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Fragmentation-Function Approach to Heavy Quarkonium Production

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Based on work done with Kang, Nayak, Sterman, and etc.

Outline

- Heavy quarkonium has two intrinsic scales
- Surprises from heavy quarkonium production
- PQCD factorization approach to heavy quarkonium production at collider energies
- Quarkonium polarization
- Summary and outlook

Heavy quarkonium has two intrinsic scales

❑ Heavy quark mass:

Heavy quark pairs are produced at a distance scale much less than fm

$$\Delta r \sim \frac{1}{2m_Q} \leq 0.1 \text{ fm} \quad \text{(for a charm-quark pair)}$$

$$\leq 0.025 \text{ fm} \quad \text{(for a b-quark pair)}$$

PQCD is "expected" to work for the production of heavy quarks

❑ Quarkonium's nonperturbative binding:

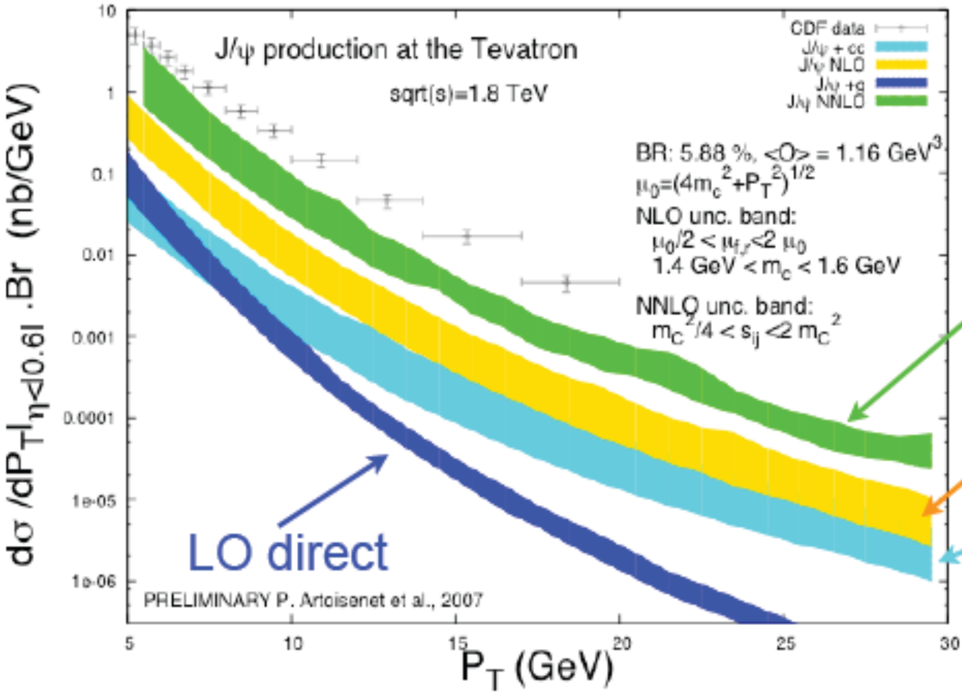
$$\frac{\left|M^2 - 4m_Q^2\right|}{4m_Q^2} \ll 1$$

for both charm and bottom quarkonia

The transition from a heavy quark pair to a quarkonium is Not perturbatively calculable

