Washington Post 1/25/2009

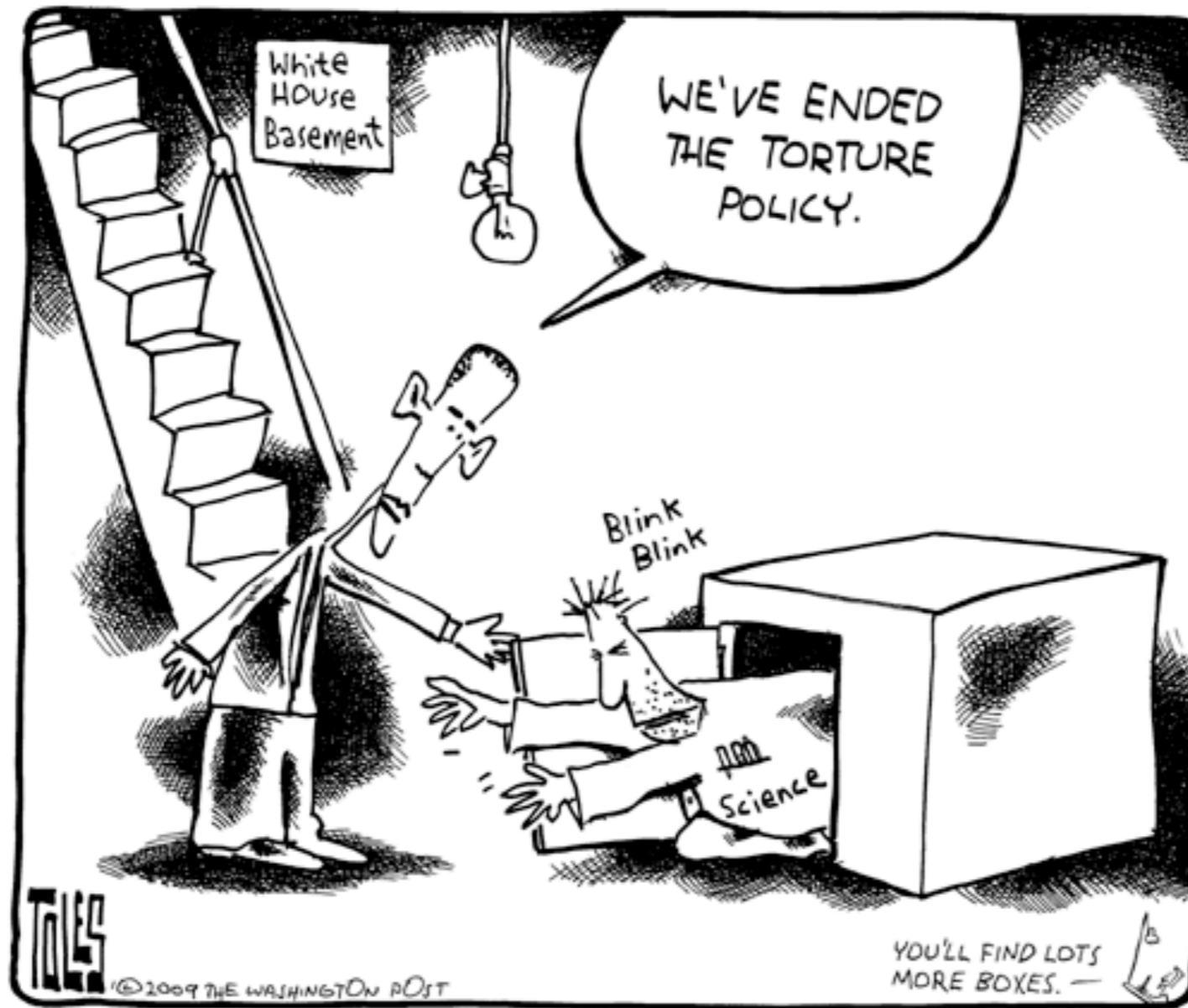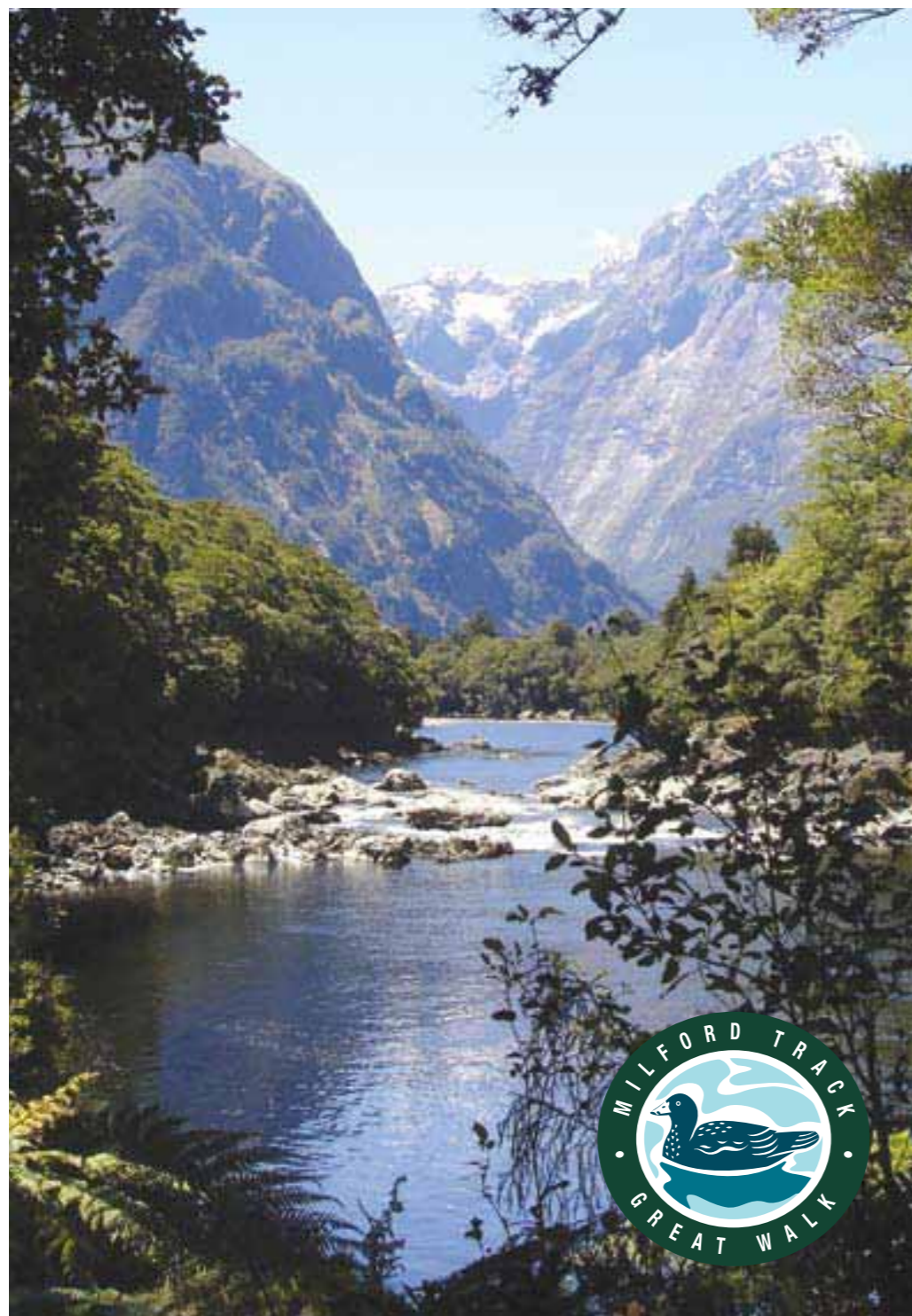http://www.washingtonpost.com/wp-dyn/content/opinions/tomtoles/?name=Toles&date=01252009

Mixing Parameters II

Stephen Parke, Fermilab
Fermilab Nu Summer School 2009

$$\sin^2 \theta_{13} \text{ from LBL:}$$

$$\nu_\mu \longrightarrow \nu_e$$

and related processes:

In Matter:

$$P_{\mu \to e} \approx |\ \sqrt{P_{atm}}\, e^{-i(\Delta_{32} \pm \delta)} + \sqrt{P_{sol}}\ |^2$$

where $\sqrt{P_{atm}} = \sin\theta_{23} \sin 2\theta_{13} \, \dfrac{\sin(\Delta_{31} \mp aL)}{(\Delta_{31} \mp aL)} \, \Delta_{31}$

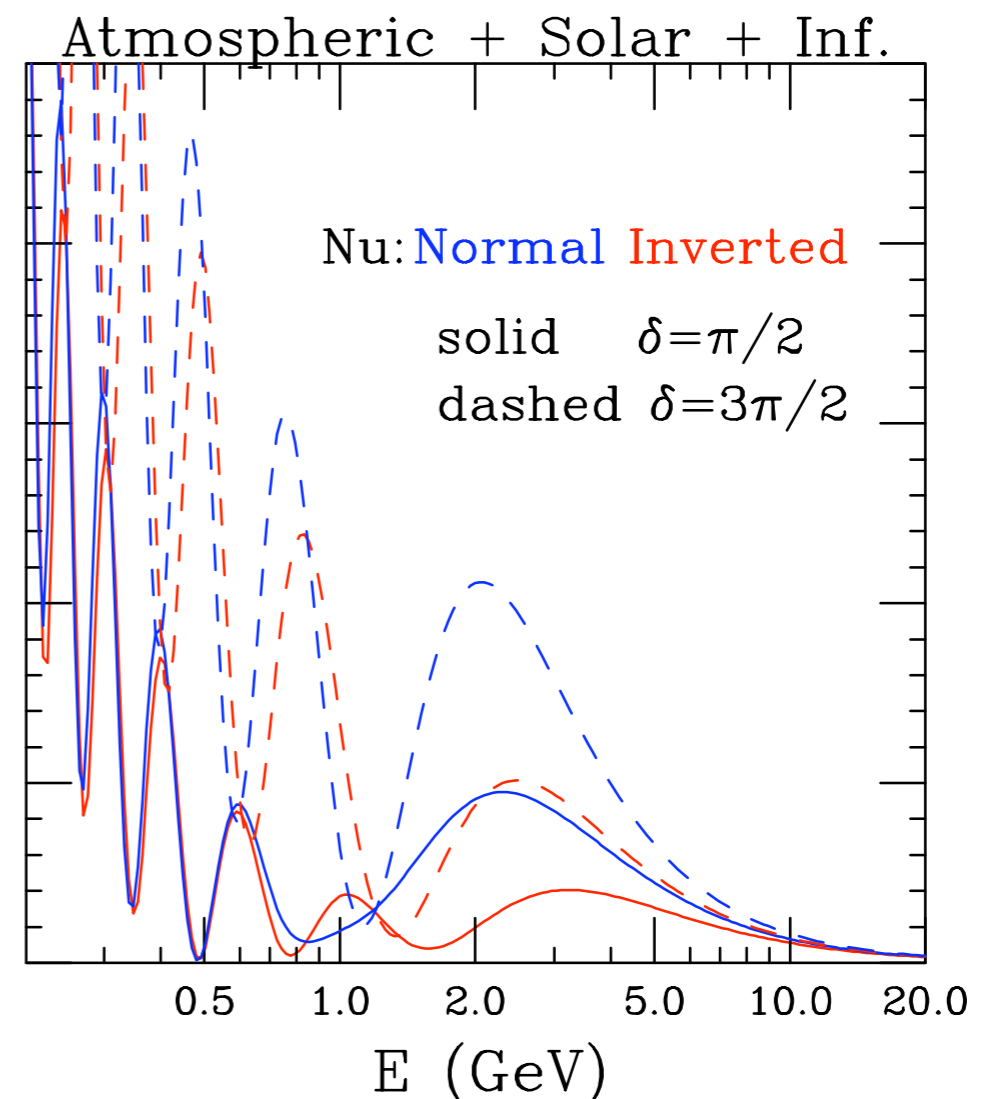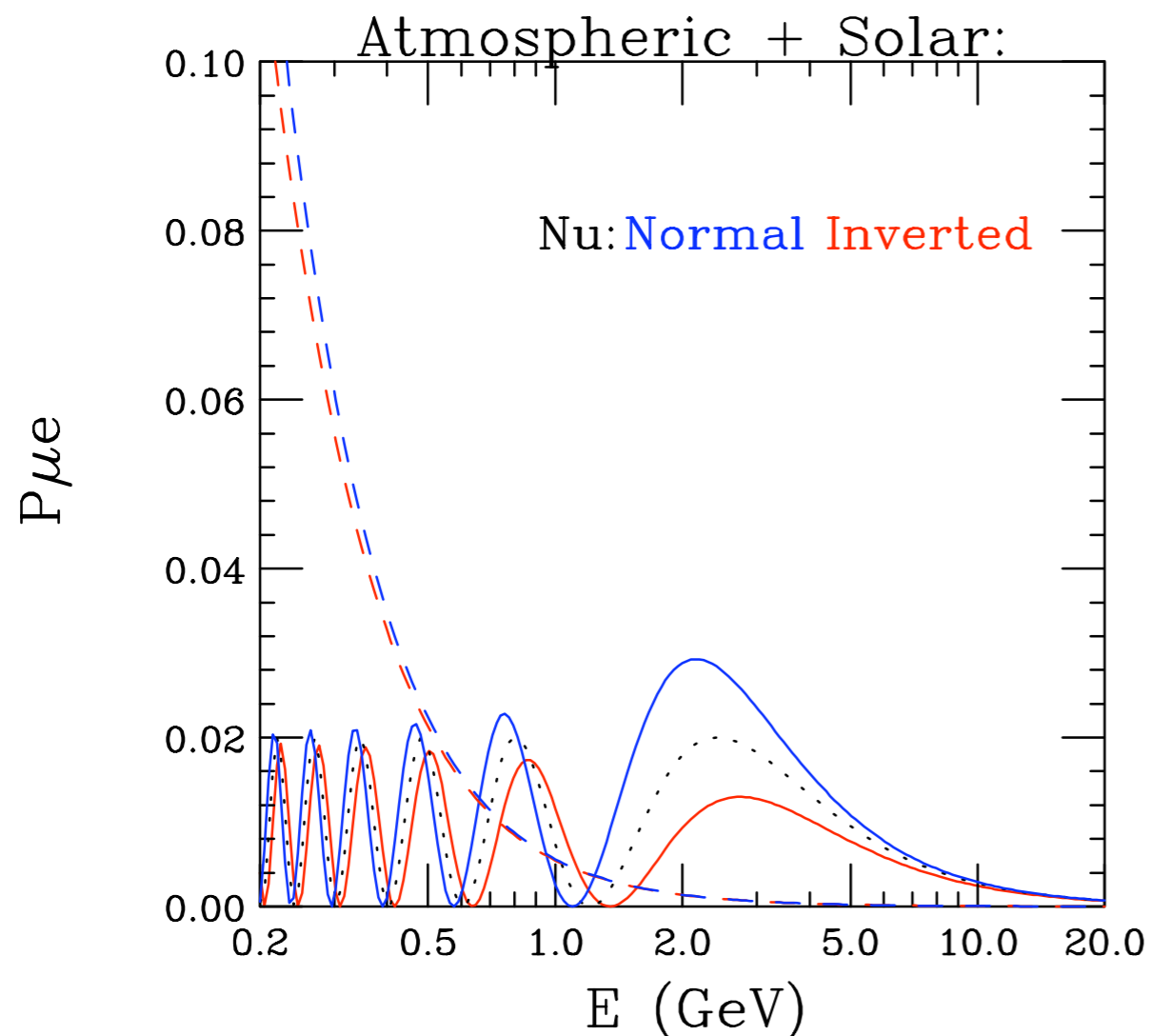and $\sqrt{P_{sol}} = \cos\theta_{23} \sin 2\theta_{12} \, \dfrac{\sin(aL)}{(aL)} \, \Delta_{21}$

For $L = 1200\ km$ and $\sin^2 2\theta_{13} = 0.04$

$$a = G_F N_e / \sqrt{2} = (4000\ km)^{-1},$$

Anti-Nu: Normal Inverted
dashes $\delta = \pi/2$
solid $\delta = 3\pi/2$
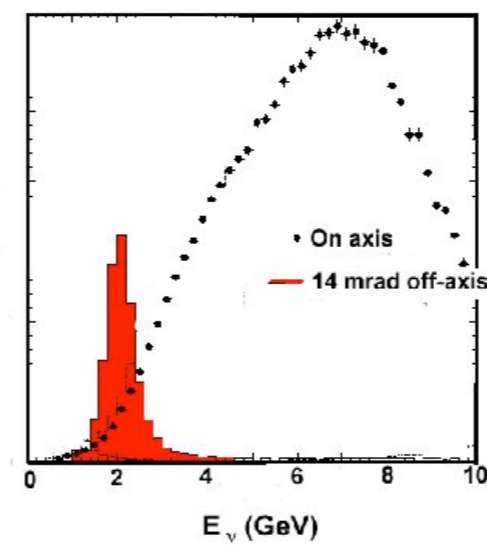

Atmospheric + Solar:

Nu: Normal Inverted

Atmospheric + Solar + Inf.

Nu: Normal Inverted

solid $\delta = \pi/2$
dashed $\delta = 3\pi/2$

$P\mu e$

E (GeV)

E (GeV)

Off-Axis Beams

BNL I

On axis

14 mrad off-axis

E_ν (GeV)

T2K

NOvA

$E_\nu = \dfrac{0.43\,\gamma\,m_\pi}{1+\gamma^2\theta^2}$

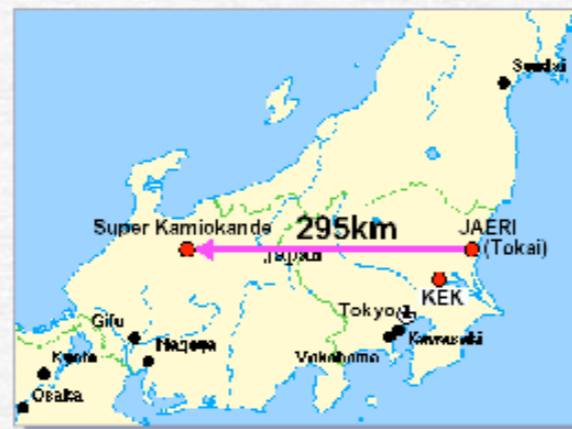## JHF → Super-Kamiokande

- 295 km baseline
- Super-Kamiokande:
  - 22.5 kton fiducial
  - Excellent e/μ ID
  - Additional π^0/e ID
- Hyper-Kamiokande
  - 20× fiducial mass of SuperK
- Matter effects small
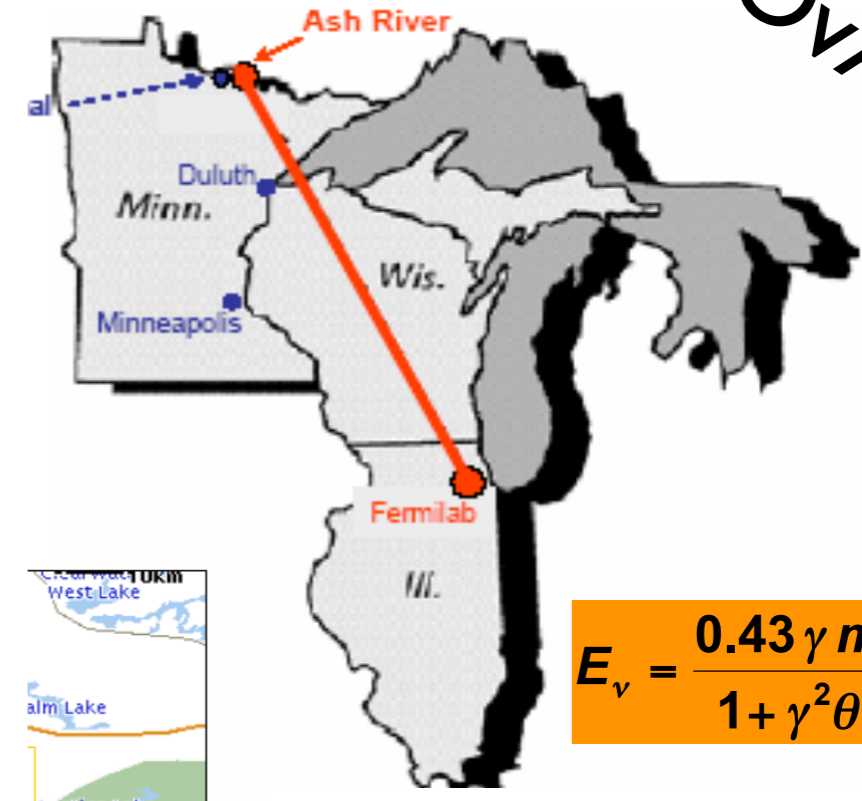- Study using fully simulated and reconstructed data

Super-Kamiokande   295km   JAERI (Tokai)

ν_μ CC events/kt/3.7E20 POT /0.2 GeV

L=295 km and

Energy at Vac. Osc. Max. (vom)

