Exact Amplitude-Based Resummation Confronts Experiment: Present and Future

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Forward

** NEW PHYSICS/PRECISION 'H' AT LHC/FCC

Must Distinguish from Higher Order SM Processes AND Must Probe Precisely to Specify Uniquely

⇒ Precision QCD⊗EW for the LHC/FCC

** UV LIMIT OF EINSTEIN'S THEORY

Can QFT Handle It?

⇒ Exact, Gauge Invariant Residual Control in Resummation for UV

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The Archetype:

*Approach to 1% Precision QCD for LHC Physics via MC Realized Amplitude-Based QED⊗QCD Resummation --

$$d\sigma = \sum_{i,j} \int dx_1 dx_2 F_i(x_1) F_j(x_2) d\hat{\sigma}_{res}$$

$$\Rightarrow \Delta\sigma_{th} = \Delta F \oplus \Delta\hat{\sigma}_{res} = \Delta\sigma_{th}(tech) \oplus \Delta\sigma_{th}(phys)$$

$$\Rightarrow$$

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$$d\hat{\sigma}_{res} = \sum_n d\hat{\sigma}_n$$

$$= e^{SUM_{IR}(QCED)} \sum_{m,n=0}^{\infty} \frac{1}{m!n!} \int \prod_{j_1=1}^{m} \frac{d^3 k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{n} \frac{d^3 k_{j_2}}{k_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy(p_1+q_1-p_2-q_2-\sum_{j_1} k_{j_1} - \sum_{j_2} k'_{j_2}) + D_{QCED}}$$

$$* \tilde{\bar{\beta}}_{m,n}(k_1,\ldots,k_m;k'_1,\ldots,k'_n) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0} \qquad (1)$$

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The Archetype:

*Approach to Feynman's Formulation of Einstein's Theory : Amplitude-Based Resummation of the Feynman Propagators therein

$$i\Delta'_F(k) = \frac{i}{k^2 - m^2 - \Sigma_s(k) + i\varepsilon}$$

$$= \frac{ie^{B''_g(k)}}{k^2 - m^2 - \Sigma'_s(k) + i\varepsilon}$$

$$\equiv i\Delta'_F(k)\big|_{\mathrm{Re}\,summed}$$

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Precision QCD for the LHC

- **Contact with Standard Resummations, SCT, SCET -- see Phys. Rev. D81(2010)076008**

Observations:

- EXACT – Compare:

  Sterman-Catani-Trentadue Threshold Resummation

  As for any f(z),

$$\left|\int_0^1 dz\, z^{n-1} f(z)\right| \leq \left(\frac{1}{n}\right) max\left|f(z)\right|,$$

  drop non-singular contributions to cross section at $z \to 1$

- SCET:

  drop $O(\lambda)$ terms, $\lambda = \sqrt{(\Lambda/Q)}$,

  $\Lambda \sim .3$ GeV, $Q \sim 100$ GeV $\Rightarrow \lambda \simeq 5.5\%$

- These methods give approximations to our $\bar{\beta}_{n,m}$

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Precision QCD for the LHC

- **Contact with Standard Resummations, CSS– RESBOS, etc.: (see 1305.0023 for details)**

$$\frac{d\sigma}{dQ^2 dy dQ_T^2} \sim \frac{4\pi^2\alpha^2}{9Q^2 s}\left\{\int \frac{d^2b}{(2\pi)^2} e^{i\vec{Q}_T\cdot\vec{b}} \sum_j e_j^2 \widetilde{W}_j(b^*;Q,x_A,x_B) e^{\{-\ln(Q^2/Q_0^2)g_1(b)-g_{j/A}(x_A,b)-g_{j/B}(x_B,b)\}}\right.$$

$$\left. + \; Y(Q_T;Q,x_A,x_B)\right\}$$

Dropped terms $O(Q_T/Q)$ in all orders of α_s :

at 5GeV, Q=M_Z, 5.5% Physical Precision Error(PPE)

Errors on the NP functions g_l also yield ~1.5% PPE,…

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- **Shower/ME Matching:**

Remove double counting between

$$\tilde{\beta}_{m,n} \rightarrow \hat{\tilde{\beta}}_{m,n}, \text{ shower - subtracted residuals}$$

$$e^{SUM_{IR}}, e^{D}, F_1(x_1)F_2(x_2)\big|_{ShowerRealization}$$

- **IR-Improved DGLAP-CS Theory(PRD81(2010)076008): New resummed scheme for P_{AB}, reduced cross section derived from (1) applied to splitting process --**

$$F_j, \hat{\sigma} \to F'_j, \hat{\sigma}' \quad \text{for}$$

$$P_{qq} \to P_{qq}^{\exp} = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+z^2}{1-z}(1-z)^{\gamma_q}, \text{etc.,}$$

giving the same value for σ, with improved MC stability

-- no need for IR cut-off (k_0) parameter

- **Complete Set (γ_A are $O(\hbar)$) :**

$$P_{qq}^{exp}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z) \right],$$

$$P_{Gq}^{exp}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q},$$

$$P_{GG}^{exp}(z) = 2C_G F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \{ \frac{1-z}{z} z^{\gamma_G} + \frac{z}{1-z}(1-z)^{\gamma_G}$$

$$+ \frac{1}{2}(z^{1+\gamma_G}(1-z) + z(1-z)^{1+\gamma_G}) - f_G(\gamma_G)\delta(1-z) \},$$

$$P_{qG}^{exp}(z) = F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \frac{1}{2} \{ z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G} \},$$

$$\gamma_q = C_F \frac{\alpha_s}{\pi} t = \frac{4C_F}{\beta_0}, \qquad \delta_q = \frac{\gamma_q}{2} + \frac{\alpha_s C_F}{\pi}(\frac{\pi^2}{3} - \frac{1}{2}),$$

$$f_q(\gamma_q) = \frac{2}{\gamma_q} - \frac{2}{\gamma_q + 1} + \frac{1}{\gamma_q + 2},$$

$$\gamma_G = C_G \frac{\alpha_s}{\pi} t = \frac{4C_G}{\beta_0}, \qquad \delta_G = \frac{\gamma_G}{2} + \frac{\alpha_s C_G}{\pi}(\frac{\pi^2}{3} - \frac{1}{2}),$$

$$f_G(\gamma_G) = \frac{n_f}{6C_G F_{YFS}(\gamma_G)} e^{-\frac{1}{2}\delta_G} + \frac{2}{\gamma_G(1+\gamma_G)(2+\gamma_G)} + \frac{1}{(1+\gamma_G)(2+\gamma_G)}$$

$$+ \frac{1}{2(3+\gamma_G)(4+\gamma_G)} + \frac{1}{(2+\gamma_G)(3+\gamma_G)(4+\gamma_G)},$$

$$F_{YFS}(\gamma) = \frac{e^{-C\gamma}}{\Gamma(1+\gamma)}, \qquad C = 0.57721566...,$$

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- KEY PART OF APPROACH: BUILD ON EXISTING PLATFORMS

$\Longrightarrow$

IR-Improved DGLAP-CS Theory

$\cup$

NLO Parton Shower MC's: MC@NLO, POWHEG, …

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- Illustration:

$$d\sigma_{MC@NLO} = \left[B + V + \int (R_{MC} - C) d\Phi_R\right] d\Phi_B [\Delta_{MC}(0) + \int (R_{MC}/B) \Delta_{MC}(k_T) d\Phi_R] + (R - R_{MC}) \Delta_{MC}(k_T) d\Phi_B d\Phi_R$$

$\Longrightarrow$ Sudakov FF

$$\Delta_{MC}(p_T) = e^{[-\int d\Phi_R \frac{R_{MC}(\Phi_B, \Phi_R)}{B} \theta(k_T(\Phi_B, \Phi_R) - p_T)]}$$

$\Longrightarrow$

$$\frac{1}{2}\hat{\bar{\beta}}_{0,0} = \bar{B} + (\bar{B}/\Delta_{MC}(0)) \int (R_{MC}/B) \Delta_{MC}(k_T) d\Phi_R$$

$$\frac{1}{2}\hat{\bar{\beta}}_{1,0} = R - R_{MC} - B\tilde{S}_{QCD}$$

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$$\bar{B} = B(1 - 2\alpha_s \Re B_{QCD}) + V + \int (R_{MC} - C) d\Phi_R$$

- **Rigorous Contact with Wilson's OPE:**
  Repeat the Gross-Wilczek-Politzer analysis of DIS.
  From

$$W^F_{\alpha\beta}(p_F,q)=\frac{1}{2\pi}\int d^4y\, e^{iqy}\langle p_F|[J_\beta(y),J_\alpha(0)]|p_F\rangle$$

$$=(2\pi)^3\sum_X \delta(q+p_F-p_X)\langle p_F|J_\beta(0)|p_X\rangle\langle p_X|J_\alpha(0)|p_F\rangle$$

(1) allows us to note that

$$\langle p_X | J_\alpha(0) | p_F \rangle = e^{\alpha_s B_{QCD}} \langle p_X | J_\alpha(0) | p_F \rangle_{\text{IRI-virt}} \quad \Rightarrow$$

$$W^F_{\alpha\beta}(p_F, q) = (2\pi)^3 \sum_X \delta(q + p_F - p_X) e^{2\alpha_s \Re B_{QCD}} \; {}_{\text{IRI-virt}}\langle p_F | J_\beta(0) | p_X \rangle \langle p_X | J_\alpha(0) | p_F \rangle_{\text{IRI-virt}} \quad \Rightarrow$$

$${}_{\text{IRI-virt}}\langle p_F | J_\beta(0) | p_X \rangle \langle p_X | J_\alpha(0) | p_F \rangle_{\text{IRI-virt}}$$

$$= \tilde{S}_{QCD}(k_1) \cdots \tilde{S}_{QCD}(k_n) \; {}_{\text{IRI-virt}}\langle p_F | J_\beta(0) | p_{X'} \rangle \langle p_{X'} | J_\alpha(0) | p_F \rangle_{\text{IRI-virt}} + \cdots +$$

$${}_{\text{IRI-virt\&real}}\langle p_F | J_\beta(0) | p_{X'}, k_1, \cdots, k_n \rangle \langle p_{X'}, k_1, \cdots, k_n | J_\alpha(0) | p_F \rangle_{\text{IRI-virt\&real}}$$

$$\Rightarrow$$

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$$W^F_{\beta\alpha}(p_F,q)=(2\pi)^3\sum_X \delta(q+p_F-p_X)e^{2\alpha_s \Re B_{QCD}}\left[\tilde{S}_{QCD}(k_1)\cdots\tilde{S}_{QCD}(k_n)\right.$$

$$_{\text{IRI-virt}}\langle p_F|J_\beta(0)|p_{X'}\rangle\langle p_{X'}|J_\alpha(0)|p_F\rangle_{\text{IRI-virt}}+\cdots$$

$$\left.+\ _{\text{IRI-virt\&real}}\langle p_F|J_\beta(0)|p_{X'},k_1,\cdots,k_n\rangle\langle p_{X'},k_1,\cdots,k_n|J_\alpha(0)|p_F\rangle_{\text{IRI-virt\&real}}\right]$$

$$=\frac{1}{2\pi}\int d^4y\sum_{X'}\sum_n\frac{1}{n!}\int\prod_{j=1}^n\frac{d^3k_j}{k_j^0}e^{\text{SUM}_{\text{IR}}(\text{QCD})}e^{iy(q+p_F-p_{X'}-\sum_j k_j)+D_{\text{QCD}}}$$

$$_{\text{IRI-virt\&real}}\langle p_F|J_\beta(0)|p_{X'},k_1,\cdots,k_n\rangle\langle p_{X'},k_1,\cdots,k_n|J_\alpha(0)|p_F\rangle_{\text{IRI-virt\&real}}$$

$$=\frac{1}{2\pi}\int d^4y\,e^{iqy}e^{\text{SUM}_{\text{IR}}(\text{QCD})+D_{\text{QCD}}}\ _{\text{IRI-virt\&real}}\langle p_F|[J_\beta(y),J_\alpha(0)]|p_F\rangle_{\text{IRI-virt\&real}},$$

$$\text{SUM}_{\text{IR}}(\text{QCD})=2\alpha_s\Re B_{\text{QCD}}+2\alpha_s\tilde{B}_{\text{QCD}}(\text{Kmax}),\quad 2\alpha_s\tilde{B}_{\text{QCD}}(\text{Kmax})=\int^{\le\text{Kmax}}\frac{d^3k}{k^0}\tilde{S}_{\text{QCD}}(k),$$

$$D_{\text{QCD}}=\frac{\int d^3k}{k}\tilde{S}_{\text{QCD}}(k)\left[e^{-iy\cdot k}-\theta(\text{Kmax}-k)\right]$$

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We use

$$W_{\beta\alpha} = \sum_a \int_0^1 \frac{dx}{x} F_a(x) W^a_{\beta\alpha}$$

to get

$$\int_0^1 dx\, x^n F_1(x, q^2) = \sum_j \tilde{\bar{C}}^{(n+1)}_{j,1}(q^2) \tilde{M}^{n+1}_j ,$$

$$\int_0^1 dx\, x^n F_2(x, q^2) = \sum_j \tilde{\bar{C}}^{(n)}_{j,2}(q^2) \tilde{M}^{n+2}_j ,$$

where

$$\tilde{\bar{C}}^{(n)}_{j,k}(q^2) = \frac{1}{2} i (q^2)^{n+1} \left(\frac{-\partial}{\partial q^2} \right)^n \int d^4 y\, e^{iqy + \mathrm{SUM}_n(\mathrm{QCD}) + D_{\mathrm{QCD}}} \frac{\tilde{C}^{(n)}_{j,k}(y^2)}{y^2 - i\epsilon y_0}$$

and

$$\left\langle p \middle| \tilde{O}^j_{\mu_1 \cdots \mu_n}(0) \middle| p \right\rangle \Big|_{\text{spin averaged}} \equiv$$

$$_{\text{IRI-virt\&real}}\left\langle p \middle| O^j_{\mu_1 \cdots \mu_n}(0) \middle| p \right\rangle_{\text{IRI-virt\&real}} \Big|_{\text{spin averaged}} = i^n \frac{1}{m_p} p_{P\mu_1} \cdots p_{P\mu_n} \tilde{M}^n_j + \cdots$$

