Lectures on Extra Dimensions
Part II

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Recapitulation

On Monday we saw that:

1) *Compact* dimensions are equivalent to new particles

Their masses (spectrum) and couplings contain information about the size, shape, etc. of the compact space

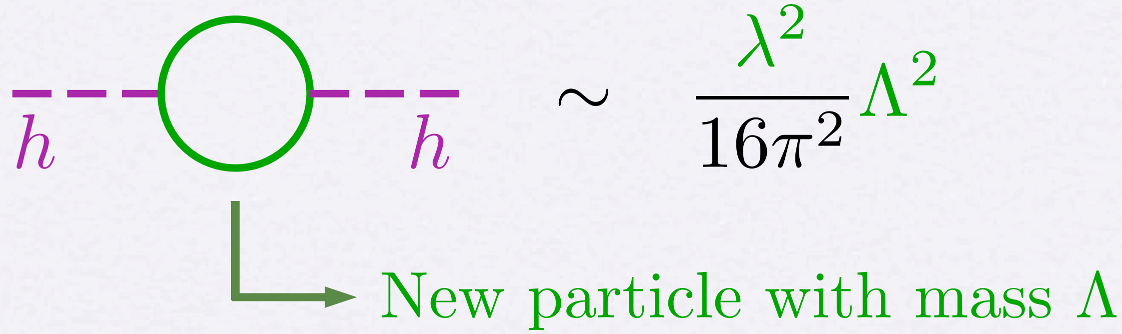
2) Particle wavefunctions can live at different place in the XD

(this is possible because the higher-d Lorentz invariance is broken by the compactification)

Localization manifested (among other places) in strength of the interactions of the SM particles (some of the parameters in the SM Lagrangian)

XD and the Hierarchy Problem

Recall that EWSB by a light scalar leads to a theoretical puzzle:



$$\sim \frac{\lambda^2}{16\pi^2}\Lambda^2$$

Λ can be the Planck or any other scale larger than the weak scale ...

A *satisfying* solution to this puzzle would involve giving a reason for why the loop should be ``cut off" near the weak scale (even if there exists heavier physics).

Do Extra Dimensions have anything to say?

XD and the Hierarchy Problem

- Generically, theories in more than 4D are intrinsically non-renormalizable.
- This is not a problem per se, but it means that they should be considered as *effective* descriptions that are relevant at scales below some scale Λ
- The effective theory itself tells us what is the highest scale it can reliably describe

E.g. if gluons propagate in 5D: $(l_5 = 24\pi^3$ is a 5D loop factor$)$

Flat space: $\Lambda \sim \dfrac{l_5}{N_c \pi R}$ RS: $\tilde{\Lambda} \sim \dfrac{l_5}{N_c (kL)} \tilde{k}$

The point is: cutoff is not far above the KK scale.

- Restate ``hierarchy problem" in XD context as:

``Why is the EW scale close to the KK scale"

- Generically, neither flat XD (e.g. UED), nor warped XD address this question in detail.
  - But it is susceptible of being answered in specific XD models (more on this later)
  - Note also that one only has to explain a ``little hierarchy" (between EW and Λ)

Gauge-Higgs Unification

(Dynamical generation of the EW scale)

We already mentioned that when A_5 obeys (+,+) b.c.'s, it leads to an IR-localized 0-mode

$$f_h(y) = \sqrt{\frac{2kL}{e^{2kL}-1}}\, e^{2ky}$$ (Higgs profile)

In the 5D theory, no terms respecting 5D gauge invariance can generate a potential for h.

But due to the compactification, a potential is induced at *loop level*

$$V(h) = \sum_r \pm \frac{N_r}{(4\pi)^2} \int_0^\infty dp\, p^3 \log \rho(-p^2)$$ (Coleman-Weinberg Potential)

Higgs is NGB

Most important: $$\rho_{\rm top}(z^2) = 1 + F_1(z^2) \sin^2\left(\frac{h}{\sqrt{2} f_h}\right) + F_2(z^2) \sin^4\left(\frac{h}{\sqrt{2} f_h}\right)$$

$f_h \propto \tilde{k}/\sqrt{kL}$

- The result is *finite* and cutoff at the scale $\tilde{k}$. Scale of $\langle h \rangle$ set by f_h!
- EW symmetry broken (or not) depending on relative strength of *interactions*

Gauge-Higgs Unification

(Model Building)

First requirement: the would-be Higgs should have the correct gauge quantum numbers

- Also, since $A_5(+,+) \quad \longleftrightarrow \quad A_\mu(--)$: cannot be a related to SM
- But interactions are, at heart, gauge interactions: non-Abelian

Look for a gauge group that contains the SM, and has extra d.o.f., e.g.

Simplest: $SU(3) \supset SU(2)_L$ $\qquad A = \begin{pmatrix} \vec{\sigma}\cdot\vec{W} & H \\ H^\dagger & 0 \end{pmatrix}$

But simplest with custodial: $SO(5) \supset SU(2)_L \times SU(2)_R$

$$T_L^i = \frac{1}{2}\begin{pmatrix} \sigma^i & 0 & 0 \\ 0 & -\sigma^{i*} & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad A_5^{\hat{a}} T^{\hat{a}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 & H^+ \\ 0 & 0 & 0 & 0 & H^0 \\ 0 & 0 & 0 & 0 & -H^- \\ 0 & 0 & 0 & 0 & -H^{0*} \\ H^- & H^{0*} & -H^+ & -H^0 & 0 \end{pmatrix}$$

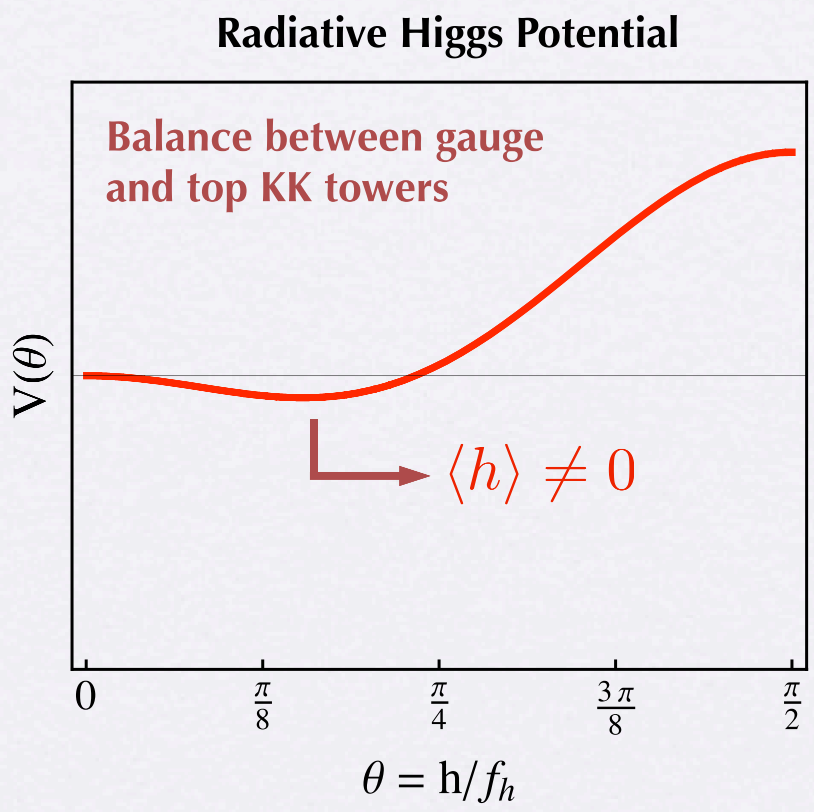
$+\, T_R^i$ generators

Gauge-Higgs Unification

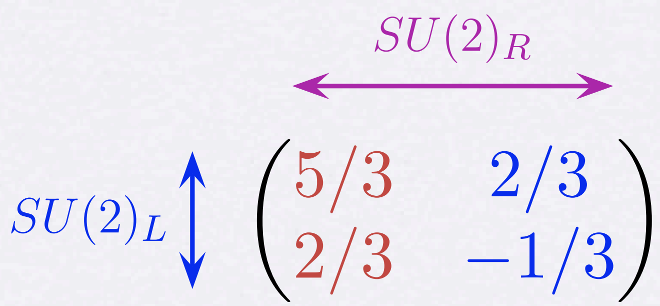
(Model Building)

