

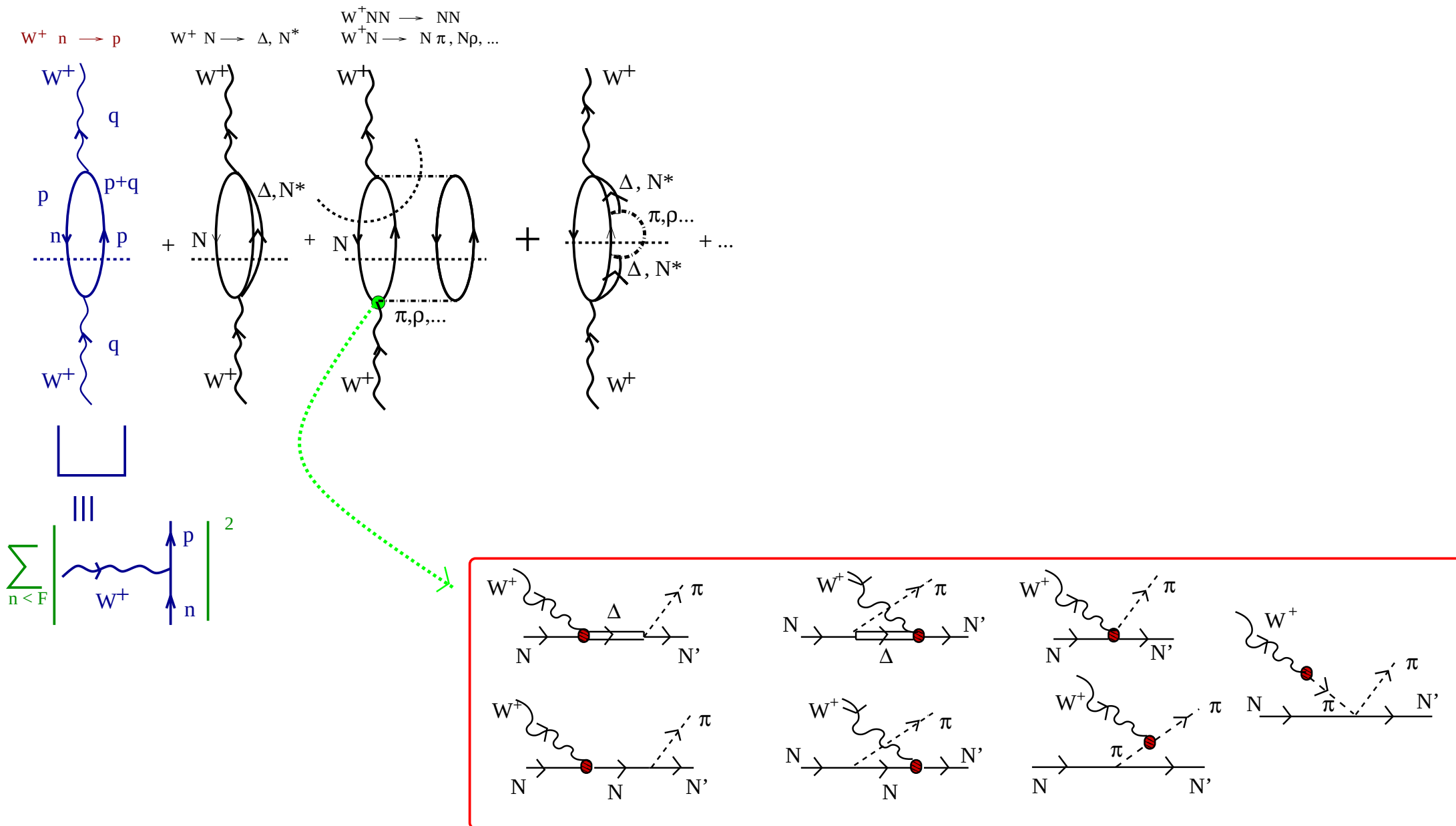
Weak Pion Production off the Nucleon

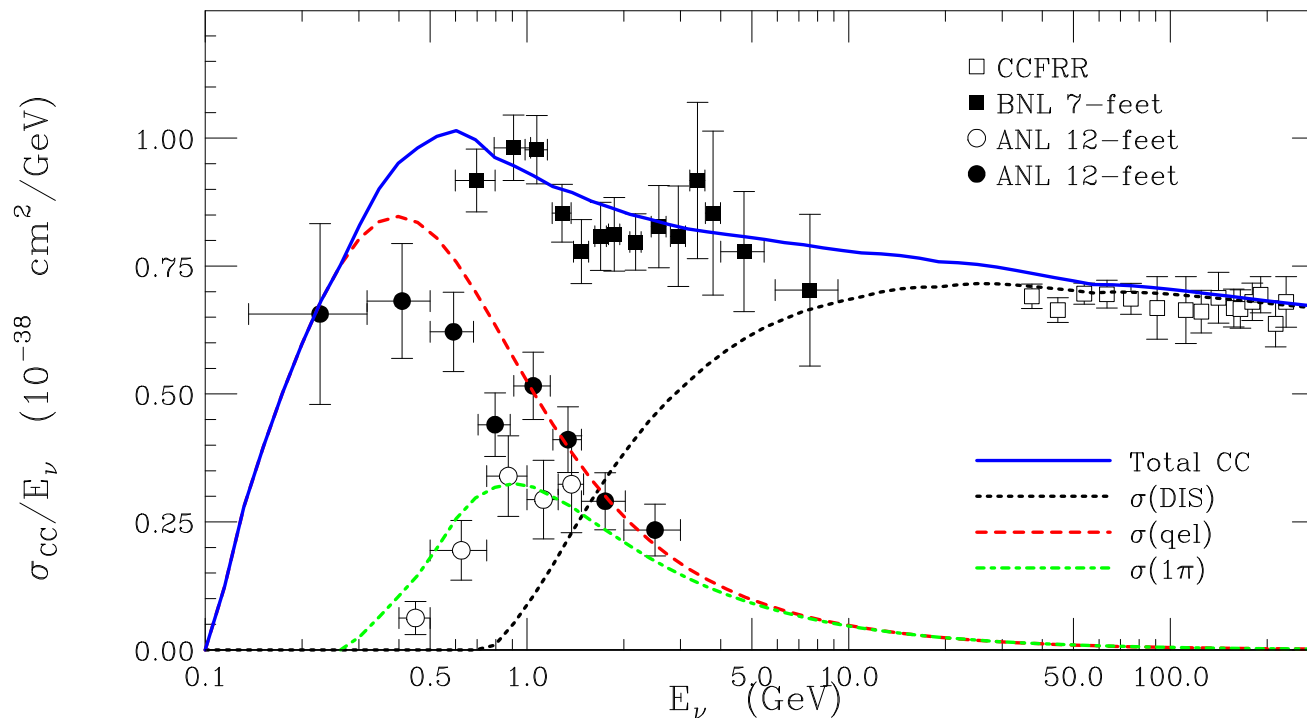
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Motivation :

1. Explore some aspects of hadron dynamics/structure, not accessible with electrons/photons, by using a **weak probe**
2. First step to study **neutrino–nucleus inclusive scattering** above the QE peak





Theoretical knowledge of the one pion cross section is important to carry out a precise data analysis... Furthermore...

Pion production $\rightarrow$ identify incorrectly one-Čerenkov-ring events, which are assumed to be **CC QE** $\nu_\alpha A \rightarrow l_\alpha A'$.

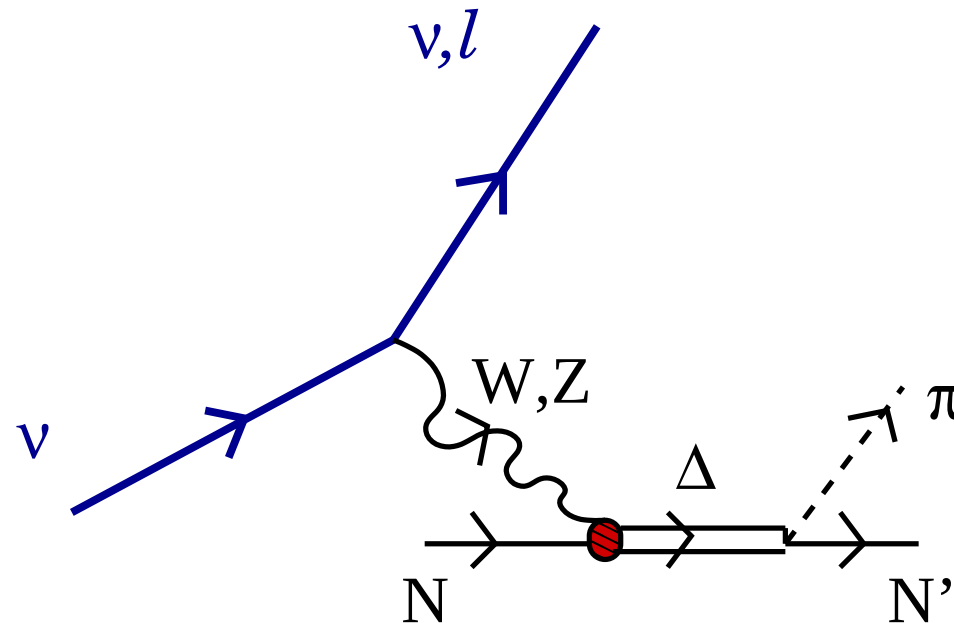
- **Appearance probability** $P(\nu_\mu \rightarrow \nu_e)$: CC QE $\nu_e A \rightarrow e A'$ signal, which is used to identify ν_e , could be confused with

that from 1π NC $\nu_\mu A \rightarrow \nu_\mu A \pi^0$ process

- **Survival probability** $P(\nu_\mu \rightarrow \nu_\mu)$: CC QE $\nu_\mu A \rightarrow \mu A'$ signal, which is used to identify ν_μ , could be confused with that from CC/NC 1π $\nu_{\tau,\mu} A \rightarrow (\nu_{\tau,\mu} \text{ ó } \tau, \mu) A' \pi$ signal, **if only one particle radiates Čerenkov light**.

For instance, $(\nu_\mu, \mu\pi)$ Incorrect E_ν re-construction $\rightarrow L/E$ analysis ?

Theoretical Model $\nu_l N \rightarrow l N' \pi$, $\nu_l N \rightarrow \nu_l N' \pi$ (C.H. Llewellyn Smith, 1972): weak excitation of the $\Delta(1232)$ resonance and its subsequent decay into $N\pi$,



$$\langle \Delta^+; p_\Delta = p + q | j_{cc+}^\mu(0) | n; p \rangle = \bar{u}_\alpha(\vec{p}_\Delta) \Gamma^{\alpha\mu}(p, q) u(\vec{p}) \cos \theta_C,$$

$$\begin{aligned} \Gamma^{\alpha\mu} = & \left[\frac{\mathbf{C}_3^A}{M} (g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu) + \frac{\mathbf{C}_4^A}{M^2} (g^{\alpha\mu} q \cdot p_\Delta - q^\alpha p_\Delta^\mu) + \mathbf{C}_5^A g^{\alpha\mu} + \frac{\mathbf{C}_6^A}{M^2} q^\mu q^\alpha \right] \\ & + \left[\frac{\mathbf{C}_3^V}{M} (g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu) + \frac{\mathbf{C}_4^V}{M^2} (g^{\alpha\mu} q \cdot p_\Delta - q^\alpha p_\Delta^\mu) + \frac{\mathbf{C}_5^V}{M^2} (g^{\alpha\mu} q \cdot p - q^\alpha p^\mu) \right. \\ & \left. + \mathbf{C}_6^V g^{\mu\alpha} \right] \gamma_5, \quad \mathbf{C}_{3,4,5,6}^A \text{ axial FF's, } \mathbf{C}_{3,4,5,6}^V \text{ vector FF's, furthermore} \end{aligned}$$

$$\mathcal{L}_{\pi N \Delta} = \frac{f^*}{m_\pi} \bar{\Psi}_\mu \vec{T}^\dagger (\partial^\mu \vec{\phi}) \Psi + \text{h.c.}, \quad f^* = 2.14$$

$$\mathbf{G}^{\mu\nu}(\mathbf{p}_\Delta) = \frac{\not{p}_\Delta + M_\Delta}{p_\Delta^2 - M_\Delta^2 + iM_\Delta \Gamma_\Delta} \left[-\mathbf{g}^{\mu\nu} + \frac{1}{3} \gamma^\mu \gamma^\nu + \frac{2}{3} \frac{\mathbf{p}_\Delta^\mu \mathbf{p}_\Delta^\nu}{\mathbf{M}_\Delta^2} - \frac{1}{3} \frac{\mathbf{p}_\Delta^\mu \gamma^\nu - \mathbf{p}_\Delta^\nu \gamma^\mu}{\mathbf{M}_\Delta} \right]$$

$eN \rightarrow e'\Delta \rightarrow e'N'\pi \Rightarrow C_{3,4,5,6}^V$ **FF's**. CVC $\Rightarrow C_6^V = 0$ and
($M_V = 0.84$ GeV)

$$\frac{C_3^V(q^2)}{2.13} = \frac{C_4^V(q^2)}{-1.51} = \frac{1 - \frac{q^2}{0.776M_V^2}}{1 - \frac{q^2}{4M_V^2}} \frac{C_5^V(q^2)}{0.48} = \frac{1}{(1 - q^2/M_V^2)^2} \times \frac{1}{1 - \frac{q^2}{4M_V^2}}$$

