Supersymmetric extensions of the Standard Model

(Lecture 4 of 4)

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Outline

Lecture 1: Introducing SUSY

Lecture 2: SUSY Higgs sectors

Lecture 3: Superpartner spectra and detection

Lecture 4: Measuring spins, couplings, and masses

SUSY makes very sharp predictions for the spins and (some) couplings of the SUSY partners. Need to measure these to test SUSY.
- Supermultiplets: partners have "opposite" spins *
- SM gauge $\leftrightarrow$ gaugino couplings

Soft SUSY breaking parameters at EW scale + RGE running reconstruct high-scale theory. Need to measure these to shed light on deeper fundamental physics.
- SUSY particle masses
- Some mixing parameters (show up in coupling strengths)

Why spin matters:

PHYSICAL REVIEW D **66**, 056006 (2002)

Bosonic supersymmetry? Getting fooled at the CERN LHC

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(Received 3 June 2002; published 23 September 2002)

Universal Extra Dimensions (UED): "partners" have same spins as corresponding SM particles.

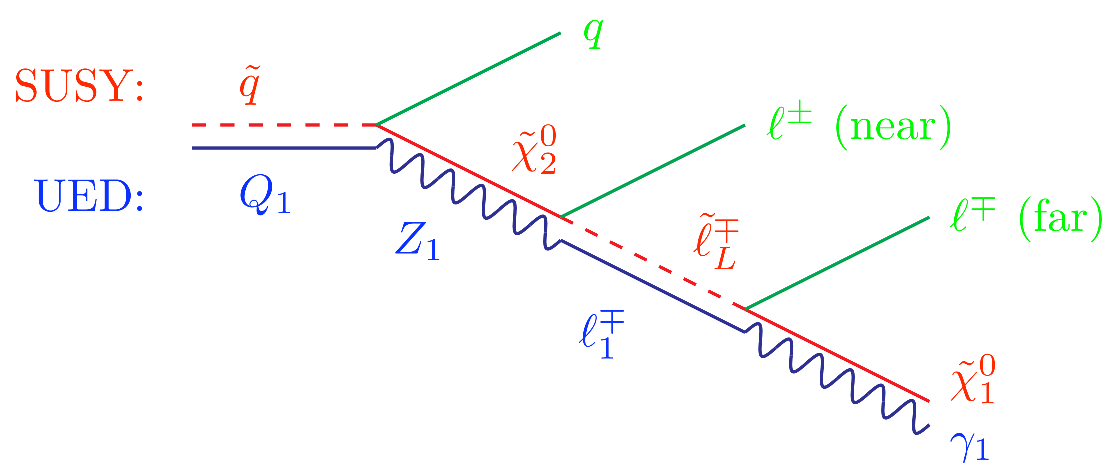

diagram from Battaglia, Datta, De Roeck, Kong, & Matchev, hep-ph/0507284

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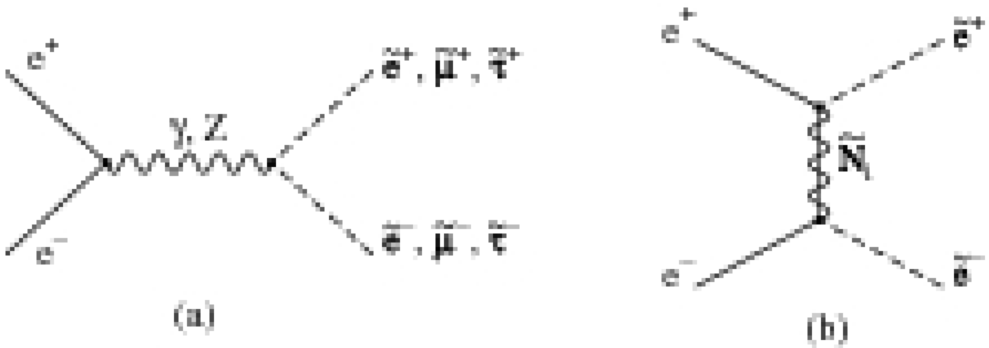
- Some mixing parameters (show up in coupling strengths)

Focus on mass extraction.

To show how ideas work, I'll start with techniques at an e^+e^- collider (ILC), then talk about LHC.

SUSY masses at ILC

Consider $e^+e^- \to$ slepton pairs



with decays $\tilde{\ell} \to \ell \widetilde{N}_1$.

How can we measure the $\tilde{\ell}$ and $\widetilde{N}_1$ masses?

One way is to scan the beam energy across the production threshold.

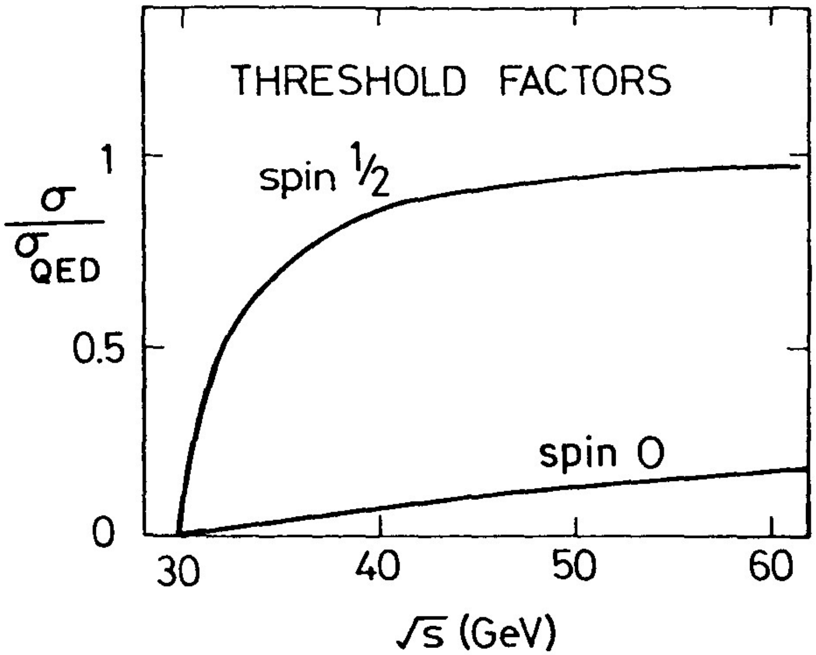


Fig. 4.7. Comparison of spin-0 and spin-$\frac{1}{2}$ particle pair production in e^+e^- collisions, for particles of mass m = 15 GeV.

Upside:
- Shape at threshold also gives you a spin measurement

Downsides:
- Takes a lot of luminosity
- Constrains what other physics you can do simultaneously (probably want to run at the highest available beam energy?)
- Can we even try to do this at LHC??

A more clever technique: use kinematic endpoints.