- Embedding of SM fermions into SO(5) more model-dependent...
- However, the general feature is that the fundamental interaction between fermions and the Higgs is through the 5D gauge coupling

$\longrightarrow$ Non-universal Yukawa couplings arise from b.c.'s and localization



- The model predicts the existence of KK quarks lighter than KK gauge bosons
- Some of these have exotic charges



Low-Energy Constraints

- KK modes can leave an imprint on low-energy observables
- Precision measurements can in principle ``see" the new physics
- The fact that observations agree well with the SM can be turned into constraints on new scenarios for physics beyond the SM

Differentiate between

- Flavor blind constraints (electroweak observables)
- Flavor-dependent constraints

Also, constraints are qualitatively different depending on whether

- KK modes couple singly at tree-level to SM fields (tree-level corrections)
- KK modes couple in pairs to SM fields (loop-level corrections)

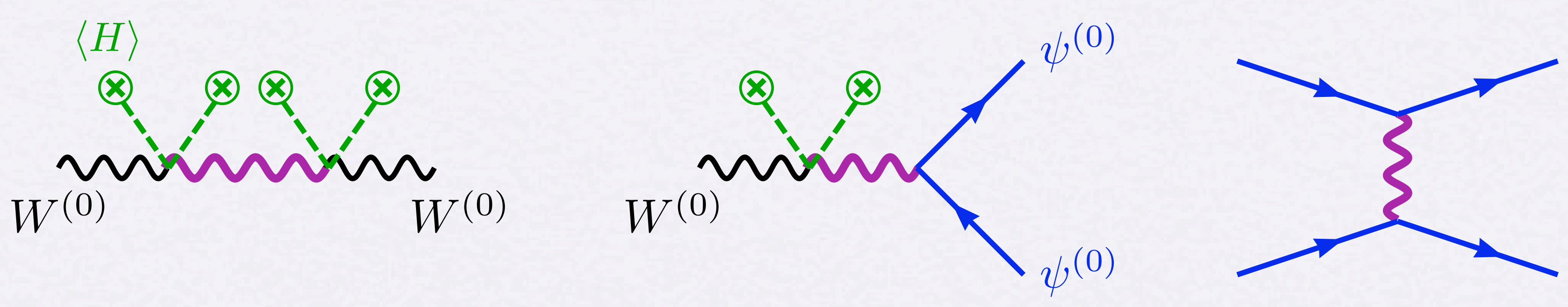
Low-Energy Constraints

Tree-level effects, e.g. Warped Scenarios

Unlike UED's, warped scenarios do not have a natural KK parity $\longrightarrow$ single KK production

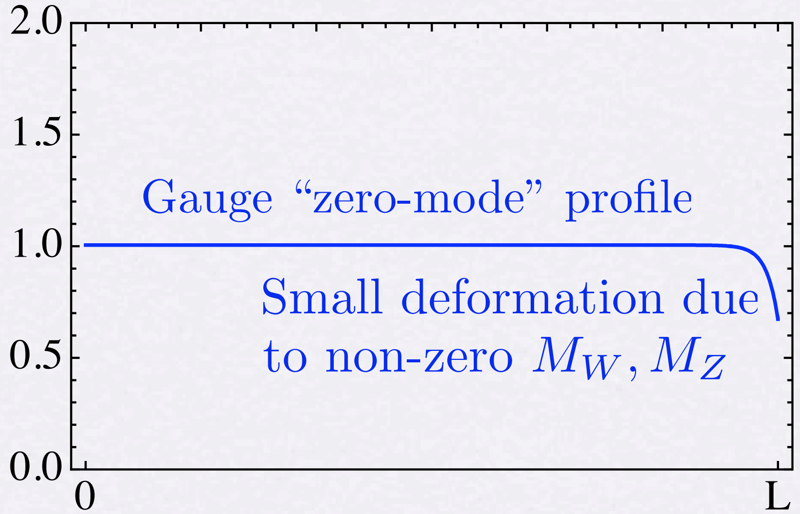
In addition, couplings to KK modes can be large due to $\sqrt{2kL}$ enhancements

Therefore, constraints on KK scale are stronger:



T-parameter   S-parameter   Fermi constant

- These effects correspond to a deformation of the gauge wavefunctions near IR brane. (Mixing with ``unperturbed" KK modes)
- Point is: these are different for W, Z and photon (no deformation here)



Low-Energy Constraints

Tree-level effects, e.g. Warped Scenarios

These constraints are strong and place the new physics in the multi-TeV range

- Effects that come via the Fermi constant are controlled if light fermions (muon) near UV
- T parameter controlled by custodial SU(2). Calculable loop effects can play a role.
- Major constraint comes from a sizable, positive contribution to S
- Other constraints such as corrections to the $Z\bar{b}b$ can also be under control

Express bound from EWPM in terms of the warped down curvature scale:

$$\tilde{k} = k\, e^{-kL} > 1.3 \text{ TeV} \quad \begin{cases} m_{\rm KK}^{\rm gauge} \approx 2.5\, \tilde{k} > 3.2 \text{ TeV} \\ m_{\rm KK}^{\rm graviton} \approx 3.8\, \tilde{k} > 5 \text{ TeV} \\ \text{KK modes of SM fermions likely heavier (mass c-dependent)} \end{cases}$$

However, many well-motivated models also predict additional fields below 1 TeV

Low-Energy Constraints

Loop-level effects, e.g. Universal Extra Dimension

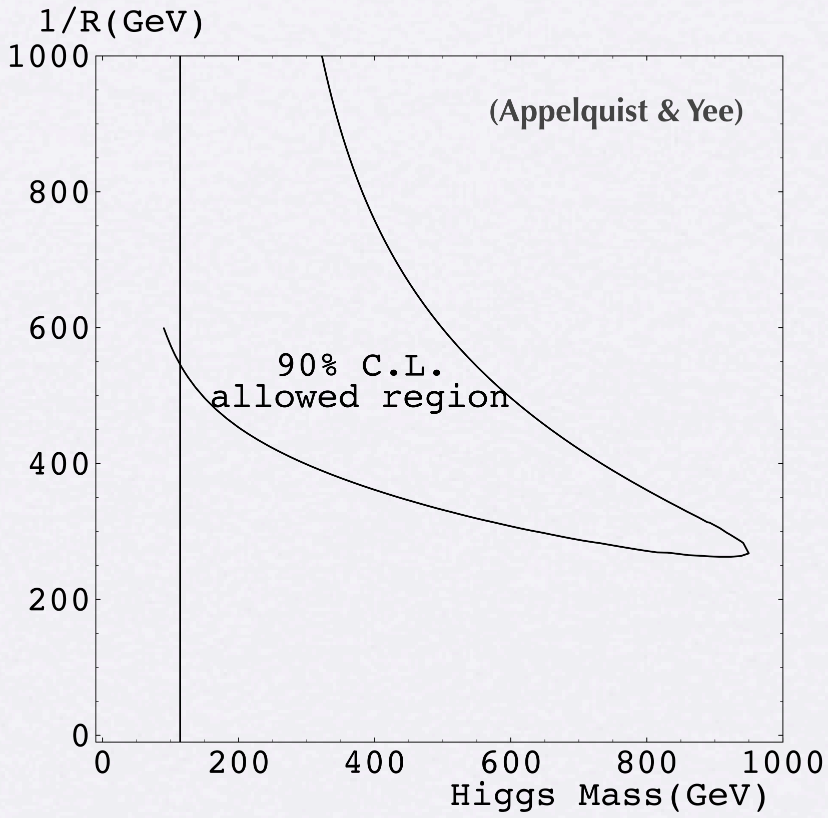
The KK parity mentioned before leads to the following expectations:

- KK modes either pair produced, or
- KK-parity even modes singly produced with loop-level strength

(This is not necessary, but is technically natural. Should be taken as definition of UED's)

UED's do not provide a theory of flavor $\longrightarrow$ Flavor structure similar to SM