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Still have Callan-Symanzik Eqn:

$$\left[\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right)\delta_{ij}-\tilde{\gamma}_{ij}^{(n)}(g)\right]\tilde{\tilde{C}}_{j,k}^{(n)}=0$$

for new matrix $\tilde{\gamma}_{ij}^{(n)}(g)$

We follow Curci, Furmanski and Petronzio(NPB175(1980)27):

For the NS operator ${}^{N}O^{F,b}(y)=\frac{1}{2}i^{N-1}S\bar{\psi}(y)\gamma_{\mu_1}\nabla_{\mu_2}\cdots\nabla_{\mu_N}\lambda^b\psi(y)-\text{trace terms},$

where $\nabla_\mu=\partial_\mu+ig\tau^a A_\mu^a$, S denotes symmetrization

$\langle p|\ {}^{N}O^{F,b}(y)|p\rangle=\ {}^{F,b}O^{N}(\alpha_s,\epsilon)p_{\mu_1}\cdots p_{\mu_N}-\text{trace terms},\qquad {}^{F,b}O^{N}(\alpha_s,\epsilon)\equiv M_{F,b}^{N}\Rightarrow$

$${}^{F,b}O^{N}(\alpha_s,\epsilon,p^2/\mu^2)=Z_O^{-1}(\alpha_s,\frac{1}{\epsilon})\ {}^{F,b}O_{\text{bare}}^{N}((\alpha_s)(\mu^2/p^2)^{\epsilon},\epsilon)$$

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Regulate collinear div. with $p^2 \neq 0$, $d=4-\varepsilon \Rightarrow$

- Use Δ, with $\Delta^2 = 0$,

$$ {}^{F,b}O^N(\alpha_s,\epsilon) = \left\langle p \middle| {}^{N}O^{F,b}_{\mu_1\cdots\mu_N}(y) \middle| p \right\rangle \Delta^{\mu_1}\cdots\Delta^{\mu_N}/(\Delta\, p)^N $$

Set $\Delta = n$, $x_{Bj} = np/np_p$, $x = nk/np$, $nA^a = 0 \Rightarrow$

$$ {}^{F,b}O^N(\alpha_s,\epsilon) = \int_{-1}^{1} dx\, x^{N-1}\ {}^{F,b}O(x,\alpha_s,\epsilon) $$

where

$$ {}^{F,b}O(x,\alpha_s,\epsilon) = Z_F\left[\delta(x-1) + x\frac{\int d^d k}{(2\pi)^d}\delta\left(x-\frac{kn}{pn}\right)\left[\frac{n}{4kn}T(p,k)\,p\right]\right. $$

T(p,k) = fully connected 4-pt fn., Z_F= field renorm., and

[**b** B denotes $b_{\alpha\alpha'}\, B^{\alpha\alpha'}_{\beta\beta'}$, etc.

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Analytic continuation to $d=4+\epsilon$, $\epsilon>0$, $p^2\to 0$ gives (CFP)

$$Z_O^{-1}(\alpha_s,\frac{1}{\epsilon})=\int_{-1}^{1}dx\,x^{N-1}\left[\Gamma_{qq}(x,\alpha_s,\frac{1}{\epsilon})\theta(x)-\Gamma_{q\bar{q}}(-x,\alpha_s,\frac{1}{\epsilon})\theta(-x)\right]$$

$\Gamma_{qq}(\Gamma_{q\bar{q}})\Leftrightarrow$ respective parton density for a quark(anti-quark) in a quark $\Rightarrow$ coefficients of $\frac{1}{\epsilon}$ give

$$-\gamma^{(N)}(\alpha_s)=2\int_{-1}^{1}dx\,x^{N-1}\left[P_{qq}(x,\alpha_s)\theta(x)-P_{q\bar{q}}(-x,\alpha_s)\theta(-x)\right]$$

$$=2\left[P_{qq}(N,\alpha_s)+(-1)^N P_{q\bar{q}}(N,\alpha_s)\right],$$

$F(N)=\int_0^1 dx\,x^{N-1}F(x)$, and

$P_{BA}\Leftrightarrow$ usual DGLAP-CS kernels in CFP convention,

with $\gamma^{(N)}(\alpha_s)\Leftrightarrow$ anomalous dimension of ${}^{N}O^{F,b}$

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Appliication to new IR-improved anomalous dimension matrix:

We IR-improve each step –

$$\langle p|\ {}^{N}O^{F,b}(y)|p\rangle \Rightarrow \langle p|\ {}^{N}\tilde{O}^{F,b}(y)|p\rangle$$

and $\quad {}^{F,b}O^{N}(\alpha_s,\epsilon) \Rightarrow\ {}^{F,b}\tilde{O}^{N}(\alpha_s,\epsilon)\ \Rightarrow$

$${}^{F,b}\tilde{O}^{N}(\alpha_s,\epsilon,p^2/\mu^2)=Z_{\tilde{O}}^{-1}(\alpha_s,\frac{1}{\epsilon})\ {}^{F,b}\tilde{O}^{N}_{bare}((\alpha_s)(\mu^2/p^2)^{\epsilon},\epsilon)\Rightarrow$$

$${}^{F,b}\tilde{O}^{N}(\alpha_s,\epsilon)=\int_{-1}^{1}dx\,x^{N-1}\ {}^{F,b}\tilde{O}(x,\alpha_s,\epsilon)\ \text{ where}$$

$${}^{F,b}\tilde{O}(x,\alpha_s,\epsilon)=Z_F[\delta(x-1)+x\frac{\int d^dk}{(2\pi)^d}\delta(x-\frac{kn}{pn})\left[\frac{\boldsymbol{n}}{4\mathrm{kn}}\tilde{T}(p,k)\boldsymbol{p}\right]$$

$\tilde{T}$(p,k) is IR-improved T(p,k) →

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IR-improved result:

$$Z_{\delta}^{-1}(\alpha_s,\frac{1}{\epsilon})=\int_{-1}^{1}dx\,x^{N-1}[\Gamma_{qq}^{exp}(x,\alpha_s,\frac{1}{\epsilon})\theta(x)-\Gamma_{q\bar{q}}^{exp}(-x,\alpha_s,\frac{1}{\epsilon})\theta(-x)]$$

where Γ_{qq}^{exp}, $\Gamma_{q\bar{q}}^{exp}$ are the respective IR-improved parton densities $\Rightarrow$

$$-\tilde{\gamma}^{(N)}(\alpha_s)=2\frac{\alpha_s}{2\pi}[P_{qq}^{exp}(N,\alpha_s)+(-1)^N P_{q\bar{q}}^{exp}(N,\alpha_s)]$$

where P_{qq}^{exp}, $P_{q\bar{q}}^{exp}$ $\Leftrightarrow$ respective IR-improved kernels $\Rightarrow$

IR-improved one-loop identifications

$$-\tilde{\gamma}^{(N)}(\alpha_s)_{ij}=2\frac{\alpha_s}{2\pi}P_{ij}^{exp}(N),$$

rigorous contact with new IR-improved DGLAP-CS theory

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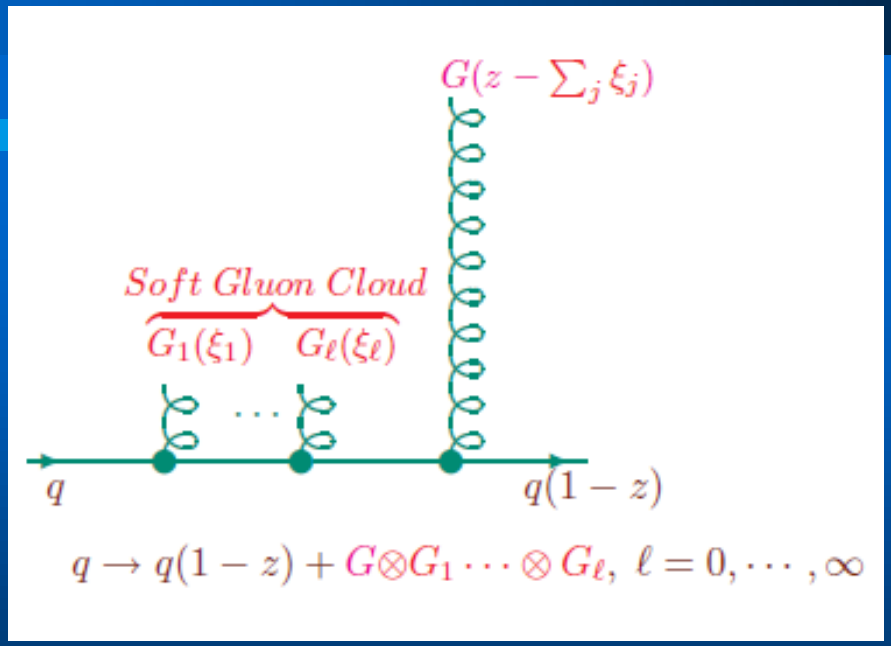
Basic Physical Idea:
Bloch-Nordsieck –

Accelerated Charge ⇒ Coherent State of Soft Gluons (Photons)

⇒ More Physical View of Splitting Process ($O(\hbar^n)$, n≥1, corrections):



Basic Physical Idea:
Bloch-Nordsiek –

More Physical View of Splitting Process in Practice:

$$P_{AB}(z) \longrightarrow P^{exp}_{AB}(z)$$

Resum terms $O((\alpha_s \ln(q^2/\Lambda^2) \ln(1-z))^n)$
for IR limit $z \longrightarrow 1 \Rightarrow$
Generate Gribov-Lipatov exponents γ_A .

•Example Direct Calculation

$$
\begin{aligned}
\int \frac{\alpha_s(t)}{2\pi} P_{BA}\, dt\, dz &= e^{\mathrm{SUM_{IR}(QCD)}(z)} \int \{ \bar{\bar{\beta}}_0 \int \frac{d^4 y}{(2\pi)^4} e^{\{ i y\cdot(p_1 - p_2) + \int^{k<K_{max}} \frac{d^3 k}{k} \bar{S}_{\mathrm{QCD}}(k) \left[e^{-i y\cdot k} - 1 \right] \}} \\
&+ \int \frac{d^3 k_1}{k_1} \bar{\bar{\beta}}_1(k_1) \int \frac{d^4 y}{(2\pi)^4} e^{\{ i y\cdot(p_1 - p_2 - k_1) + \int^{k<K_{max}} \frac{d^3 k}{k} \bar{S}_{\mathrm{QCD}}(k) \left[e^{-i y\cdot k} - 1 \right] \}} \\
&+ \cdots \} \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0} \\
&= e^{\mathrm{SUM_{IR}(QCD)}(z)} \int \{ \bar{\bar{\beta}}_0 \int_{-\infty}^{\infty} \frac{dy}{(2\pi)} e^{\{ i y\cdot(E_1 - E_2) + \int^{k<K_{max}} \frac{d^3 k}{k} \bar{S}_{\mathrm{QCD}}(k) \left[e^{-i y k} - 1 \right] \}} \\
&+ \int \frac{d^3 k_1}{k_1} \bar{\bar{\beta}}_1(k_1) \int_{-\infty}^{\infty} \frac{dy}{(2\pi)} e^{\{ i y\cdot(E_1 - E_2 - k_1^0) + \int^{k<K_{max}} \frac{d^3 k}{k} \bar{S}_{\mathrm{QCD}}(k) \left[e^{-i y\cdot k} - 1 \right] \}} \\
&+ \cdots \} \frac{d^3 p_2}{p_2^0 q_2^0}
\end{aligned}
$$

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$$I_{YFS}(zE,0) = \int_{-\infty}^{\infty} \frac{dy}{2\pi} e^{[iy(zE)+\int^{k<zE} \frac{d^3k}{k} \bar{S}_{QCD}(k)(e^{-iyk}-1)]}$$

$$= F_{YFS}(\gamma_q)\frac{\gamma_q}{zE}$$

$$I_{YFS}(zE,k_1) = \int_{-\infty}^{\infty} \frac{dy}{2\pi} e^{[iy(zE-k_1)+\int^{k<zE} \frac{d^3k}{k} \bar{S}_{QCD}(k)(e^{-iyk}-1)]}$$

$$= (\frac{zE}{zE-k_1})^{1-\gamma_q} I_{YFS}(zE,0)$$

$$\int \left(\bar{\bar{\beta}}_0 \frac{\gamma_q}{zE} + \int dk_1 k_1 d\Omega_1 \bar{\bar{\beta}}_1(k_1) (\frac{zE}{zE-k_1})^{1-\gamma_q} \frac{\gamma_q}{zE} \right) \frac{d^3p_2}{E_2 q_2^0} = \int dt \frac{\alpha_s(t)}{2\pi} P_{BA}^0 dz + \mathcal{O}(\alpha_s^2).$$

so that differentiation yields

$$P_{BA} = P_{BA}^0 z^{\gamma_q} F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q}$$

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• Herwiri1.031(PRD81(2010)076008):

w consultation from Bryan Webber, Stefano Frixione, and Mike Seymour, implementation of IR-improved kernels in Herwig 6.5 environment to get Herwiri1.031, MC@NLO/Herwiri1.031.

Observations:

1. SUM_{IR} is an IR effect – It contains as designed only the IR part of the LL, the rest of the LL is in D and the residuals $\tilde{\bar{\beta}}_m$, as we show in PRD81(2010)076008.
2. Herwiri is just as general as Herwig6.5, as they run the same set of processes

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- Herwiri++, Herwiri++/Powheg, Herwiri++/MC@NLO -- running but still undergoing check-out

- Pythia8(Tjorborn Sjostrand, Peter Skands, Jesper Christiansen) , in progress

- Sherpa(JanWinter), in progress

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- Important Technical Point on MC@NLO vs POWHEG Hardest Emission Sudakov in POWHEG uses full O(α_s) emission result (we follow 0803.0883)

$$\Delta_{\hat{R}}(p_T) = \exp\left[-\int \mathrm{d}\Phi_R \frac{\hat{R}(\Phi_B, \Phi_R)}{B(\Phi_B)} \theta\left(k_T(\Phi_B, \Phi_R) - p_T\right)\right]$$

$\Rightarrow$ must synthesize this with (1) as well

- Some initial illustrative results follow first.