Different treatment ⟶ Different production model

Color Singlet Model – Huge high order corrections



NNLO

NLO

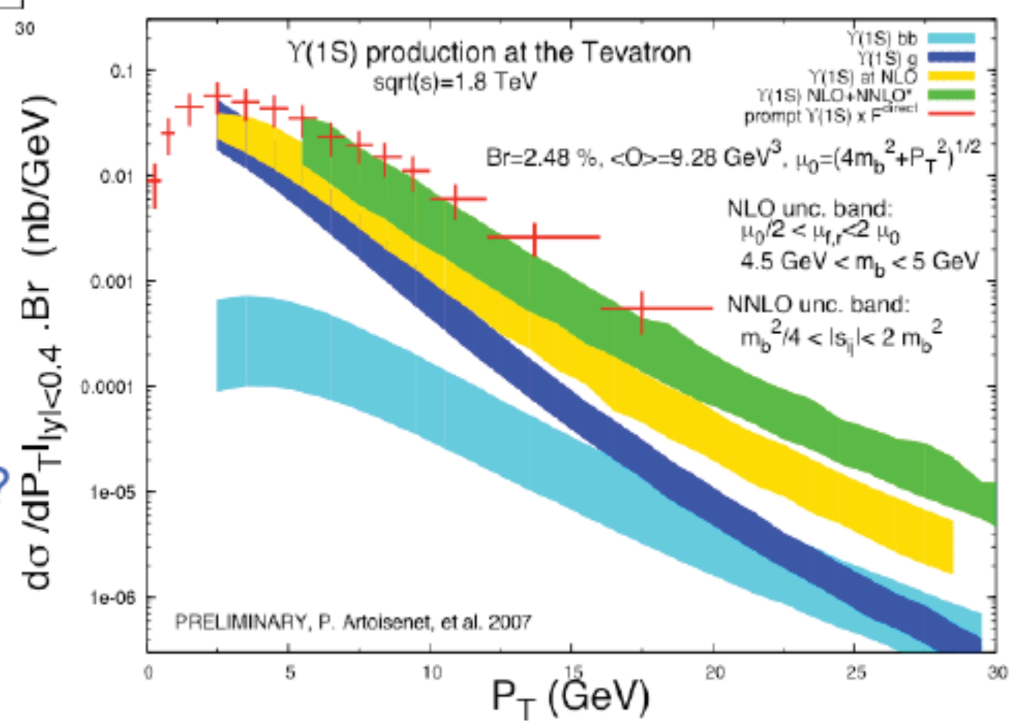
LO associate

Color-singlet contribution for J/ψ and Upsilon production at Tevatron

P. Artoisenet, F. Maltoni, et.al. 2007

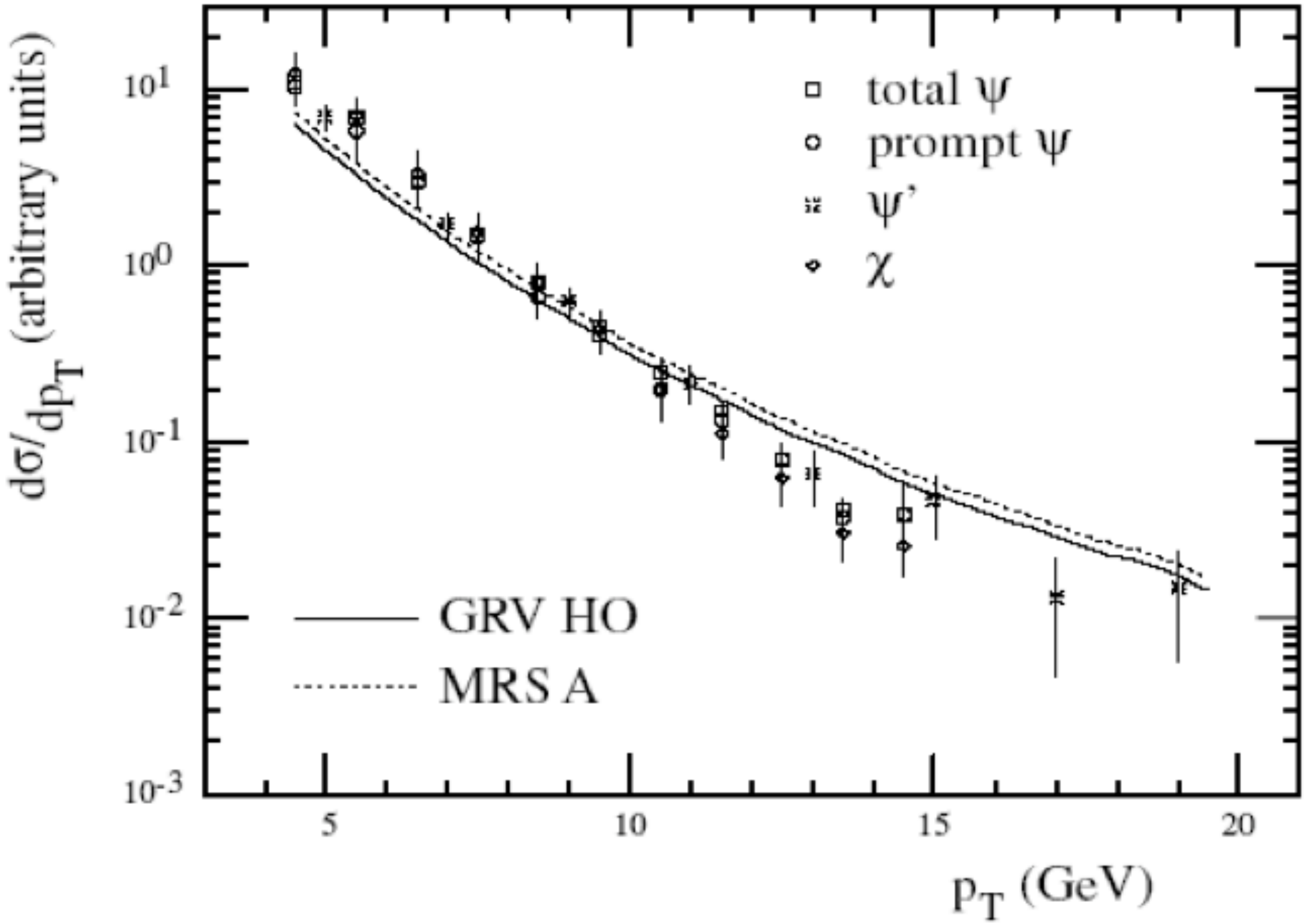
Large uncertainty band
⇒strong scale dependence

Large NLO, NNLO contribution
⇒ how perturbative series converge?