Requires assumptions about the flavor structure of higher-dimension operators (presumably related to Yukawa couplings)



EW constraints mainly from S and T parameters, mainly due to top KK tower:

$$T_j^t \sim \frac{1}{\alpha}\frac{3m_t^2}{8\pi^2 v^2}\frac{2}{3}\frac{m_t^2}{M_j^2} \quad , \qquad S_j^t \sim \frac{1}{6\pi}\frac{m_t^2}{M_j^2}$$

T-parameter can be compensated by heavy Higgs

Remarks on Collider Phenomenology

RS Collider Phenomenology

Features of signals determined by how the fields are localized along the extra dimension

- Resolution of flavor puzzle (fermion mass hierarchies and FCNC suppression) suggests fermions (therefore SM gauge fields) arise from *bulk* fields
    - Light quarks and leptons localized near UV brane
    - $Q_L^3 = (t_L, b_L)$ almost flat (but somewhat towards IR brane)
    - t_R localized near IR brane (hence top mass larger than other fermion masses)
    - Higgs localized near the IR brane
    - *All* KK modes localized near the IR brane
- Pattern of localization leads to
    - Couplings between light quarks and leptons to KK physics suppressed (by $\frac{1}{kL}$ for gauge KK's, by Yukawa's for KK gravitons)
    - Couplings of t_R, photon and transverse W's & Z's to KK modes order 1
    - Couplings of t_R, Higgs and W_L, Z_L enhanced by kL

RS Collider Phenomenology

In general, KK production suppressed due to small couplings to proton constituents

- KK gluons have largest production cross sections
- Decays into third generation fermions, mostly tops

  (but can change if other ``light" KK fermions present)

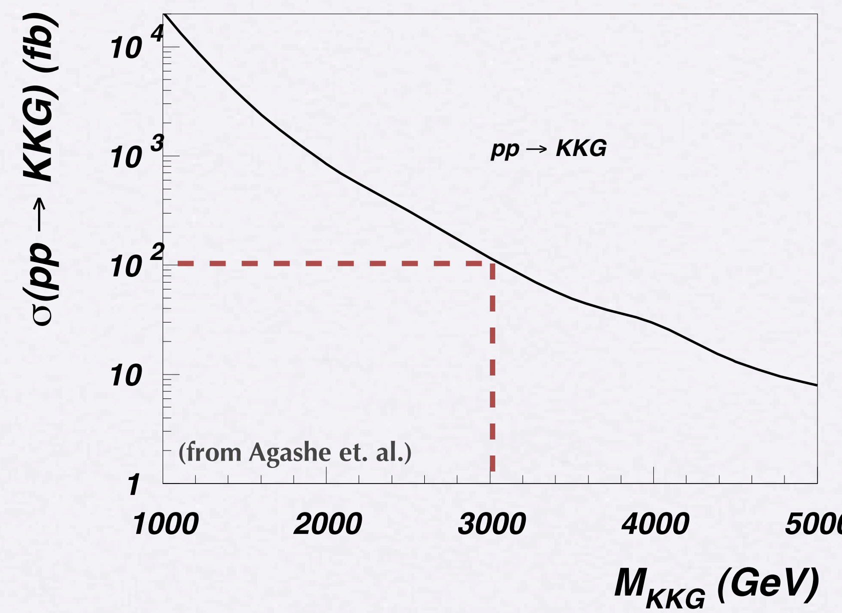
- KK gluon are relatively *broad* resonances

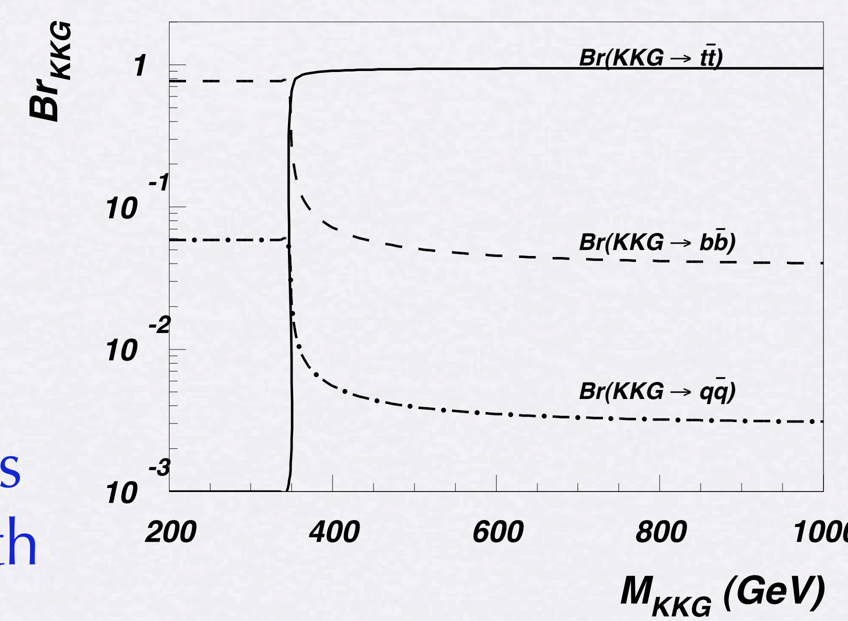
  (e.g. somewhat strong couplings to top)

- Preliminary studies indicate LHC reach up to 4 TeV with 100 fb^{-1}
- KK EW gauge bosons have smaller production XS
- Decays into leptons suppressed

  (i.e. non-standard Z's: decays into (top, W, Z) preferred)

- In many models, there are also new top-like fermions (but vector-like): LHC reach can be up to 1.5 TeV with 300 fb^{-1}, and more if they are produced singly





BR's can change if additional ``light" KK fermions

KK Gravitons

The graviton (spin-2) resonances with couplings to SM particles set by the TeV scale are very characteristic of the RS framework.

Clear signal... at least when fermion localized on IR brane

But this scenario is not favored by EW or flavor constraints!

More likely, both gauge bosons and fermions in the bulk, with light fermions near the UV brane

$\longrightarrow$ KK graviton couplings to fermions suppressed

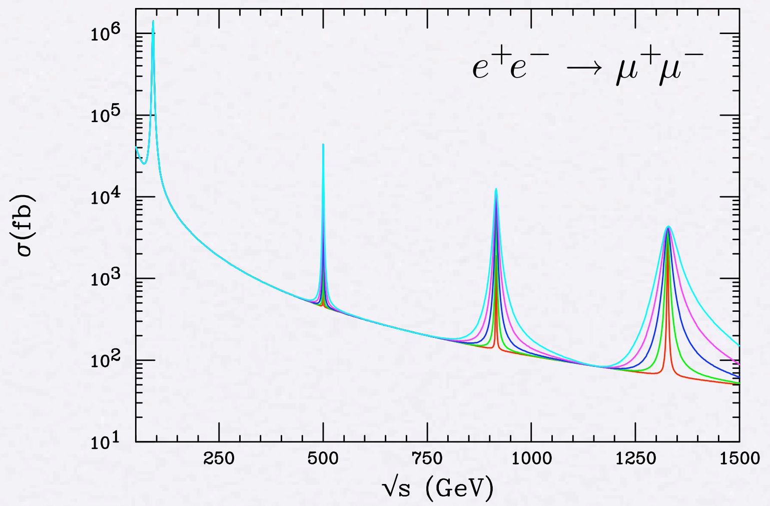
$\longrightarrow$ Decays into longitudinal gauge bosons, top and Higgs dominant

$\longrightarrow$ Couplings to massless gauge bosons somewhat suppressed, but gluon fusion possible