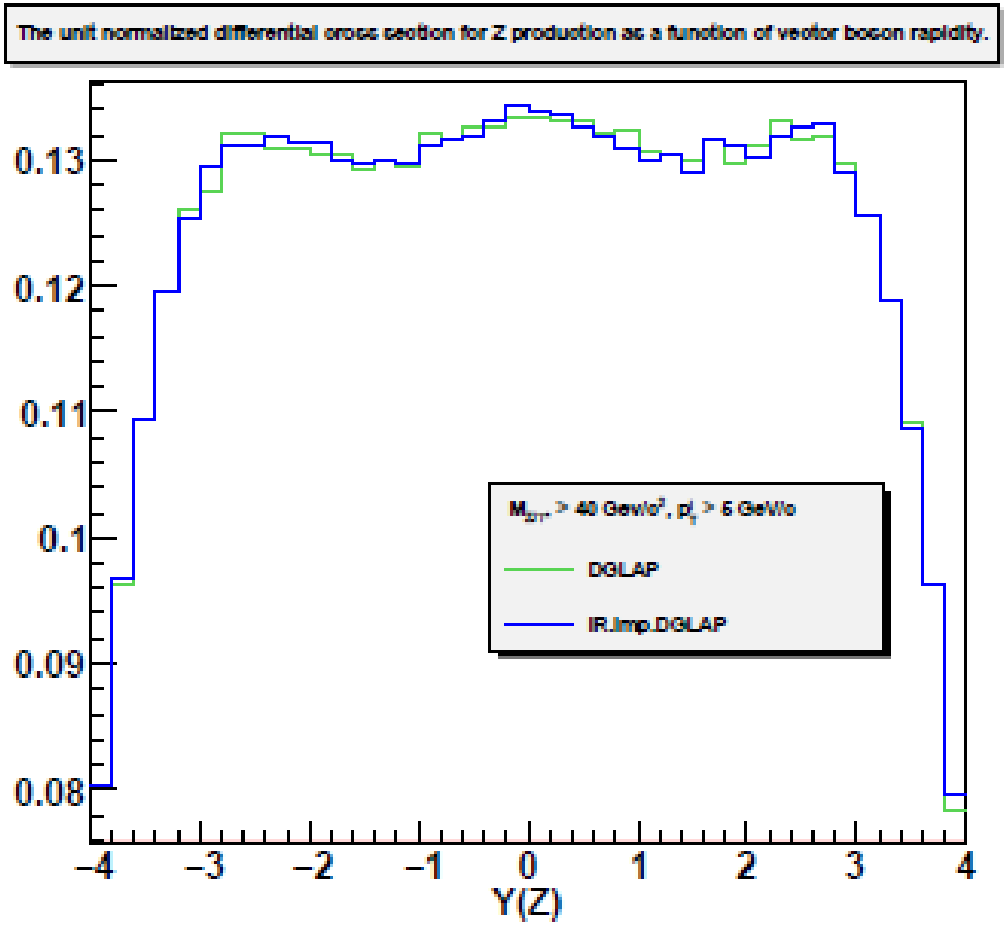

Figure 9: The Z rapidity-distribution(ISR parton shower) comparison in HERWIG6.5.

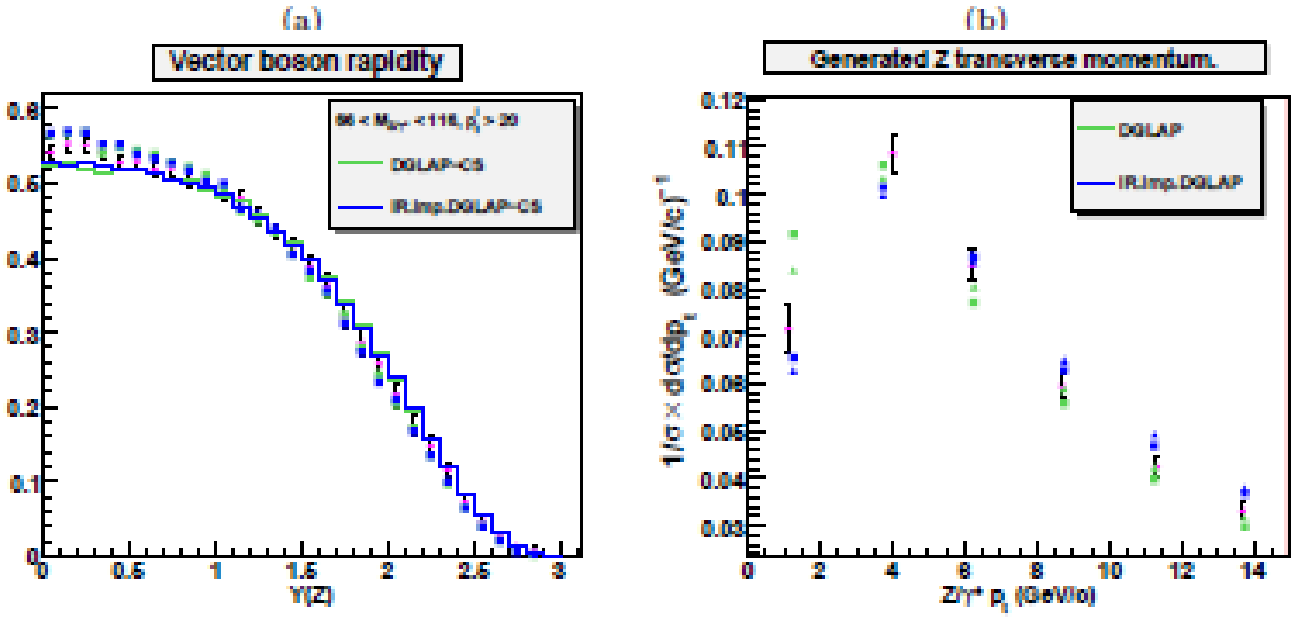

Figure 11: Comparison with FNAL data: (a), CDF rapidity data on (Z/γ^*) production to e^+e^- pairs, the circular dots are the data, the green(blue) lines are HERWIG6.510(HERWIRI1.031); (b), D0 p_T spectrum data on (Z/γ^*) production to e^+e^- pairs, the circular dots are the data, the blue triangles are HERWIRI1.031, the green triangles are HERWIG6.510. In both (a) and (b) the blue squares are MC@NLO/HERWIRI1.031, and the green squares are MC@NLO/HERWIG6.510, where we use the notation (see the text) MC@NLO/X to denote the realization by MC@NLO of the exact $\mathcal{O}(\alpha_s)$ correction for event generator X. Note that these are untuned theoretical results.

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- **HERWIRI1.031 -- PRD81 (2010) 076008:**

  - For the CDF rapidity data, HERWIRI1.031 is closer to the data than is HERWIG6.510 (1.54 vs 1.77 for χ^2/d.o.f. resp.);
    for MC@NLO/HERWIRI1.031 and MC@NLO/HERWIG6.510
    the χ^2/d.o.f are 1.42 and 1.40 resp., both are within 10% of the data
    $\Rightarrow$ Need NNLO level, in progress.
  - For the D0 p_T data, HERWIRI1.031 gives a better fit
    to the data compared to HERWIG6.5 for low p_T,
    for $p_T < 12.5$GeV, the χ^2/d.o.f. are $\sim$ 2.5 and 3.3 respectively
    - we add the statistical and systematic errors,
    showing that the IR-improvement makes a better representation
    of QCD in the soft regime for a given fixed order in perturbation theory.

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• LHC DATA: CMS Rapidity & ATLAS PT Spectrum for Z/γ* Production

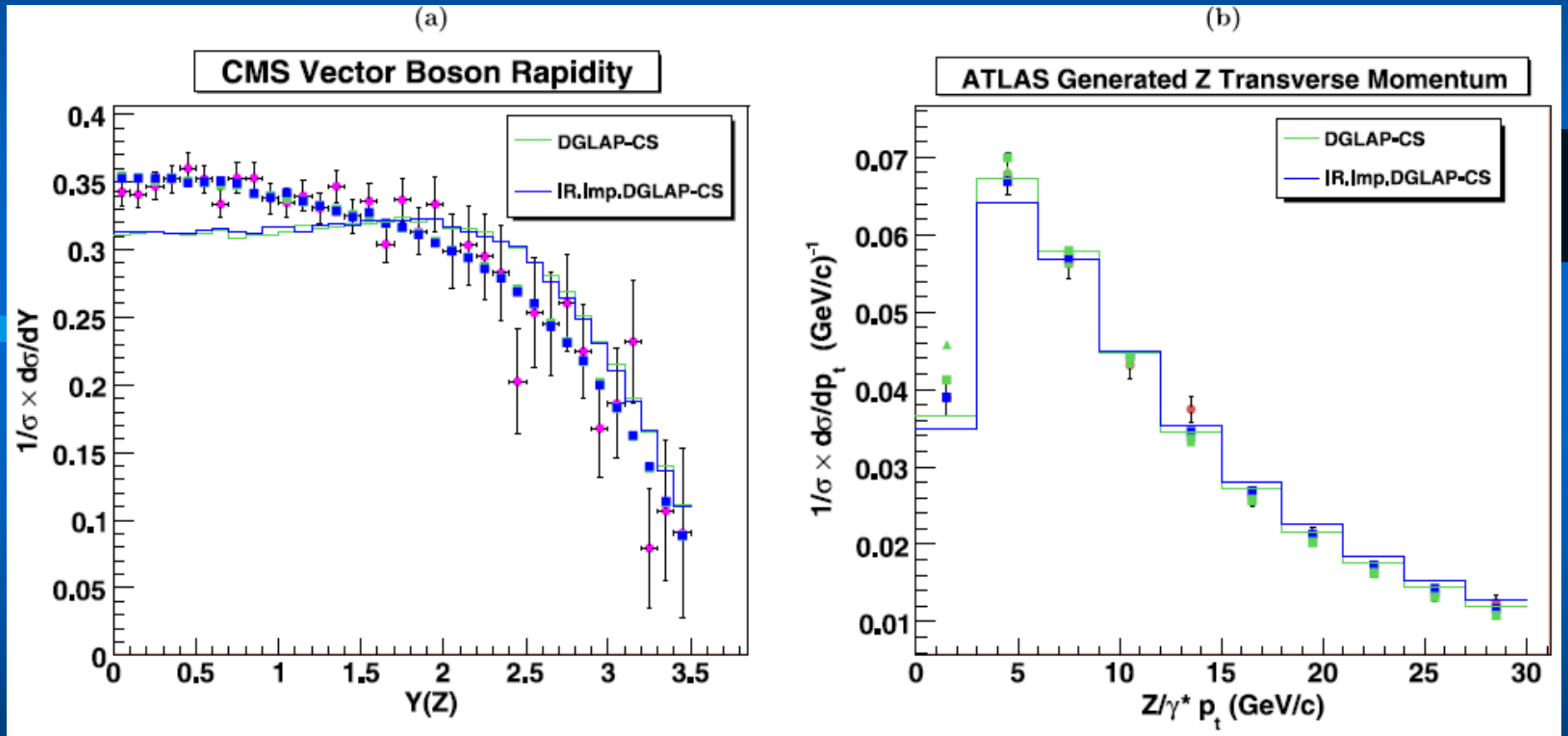
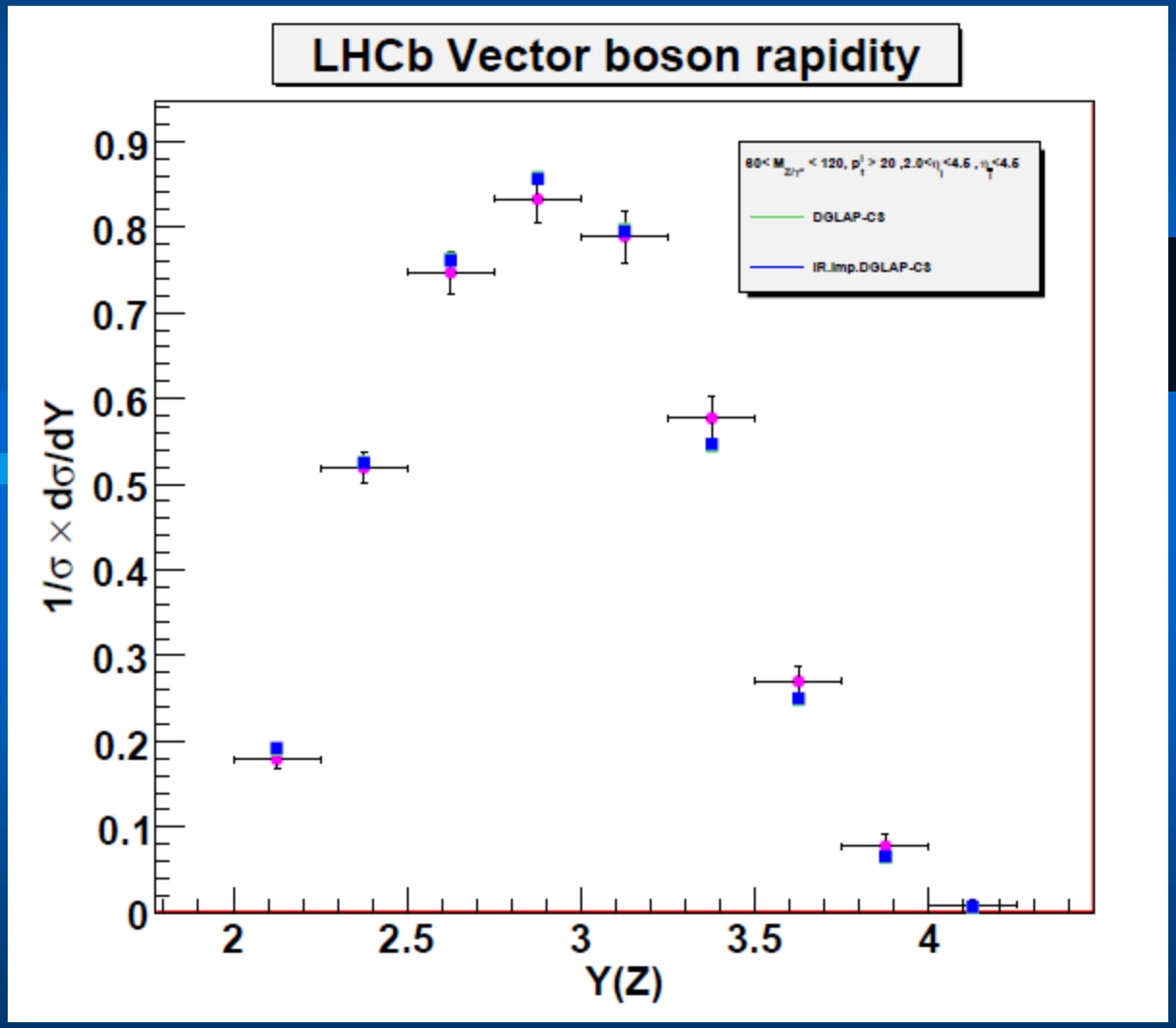


