Problems of charge compensation in a ring e+e- higgs factory

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Introduction

Observation of the Higgs(126) have triggered proposals of e+e- ring Higgs factories on 2E=240 GeV (A.Blondel and F.Zimmermann, arXiv:1112.2518) and 2E=240-500 GeV (K.Oide, Super-Tristan, Feb.2012) and then many others.

There were hopes that using a crab-waist scheme (as was proposed for Super B factory) the luminosity of the ring e+e- collider could be higher than at linear colliders by a factor 20 at 2E=240 GeV and similar at 2E=500 GeV.

However, it turned out that the luminosity of high energy e+e- storage rings is limited by beamstrahlung (radiation in the field of the opposing beam), V.Telnov, arXiv:1203.6563, PRL 110,114801 (2013). At high energy storage rings emission of single high energy photons in the tail of the beamstrahlung spectra determines the beam lifetime, this put the limitation on beam parameters ($N/\sigma_x\sigma_z$) and thus on luminosity.

Beam lifetime due to beamstrahlung

The electron loses the beam after emission of beamstrahlung photon with an energy greater than the threshold energy E_{th}=ηE_0, where a *ring energy acceptance* η~0.01.

These photons have energies mach larger than the critical energy

$$E_c = \hbar\omega_c = \hbar\frac{3\gamma^3 c}{2\rho};$$

The spectrum per unit length at $u = E_\gamma / E_c >> 1$

$$\frac{dn}{dx} = \sqrt{\frac{3\pi}{2}}\frac{\alpha\gamma}{2\pi\rho}\frac{e^{-u}}{\sqrt{u}}du, \qquad \alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

The number of photons on collision length *l* with $E_\gamma > \eta E_0$

$$n_\gamma(E_\gamma \geq \eta E_0) \approx \frac{\alpha^2 \eta l}{\sqrt{6\pi} r_e \gamma u^{3/2}} e^{-u}; \quad u = \frac{\eta E_0}{E_c},$$

$l \approx \sigma_z / 2$ for head-on and $l \approx \beta_y / 2$ for crab-waist collisions

The corresponding beam lifetime depends exponentially on the critical energy (which is prop. to the beam field).

Using these simple formulas, one can estimate the critical energy of beamstrahlung photons (for the maximum beam field) corresponding to a beam lifetime of ~30 minutes:

$$u = \eta E_0 / E_{\mathrm{c}} \approx 8.5; \quad E_{\mathrm{c}} \approx 0.12 \eta E_0 \sim 0.1 \eta E_0.$$

This estimate is done for typical collider parameters (R, E_0, σ_z) , but the accuracy of this expression is quite good for any ring collider, because it depends logarithmically on these parameters (as well as on the lifetime).

The critical energy is related to the beam parameters as follows:

$$\frac{E_{\mathrm{c}}}{E_0} = \frac{3\gamma {r_e}^2 N}{\alpha \sigma_x \sigma_z}. \qquad \text{where } r_e = e^2/mc^2.$$

This imposes a new restriction on the beam parameters

$$\boxed{\frac{N}{\sigma_x \sigma_z} < 0.1 \eta \frac{\alpha}{3\gamma {r_e}^2}} \qquad *$$

This additional constraint on beam parameters should be taken into account in luminosity optimization.

Head-on and "crab-waist" collision schemes

Below we consider two collision schemes: head-on and crab-waist.
In the crab-waist scheme the beams collide at an angle $\theta >> \sigma_x / \sigma_z$
This scheme allows a higher luminosity, if it is determined by the tune shift (beam-beam strength parameter characterizing instabilities).
For head-on collisions the tune shift ($\xi_y \leq 0.1 - 0.15$) and the luminosity

(1) $$\xi_y = \frac{N r_e \beta_y}{2\pi\gamma\sigma_x\sigma_y} \approx \frac{N r_e \sigma_z}{2\pi\gamma\sigma_x\sigma_y} \text{ for } \beta_y \approx \sigma_z \quad \mathcal{L} \approx \frac{N^2 f}{4\pi\sigma_x\sigma_y} \approx \frac{N f \gamma \xi_y}{2 r_e \sigma_z}$$

For the crab-waist scheme

(2) $$\xi_y = \frac{N r_e \beta_y^2}{\pi\gamma\sigma_x\sigma_y\sigma_z} \text{ for } \beta_y \approx \sigma_x/\theta \quad \mathcal{L} \approx \frac{N^2 f}{2\pi\sigma_y\sigma_z\theta} \approx \frac{N^2 \beta_y f}{2\pi\sigma_x\sigma_y\sigma_z} \approx \frac{N f \gamma \xi_y}{2 r_e \beta_y}$$