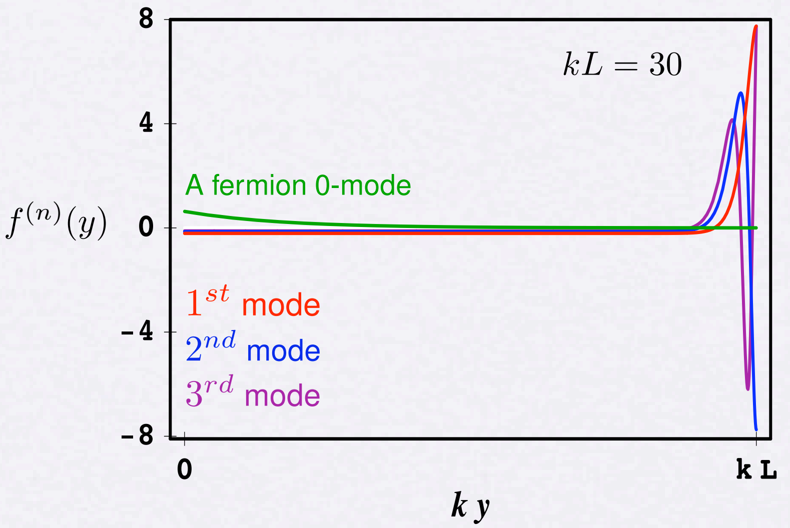
Unfortunately, KK gravitons expected to be above ~ 4 TeV

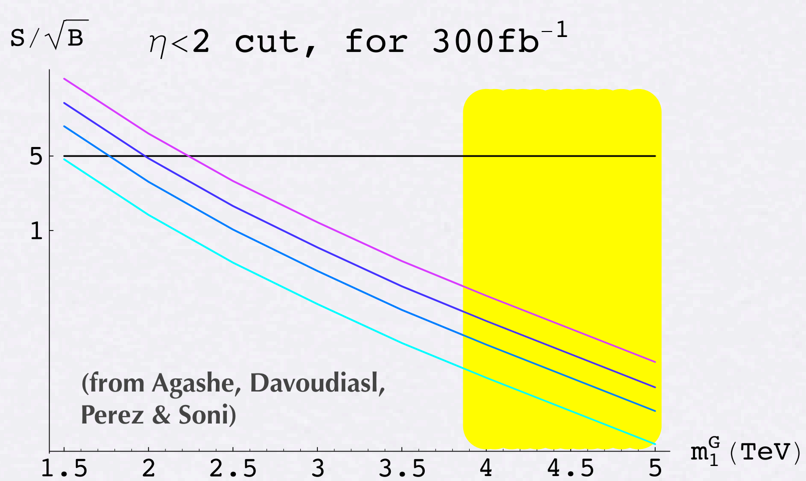
$\longrightarrow$ Better prospects for $gg \to G \to ZZ \to 4l$ but discovery unlikely

(from Davoudiasl, Hewett & Rizzo)



KK wavefunctions

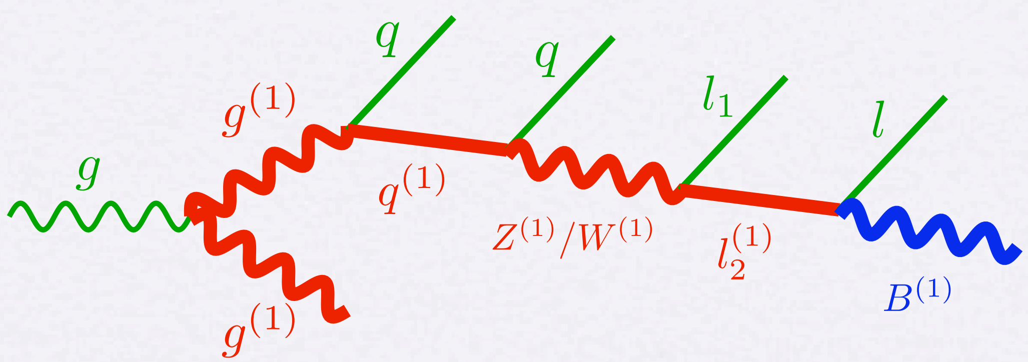




(from Agashe, Davoudiasl, Perez & Soni)

UED Collider Phenomenology

- Expect colored particles (quarks and gluons) to be heavier than leptons and weak gauge bosons
- Assume KK parity, so lightest Kaluza-Klein particle (LKP) is stable
- At a hadron collider, produce gluon or quark pairs, then cascade decays to LKP



- Similar to cascade decays in SUSY (replacing KK modes by superpartners)
- Recall also that KK mass splittings arise at loop level $\longrightarrow$ soft jets and leptons (not easy)

  LHC reach with 100 fb^{-1} about 1.5 TeV in the ``golden" $4l + \not{E}_T$ channel

- Second level KK modes are about 2 times as heavy in 5D (and $\sqrt{2}$ heavier in 6D)

  These are KK-parity even, can decay into SM particles and lead to (narrow) resonances

  (pair production, or loop suppressed single production)

The RS Radion

Radion is part of the 5D gravity field:

$$ds^2 = e^{-2(ky+F)}\eta_{\mu\nu}dx^\mu dx^\nu - (1+2F)^2 dy^2 \qquad F(x,y) = r(x)R(y)$$

Linear couplings through energy-momentum tensor:

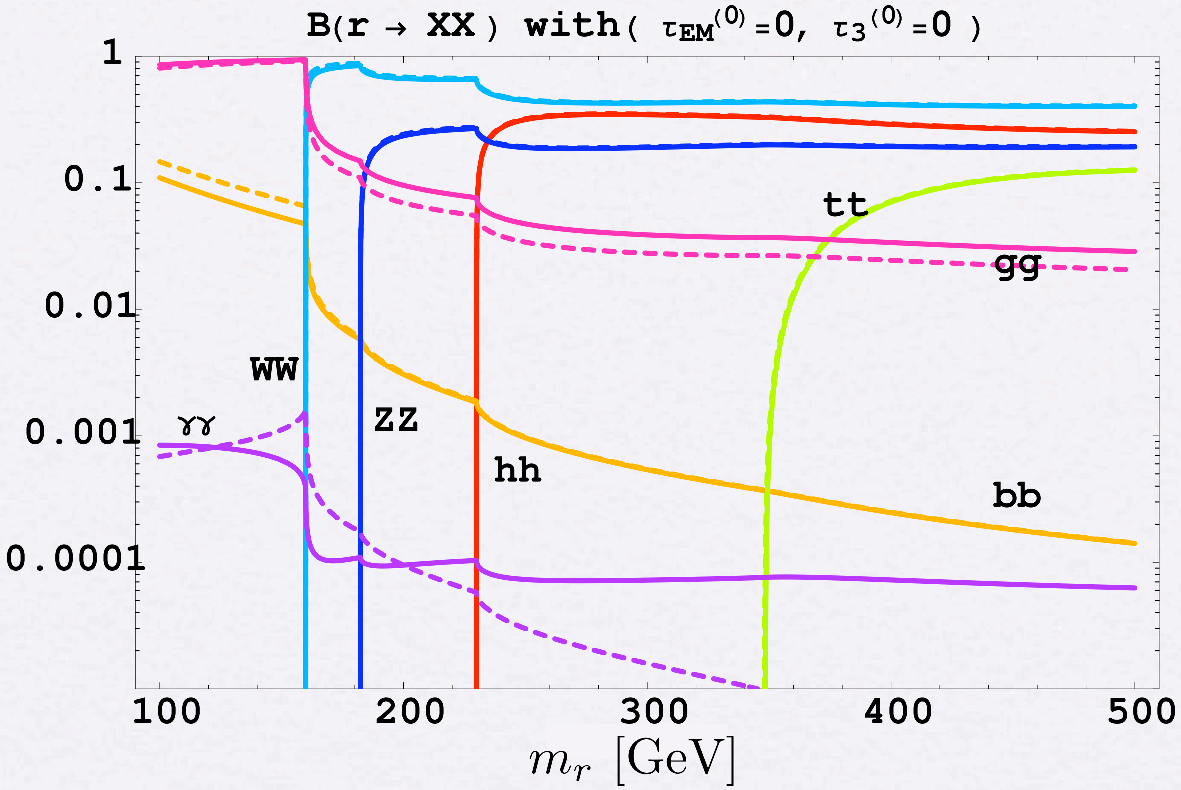
$$S_{\rm radion} = \int d^5x\sqrt{g}\left[F\left(\mathrm{Tr}\,T^{MN} - 3\,T^{55}g_{55}\right)\right]$$

In general, couplings proportional to mass, similar to Higgs!