$C_{3,4,5,6}^A$ **Axial FF's** : Δ^{++} ($\nu_\mu p \rightarrow \mu^- p \pi^+$) data taken in
the ANL and BNL bubble chambers (filled in with
deuterium)

Dominant form factor: $C_5^A(q^2)$. $C_3^A(q^2)$ and $C_4^A(q^2)$
contributions are small and we have taken (**Adler's model**
1968)

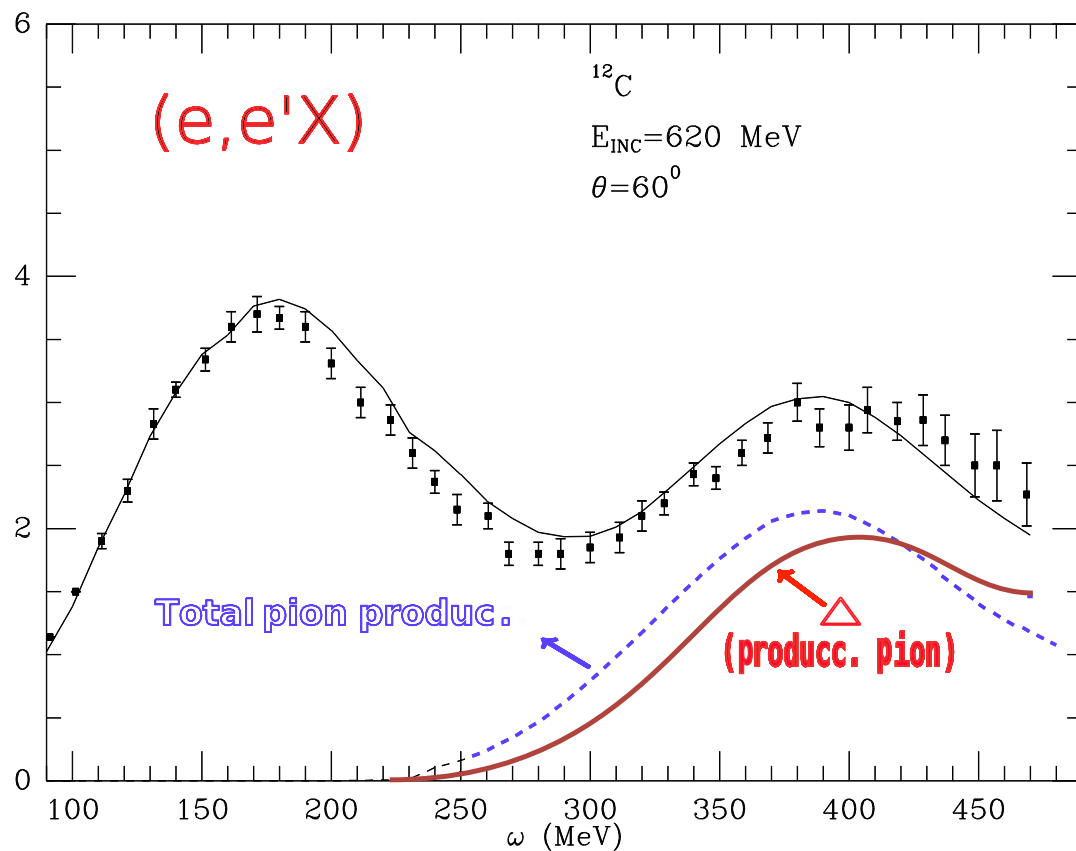
$$C_4^A(q^2) = -\frac{C_5^A(q^2)}{4}, \quad C_3^A(q^2) = 0$$

Furthermore **PCAC** ($\partial_\mu A^\mu \propto m_\pi^2$) and **Goldberger–Treiman** $C_5^A(0) \sim \sqrt{\frac{2}{3}} \frac{f_\pi}{m_\pi} f^* = 1.2$

$$C_5^A(q^2) = \frac{1.2}{(1 - q^2/M_{A\Delta}^2)^2} \times \frac{1}{1 - \frac{q^2}{3M_{A\Delta}^2}}, \quad \underbrace{C_6^A(q^2) = C_5^A(q^2) \frac{M^2}{m_\pi^2 - q^2}}_{\text{PCAC}}$$

$M_{A\Delta}$ fitted to the q^2 dependence of the $\nu_\mu p \rightarrow \mu^- p \pi^+$ cross section (neutrino energy averaged) **with** ($M(\pi N) < 1.4$ GeV) **measured at ANL and BNL**. It varies in the range 0.95 GeV (ANL) – 1.28 GeV (BNL). We set

$$M_{A\Delta} \sim 1.05 \text{ GeV} \quad (\text{axial nucleon mass})$$

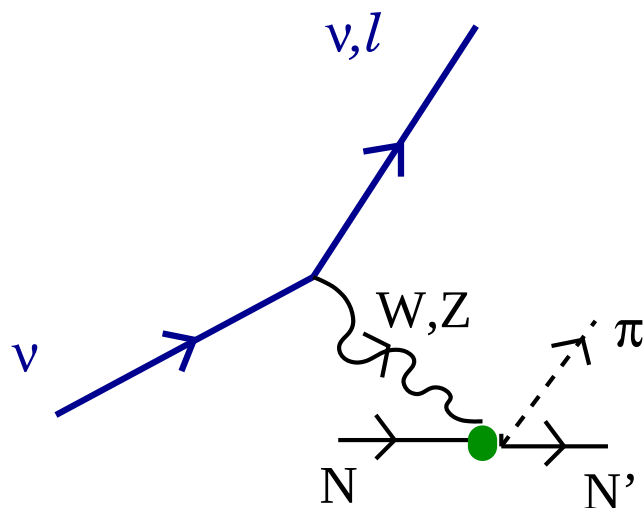


... but **only the Δ pole** contribution turns out to be **an insufficient** model, even at the Δ peak, and **specially close to pion threshold**. Close to pion threshold, the pion from the $(\nu_\mu, \mu\pi)$ **reaction will not radiate Čerenkov light** and thus it would be necessary an improved theoretical model to carry out a proper **L/E oscillation analysis**.

Such model for the $\nu_l N \rightarrow l N' \pi$, $\nu_l N \rightarrow \nu_l N' \pi$ reactions should include **non resonant terms** $\Rightarrow$ **Realization of the axial and vector currents, which couple to the W, Z^0 bosons, for a system of pions and nucleons.**

Non-linear σ -Model: EFT involving pions and nucleons which implements spontaneous chiral symmetry breaking.

If $\Psi_q = \begin{pmatrix} \Psi_u \\ \Psi_d \end{pmatrix}$, the CC and NC, which induce $W(Z^0)N \rightarrow N'\pi$



$$j_{cc\pm}^\mu = \cos \theta_C \bar{\Psi}_q \gamma^\mu (1 - \gamma_5) \left(-\frac{\tau_{\pm 1}^1}{\sqrt{2}} \right) \Psi_q$$

$$j_{nc}^\mu = \bar{\Psi}_q \gamma^\mu (1 - 2 \sin^2 \theta_W - \gamma_5) \tau_0^1 \Psi_q$$

$$- \left[4 \sin^2 \theta_W \mathbf{s}_{em,IS}^\mu - \bar{\Psi}_s \gamma^\mu (1 - \gamma_5) \Psi_s \right]$$

$$s_{em}^\mu = \underbrace{\frac{1}{6} \bar{\Psi}_q \gamma^\mu \Psi_q - \frac{1}{3} \bar{\Psi}_s \gamma^\mu \Psi_s}_{\mathbf{s}_{em,IS}^\mu} + \frac{1}{\sqrt{2}} \bar{\Psi}_q \gamma^\mu \frac{\tau_0^1}{\sqrt{2}} \Psi_q$$

$$\langle N' \pi | \mathbf{j}_{cc+}^\mu(0), \mathbf{j}_{cc-}^\mu(0), \mathbf{j}_{nc}^\mu(0) | N \rangle = ? \Leftarrow \underline{\text{QCD and its pattern of } S_\chi \text{SB}}$$

If $\Psi = \begin{pmatrix} p \\ n \end{pmatrix}$, $U = \frac{f_\pi}{\sqrt{2}} e^{i\vec{\tau} \cdot \boxed{\vec{\phi}}/f_\pi} = \frac{f_\pi}{\sqrt{2}} \xi^2$, with $f_\pi \sim 93$ MeV,

$$\begin{aligned} \mathcal{L}_{N\pi} &= \bar{\Psi} i \gamma^\mu [\partial_\mu + \mathcal{V}_\mu] \Psi - M \bar{\Psi} \Psi + g_A \bar{\Psi} \gamma^\mu \gamma_5 \mathcal{A}_\mu \Psi \\ &+ \frac{1}{2} \text{Tr} [\partial_\mu U^\dagger \partial^\mu U] \boxed{+ m_\pi^2 \frac{f_\pi}{2\sqrt{2}} \text{Tr}(U + U^\dagger - \sqrt{2} f_\pi)} \end{aligned}$$

$$\mathcal{V}_\mu = \frac{1}{2} (\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi) \quad \mathcal{A}_\mu = \frac{i}{2} (\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi)$$

Isospin rotat. $\xi \rightarrow \mathbf{T}_V \xi \mathbf{T}_V^\dagger$, $\Psi \rightarrow \mathbf{T}_V \Psi$, $T_V = e^{-i \frac{\vec{\tau} \cdot \vec{\theta}_V}{2}}$

Axial rotat. $\xi \rightarrow \mathbf{T}_A^\dagger \xi \mathbf{T}_A^\dagger = \mathbf{T}_A \xi \mathbf{T}_A^\dagger$, $\Psi \rightarrow \mathbf{T}_A \Psi$, $T_{A,A} = e^{-i \frac{\vec{\tau} \cdot \vec{\theta}_{A,A}}{2}}$

Isospin rotat. $\Rightarrow \delta \mathcal{L}_{N\pi} = 0$, Axial rotat. $\Rightarrow \delta \mathcal{L}_{N\pi} \boxed{\propto m_\pi^2 \neq 0}$

Up to order $\mathcal{O}(1/f_\pi^4)$, $\mathcal{L}_{N\pi}$ reads,

$$\begin{aligned} \mathcal{L}_{N\pi} = & \bar{\Psi}[\mathrm{i}\not{\partial} - M]\Psi + \frac{1}{2}\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - \frac{1}{2}m_\pi^2\vec{\phi}^2 \quad (\text{kinetic}) + \\ & \frac{g_A}{f_\pi}\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}(\partial_\mu\vec{\phi})\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma_\mu\vec{\tau}\left(\vec{\phi}\times\partial^\mu\vec{\phi}\right)\Psi - \frac{g_A}{6f_\pi^3}\bar{\Psi}\gamma^\mu\gamma_5\left[\vec{\phi}^2\frac{\vec{\tau}}{2}\partial_\mu\vec{\phi} - (\vec{\phi}\partial_\mu\vec{\phi})\frac{\vec{\tau}}{2}\vec{\phi}\right]\Psi \\ & - \frac{1}{6f_\pi^2}(\vec{\phi}^2\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - (\vec{\phi}\partial_\mu\vec{\phi})(\vec{\phi}\partial^\mu\vec{\phi})) + \frac{m_\pi^2}{24f_\pi^2}(\vec{\phi}^2)^2 + \mathcal{O}(1/f_\pi^4) \end{aligned}$$

Contact interactions $NN\pi$, $\underbrace{NN\pi\pi}_{\text{WT}}$, $NN\pi\pi\pi$ and $\pi\pi\pi\pi$.