Consider $e^+e^- \to \tilde{\ell}_R \tilde{\ell}_R^* \to \ell^- \widetilde{N}_1 \ell^+ \widetilde{N}_1$.
- Measure maximum and minimum values of ℓ energies
- Extract $m_{\tilde{\ell}_R}$ and $m_{\widetilde{N}_1}$

Here's how it works.
(1) Consider the rest frame of one $\tilde{\ell}$. Energy and momentum conservation:

$$E_\ell + E_{\widetilde{N}} = m_{\tilde{\ell}}, \qquad \vec{p}_\ell = -\vec{p}_{\widetilde{N}}$$

Neglect the mass of ℓ. Then $E_\ell = |\vec{p}_\ell|$.
Also have $E_{\widetilde{N}} = \sqrt{m_{\widetilde{N}}^2 + \vec{p}_{\widetilde{N}}^{\,2}} = \sqrt{m_{\widetilde{N}}^2 + E_\ell^2}$.
Plug in to energy conservation equation, rearrange, and square both sides:

$$m_{\widetilde{N}}^2 + E_\ell^2 = m_{\tilde{\ell}}^2 - 2m_{\tilde{\ell}} E_\ell + E_\ell^2$$

$$\text{or} \qquad E_\ell = |\vec{p}_\ell| = \frac{m_{\tilde{\ell}}^2 - m_{\widetilde{N}}^2}{2m_{\tilde{\ell}}}$$

(2) Now we'll boost the $\tilde{\ell}$ to the collider center-of-mass frame.

$$E_{\tilde{\ell}_{R1}} + E_{\tilde{\ell}_{R2}} = \sqrt{s}, \qquad \vec{p}_{\tilde{\ell}_{R1}} = -\vec{p}_{\tilde{\ell}_{R2}}$$

Use the fact that two particles of the same mass $m_{\tilde{\ell}}$ are produced:

$$\begin{aligned} E_{\tilde{\ell}_{R1}} &= \sqrt{m_{\tilde{\ell}}^2 + \vec{p}_{\tilde{\ell}_{R1}}^{\,2}} = E_{\tilde{\ell}_{R2}} = \frac{\sqrt{s}}{2} = \gamma m_{\tilde{\ell}} \\ |\vec{p}_{\tilde{\ell}_{R1}}| &= \sqrt{\frac{s}{4} - m_{\tilde{\ell}}^2} = \gamma m_{\tilde{\ell}} |\vec{v}| \end{aligned}$$

(3) Compute E_ℓ^{CM} in the CM frame by doing the boost:

($\cos\theta^*$ is defined in $\tilde{\ell}$ rest frame)

$$E_\ell^{CM} = \gamma\left(E_\ell + \beta p_{\ell z}\right) = \gamma\left(E_\ell + \beta|\vec{p}_\ell|\cos\theta^*\right) = E_\ell\left(\gamma + \gamma|\vec{v}|\cos\theta^*\right)$$

From above we have

$$\gamma = \frac{\sqrt{s}}{2m_{\tilde{\ell}}}, \qquad \gamma|\vec{v}| = \frac{\sqrt{s - 4m_{\tilde{\ell}}^2}}{2m_{\tilde{\ell}}}$$

Put it all together:

$$E_\ell^{CM} = \frac{m_{\tilde{\ell}}^2 - m_{\tilde{N}}^2}{4m_{\tilde{\ell}}^2}\left(\sqrt{s} + \sqrt{s + 4m_{\tilde{\ell}}^2}\cos\theta^*\right)$$

Max (min) lepton energy corresponds to $\cos\theta^* = 1$ (-1).
$\sqrt{s}$ is known: collider CM energy.

Measure E_ℓ^{max} and E_ℓ^{min} from lepton kinematic distributions.
Solve for $m_{\tilde{\ell}}$ and $m_{\tilde{N}}$! A little algebra gives:

$$m_{\tilde{\ell}}^2 = \frac{s}{4}\left[1 - \left(\frac{E^{max} - E^{min}}{E^{max} + E^{min}}\right)^2\right]$$

$$m_{\tilde{N}}^2 = m_{\tilde{\ell}}^2\left[1 - \frac{2(E^{max} + E^{min})}{\sqrt{s}}\right]$$

Need to isolate data sample with only $\tilde{\ell}_R\tilde{\ell}_R$ pair production: can use e^+e^- beam polarization to suppress $\tilde{\ell}_L\tilde{\ell}_L$ and W^+W^- background.

In practice, things are a little more complicated.

Ex: $e^+e^- \to \tilde{\mu}^+_{L,R}\tilde{\mu}^-_{L,R}$ with $m_{\tilde{\mu}_R} = 178$ GeV, $m_{\tilde{\mu}_L} = 287$ GeV
Note the muon energy edges at about 65 and 220 GeV.

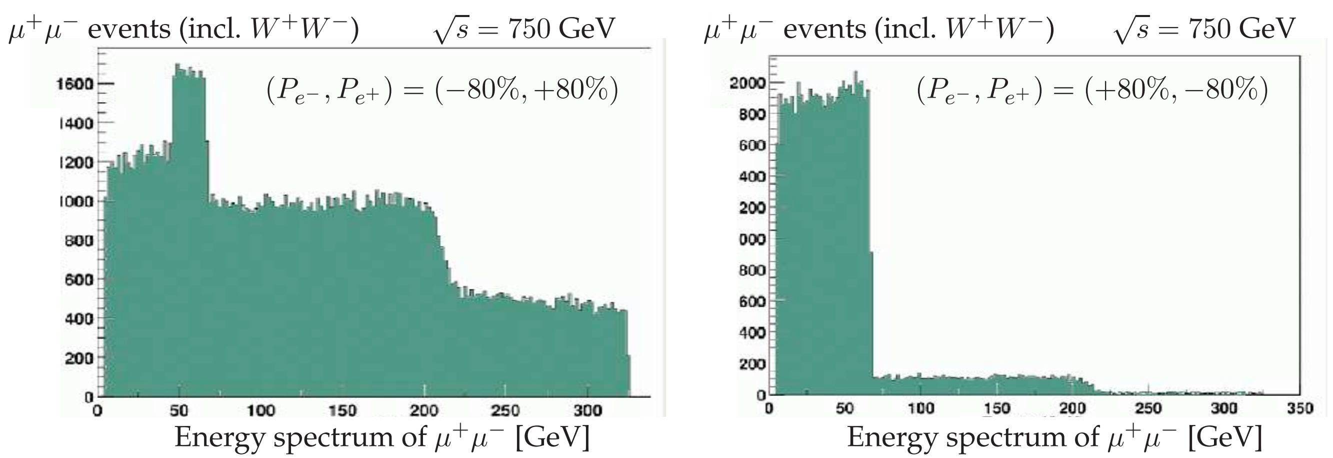


Figure 3.4: Energy spectrum of muons from $\tilde{\mu}_{L,R}$ decays into $\mu\tilde{\chi}^0_1$ final states, including the W^+W^- background decaying into $\mu\nu$ final states in the scenario S3, cf. table 3.1, for two combinations of beam polarizations for $\sqrt{s} = 750$ GeV and $\mathcal{L}_{\rm int} = 500$ fb^{-1} [87].

from hep-ph/0507011

These plots also demonstrate effect of beam polarization:
RH e^- and LH e^+ eliminate large t-channel W^+W^- background.
Beam pol also changes the strength of the Z^* contribution:
different effect on $\tilde{\mu}_L$ and $\tilde{\mu}_R$ pair production.