- Couplings to fermions: $f(c_L,c_R)m_\psi \frac{r}{\Lambda_r}\bar{\psi}_L\psi_R$ $\quad [f(c_L,c_R)=1 \text{ for IR localized fermions}]$
- Couplings to massive gauge fields: $-\frac{r}{\Lambda_r}\left(2M_W^2\,W_\mu^+\,W^{\mu-} + M_Z^2 Z_\mu\,Z^\mu\right) + \text{ corrections}$
- Massless gauge: $-\frac{r}{\Lambda_r}\frac{1}{4(kL)}\left[1 - 4\pi\alpha\left(\tau_{\rm UV}^{(0)} + \tau_{\rm IR}^{(0)}\right) + \frac{\alpha}{2\pi}\left(b - \sum_i \kappa_i F_i(\tau_i)\right)kL\right]F_{\mu\nu}F^{\mu\nu}$

  loops

Suppression is $\Lambda_r \equiv \sqrt{6}\,\tilde{k} = \sqrt{6}\,k\,e^{-kL}$ (order TeV)



Compare to Higgs discovery potential via

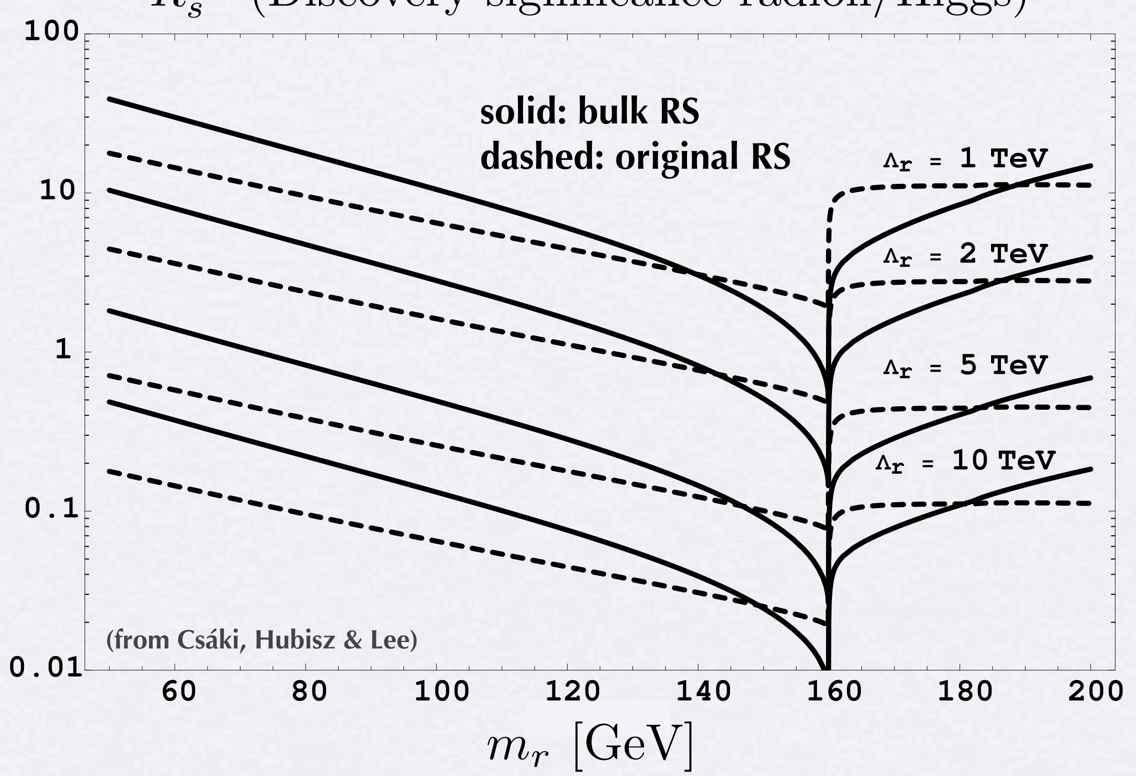
$$gg \to r \to \gamma\gamma$$
$$gg \to r \to ZZ \to 4l$$

Discovery significance:

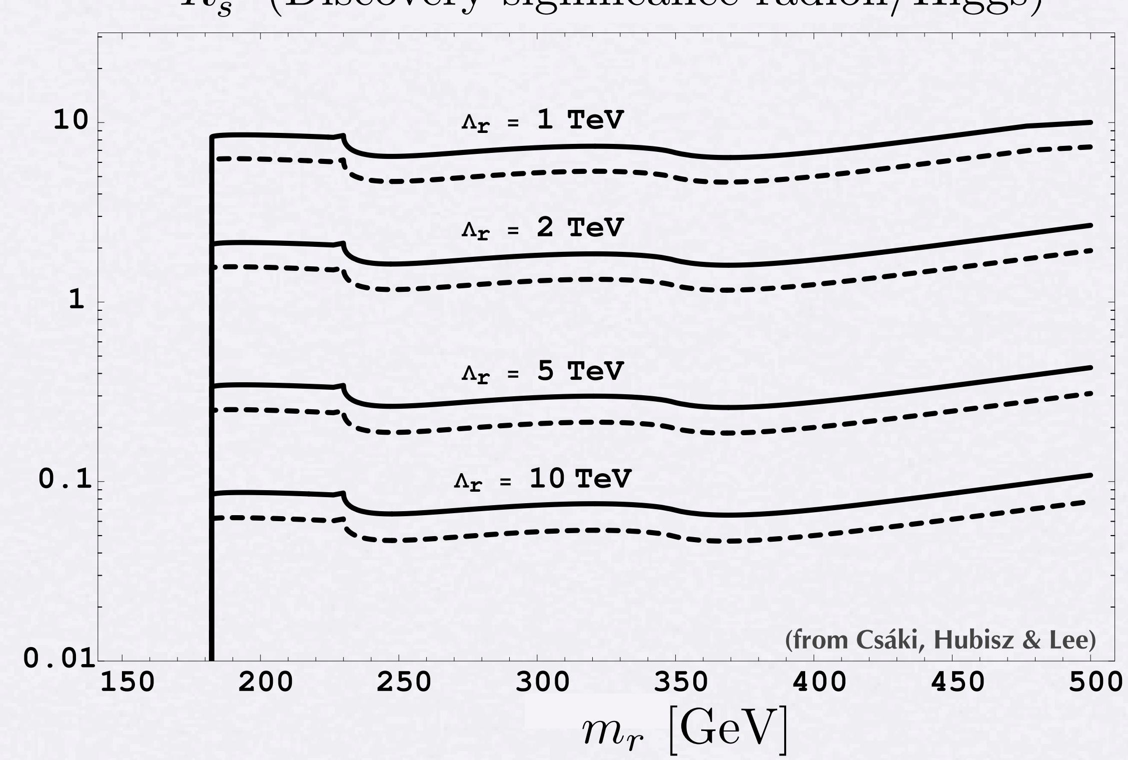
$$R_S^{\gamma\gamma} \equiv \frac{S(r)}{S(h_{SM})} = \frac{\Gamma(r \to gg)\, B(r \to \gamma\gamma))}{\Gamma(h_{SM} \to gg)\, B(h_{SM} \to \gamma\gamma))} \sqrt{\frac{max(\Gamma_{tot}(h_{SM}), \Delta M_{\gamma\gamma})}{max(\Gamma_{tot}(r), \Delta M_{\gamma\gamma})}}$$

and similar for $4l$

$R_s^{\gamma\gamma}$ (Discovery significance radion/Higgs)



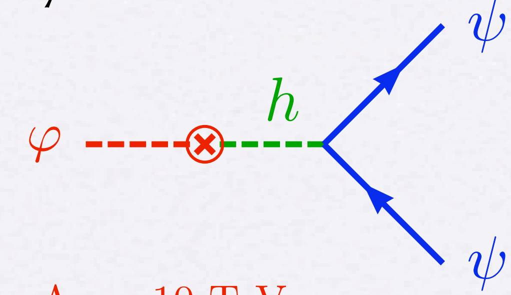
R_s^{4l} (Discovery significance radion/Higgs)