In the crab-waist scheme one can make $\beta_y \sim \sigma_y/\theta << \sigma_z$, therefore the luminosity is higher. Nf is determined by SR power. The only free parameters in L are σ_z (for head-on) and β_y (crab-waist), they are constrained by beamstrahlung condition

(3) $$\boxed{\frac{N}{\sigma_x\sigma_z} < 0.1\eta\frac{\alpha}{3\gamma r_e{}^2}}$$

Comparing (1),(2),(3) one can find the minimum beam energy when beamstrahlung becomes important.

For head-on collisions

$$\gamma_{\min} = \left(\frac{0.1\eta\alpha\sigma_z^2}{6\pi r_e \xi_y \sigma_y} \right)^{1/2} \propto \frac{\sigma_z^{3/4}}{\xi_y^{1/2} \varepsilon_y^{1/4}}$$

For "crab-waist" collisions

$$\gamma_{\min} = \left(\frac{0.1\eta\alpha\beta_y^2}{3\pi r_e \xi_y \sigma_y} \right)^{1/2} \propto \frac{2^{1/2} \beta_y^{3/4}}{\xi_y^{1/2} \varepsilon_y^{1/4}}$$

In the crab-waist scheme the beamstrahlung becomes important at much low energies because $\beta_y << \sigma_z$. For typical values of parameters E_{min}>70 GeV for head-on collisions and E_{min}>20 GeV for "crab-waist".

For considered colliders with $2E_0$>240 GeV beamstrahlung is important in both schemes.

Luminosities with account of beamstrahlung

For head-on collisions

$$\mathcal{L} \approx \frac{(Nf)N}{4\pi\sigma_x\sigma_y}, \; \xi_y \approx \frac{N r_e \sigma_z}{2\pi\gamma\sigma_x\sigma_y}, \; \frac{N}{\sigma_x\sigma_z} \equiv k \approx 0.1\eta \frac{\alpha}{3\gamma {r_e}^2} \qquad \sigma_y \approx \sqrt{\varepsilon_y\sigma_z}$$

Together these equations give

$$\mathcal{L} \approx \frac{Nf}{4\pi}\left(\frac{0.1\eta\alpha}{3}\right)^{2/3}\left(\frac{2\pi\xi_y}{\gamma r_e^5\varepsilon_y}\right)^{1/3}$$

$$\sigma_{z,\mathrm{opt}} = \varepsilon_y^{1/3}\left(\frac{6\pi\gamma^2 r_e\xi_y}{0.1\eta\alpha}\right)^{2/3}$$

Luminosities with account of beamstrahlung

Similarly for the crab-waist collisions

$$\mathcal{L} \approx \frac{(Nf)N\beta_y}{2\pi\sigma_x\sigma_y\sigma_z}, \; \xi_y \approx \frac{Nr_e\beta_y^2}{\pi\gamma\sigma_x\sigma_y\sigma_z}, \; \frac{N}{\sigma_x\sigma_z} \equiv k \approx 0.1\eta\frac{\alpha}{3\gamma {r_e}^2} \qquad \sigma_y \approx \sqrt{\varepsilon_y\beta_y}$$

The corresponding solutions are

$$\mathcal{L} \approx \frac{Nf}{4\pi}\left(\frac{0.2\eta\alpha}{3}\right)^{2/3}\left(\frac{2\pi\xi_y}{\gamma r_e^5\varepsilon_y}\right)^{1/3}$$

$$\beta_{y,\mathrm{opt}} = \varepsilon_y^{1/3}\left(\frac{3\pi\gamma^2 r_e\xi_y}{0.1\eta\alpha}\right)^{2/3}$$

In the beamstrahlung dominated regime the luminosities in crab-waist and head-on collisions are practically the same! (difference $2^{2/3}$~1)

As soon as the crab-waist gives no profit at high energies, further we will consider only the head-on scheme.

The maximum luminosity with account of beamstrahlung

$$\mathcal{L} \approx h\frac{N^2 f}{4\pi\sigma_x\sigma_y} = h\frac{Nf}{4\pi}\left(\frac{0.1\eta\alpha}{3}\right)^{2/3}\left(\frac{2\pi\xi_y}{\gamma r_e^5\varepsilon_y}\right)^{1/3}$$

where h is the hourglass loss factor, $f=n_b c/2\pi R$.