Parameters: f_π and g_A . Noether's currents

$$j^\mu = \frac{\partial\mathcal{L}_{N\pi}}{\partial(\partial_\mu\varphi_a)}\delta\varphi_a, \quad a = 1, 2, \dots$$

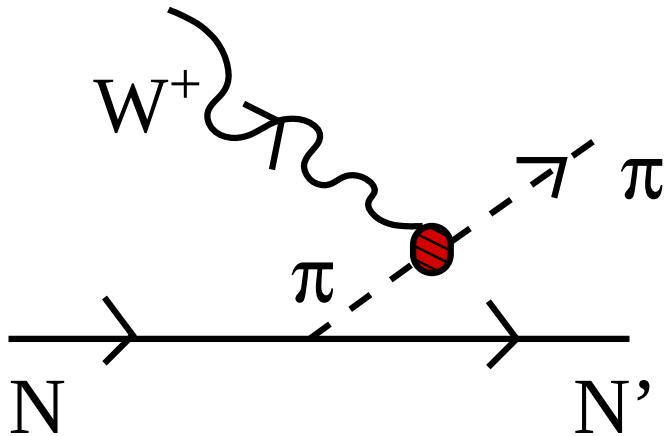
up to order $\mathcal{O}(1/f_\pi^3)$...

$$\begin{aligned}
\vec{V}^\mu &= \vec{\phi} \times \partial^\mu \vec{\phi} + \frac{g_A}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 (\vec{\phi} \times \vec{\tau}) \Psi + \bar{\Psi} \gamma^\mu \frac{\vec{\tau}}{2} \Psi - \frac{1}{4f_\pi^2} \bar{\Psi} \gamma^\mu \left[\vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi \\
&\quad - \frac{\vec{\phi}^2}{3f_\pi^2} (\vec{\phi} \times \partial^\mu \vec{\phi}) + \mathcal{O}\left(\frac{1}{f_\pi^3}\right), \quad \partial_\mu \vec{V}^\mu = \mathbf{0} \\
\vec{A}^\mu &= f_\pi \partial^\mu \vec{\phi} + \frac{1}{2f_\pi} \bar{\Psi} \gamma^\mu (\vec{\phi} \times \vec{\tau}) \Psi + g_A \bar{\Psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}}{2} \Psi + \frac{2}{3f_\pi} \left[\vec{\phi} (\vec{\phi} \cdot \partial^\mu \vec{\phi}) - \vec{\phi}^2 \partial^\mu \vec{\phi} \right] \\
&\quad - \frac{g_A}{4f_\pi^2} \bar{\Psi} \gamma^\mu \gamma_5 \left[\vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi + \mathcal{O}\left(\frac{1}{f_\pi^3}\right), \quad \underbrace{\partial_\mu \vec{A}^\mu \propto m_\pi^2 \dots}_{\text{PCAC}}
\end{aligned}$$

+ isospin relations $\Rightarrow$ evaluate CC $\langle N' \pi | j_{cc+}^\mu(0), j_{cc-}^\mu(0) | N \rangle$

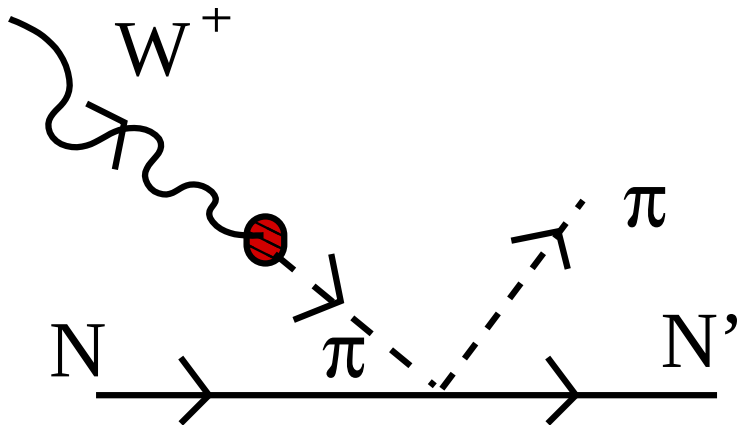
$$\begin{aligned}
\langle p \pi^0 | j_{cc+}^\mu(0) | n \rangle &= -\frac{1}{\sqrt{2}} \left[\langle \mathbf{p} \pi^+ | \mathbf{j}_{cc+}^\mu(\mathbf{0}) | \mathbf{p} \rangle - \langle \mathbf{n} \pi^+ | \mathbf{j}_{cc+}^\mu(\mathbf{0}) | \mathbf{n} \rangle \right] \\
\langle p \pi^- | j_{cc-}^\mu(0) | p \rangle &= \langle \mathbf{n} \pi^+ | \mathbf{j}_{cc+}^\mu(\mathbf{0}) | \mathbf{n} \rangle \\
\langle n \pi^- | j_{cc-}^\mu(0) | n \rangle &= \langle \mathbf{p} \pi^+ | \mathbf{j}_{cc+}^\mu(\mathbf{0}) | \mathbf{p} \rangle \\
\langle n \pi^0 | j_{cc-}^\mu(0) | p \rangle &= -\langle p \pi^0 | j_{cc+}^\mu(0) | n \rangle = \frac{1}{\sqrt{2}} \left[\langle \mathbf{p} \pi^+ | \mathbf{j}_{cc+}^\mu(\mathbf{0}) | \mathbf{p} \rangle - \langle \mathbf{n} \pi^+ | \mathbf{j}_{cc+}^\mu(\mathbf{0}) | \mathbf{n} \rangle \right]
\end{aligned}$$

$$\begin{aligned}
\vec{V}^\mu &= \vec{\phi} \times \partial^\mu \vec{\phi} + \frac{g_A}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 (\vec{\phi} \times \vec{\tau}) \Psi + \bar{\Psi} \gamma^\mu \frac{\vec{\tau}}{2} \Psi - \frac{1}{4f_\pi^2} \bar{\Psi} \gamma^\mu \left[\vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi \\
&\quad - \frac{\vec{\phi}^2}{3f_\pi^2} (\vec{\phi} \times \partial^\mu \vec{\phi}) + \mathcal{O}\left(\frac{1}{f_\pi^3}\right) \\
\vec{A}^\mu &= f_\pi \partial^\mu \vec{\phi} + \frac{1}{2f_\pi} \bar{\Psi} \gamma^\mu (\vec{\phi} \times \vec{\tau}) \Psi + g_A \bar{\Psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}}{2} \Psi + \frac{2}{3f_\pi} \left[\vec{\phi} (\vec{\phi} \cdot \partial^\mu \vec{\phi}) - \vec{\phi}^2 \partial^\mu \vec{\phi} \right] \\
&\quad - \frac{g_A}{4f_\pi^2} \bar{\Psi} \gamma^\mu \gamma_5 \left[\vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi + \mathcal{O}\left(\frac{1}{f_\pi^3}\right)
\end{aligned}$$



$$\begin{aligned}
j_{cc+}^\mu \Big|_{PF} &= \mp i \mathbf{F}_{PF}(\mathbf{q}^2) \frac{\sqrt{2} M g_A}{f_\pi} \cos \theta_C \frac{(2k_\pi - q)^\mu}{(k_\pi - q)^2 - m_\pi^2} \bar{u}(\vec{p}') \gamma_5 u(\vec{p}) \\
&\quad (- \Rightarrow W^+ p \rightarrow p \pi^+, + \Rightarrow W^+ n \rightarrow n \pi^+)
\end{aligned}$$

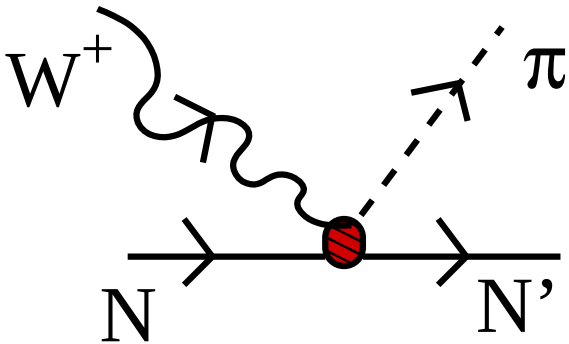
$$\begin{aligned}
\vec{V}^\mu &= \vec{\phi} \times \partial^\mu \vec{\phi} + \frac{g_A}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 (\vec{\phi} \times \vec{\tau}) \Psi + \bar{\Psi} \gamma^\mu \frac{\vec{\tau}}{2} \Psi - \frac{1}{4f_\pi^2} \bar{\Psi} \gamma^\mu \left[\vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi \\
&\quad - \frac{\vec{\phi}^2}{3f_\pi^2} (\vec{\phi} \times \partial^\mu \vec{\phi}) + \mathcal{O}\left(\frac{1}{f_\pi^3}\right) \\
\vec{A}^\mu &= \mathbf{f}_\pi \partial^\mu \vec{\phi} + \frac{1}{2f_\pi} \bar{\Psi} \gamma^\mu (\vec{\phi} \times \vec{\tau}) \Psi + g_A \bar{\Psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}}{2} \Psi + \frac{2}{3f_\pi} \left[\vec{\phi} (\vec{\phi} \cdot \partial^\mu \vec{\phi}) - \vec{\phi}^2 \partial^\mu \vec{\phi} \right] \\
&\quad - \frac{g_A}{4f_\pi^2} \bar{\Psi} \gamma^\mu \gamma_5 \left[\vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi + \mathcal{O}\left(\frac{1}{f_\pi^3}\right)
\end{aligned}$$



$$\begin{aligned}
j_{cc+}^\mu \Big|_{PP} &= \mp i \mathbf{F}_\rho \left((\mathbf{q} - \mathbf{k}_\pi)^2 \right) \frac{\cos \theta_C}{\sqrt{2} f_\pi} \frac{q^\mu}{q^2 - m_\pi^2} \bar{u}(\vec{p}') \not{q} u(\vec{p}) \\
& \quad (- \Rightarrow W^+ p \rightarrow p \pi^+, + \Rightarrow W^+ n \rightarrow n \pi^+)
\end{aligned}$$

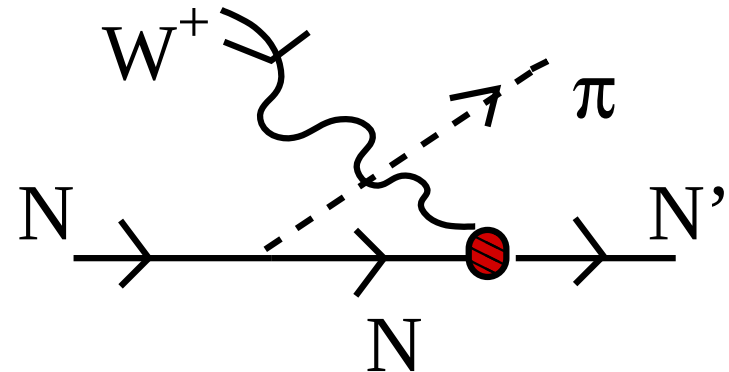
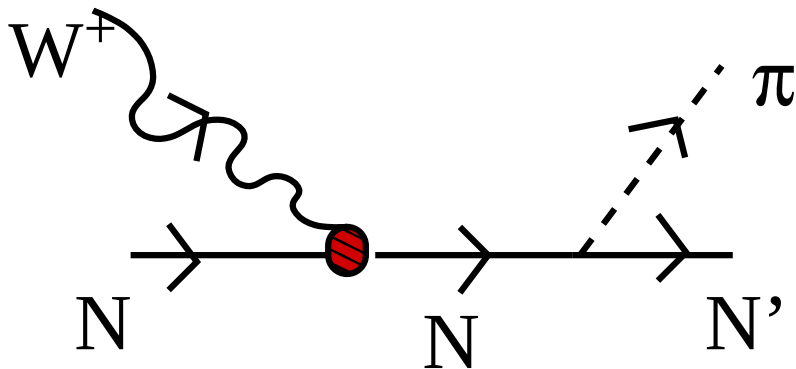
$$\mathbf{F}_\rho(\mathbf{t}) = \frac{1}{1 - \mathbf{t}/m_\rho^2}$$

$$\begin{aligned}
\vec{V}^\mu &= \vec{\phi} \times \partial^\mu \vec{\phi} + \frac{g_A}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 (\vec{\phi} \times \vec{\tau}) \Psi + \bar{\Psi} \gamma^\mu \frac{\vec{\tau}}{2} \Psi - \frac{1}{4f_\pi^2} \bar{\Psi} \gamma^\mu \left[\vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi \\
&\quad - \frac{\vec{\phi}^2}{3f_\pi^2} (\vec{\phi} \times \partial^\mu \vec{\phi}) + \mathcal{O}\left(\frac{1}{f_\pi^3}\right) \\
\vec{A}^\mu &= f_\pi \partial^\mu \vec{\phi} + \frac{1}{2f_\pi} \bar{\Psi} \gamma^\mu (\vec{\phi} \times \vec{\tau}) \Psi + g_A \bar{\Psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}}{2} \Psi + \frac{2}{3f_\pi} \left[\vec{\phi} (\vec{\phi} \cdot \partial^\mu \vec{\phi}) - \vec{\phi}^2 \partial^\mu \vec{\phi} \right] \\
&\quad - \frac{g_A}{4f_\pi^2} \bar{\Psi} \gamma^\mu \gamma_5 \left[\vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi + \mathcal{O}\left(\frac{1}{f_\pi^3}\right)
\end{aligned}$$



$$\begin{aligned}
j_{cc+}^\mu \Big|_{CT} &= \mp i \frac{\cos \theta_C}{\sqrt{2} f_\pi} \bar{u}(\vec{p}') \gamma^\mu \left(g_A \mathbf{F}_{CT}^V(\mathbf{q}^2) \gamma_5 - \mathbf{F}_\rho \left((\mathbf{q} - \mathbf{k}_\pi)^2 \right) \right) u(\vec{p}) \\
&\quad (- \Rightarrow W^+ p \rightarrow p \pi^+, + \Rightarrow W^+ n \rightarrow n \pi^+)
\end{aligned}$$

$$\begin{aligned}
\vec{V}^\mu &= \vec{\phi} \times \partial^\mu \vec{\phi} + \frac{g_A}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 (\vec{\phi} \times \vec{\tau}) \Psi + \bar{\Psi} \gamma^\mu \frac{\vec{\tau}}{2} \Psi - \frac{1}{4f_\pi^2} \bar{\Psi} \gamma^\mu \left[\vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi \\
&\quad - \frac{\vec{\phi}^2}{3f_\pi^2} (\vec{\phi} \times \partial^\mu \vec{\phi}) + \mathcal{O}\left(\frac{1}{f_\pi^3}\right) \\
\vec{A}^\mu &= f_\pi \partial^\mu \vec{\phi} + \frac{1}{2f_\pi} \bar{\Psi} \gamma^\mu (\vec{\phi} \times \vec{\tau}) \Psi + \mathbf{g_A} \bar{\Psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}}{2} \Psi + \frac{2}{3f_\pi} \left[\vec{\phi} (\vec{\phi} \cdot \partial^\mu \vec{\phi}) - \vec{\phi}^2 \partial^\mu \vec{\phi} \right] \\
&\quad - \frac{g_A}{4f_\pi^2} \bar{\Psi} \gamma^\mu \gamma_5 \left[\vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi + \mathcal{O}\left(\frac{1}{f_\pi^3}\right)
\end{aligned}$$