$E_{vom} = 0.6\ GeV\left\{\dfrac{\delta m^2_{32}}{2.5\times 10^{-3}\ eV^2}\right\}$
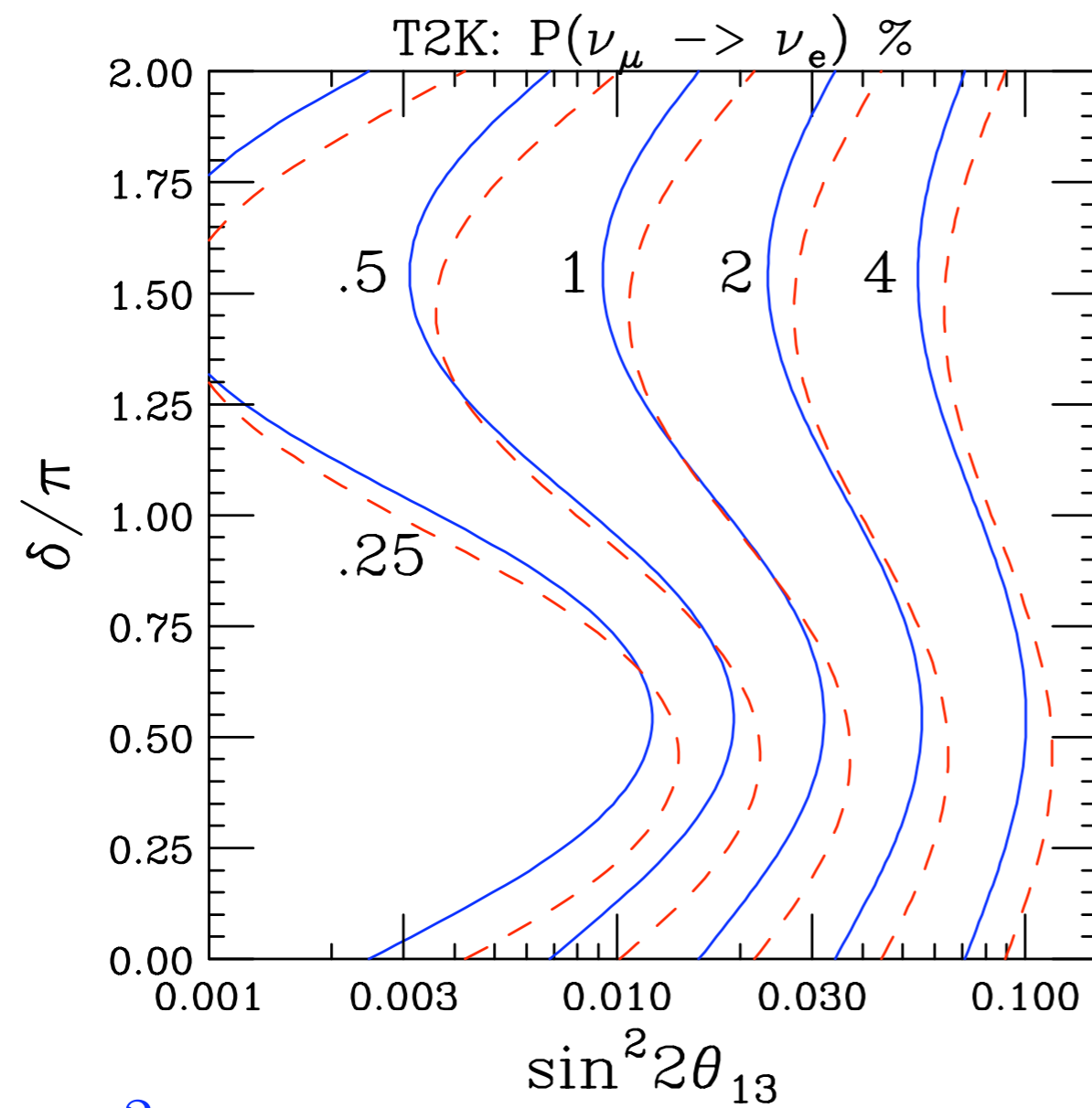
0.75 upgrade to 4 MW

L=700 – 1000 km and

Energy near 2 GeV

$E_{vom} = 1.8\ GeV\left\{\dfrac{\delta m^2_{32}}{2.5\times 10^{-3}\ eV^2}\right\}$

$\times\left\{\dfrac{L}{820\ km}\right\}$

0.4 upgrade to 2 MW

T2K:

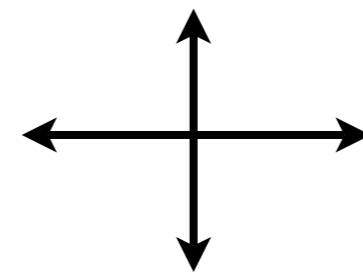T2K: $P(\nu_\mu \rightarrow \nu_e)$ %

.5   1   2   4

.25

δ/π

$\sin^2 2\theta_{13}$

VOM: $\Delta_{31} \neq \pi/2$

Matter Effect:

$\delta m^2_{31} > 0$

$\delta m^2_{31} < 0$

Beam 0.5%

T2K

Search for ν_e appearance

For LARGE δm_{31}^2

$$\langle P(\nu_\mu \to \nu_e)\rangle \quad = \quad \frac{1}{2}\sin^2\theta_{23}\sin^2 2\theta_{13} - \frac{1}{2}J.\Delta_{21} + \cos^2\theta_{23}\sin^2 2\theta_{12}\Delta_{12}^2$$

$$J = \sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta$$

$$\Delta_{21} = \delta m_{21}^2 L/4E$$

At $\delta = 0 \; or \; \pi$

90% C.L. sensitivity
(0.75 kW beam x 5yr)

×~20 improvement

$\sin^2 2\theta_{13} \sim 0.008$ ($\delta_{CP} = 0,\pi$)

$$\langle P(\nu_\mu \to \nu_e)\rangle \quad = \quad \frac{1}{2}\sin^2\theta_{23}\sin^2 2\theta_{13} + \cos^2\theta_{23}\sin^2 2\theta_{12}\Delta_{12}^2$$

$$\approx \quad 0.5\%$$

$$\langle P(\nu_\mu \to \nu_e)\rangle_{T2K} \approx 0.5\%$$

0.5% ν_e in beam

NOνA: $\delta m_{31}^2 > 0$ $\delta m_{31}^2 < 0$

NOνA: $P(\nu_\mu \rightarrow \nu_e)$ %

NOνA: $P(\overline{\nu}_\mu \rightarrow \overline{\nu}_e)$ %
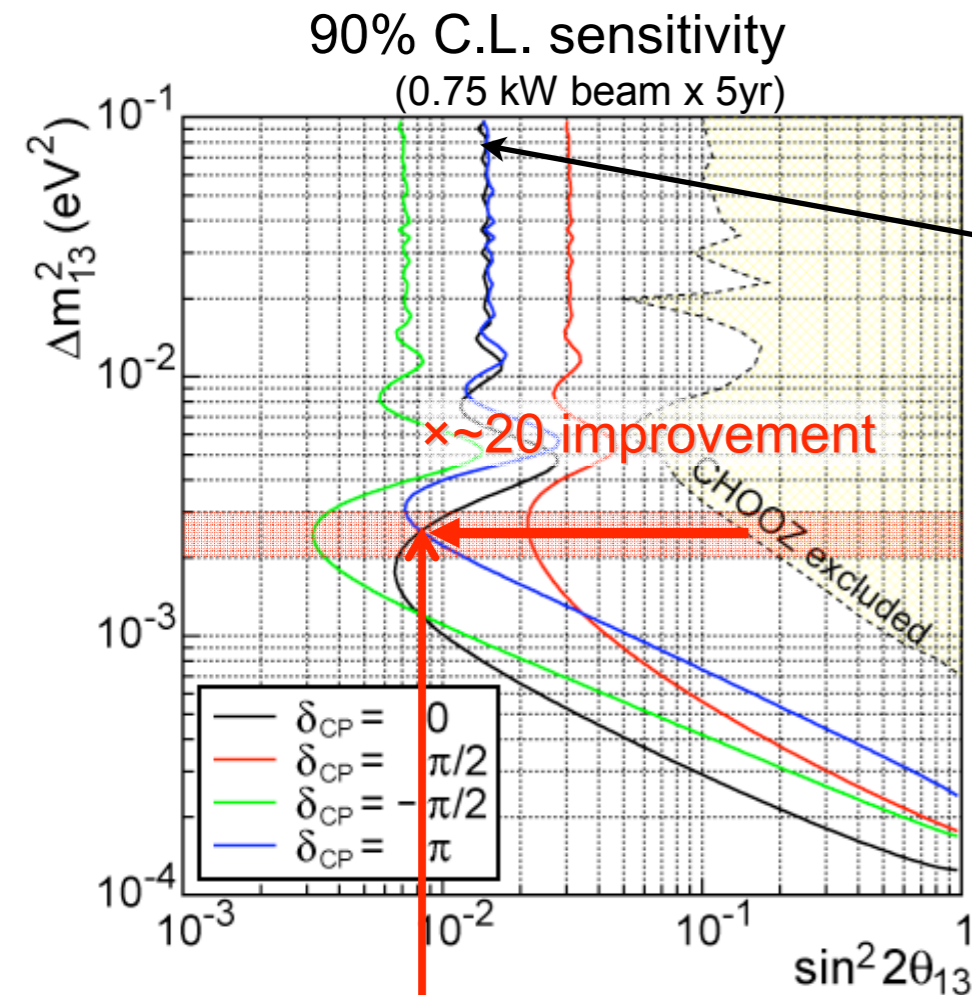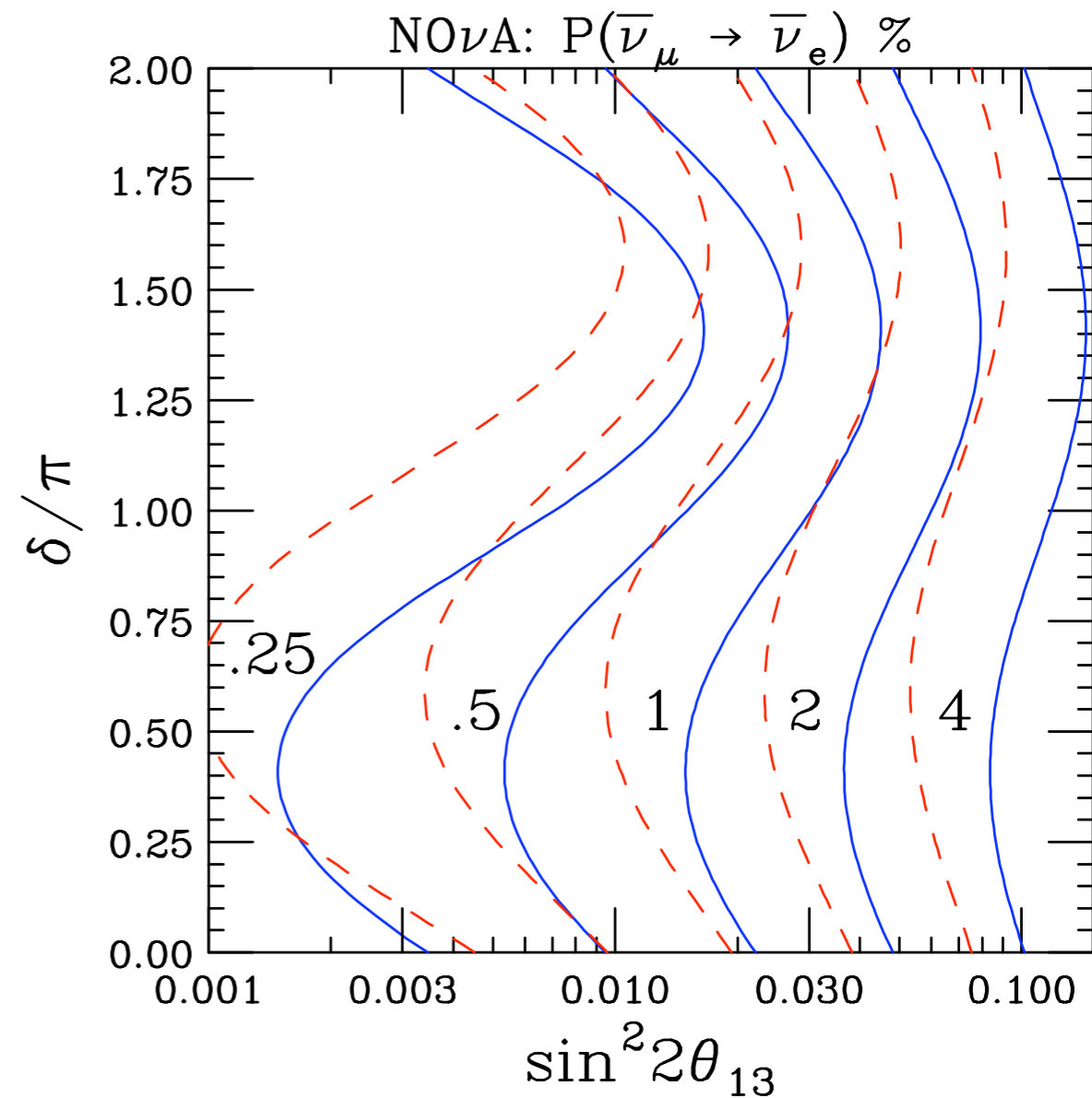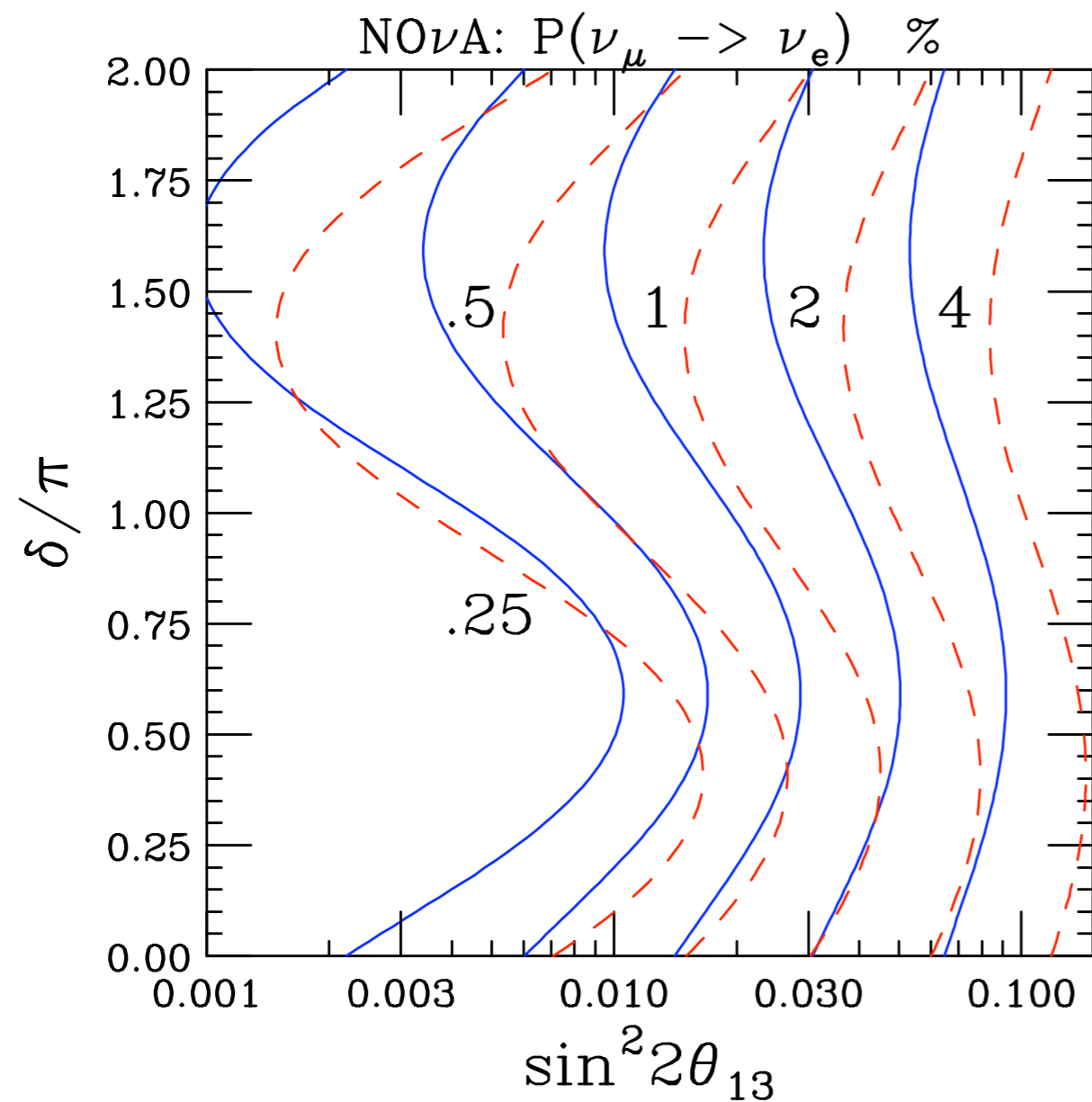
Beam 0.5-1%

Phase I
Sensitivity approx 0.5-1%

Correlations between Neutrinos and Antineutrinos:

Sutherland Falls
580m #5

$\langle E_\nu \rangle = 2.3 \ \text{GeV} \quad L = 820.\text{km}$

$\sim \sin \Delta_{32} \sin \delta$

vac

$\sim \cos \Delta_{32} \cos \delta$

$\langle P(\overline{\nu}_\mu \rightarrow \overline{\nu}_e) \rangle \ \%$

$\delta =$

✛	0
⊞	$\pi/2$
◇	π
✳	$3\pi/2$

$\sin^2 2\theta_{13} = 0.05$

$\langle P(\nu_\mu \rightarrow \nu_e) \rangle \ \%$

NOvA:

NOνA: E=2.3GeV and L=810km

$\delta m^2 < 0$

$\delta =$ +   0
  × $\pi/2$
  ◇ π
  ✳ $3\pi/2$

critical

$\delta m^2 > 0$

$\sin^2 2\theta_{13} = 0.05$

$\langle P(\overline{\nu}_\mu \rightarrow \overline{\nu}_e) \rangle$ %

$\langle P(\nu_\mu \rightarrow \nu_e) \rangle$ %
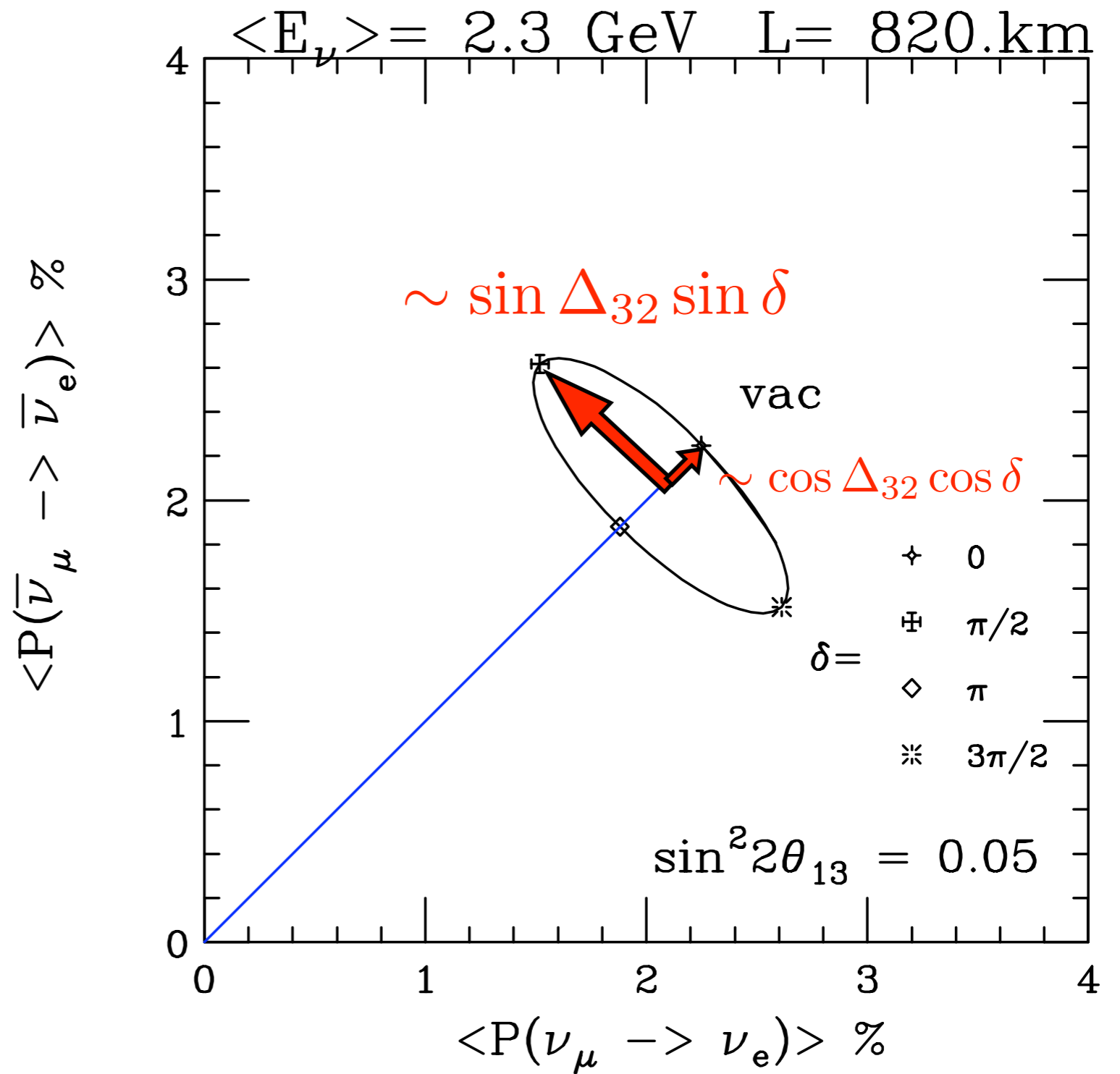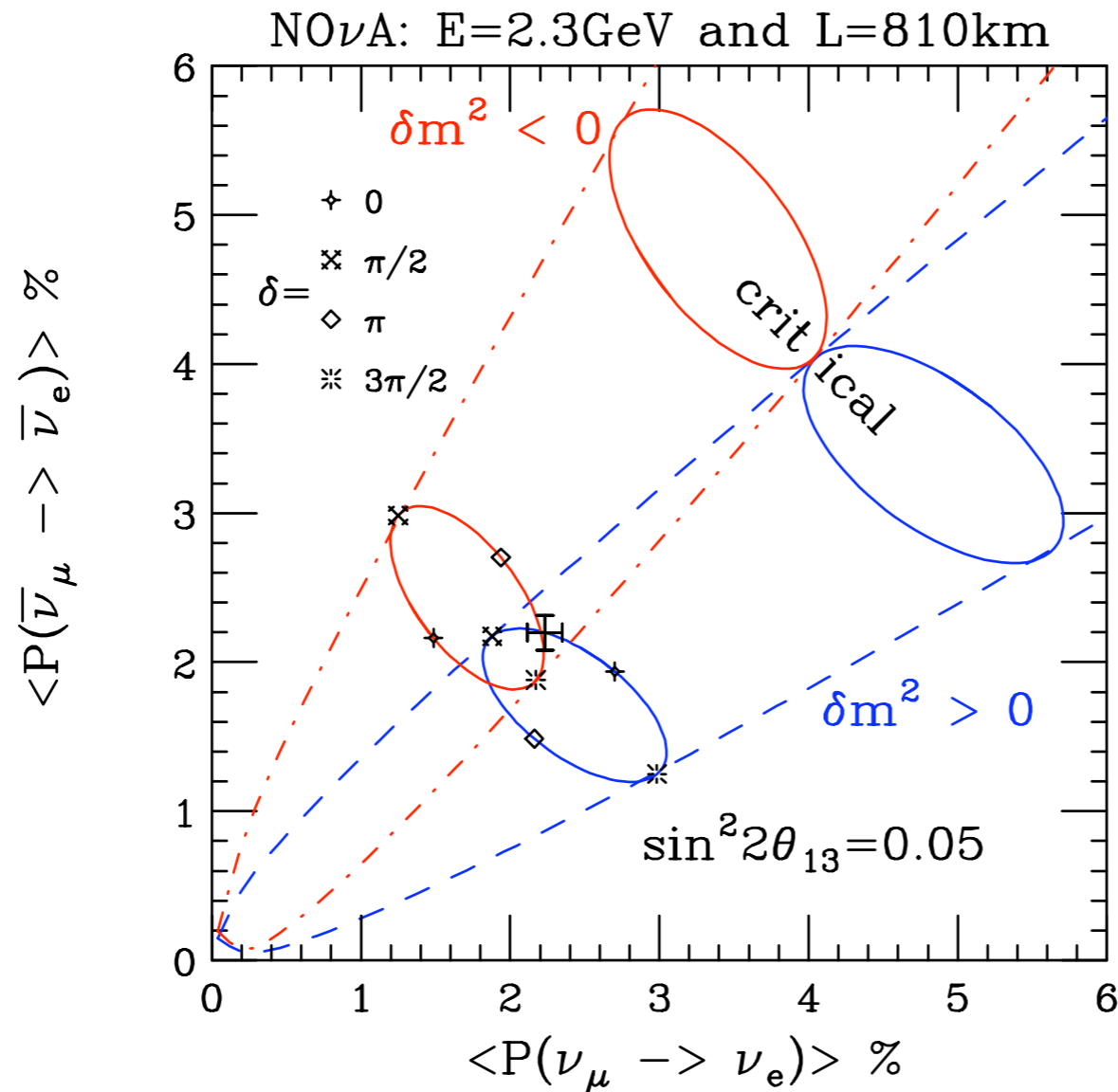
in the overlap region

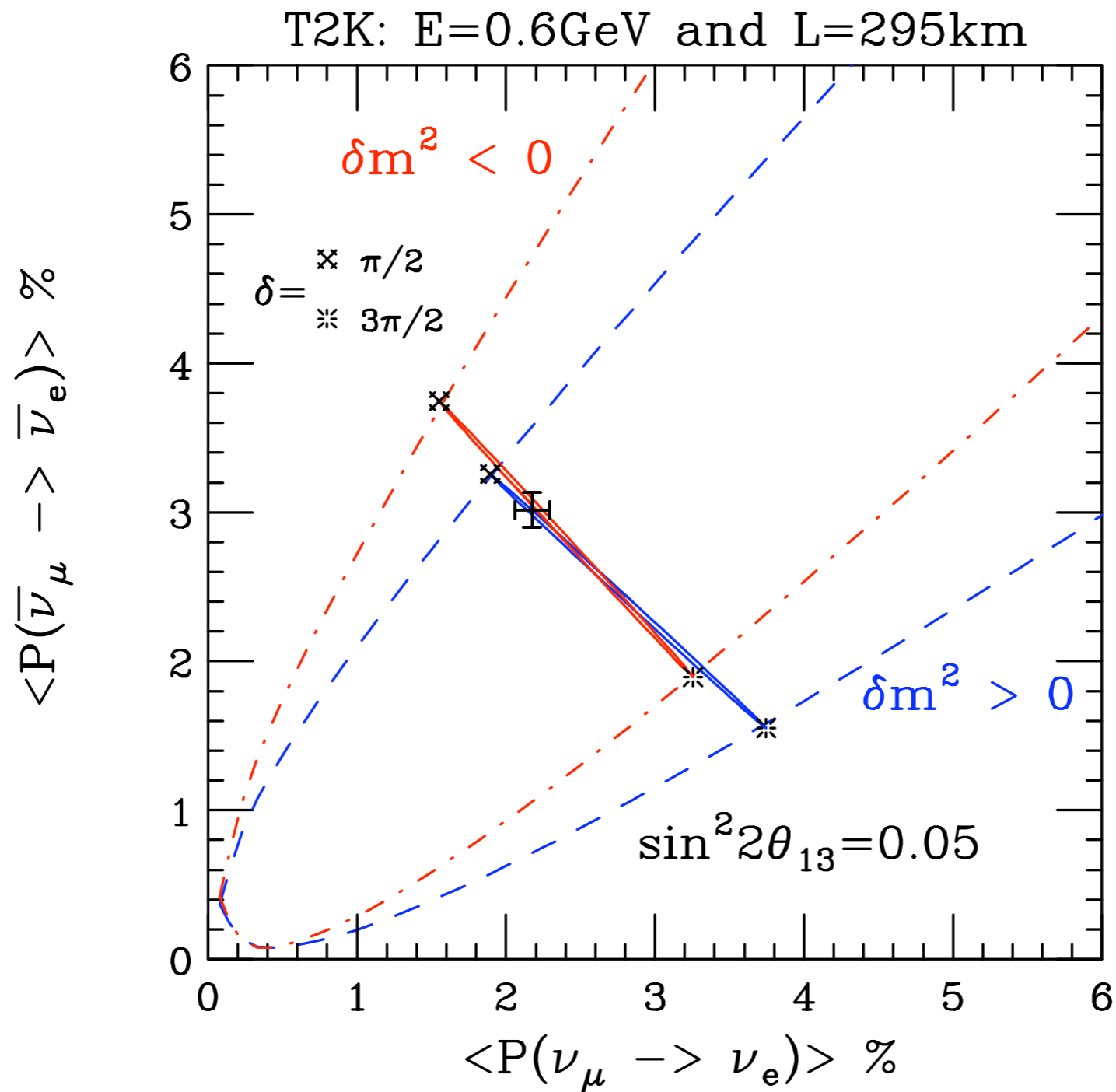$$\langle \sin \delta \rangle_+ - \langle \sin \delta \rangle_- = 2 \langle \theta \rangle / \theta_{crit} \approx 1.4 \sqrt{\frac{\sin^2 2\theta_{13}}{0.05}}$$

exact along diagonal --- approximately true throughout the overlap region!!!

$$\theta_{crit} = \frac{\pi^2}{8} \frac{\sin 2\theta_{12}}{\tan \theta_{23}} \frac{\delta m_{21}^2}{\delta m_{31}^2} \left(\frac{4\Delta^2/\pi^2}{1 - \Delta \cot \Delta} \right) /(aL) \sim 1/6$$
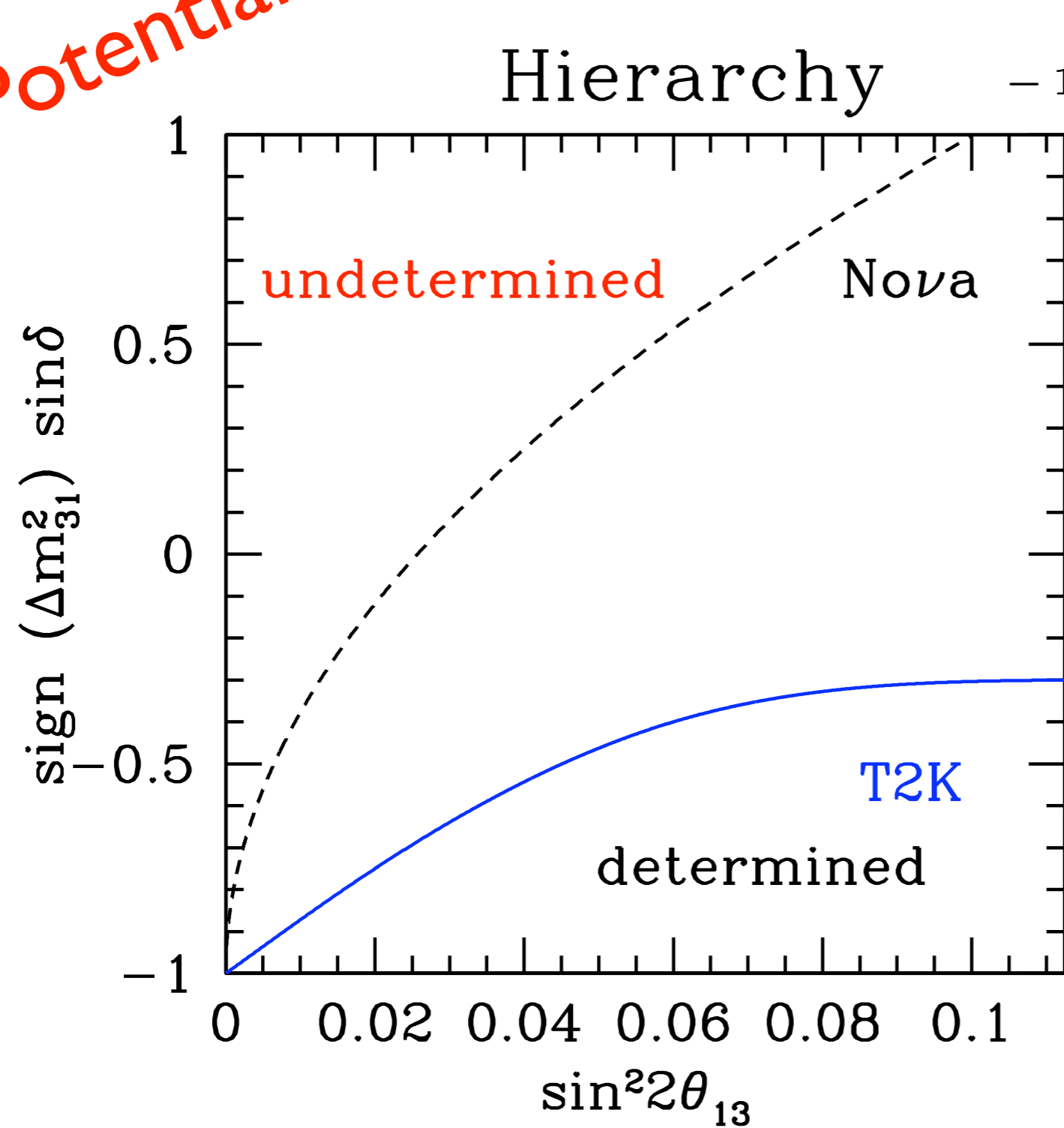
i.e. $\sin^2 2\theta_{crit} = 0.10$

T2K:

T2K: E=0.6GeV and L=295km

$\delta m^2 < 0$

$\delta = $ ✕ $\pi/2$ ✳ $3\pi/2$

$\delta m^2 > 0$

$\sin^2 2\theta_{13} = 0.05$

$\langle P(\overline{\nu}_\mu \to \overline{\nu}_e) \rangle$ %

$\langle P(\nu_\mu \to \nu_e) \rangle$ %

$$\langle \sin \delta \rangle_+ - \langle \sin \delta \rangle_- = 2\langle \theta \rangle / \theta_{crit} \approx 0.47 \sqrt{\frac{\sin^2 2\theta_{13}}{0.05}}$$

Potential

Hierarchy

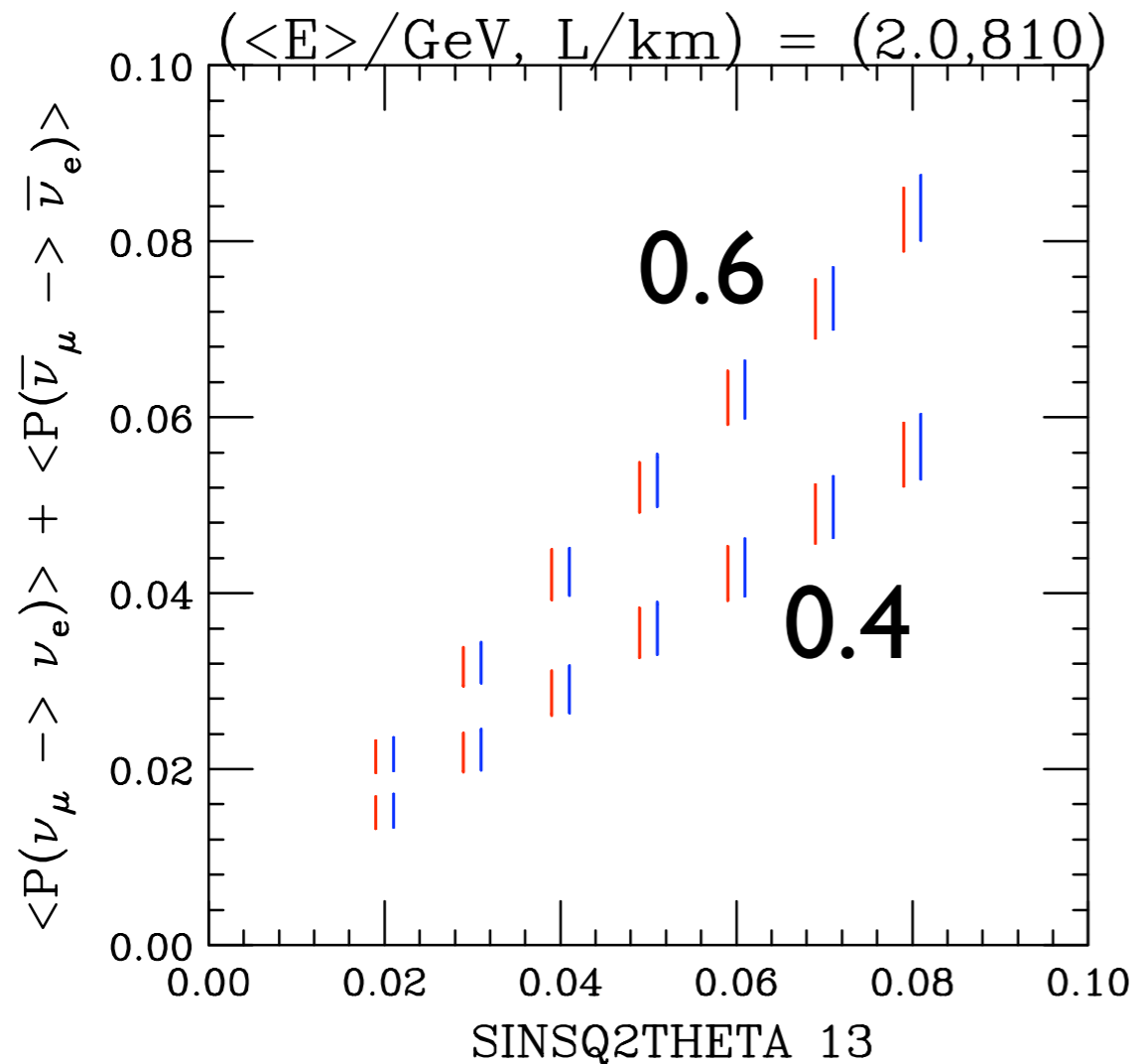$$-1 \;+\; 1.4 \sqrt{\frac{\sin^2 2\theta_{13}}{0.05}}$$

undetermined

Noνa

T2K

determined

$$-1 \;+\; 0.47 \sqrt{\frac{\sin^2 2\theta_{13}}{0.05}}$$

sign (Δm^2_{31}) sinδ

$\sin^2 2\theta_{13}$

At Vac. Osc. Max. $(\Delta_{31} = \frac{\pi}{2})$

$$P(\nu_\mu \to \nu_e) + P(\bar{\nu}_\mu \to \bar{\nu}_e) \approx 2 \sin^2 \theta_{23} \sin^2 2\theta_{13}$$