The SR power in rings $$P = 2\delta E\frac{cNn_{\mathrm{b}}}{2\pi R} = \frac{4e^2\gamma^4 cNn_{\mathrm{b}}}{3RR_{\mathrm{b}}}$$

Finally, the luminosity

$$\mathcal{L} \approx h\frac{(0.1\eta\alpha)^{2/3}PR}{32\pi^2\gamma^{13/3}r_e^3}\left(\frac{R_{\mathrm{b}}}{R}\right)\left(\frac{6\pi\xi_y r_e}{\varepsilon_y}\right)^{1/3}$$

In practical units

$$\frac{\mathcal{L}}{10^{34}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}} \approx \frac{100h\eta^{2/3}\xi_y^{1/3}}{(E_0/100\,\mathrm{GeV})^{13/3}(\varepsilon_y/\,\mathrm{nm})^{\frac{1}{3}}}\left(\frac{P}{100\,\mathrm{MW}}\right)\left(\frac{2\pi R}{100\,\mathrm{km}}\right)\frac{R_{\mathrm{b}}}{R}$$

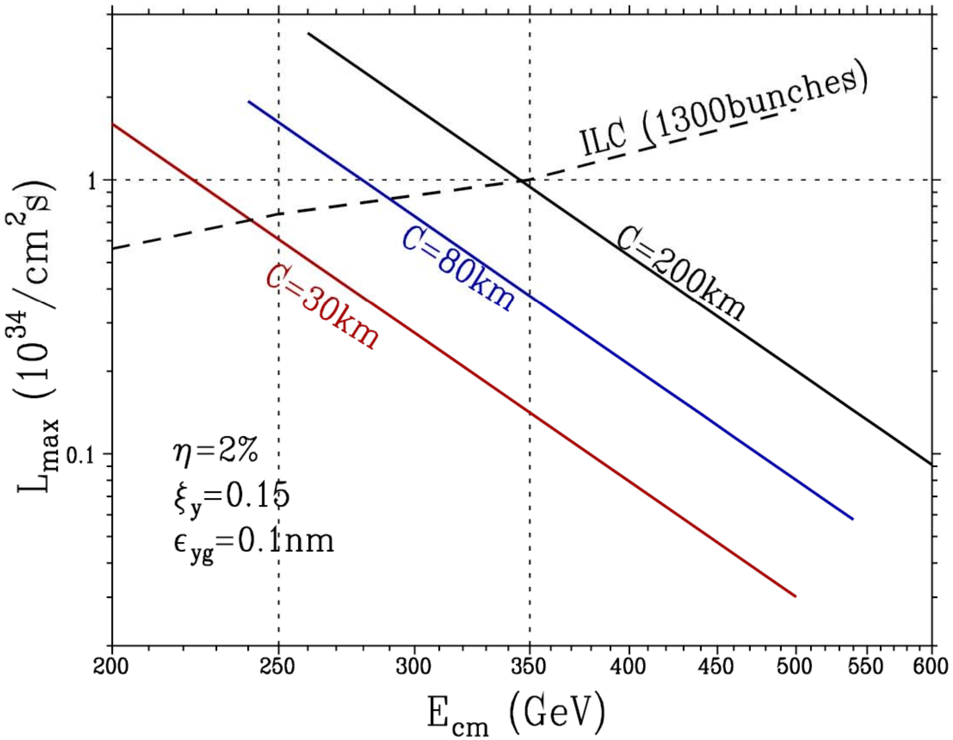
The beamstrahlung suppresses the luminosity by a factor $\sigma_z/\sigma_{opt}=(E_{min}/E_0)^{4/3}$ for the energies above E_{min}, which is about 70 GeV for head-on and 20 GeV for crab-waist schemes.

Luminosity vs. Energy

example with

- η=2%
- ξ_y=0.15
- ε_{gy}=0.1nm

P_{SR}=100 MW



For 2E=240 GeV the luminosities (per one IP) of ring and linear colliders are compa-rable. But large ring colliders with 4 IP can provide by one order higher luminosity.

Charge compensation: history

The idea to collide 4 beams (e+e- with e+e-) is more than 40 years old. Beams are neutral, there are no collision effects, sound nice.