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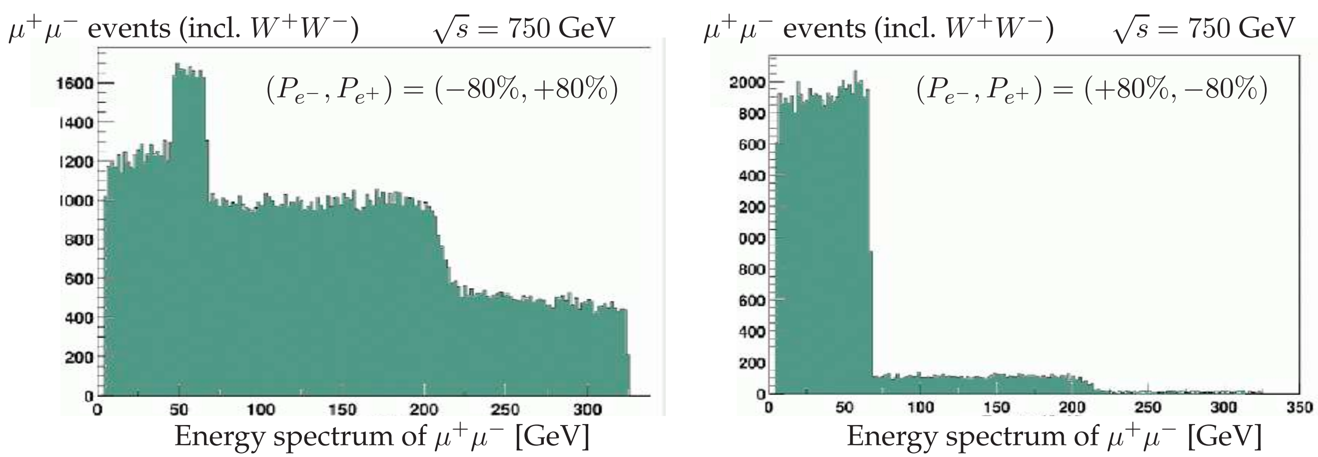

Figure 3.4: Energy spectrum of muons from $\tilde{\mu}_{L,R}$ decays into $\mu\tilde{\chi}^0_1$ final states, including the W^+W^- background decaying into $\mu\nu$ final states in the scenario S3, cf. table 3.1, for two combinations of beam polarizations for $\sqrt{s} = 750$ GeV and $\mathcal{L}_{\text{int}} = 500$ fb^{-1} [87].

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Eyeballing the endpoints:

$\tilde{\mu}_L$: $E^{max} \approx 220$ GeV, $E^{min} \approx 65$ GeV (note pol'n dep $\to \tilde{\mu}_L$)

$\tilde{\mu}_R$: $E^{max} \approx 65$ GeV, E^{min} not visible!

Solve: get $m_{\tilde{\mu}_L}$ and $m_{\tilde{N}}$ from $\tilde{\mu}_L$ endpoints; plug in $m_{\tilde{N}}$ to get $m_{\tilde{\mu}_R}$ from E^{max}

$m_{\tilde{\mu}_L} \approx 282$ GeV (compare input 287 GeV)

$m_{\tilde{N}_1} \approx 153$ GeV

$m_{\tilde{\mu}_R} \approx 167$ GeV (compare input 178 GeV)

Why are the lepton energy distributions flat?

Take another look at the formula:

$$E_\ell^{CM} = \frac{m_{\tilde{\ell}}^2 - m_{\tilde{N}}^2}{4m_{\tilde{\ell}}^2}\left(\sqrt{s} + \sqrt{s + 4m_{\tilde{\ell}}^2}\cos\theta^*\right)$$

We're asking about the differential cross section,

$$\frac{d\sigma}{dE_\ell^{CM}} = \frac{d\sigma}{d\cos\theta^*}\frac{d\cos\theta^*}{dE_\ell^{CM}}$$

$d\cos\theta^*/dE_\ell^{CM}$ is a constant.

$d\sigma/d\cos\theta^*$ is the $\tilde{\ell}$ decay distribution in the $\tilde{\ell}$ rest frame.

- $\tilde{\ell}$ is a scalar: it can't single out any direction.
  $\rightarrow$ uniform decay distribution over the solid angle:

$$\frac{d\sigma}{d\cos\theta^* d\phi^*} = \text{const}$$

Integrating over the ϕ^* angle gives us what we want to know:
$d\sigma/dE_\ell^{CM}$ is flat (with endpoints).

SUSY masses at the LHC

Difficult:

- Missing p_T: don't know boost of CM along beam direction.
- *Two* invisible particles: know only the sum of their missing p_T.

But: LHC can produce heavy sparticles: long decay chains, many kinematic variables to play with.
Since we don't know the boost of individual events, want to use kinematic invariants, like invariant masses.

Consider the decay chain $\widetilde{N}_2 \to \tilde{\ell}_R^{\pm} \ell^{\mp} \to \widetilde{N}_1 \ell^+ \ell^-$
(First need to select events that contain a $\widetilde{N}_2$ and identify the $\ell^+\ell^-$ coming from the $\widetilde{N}_2$ decay.)
Invariant observable: invariant mass of $\ell^+\ell^-$: $M_{\ell\ell}$

How is this related to the SUSY masses?

Considering the decay chain $\widetilde{N}_2 \to \tilde{\ell}_R^{\pm}\ell^{\mp} \to \widetilde{N}_1 \ell^+ \ell^-$
Momentum and energy conservation in each decay:

$$p_{\widetilde{N}_2} = p_{\ell_1} + p_{\tilde{\ell}} \qquad\qquad p_{\tilde{\ell}} = p_{\ell_2} + p_{\widetilde{N}_1}$$

Combine and rearrange:

$$M_{\ell\ell}^2 = (p_{\ell_1} + p_{\ell_2})^2 = (p_{\widetilde{N}_2} - p_{\widetilde{N}_1})^2 = m_{\widetilde{N}_2}^2 + m_{\widetilde{N}_1}^2 - 2\vec{p}_{\widetilde{N}_2} \cdot \vec{p}_{\widetilde{N}_1}$$

What is this? Let's work in the $\widetilde{N}_2$ rest frame (can do that because we're calculating kinematic invariants!)

$\to$ $p_{\widetilde{N}_2} \cdot p_{\widetilde{N}_1} = m_{\widetilde{N}_2} E_{\widetilde{N}_1}$ where $E_{\widetilde{N}_1}$ is energy in the $\widetilde{N}_2$ rest frame, so

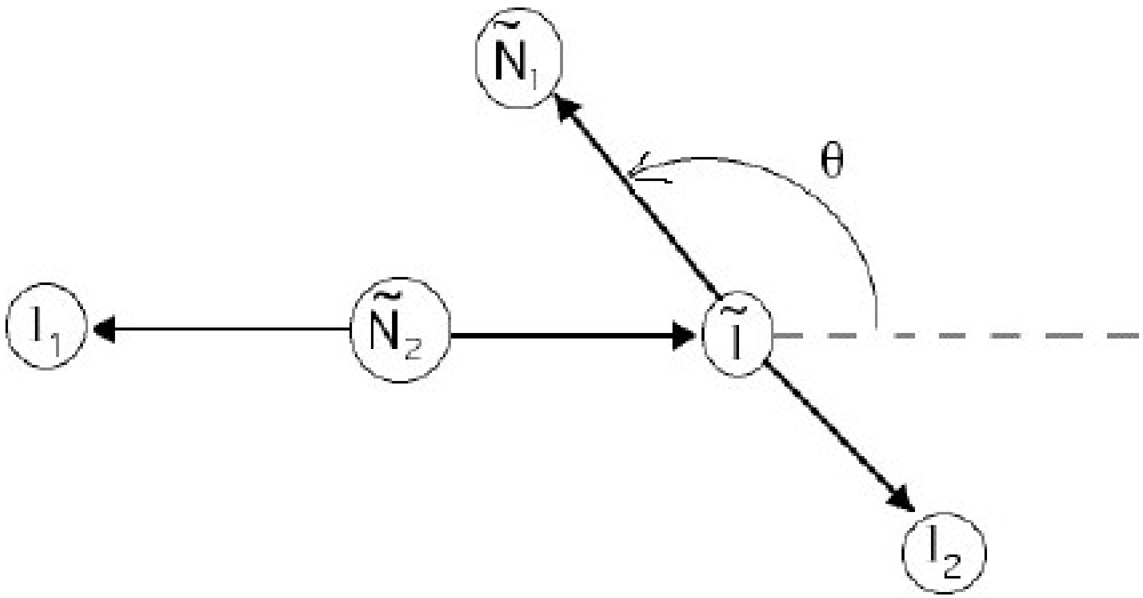
$$M_{\ell\ell}^2 = m_{\widetilde{N}_2}^2 + m_{\widetilde{N}_1}^2 - 2m_{\widetilde{N}_2} E_{\widetilde{N}_1}$$

Now we need to find the kinematic endpoint(s) of $E_{\widetilde{N}_1}$ in the $\widetilde{N}_2$ rest frame in terms of the SUSY masses.