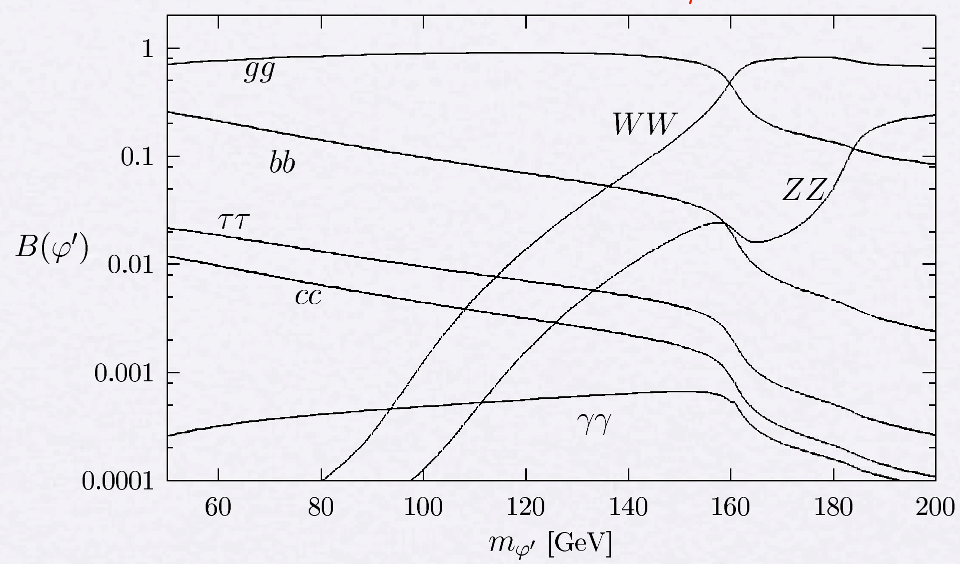
Radion-Higgs Mixing

Previous slide neglects radion-Higgs mixing (a good approx. in many models)

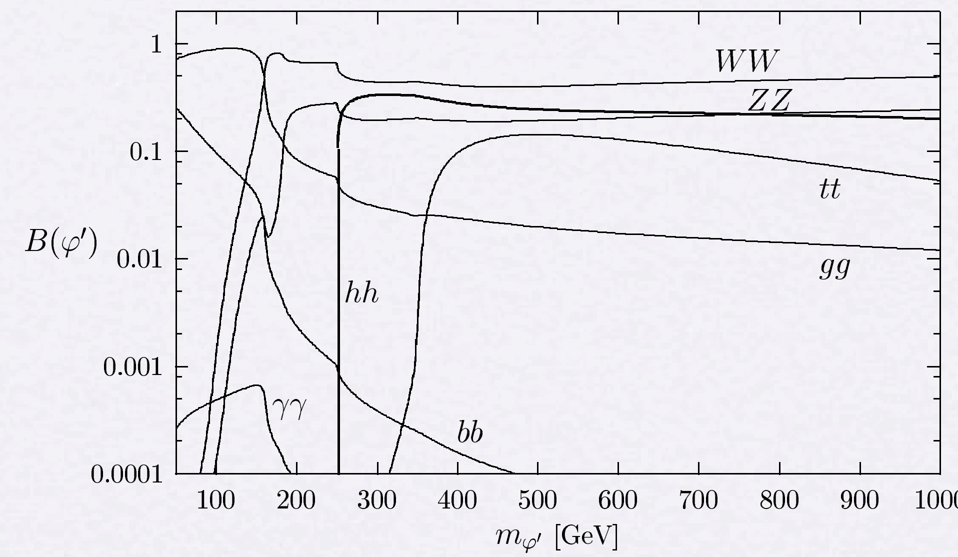
Coupling to curvature: $-\xi\,\mathcal{R}[g]H^\dagger H \longrightarrow -\frac{6\xi v}{\Lambda_\varphi}\varphi\Box h$



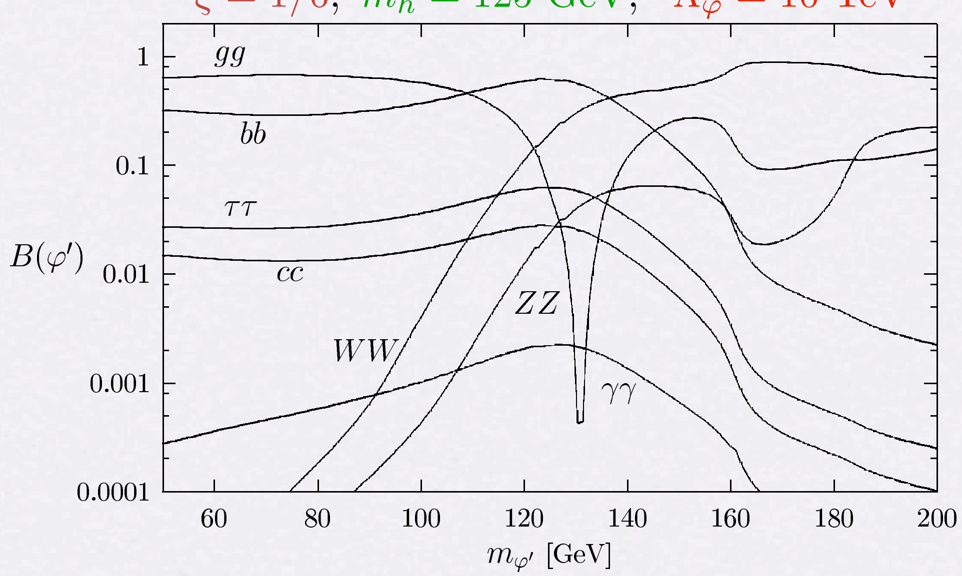
$\xi = 0,\ m_h = 125\text{ GeV},\ \ \Lambda_\varphi = 10\text{ TeV}$



$\xi = 0,\ m_h = 125\text{ GeV},\ \ \Lambda_\varphi = 10\text{ TeV}$



$\xi = 1/6,\ m_h = 125\text{ GeV},\ \ \Lambda_\varphi = 10\text{ TeV}$



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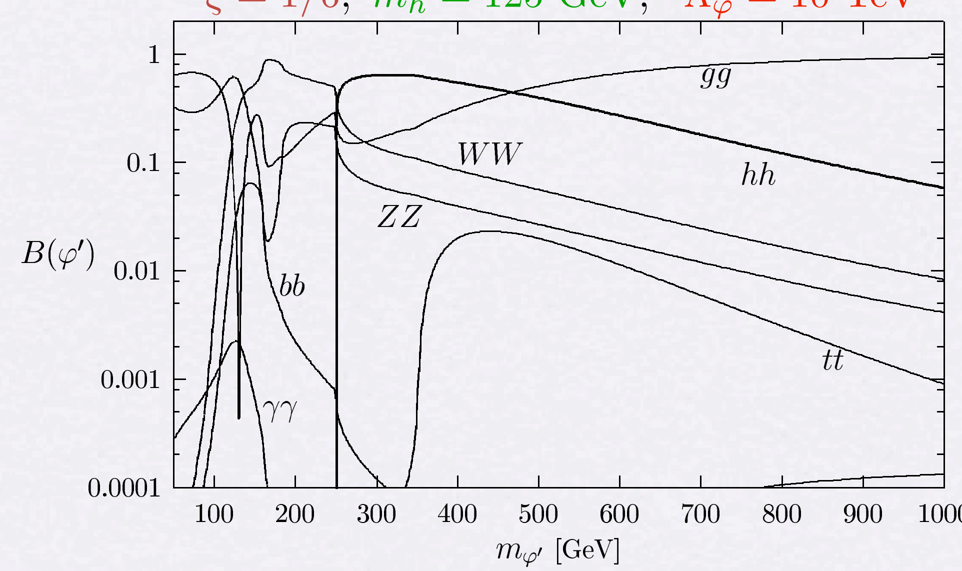


(from Giudiice, Rattazzi & Wells)

Black Hole Production?