Such 4-beam e+e- collider on the energy 2E~2 GeV, DCI, was build in 1970th in Orsay. There were hopes to increase the luminosity by a factor of 100 compared to the normal 2-beam e+e- case. But the result was confusing: the maximum luminosity was approximately the same. The reason - instability of neutral e+e- beams: small displacement of charges leads to the charge separation in opposing beam and thus to development of instability and the loss of the beam neutrality, appearance of tune shifts and corresponding resonances. The attainable beam-beam parameter ξ was approximately the same as without neutralization.

Due to these reasons the idea of charge neutralization in e+e- collisions was dismissed.

Charge compensation at e+e- ring Higgs factories

The above exercises with 4-beam neutralization of beam collisions have shown that this method does not help to increase the beam-beam parameter ξ_y, however, it should work at ξ_y similar to those in 2-beam collisions.

We have such case at e+e- Higgs factory. Their luminosities are limited by beamstrahlung. Using 4-beam charge compensation one can, in principle, to suppress the beamstrahlung and increase the luminosity up to the limit determined by the parameters ξ_y .

For head-on collisions it give only a factor of 2-3 at 2E=240 GeV, no big sense.

However, in the crab-waist scheme the situation is much more attractive. Comparing luminosities for crab-waist colliders suggested by Oide with those corrected on beamstrahlung (tables I and II in V.T. paper) one can see possible profits

2E(GeV)	240	400	400	500
L_{nb}/L_b	16	33	43	25

Here nb-no beamstrahlung, b- with beamstrahlung

This already deserves a consideration!

Required degree of charge compensation

Let us find the charge compensation ``quality'' required to increase the luminosity by a factor of L_c/L. From above (V.Telnov's) equations

$$\mathscr{L} \propto k^{2/3} \xi_y^{1/3},$$

where $k \equiv N/(\sigma_x \sigma_z) \approx 0.1\eta\alpha/(3\gamma r_e{}^2)$. For the case of charge-compensated beams, the beamstrahlung condition should be rewritten as follows: $\Delta N_c/(\sigma_{x,c}\sigma_{z,c}) \equiv k$ or $N_c/(\sigma_{x,c}\sigma_{z,c}) \equiv k' = k(N_c/\Delta N_c)$. The expression for luminosity will be similar to Eq. 7.1, with k replaced by k':

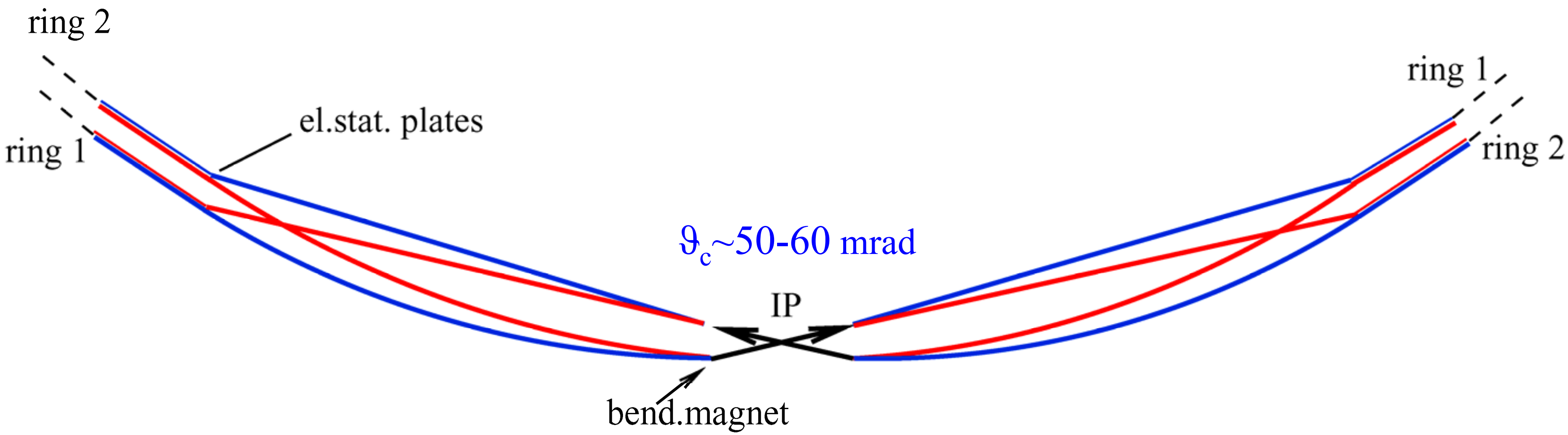
$$\mathscr{L}_c \propto \left(k \frac{N_c}{\Delta N_c}\right)^{2/3} \xi_{y,c}^{1/3}.$$

From these two equations, we get the required degree of charge compensation

$$\frac{\Delta N_c}{N_c} = \left(\frac{\mathscr{L}}{\mathscr{L}_c}\right)^{3/2} \left(\frac{\xi_c}{\xi}\right)^{1/2}.$$

For $\xi_c = \xi$, the increase of the luminosity by a factor of 10 one needs $\Delta N/N=0.03$.

