LFV

Observable lepton EDMs

Impact on EDM

- Requires non-minimal field content (beyond ν_R's): need to communicate CP violation to TeV-scale particles

- Examples:

    - SUSY see-saw: CPV (and LFV) imprinted in the slepton mass matrices by radiative corrections

    - Soft leptogenesis (sneutrino out-of-eq. decays)

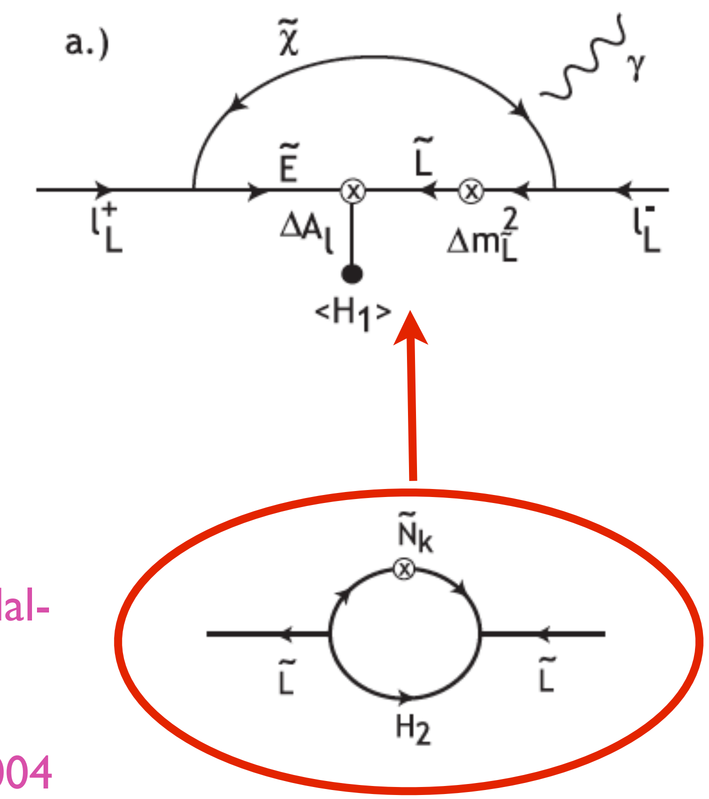
    - Low scale see-saw (LRSM, ...)

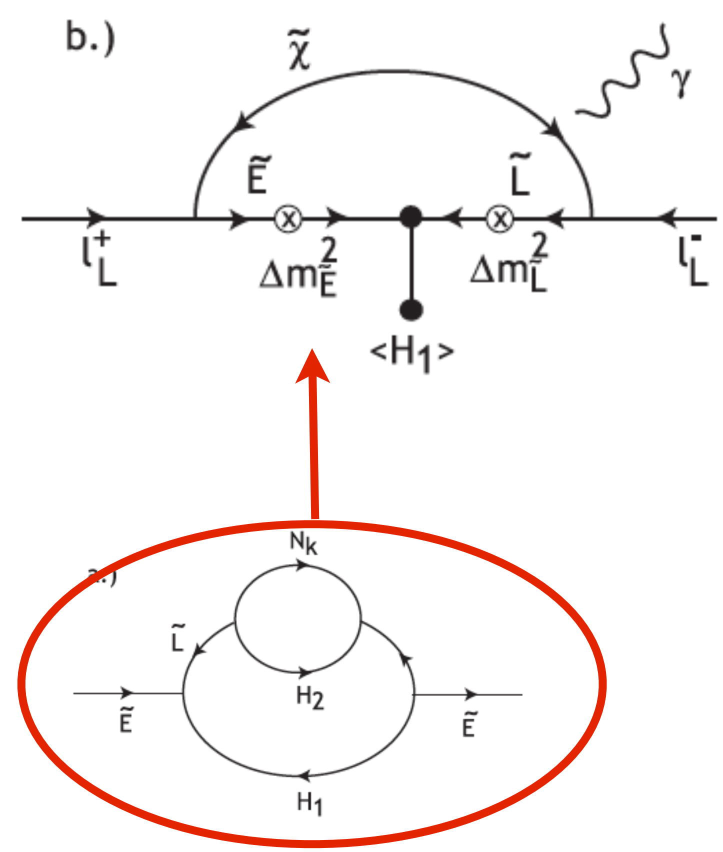
- Correlations in general weaker than in Electroweak Baryogenesis