If particles are scattered with impact parameter below their Schwarzschild radius

$$r_S(E) = k(n) \left(\frac{E}{M_\star} \right)^{\frac{1}{n+1}} \frac{1}{M_\star} \qquad k(n) = \left(\frac{2\,(2\pi)^n}{(n+2)\Omega_{2+n}} \right)^{\frac{1}{n+1}} = 0.8(n=1) - 2.4(n=6)$$

a black hole is formed! Here $M_\star$ is the effective scale of quantum gravity

- If $M_\star \sim$ TeV we could be producing mini-BH's at the LHC!
- The cross section for BH formation is geometrical:

$$\sigma_{\rm BH} \sim \pi r_S^2(E)$$

- Notice it is *not* suppressed by small couplings nor phase space, and increases with energy
- Black holes decay democratically and promptly into all SM particles (taking spin multiplicities into account)

There has been a great deal of work studying the dynamics of BH formation, how energy and angular momentum are radiated, flat space versus curved space BH's, etc.

Black Hole Production?

However, the previous picture assumes that we produce *thermal* black holes. This would be true if produced well-above threshold $(E \gg M_\star)$

Determining the *threshold* for BH production, and how it relates to the fundamental scale in a model is difficult (not least because we do not know the quantum gravity theory!)

$\longrightarrow$ Hard to translate observation into bounds of a fundamental scale

In addition:

- Threshold for BH production expected to be a factor of a few above $M_\star$
- Not all parton energy goes into the BH mass (``inelasticity")
- Rapidly falling pdf's imply that production at threshold is dominant

Thus, do not expect many truly thermal BH, if any...

Upshot: initial estimates with large cross section for BH production and spectacular multi-particle and isotropic decays were probably too optimistic...

Other Quantum/Strong Gravity Signals

However, the presence of a nearby gravity scale (if sufficiently low) can have observable effects:

The $2 \to 2$ production cross section is expected to increase as the energy approaches the d-dimensional Planck scale (either through virtual BH's or strong gravity effects)

Effect may be observable even in dijets, via

$$R_\eta = \frac{N_{\rm events}(0 < |\eta| < 0.5)}{N_{\rm events}(0.5 < |\eta| < 1)} \quad \xrightarrow{\text{QCD high E}} \quad \sim \; 0.6$$

since larger transverse momentum than in QCD is expected.

This would be relevant in ADD type of scenarios, or RS with the SM on the IR brane.

For the better motivated RS models discussed before, the partons *see* a much larger Planck scale and BH production at the LHC is hopeless

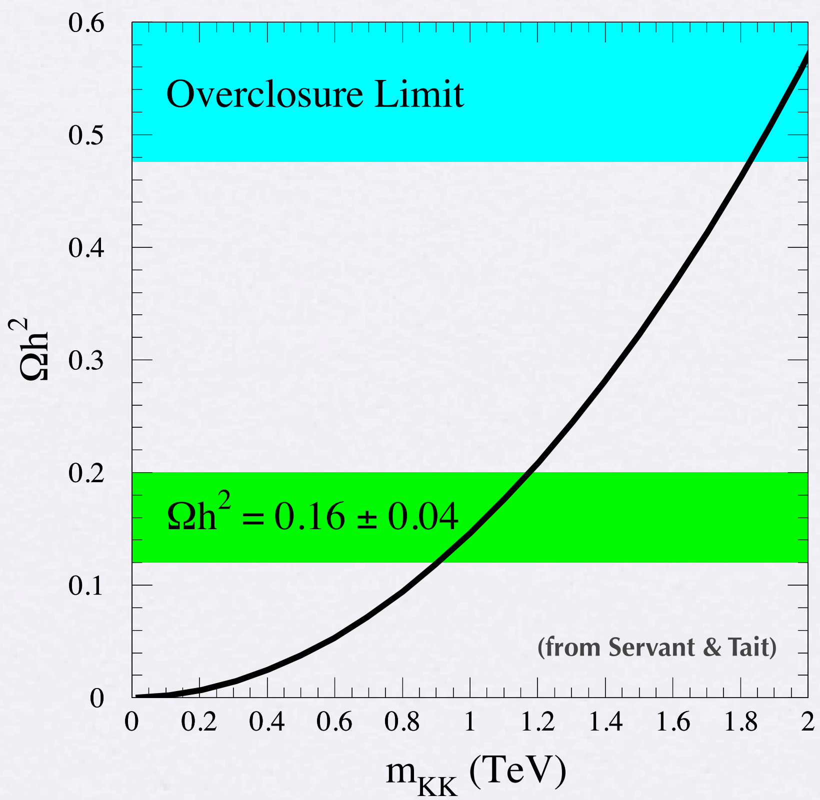
Dark Matter in UED

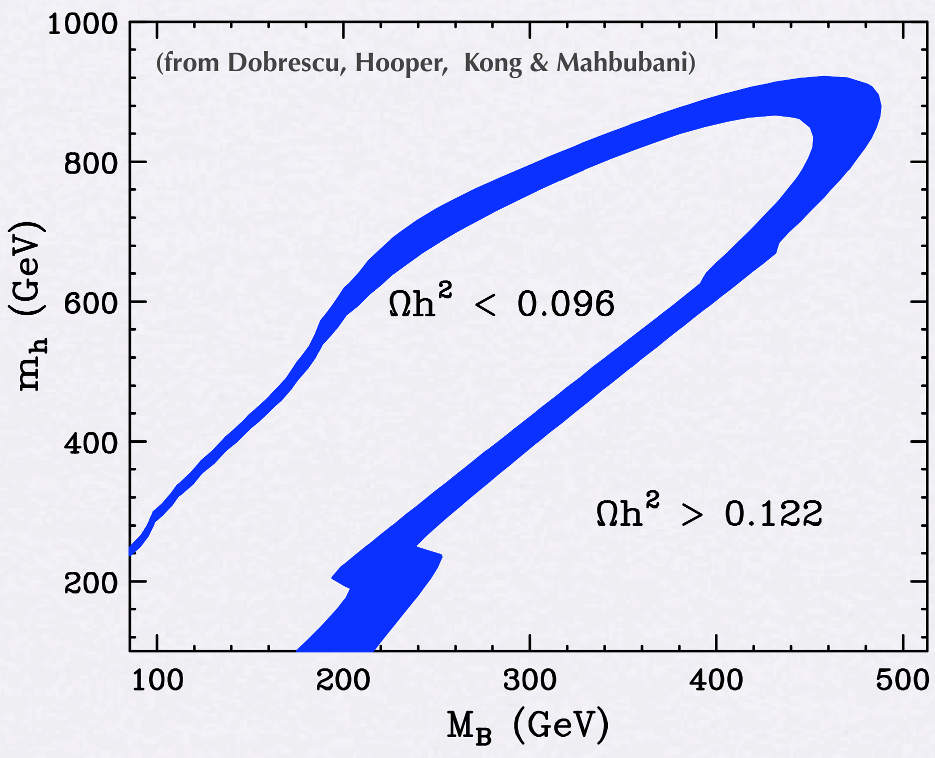
Already mentioned that odd-level KK modes are KK-parity odd $\longrightarrow$ lightest is stable

These states have EW scale masses and interactions $\longrightarrow$ correct order for $\Omega_{\rm DM}$

There are also interesting differences in 5D versus 6D, arising from:

- spin-1 versus spin-0 nature of DM candidate (as suggested by calculable loops)
- Couplings to fermions proportional to m_f when spin-0, not so when spin-1
- $\longrightarrow$ Relic density obtained for lower masses in 6D than in 5D