Fig. 2. Comparison with LHC data: (a) CMS rapidity data on (Z/γ^*) production to e^+e^-, $\mu^+\mu^-$ pairs, the circular dots are the data, the green (blue) lines are HERWIG6.510 (HERWIRI1.031); (b) ATLAS p_T spectrum data on (Z/γ^*) production to (bare) e^+e^- pairs, the circular dots are the data, the blue (green) lines are HERWIRI1.031 (HERWIG6.510). In both (a) and (b) the blue (green) squares are MC@NLO/HERWIRI1.031 (HERWIG6.510 (PTRMS = 2.2 GeV)). In (b), the green triangles are MC@NLO/HERWIG6.510 (PTRMS = 0). These are otherwise untuned theoretical results. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

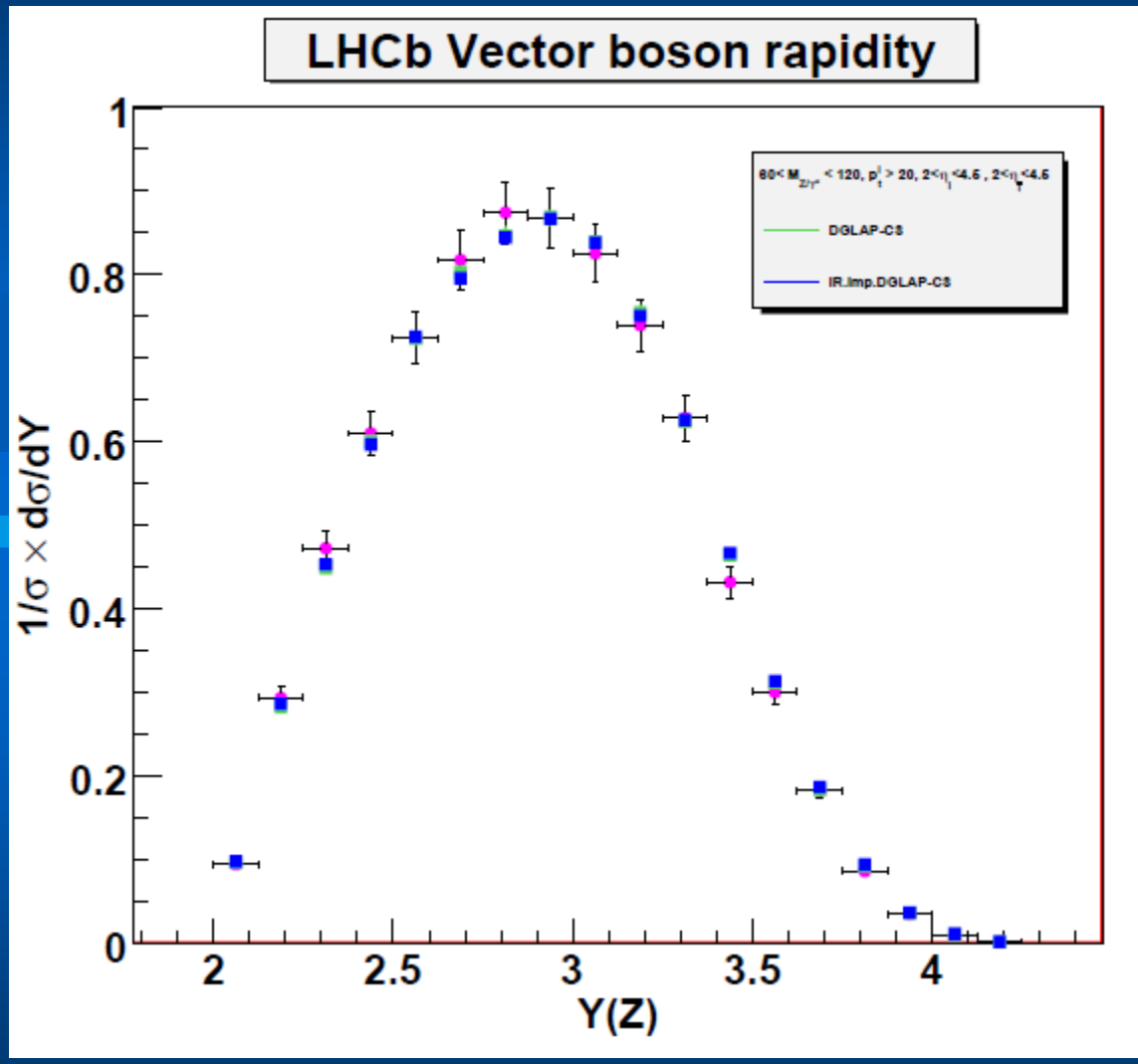
- LHC DATA: LHCb Rapidity for Z/γ^* Production(e^+e^-)

χ^2/d.o.f:
.746, .814, .836 for MC@NLO/Herwiri MC@NLO/Herwig (PTRMS=0), MC@NLO/Herwig (PTRMS=2.2GeV), respectively



- LHC DATA: LHCb Rapidity for Z/γ* Production ($\mu^+\mu^-$)

χ^2/d.o.f:
.773, .555, .537 for
MC@NLO/Herwiri
MC@NLO/Herwig
(PTRMS=0),
MC@NLO/Herwig
(PTRMS=2.2GeV),
respectively



• Observations

1. For the unimproved case, the data suggest that we need a GAUSSIAN (intrinsic) PTRMS ≅ 2.2 GeV

[Herwiri1.031(blue line), Herwig6510(green line(PTRMS=2.2GeV)), MC@NLO/Herwiri1.031(blue squares), MC@NLO/Herwig6510(green squares (PTRMS=2.2GeV), green triangles(PTRMS=0))] (similar to what holds at FNAL)

2. This same shape is provided from fundamental principles by the MC@NLO/Herwiri1.031 with PTRMS ≅ 0 GeV (similar to what holds at FNAL)

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- Observations (Quantitative)

1. Unimproved case, the respective χ^2 /d.o.f.'s are 1.37, 0.70 (MC@NLO/Herwig6510(PTRMS=2.2GeV)) for the p_T and CMS rapidity data
2. IR-improved case, the respective χ^2 /d.o.f.'s are 0.72, 0.72 (MC@NLO/Herwiri1.031) for the p_T and CMS rapidity data
3. Unimproved case, the respective χ^2 /d.o.f.'s are 2.23, 0.70 (MC@NLO/Herwig6510(PTRMS=0)) for the p_T and CMS rapidity data
4. So far, LHCb rapidity data support these results. p_T data for LHCb still under study: need unfolding matrix process, etc.

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• Which is Better for Precision QCD Theory?

1. Precocious Bjorken Scaling in SLAC-MIT Experiments: already at $Q^2 \cong 1_+$ GeV^2

$\Rightarrow$ PTRMS^2 small compared to 1_+ GeV^2

See R.P. Feynman, M. Kislinger and F. Ravndal, Phys. Rev. D**3** (1971) 2706; R. Lipes, *ibid*.**5** (1972) 2849; F.K. Diakonas, N.K. Kaplis and X.N. Mawita, *ibid*. **78** (2008) 054023; K. Johnson, Proc. Scottish Summer School Phys. **17** (1976) p. 245; A. Chodos *et al*., Phys. Rev. D**9** (1974) 3471; *ibid*. **10** (1974) 2599; T. DeGrand *et al*., *ibid*. **12** (1975) 2060; …. – all have PTRMS^2 << 1_+ GeV^2

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- Which is Better for Precision QCD Theory?

2. The first principles approach is not subject to arbitrary functional variation to determine its $\Delta\sigma_{th}$

3. Experimentally, in principle, events in the two cases should look different in terms of the properties of the rest of the particles in the events – this is under study.

Here, we show the following.

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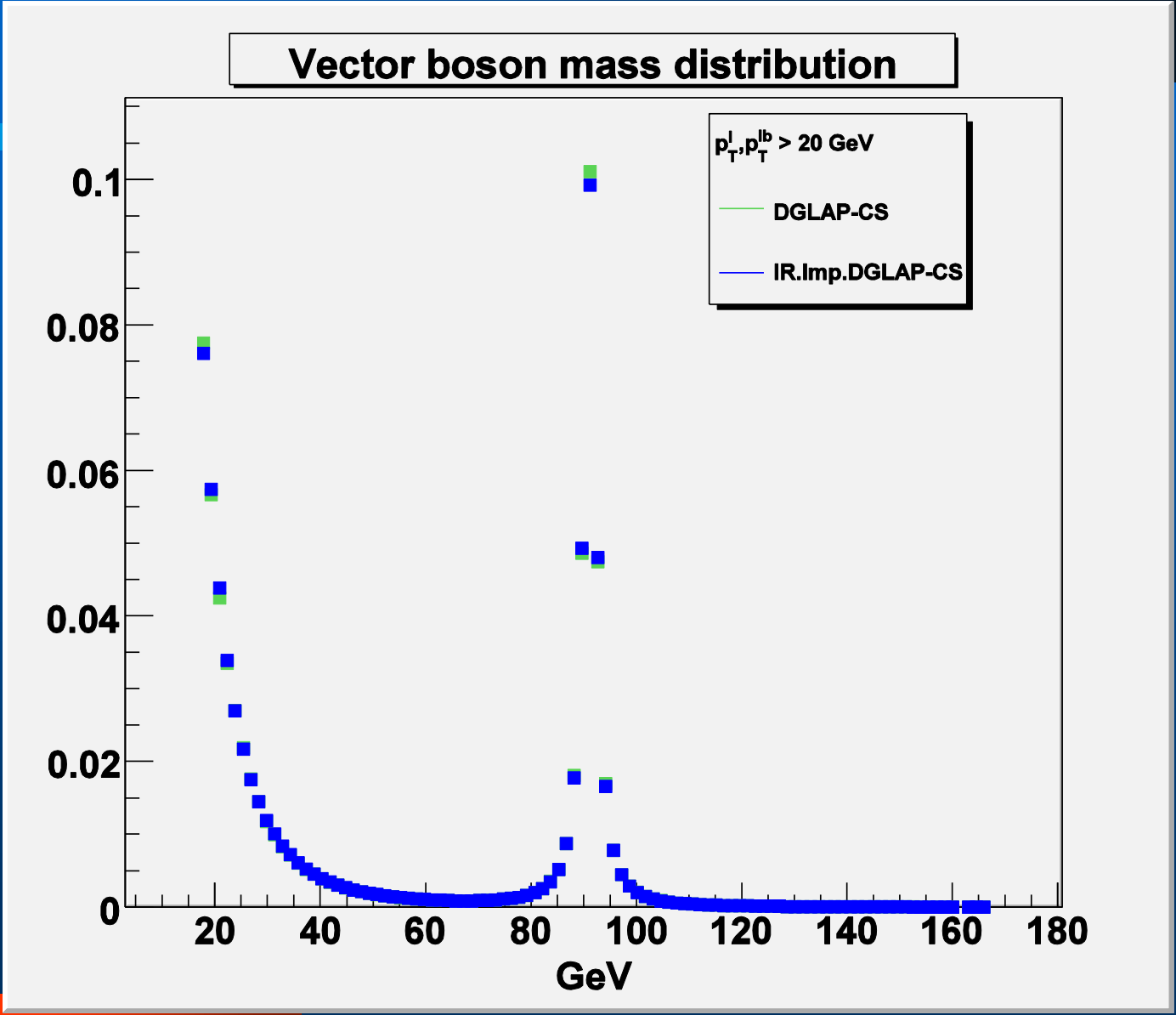
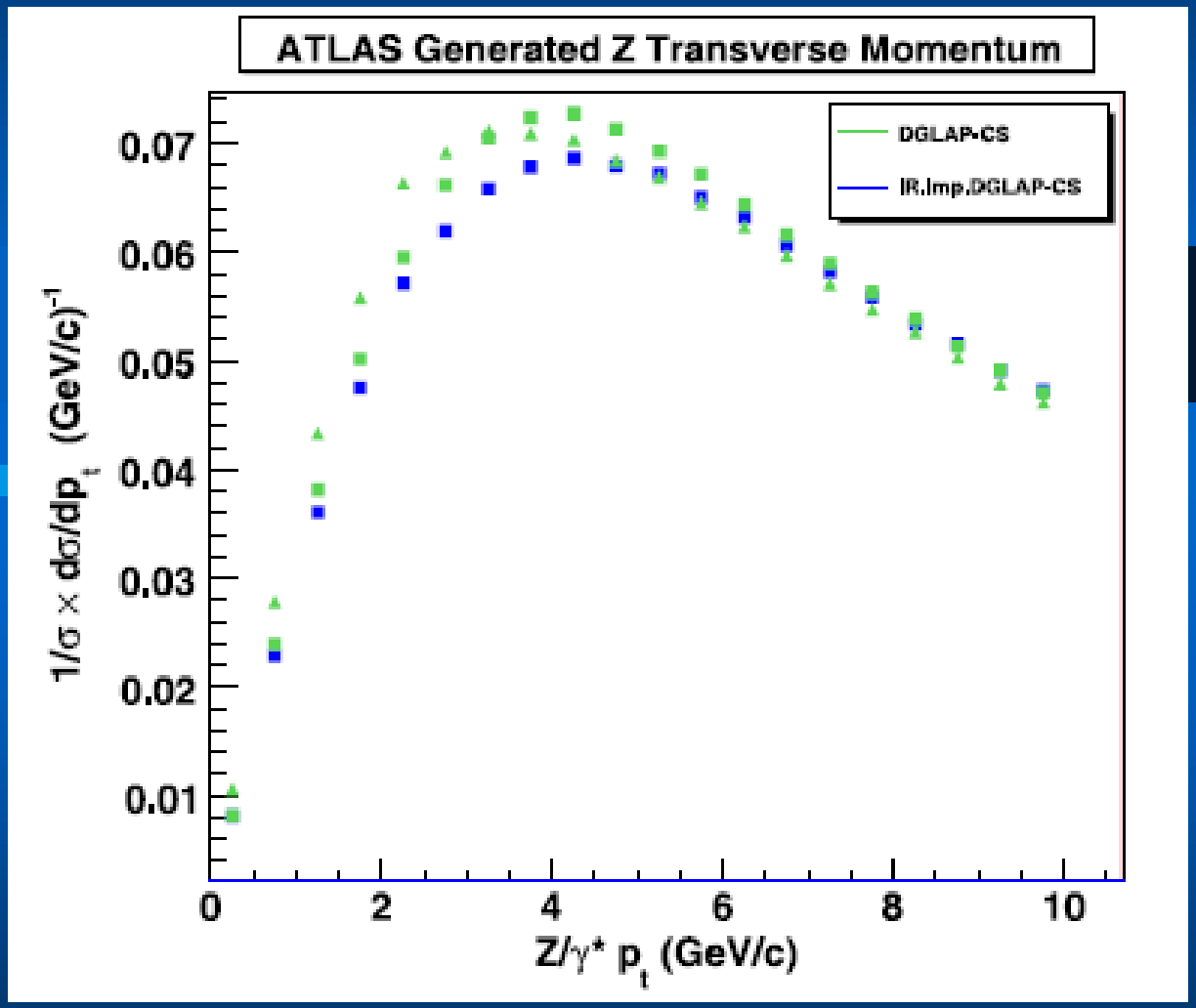
For example: 2.2% Peak Effect



THEORY COMPARISON: FINER BINS -- 0.5GeV vs 3.0GeV



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OBSERVATIONS

* IR-Improved DGLAP-CS Theory Increases Definiteness of Precision Determination of NLO Parton Shower MC's and Improves Such.
* More Potential Checks Against Experiment Are Being Pursued.