... improve the WNN transition vertex

$$\langle p; \vec{p}' = \vec{p} + \vec{q} | \mathbf{j}_{\mathbf{cc}+}^{\alpha}(\mathbf{0}) | n; \vec{p} \rangle = \cos \theta_C \bar{u}(\vec{p}') (\mathbf{V}_{\mathbf{N}}^{\alpha}(\mathbf{q}) - \mathbf{A}_{\mathbf{N}}^{\alpha}(\mathbf{q})) u(\vec{p})$$

$$\mathbf{V}_{\mathbf{N}}^{\alpha}(\mathbf{q}) = 2 \times \left(\mathbf{F}_1^{\mathbf{V}}(\mathbf{q}^2) \gamma^{\alpha} + i \mu_{\mathbf{V}} \frac{\mathbf{F}_2^{\mathbf{V}}(\mathbf{q}^2)}{2M} \sigma^{\alpha\nu} q_{\nu} \right)$$

$$\mathbf{A}_{\mathbf{N}}^{\alpha}(\mathbf{q}) = \underbrace{\frac{g_A}{(1 - q^2/M_A^2)^2}}_{\mathbf{G}_A(\mathbf{q}^2)} \times \left(\gamma^{\alpha} \gamma_5 + \underbrace{\frac{\not{q}}{m_{\pi}^2 - q^2} q^{\alpha} \gamma_5}_{\text{PCAC}} \right), \quad \begin{cases} g_A = 1.26 \\ M_A = 1.05 \text{ GeV} \end{cases}$$

$$\mathbf{F}_1^{\mathbf{V}}(\mathbf{q}^2) = \frac{1}{2} (\mathbf{F}_1^{\mathbf{p}}(\mathbf{q}^2) - \mathbf{F}_1^{\mathbf{n}}(\mathbf{q}^2)), \quad \mu_{\mathbf{V}} \mathbf{F}_2^{\mathbf{V}}(\mathbf{q}^2) = \frac{1}{2} (\mu_{\mathbf{p}} \mathbf{F}_2^{\mathbf{p}}(\mathbf{q}^2) - \mu_{\mathbf{n}} \mathbf{F}_2^{\mathbf{n}}(\mathbf{q}^2)),$$

furthermore $\mathbf{CVC} \Rightarrow \boxed{F_{PF}(q^2) = F_{CT}^V(q^2) = 2F_1^V(q^2) = F_1^p - F_1^n}$

$$\begin{aligned}
\vec{V}^\mu &= \vec{\phi} \times \partial^\mu \vec{\phi} + \frac{g_A}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 (\vec{\phi} \times \vec{\tau}) \Psi + \bar{\Psi} \gamma^\mu \frac{\vec{\tau}}{2} \Psi - \frac{1}{4\mathbf{f}_\pi^2} \bar{\Psi} \gamma^\mu \left[\vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi \\
&\quad - \frac{\vec{\phi}^2}{3\mathbf{f}_\pi^2} (\vec{\phi} \times \partial^\mu \vec{\phi}) + \mathcal{O}\left(\frac{1}{\mathbf{f}_\pi^3}\right) \\
\vec{A}^\mu &= f_\pi \partial^\mu \vec{\phi} + \frac{1}{2f_\pi} \bar{\Psi} \gamma^\mu (\vec{\phi} \times \vec{\tau}) \Psi + g_A \bar{\Psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}}{2} \Psi + \frac{2}{3\mathbf{f}_\pi} \left[\vec{\phi} (\vec{\phi} \cdot \partial^\mu \vec{\phi}) - \vec{\phi}^2 \partial^\mu \vec{\phi} \right] \\
&\quad - \frac{g_A}{4\mathbf{f}_\pi^2} \bar{\Psi} \gamma^\mu \gamma_5 \left[\vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi + \mathcal{O}\left(\frac{1}{\mathbf{f}_\pi^3}\right)
\end{aligned}$$

$\nu_l N \rightarrow l N' \pi \pi$, $\nu_l N \rightarrow \nu_l N' \pi \pi$ close to threshold. $N(1440)$

degrees of freedom?

Evaluation of NC $\langle N'\pi | j_{\text{nc}}^\mu(0) | N \rangle$:

$$j_{\text{nc}}^\mu = \bar{\Psi}_q \gamma^\mu (1 - 2 \sin^2 \theta_W - \gamma_5) \boxed{\tau_0^1} \Psi_q - \boxed{4 \sin^2 \theta_W \mathbf{s}_{\text{em,IS}}^\mu - \bar{\Psi}_s \gamma^\mu (1 - \gamma_5) \Psi_s}$$

$$s_{\text{em}}^\mu = \underbrace{\frac{1}{6} \bar{\Psi}_q \gamma^\mu \Psi_q - \frac{1}{3} \bar{\Psi}_s \gamma^\mu \Psi_s}_{\mathbf{s}_{\text{em,IS}}^\mu} + \frac{1}{\sqrt{2}} \bar{\Psi}_q \gamma^\mu \frac{\boxed{\tau_0^1}}{\sqrt{2}} \Psi_q$$

- ME's $j_{\text{cc}+}^\mu \Rightarrow$ ME's **isovector** (τ_0^1) j_{nc}^μ **contribution**
- Δ **does not contribute to the isoscalar j_{nc}^μ part**
- $\langle n\pi^+ | s_{\text{em,IS}}^\mu | p \rangle = \langle p\pi^- | s_{\text{em,IS}}^\mu | n \rangle = \sqrt{2} \langle \mathbf{p}\pi^0 | \mathbf{s}_{\text{em,IS}}^\mu | \mathbf{p} \rangle = -\sqrt{2} \langle n\pi^0 | s_{\text{em,IS}}^\mu | n \rangle$

$$\langle p\pi^0 | s_{\text{em,IS}}^\mu | p \rangle = -\frac{\langle n\pi^0 | s_{\text{em}}^\mu(0) | n \rangle - \langle p\pi^0 | s_{\text{em}}^\mu(0) | p \rangle}{2}$$

$$\mathbf{s}_{\text{em}}^\mu = \underbrace{\bar{\Psi} \gamma^\mu \left(\frac{1 + \tau_z}{2} \right) \Psi}_{\text{PN, PNC}} + \underbrace{\frac{ig_A}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 (\tau_{-1}^1 \phi^\dagger + \tau_{+1}^1 \phi) \Psi}_{\text{CT}} + \underbrace{i (\phi^\dagger \partial^\mu \phi - \phi \partial^\mu \phi^\dagger)}_{\text{PF}} + \dots$$

CT, PF do not contribute $\Rightarrow$ PN and PNC $\Rightarrow$ ME's of $s_{\text{em}, IS}^\mu$

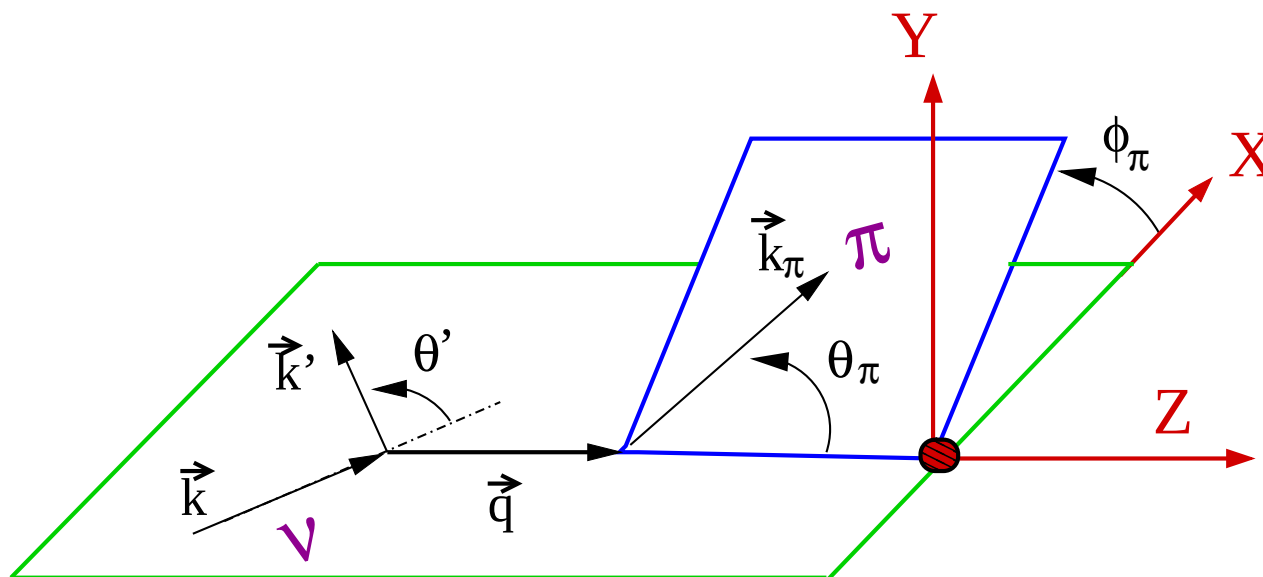
- ME's $\mathbf{j}_{\text{nc, str}}^\mu = \bar{\Psi}_s \gamma^\mu (1 - \gamma_5) \Psi_s$: nucleon strange content

$$\begin{aligned} \langle p\pi^0 | \mathbf{j}_{\text{nc, str}}^\mu(\mathbf{0}) | p \rangle = & -i \frac{g_A}{2f_\pi} \bar{u}(\vec{p}') \left\{ k_\pi \gamma_5 \frac{\not{p} + \not{q} + M}{(p+q)^2 - M^2 + i\epsilon} \left[\mathbf{V}_{\text{N},s}^\mu(\mathbf{q}) - \mathbf{A}_{\text{N},s}^\mu(\mathbf{q}) \right] \right. \\ & \left. + \left[\mathbf{V}_{\text{N},s}^\mu(\mathbf{q}) - \mathbf{A}_{\text{N},s}^\mu(\mathbf{q}) \right] \frac{\not{p}' - \not{q} + M}{(p'-q)^2 - M^2 + i\epsilon} k_\pi \gamma_5 \right\} u(\vec{p}) \quad \Leftarrow \text{PN} + \text{PNC} \end{aligned}$$

$$\mathbf{V}_{\text{N},s}^\mu(\mathbf{q}) = \underbrace{F_1^s(q^2)}_{\approx 0} \gamma^\mu + i\mu_s \overbrace{\frac{F_2^s(q^2)}{2M}}^{\approx 0} \sigma^{\mu\nu} q_\nu, \quad \mathbf{A}_{\text{N},s}^\mu(\mathbf{q}) = \boxed{g_S} (1 - \mathbf{q}^2/M_A^2)^{-2} \underbrace{G_A^s(q^2)}_{\text{don't contr.}} \gamma^\mu \gamma_5 + \underbrace{G_P^s}_{\text{don't contr.}} q^\mu \gamma_5$$

Results :

$$\Rightarrow \text{CC} : \nu_l(k) + N(p) \rightarrow l^-(k') + N(p') + \pi(k_\pi)$$



$$\frac{d^5\sigma_{\nu l1}}{d\Omega(\hat{\mathbf{k}}')dE'd\Omega(\hat{\mathbf{k}}_\pi)} = \frac{|\vec{k}'|}{|\vec{k}|} \frac{G^2}{4\pi^2} \int_0^{+\infty} \frac{d|\vec{k}_\pi||\vec{k}_\pi|^2}{E_\pi} \boxed{L_{\mu\sigma}^{(\nu)} (W_{\text{CC}\pi}^{\mu\sigma})^{(\nu)}}$$

$$(W_{\text{CC}\pi}^{\mu\sigma})^{(\nu)} = \frac{1}{4M} \sum_{\text{spins}} \int \frac{d^3 p'}{(2\pi)^3} \frac{1}{2E'_N} \delta^4(p' + k_\pi - q - p) \langle \mathbf{N}' \pi | \mathbf{j}_{\text{cc}+}^\mu(\mathbf{0}) | \mathbf{N} \rangle \langle \mathbf{N}' \pi | \mathbf{j}_{\text{cc}+}^\sigma(\mathbf{0}) | \mathbf{N} \rangle^*$$

$$\mathbf{L}_{\mu\sigma}^{(\nu)} = (\mathbf{L}_{\mathbf{s}}^{(\nu)})_{\mu\sigma} + i(\mathbf{L}_{\mathbf{a}}^{(\nu)})_{\mu\sigma} = k'_\mu k_\sigma + k'_\sigma k_\mu - g_{\mu\sigma} k \cdot k' + i\epsilon_{\mu\sigma\alpha\beta} k'^\alpha k^\beta$$

$$\Rightarrow \text{CC} : \bar{\nu}_l(k) + N(p) \rightarrow l^+(k') + N(p') + \pi(k_\pi)$$

$$\mathbf{L}_{\mu\sigma}^{(\bar{\nu})} = \mathbf{L}_{\sigma\mu}^{(\nu)}, \quad \mathbf{j}_{\text{cc}+}^\sigma \leftrightarrow \mathbf{j}_{\text{cc}-}^\sigma$$