Scheme of a charge compensated crab-waist e^+e^- ring collider

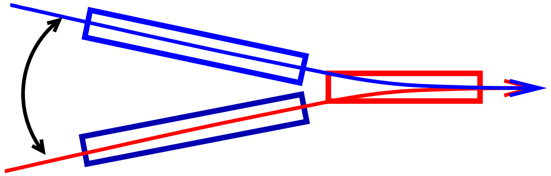


Crab-waist collisions assume collisions at some horizontal angle, that requires 2 rings with electrostatic separators near IP and inside rings, plus one rings for injection, 3 rings in total.

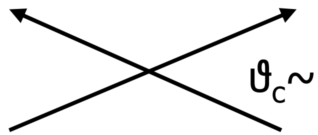
The main problem: combining (merging) e+e- beams at the IP

After the focusing e+ and e- beam (travelling from one direction) should perfectly overlap at the IP. This can be done using a bending magnet between final quads and the IP.

$\vartheta_m \sim 10$ mrad



The problem: SR radiation in the bending magnet leads to the energy spread which (due to beam collisions in the solenoid field at $\vartheta_c \approx 50\text{-}60$ mrad) causes the increase of the vertical beam size at the IP.



Remark: the beam energy spread upstream the final focus system is compensated by the chromatic correction system, but the energy arising after FF system is uncompensated and give contribution to σ_y .

The energy spread in the magnet with the length l and the bending angle ϑ

$$\frac{\sigma_E}{E} = \sqrt{\frac{55}{24\sqrt{3}\alpha}\frac{r_e}{l}\vartheta^{3/2}\gamma^{5/2}} = \left(\frac{\vartheta}{5\text{ mrad}}\right)^{3/2}\frac{(E/120\text{ GeV})^{5/2}}{l(\text{m})}$$

For l =1 m, E=120 GeV, $\vartheta=\vartheta_m/2=5$ mrad $\sigma_E/E=3.5\times10^{-4}$.

The increase of the vertical beam size $\Delta\sigma_y \approx \frac{L^2}{2R}\frac{\sigma_E}{E}$, $\mathrm{R} = \frac{\mathrm{E}}{\mathrm{eB_S}(\vartheta_c/2)}$

For L=300 cm, B_s=3T, ϑ_c=60 mrad $\Delta\sigma_y = 3.5\cdot10^{-5}$ cm

The expected σ_y In the crab-waist scheme is ~ 5×10^{-6} cm, i.e. << $\Delta\sigma_y$

One can see that synchrotron radiation in the merging bending magnet is a stopper for the charge compensation !

Beam combining without the bending magnet

If the merging angle ϑ_m is much smaller than the collision angle ϑ_c , then e+ and e- beams traveling in one direction overlap at the IP and therefore beamstrahlung is reduced. May be the combining bending magnet is not needed at all?

My simulation of the beam lifetime has shown that the effective beam field is reduced by a factor $F=(0.63\text{-}0.7)(\vartheta_c/\vartheta_m)$.
That means that the beamstrahlung condition changes as follows:

$$N/\sigma_x\sigma_z<0.1\eta\alpha/3\gamma r_e \rightarrow N/\sigma_x\sigma_z< F\bullet\, 0.1\eta\alpha/3\gamma r_e$$

This gives the luminosity

$$L_c=(0.73\text{-}0.79)\ (\vartheta_c/\vartheta_m)^{2/3}\ L.$$

For optimistic angles ϑ_c=60 mrad, ϑ_m=10 mrad $L_c/L=2.47$

ϑ_c=100 mrad, ϑ_m=10 mrad $L_c/L=3.5$

Not much…

Conclusion

❖ On first sight, the charge compensation in 4-beam collisions allows to suppress beamstrahlung and thus to increase the luminosity in the crab-waist scheme by one order of magnitude.

❖However, it looks impossible to produce charge compensated beam due to SR radiation in the combining bending magnet and following increase of the vertical beam size

❖In the scheme without combining magnet the luminosity can be increased only by a factor of ~2.5.

At present, I do not see any solutions of the problem.