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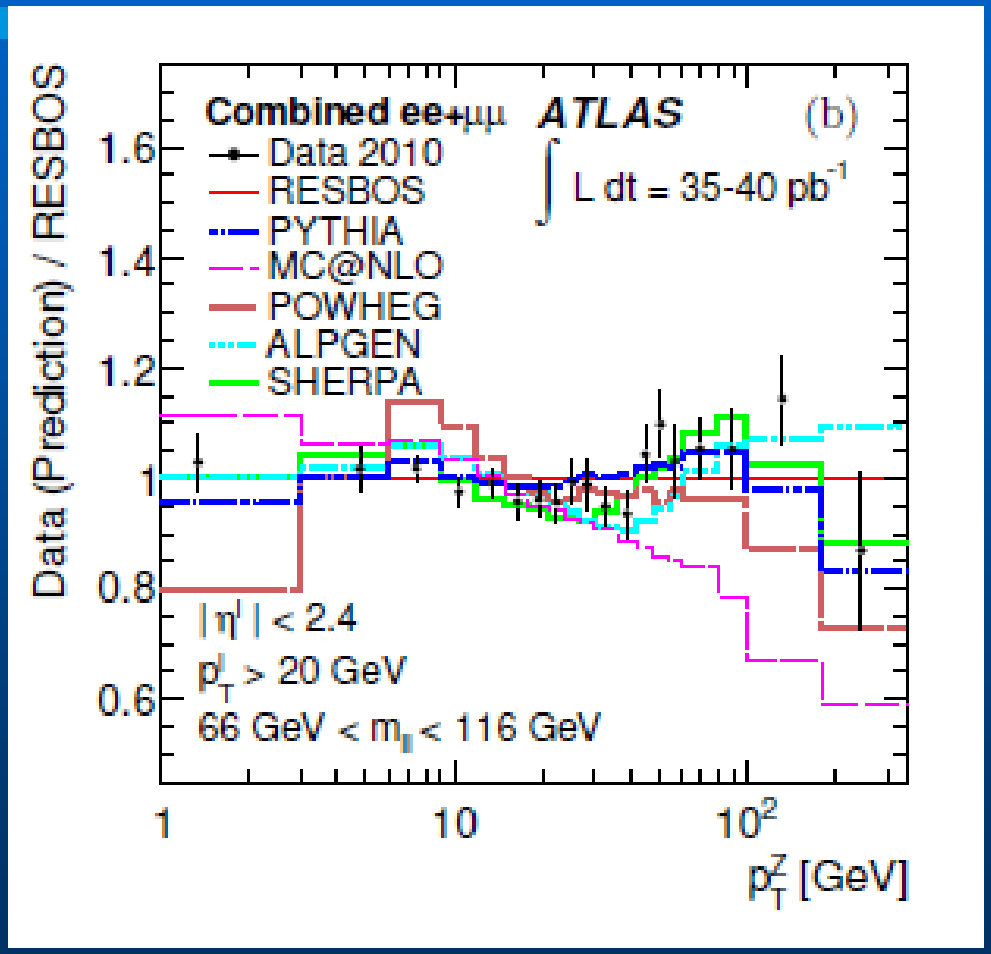
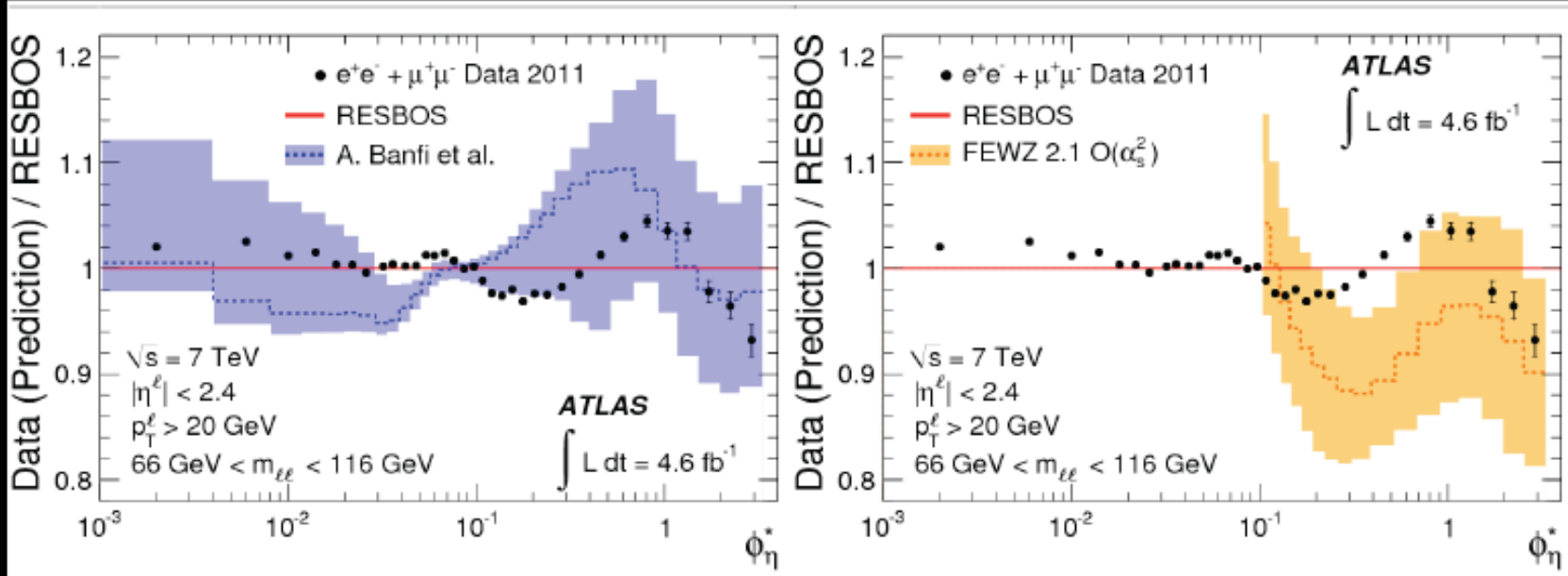
- **MORE THEORY COMPARISONS:ATLAS(1107.2381)**



- **MORE THEORY COMPARISONS:Hassani(EW Moriond, 2013) – { $\phi^*_\eta = \tan(\frac{1}{2}(\pi - \Delta\phi))\sin\theta^*$ }**

Z/γ* transverse momentum (dσ/dφ*$_\eta$(ll))



- Calculations from A. Banfi et al. (resummed QCD predictions+fixed-order pQCD) is less good than ResBos
- **Measurement precision about one order of magnitude lower than the present theoretical uncertainties**
- FEWZ predictions undershoot the data by ~10% which confirm previous CDF observation (PRD 86,052010) See Julia's and Stefan's talks

Near Future

* Herwig++(soon, running , under cross checks)
* Pyhtia 8,6 (w consultation from Peter Skands and Torbjorn Sjostrand, Jesper Christiansen)
* Sherpa (w consultation from Jan Winter)

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Near Future

*New Observables: ϕ_η^* (w p_T cuts, etc.)

*New Data: ATLAS & CMS,

EACH > 10^7 lepton pairs

$\Longrightarrow$ COMPLETE INTRINSIC P_T TESTS

*HERWIRI2.1 (w S.Yost, M. Hjena, V. Halyo)

HERWIG6.5 $\cup$ KK MC 4.22

KK MC 4.22 (w S. Jadach, Z.Was),

PRD **88** (2013) 114022

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. . . *MC@NNLO(see also Julia's and Stefan's talks)

KK MC 4.22

v_{max}	KKsem Refer.	$\mathcal{O}(\alpha^3)_{EEX3}$	$\mathcal{O}(\alpha^2)_{CEEX}$ intOFF	$\mathcal{O}(\alpha^2)_{CEEX}$
	$\sigma(v_{max})$ [pb]			
0.01	0.9145 ± 0.0000	0.9150 ± 0.0004	0.9150 ± 0.0004	0.9323 ± 0.0004
0.10	1.0805 ± 0.0000	1.0807 ± 0.0004	1.0808 ± 0.0004	1.0920 ± 0.0004
0.30	1.1612 ± 0.0000	1.1615 ± 0.0004	1.1616 ± 0.0004	1.1691 ± 0.0004
0.50	1.1974 ± 0.0000	1.1977 ± 0.0004	1.1981 ± 0.0004	1.2036 ± 0.0004
0.70	1.2310 ± 0.0000	1.2312 ± 0.0004	1.2317 ± 0.0004	1.2357 ± 0.0004
0.90	1.6104 ± 0.0000	1.6128 ± 0.0003	1.6114 ± 0.0004	1.6148 ± 0.0004
0.99	1.6218 ± 0.0000	1.6254 ± 0.0003	1.6244 ± 0.0004	1.6277 ± 0.0004
	$A_{FB}(v_{max})$			
0.01	0.5883 ± 0.0000	0.5883 ± 0.0005	0.5883 ± 0.0005	0.6033 ± 0.0005
0.10	0.5882 ± 0.0000	0.5881 ± 0.0004	0.5881 ± 0.0004	0.5966 ± 0.0004
0.30	0.5879 ± 0.0000	0.5879 ± 0.0004	0.5879 ± 0.0004	0.5932 ± 0.0004
0.50	0.5875 ± 0.0000	0.5874 ± 0.0004	0.5875 ± 0.0004	0.5912 ± 0.0004
0.70	0.5848 ± 0.0000	0.5845 ± 0.0004	0.5846 ± 0.0004	0.5868 ± 0.0004
0.90	0.4736 ± 0.0000	0.4722 ± 0.0003	0.4728 ± 0.0003	0.4748 ± 0.0003
0.99	0.4710 ± 0.0000	0.4691 ± 0.0003	0.4697 ± 0.0003	0.4716 ± 0.0003

TABLE II. Study of total cross section $\sigma(v_{max})$ and charge asymmetry $A_{FB}(v_{max})$, $d\bar{d} \to \mu^-\mu^+$, at $\sqrt{s}$ =189GeV. See Table I for definition of the energy cut v_{max}, scattering angle and M.E. type,

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v=1- s'/s (EW lib. = DIZET6.21)

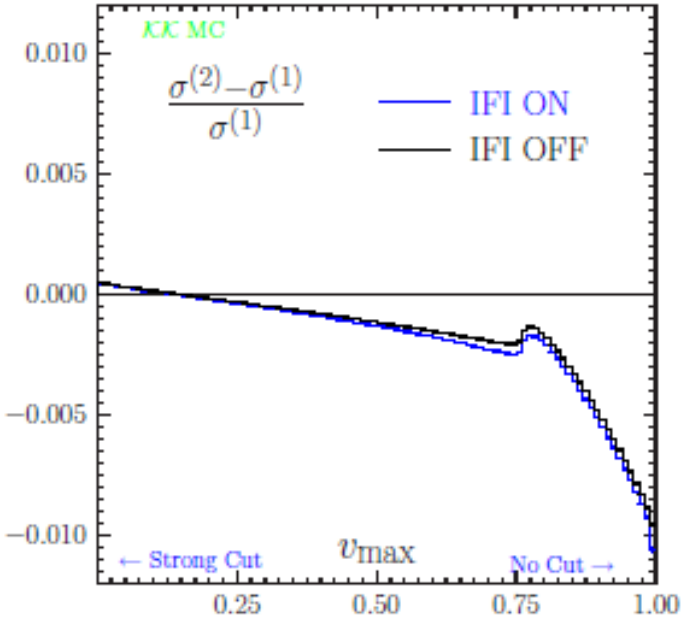
Physical Precision of CEEX ISR

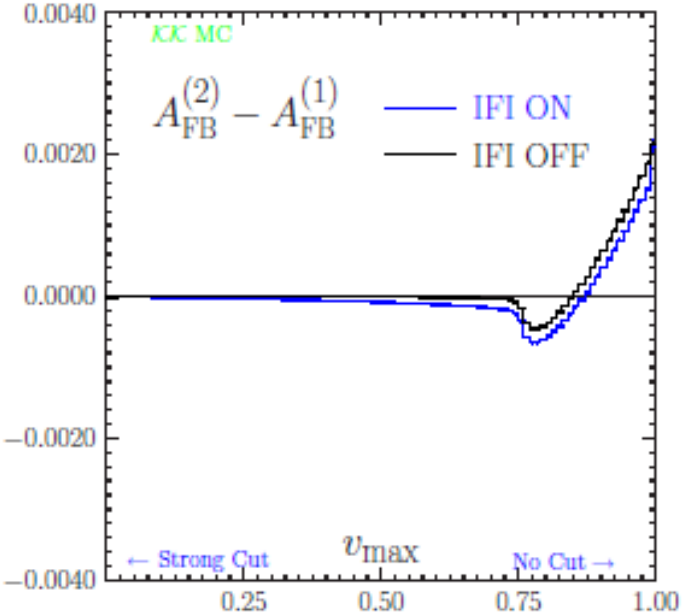
The difference between second and first order CEEX results for at 189GeV.