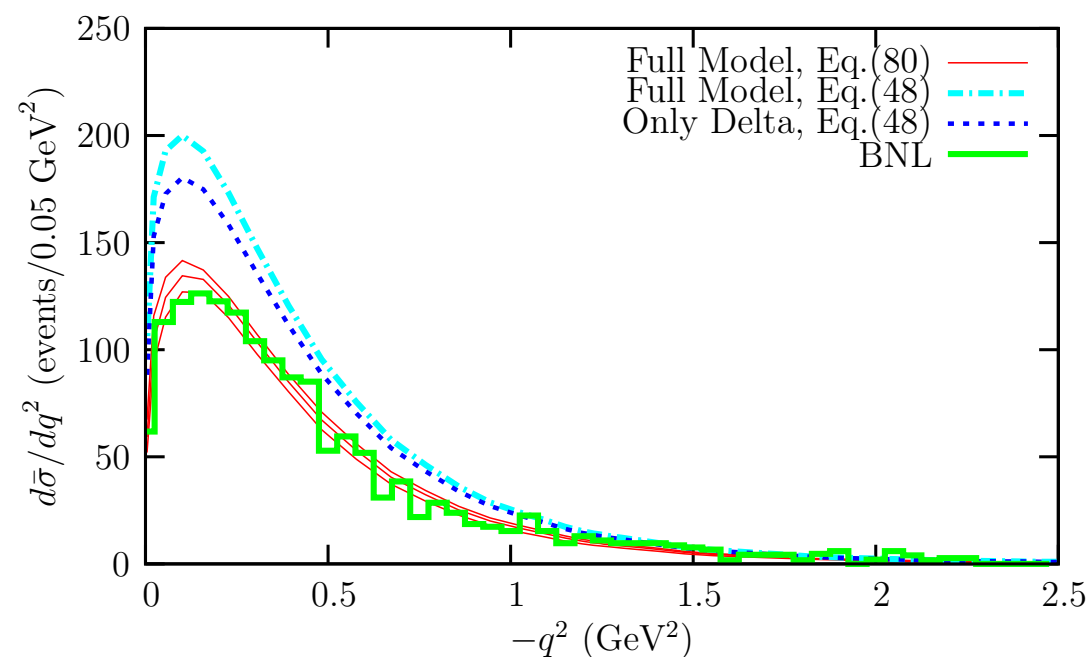
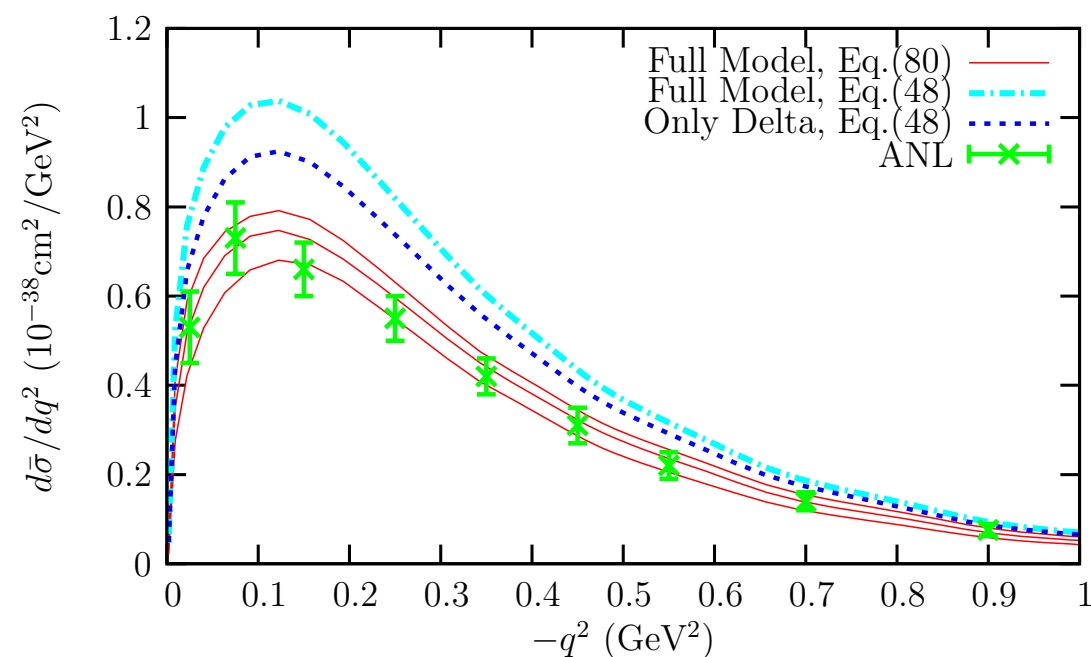
$$\Rightarrow \text{NC} : \nu(k) + N(p) \rightarrow \nu(k') + N(p') + \pi(k_\pi)$$

$$\mathbf{j}_{\text{cc}+}^\sigma \leftrightarrow \frac{1}{2} \mathbf{j}_{\text{nc}}^\sigma, \quad (\mathbf{W}_{\text{NC}\pi}^{\mu\sigma})^{(\nu)} = (\mathbf{W}_{\text{NC}\pi}^{\mu\sigma})^{(\bar{\nu})}$$

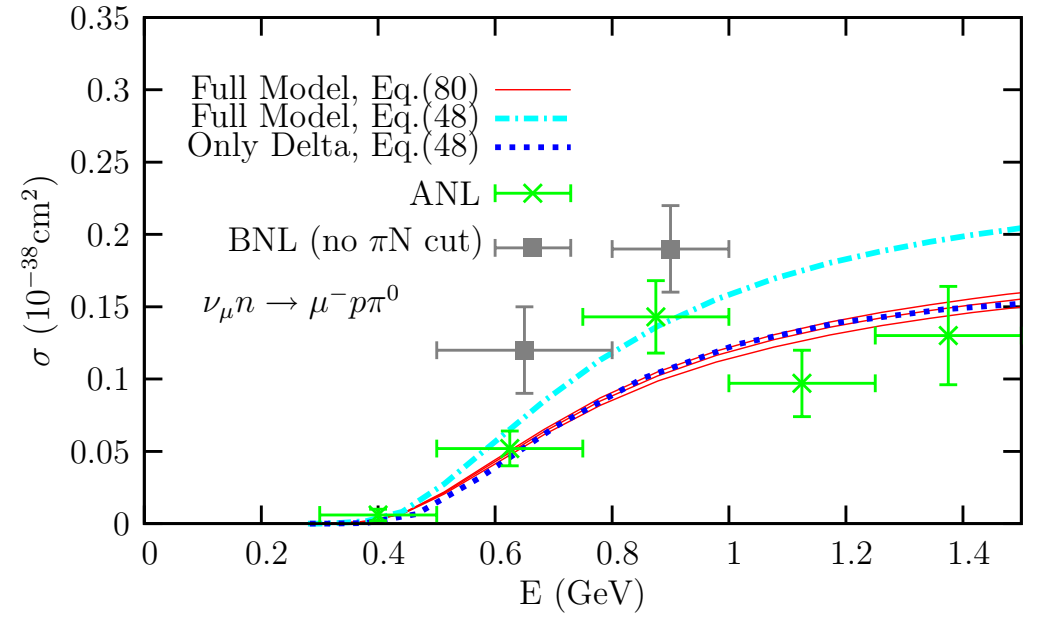
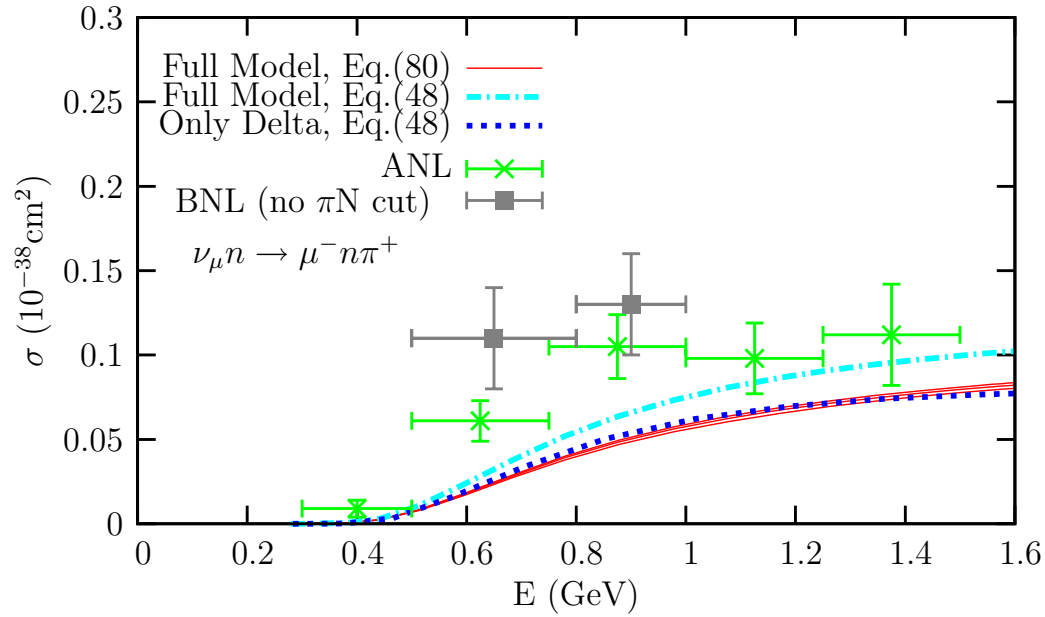
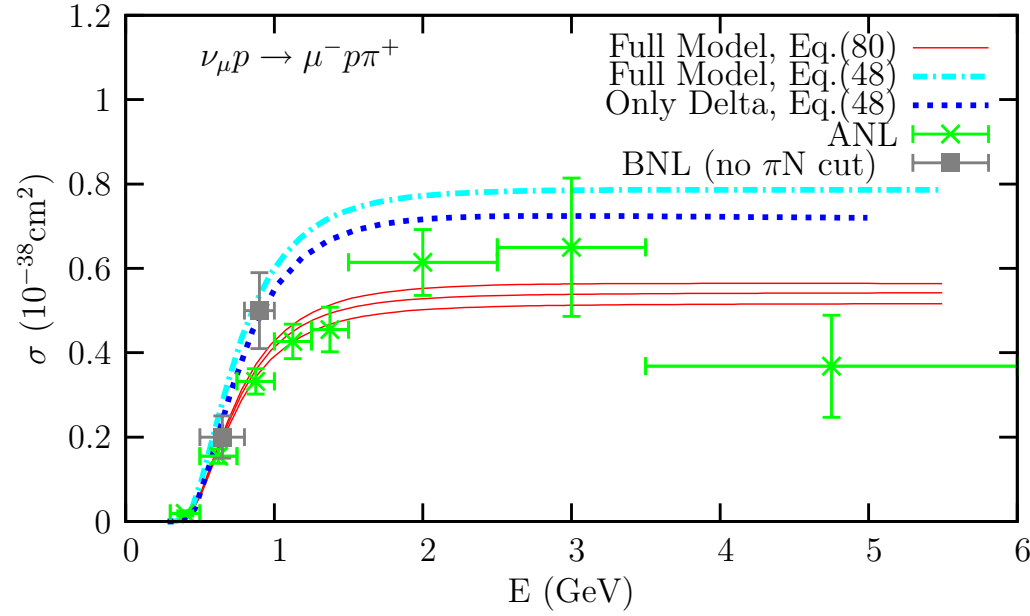
Note $\underbrace{(E', \theta')}_{\text{outgoing lepton}} \leftrightarrow q^2, \underbrace{W^2 = (p + q)^2}_{\pi\text{N inv. mass}}$