CEM: OK for inclusive production

- **Good for total cross section, ok for P_T distribution:**



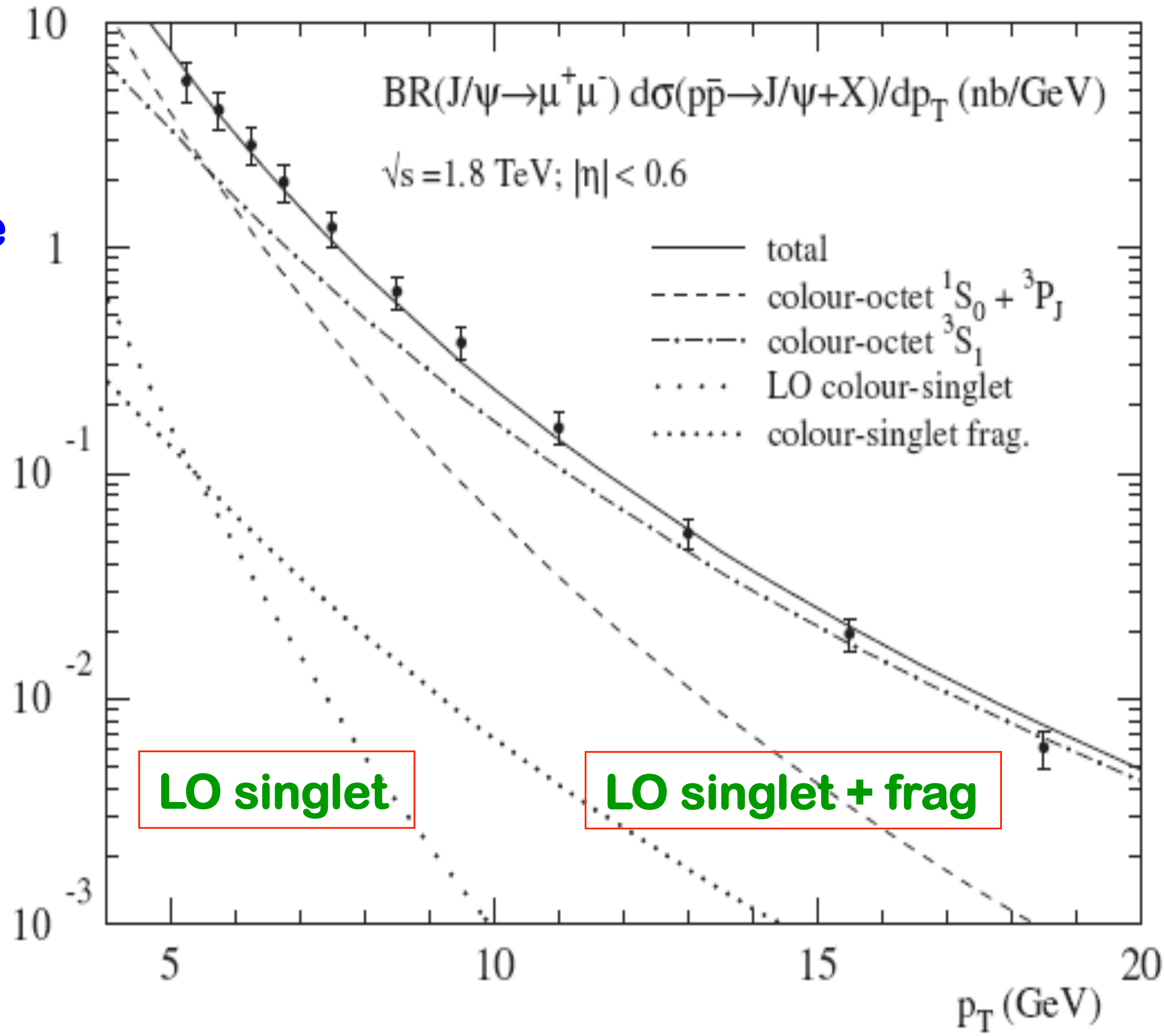
Amundson et al, PLB 1997

NRQCD Model: Best fit to Tevatron data

❑ **Unpolarized J/ψ at the Tevatron:**

✧ Good description for various inclusive heavy quarkonium production

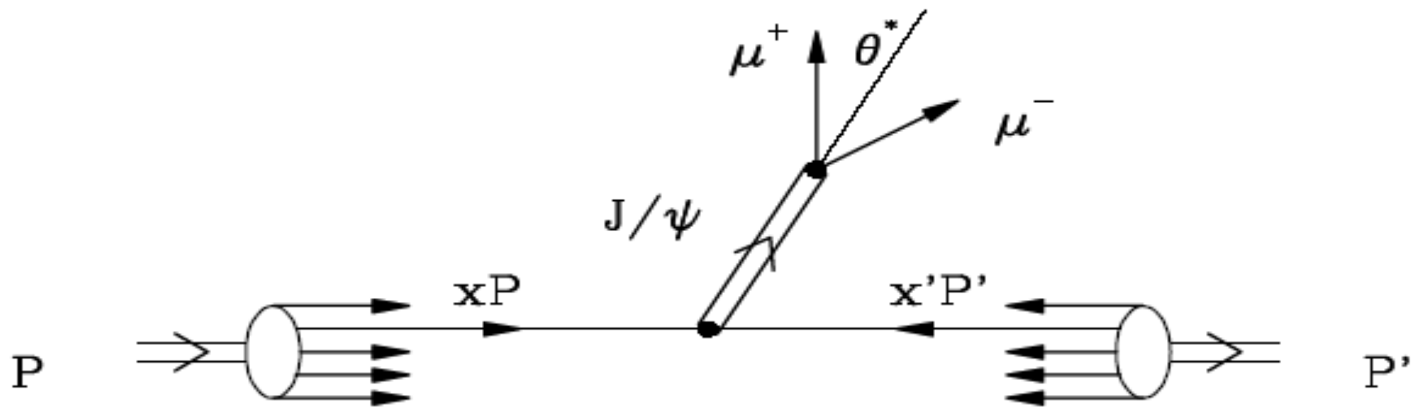
✧ NRQCD matrix elements fixed by data



M. Kramer, 2001

Polarization of quarkonium at Tevatron

- Measure angular distribution of $\mu^+\mu^-$ in J/ψ decay



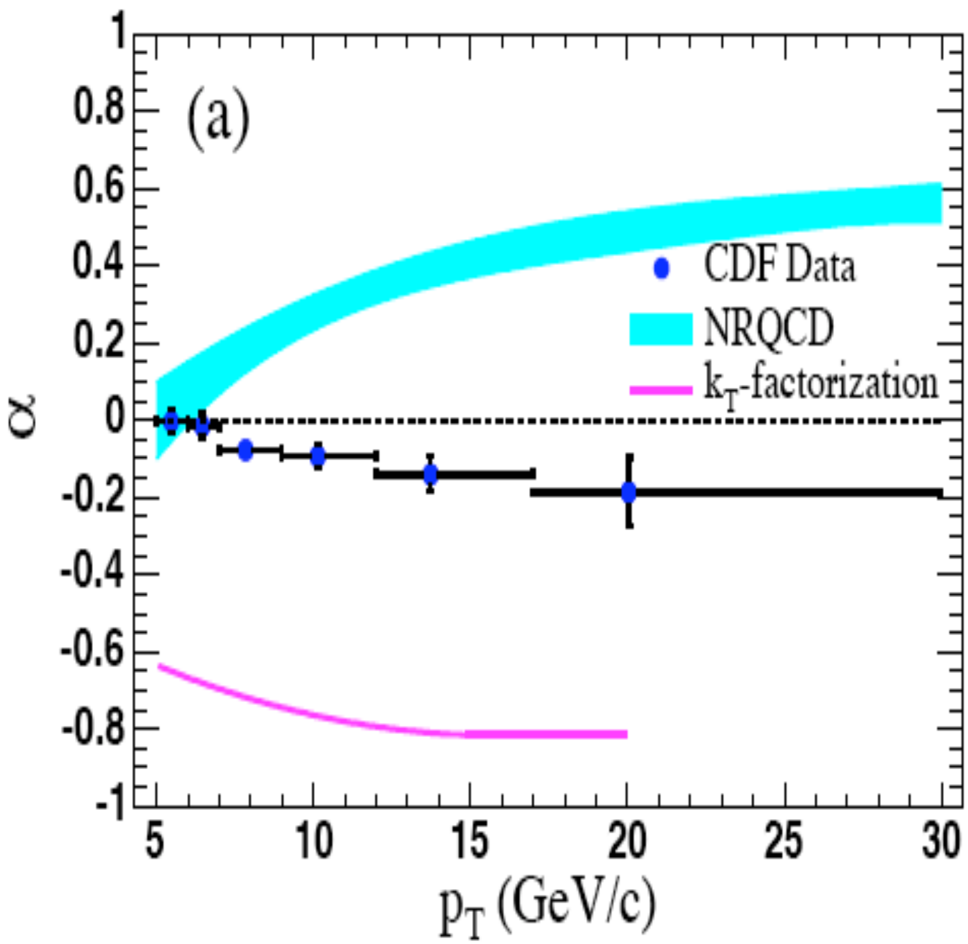
- Normalized distribution:

$$I(\cos\theta^*) = \frac{3}{2(\alpha+3)}(1+\alpha\cos\theta^*)$$

$$\alpha = \begin{cases} +1 & \text{fully transverse} \\ 0 & \text{unpolarized} \\ -1 & \text{fully longitudinal} \end{cases}$$