The energy cut is on s'/s, where $s' = m^2_{f\bar{f}}$.

Scattering angle is $\theta = \theta^{\bullet}$.

[Angle $\theta^{\bullet}$ is defined in Phys. Rev. **D41**, 1425 (1990)]





More on KKMC version 4.22 (2013)
Physics extensions, 3rd step: PDFs for quark beam

Quark beams at energies varying according to PDFs.

Main problem in the code: variable $\sqrt{s}$ from one MC event to another.
Luckily already solved for beamstrahlung.

Test for $\boxed{u\bar{u} \to e^- e^+ + n\gamma}$ at $\sqrt{s} = M_Z$.

KKMC vs. KKsem not available :
Only kinematics was tested, see event printout next slide.

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More on KKMC version 4.22 (2013)
Physics extensions, 3rd step: PDFs for quark beam

```
*****************************************************************************
*                         KK Monte Carlo                                    *
*            Version        4.22          May 2013                          *
*    7000.00000000                 CMS energy average       CMSene     a1 *
*       0.00000000                 Beam energy spread       DelEne     a2 *
*              100                 Max. photon mult.        npmax      a3 *
*                0                 wt-ed or wt=1 evts.      KeyWgt     a4 *
*                1                 ISR switch               KeyISR     a4 *
*                1                 FSR switch               KeyFSR     a5 *
*                2                 ISR/FSR interferenc      KeyINT     a6 *
*                1                 New exponentiation       KeyGPS     a7 *
*                0                 Hadroniz.  switch        KeyHad     a7 *
*       0.20000000                 Hadroniz. min. mass      HadMin     a9 *
*       1.00000000                 Maximum weight           WTmax     a10 *
*              100                 Max. photon mult.        npmax     a11 *
*                2                 Beam ident               KFini     a12 *
*       0.03500000                 Manimum phot. ener.      Ene       a13 *
*   0.10000000E-59                 Phot.mass, IR regul      MasPho    a14 *
*    1.2500000                     Phot. mult. enhanc.      Xenph     a15 *
*       0.00000000                    PolBeam1(1)           Pol1x     a17 *
*       0.00000000                    PolBeam1(2)           Pol1y     a18 *
*       0.00000000                    PolBeam1(3)           Pol1z     a19 *
*       0.00000000                    PolBeam2(1)           Pol2x     a20 *
*       0.00000000                    PolBeam2(2)           Pol2y     a21 *
*       0.00000000                    PolBeam2(3)           Pol2z     a22 *
*****************************************************************************
                           Event listing (summary)
  I particle/jet KS     KF  orig    p_x      p_y      p_z       E        m
  1 !u!          21      2    0    0.000    0.000   22.668   22.668    0.005
  2 !ubar!       21     -2    0    0.000    0.000 -245.458  245.458    0.005
  3 (Z0)         11     23    1   23.016   18.370  -80.068  115.249   77.487
  4 gamma         1     22    1  -30.989   -6.132 -128.905  132.719    0.000
  5 gamma         1     22    1    0.000    0.000    0.031    0.031    0.000
  6 gamma         1     22    1    7.973  -12.238  -13.848   20.127    0.000
  7 gamma         1     22    1    0.000    0.000 3477.332 3477.332    0.000
  8 gamma         1     22    1    0.000    0.000-3254.542 3254.542    0.000
  9 tau-          1     15    3  -24.701   21.657  -20.217   38.613    1.777
 10 tau+          1    -15    3   47.716   -3.287  -59.851   76.635    1.777
                sum:  0.00         0.000    0.000    0.000 7000.000 7000.000
```

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Cont.

```
                          Event listing (summary)
 I particle/jet KS     KF  orig    p_x      p_y      p_z       E        m
 1 !u!          21      2    0    0.000    0.000  271.908  271.908    0.005
 2 !ubar!       21     -2    0    0.000    0.000   -6.542    6.542    0.005
 3 (Z0)         11     23    1    0.047    1.133  244.401  257.454   80.928
 4 gamma         1     22    1   -0.047   -1.133   20.965   20.996    0.000
 5 gamma         1     22    1    0.000    0.000 3228.092 3228.092    0.000
 6 gamma         1     22    1    0.000    0.000-3493.458 3493.458    0.000
 7 mu-           1     13    3    0.601   14.537    2.005   14.687    0.106
 8 mu+           1    -13    3   -0.554  -13.404  242.396  242.767    0.106
                sum:  0.00         0.000    0.000    0.000 7000.000 7000.000
                          Event listing (summary)
 I particle/jet KS     KF  orig    p_x      p_y      p_z       E        m
 1 !u!          21      2    0    0.000    0.000 1816.851 1816.851    0.005
 2 !ubar!       21     -2    0    0.000    0.000   -1.137    1.137    0.005
 3 (Z0)         11     23    1    0.011    0.003 1810.259 1812.532   90.760
 4 gamma         1     22    1   -0.012   -0.002    5.371    5.371    0.000
 5 gamma         1     22    1    0.000    0.000 1683.149 1683.149    0.000
 6 gamma         1     22    1    0.000    0.000-3498.863 3498.863    0.000
 7 mu-           1     13    3   12.468  -25.466 1612.743 1612.992    0.106
 8 mu+           1    -13    3  -12.457   25.469  197.516  199.540    0.106
                sum:  0.00        -0.001    0.001   -0.084 6999.916 6999.916

*******************************************************************************
*                        KK2f_Finalize  printouts                            *
*    7000.00000000                cms energy total          cmsene     a0 *
*             5000                total no of events        nevgen     a1 *
*                ** principal info on x-section **                         *
*     233.95163953  +- 1.04896414  xs_tot MC R-units        xsmc       a1 *
*       0.41468908                 xs_tot    picob.         xSecPb     a3 *
*       0.00185933                 error     picob.         xErrPb     a4 *
*       0.00448368                 relative error           erel       a5 *
*       0.82048782                 WTsup, largest WT        WTsup     a10 *
*                        ** some auxiliary info **                         *
*       0.00219522                 xs_born   picobarns       xborn    a11 *
*       0.73760000                 Raw phot. multipl.                 === *
*       5.00000000                 Highest phot. mult.                === *
*                          End of KK2f_Finalize                            *
*******************************************************************************
```

KKMC 4.22

NON-ZERO PT H.O. EW CORRECTIONS,

.2% = $\Delta\sigma_{th}$

- OTHER EFFORTS TO IMPROVE RESUMMATION IN PROGRESS: EW COLLINEAR REGIME – BARZE ET AL., …
- NEW NLO and NNLO RESULTS: multi-leg, tt , …
- WHAT WE CAN SAY IS THIS: FULL EXPLOITATION OF LHC/FCC DISCOVERY POTENTIAL WILL NEED SUCH EFFORTS

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Possible other extensions?

Could one include/improve QCD correction for the incoming beams?
Yes, for example classic NLO corrs, Powheg style etc.

In this case the upper level of KKMC would be replaced by C++ code, which is already in place in some simple form.

The extension to $q\bar{q} \to W \to l + \nu$ is thinkable, but would require update of the QED matrix element (EW corrs. ?)

NB. t-canel W exchange with h.o. QED is already there for $\nu\bar{\nu}$ chanel and could be exploited as a starting point.

However, the bottom line still vailid is:
Let us exploit the existing KKMC as much as we can for LHC/FCC(-ee)!!!

Z invisible width from $\sigma(e^+e^- \to \nu\bar{\nu}\gamma)/\sigma(e^+e^- \to \mu\bar{\mu}\gamma)$

-- Subtitle:

Study on theoretical uncertaincies (QED) in the measurement of Z the invisible width from $e^-e^+ \to \nu + \bar{\nu} + \gamma$ using KKMC

S. Jadach, B.F.L. Ward and Z. Wąs

IFJ-PAN, Kraków, Poland

Partly supported by Polish Government grant
Narodowe Centrum Nauki DEC-2011/03/B/ST2/02632

To be presented at FCCee Physics study meeting
CERN, March 10th, 2014

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Z invisible width from $\sigma(e^+e^- \to \nu\bar{\nu}\gamma)/\sigma(e^+e^- \to \mu\bar{\mu}\gamma)$

, S. Jadach et al., in progress:

- Z invisible width in terms of number of neutrinos from LEP $N_\nu = 2.984 \pm 0.008$
- According to "The TLEP Design Study...", page 29 `http://arxiv.org/abs/arXiv:1308.6176` could be measured 10 times better.
- TLEP run near WW threshold 5pb would ensure 3M events with visible photon and invisible $Z \to \nu\bar{\nu}$ decay.
- No reliable estimate of the theoretical (QED) uncertainties at this precision level – only hope that this process is possibly better that Z peak cross section.
- Let us make 1st step in working out such an estimate...

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Acceptance criteria for $e^- e^+ \to \nu + \bar{\nu} + \gamma$

Acceptance criteria:

- Minimum photon angle $\Theta_{\text{min}} = 15^o$,
- Minimum photon energy $x_\gamma = 0.3, \quad E_\gamma > x_\gamma E_{beam}$,
- Minimum phot. transv. mom. $x_T = 0.3, \quad k_\gamma^T > x_\gamma E_{beam}$,
- Only one photon within the above restrictions.

Variable $v = E_\gamma / E_{beam}$ will be used in the histograms.
MC results will come from KKMC version 4.22, see above.

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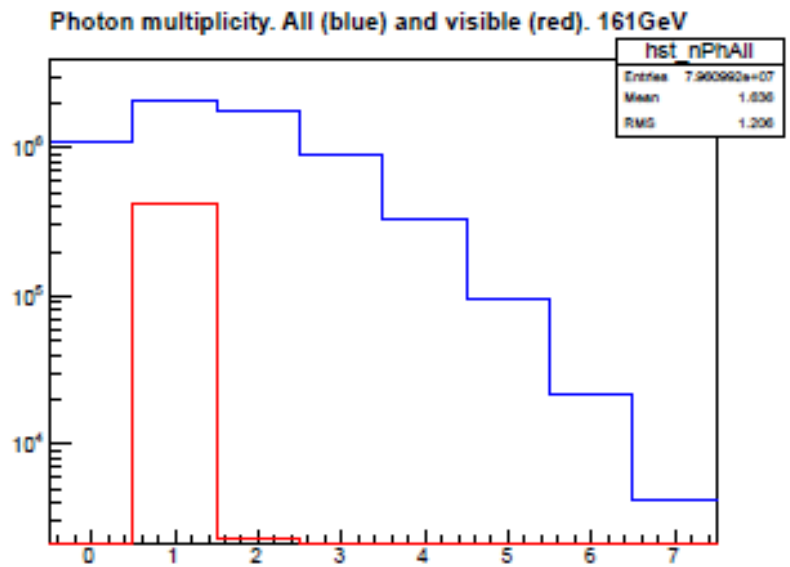
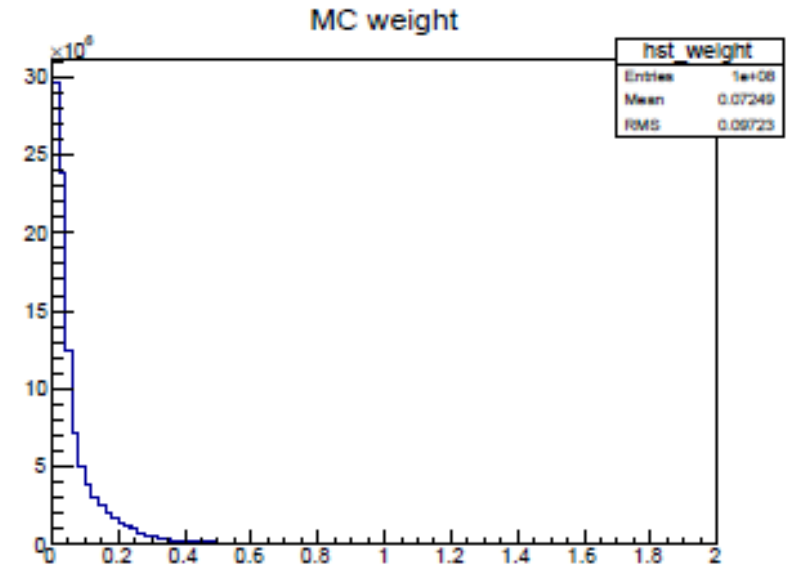
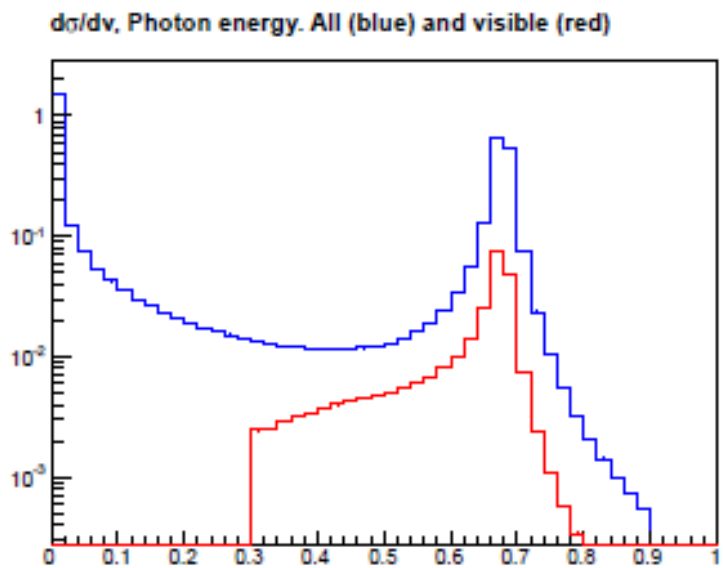
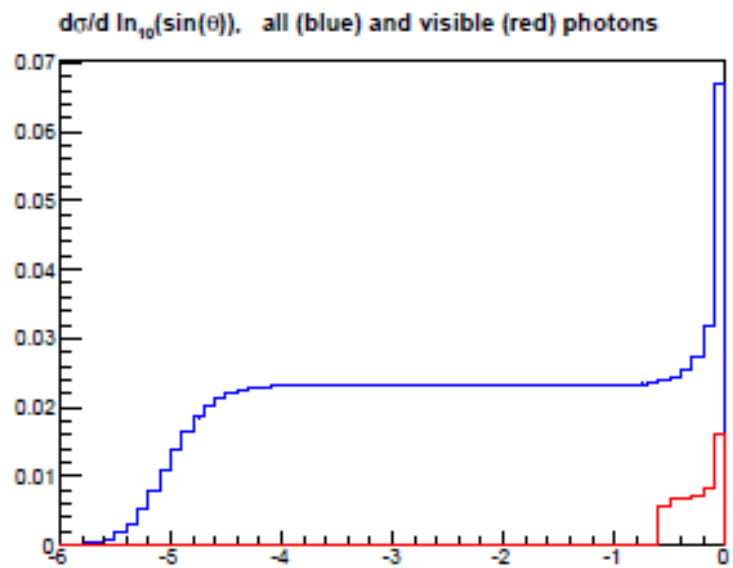
Acceptance criteria at work, 161GeV