Strategy:

Relate the energies to masses and the $\tilde{\ell}$ decay angle θ

Relate the energies to masses and the $\tilde{\ell}$ decay angle θ in $\widetilde{N}_2$ rest frame.



Look at $\widetilde{N}_2$ decay: $\quad m_{\widetilde{N}_2} = E_{\ell_1} + E_{\tilde{\ell}}, \qquad \vec{p}_{\ell_1} = -\vec{p}_{\tilde{\ell}}$

Solve using four-momentum conservation (with $m_\ell \simeq 0$):

$$E_{\ell_1} = \frac{1}{2m_{\widetilde{N}_2}}\left(m^2_{\widetilde{N}_2} - m^2_{\tilde{\ell}}\right) \qquad |\vec{p}_{\ell_1}| = E_{\ell_1}$$

$$E_{\tilde{\ell}} = \frac{1}{2m_{\widetilde{N}_2}}\left(m^2_{\widetilde{N}_2} + m^2_{\tilde{\ell}}\right) \qquad |\vec{p}_{\tilde{\ell}}| = |\vec{p}_{\ell_1}| = E_{\ell_1}$$

Now let's do the $\tilde{\ell}$ decay in the $\tilde{\ell}$ rest frame (denoted by a star – will need to boost back to the $\widetilde{N}_2$ rest frame at the end!)

4-momentum conservation: $m_{\tilde{\ell}} = E^*_{\ell_2} + E^*_{\widetilde{N}_1}$, $\qquad \vec{p}^*_{\ell_1} = -\vec{p}^*_{\widetilde{N}_1}$

$$E^*_{\ell_2} = \frac{1}{2m_{\tilde{\ell}}}\left(m^2_{\tilde{\ell}} - m^2_{\widetilde{N}_1}\right) \qquad |\vec{p}^*_{\ell_2}| = E^*_{\ell_2}$$

$$E^*_{\widetilde{N}_1} = \frac{1}{2m_{\tilde{\ell}}}\left(m^2_{\tilde{\ell}} + m^2_{\widetilde{N}_1}\right) \qquad |\vec{p}^*_{\widetilde{N}_1}| = |\vec{p}^*_{\ell_2}| = E^*_{\ell_2}$$

Have $E^*_{\widetilde{N}_1}$ in the $\tilde{\ell}$ rest frame; need to boost to $\widetilde{N}_2$ rest frame.

Work out the kinematic boost from the $\tilde{\ell}$ energy and momentum:

$$\gamma = \frac{E_{\tilde{\ell}}}{m_{\tilde{\ell}}} = \frac{m^2_{\widetilde{N}_2} + m^2_{\tilde{\ell}}}{2m_{\widetilde{N}_2} m_{\tilde{\ell}}}, \qquad \gamma\beta = \frac{|\vec{p}_{\tilde{\ell}}|}{m_{\ell}} = \frac{m^2_{\widetilde{N}_2} - m^2_{\tilde{\ell}}}{2m_{\widetilde{N}_2} m_{\tilde{\ell}}}$$

Now do the boost:

$$E_{\widetilde{N}_1} = \gamma\left(E^*_{\widetilde{N}_1} + \beta|\vec{p}^*_{\widetilde{N}_1}|\cos\theta^*\right)$$

where θ^* is the angle between the $\tilde{\ell}$ decay direction and the $\tilde{\ell}$ boost (in the $\tilde{\ell}$ rest frame)

Plug in γ and $\gamma\beta$:

$$E_{\widetilde{N}_1} = \frac{1}{4m_{\widetilde{N}_2} m^2_{\widetilde{\ell}}} \left[\left(m^2_{\widetilde{N}_2} + m^2_{\widetilde{\ell}} \right) \left(m^2_{\widetilde{\ell}} + m^2_{\widetilde{N}_1} \right) + \left(m^2_{\widetilde{N}_2} - m^2_{\widetilde{\ell}} \right) \left(m^2_{\widetilde{\ell}} - m^2_{\widetilde{N}_1} \right) \cos\theta^* \right]$$

Remember our original formula for the $\ell\ell$ invariant mass:

$$M^2_{\ell\ell} = m^2_{\widetilde{N}_2} + m^2_{\widetilde{N}_1} - 2m_{\widetilde{N}_2} E_{\widetilde{N}_1}$$

Kinematic endpoint: the maximum of $M_{\ell\ell}$ corresponds to the minimum of $E_{\widetilde{N}_1}$, which occurs for $\cos\theta^* = -1$:

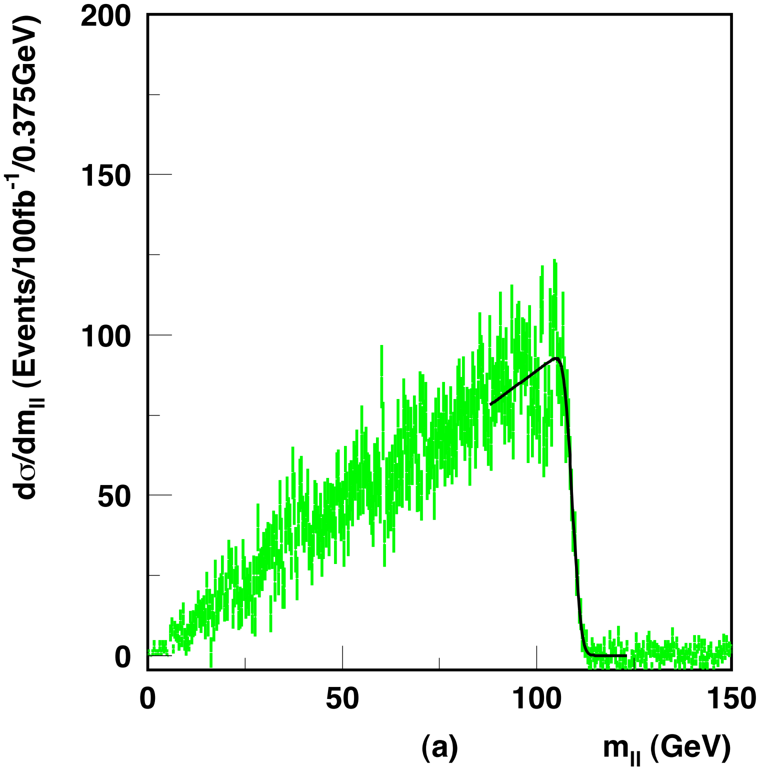
$$E_{\widetilde{N}_1} \Big|^{\text{min}} = \frac{1}{2m_{\widetilde{N}_2} m^2_{\widetilde{\ell}}} \left(m^4_{\widetilde{\ell}} + m^2_{\widetilde{N}_2} m^2_{\widetilde{N}_1} \right)$$

Plugging in to $M^2_{\ell\ell}$ formula and simplifying gives

$$M_{\ell\ell}|^{\text{max}} = \left[\frac{\left(m^2_{\widetilde{N}_2} - m^2_{\widetilde{\ell}} \right) \left(m^2_{\widetilde{\ell}} - m^2_{\widetilde{N}_1} \right)}{m^2_{\widetilde{\ell}}} \right]^{1/2} .$$

One endpoint measurement constrains a combination of three SUSY masses.

$$M_{\ell\ell}|^{\mathrm{max}} = \left[\frac{\left(m^2_{\widetilde{N}_2} - m^2_{\widetilde{\ell}}\right)\left(m^2_{\widetilde{\ell}} - m^2_{\widetilde{N}_1}\right)}{m^2_{\widetilde{\ell}}}\right]^{1/2}$$



from Paige, hep-ph/0211017

LHC can do more if we look at longer decay chains:
$\rightarrow$ more kinematic invariants to play with.