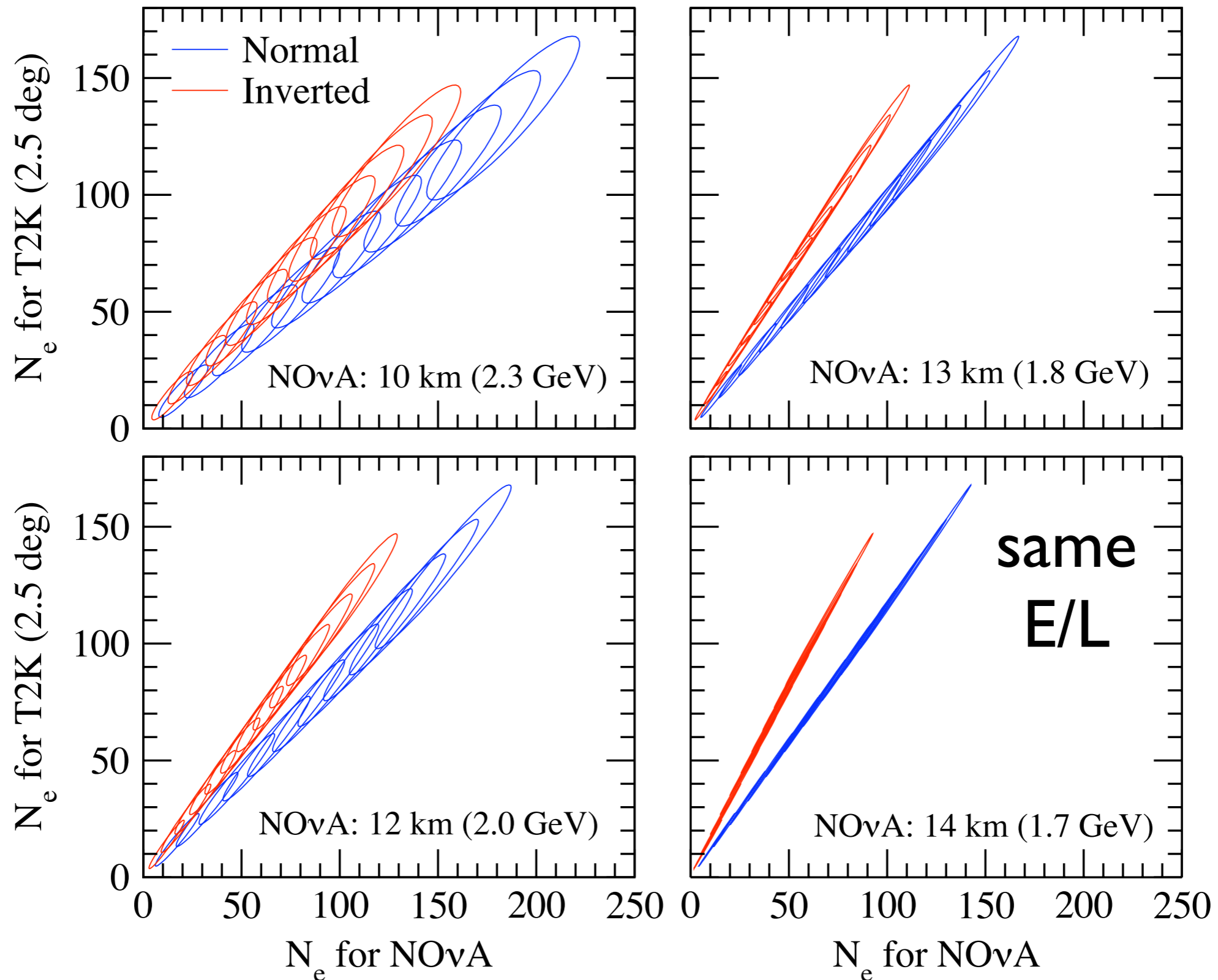directly comparable to reactor

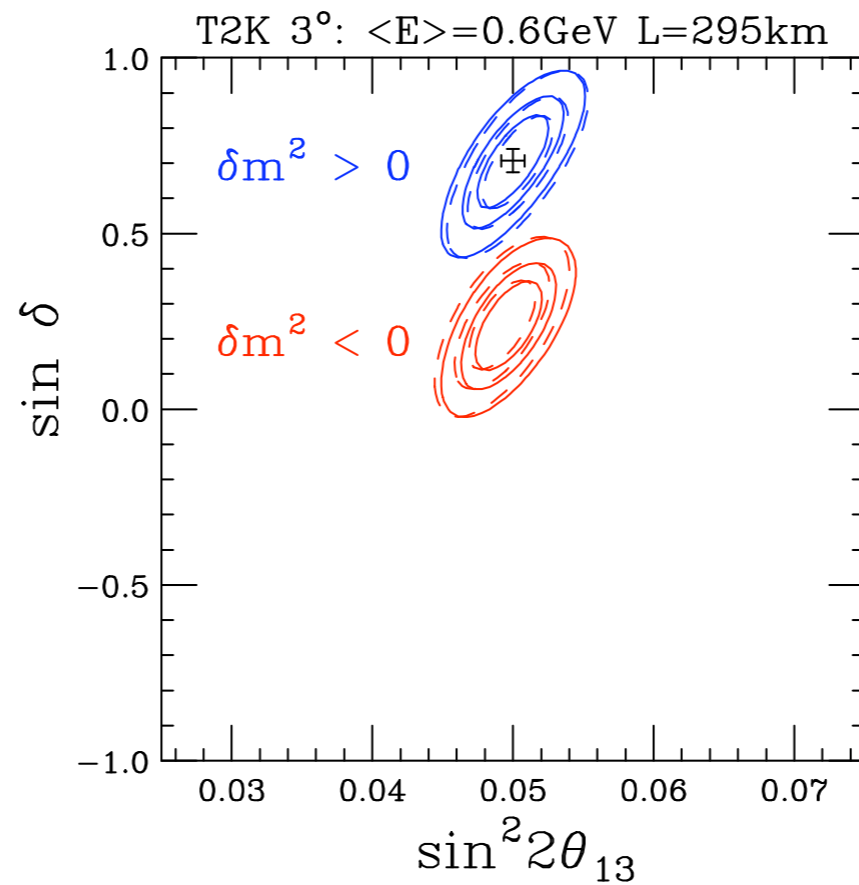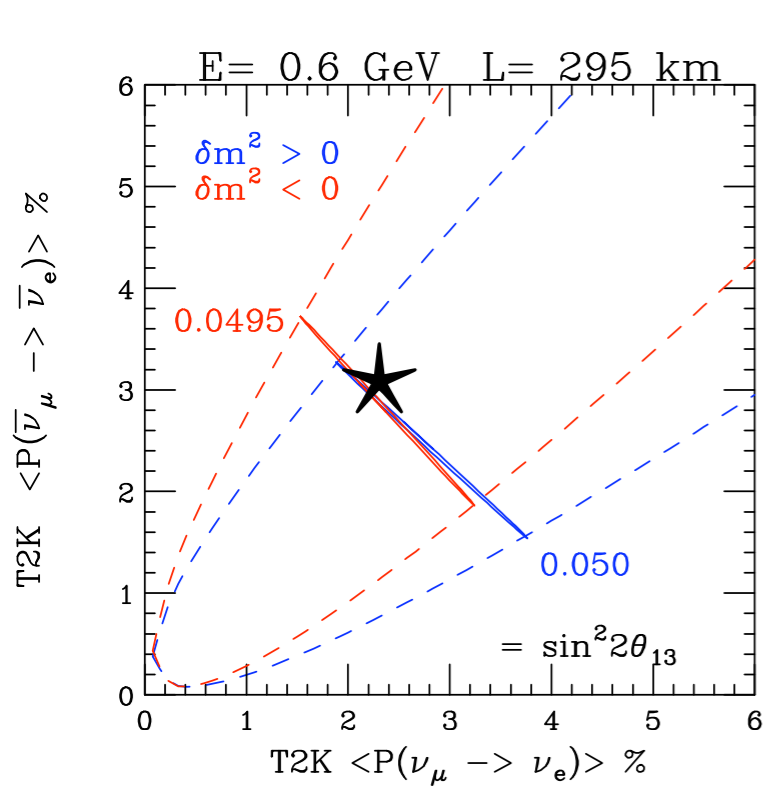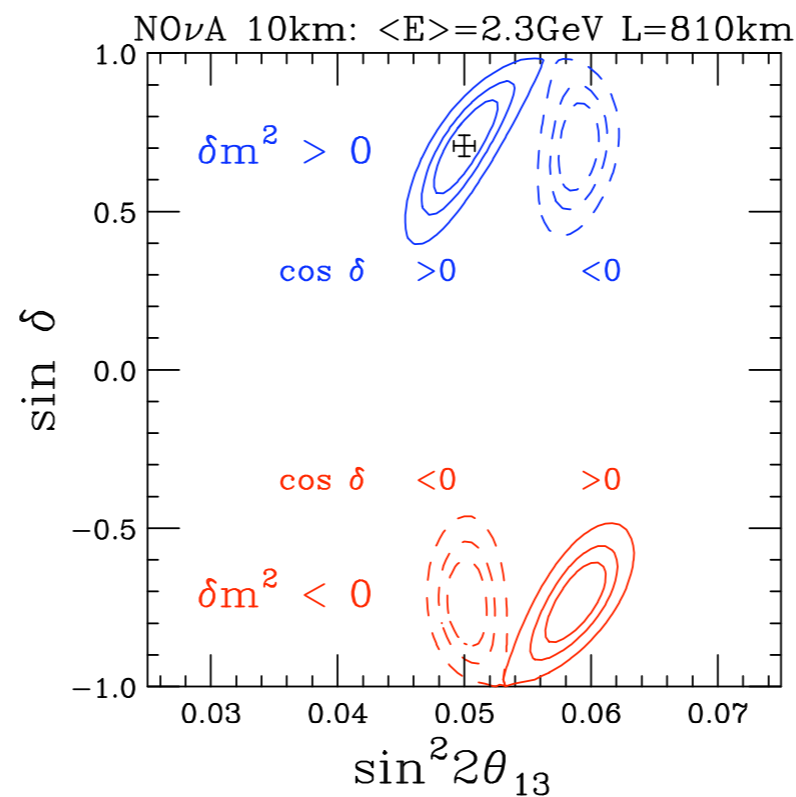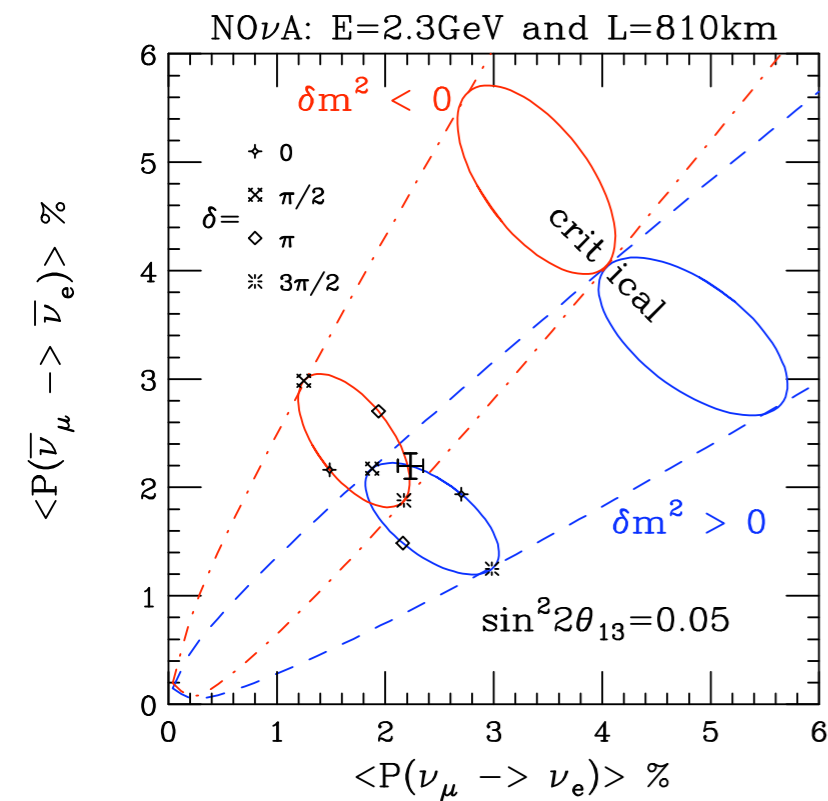$$1 - P(\bar{\nu}_e \to \bar{\nu}_e) = \sin^2 2\theta_{13}$$

$\sin^2 2\theta_{23} = 0.96$

$\sin^2 \theta_{23} = 0.4$ ơ 0.6

(4*0.4*0.6=0.96)

What about combining T2K and NOvA?
Neutrinos Only

E= 0.6 GeV   L= 295 km

$\delta m^2 > 0$

$\delta m^2 < 0$

0.0495

0.050

$= \sin^2 2\theta_{13}$

T2K $\langle P(\bar{\nu}_\mu \to \bar{\nu}_e) \rangle$ %

T2K $\langle P(\nu_\mu \to \nu_e) \rangle$ %

T2K 3°: <E>=0.6GeV L=295km

$\delta m^2 > 0$

$\delta m^2 < 0$

$\sin \delta$

$\sin^2 2\theta_{13}$

$$\langle \sin \delta \rangle_+ \quad - \quad \langle \sin \delta \rangle_-$$

$$\approx \quad 0.47 \sqrt{\frac{\sin^2 2\theta_{13}}{0.05}}$$

NOνA: E=2.3GeV and L=810km

$\delta m^2 < 0$

$+$ 0

$\times$ $\pi/2$

$\delta =$ $\diamond$ π

$*$ $3\pi/2$

critical

$\delta m^2 > 0$

$\sin^2 2\theta_{13} = 0.05$

$\langle P(\bar{\nu}_\mu \to \bar{\nu}_e) \rangle$ %

$\langle P(\nu_\mu \to \nu_e) \rangle$ %

NOνA 10km: <E>=2.3GeV L=810km

$\delta m^2 > 0$

$\cos \delta$ >0   <0

$\cos \delta$ <0   >0

$\delta m^2 < 0$

$\sin \delta$

$\sin^2 2\theta_{13}$

$$\langle \sin \delta \rangle_+ \quad - \quad \langle \sin \delta \rangle_-$$

$$\approx \quad 1.4 \sqrt{\frac{\sin^2 2\theta_{13}}{0.05}}$$

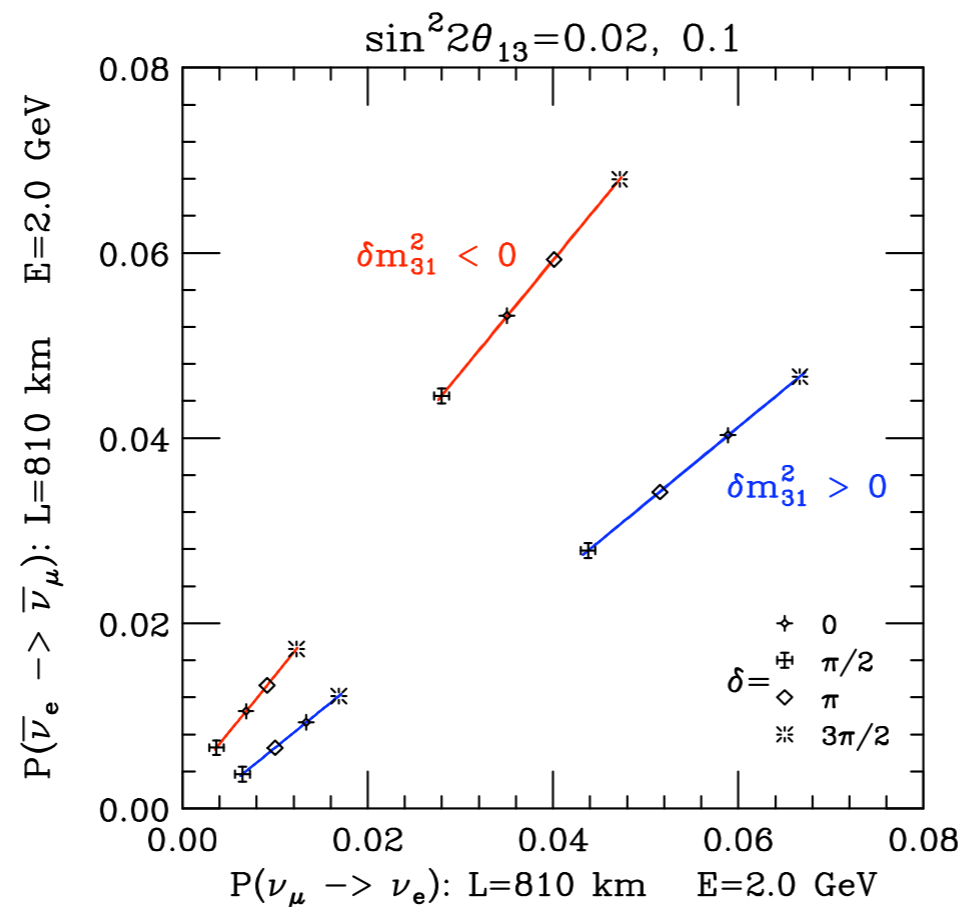(ρL) for NOvA <u>three</u> times larger than (ρL) than T2K.

BetaBeams (nubar_e) at Fermilab:

$^6He^{2+}$, $^8Li^{3+}$

$$\nu_\mu \longrightarrow \nu_e \qquad \vee \qquad \bar{\nu}_e \longrightarrow \bar{\nu}_\mu$$

$$P(\nu_\mu \to \nu_e) > P(\bar{\nu}_e \to \bar{\nu}_\mu) \quad \text{for Normal Hierarchy}$$

$$\text{and} \quad P(\nu_\mu \to \nu_e) < P(\bar{\nu}_e \to \bar{\nu}_\mu) \quad \text{for Inverted Hierarchy,}$$

$\sin^2 2\theta_{13} = 0.02,\ 0.1$

$\delta m_{31}^2 < 0$

$\delta m_{31}^2 > 0$

$\delta = \begin{cases} 0 \\ \pi/2 \\ \pi \\ 3\pi/2 \end{cases}$

$P(\bar{\nu}_e \to \bar{\nu}_\mu)$: L=810 km   E=2.0 GeV

$P(\nu_\mu \to \nu_e)$: L=810 km   E=2.0 GeV

$\delta m_{31}^2 > 0$

E=2.0 GeV

Olga Mena + SP hep-ph/0709nnn

Way Forward:

Signal Events =
      Fid. Mass
               *   P.O.T. (beam power*time)
                * Efficiency

2nd Oscillation Maximum

Broadband Beam: Same L, Lower E Fermilab to DUSEL
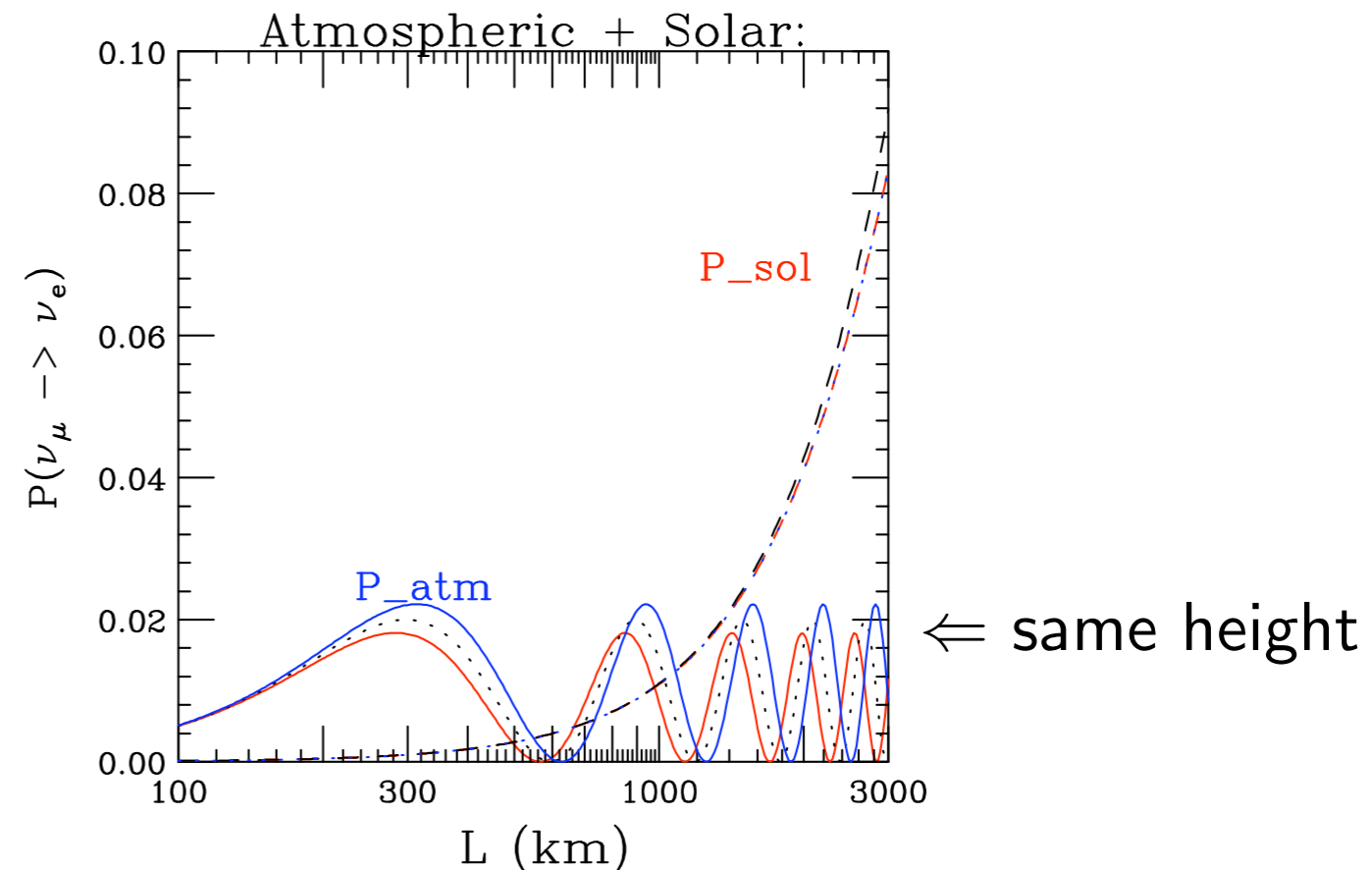
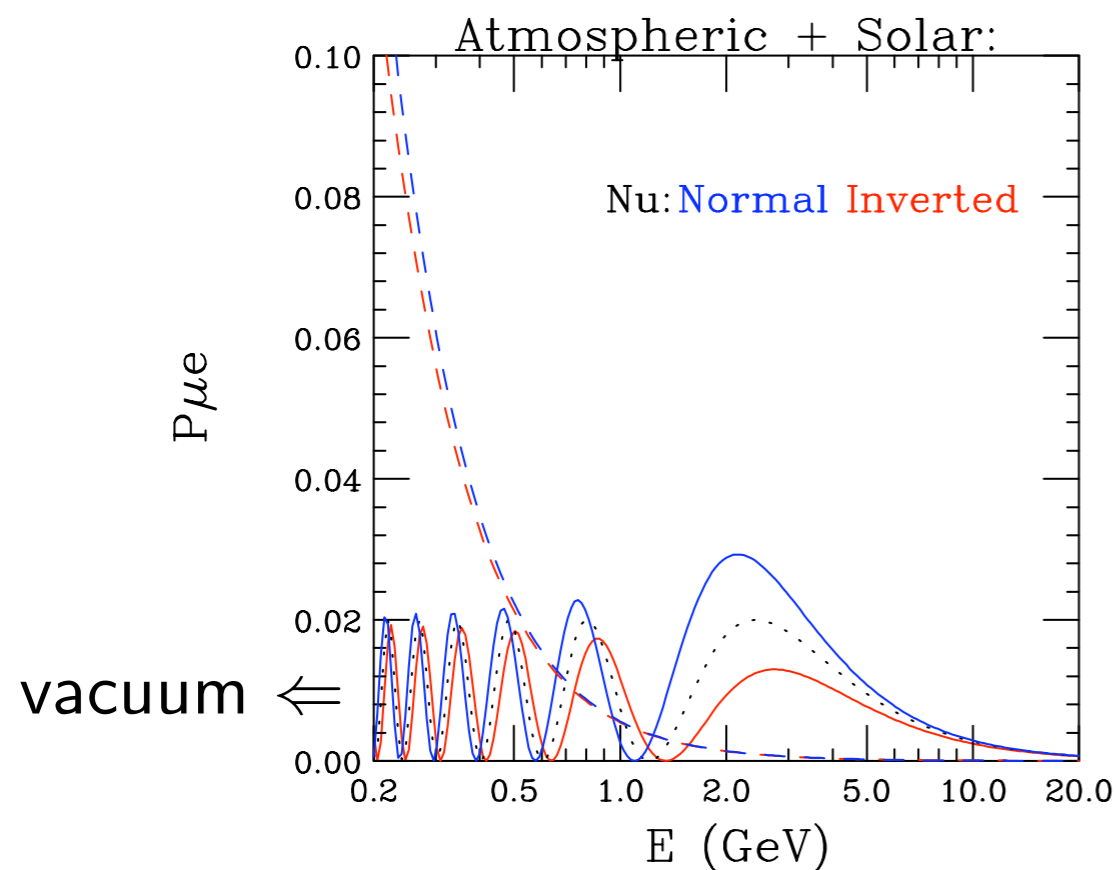Narrow Band Beam: Same E, Longer L T2KK