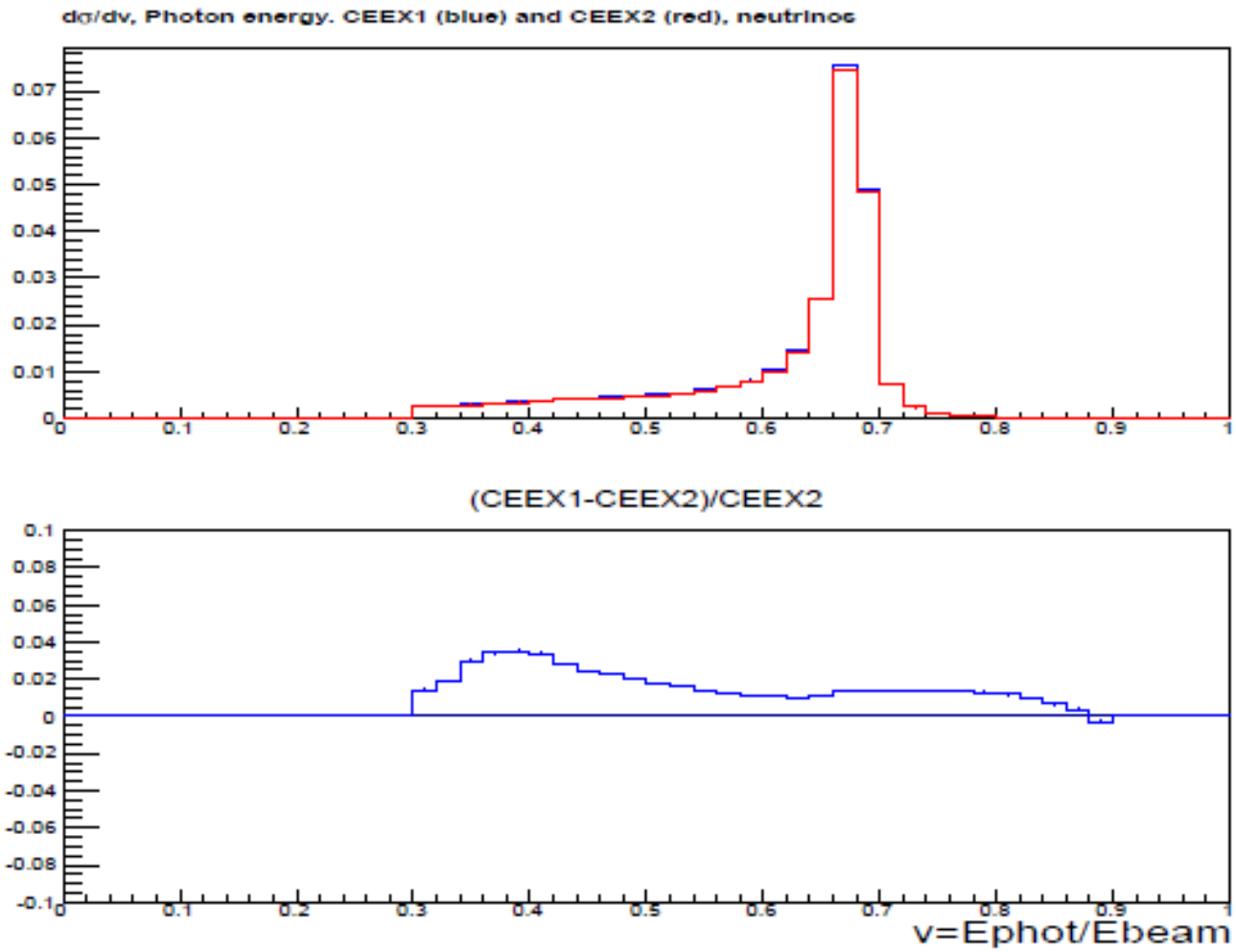








H.O. QED corrections estimate, neutrino channel.



Defining $e^- e^+ \to \nu + \bar{\nu} + \gamma$ as Born, CEEX1 is Born with soft photon resummation and CEEX2 is 1st order soft photon resummation.

QED uncertainty $\sim 1 - 2\%$.

Normalization from muon channel?

- For calculating invisible width from $e^- e^+ \to \nu\bar{\nu}\gamma$ process we need to get normalization from somewhere.
- One possibility is to use similar SM process $e^- e^+ \to \mu^- \mu^+ \gamma$ with the muonic decay of Z.
- We require only angular cut on both muons: $\cos\theta_\mu < 0.95$.
- Selection of single radiative photon exactly as for neutrinos.
- We examine QED corrections in the same way.

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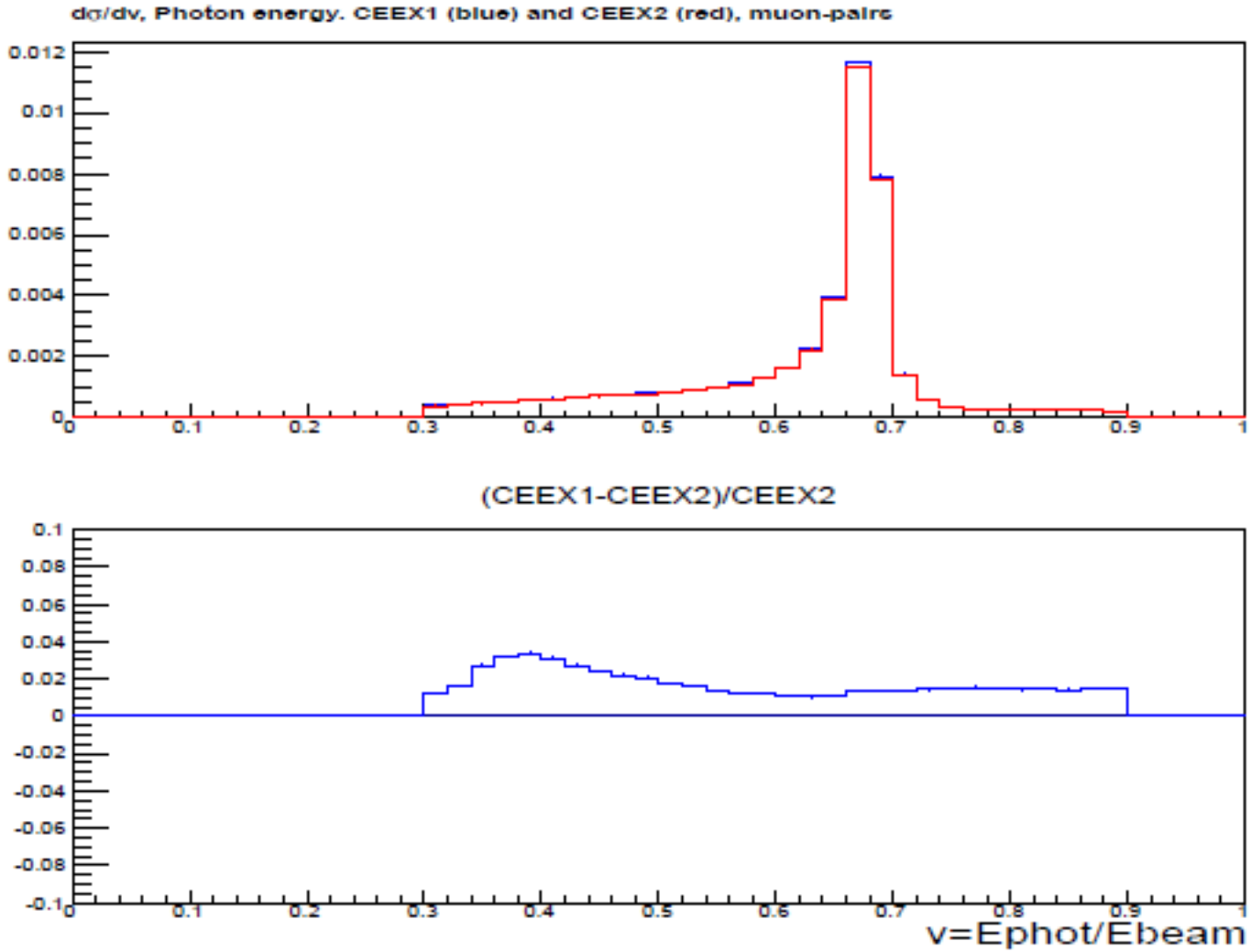
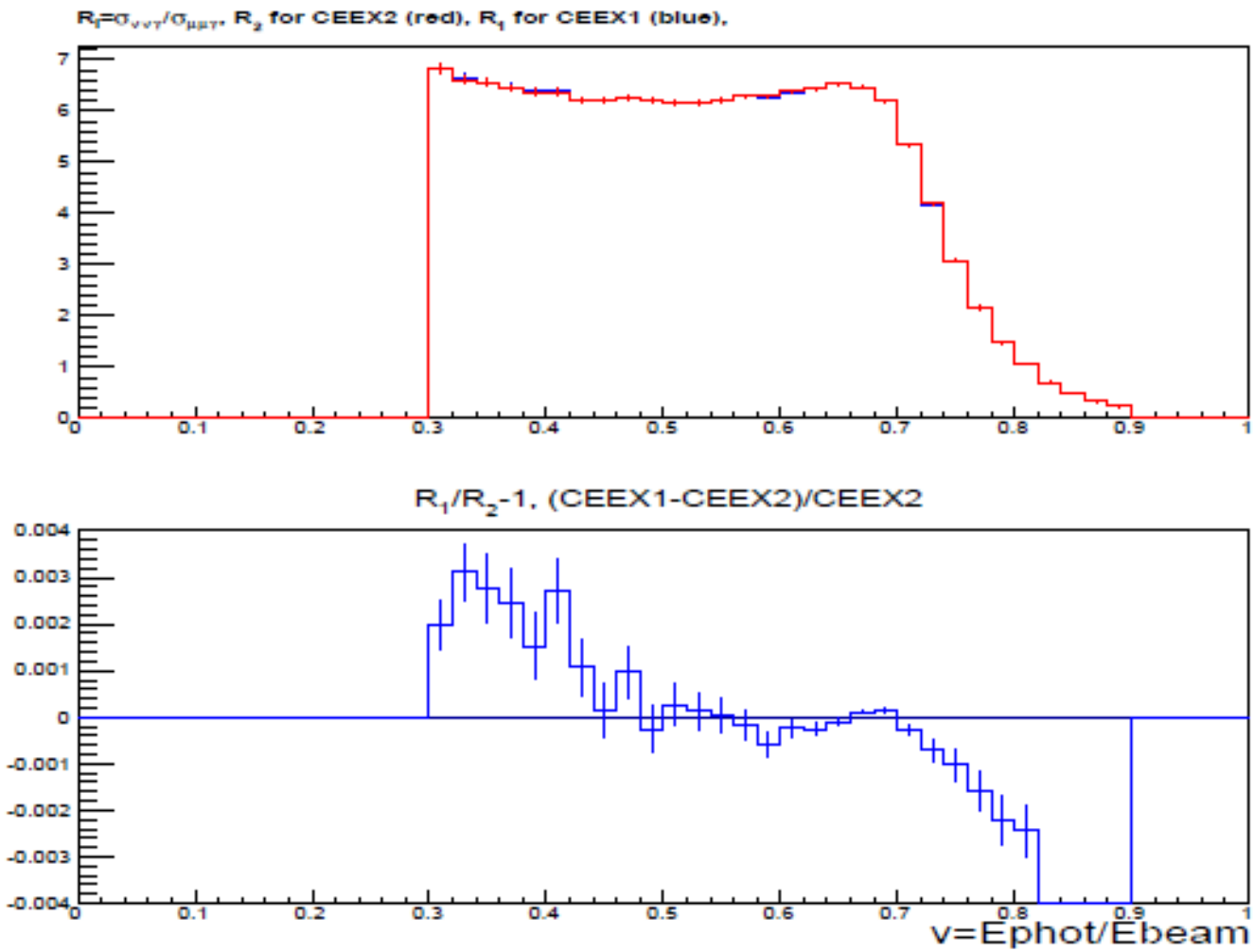
H.O. QED corrections estimate, muon channel.



Defining $e^- e^+ \to \mu^- \mu^+ \gamma$ as Born, CEEX1 is Born with soft photon resummation and CEEX2 is 1st order soft photon resummation.