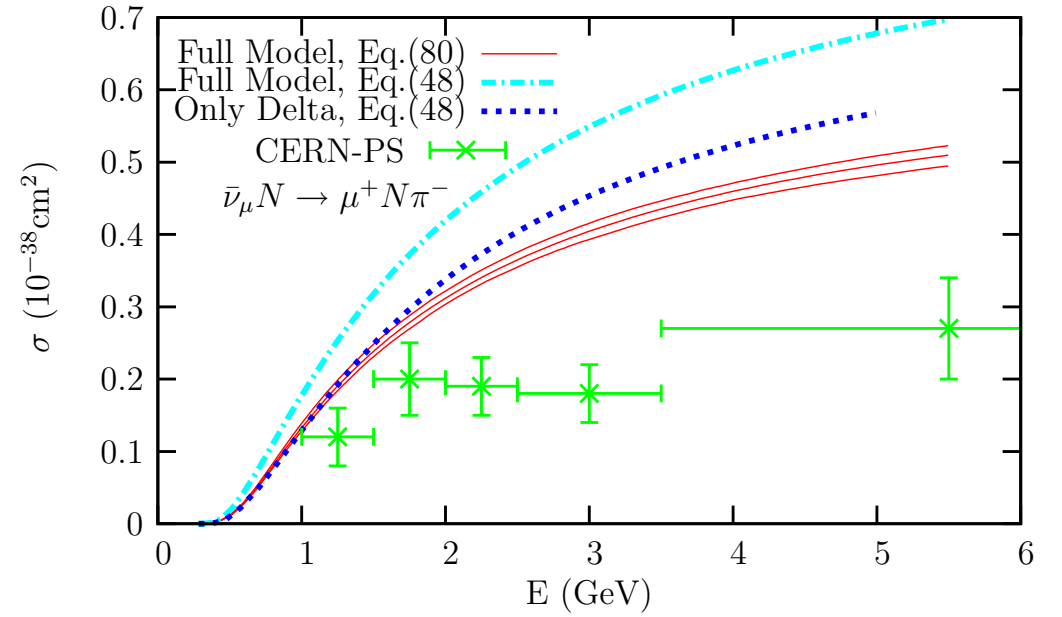
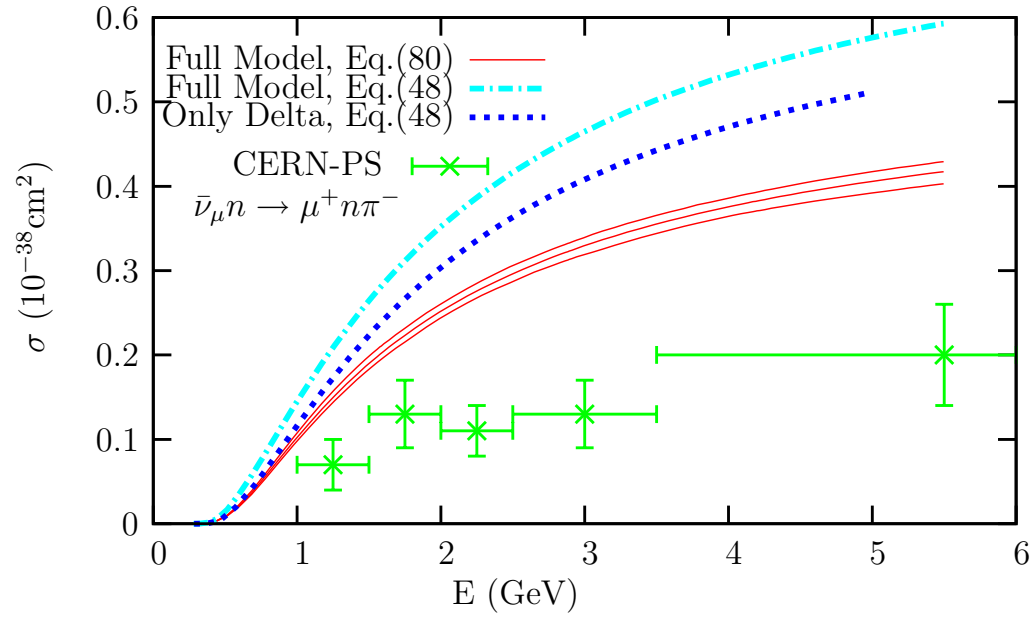
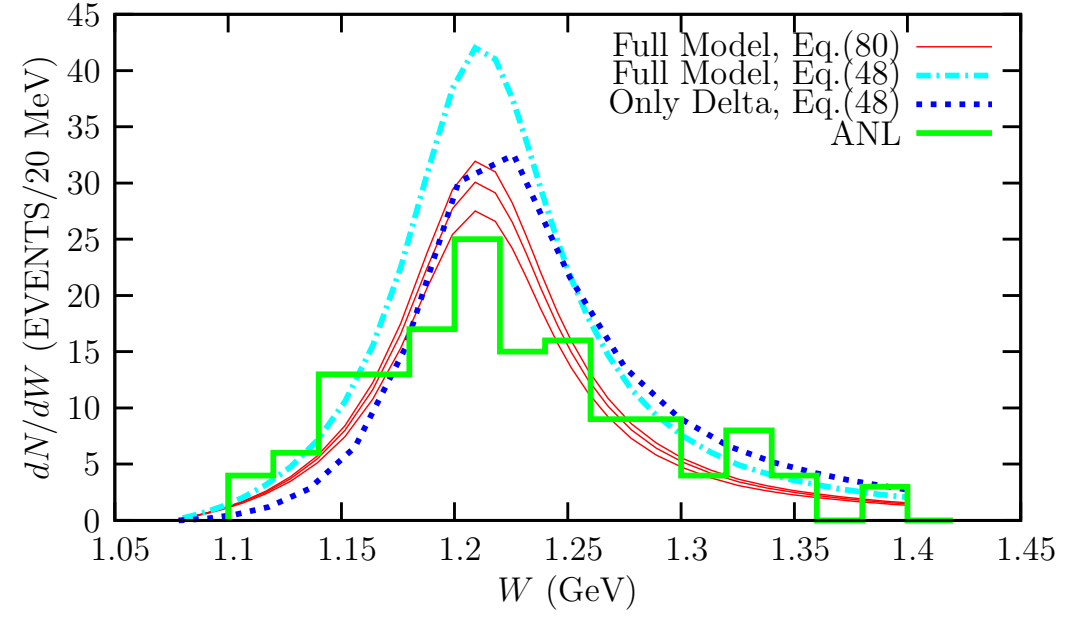
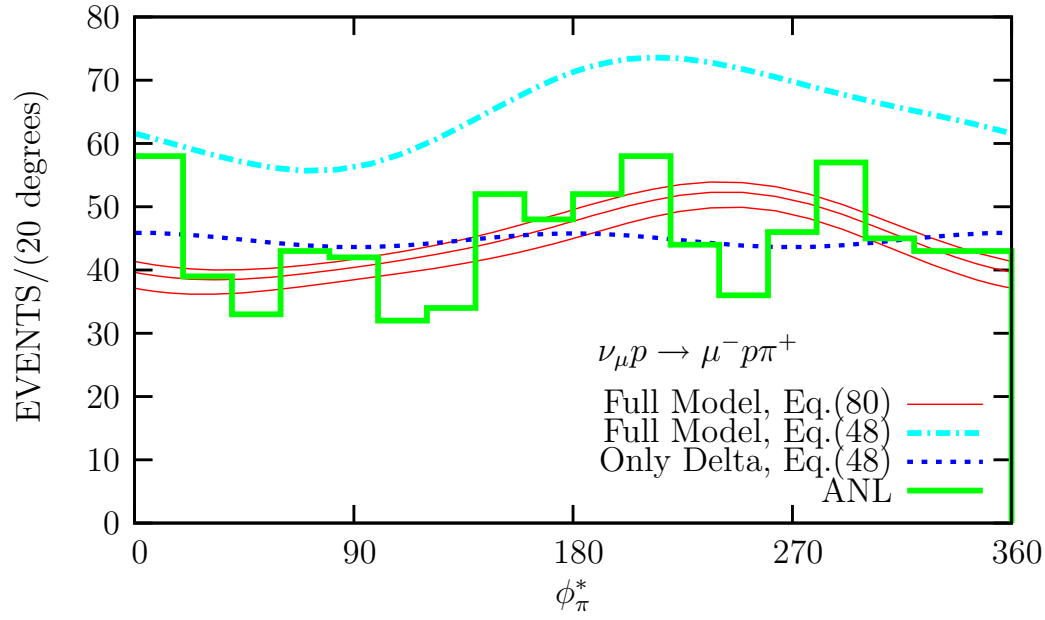
Differential cross section (flux averaged)

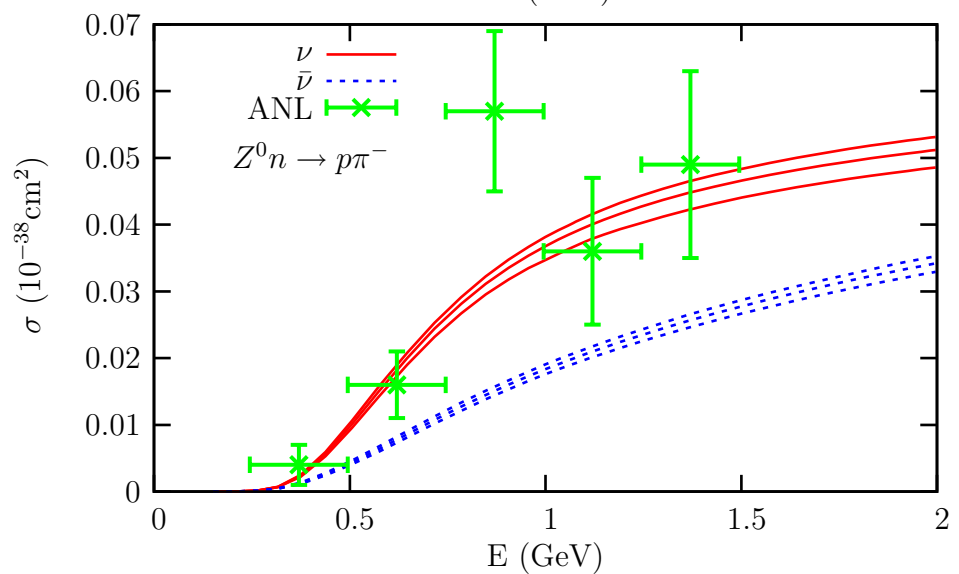
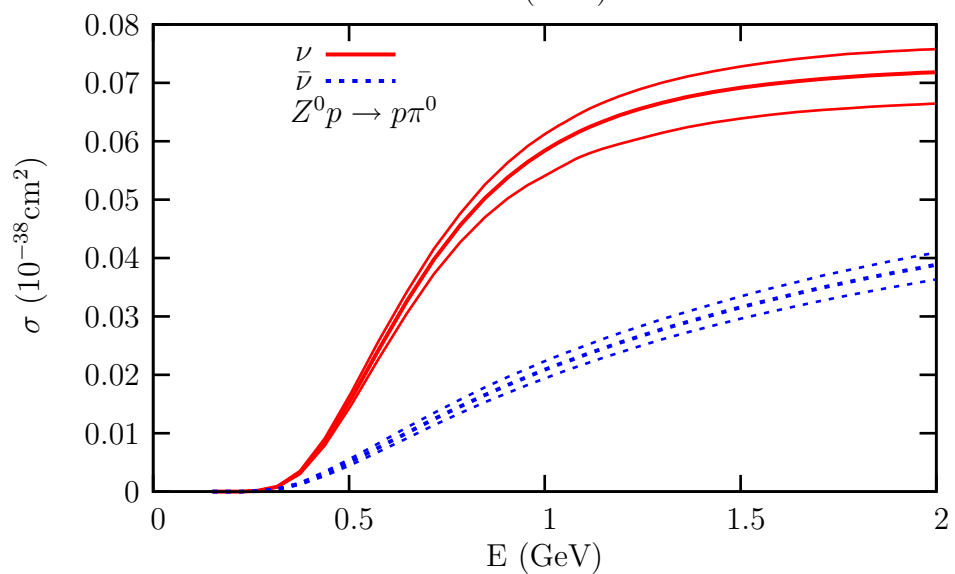
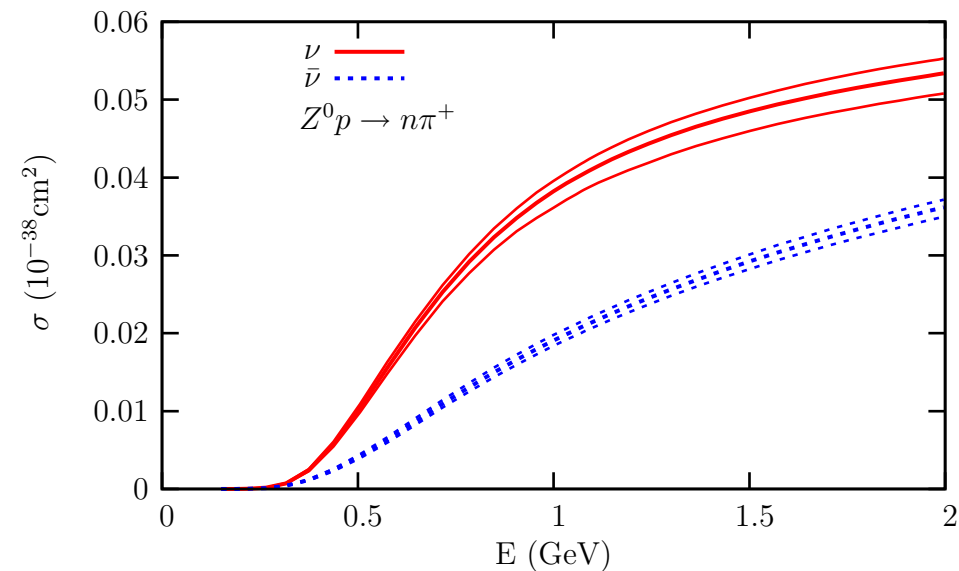
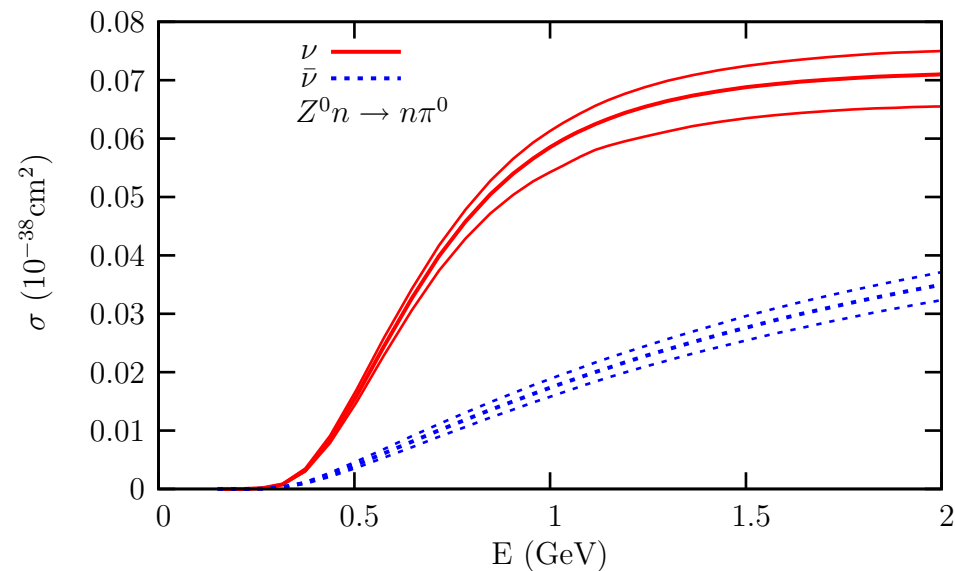
$$\int_{M+m_\pi}^{1.4 \text{ GeV}} dW \frac{d\bar{\sigma}_{\nu_\mu \mu^-}}{dq^2 dW}, \quad \nu_\mu \mathbf{p} \rightarrow \mu^- \mathbf{p} \pi^+$$



Fit to ANL : $C_5^A(0) = 0.869 \pm 0.075$, $M_{A\Delta} = 0.981 \pm 0.081 \text{ GeV}$







Below the τ prod. threshold, Distinguish ν_τ from $\bar{\nu}_\tau$?