Add a squark to the top of our decay chain:

$$\tilde{q} \rightarrow \widetilde{N}_2 q \rightarrow \tilde{\ell}^{\pm}\ell^{\mp} q \rightarrow \widetilde{N}_1 \ell^+ \ell^- q$$

Invariant mass of q and the first lepton emitted (ℓ_1) has an endpoint analogous to the $\ell\ell$ endpoint:

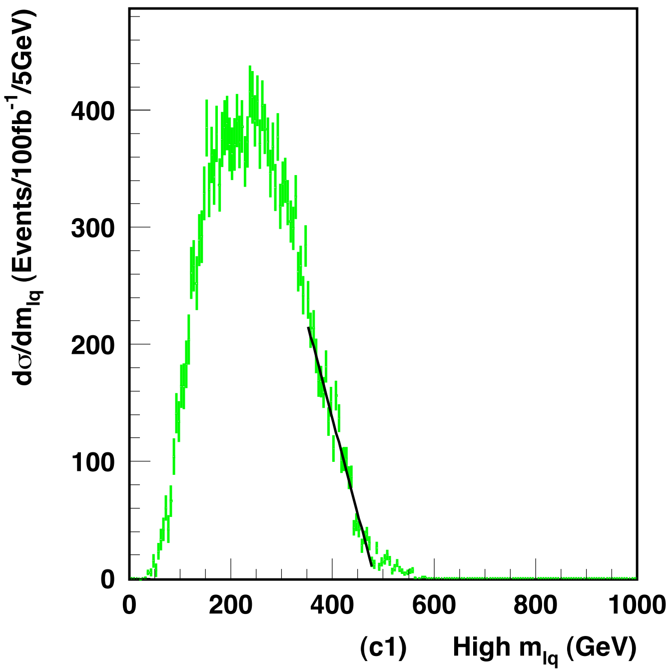
$$M_{q\ell_1}\Big|^{\max} = \left[\frac{\left(m^2_{\tilde{q}} - m^2_{\widetilde{N}_2}\right)\left(m^2_{\widetilde{N}_2} - m^2_{\tilde{\ell}}\right)}{m^2_{\widetilde{N}_2}}\right]^{1/2}$$

How to distinguish ℓ_1 from ℓ_2?

$\rightarrow$ ℓ_1 likely to have higher energy.

With $M_{q\ell_1}|^{\max}$ and $M_{\ell\ell}|^{\max}$ we have 2 measurements and 4 unknowns.

Not doing better than before... yet.

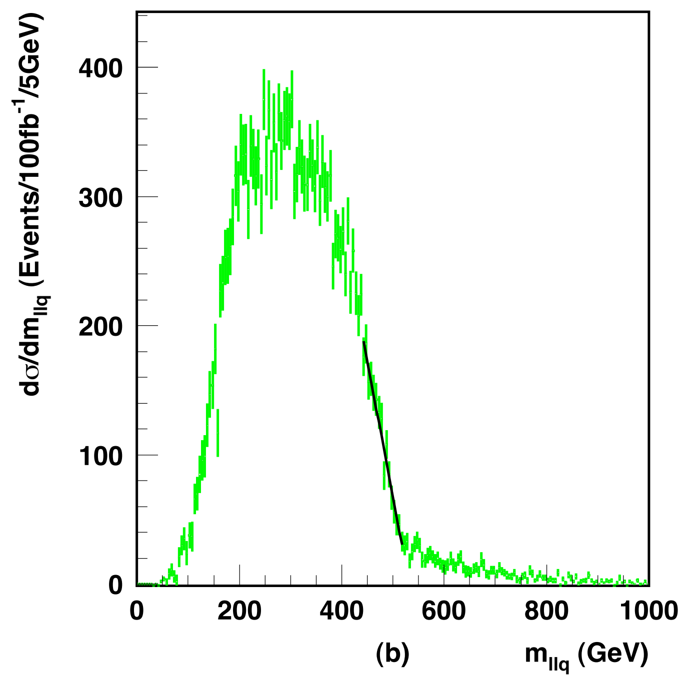


from Paige, hep-ph/0211017

Decay chain has an extra kinematic invariant:

Invariant mass of $q\ell^+\ell^-$.

$$M_{q\ell\ell}|^{\max} = \left[\frac{\left(m_{\tilde{q}}^2 - m_{\tilde{N}_2}^2\right)\left(m_{\tilde{N}_2}^2 - m_{\tilde{N}_1}^2\right)}{m_{\tilde{N}_2}^2}\right]^{1/2}$$

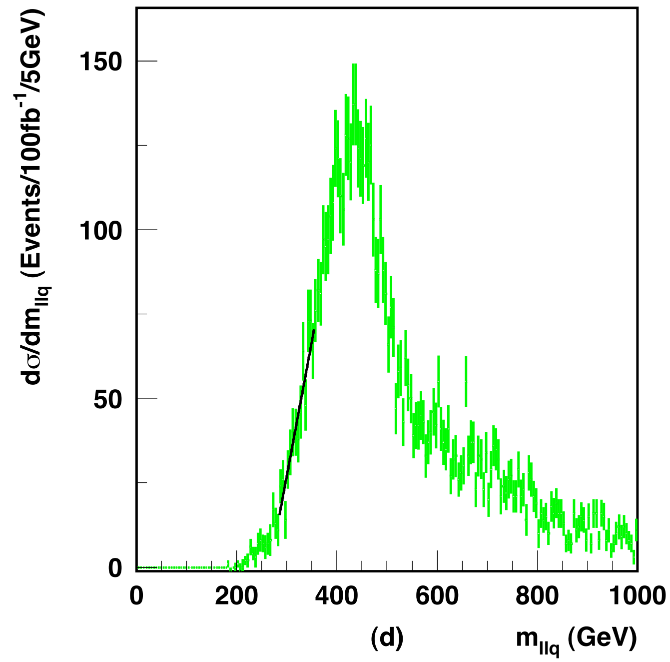


from Paige, hep-ph/0211017

3 measurements and 4 unknowns.

Doing better!

There are also lower kinematic edges:



from Paige, hep-ph/0211017

After applying a cut $M_{\ell\ell} > M_{\ell\ell}^{\text{max}}/\sqrt{2}$, get a complicated formula for a lower kinematic endpoint for $M_{q\ell\ell}$.

Can also consider the decay chain $\tilde{q} \to \widetilde{N}_2 q \to \widetilde{N}_1 hq$ with $h \to b\bar{b}$

[The Higgs mass can be measured elsewhere]

Then M_{hq} has a threshold (lower kinematic edge)

Get enough measurables to extract all the masses!

Uncertainties from blurring of the kinematic endpoints by backgrounds, wrong jet/lepton combinations, also gluon radiation off the jet at NLO.

Kinematic endpoints:

- Need long decay chains, good statistics
- Subject to background, resolution, QCD radiation smearing

Can we do better? Lots of recent progress:

Review: Barr & Lester, arXiv:1004.2732

Exact kinematic relations:

Completely solve the kinematics of each SUSY cascade decay.
Need on-shell intermediates, reasonably long decay chains.

Kawagoe, Nojiri, Polesello, PRD 71, 035008 (2005), Cheng et al, PRL 100, 252001 (2008)

Minima, maxima, kinks, and cusps:

Find mass relations, upper and lower bounds from dependence of new observables on unknown fit variables.
MT2, MT2 kinks, M_{2C}, $\sqrt{\hat{s}}_{\text{min}}$, etc.