SUSY see-saw

- Asymmetries controlling baryogenesis:

- Contributions to EDMs:

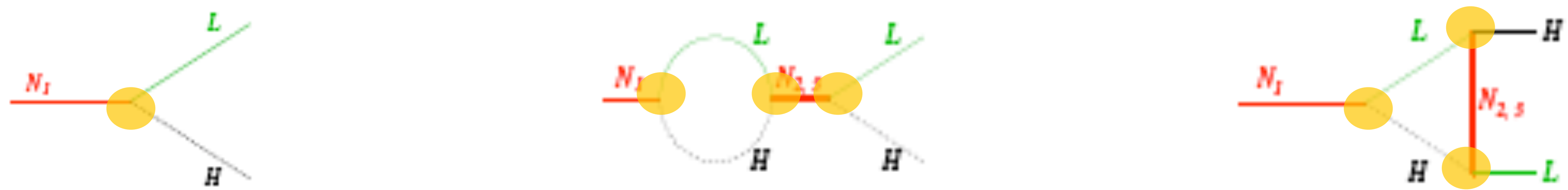




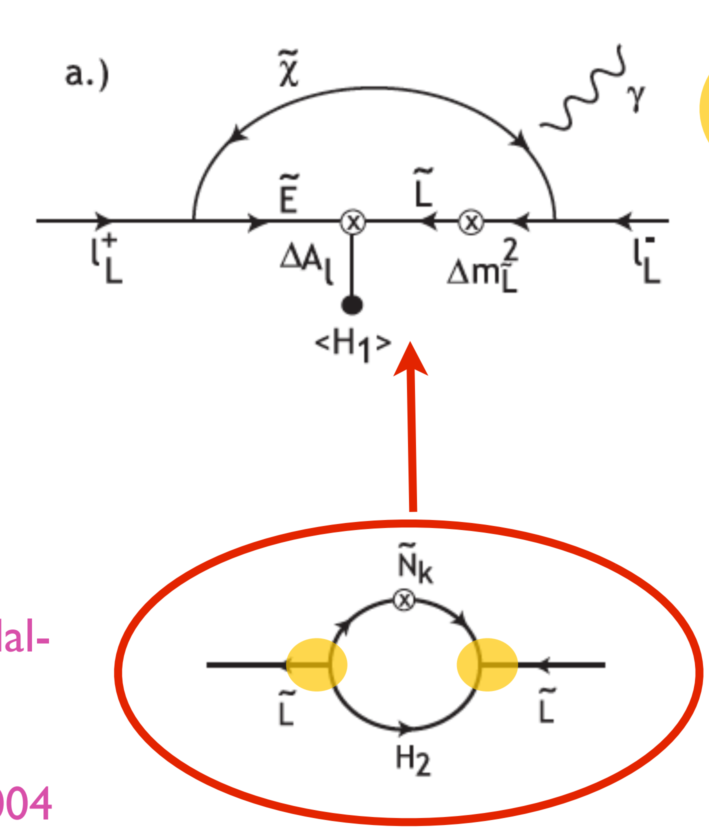
Ellis-Hisano-Raidal-Shimizu 2002
Masina 2003
Farzan-Peskin 2004