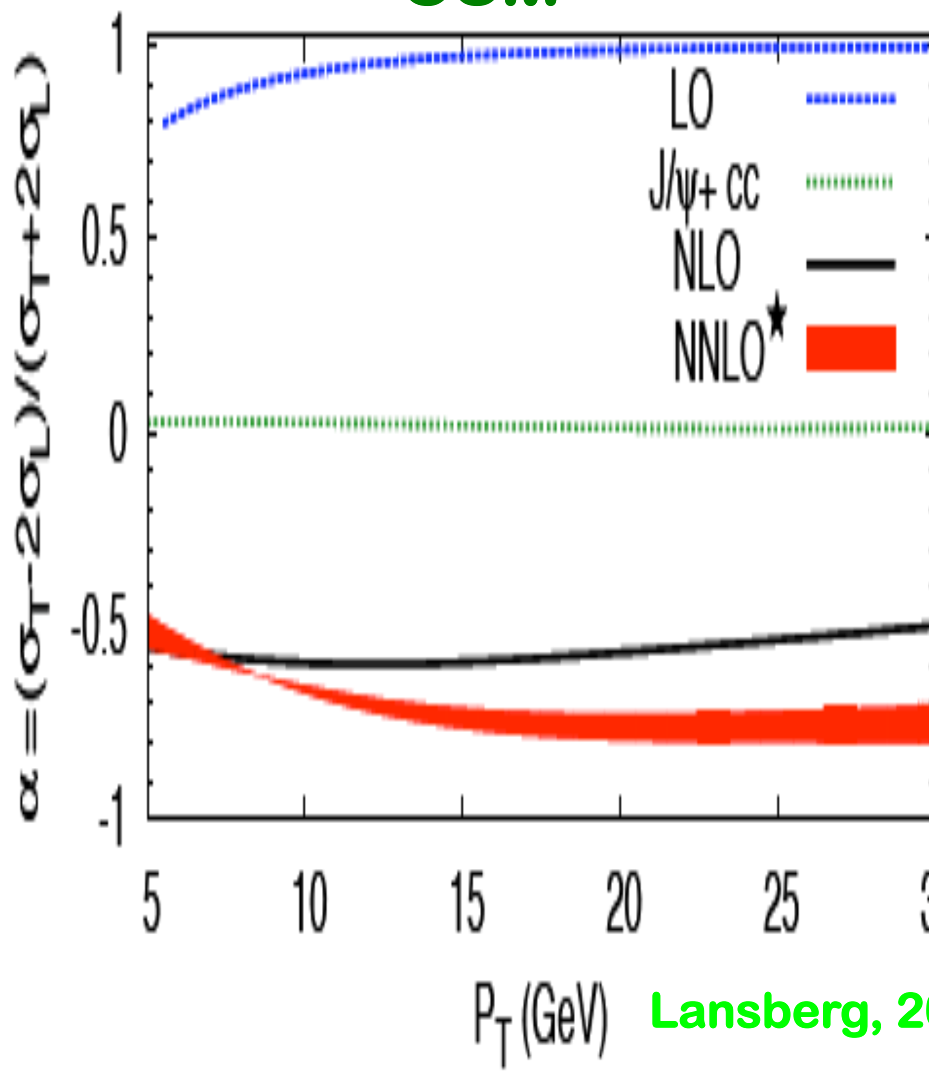
Surprises from J/ψ polarization

NRQCD



Cho & Wise, Beneke & Rothstein, 1995, …

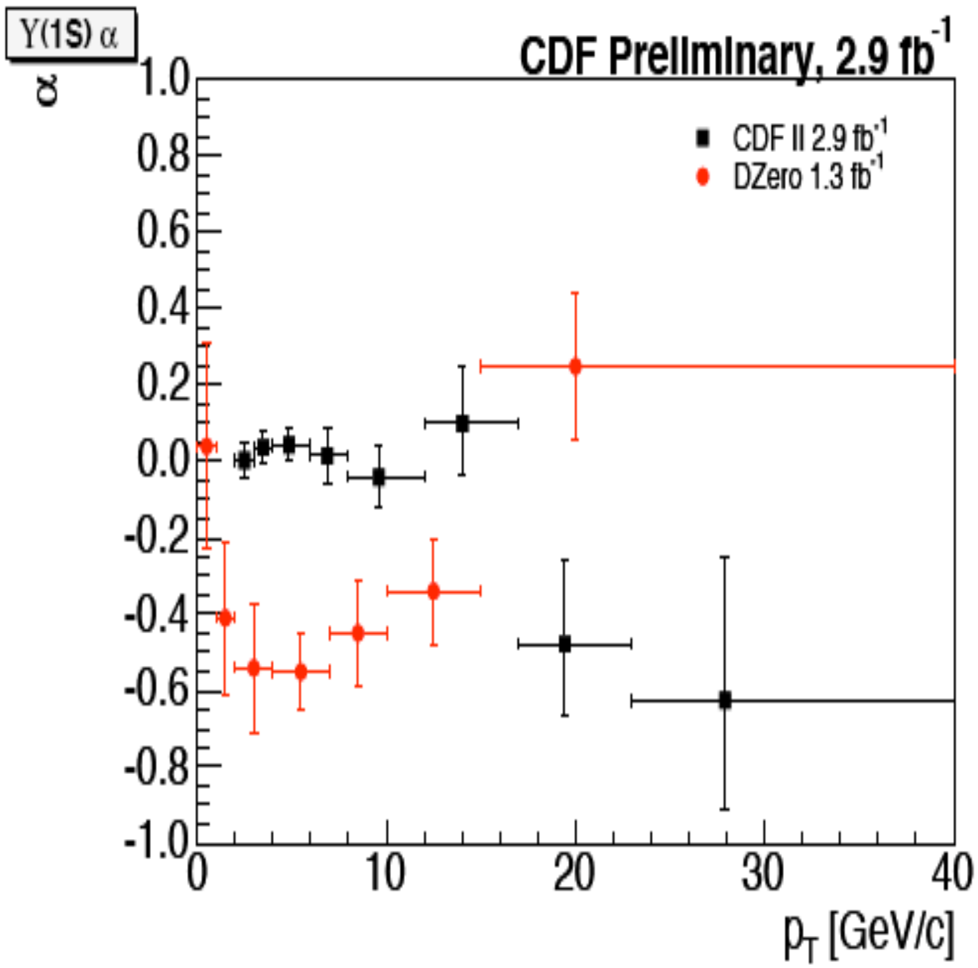
CSM



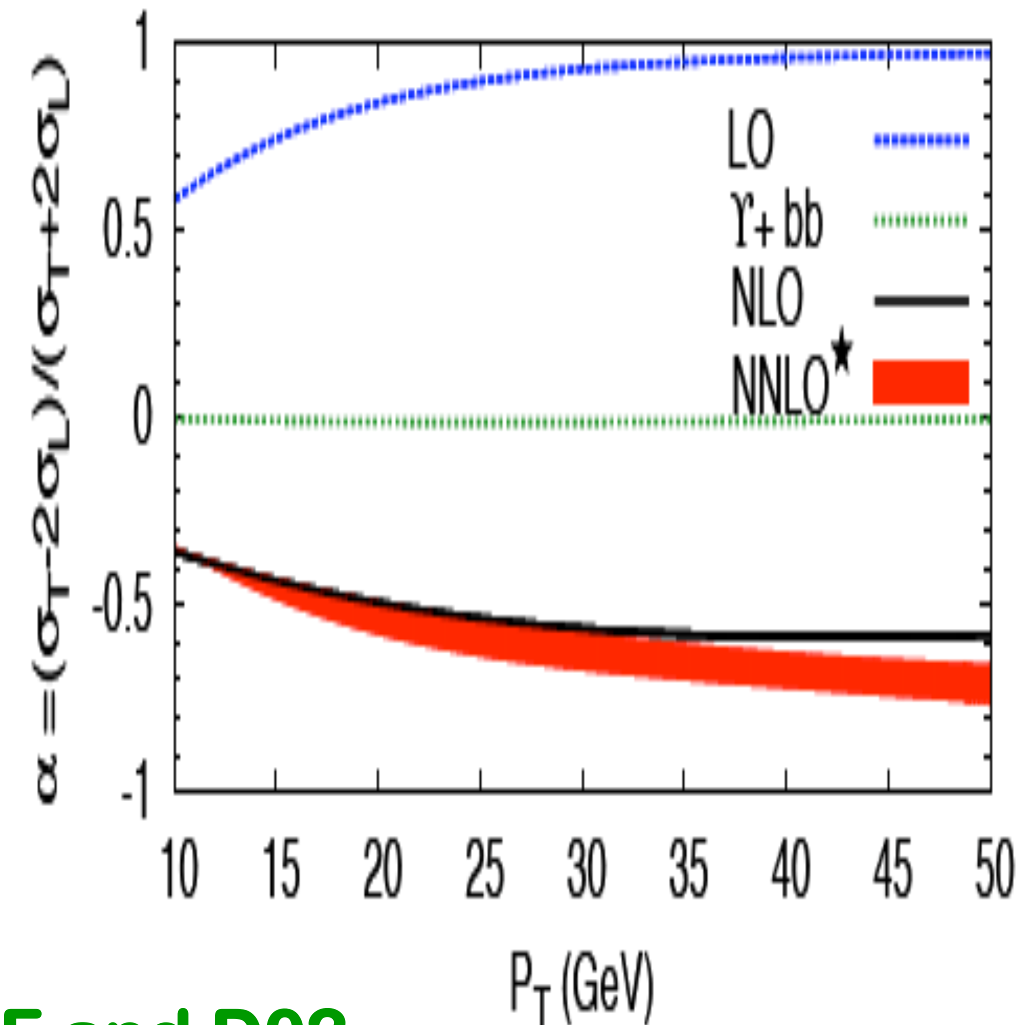
Lansberg, 2009

Role of higher order contribution?

Confusions from Upsilon polarization



Singlet model



Resolution between CDF and D0?

P. Artoisenet, et al. 2008

Questions

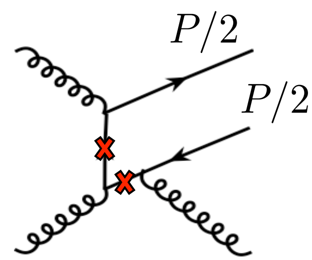
- ❑ Why the high order correction in CSM is so large? How many orders should we calculate?

- ❑ Why the high order CSM predicts the longitudinally polarized J/psi?