Exact kinematic relations Kawagoe, Nojiri, & Polesello, PRD 71, 035008 (2005)

Completely solve the kinematics of each SUSY cascade decay.
- Selected events must be from one particular decay chain
- SUSY particles in the decay chain must be on mass shell

Each event gives you the 4-momenta of all the decay products except $\widetilde{N}_1$.

Have to consider a longer decay chain: $\tilde{g} \to q\tilde{q} \to qq\widetilde{N}_2 \to qq\ell\tilde{\ell} \to qq\ell\ell\widetilde{N}_1$. 5 sparticles involved $\to$ 5 mass-shell conditions:

$$m^2_{\widetilde{N}_1} = p^2_{\widetilde{N}_1} \qquad m^2_{\tilde{\ell}} = (p_{\widetilde{N}_1} + p_{\ell_1})^2 \qquad m^2_{\widetilde{N}_2} = (p_{\widetilde{N}_1} + p_{\ell_1} + p_{\ell_2})^2$$

$$m^2_{\tilde{q}} = (p_{\widetilde{N}_1} + p_{\ell_1} + p_{\ell_2} + p_{q_1})^2 \qquad m^2_{\tilde{g}} = (p_{\widetilde{N}_1} + p_{\ell_1} + p_{\ell_2} + p_{q_1} + p_{q_2})^2$$

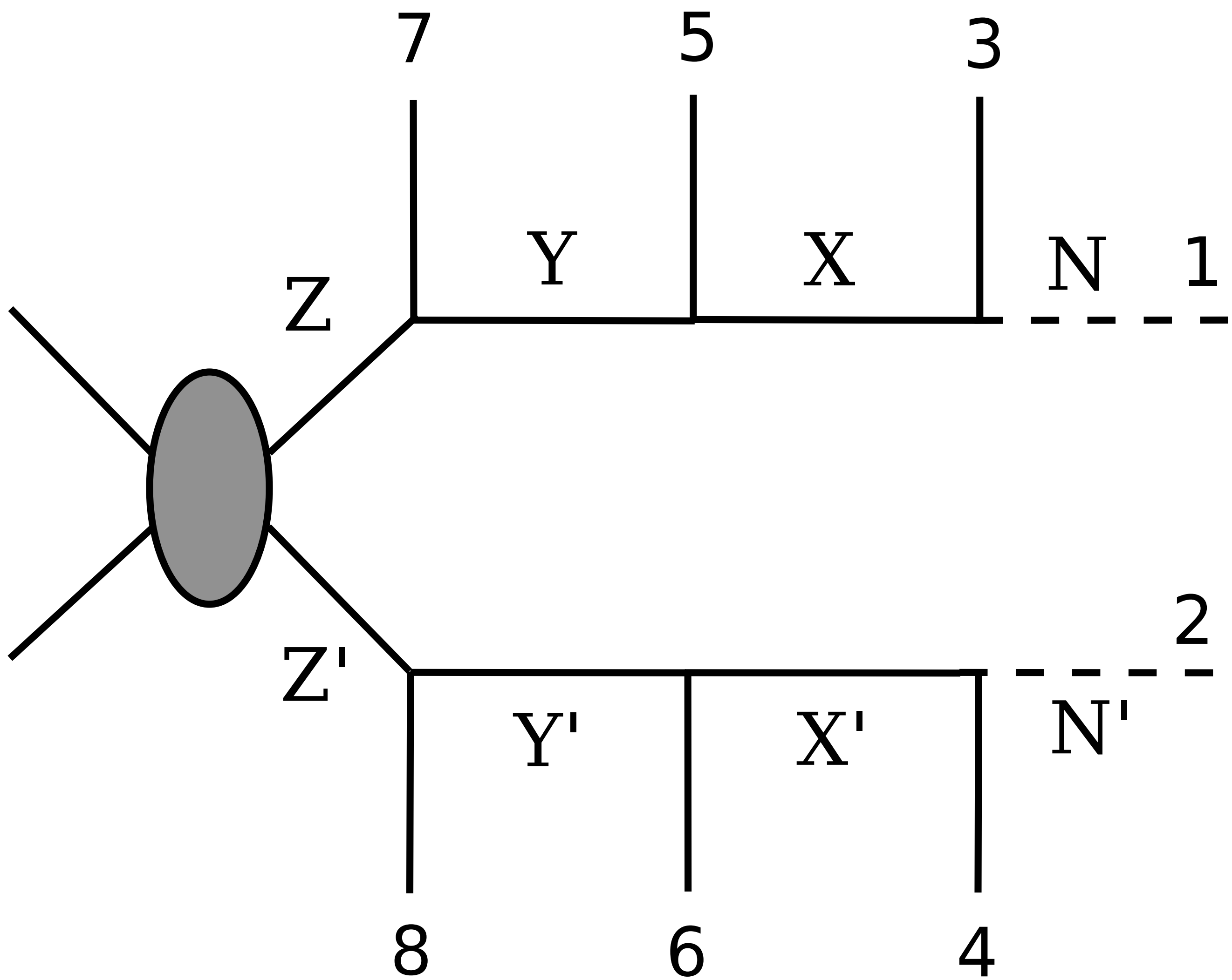
Each $qq\ell\ell\widetilde{N}_1$ event contains 4 unmeasured degrees of freedom, the 4 components of the $\widetilde{N}_1$ 4-momentum.

$\to$ Each event picks out a 4-dimensional hypersurface in a 5-dimensional mass parameter space.

Overlap multiple events in this hyperspace $\to$ find a discrete set of solutions from overlap of different hypersurfaces.

Exact kinematic relations II Cheng et al, PRL 100, 252001 (2008)

Solve shorter chains by using both sides of the event.



6 constraint equations from one event:

$$\begin{aligned}
(M_Z^2 \;=)\quad (p_1+p_3+p_5+p_7)^2 &= (p_2+p_4+p_6+p_8)^2, \\
(M_Y^2 \;=)\quad (p_1+p_3+p_5)^2 &= (p_2+p_4+p_6)^2, \\
(M_X^2 \;=)\quad (p_1+p_3)^2 &= (p_2+p_4)^2, \\
(M_N^2 \;=)\quad p_1^2 &= p_2^2.
\end{aligned}$$

$$p_1^x + p_2^x = p_{miss}^x, \quad p_1^y + p_2^y = p_{miss}^y.$$

8 unknown components of missing (invisible) particle 4-momenta (p_1 and p_2)

Still 2 unknowns: cannot solve.