In VACUUM the SAME but NOT in MATTER
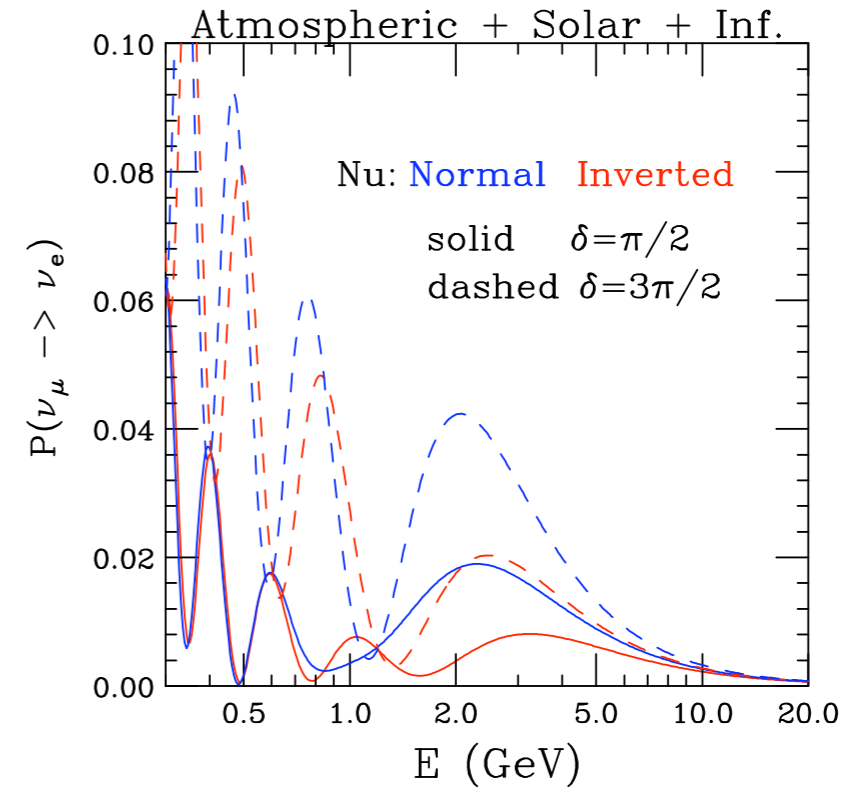
$$\sin^2 2\theta_{13} = 0.04$$

L=1200km

E=0.6 GeV

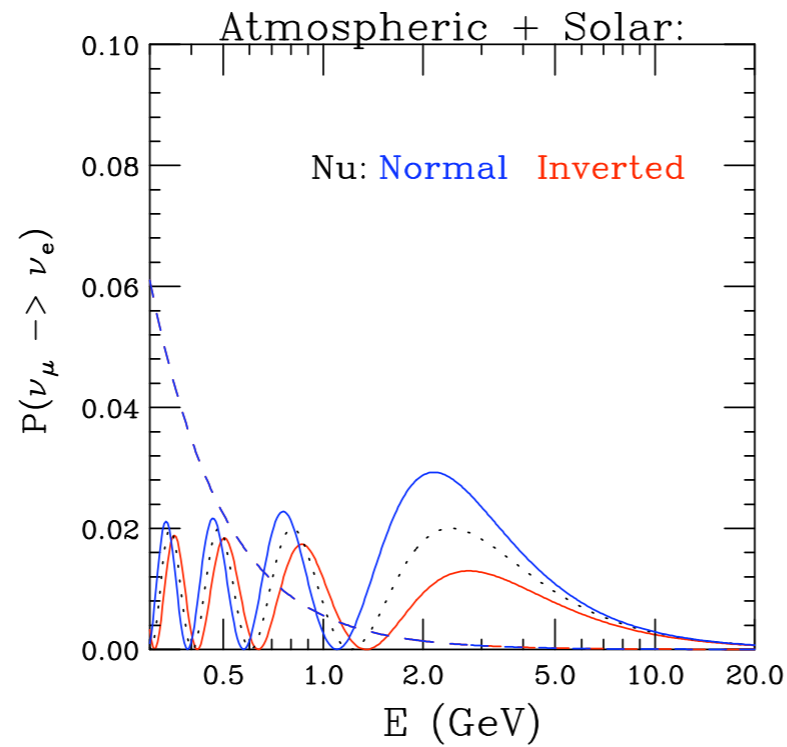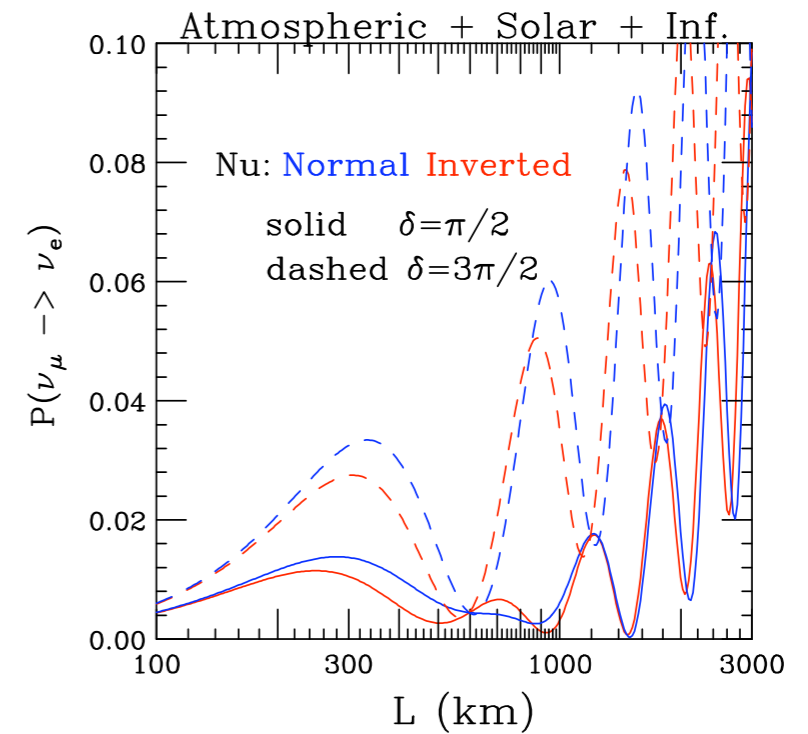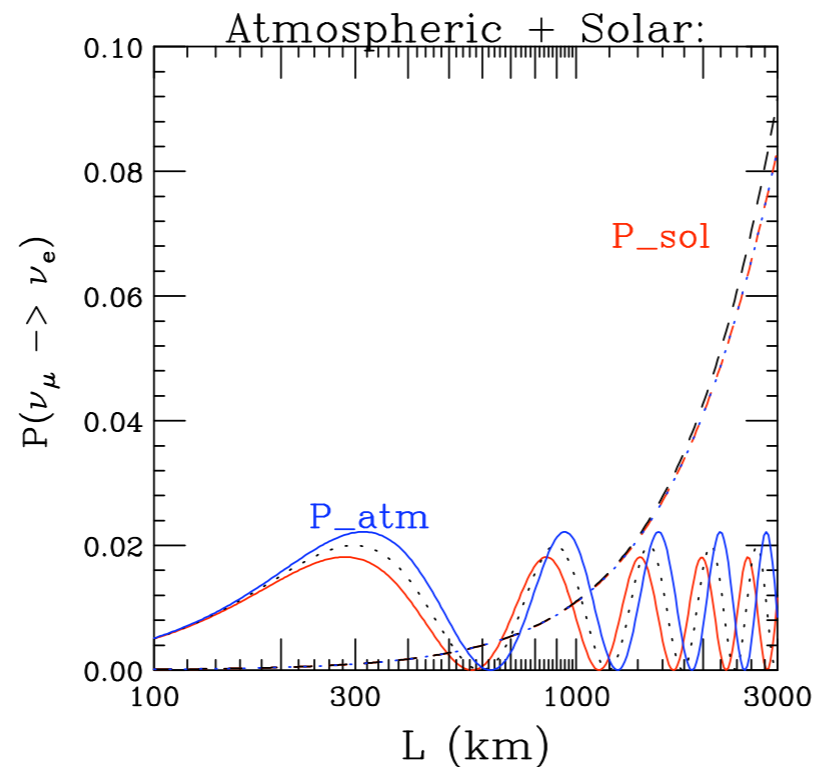$$P_{\mu \to e} \approx \mid \sqrt{P_{atm}} e^{-i(\Delta_{32} \pm \delta)} + \sqrt{P_{sol}} \mid^2$$

$$P_{\mu \to e} \approx | \textcolor{blue}{\sqrt{P_{atm}}} e^{-i(\Delta_{32} \pm \delta)} + \textcolor{red}{\sqrt{P_{sol}}} |^2$$

L=1200 km to "DUSEL"

E=0.6 GeV T2KK

where $\sqrt{P_{atm}} = \sin\theta_{23} \sin 2\theta_{13} \frac{\sin(\Delta_{31} \mp aL)}{(\Delta_{31} \mp aL)} \Delta_{31}$

and $\sqrt{P_{sol}} = \cos\theta_{23} \sin 2\theta_{12} \frac{\sin(aL)}{(aL)} \Delta_{21}$

$$P_{\mu \to e} \approx |\sqrt{P_{atm}}e^{-i(\Delta_{32} \pm \delta)} + \sqrt{P_{sol}}|^2$$

depends on θ_{13}
amplification or suppression
by matter (E)

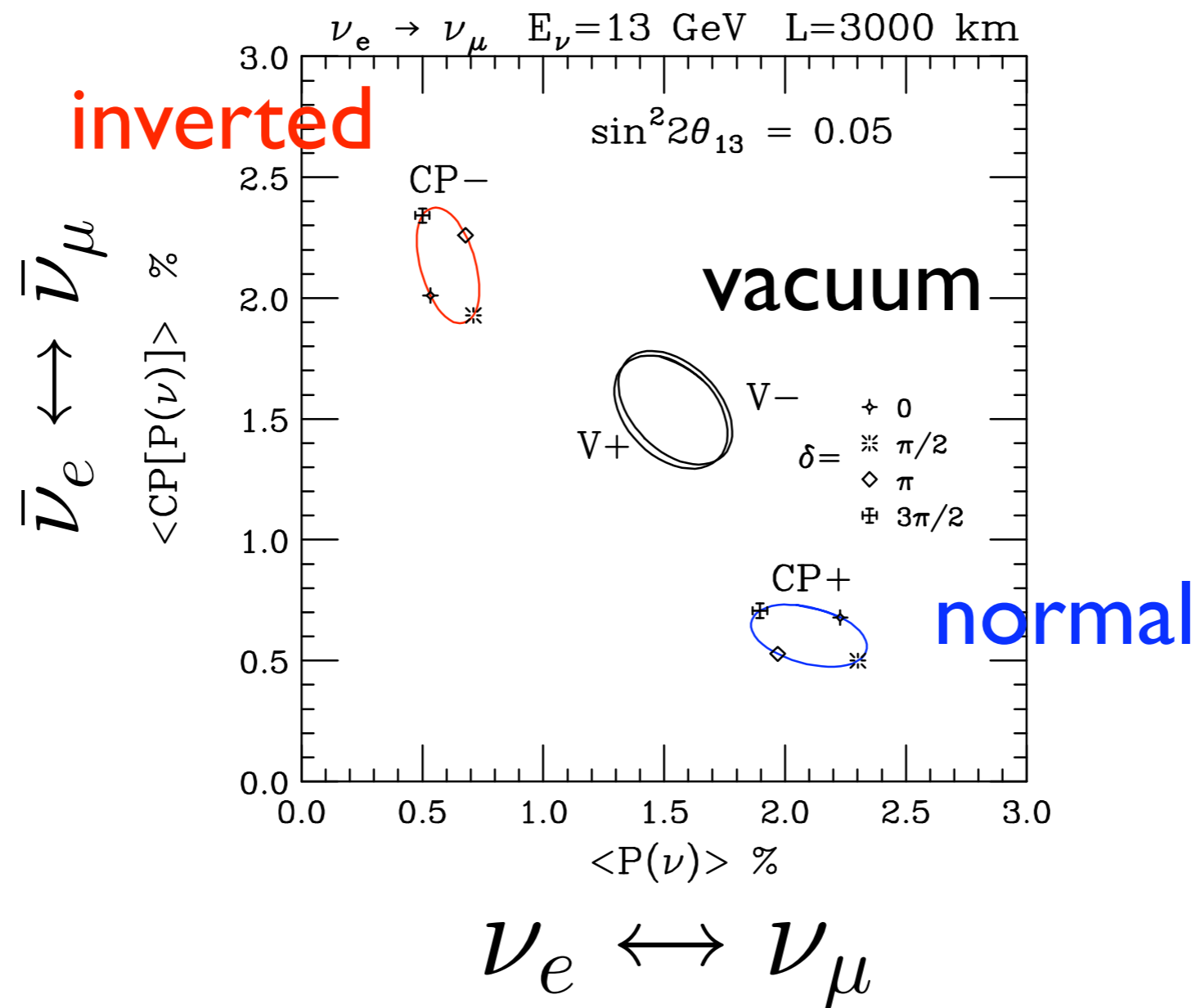independent of θ_{13}
$\approx$ independent of
matter effect

Suppression $\geq$ Enhancement

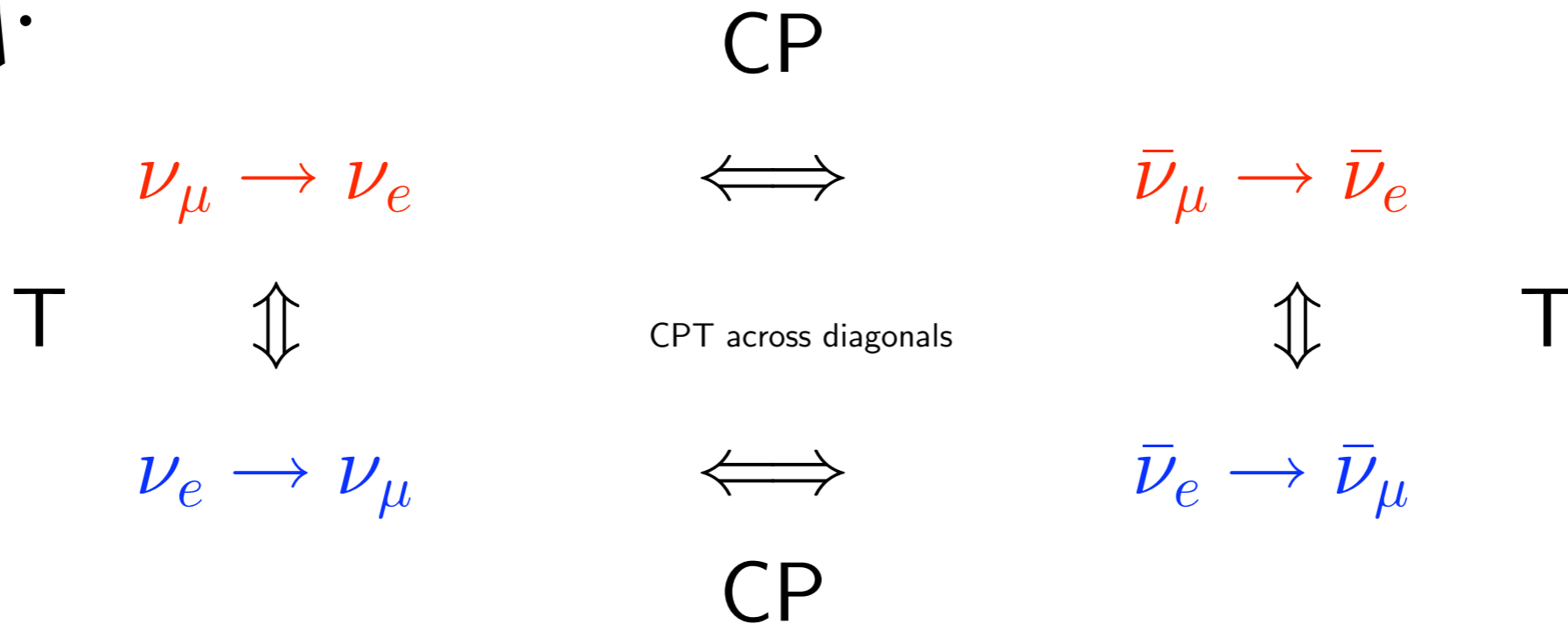except when L > 4000 km.
$\sin(aL) = 0!$ when L$\approx$ 7500 km

L/E $\geq$ significant fraction of 500 km/GeV

Event rate: $E(E/L)^2$

Neutrino Factory: muon storage ring

Summary:

$$\nu_\mu \longrightarrow \nu_e \qquad\qquad \overset{\text{CP}}{\Longleftrightarrow} \qquad\qquad \bar\nu_\mu \longrightarrow \bar\nu_e$$

T $\qquad\qquad \Updownarrow \qquad\qquad$ CPT across diagonals $\qquad\qquad \Updownarrow \qquad\qquad$ T

$$\nu_e \longrightarrow \nu_\mu \qquad\qquad \overset{\text{CP}}{\Longleftrightarrow} \qquad\qquad \bar\nu_e \longrightarrow \bar\nu_\mu$$

- First Row: Superbeams where ν_e contamination ~ 1 %

- Second Row: ν-Factory or β-Beams, no beam contamination

$|U_{\alpha j}|^2$

ν_e ■ ν_μ ▩ ν_τ ■

Neutrino Mass Squared

Fractional Flavor Content varying $\cos \delta$

CPT $\Rightarrow$ invariant $\delta \leftrightarrow -\delta$
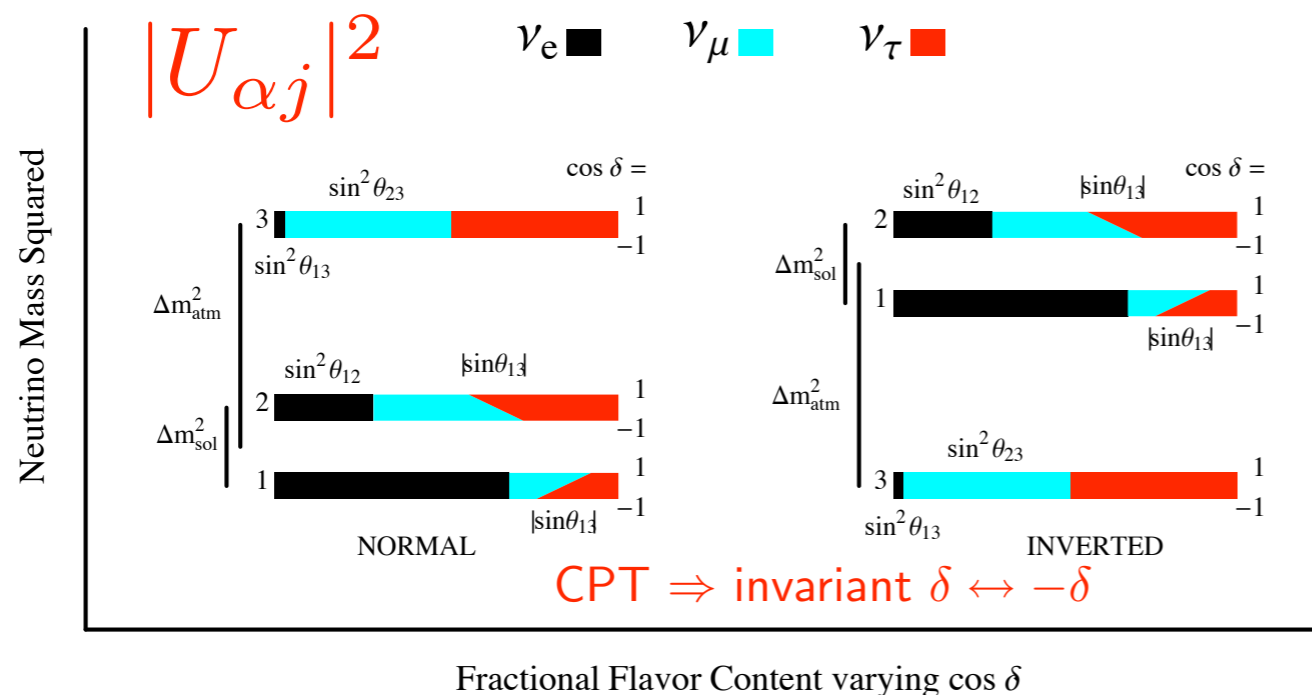
- Size of $|U_{e3}|^2$

- Hierarchy ?