$\sigma_{\text{NC}}/\sigma_{\text{CC}}$ ANL cross sections at $E = 0.6 - 1.2$ GeV

	ANL	Our results
$R_+ = \sigma(\nu p \rightarrow \nu n \pi^+)/\sigma(\nu p \rightarrow \mu^- p \pi^+)$	0.12 ± 0.04	$0.12 - 0.10$
$R_0 = \sigma(\nu p \rightarrow \nu p \pi^0)/\sigma(\nu p \rightarrow \mu^- p \pi^+)$	0.09 ± 0.05	$0.18 - 0.14$
$R_- = \sigma(\nu n \rightarrow \nu p \pi^-)/\sigma(\nu p \rightarrow \mu^- p \pi^+)$	0.11 ± 0.022	$0.12 - 0.09$

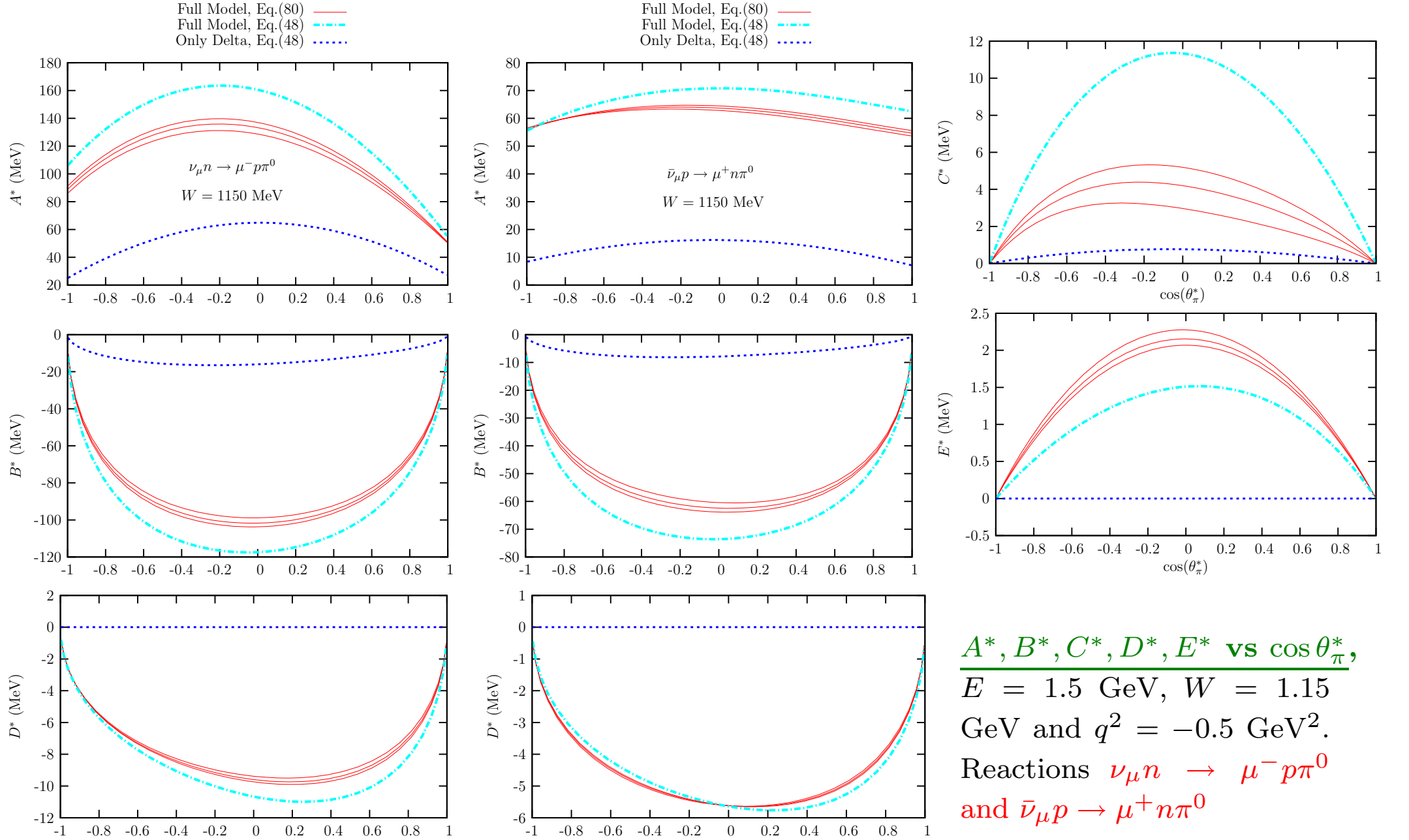
NC: Cross sections (10^{-38}cm^2) for $\langle E \rangle = 2.2$ GeV (no cut in W)

	CERN	Our results
$\sigma(\nu p \rightarrow \nu p \pi^0)$	0.130 ± 0.020	0.105 ± 0.006
$\sigma(\nu p \rightarrow \nu n \pi^+)$	0.080 ± 0.020	0.091 ± 0.003
$\sigma(\nu n \rightarrow \nu n \pi^0)$	0.080 ± 0.020	0.104 ± 0.006
$\sigma(\nu n \rightarrow \nu p \pi^-)$	0.110 ± 0.030	0.082 ± 0.003

Dependence on θ_π^* (CM πN pion polar angle). Lorentz invariance (f.i., CC) $\Rightarrow$

$$\frac{d^5\sigma_{\nu l}}{d\Omega(\hat{k}')dE'd\Omega^*(\hat{k}_\pi)} = \frac{|\vec{k}'|}{|\vec{k}|} \frac{G^2}{4\pi^2} \left\{ \overbrace{A^* + B^* \cos \phi_\pi^* + C^* \cos 2\phi_\pi^*}^{\text{Similar to } eN \rightarrow e' N \pi} + \underbrace{D^* \sin \phi_\pi^* + E^* \sin 2\phi_\pi^*}_{\text{parity violating}} \right\}$$

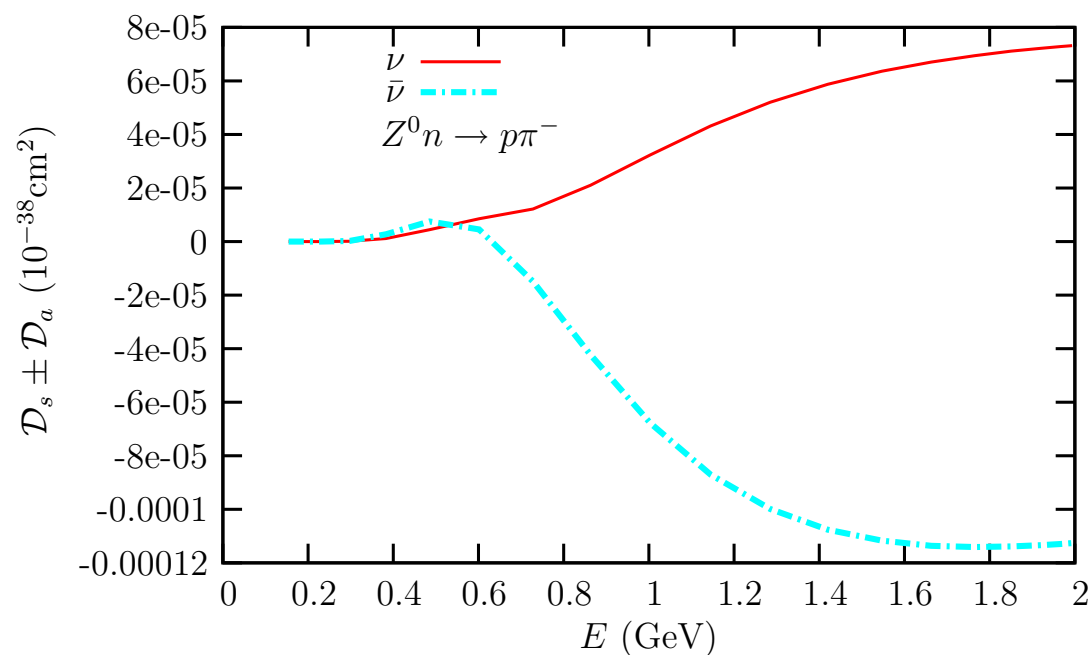
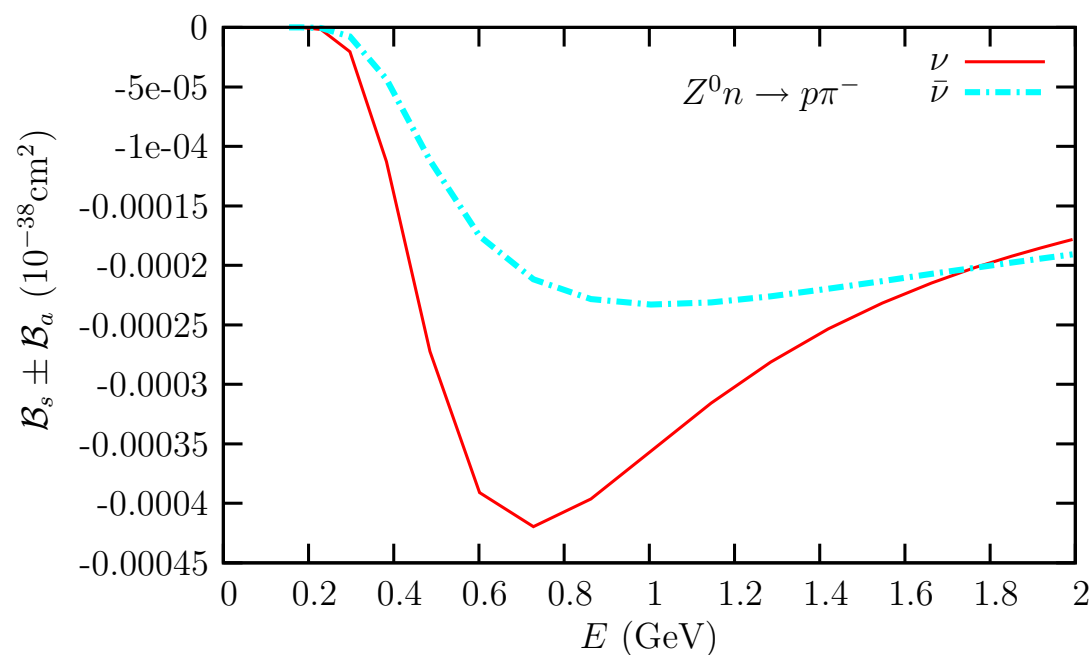
- Explicit ϕ_π dependence
- A^*, B^*, C^*, D^*, E^* functions of E, q^2, W, θ_π^*
- C^* and E^* are the same ν and $\bar{\nu}$, when $(W^{\mu\sigma})^{(\bar{\nu})} = (W^{\mu\sigma})^{(\nu)}$