Add a second event: 8 more unknowns (q_1 and q_2) but 10 more equations:

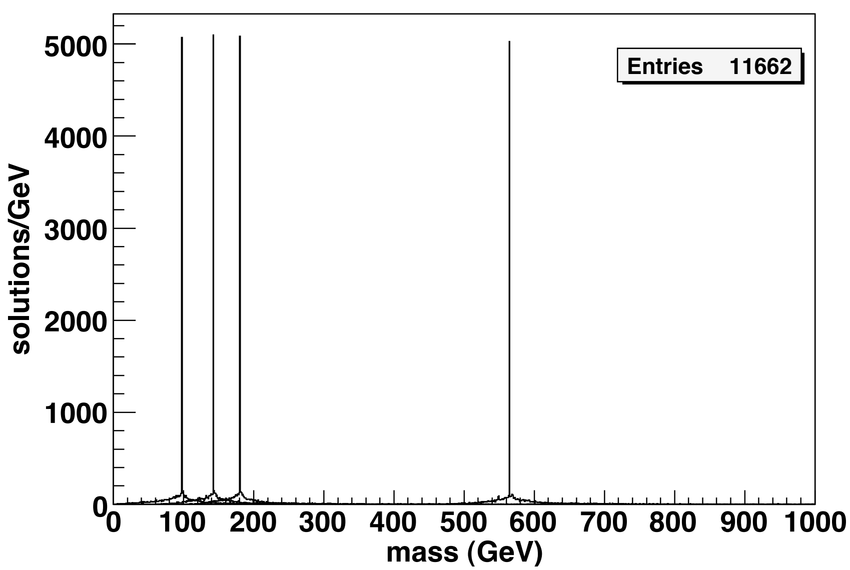
$$\begin{aligned}
q_1^2 &= q_2^2 &&= p_2^2, \\
(q_1 + q_3)^2 &= (q_2 + q_4)^2 &&= (p_2 + p_4)^2, \\
(q_1 + q_3 + q_5)^2 &= (q_2 + q_4 + q_6)^2 &&= (p_2 + p_4 + p_6)^2, \\
(q_1 + q_3 + q_5 + q_7)^2 &= (q_2 + q_4 + q_6 + q_8)^2 &&= (p_2 + p_4 + p_6 + p_8)^2,
\end{aligned}$$

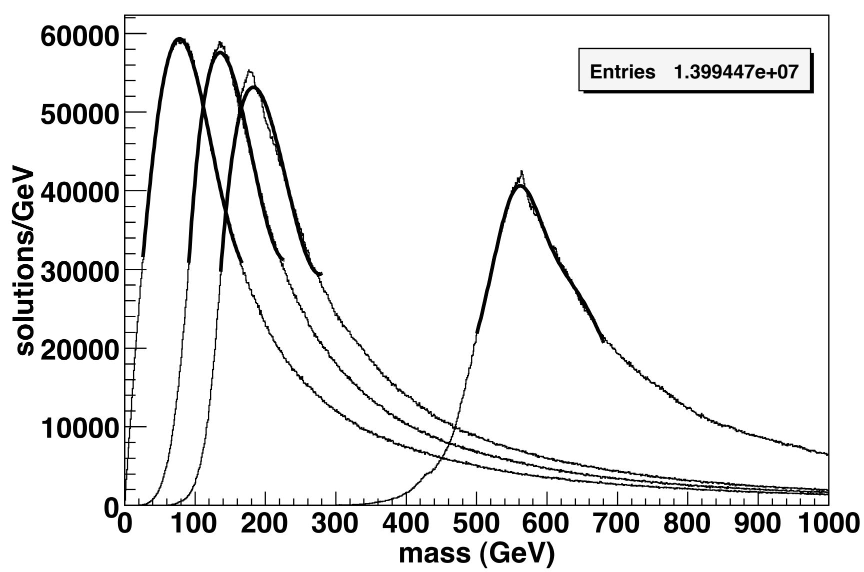
$$q_1^x + q_2^x = q_{miss}^x, \qquad q_1^y + q_2^y = q_{miss}^y.$$

Can invert for the masses directly!

SPS1a: Ideal from 100 events (no combinatorics or resolution)

300 fb^{-1} after ATLFAST, combinatorics, some cuts to reduce wrong combinations





Cheng et al, PRL 100, 252001 (2008)

Can reconstruct genuine mass peaks!

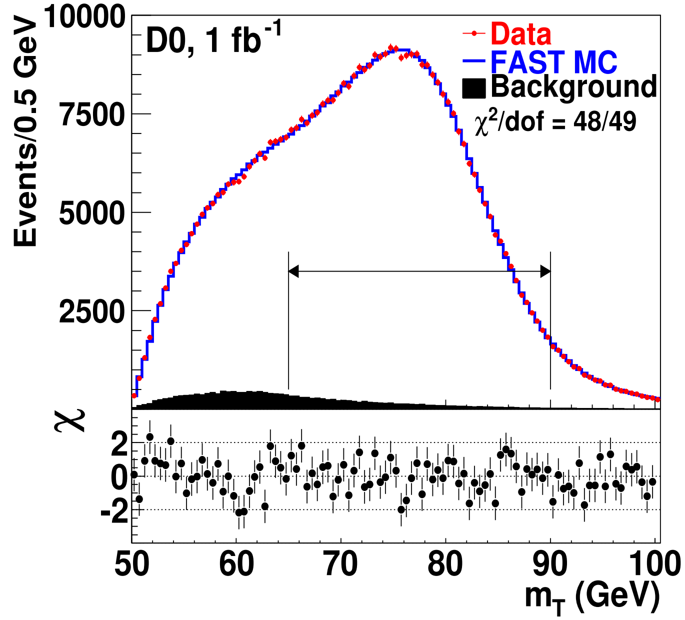
Relies on all decays being 2-body decays.

Other new techniques

- How to reconstruct masses in shorter decay chains?
- How to reconstruct masses in chains with 3-body decays?
- How to quickly determine overall new physics mass scale?

What about more inclusive observables?

MT2



Based on the transverse mass:

Define $\alpha_i = (E_{Ti}, p_{xi}, p_{yi}, 0)$,
$E_{Ti} = \sqrt{p_{xi}^2 + p_{yi}^2 + m_i^2}$
Then $M_T^2 \equiv (\alpha_\ell + \alpha_\nu)^2$.

(Depends on guessing right for m_i...)

Classic use is W mass measurement.

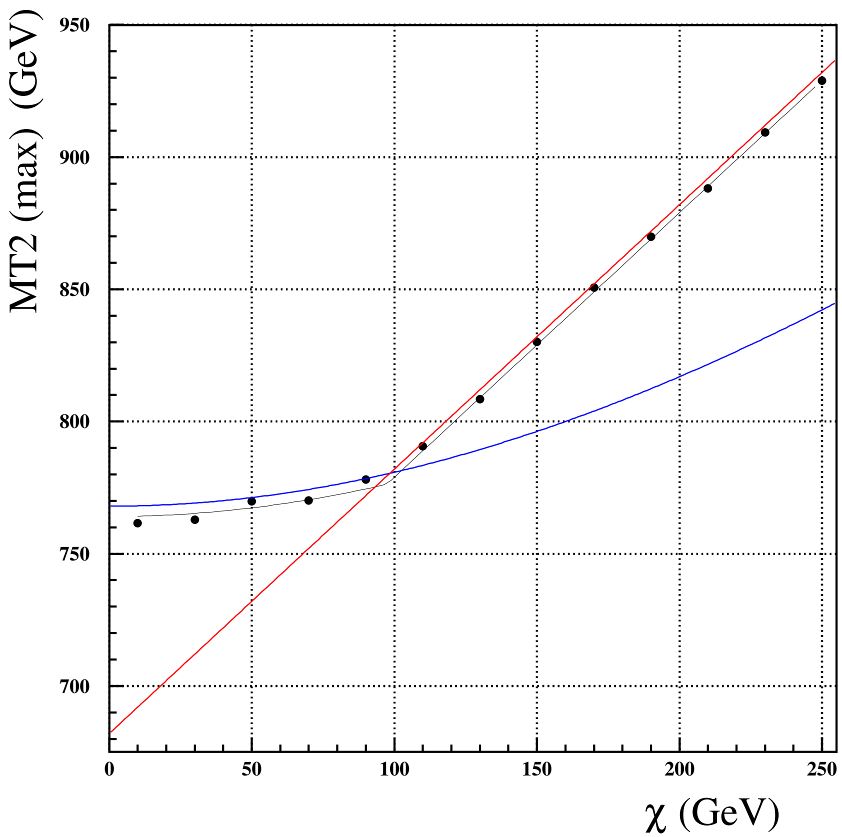
What about events with 2 invisible particles?
Don't know $\vec{p}_T$ of each invisible particle; only know their sum.