- CPV ?

- Maximal {23} Mixing ?

- .....

- New Interactions and Surprises !!!

Mossbauer Neutrinos Review:

Mossbauer effect with Neutrinos in the $^3H - ^3He$ system:

Source: $^3H \rightarrow (^3He + e_B^-) + \bar{\nu}_e$

Detector: $\bar{\nu}_e + (^3He + e_B^-) \rightarrow ^3H$

count via decay or mass spectro.

$Q = 18.6$ keV and $\Gamma_{^3H} = 1.2 \times 10^{-24}$ eV

Various line broadening effects which significantly increase Γ_{eff}

Serious technical difficulties exist but it is not impossible (Raghaven, Potzel)

For $\Gamma_{eff} \sim 10^{-11}$ eV ($\Delta E/E \sim 10^{-15}$) then $\sigma \sim 10^{-33} cm^2$!!!

Do Mossbauer Neutrinos Oscillate? YES

(Akhmedov, Kopp, Lindner 0802.2513, 0803.1424)

(see also Bilenky, Feilitzsch, Potzel)

ν_e Disappearance

solar osc. (first min 270m) $P_\odot$

atm osc. (first min 9m)

$$P(\bar{\nu}_e \to \bar{\nu}_e) \;\; = \;\; 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21}$$

$$-\sin^2 2\theta_{13}\left[\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}\right]$$

$\Delta_{ij} \equiv \frac{\Delta m_{ij}^2 L}{4E}$ (kinematic phase).

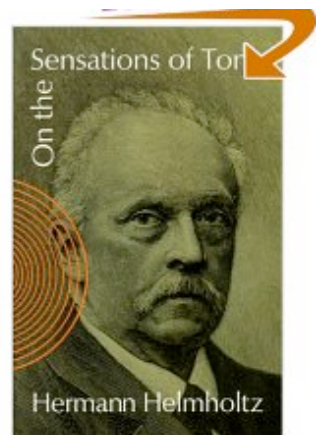$\Delta_{21} = \Delta_{31} - \Delta_{32}.$

$$\cos^2\theta_{12} > \sin^2\theta_{12}$$

Sensations of Tor
On the
Hermann Helmholtz

1875

- for Normal Hierarchy (NH): $|\Delta_{31}| > |\Delta_{32}|$

  phase of atmospheric oscillation ADVANCES by $2\pi\sin^2\theta_{12}$
  for every solar osc.

- for Inverted Hierarchy (IH): $|\Delta_{31}| < |\Delta_{32}|$

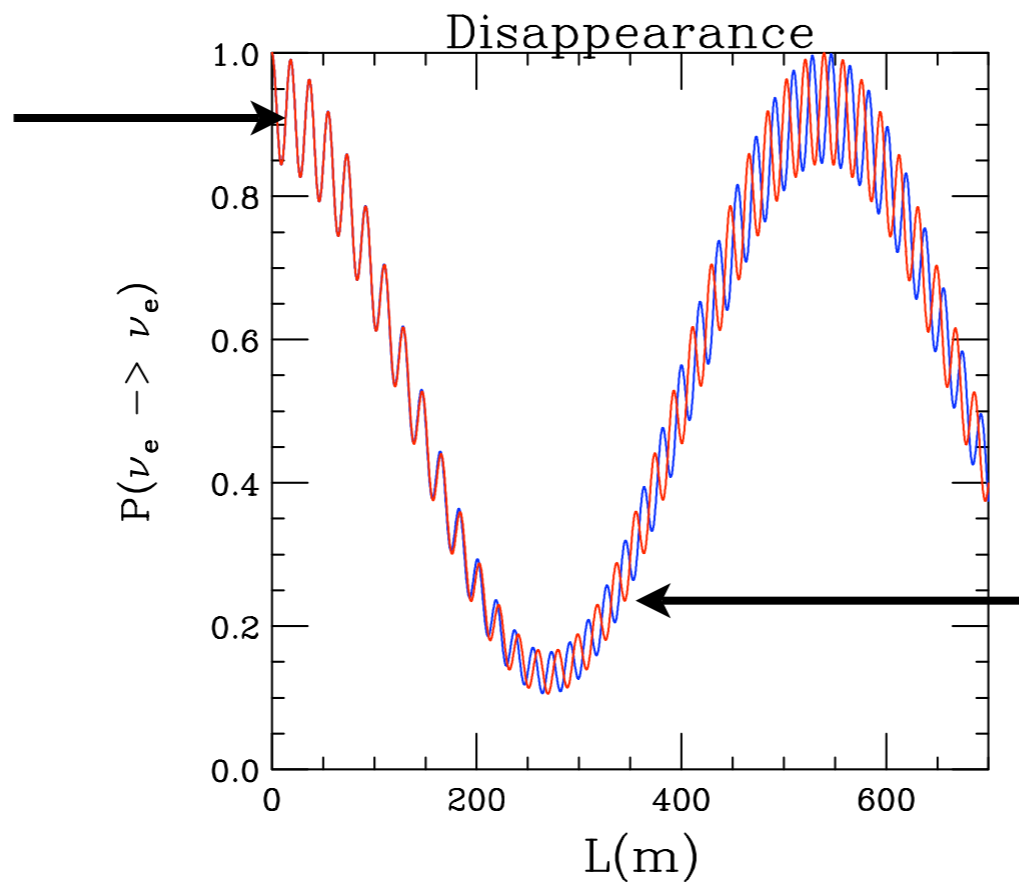 phase of atmospheric oscillation RETARDED by $2\pi\sin^2\theta_{12}$
  for every solar osc.

What about when $\sin^2\theta_{12} = \frac{1}{2}$?
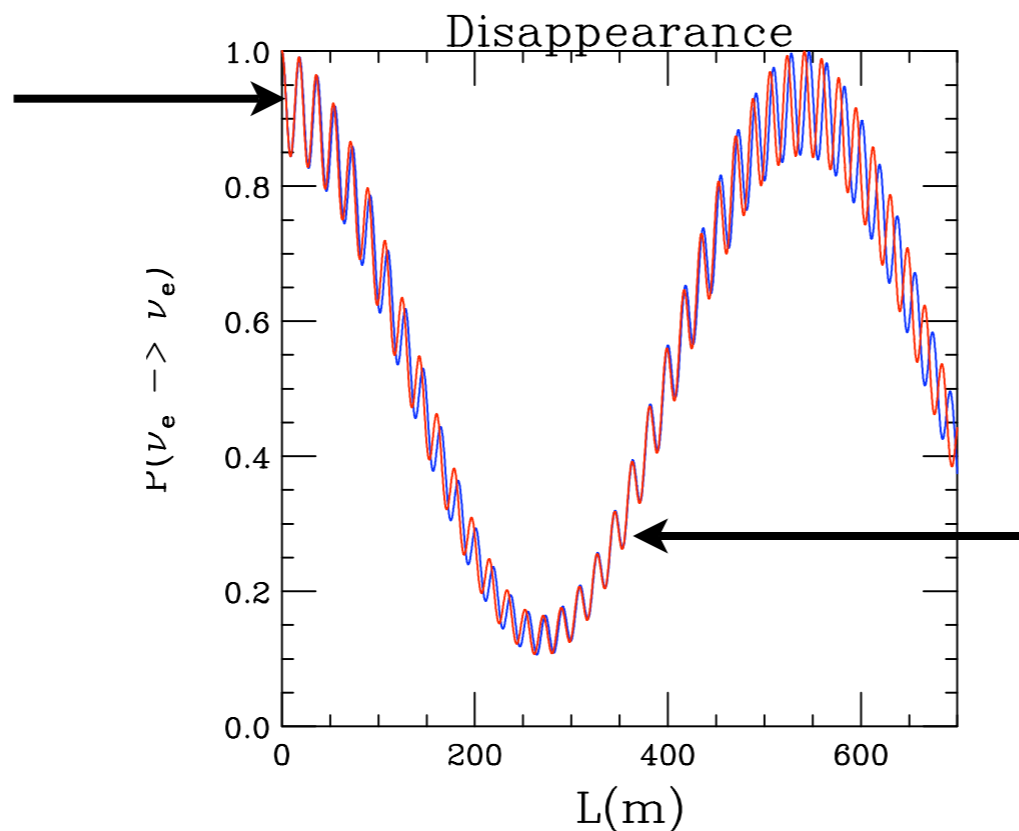
in phase

Disappearance

$P(\nu_e \to \nu_e)$

L(m)

out of phase by pi/2

$\delta m^2_{31} > 0$

$\delta m^2_{31} < 0$

$\delta m^2_{IH} = 1.03 \times \delta m^2_{NH}$

Disappearance

$P(\nu_e \to \nu_e)$

L(m)

in phase

Combining the Atm Osc:

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \quad = \quad 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21}$$

$$-\frac{1}{2} \sin^2 2\theta_{13} \left\{ 1 - \sqrt{(1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21})} \cos(2\Delta_{ee} \pm \phi_\odot) \right\}$$

- $\frac{1}{2} \sin^2 2\theta_{13} (1 \mp \sqrt{1 - \sin^2 2\theta_{13} \sin^2 \Delta_{21}})$ gives the amplitude modulation.

- the $(2\Delta_{ee} \pm \phi_\odot)$ part:

  - $\pm$ Hierarchy: $+$ Normal and - Inverted.

  - linear term $2\Delta_{ee} \equiv \Delta m_{ee}^2 L / 2E$:

$$\Delta m_{ee}^2 \quad = \quad c_{12}^2 |\Delta m_{31}^2| + s_{12}^2 |\Delta m_{32}^2| > 0$$
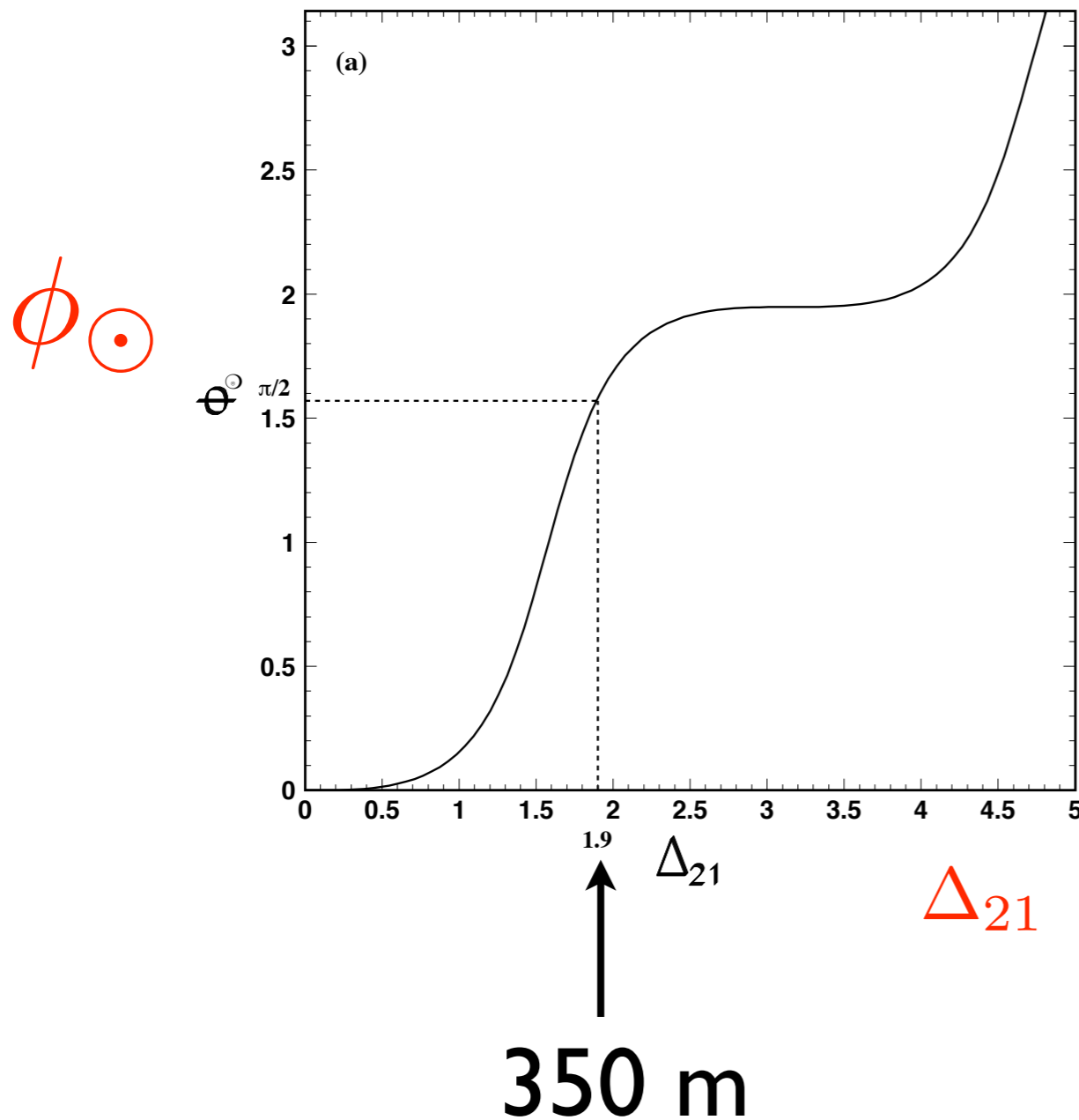$$= \quad |m_3^2 - (c_{12}^2 m_1^2 + s_{12}^2 m_2^2)|$$
$$\updownarrow$$
$$\nu_e \text{ weighted average of } m_1^2 \text{ and } m_2^2$$

  - everything else $\phi_\odot$: and only depends on Δ_{21} and θ_{12}.

The Phase:

- $\phi_\odot = \arctan(\cos 2\theta_{12} \tan \Delta_{21}) - \Delta_{21} \cos 2\theta_{12}$

L(m)

(a)

$\phi_\odot$

Δ_{21}

1.9

Δ_{21}

350 m

$\phi_\odot(\Delta_{21} + \pi) = \phi_\odot(\Delta_{21}) + 2\pi \sin^2 \theta_{12},$

$$\frac{d\phi_\odot}{d\Delta_{21}}\Big|_{n\pi} = 0$$

for $n = 0, 1, 2, \ldots$

Strategy:

(I) Precision (<1%) measurement of δm^2_{ee} at L around 10 m

Disappearance
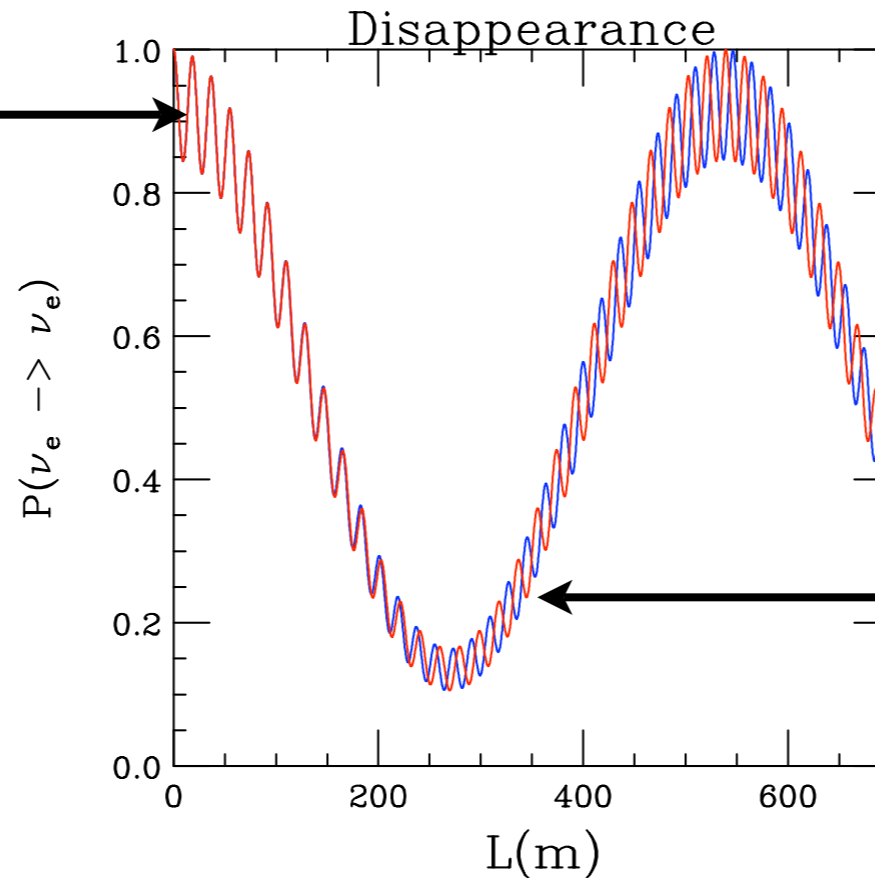
(II) determination of phase at L=350 m

But this is after 20 or so oscillation !!!
What about smearing in the L/E?

E ok, as $\Delta E/E \sim 10^{-15}$

Smearing L:

$P_\odot = 0$

| Disappearance | Disappearance | Disappearance |

$\sigma = 0.1\%$ $\Delta m^2 > 0$ (blue), $\Delta m^2 < 0$ (green) $\sigma = 0.5\%$ $\sigma = 2\%$
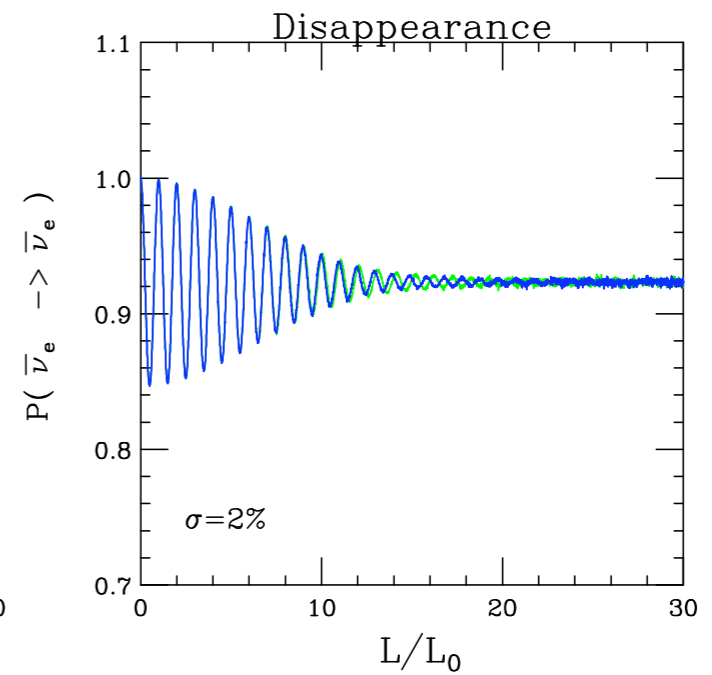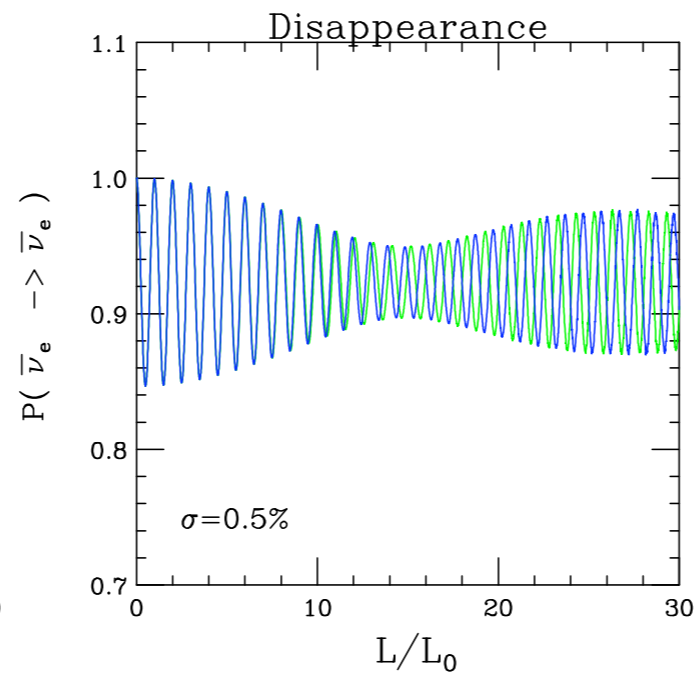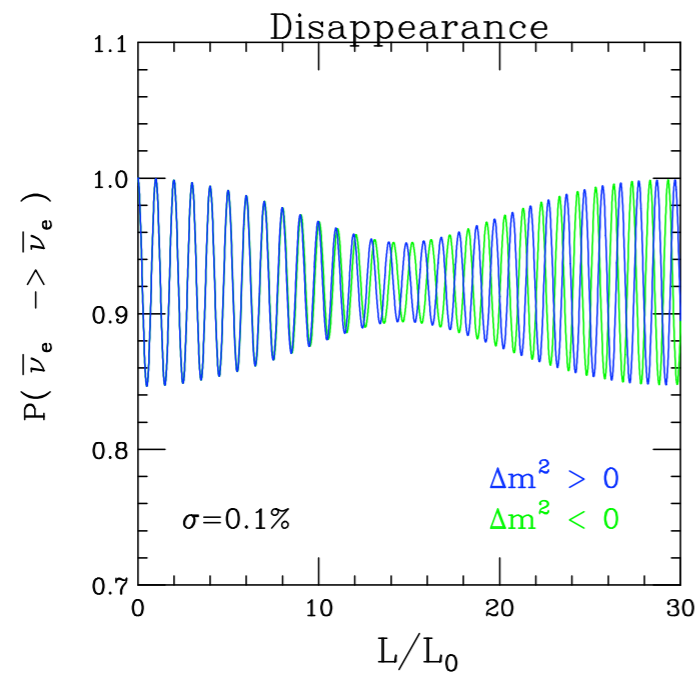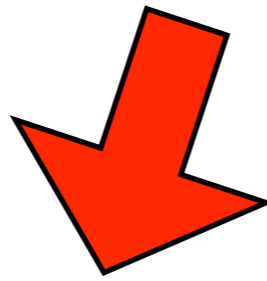
(note: amplitude modulation, 40% at solar minimum!)