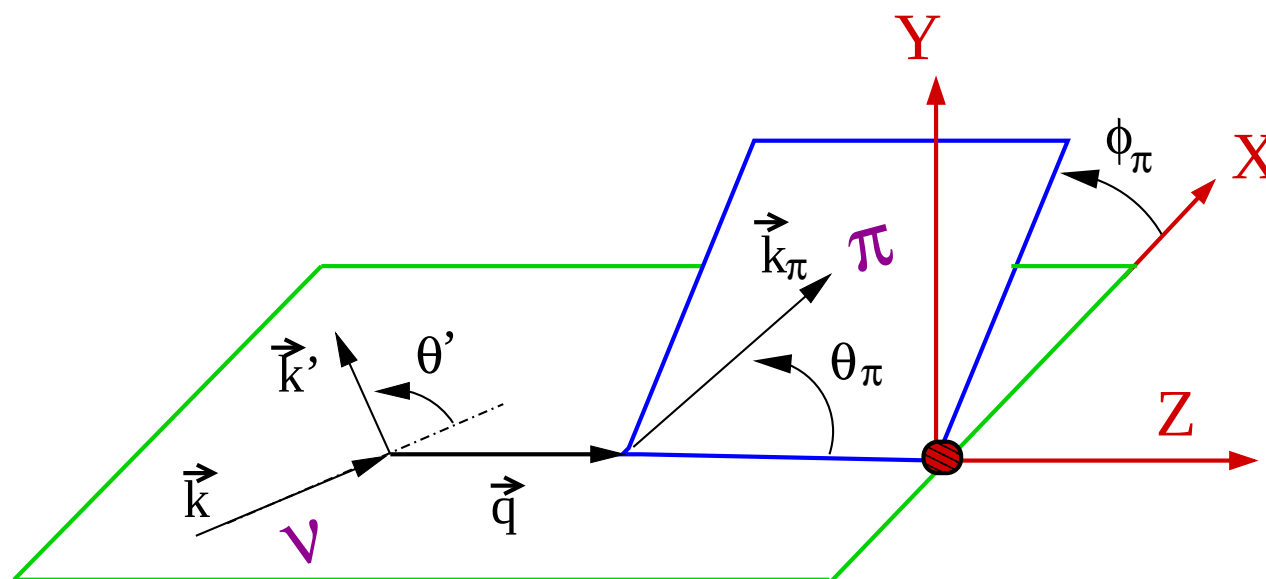
... new NC neutrino–antineutrino asymmetries

$$\frac{1}{2} \left(\frac{d\sigma(\phi_\pi)}{d\phi_\pi} - \frac{d\sigma(\phi_\pi + \pi)}{d\phi_\pi} \right) \Big|_{\nu} = (\mathcal{B}_s + \mathcal{B}_a) \cos \phi_\pi + (\mathcal{D}_s + \mathcal{D}_a) \sin \phi_\pi$$

$$\frac{1}{2} \left(\frac{d\sigma(\phi_\pi)}{d\phi_\pi} - \frac{d\sigma(\phi_\pi + \pi)}{d\phi_\pi} \right) \Big|_{\bar{\nu}} = (\mathcal{B}_s - \mathcal{B}_a) \cos \phi_\pi + (\mathcal{D}_s - \mathcal{D}_a) \sin \phi_\pi$$



Parity violation



$$\begin{aligned}
 \frac{d^5 \sigma_{\nu l}}{d\Omega(\hat{k}') dE' d\Omega^*(\hat{k}_\pi)} &= \frac{|\vec{k}'|}{|\vec{k}|} \frac{G^2}{4\pi^2} \left\{ \overbrace{A^* + B^* \cos \phi_\pi^* + C^* \cos 2\phi_\pi^*}^{\text{Similar to } eN \rightarrow e' N \pi} \right. \\
 &\quad \left. + \underbrace{D^* \sin \phi_\pi^* + E^* \sin 2\phi_\pi^*}_{\text{parity violating}} \right\}
 \end{aligned}$$

$$L_{\mu\sigma}^{(\nu)} = (\mathbf{L}_s^{(\nu)})_{\mu\sigma} + i(\mathbf{L}_a^{(\nu)})_{\mu\sigma} = k'_\mu k_\sigma + k'_\sigma k_\mu - g_{\mu\sigma} k \cdot k' + i\epsilon_{\mu\sigma\alpha\beta} k'^\alpha k^\beta$$

By construction (similar for both CC and NC),

$$W^{\mu\sigma} = \mathbf{W}_s^{\mu\sigma} + i\mathbf{W}_a^{\mu\sigma}, \quad W_{s,a}^{\mu\nu} = (W_{s,a}^{\mu\nu})^{\text{PC}} + (\mathbf{W}_{s,a}^{\mu\nu})^{\text{PV}}$$

$$(W_s^{\mu\nu})^{\text{PC}} = W_1 g^{\mu\nu} + W_2 p^\mu p^\nu + W_3 q^\mu q^\nu + W_4 k_\pi^\mu k_\pi^\nu + \dots$$

$$(W_a^{\mu\nu})^{\text{PC}} = W_{14} \epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta + W_{15} \epsilon^{\mu\nu\alpha\beta} p_\alpha k_{\pi\beta} + W_{16} \epsilon^{\mu\nu\alpha\beta} q_\alpha k_{\pi\beta} + \dots$$

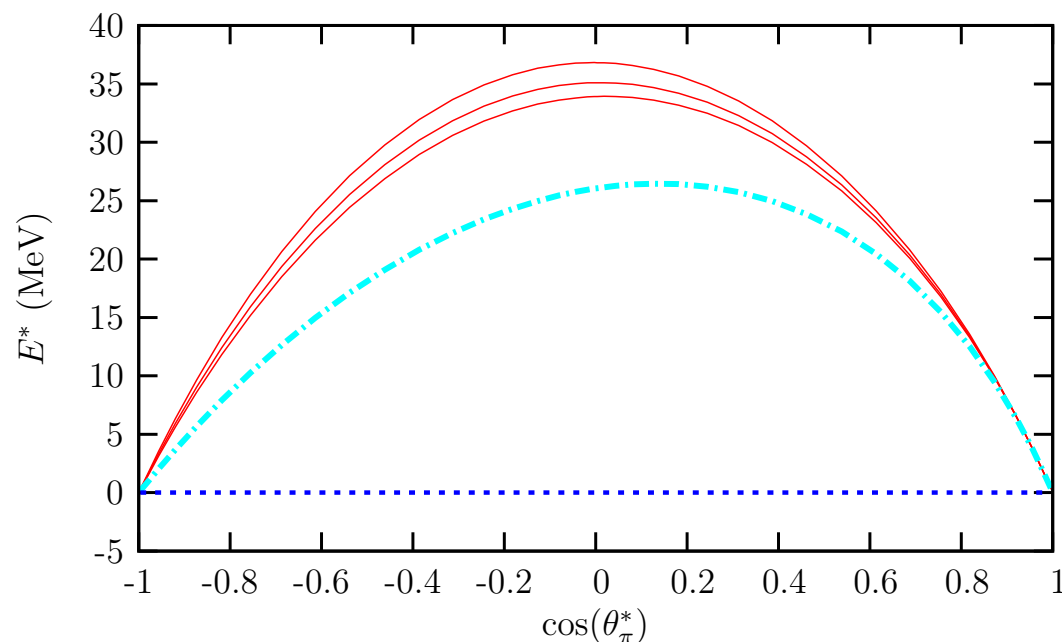
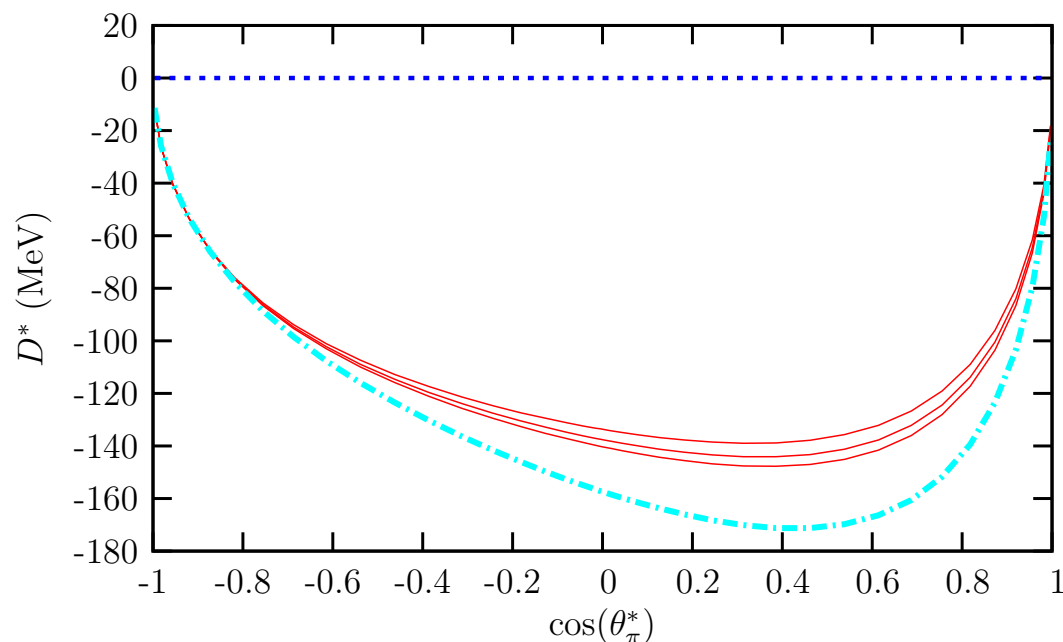
$$(\mathbf{W}_s^{\mu\nu})^{\text{PV}} = \mathbf{W}_8 \left(\mathbf{q}^\mu \epsilon_{\alpha\beta\gamma}^\nu \mathbf{k}_\pi^\alpha \mathbf{p}^\beta \mathbf{q}^\gamma + \mathbf{q}^\nu \epsilon_{\alpha\beta\gamma}^\mu \mathbf{k}_\pi^\alpha \mathbf{p}^\beta \mathbf{q}^\gamma \right) + \dots$$

$$(\mathbf{W}_a^{\mu\nu})^{\text{PV}} = \mathbf{W}_{11} (\mathbf{q}^\mu \mathbf{p}^\nu - \mathbf{q}^\nu \mathbf{p}^\mu) + \mathbf{W}_{12} (\mathbf{q}^\mu \mathbf{k}_\pi^\nu - \mathbf{q}^\nu \mathbf{k}_\pi^\mu) + \dots$$

Under Parity

$$L_{\mu\nu}^{(\nu)} \rightarrow (L^{\nu\mu})^{(\nu)}, \quad (W_{\mu\nu})^{\text{PC}} \rightarrow (W^{\nu\mu})^{\text{PC}}, \quad (\mathbf{W}_{\mu\nu})^{\text{PV}} \rightarrow -(\mathbf{W}^{\nu\mu})^{\text{PV}}$$

- $d^5\sigma/d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)$ is not inv. under parity, since the pseudovector $\vec{k} \times \vec{k}'$ is used to define the Y axis.
- $d^3\sigma/d\Omega(\hat{k}')dE'$ scalar, except for the factor $|\vec{k}'|/|\vec{k}| \Rightarrow$ parity violation **disappears** when performing the $\int d\Omega^*(\hat{k}_\pi)$

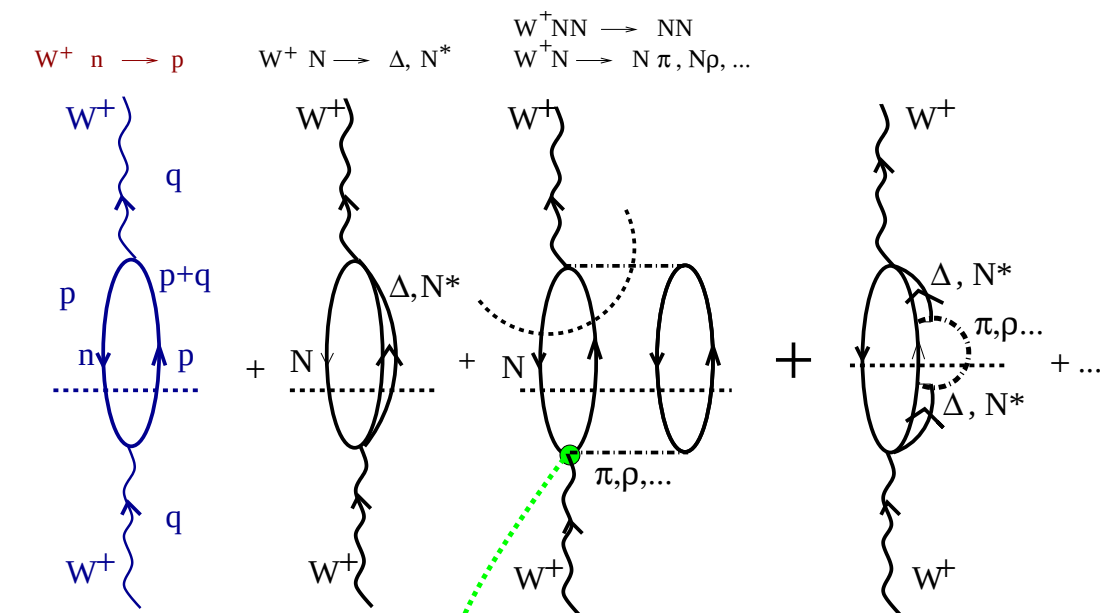


$$\nu_\mu n \rightarrow \mu^- p \pi^0 \quad (E = 1.5 \text{ GeV}, W = M_\Delta, q^2 = -0.5 \text{ GeV}^2)$$

- Non-resonant terms are needed to produce non-vanishing parity violating structure functions

Conclusions: We have derived a model for CC and NC weak pion production off the nucleon

1. In addition to the Δ resonance, we include **non-resonant contributions** $\Leftarrow$ QCD $S\chi$ SB.
2. Non resonant contributions are important $\Rightarrow$ re-adjust of $C_5^A(q^2)$.
3. $\nu - \bar{\nu}$ Asymmetries, distinguish ν_τ from $\bar{\nu}_\tau$?
4. Parity violation effects due to the interferences between the non resonant and Δ contributions.
5. Starting point to study **inclusive and exclusive neutrino-nucleus scattering** above the QE region.



$$\sum_{n < F} \left| \text{Diagram} \right|^2$$