- ❑ What happen to the octet channel of quarkonium production and the NRQCD formalism?

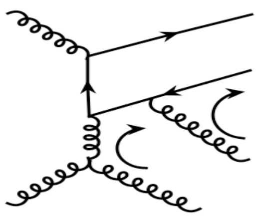
CSM: explicit high order calculation

- Leading order in α_s



$$\sim \mathcal{O}\left(\frac{1}{P_T^8}\right)$$

- Next-to-leading order (NLO): Artoisenet et al. 2007



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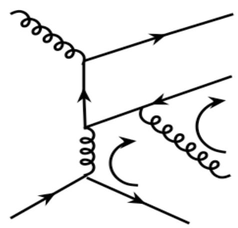
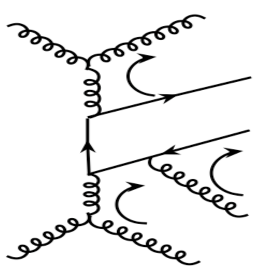


$+ \ldots \quad > 10$ times LO

- Next-to-next-to leading order (NNLO): Artoisenet et al. 2008

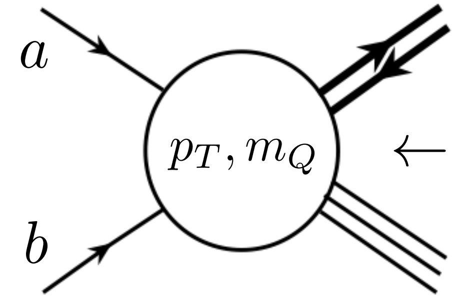


$+ \ldots \quad > 10$ times NLO

- What happens to NNNLO … ?