Source: Detector:
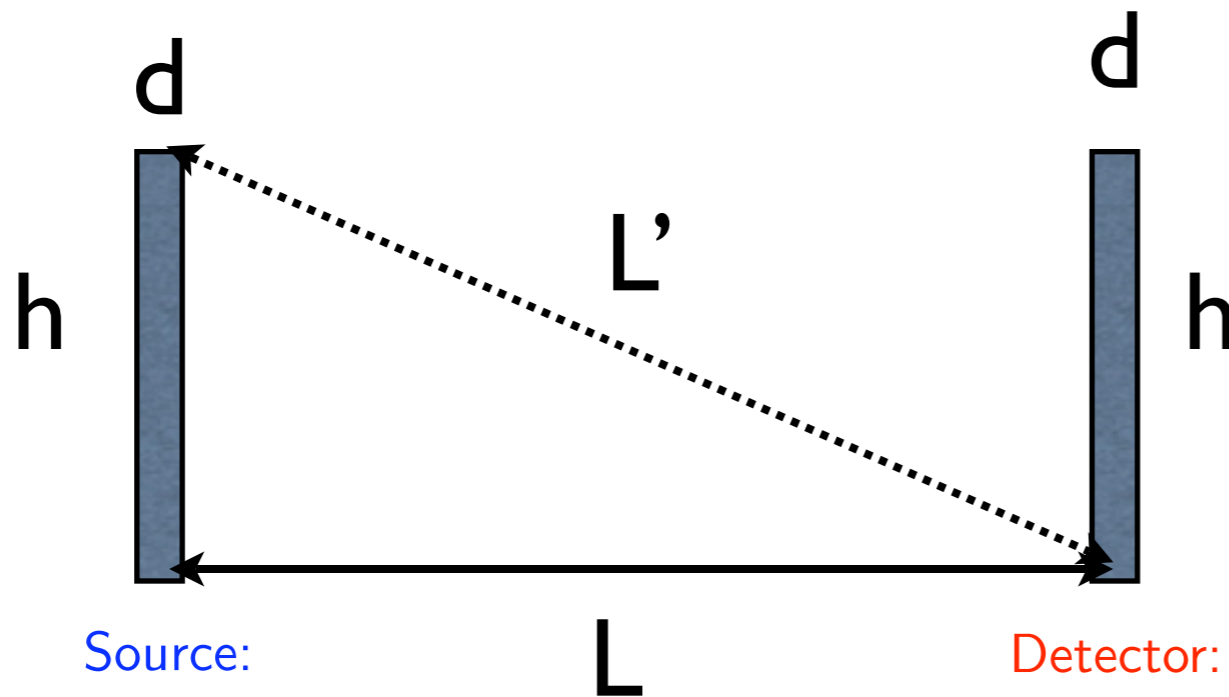
$d < L/200$

$L' \approx L(1 + \frac{1}{2}\frac{h^2}{L^2})$ so $h < L/10$

OK

Phase I: Measurement of δm^2_{ee}

(the atm δm^2 near the first osc. minima for a $\bar{\nu}_e$ disapp. exp.)

Event Rate:

$$R_{ench} = 3 \times 10^5 \left(\frac{S}{1MCi} \right) \left(\frac{M_T}{100g} \right) \left(\frac{L}{10m} \right)^{-2} day^{-1}$$

Minakata and Uchinami: hep/0602046

- Run IIB $= 10$ measurement points at $(1/5,\ 3/5,\ \cdots\ 19/5)L_{OM}$

- 10^6 events each, $\sigma_{usys} = 0.2\%$, $\sigma_c = 10\%$

- Sensitivity in $\delta m^2_{ee} \approx 0.3 \left(\frac{\sin^2 2\theta_{13}}{0.1} \right)^{-1} \%$

Phase II: phase at 350 m

Event Rate:

$$R_{ench} = 2 \times 10^2 \left(\frac{S}{1MCi}\right) \left(\frac{M_T}{100g}\right) \left(\frac{L}{350m}\right)^{-2} day^{-1}$$

5 Baselines: $L = 350 \pm 5 \pm 10$ m

$\sin^2\Theta_{12} = 0.31$ $\Delta m_{21}^2 = 8.0 \times 10^{-5}$ eV^2 $\sin^2 2\Theta_{13} = 0.1$

NH IH
$\Delta m_{ee}^2 = 2.5 \times 10^{-3}$ eV^2
$\Delta m_{ee}^2 = 2.5 \, (1.+0.5\%) \times 10^{-3}$ eV^2
$\Delta m_{ee}^2 = 2.5 \, (1.-0.5\%) \times 10^{-3}$ eV^2

(a)
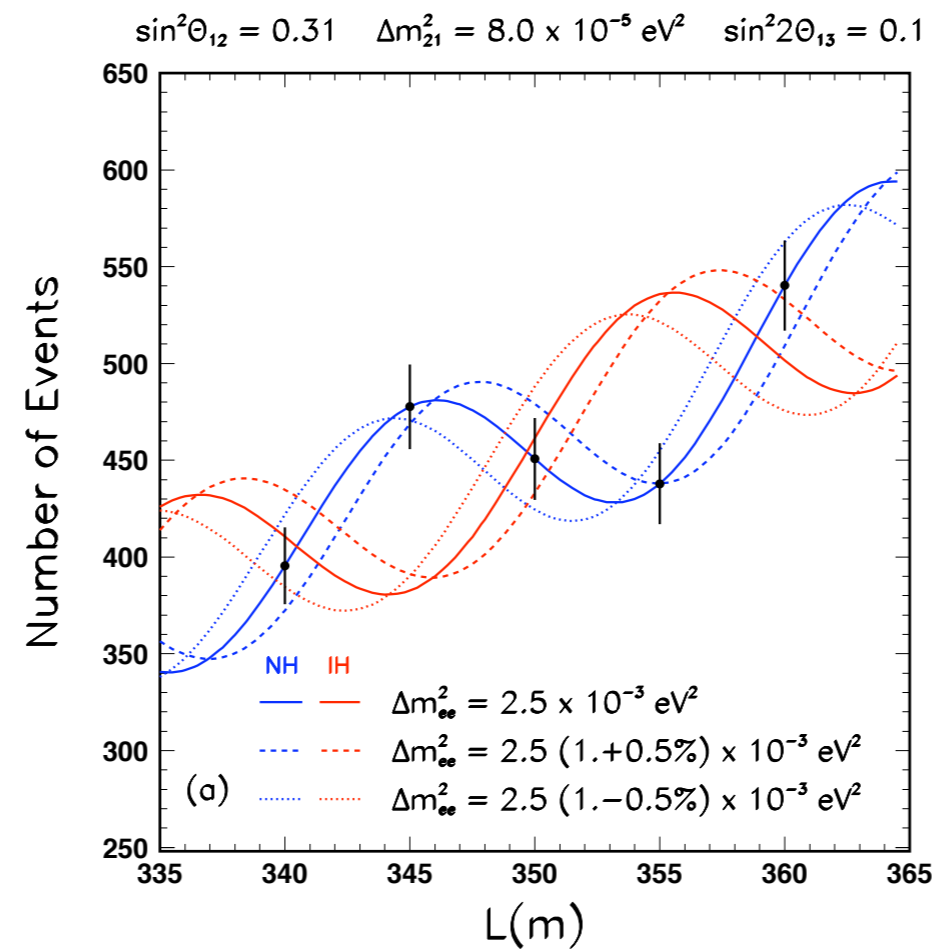
Phase I

Phase II

Phase I+II

Normal Hierarchy | Inverted Hierarchy

input $\Delta m^2_{ee} = 2.5 \times 10^{-3} eV^2$
$\sin^2 2\theta_{13} = 0.1$

$\sigma_c = 10\%$
$\sigma_{usys,i} = 0.2\%$

Δm^2_{ee} ($\times 10^{-3}$ eV^2)

$\sin^2 2\theta_{13}$

first phase alone

second phase alone

1st and 2nd phases combined

90% CL
95% CL
99% CL
Input

Sensitivity:

$\sigma_{usys} = 0.2\%$

$\sigma_{usys} = 1.0\%$

Number of Events/Detector

$\sin^2 2\theta_{13}$

99 % CL
95 % CL
90 % CL
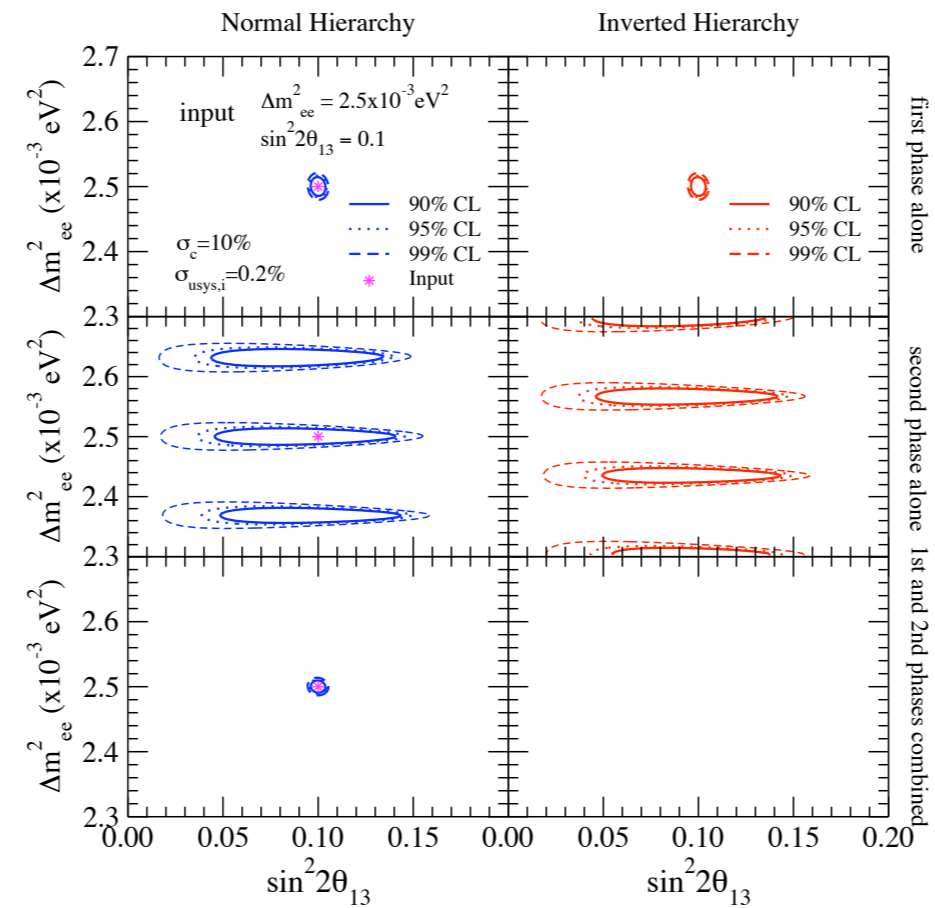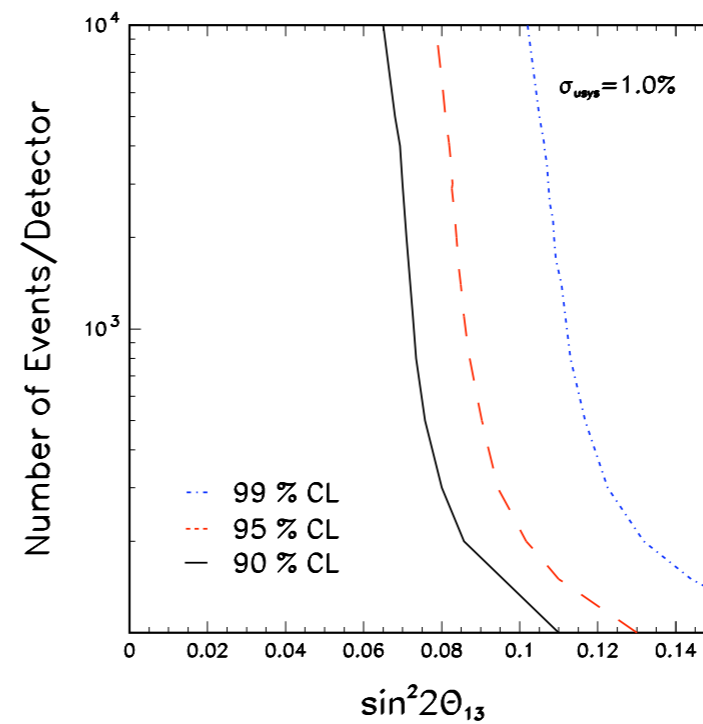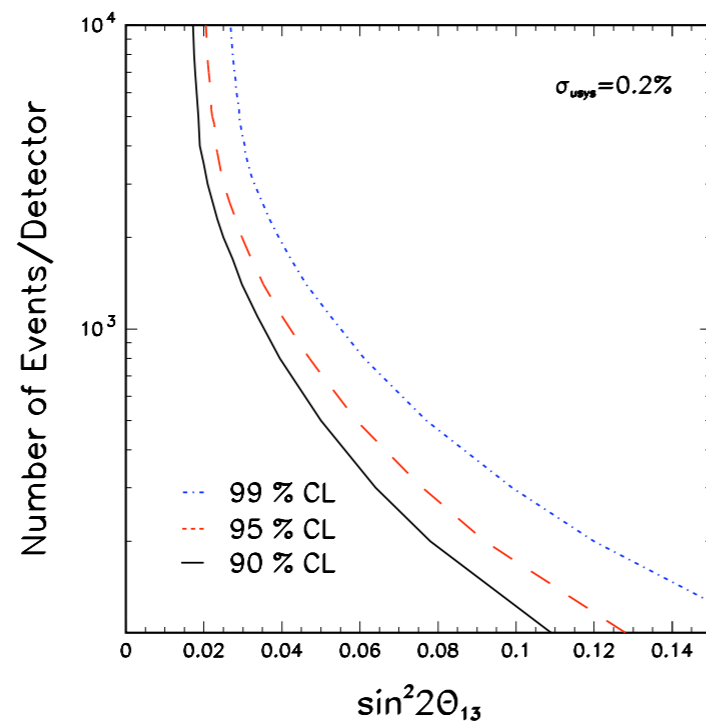
Reactor Neutrinos:

Detected Reactor Spectrum

Detected Spectrum

Reactor $\overline{\nu}_e$ from neutron rich fission fragments

$\overline{\nu}_e + p \rightarrow e^+ + n$
Cross section

Detected Spectrum

- In practice, only 1.5 neutrinos/fission detectable
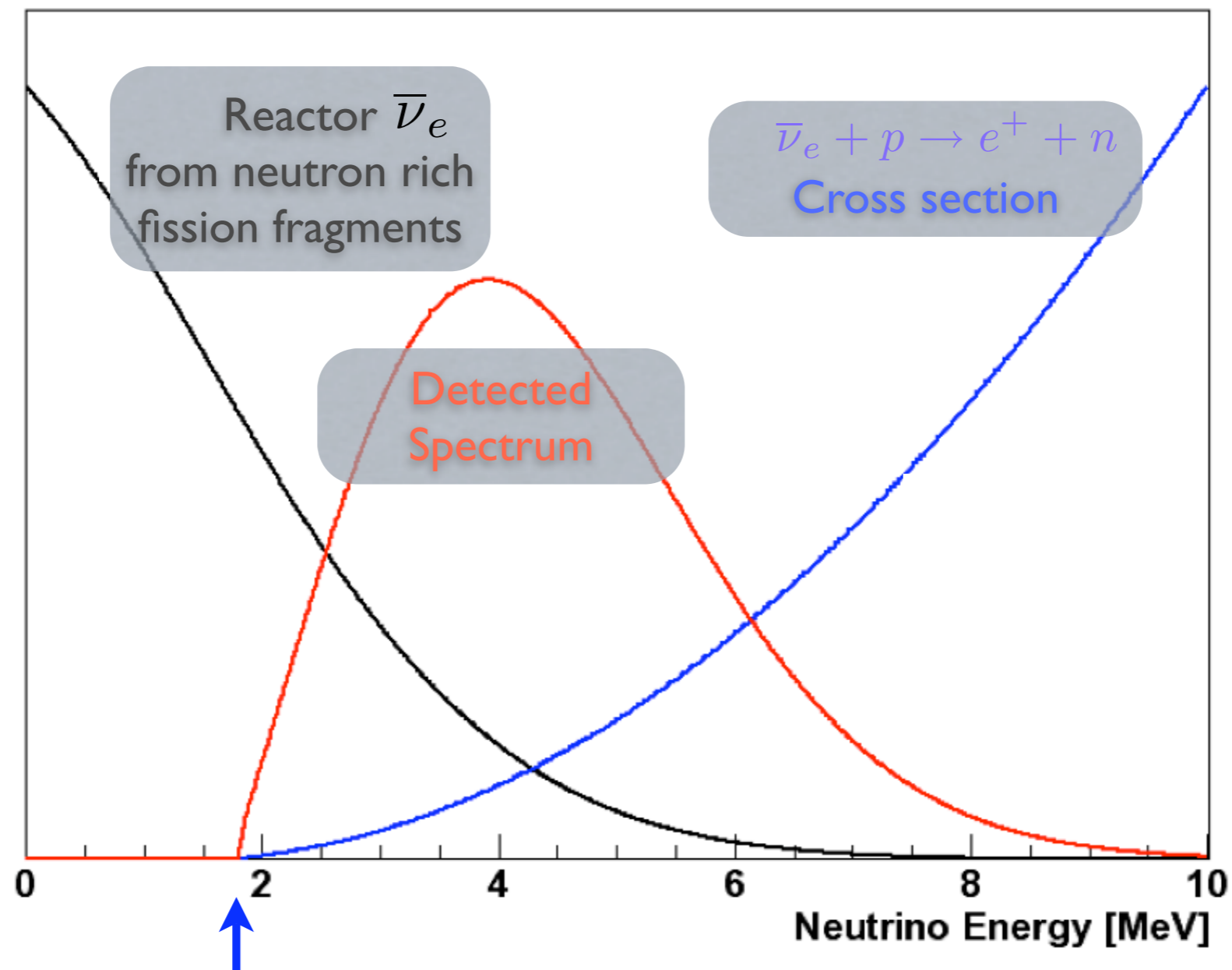
Hawaii Antineutrino Observatory
Hanohano