Dark Matter in UED

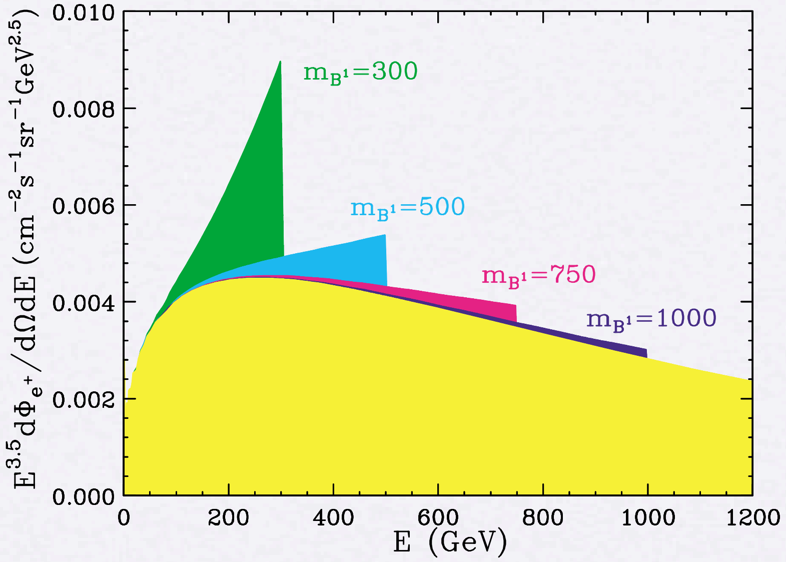
The prospects for direct and indirect detection in 5D and 6D are also different:

Indirect detection:

In 5D with KK-photon LKP, self-annihilation into e^+e^- pairs is sizable (not helicity suppressed)

Also, continuum photon signals (from annihilation products) can also be detectable

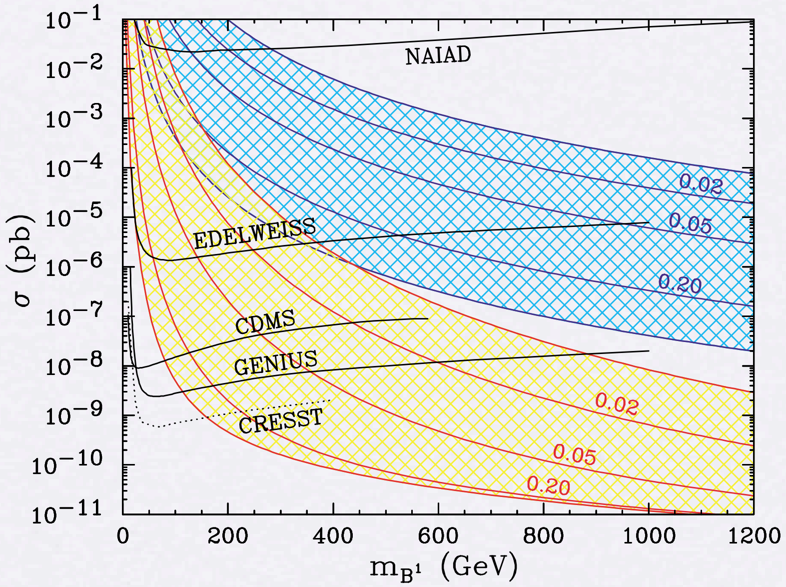
In contrast, helicity suppression of the "*spinless photon*" into fermions in 6D, makes such searches much harder.



(from Cheng, Feng & Matchev)

Direct detection:

Feasible in 5D, harder in 6D...



Notice: 6D UED's with a scalar DM candidate is more similar to neutralino in SUSY!

Dark Matter in Warped Scenarios

There has been recent interest in describing DM within the RS framework

- One option is to try to mimic KK parity by *doubling* the space so it is symmetric about the center (could also lead to light states consistent with EWPM)

- Recently a *fully realistic* model (no KK-parity) constructed with

  - A spin-1 DM candidate of mass

    $$m_{X^-} = 300 - 600 \text{ GeV}$$

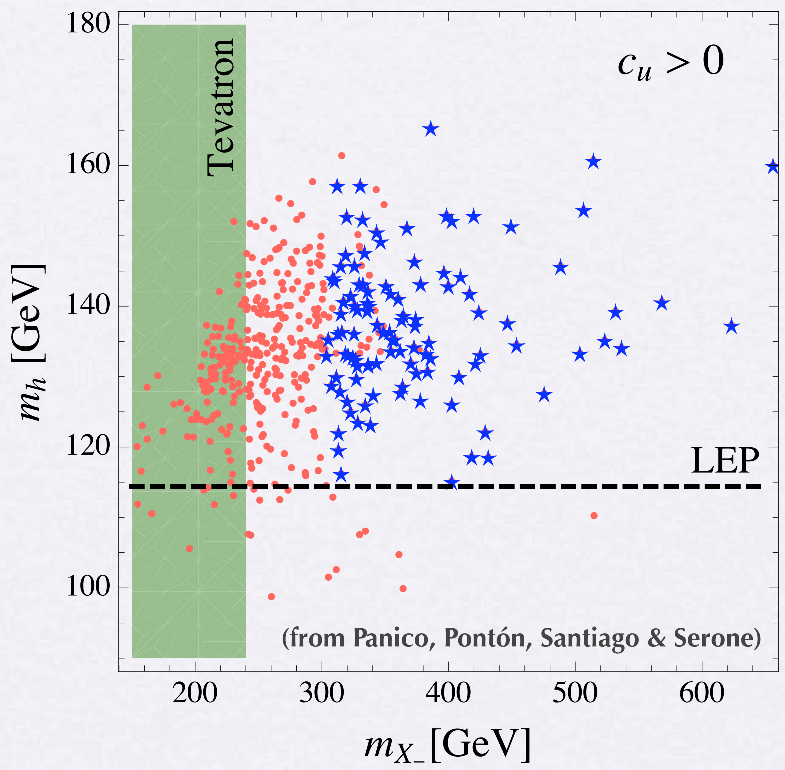
  - Decays into tops (and cousins)

  - Predicts new fermions nearly degenerate with X^- (these play a role in coannihilations)

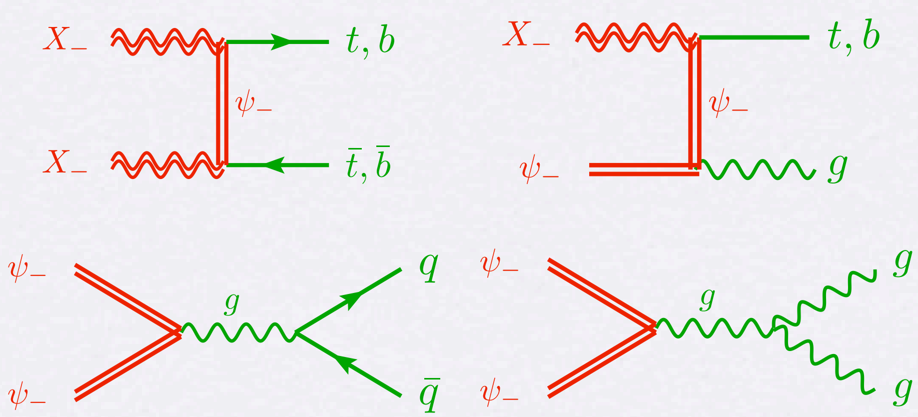
  - Strong connection to EWSB (gauge-Higgs unif.)

Associated fermions easily detectable at colliders.

Direct and indirect searches hard



Self- and co-annihilation channels



Other Topics Not Covered Here

- ``Large extra dimensions" (ADD)
- Flat TeV-scale extra dimensions with some fields on the brane, other in the bulk
- ``Higgsless" theories (no Higgs scalar, unitarization via KK gauge bosons)
- *Really* small extra dimensions (e.g. at the GUT scale). Not directly observable at the LHC but could leave imprints for instance in the characteristics of the soft breaking parameters in a SUSY theory

Following is a very incomplete set of references. When the subject has evolved over time I chose to present the most recent reference. Thus, there is no pretension of historical accuracy.

[1] L. Randall and R. Sundrum, "A large mass hierarchy from a small extra dimension," [arXiv:hep-ph/9905221].

[2] W. D. Goldberger and M. B. Wise, "Modulus stabilization with bulk fields," [arXiv:hep-ph/9907447].

[3] B. A. Dobrescu and E. Pontón, "Chiral compactification on a square," [arXiv:hep-th/0401032]. G. Burdman, B. A. Dobrescu and E. Pontón, "Six-dimensional gauge theory on the chiral square," [arXiv:hep-ph/0506334].

Localization of fields

[4] N. Arkani-Hamed and M. Schmaltz, "Hierarchies without symmetries from extra dimensions," [arXiv:hep-ph/9903417].

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Low-energy constraints

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