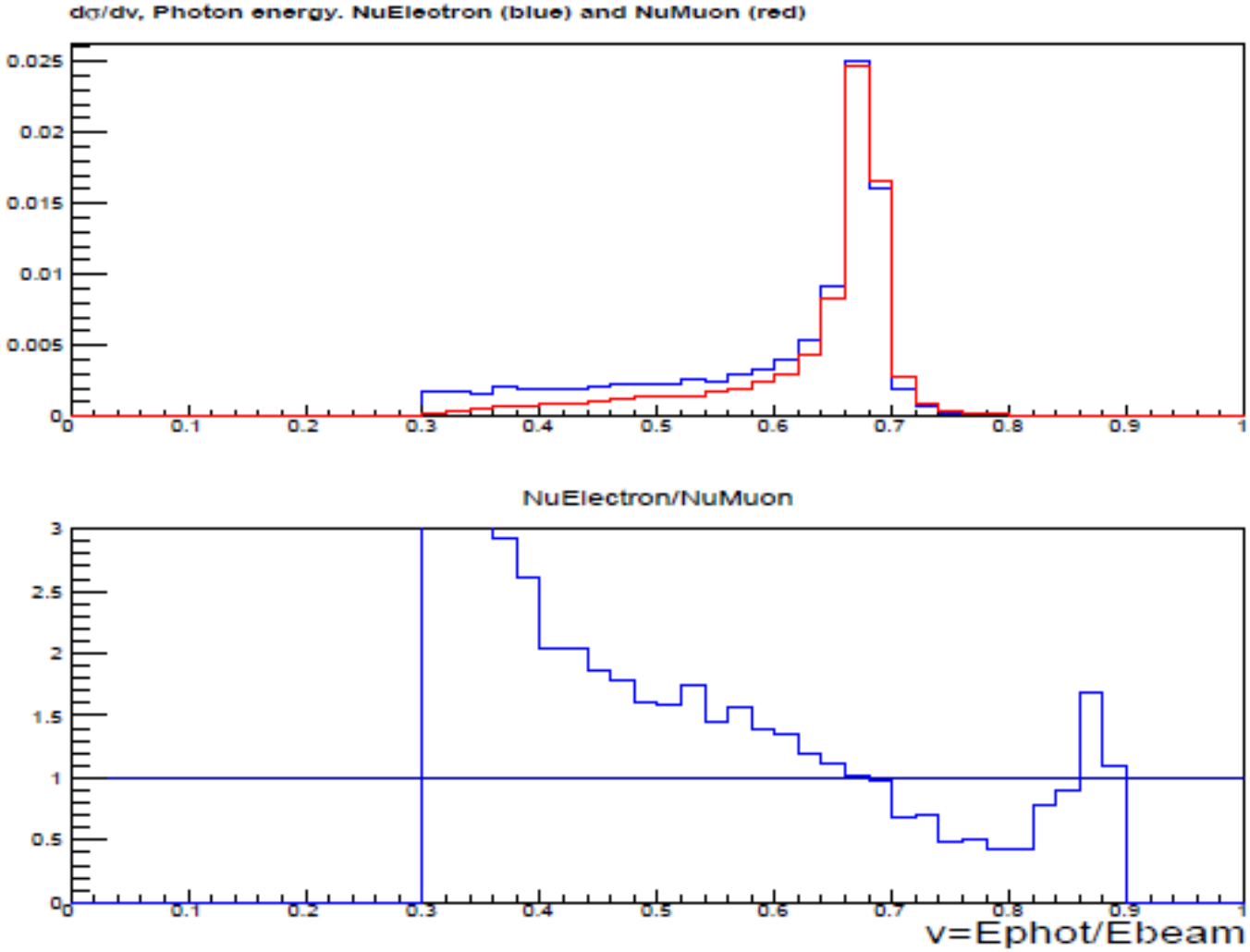
QED uncertainty again $\sim 1 - 2\%$.

QED corr. estimate in $\sigma(\nu\bar{\nu}\gamma)/\sigma(\mu^-\mu^+\gamma)$



QED corrections seems to cancell dramaticaly in the ratio $\sigma(\nu\bar{\nu}\gamma)/\sigma(\mu^-\mu^+\gamma)$. They drop to $\sim$ 0.03% !!!.
This is PRELIMINARY result requiring further tests.

Importance of *t*-channel exchange



The *t*-channel exchange is present in electron neutrino channel.

It is of order of 10% It cannot be easily minimized by cutoffs, it has to be reliably calculated and subtracted!

$\Gamma_{Z,inv}$ SUMMARY

Conclusions-I

From this limited study using KKMC at 161GeV we conclude that:

- QED corrections are sizeable, their uncertainty in $\sigma(\nu\bar{\nu}\gamma)$ is estimated $\sim 2\%$
- QED uncertainty seems to drop dramaticaly in the ratio $\sigma(\nu\bar{\nu}\gamma)/\sigma(\mu^-\mu^+\gamma)$, down to $3 \cdot 10^{-4}$! $\Rightarrow \Delta N_\nu^{th} \cong 0.00045$ (prelm.)
- t-channel contrubution is $\sim 10\%$ near Z peak in photon energy. Possibly the bigest source of theoretical uncertainty in N_ν measurement from ratiative return.

To be studied further most urgently:

- The dependence on $\sqrt{s}$
- The dependence on $\theta_{\min}$ and other cutoffs
- Dont give up on N_ν from Z-peak cross section!

- Here, one has to take care of the t-channel charge flow when the W is exchanged when one is exponentiating using YFS theory. YFS theory states that only the external line radiation exponentiates (as seen in PRD**73**(2006)073001 in the virtual corrections to bremsstrahlung) and we currently see no reason why this theory will not hold for the W exchange graphs, wherein the W is off-shell and internal.
- The t-channel exchange of W does not bring Bhabha-style collinear enhancement as W is massive, but cancellation of gauge dependencies is far more complex.
- Predictions for $\nu\bar{\nu}\gamma$ production at LEP by D. Bardin, S. Jadach, T. Riemann, Z. Was, Eur. Phys.J. **C24** (2002) 373 was devoted to match of first order matrix element with exponentiation -- initial state bremsstrahlung style. Real and virtual corrections were discussed.
- Gauge invariance, infrared / collinear singularities and tree level matrix element for $e^+ e^- \to \nu(e)\, \bar{\nu}(e)\, \gamma\gamma$, Z. Was, Eur. Phys. J.**C44** (2005) 489, was devoted to second order matrix elements.
- The two publications together with basic references of KKMC were sufficient for the precision requirements of LEP time.

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$\Gamma_{Z,inv}$ SUMMARY

- In general results were however not explored to the limits and work for evaluation/improvements of the precision estimation is required.
- This may be specially important effort for virtual corrections for the respective single photon emission amplitudes.
- On the other hand, the actual steps for the work are rather clear.
- No limitations of principle are to be expected.
- Need to compare with other approaches

Conclusions-II

KK MC is still alive and possibly still useful!

Resummed Quantum Gravity

- Recent Progress: Cosmological Constant Λ

In Phys.Dark Univ. **2**(2013)97, using

$$B_g''(k) = \frac{\kappa^2 |k^2|}{8\pi^2} \ln\left(\frac{m^2}{m^2 + |k^2|}\right)$$

we show that we get the UV limit

$$k^2 G_N(k) \rightarrow .0442$$

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and the scalar contribution to Λ as

$$\Lambda_s = -8\pi G_N \frac{\int d^4k}{2(2\pi)^4} \frac{(2k_0^2) e^{-\lambda_c (k^2/(2m^2)) \ln(k^2/m^2+1)}}{k^2+m^2}$$
$$\cong -8\pi G_N [\frac{1}{G_N^2 64\rho^2}], \quad \rho = \ln \frac{2}{\lambda_c}$$

for $\lambda_c = \frac{2m^2}{M_{Pl}^2}$.

A Dirac fermion gives -4 times Λ_s.

$\Rightarrow$ UV limit

$$\Lambda(k) \underset{k^2 \to \infty}{\longrightarrow} k^2 \lambda_*,$$
$$\lambda_* = -\frac{c_{2,eff}}{2880} \sum_j (-1)^{F_j} n_j / \rho_j^2$$
$$\cong 0.0817$$

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Comparison with EFRG(Reuter et al., Percacci et al, Litim ,….):

Illustration(Laucsher&Reuter(PRD65(2002)025013))--

UV Fixed Point:

$$\beta_\lambda(\lambda_k, g_k; \alpha, d) = -2\lambda_k + \nu_d\, d\, g_k + \left[2d(d-1+2\alpha)(4\pi\ \ )^{1-\frac{d}{2}}\, \Phi^2_{d/2}(0) - (d-2)\,\omega_d\right] \lambda_k\, g_k$$
$$+\frac{1}{2} d(d+1)(d-2)(4\pi)^{1-\frac{d}{2}} \omega_d\, \Phi^1_{d/2}(0)\, g_k^2 + \mathcal{O}\left(g^3\right) \ ,$$

$$\beta_g(\lambda_k, g_k; \alpha, d) = (d-2)\, g_k - (d-2)\omega_d\, g_k^2 + \mathcal{O}\left(g^3\right)$$

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For d=4, cut-off profile

$R^{(0)}(y) = y/(e^y - 1)$,

$g_* \cong \pi/(13\pi^2/144+55/24+\alpha)$

$\lambda_* \cong 3\zeta(3)/(13\pi^2/144+19/24)$

Evidently, for appropriate α and $R^{(0)}(y)$ we can have qualitative agreement with our pure gravity results

$g_* \cong 0.0533$

$\lambda_* \cong -0.000189$

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An Estimate of Λ:

Planck Scale Cosmology --

(Bonanno&Reuter(J.Phys.Conf.Series140(2008)012008))

Transition between Planck regime and classical FRW regime at

$$t_{tr} \cong 25 t_{Pl}$$

$\Rightarrow$

$$\rho_\Lambda(t_{tr}) \equiv \frac{\Lambda(t_{tr})}{8\pi G_N(t_{tr})} = \frac{-M_{Pl}^4(k_{tr})}{64} \sum_j \frac{(-1)^F n_j}{\rho_j^2}$$

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For t_{eq} = time of radiation matter equality

we get (see Branchina&Zappala (G.R.Grav.42(2010)141))

$$\rho_\Lambda(t_0) \cong \frac{-M_{Pl}^4(1+c_{2,eff}k_{tr}^2/(360\pi M_{Pl}^2))^2}{64}\sum_j \frac{(-1)^F n_j}{\rho_j^2} \times \frac{t_{tr}^2}{t_{eq}^2} \times \left(\frac{t_{eq}^{2/3}}{t_0^{2/3}}\right)^3$$

$$\cong \frac{-M_{Pl}^2(1.0362)^2(-9.197\times 10^{-3})}{64}\frac{(25)^2}{t_0^2}$$

$$\cong (2.400\times 10^{-3} eV)^4.$$

Compare:

$$\rho_\Lambda(t_0)|_{exp} \cong (2.368\times 10^{-3} eV(1\pm 0.023))^4.$$

CONSISTENCY CHECKS

* What About EW, QCD, GUT Symmetry Breaking Scales?

Consider GUT symmetry breaking:
It gives a $M_{GUT}{}^4/(.01\ M_{Pl}{}^4/64) < 10^{-6}$ correction, which we drop here.
The other breaking scales are even smaller and hence their corrections are even less significant in our result for ρ_Λ.

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CONSISTENCY CHECKS

* What About BBN Constraint?

B-R BDY CONDITION: $H(t_{tr-})=H(t_{tr+})$

$\Rightarrow$ Gauge Transformation Between Planck Scale Regime and usual FRW Regime

B-R: $t \rightarrow t' = t - t_{as} \Rightarrow \alpha/t_{tr} = 1/(2(t_{tr}-t_{as}))$

$\Rightarrow t_{as} = (1-1/(2\alpha))t_{tr}$, with $t_{tr} = \alpha/M_{Pl}$,

$\alpha = 25$.

RQG: $t \rightarrow t' = \gamma t$, as part of a dilatation $\Rightarrow$

$\alpha/t_{tr} = 1/(2\gamma t_{tr}) \Rightarrow \gamma = 1/(2\alpha)$

$\Rightarrow$

$$\Omega_\Lambda(t_{BBN}) = \frac{M_{Pl}^2(1.0362)^2 9.194\times 10^{-3}(25)^2/\left(64 t_{BBN}^2\right)}{(3/(8\pi G_N))\left(1/(2\gamma t_{BBN})^2\right)}$$

$$\cong \frac{\pi 10^{-2}}{24}$$

$$= 1.31\times 10^{-3}.$$

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CONSISTENCY CHECKS

* What About SUSY GUTS?

Note

$$< 0|\mathcal{H}|0 > \sim \int^{M_{Pl}} \frac{d^3k}{(2\pi)^3}\frac{1}{2}\omega(k) = \int^{M_{Pl}} \frac{d^3k}{(2\pi)^3}\frac{1}{2}\sqrt{k^2+m_t^2}$$

Raises the question of GUTS: Use SO(10) SUSY GUT Approach of Dev & Mohapatra (PRD82(2010)035014):

Intermediate Stage:

$SU_{2L}xSU_{2R}xU_1XSU(3)^c$

SM Stage at ~2TeV = M_R:

$SU_{2L}XU_1XSU(3)^c$

SUSY Breaking at EW scale M_S:

$U_1XSU(3)^c$

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CONSISTENCY CHECKS

* What About SUSY GUTS?

- Possible spectrum

$$m_{\tilde{g}} \cong 1.5(10)\text{TeV}$$
$$m_{\tilde{G}} \cong 1.5\text{TeV}$$
$$m_{\tilde{q}} \cong 1.0\text{TeV}$$
$$m_{\tilde{l}} \cong 0.5\text{TeV}$$
$$m_{\tilde{\chi}_i^0} \cong \begin{cases} 0.4\text{TeV}, & i = 1 \\ 0.5\text{TeV}, & i = 2,3,4 \end{cases}$$
$$m_{\tilde{\chi}_i^\pm} \cong 0.5\text{TeV}, \; i = 1,2$$
$$m_S = .5\text{TeV}, \; S = A^0, \; H^\pm, \; H_2,$$

$\Longrightarrow$

$$\Delta_{\text{GUT}} = \sum_{j \in \{\text{MSSM low energy susy partners}\}} \frac{(-1)^F n_j}{\rho_j^2} \cong 1.13(1.12) \times 10^{-2}$$

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CONSISTENCY CHECKS

* What About SUSY GUTS?

$\Longrightarrow$

- Compensate by either (A) adding new susy families with scalars lighter than fermions or (B) allowing the gravitino mass to go to ~.05 M_{GUT} ~ 2x 10^{15} GeV.
- For approach (A), new quarks and leptons at M_{High}~ 3.4(3.3) x 10^3 TeV, scalar partners at ~.5TeV =M_{Low}

CONCLUSIONS

* Herwiri1.031 Just as General Herwig6.5, No Tweaking, Should Be Better in IR Due to Bloch-Nordsieck Effect – Await New Data
* KKMC4.22: New Precision EW Results at LHC, TLEP
* Real Progress on Λ in QFT

(Resummed Quantum Gravity Realization of Feynman's Approach Einstein-Hilbert Theory)

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