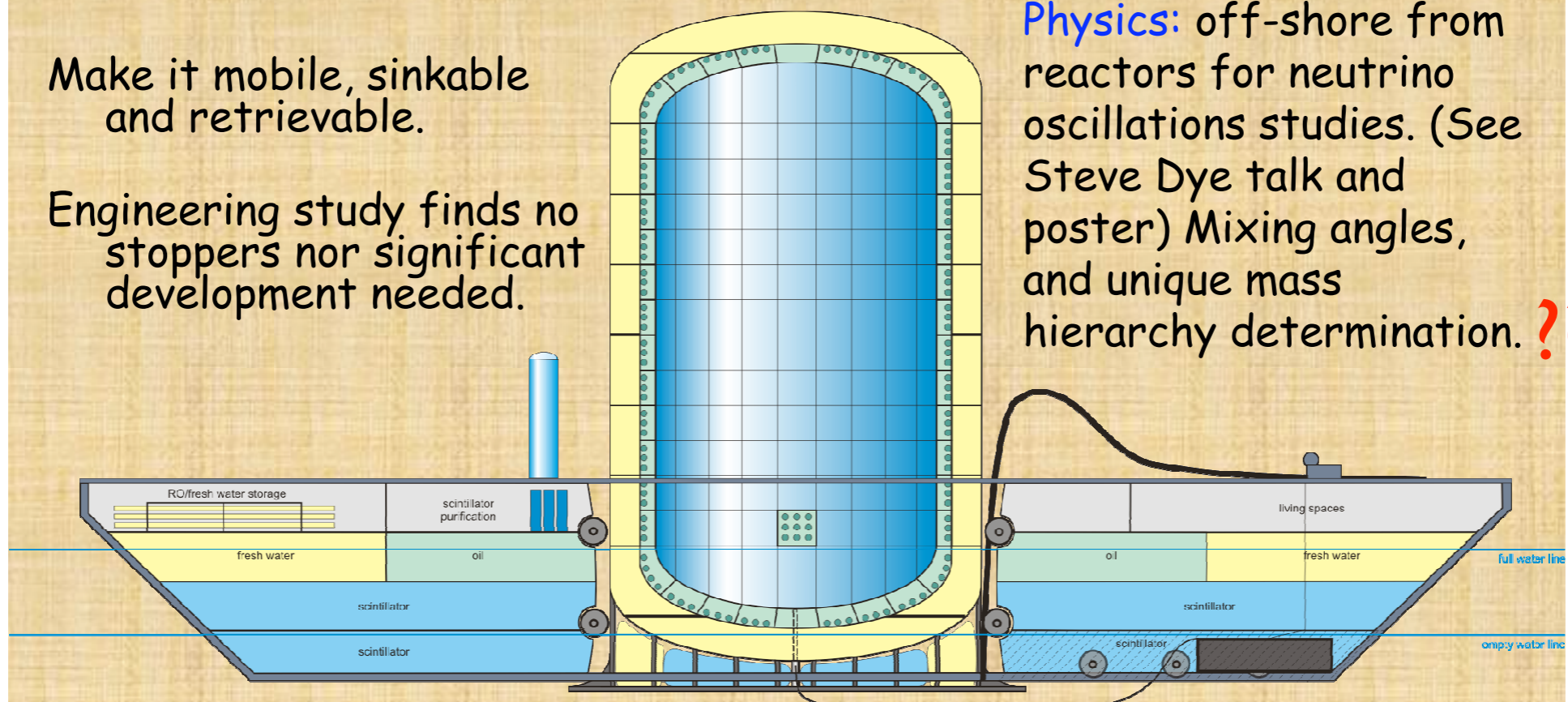**Idea:** detector based on KamLAND technology adapted for deep ocean, but >10 x larger (for good counting rate)

Make it mobile, sinkable and retrievable.

Engineering study finds no stoppers nor significant development needed.

Geology: mid-Pacific and elsewhere for geo-neutrinos from mantle.

Physics: off-shore from reactors for neutrino oscillations studies. (See Steve Dye talk and poster) Mixing angles, and unique mass hierarchy determination. **???**
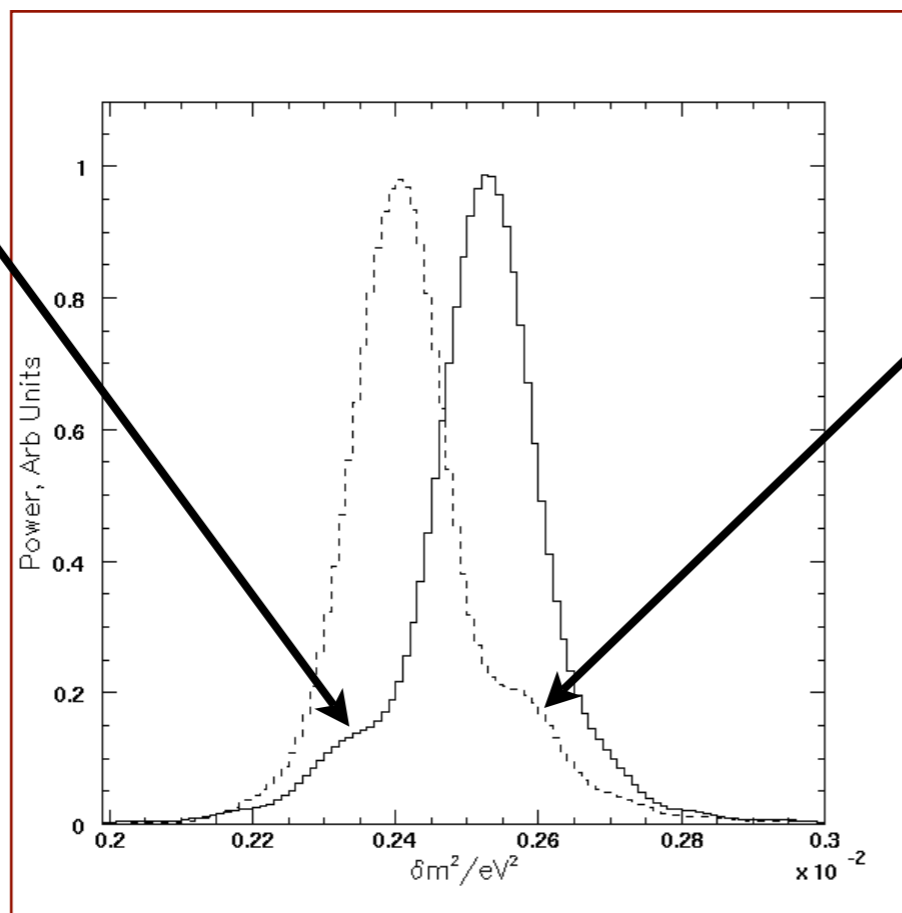

RO/fresh water storage

scintillator purification

fresh water

oil

scintillator

scintillator

living spaces

oil

fresh water

scintillator

scintillator

full water line

empty water line

Fourier Transforms: Hanohano

$$P(\bar{\nu}_e \to \bar{\nu}_e) \;=\; 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21}$$

$$-\sin^2 2\theta_{13}\left[\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}\right]$$

dominant frequency

sub-dominant frequency
(1/5 the power)

NH:
shoulder at
smaller freq.

IH:
shoulder at
higher freq.

$\sin^2 2\theta_{13} > 0.05$ for 10 Kton-yr

$\sin^2 2\theta_{13} > 0.02$ for 100 Kton-yr

NH v IH :

Disappearance

L= 55.km σ= 3.5%/sqrt(E/MeV)

$P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$

$\Delta m^2_{ee}(IH) = 1.008 * \Delta m^2_{ee}(NH)$

E (MeV)

Disappearance

L= 55.km σ= 3.5%/sqrt(E/MeV)

< 2 %

% Difference

$\Delta m^2_{ee}(IH) = 1.008 * \Delta m^2_{ee}(NH)$

E (MeV)
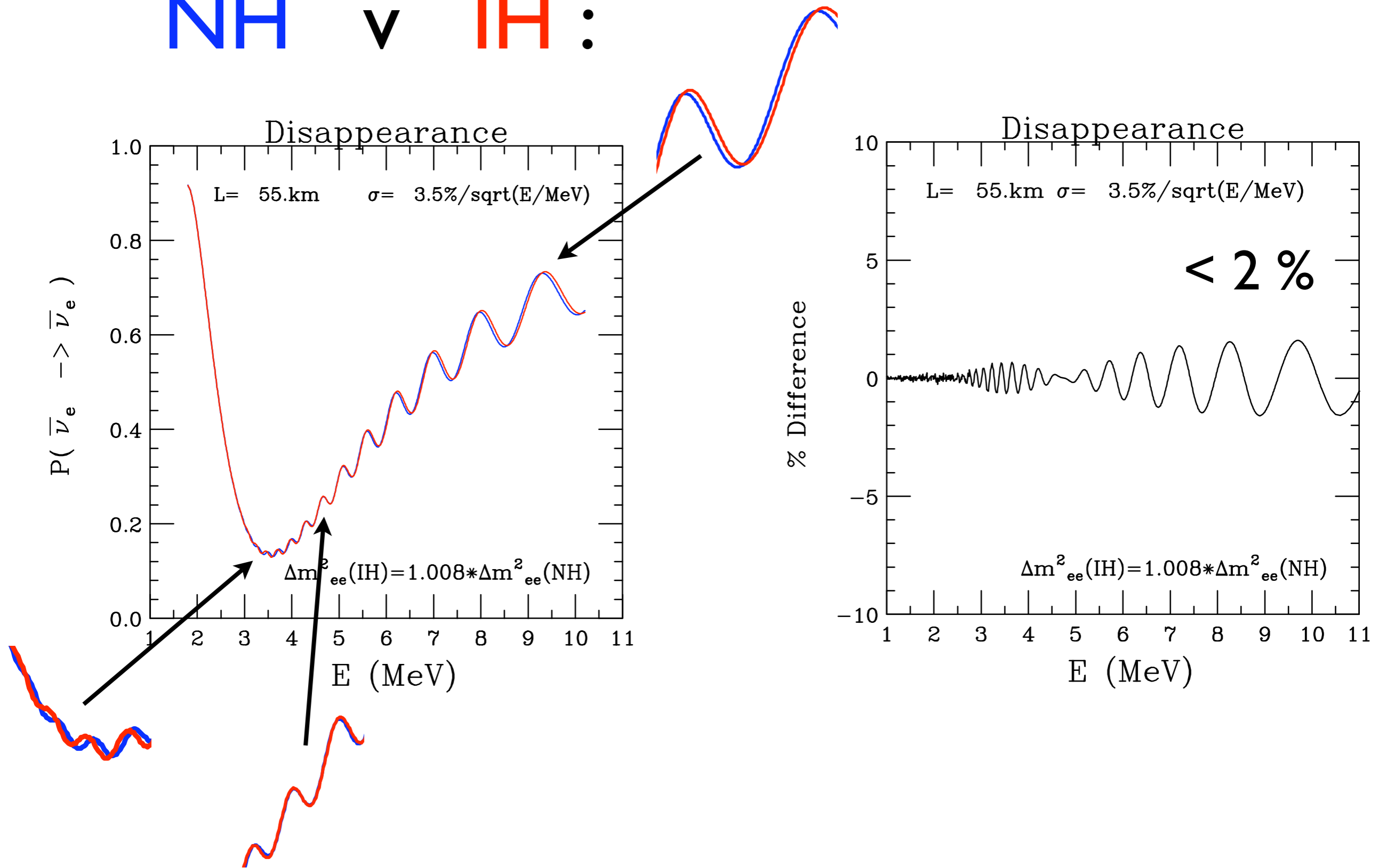
Uncertainty in E scale ???
between 2 and 8 MeV !!!

$$\frac{P_{IH}(E_{obs}) - P_{NH}(E_{true})}{P_{NH}(E_{true})}\%$$

E scale

NH -> IH   IH -> NH

+/-2%   +/-0.6%

$E_{obs} - E_{true}$ (KeV)

E_{true} (MeV)

Disappearance

L= 55.km σ= 3.5%/sqrt(E/MeV)

< 0.5%

% Difference

$\Delta m^2_{ee}(IH) = 1.008 * \Delta m^2_{ee}(NH)$

E (MeV)
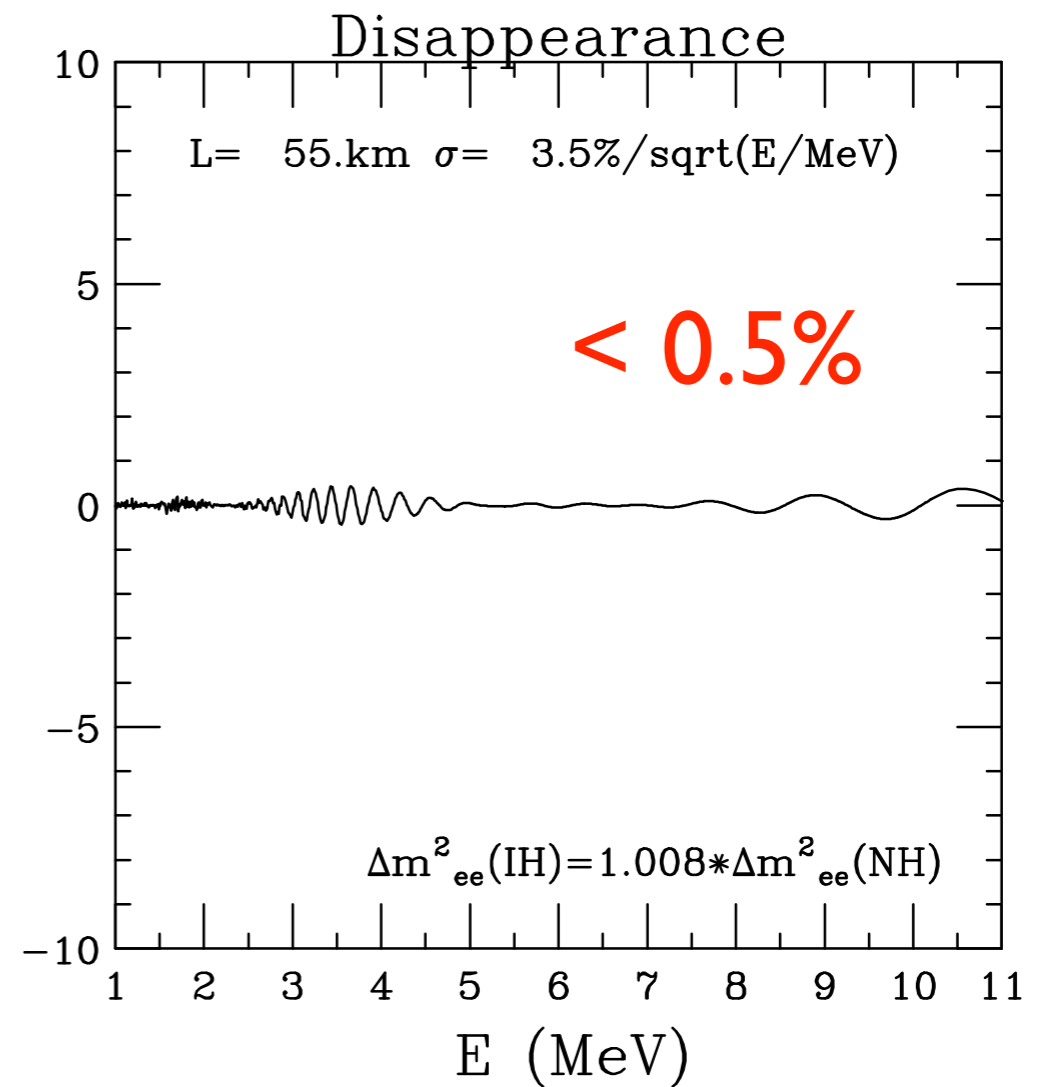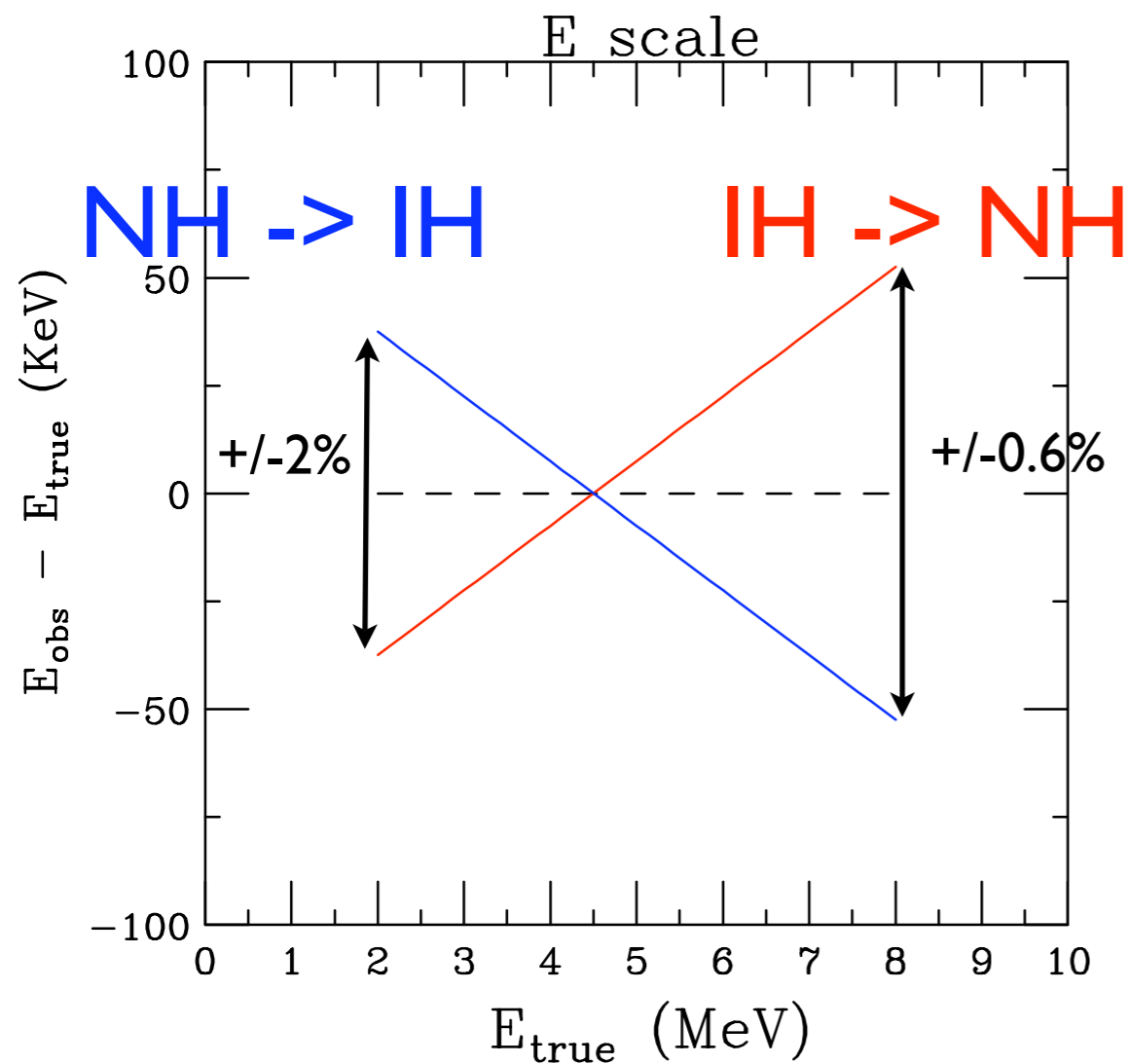
$E_{obs} = E_{true} + 0.015 \times (E_{true} - 4.5)$

$E_{obs} = E_{true} - 0.015 \times (E_{true} - 4.5)$

Summary & Conclusions

The phase advancement or retardation of the atmospheric oscillation allows for the possibly determination of the neutrino mass hierarchy in $\bar{\nu}$ disappearance experiments: but it's quite a challenge:

- Even for monochromatic $\bar{\nu}$ beams (Mossbauer) this would require a high precision measurement of $\delta m^2_\square$ around the first oscillation minimum as well as a determination of the phase 20 or so oscillations out !

  Challenging, but the high event rate that maybe possible with Mossbauer neutrinos could make this possible with modest size detectors.

- Reactor neutrinos using multi-cycle analyses (Fourier) requires high precision relative determination of the neutrino energy from 2 to 8 MeV.

  E.g. what you call a "6 MeV neutrino" must have twice the energy of what you call a "3 MeV neutrino" to about 1%, otherwise the hierarchies can be confused. This requirement is very challenging for reactor neutrinos.

For 3 neutrinos the CP violating term is
$$\Delta P_{CP} = J \sin \Delta_{21} \sin \Delta_{32} \sin \Delta_{31}$$
The form of this expression is the same in vacuum and uniform matter except for replacement of the vacuum parameters with their matter counter parts.

In the limit $L \to 0$ these two expression must be are identical !!!

Use this fact to derive a relationship between
the vacuum parameters and the matter counter parts
similar to what we have for 2 neutrinos
$$\delta m_0^2 \, \sin 2\theta_0 = \delta m_N^2 \, \sin 2\theta_N?$$

1966

ISAAC ASIMOV
The Neutrino

And yet the nothing-particle is not a nothing at all

We are "due" for a supernova anytime now we can only hope that it will hold off until the science of neutrino astronomy is further advanced.