SUSY see-saw

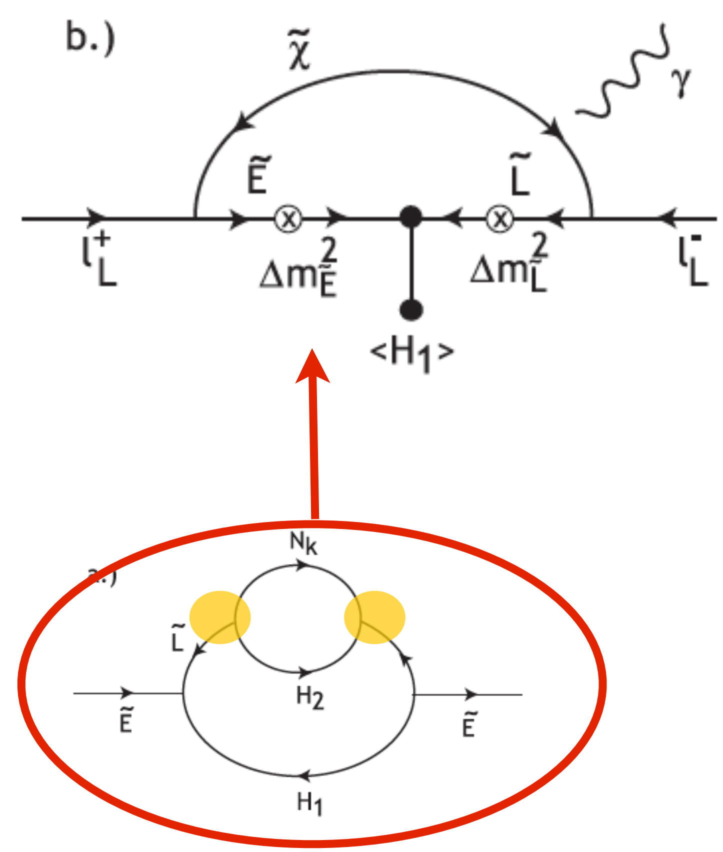
- Asymmetries controlling baryogenesis:



- Contributions to EDMs:



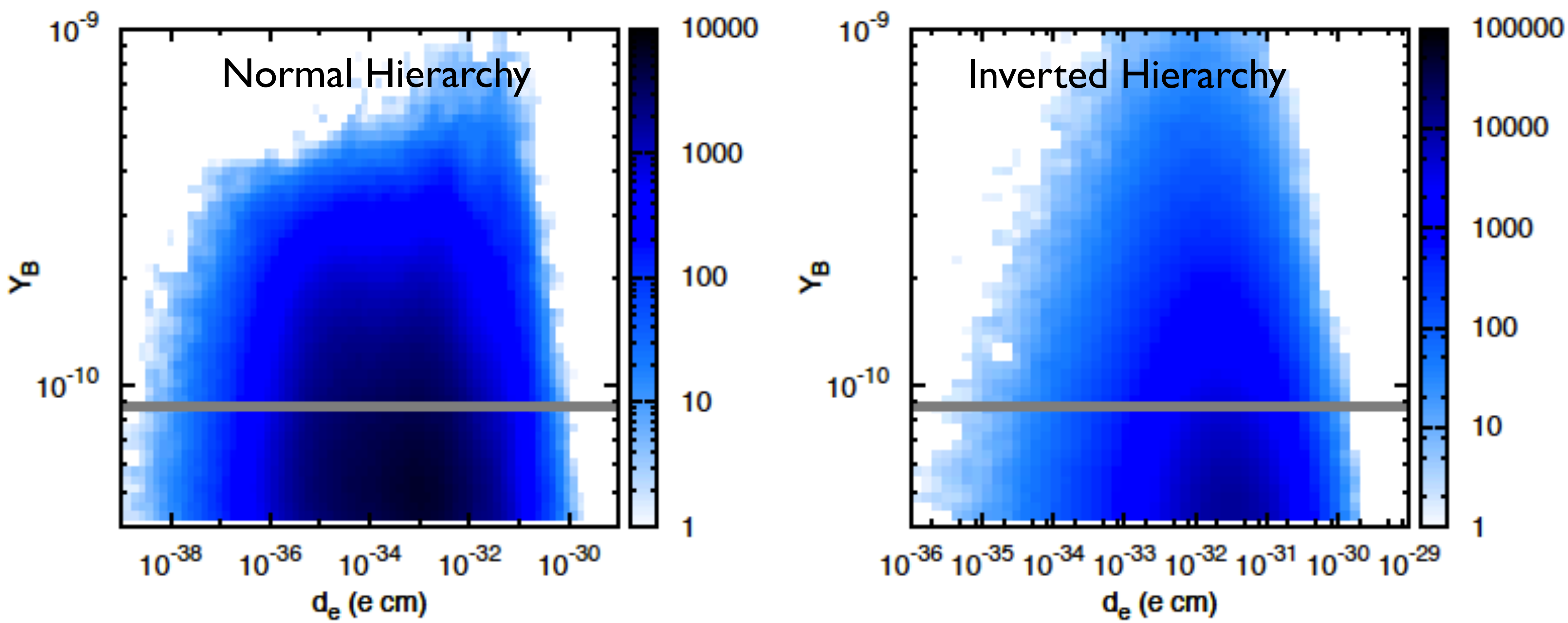
λ_ν : neutrino Yukawa couplings



Ellis-Hisano-Raidal-Shimizu 2002
Masina 2003
Farzan-Peskin 2004

- λ_ν cannot be reconstructed in terms of measured low-energy parameters. Numerical scan over see-saw parameters shows no correlation between BAU and CPV phases of U_{PMNS}
- No correlation between BAU and charged lepton EDMs
- Typically $d_e < 10^{-30}$ e cm

Davidson-Garayoa-Palorini-Rius 2008
....



Conclusions

- Baryogenesis mechanisms are intimately tied to BSM physics
- Some (but not all!) mechanisms leave imprint on TeV scale dynamics (EWB, leptogenesis, ...) and can be probed through a combination of energy and intensity frontier searches
- EDMs play a key role in probing new sources of CP violation that may be relevant to explain the BAU. However, no "theorem" links

  Phases responsible for BAU $\leftrightarrow$ phases responsible for EDMs

  In this talk I provided examples illustrating this point.
- EDM searches are part of a larger effort to understand whether TeV-scale physics had anything to do with generating the BAU

Backup

SUSY see-saw

- Asymmetries controlling baryogenesis

$$\epsilon_{\alpha\alpha} \simeq \frac{3M_1}{8\pi v_u^2 \, [\lambda^\dagger \lambda]_{11}} \mathrm{Im} \left\{ [\lambda]_{\alpha 1} [m_\nu^\dagger \lambda]_{\alpha 1} \right\}$$

- Contributions to EDMs

Ellis-Hisano-Raidal-Shimizu 2002
Masina 2003
Farzan-Peskin 2004
Joaquim-Masina-Riotto 2007

$$d_e \sim \frac{4\alpha}{(4\pi)^5} \frac{m_e^2}{m_{SUSY}^2} \mathrm{Im}[H \ C]_{ee} (1.9 \ 10^{-11} \text{ e cm})$$

$$H \equiv \lambda \lambda^\dagger$$

$$d_e \simeq \frac{-e}{2} \frac{8\alpha}{(4\pi)^7} \frac{10 m_e \tan\beta}{m_{SUSY}^2} \frac{\mathrm{Im}[\lambda_{ek}^* \lambda_{\alpha k} m_{\ell_\alpha}^2 \lambda_{\alpha m}^* \lambda_{em}]}{v^2 \cos^2 \beta} F(M_k^2)$$

$$C^{(n)} \equiv \lambda \log^n \left(\frac{M M^\dagger}{M_X^2} \right) \lambda^\dagger$$