For each event:
- Construct both M_T^2 variables, with a guess for $\vec{p}_{T1}$ and $\vec{p}_{T2}$ that gives correct total missing p_T.
- Vary the guess until the larger M_T^2 is minimized.
This value is MT2. Lester & Summers, PLB463, 99 (1999)
Upper endpoint of MT2 distribution is the parent particle mass...
assuming that the invisible particle's mass was guessed correctly!

MT2 kinks

MT2 really just gets you mass differences.

But features in the MT2 dependence in the plane of the two masses can—in some circumstances—get you the actual masses.



Similarly for kinematic endpoint observables: each event really defines a boundary for the allowed region in the space of unknown masses.

Put together many observables to nail down the true masses.

$\sqrt{\hat{s}}_{\rm min}$

Right after discovery, we don't have a lot of events, we haven't identified decay chains, we just want to know as much about the new physics as possible.

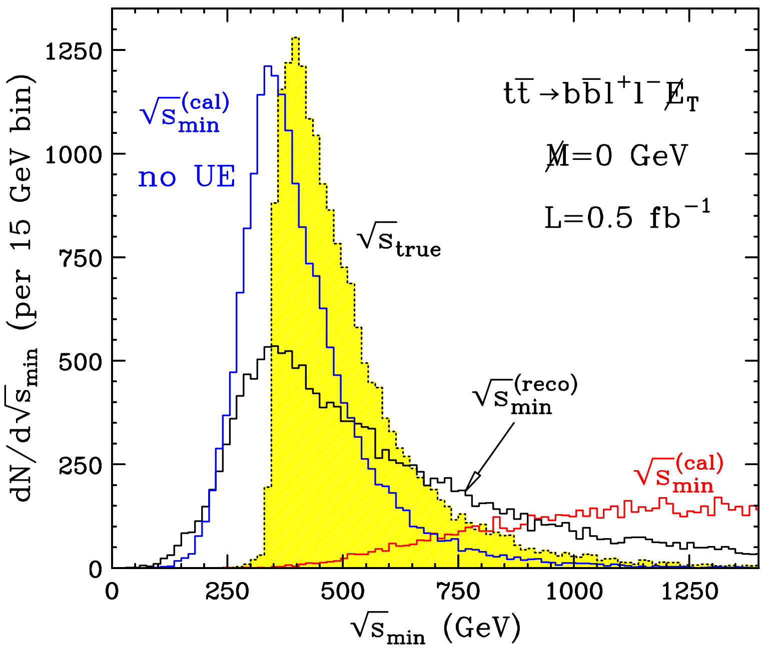
What is the mass scale???

Define another variable:

$$\sqrt{\hat{s}}_{\rm min} = \sqrt{E^2 - P_z^2} + \sqrt{E_{T\,\rm miss}^2 + M_{\rm invis}^2}$$ Konar et al, JHEP 0903, 085 (2009)

$E =$ total calorimeter energy
$\vec{P} =$ total visible momentum
$M_{\rm invis} =$ total mass of all invisible particles: a guess

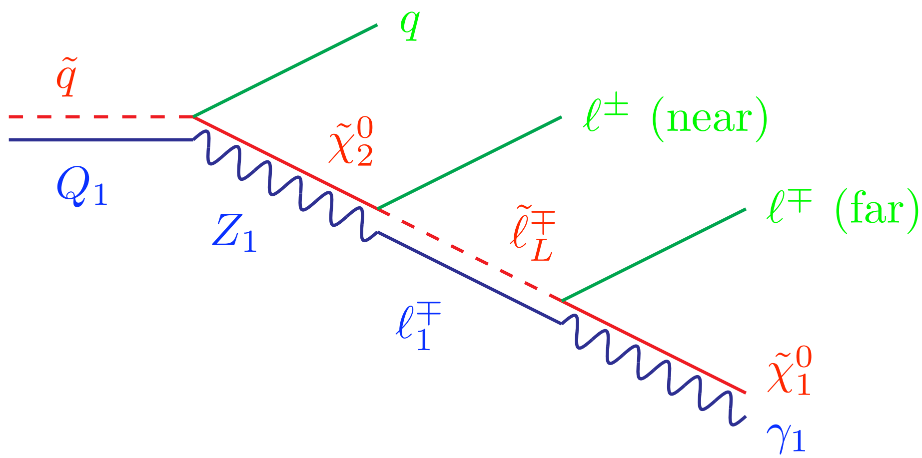
$\sqrt{\hat{s}}_{\text{min}}$ gives the approximate kinematic threshold for the new physics production.



Konar et al, arXiv:1006.0653

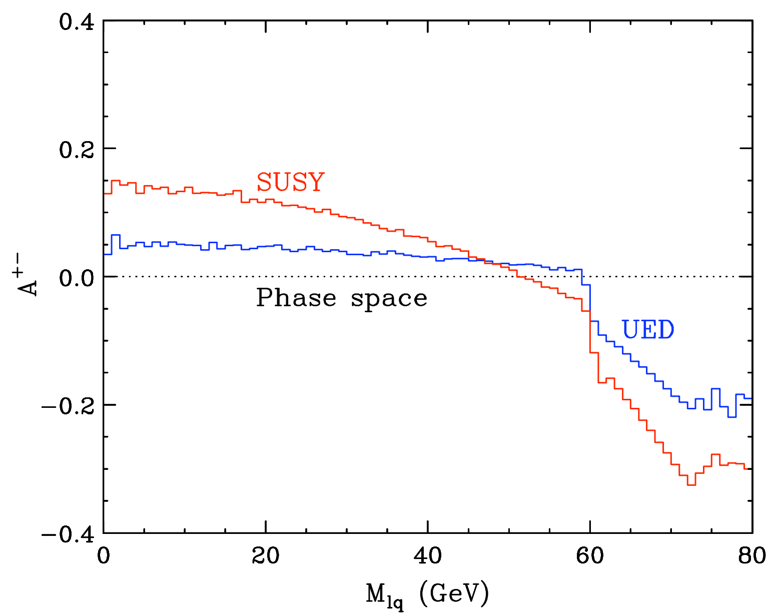
Plot: dilepton events from $t\bar{t}$ production. Assumes $M_{\text{invis}} = 0$.

How to measure spins



figs from Battaglia, Datta, De Roeck, Kong, & Matchev, hep-ph/0507284

- Spins control angular decay distribution in parent's rest frame.
- Polar angle of intermediate particle decay related to invariant masses of visible particle pairs: e.g., $q\,\ell_{\text{near}}$.
- Charge asymmetry to pick the right lepton.



What about top and bottom of chain?

If we can reconstruct the *full* kinematics of decay chain, can boost to any particle's rest frame and examine angular distributions of production and decay. Cheng et al, arXiv:1008.0405

Can do it if there are enough mass-shell constraints (long enough chain) and masses are known (from mass extraction techniques).

- Reconstruct full kinematics (3 visible daughters are enough)
- Boost to a particle's rest frame
- Look at decay distribution: polynomial in $\cos\theta$ of degree $2S$
  $\rightarrow$ Get particle spin
- Measure polarization axis relative to boost direction
  $\rightarrow$ Spin correlation between 2 chains in event

LSP is harder, but can tell whether it's a fermion or boson by angular momentum conservation in its parent's decay.

Summary

Reconstructing SUSY masses requires sophisticated techniques

Tremendous progress in past ~ 5 years

Useful not just for SUSY but for any theory with pair production and decays to an invisible particle (generic models of dark matter from a new parity-odd sector)

Once masses are found, missing-momentum reconstruction is a valuable tool for spin determination