Why high orders in CSM are so large?

- **Partonic hard part in CSM:**



  Expansion in power of α_s

$$\hat{\sigma}_{ab\to Q\bar{Q}}(p_T, m_Q, \mu, \alpha_s(\mu)) = \sum_n \hat{\sigma}^{(n)}_{ab\to Q\bar{Q}}(p_T, m_Q, \mu) \left(\frac{\alpha_s(\mu)}{2\pi}\right)^n$$

- **Complication with two observed momentum scales:**

  ✧ **IF $p_T^2 \gg m_Q^2$, high order in α_s is not necessary small!**
  Ambiguity in choosing the scale μ

  ✧ **the size of the hard coefficients depends where the singlet pair was produced!**

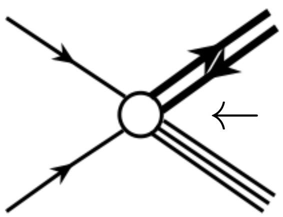
QCD power counting

❑ Non-relativistic s=1 projection operator:

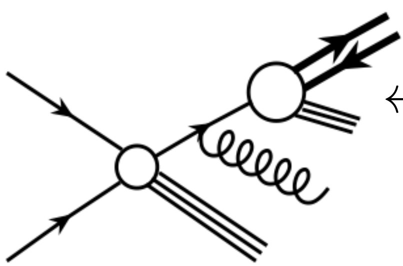
$$\mathcal{P}(P, q \to 0, s=1) = \frac{1}{\sqrt{2m_Q}}\,\gamma_\mu \epsilon^\mu_\lambda(P)\left[\gamma\cdot\frac{P}{2} + m_Q\right]$$

Does not pick up leading power in m_Q/p_T

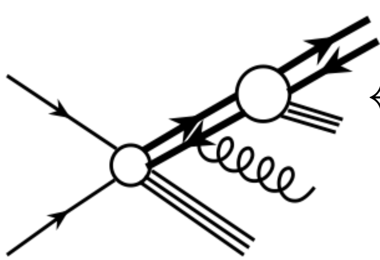
❑ Power counting for the hard part:



$$\leftarrow\ \Delta r \sim \frac{1}{p_T} \quad \Rightarrow \quad \frac{1}{p_T^4}\left[\frac{m_Q^4}{p_T^4}, \dots\right]$$



$$\leftarrow\ \frac{1}{m_Q} \quad \Rightarrow \quad \frac{1}{p_T^4}\left[\log(\frac{p_T^2}{\mu_0^2})\right]^n\left[\frac{m_Q^2}{\mu_0^2}, \dots\right] \qquad \mu_0 \sim 2m_Q$$

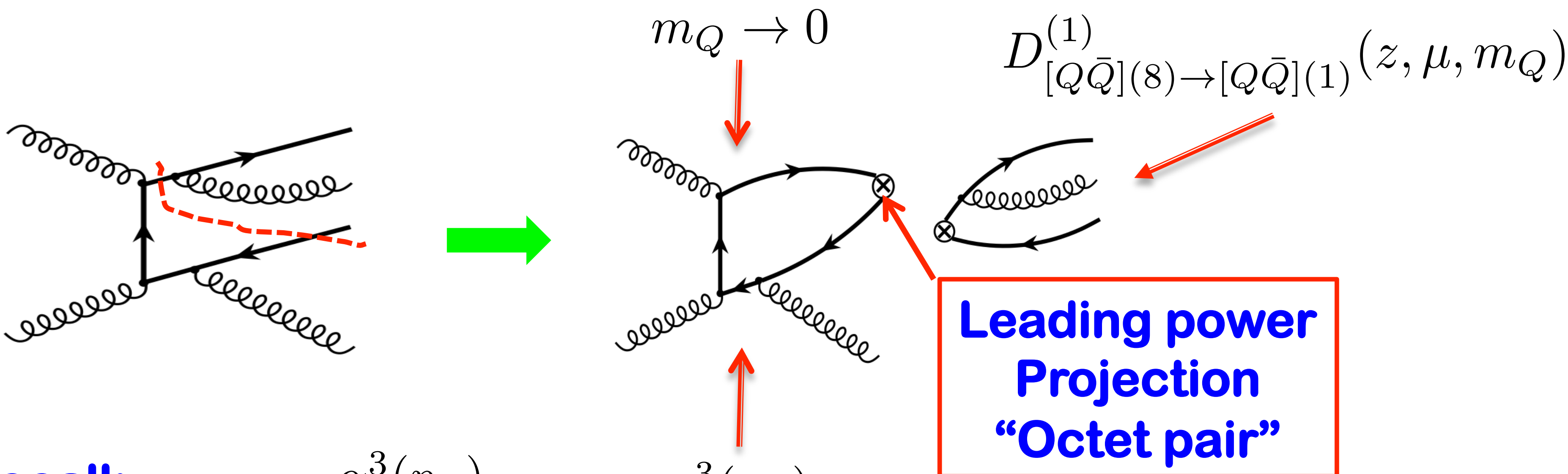


$$\leftarrow\ \frac{1}{m_Q} \quad \Rightarrow \quad \frac{1}{p_T^6}\left[\log(\frac{p_T^2}{\mu_0^2})\right]^n\left[m_Q^2\left(\log\frac{\mu_0^2}{m_Q^2}, \frac{m_Q^2}{\mu_0^2}, \dots\right)\right]$$

Neutralize the color at $1/2m_Q$

Leading power quark-antiquark channel

- NLO in α_s may be leading in m_Q/p_T power:



Recall: $\hat{\sigma}^{\rm LO} \propto \frac{\alpha_s^3(p_T)}{p_T^8}$ $\qquad \frac{\alpha_s^3(p_T)}{p_T^6}$

- LO quark-antiquark fragmentation – no evolution yet:

$$D^{(1)}_{[c\bar{c}(a8)]\to J/\psi}(z,m_c,\mu) = \frac{\alpha_s\langle\mathcal{O}^1(^3S_1)\rangle}{27\pi m_c}\left\{z(1-z)\ln\left[\frac{z}{1-z}\left(\frac{\mu^2}{4m_c^2}-1\right)\right]+(1-z)^2\left[\frac{z}{1-z}-\frac{1}{\frac{\mu^2}{4m_c^2}-1}\right]\right\}$$

Log's $\qquad\qquad$ power

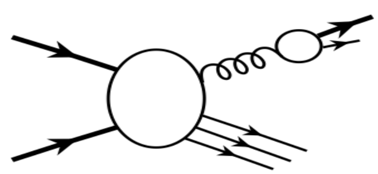
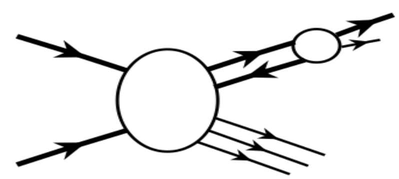
PQCD factorization

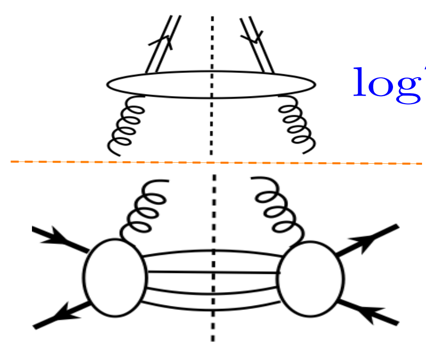
Kang, Qiu and Sterman, 2010

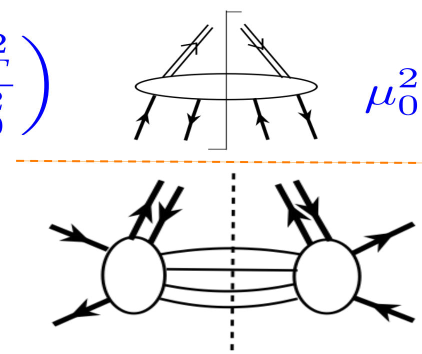
❑ Idea:

- ✧ Reorganize partonic hard part in terms of power expansion $\frac{m_Q^2}{p_T^2}$
- ✧ Resum logarithm contribution into "fragmentation functions"
- ✧ "Direct" contribution by the subtraction

❑ Factorization:

$E\frac{d\sigma_{J/\psi}}{d^3P}$: $\Big|$  $+$  $+\ldots\Big|^2$

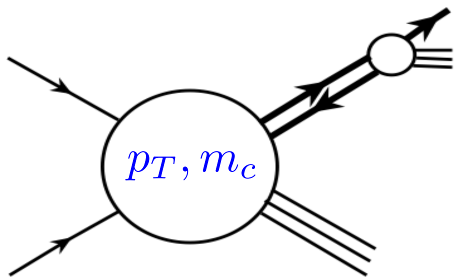
⟹  $\log^n\left(\frac{P_T^2}{\mu_0^2}\right)$ $\mathcal{O}\left(\frac{1}{P_T^4}\right)$

$+$  $\mu_0^2\log^n\left(\frac{P_T^2}{\mu_0^2}\right)$ $\mu_0\sim 2m_Q$ $\mathcal{O}\left(\frac{1}{P_T^6}\right)$

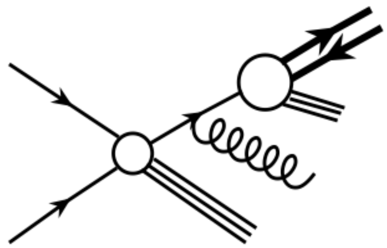
$+$ **Nonlogarithmic high order and high power terms**

PQCD factorization formula

Kang, Qiu and Sterman, 2010



$$E\frac{d\sigma_{J/\psi}}{d^3P}\;:$$

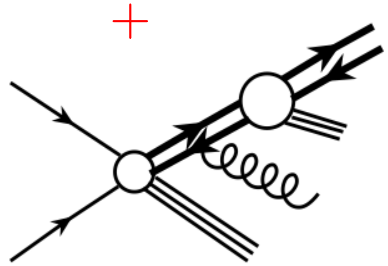


$\frac{1}{p_T^4}$

High p_T

$$\sigma^{\rm LP}_{ab\to f}(z,p_T,\alpha_s)\otimes D_{f\to J/\psi}(z,m_c)$$

$+$



$\frac{1}{p_T^6}$

Medium p_T

$$\sigma^{\rm NLP}_{ab\to[c\bar{c}(\kappa)]}(z,p_T,\alpha_s)\otimes D_{[c\bar{c}(\kappa)]\to J/\psi}(z,m_c)$$

$+ \ldots$

$+$

Low p_T

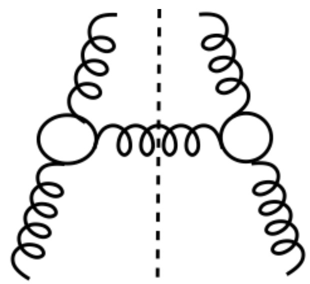
$$\sigma^{\text{“Direct”}}_{ab\to[c\bar{c}(\kappa)]}(p_T,m_c)\otimes\langle\mathcal{O}[c\bar{c}(\kappa)](0)\rangle$$

Non-logarithmic high order and high power terms

PQCD evolution of fragmentation functions

Kang, Qiu and Sterman, 2010

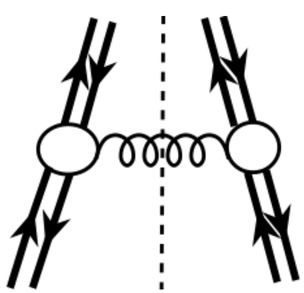
- Single parton:



Same as normal DGLAP
Difference from input distribution

$$D_{g\to J/\psi}(z,\mu_0,m_Q)$$

- Heavy quark pair:



Differ from DGLAP – still logarithmic
Spin-color sensitive
Infrared safe evolution
Nonperturbative input distribution

$$\mu^2\frac{\partial}{\partial\mu^2}D_{[Q\bar{Q}(a8)]\to H}(z,\mu,m_Q)=\int_z^1\frac{dz'}{z'}D_{[Q\bar{Q}(a8)]\to H}(z',\mu,m_Q)\mathcal{K}^{(1)}_{a8\to a8}(z/z',\alpha_s)$$

$$\mathcal{K}^{(1)}_{a8\to a8}(z,\alpha_s)=\frac{\alpha_s}{2\pi}(2C_A)\left(\left[\frac{z}{1-z}\right]+\frac{1}{4}\delta(1-z)\right)$$

Evolution dominated by octet channels

NRQCD or CSM for input distributions

- **Input distributions are universal and non-perturbative:**

  Should, in principle, be extracted from experimental data.

- **Single parton:** Nayak, Qiu and Sterman, 2005

$$D_{g\to J/\psi}(z,\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{g\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\langle\mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle|_{\mathrm{NRQCD}}$$

Dominated by transverse polarization

- **Heavy quark pair:** Kang, Qiu and Sterman, 2010

$$D_{[Q\bar{Q}(\kappa)\to J/\psi}(z,\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{[Q\bar{Q}(\kappa)\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\langle\mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle|_{\mathrm{NRQCD}}$$

Dominated by longitudinal polarization

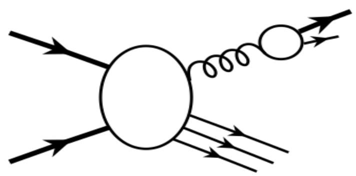
Above factorizations have not been proved

Heavy quarkonium polarization

❑ Competition between LP and NLP:

Kang, Qiu and Sterman, 2010

$$E\frac{d\sigma_{J/\psi}}{d^3P}:$$



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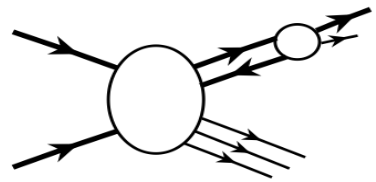


$+\ldots$

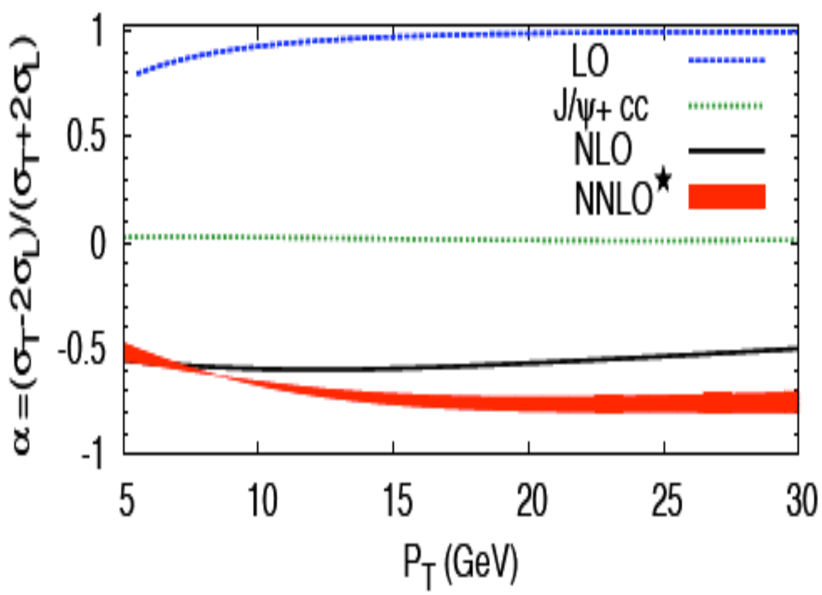
$$\frac{\langle \mathcal{O}_{\rm octet}\rangle}{p_T^4} \quad \text{vs.} \quad \frac{\langle \mathcal{O}_{\rm singlet}\rangle}{p_T^6}$$

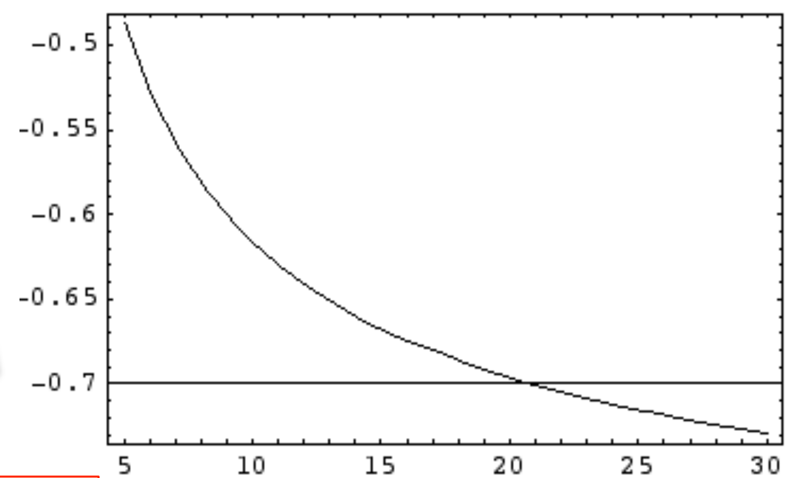
Transverse polarization

Longitudinal polarization

At $1/p_T$, both leading channels dominated by octet channels!

❑ Polarization:





Scale!

FF reproduces the polarization of CSM at NLO

Summary and outlook

- **Proposed a new pQCD factorization formula for heavy quarkonium production in power expansion $\frac{m_Q^2}{p_T^2}$:**

  $\sigma(p_T)$ = LP + NLP + "Direct"

- **Both LP and NLP in hard production dominated by octet channels (octet single parton & octet QQbar pair)**

- **Observed polarization of prompt quarkonium is a result of competition between LP and NLP**

- **A global analysis of existing data in this formalism is needed to fix the nonperturbative matrix elements**

Make predictions for LHC!

Thank you!